Investigation of Transmission Error, Friction and Wear in Anti-Backlash Involute Gear Transmissions: A Finite Element Approach

Jesper Brauer
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Doctoral thesis

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Abstract

This thesis investigates some of the most important properties of anti-backlash involute gear transmissions. The approach blends mathematical modelling and analysis and numerical simulation using the finite element (FE) method.

Anti-backlash involute gear transmissions are frequently used in mechatronic products since backlash can impair their performance. The most common way to eliminate both constant and variable backlash is to use some kind of preloaded spring that forces the driving gear into continuous contact with the driven gear. But this approach means that the gears suffer from flank interference (or double-flank action), which increases friction and wear. In addition, the extra degree of freedom that is added to the meshing gear pair may have a significant effect on the transmission error. All these effects may impair the performance of a mechatronic product and therefore need to be investigated.

Two parameterised finite element contact models have been developed. One of these is a 2D model of a meshing spur gear pair and has wear features while the other is a 3D model of a meshing conical involute gear pair. Both models are capable of simulating flank interference and are based on a global-local FE method that gives a dense FE mesh in the contact regions and a coarse mesh in the rest of the teeth.

Wear simulation with the 2D FE model is based on a mixed FE and analytical approach. This method uses FEM to determine the load distribution between the interacting teeth, and then determines the contact pressure and sliding distance using analytical expressions based on Hertzian theory. This approach makes it possible to have a fairly coarse FE discretisation that reduces the time otherwise required to solve the FE contact problem. The results of the simulation show that the friction torque becomes high and very non-linear when involute teeth are subject to flank interference, but that it reduces as wear decreases the flank interference.

The development of the 3D FE model began with derivation of the mathematical equations and intervals that define the geometry of a conical involute gear. These equations were then used to develop a general parameterised FE model of involute gears. The meshing of conical involute gears mounted on parallel axes was also analysed and analytical expressions were derived for the flank interference, sliding velocity and flank forces between the mating tooth flanks. The FE model was then used to investigate a spring-loaded anti-backlash conical involute gear transmission. The simulation reveals that the axial motion of the spring-loaded gear has a significant influence on the transmission error since it introduces a rigid component into the error. This finding is verified by a derived analytical expression. Friction was shown to cause axial oscillations of the spring-loaded gear. Self-locking effects may also arise when coefficient of frictions are larger than or in the vicinity of a derived critical value.
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Stockholm, September 2003
Appended papers

C. J. Brauer, A general finite element model of involute gears, submitted to a scientific journal
D. J. Brauer, Transmission error and friction in anti-backlash conical involute gear transmissions: A global-local FE approach, submitted to a scientific journal
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Appended papers:

Paper A: Analytical geometry of straight conical involute gears
Paper B: Simulation of wear in gears with flank interference –
      A mixed FE and analytical approach
Paper C: A general finite element model of involute gears
Paper D: Transmission error and friction in anti-backlash conical
      involute gear transmissions: A global-local FE approach
1 Introduction

Systems that transmit power from one system to another are an essential part of many technical systems. The main purpose of many such transmission systems is to change the form of the power from the emitting system into a form that suits the receiving system. Ideally this should be done without introducing any disturbances or noise that may affect the performance of the technical system. One of the most common forms of power transmission is geared transmission, which is found in numerous products including cars, CD players and photocopying machines. Gears are not only capable of transmitting very large power loads and operating at high speeds [1], but they can do so with very uniform motion, and in relatively compact configurations [2].

1.1 Involute gears

Gears can be classified into two general categories: those where the axes of the gear pair are parallel and those where the axes are non-parallel (usually at 90° to each other [3]). Gears in the first category almost always have the involute tooth form [4] that was developed by Philippe de la Hire in 1696. Leonard Euler developed the mathematics for these gears in the eighteenth century, making him the father of involute gearing [5]. If a string is attached to a point of a curve and stretched such that it is tangent to the curve at the point of attachment, then the involute of the curve is described by the free endpoint of the string as it winds around the curve (which is the evolute of its involute) while held taut. Involute gearings are normally based on the involute of a circle (see Fig. 1).

![Involute of a circle](image)

**Fig. 1. The involute of a circle.**

According to reference [6] the involute tooth form offers three major advantages:

- It is insensitive to mounting errors since conjugate action is independent of changes in centre distance.
- The flanks of the basic rack are straight-sided and, and can therefore be produced relatively simply and accurately.
- One cutter can generate all the gear teeth with the same pitch.
1.2 Backlash

One of the major disadvantages of involute gears transmissions (and of most other types of gears) in mechatronic applications, and in particular precision machines, is the presence of backlash. Backlash can be defined as the amount of angular motion in a gear when its mating gear is fixed (see Fig. 2), or as the shortest distance between the surfaces of the trailing flanks when the driving flanks are in contact [7]. The motion of the driving gear will not move the driven gear until the gap between the tooth flanks is removed (see Fig. 2 and reference [8]).

![Diagram of backlash](image)

Fig. 2. Hysteresis loop caused by angular backlash. The driven gear (gear 2) has a high friction to inertia ratio such that when the driving gear starts to decelerate the tooth flanks will not separate.

This deadband behaviour (or lost motion), which is dependent on the friction between the mating tooth flanks and the relative inertias of the input and output, can cause discontinuities in velocity and acceleration, leading to high mechanical stresses on the components [9]. In addition, it introduces hysteresis nonlinearity into the technical system, which often results in steady state errors or oscillations in the control system. There are methods of compensating for backlash, but it is very difficult to predict when the driving gear will lose contact with the driven gear, and when it does the driven gear and the technical system are to some extent uncontrollable [10]. Therefore, mechatronic products that include geared transmissions often require an anti-backlash function to remove or reduce the backlash between the meshing gear flanks.

2
2 Anti-backlash involute gear transmissions

“Ideally, meshing gears do not have any backlash nor do they need any to operate satisfactory” [11]. However, manufacturing and mounting errors and the effects of load and temperature inevitably result in backlash. Some of this backlash is even intentional to prevent gear interference. The sources of backlash fall into two categories. Some have a constant magnitude, such as centre distance allowance and tolerance, while in others the magnitude of the backlash varies with gear rotation, as with total composite error (see Fig. 3) and shaft runout at the gear position. Backlash in this second category is largely due to eccentricity between the gear axis and the centre of rotation, which results in both addition and subtraction of backlash. Backlash resulting from this category of sources is also responsible for a large component of the transmission error.

![Image of backlash components](image)

*Fig. 3. Typical total composite error plot (see [11]). The gear is meshed with a high-quality master gear on a variable centre fixture. The gears are forced together by a preload.*

2.1 Anti-backlash methods

The negative effects of backlash have led to considerable design efforts to eliminate and control it [11], [12] and [13]. One simple way to eliminate it is by meshing the gear teeth tightly, but this may give rise to excessive amounts of friction. Very precise gears and mountings minimise the problem, but high-precision gearing is extremely expensive. Instead, special design, fabrication and assembly methods have been developed to permit the use of imperfect gears and associated parts without generating significant backlash. The most common way to eliminate both constant and variable backlash is to use some kind of preloaded spring, which forces the driving gear into continuous contact with the driven gear. There are several ways in which this can be done, including

- “splitting” one of the gears in a gear pair into two sections that rotate freely in relation to one another but are constrained axially. A preloaded spring can eliminate backlash
by forcing the two sections to shift in relation to each other so that the tooth space on
the mating gear is completely filled.

- allowing one of the gear centres in a gear pair to move or float in such a way that the
distance between the gear centres can be adjusted. A preloaded spring can eliminate
backlash by forcing the floating gear onto the other gear.

- using conical involute gears, which have tapered tooth thickness, a tapered root and a
tapered outside diameter. A preloaded spring can eliminate backlash by forcing one of
the gears onto the other in the axial direction.

2.2 Problems

In an anti-backlash gear transmission, the axis of the driving gear should ideally never lose
contact with the axis of the driven gear. Consequently, for most anti-backlash involute gear
transmissions, both a driving and a non-driving flank of the driving gear must be in contact
with their mating flanks at rotation reversals to ensure that the load is handed over without any
deadband movement. This **flank interference** (see Fig. 4) give rise to forces between the
contacting gear flanks, though no torque is transmitted between the driving gear axis and the
driven gear axis. As a result, gears and bearings are constantly loaded which increases friction
and wear and decreases their life. Friction is a highly non-linear phenomenon [14] and may
result in steady state errors, limit cycles and poor performance of the control system [15].
Gears subjected to flank interference (double-flank action) are also sensitive to profile quality
since irregularity in profiles is doubly sensed [11]. One other important issue is how the
transmission error is affected by the extra degree (or degrees) of freedom that is often added to
the meshing gear pair in order to eliminate backlash.

![Fig. 4. Flank interference between a meshing spur gear pair.](image)
3 Research approach

The overarching research question for this thesis may be formulated as:

*How are the transmission properties affected by the anti-backlash function in anti-backlash involute gear transmissions?*

Most anti-backlash involute gear transmissions are characterised by flank interference and one or several additional degrees of freedom intended to eliminate the backlash between the gear flanks. To limit the scope of the work, the following hypotheses have been formulated. (Note that according to Beach and Alvager [16] hypotheses may be stated in question form.)

1. How is the friction torque affected by flank interference and mild wear in (involute) spur gears? (See appended paper B.)

2. How are the friction torque and transmission error affected by the flank interference and the additional degree of freedom in anti-backlash conical involute gear transmissions? (See appended paper D.)

*Fig. 5. An FE model of a conical involute gear pair.*
Fig. 6. A 2D FE model of two long cylinders in contact where one of the two symmetrical properties of the problem has been utilised. Plane strain is assumed and the cylinders are represented by 8-noded isoparametric quadrilateral elements (ANSYS: PLANE82). The contact interaction is represented by 3-noded surface-to-surface elements (ANSYS: CONTA172 and TARGE169).

The approach chosen to deal with these questions blends mathematical modelling and analysis and numerical simulation using the FE software ANSYS. One parameterised 2D FE contact model with wear features of a meshing spur gear pair and one parameterised 3D FE contact model of a meshing conical involute gear pair (see Fig. 5) were developed. Both models are capable of simulating flank interference. The first step in the development of the 3D FE model was derivation of the mathematical equations and intervals that define the geometry of a conical involute gear (see appended papers A and C). These mathematical equations were verified by simulating a gear-generating process using the CAD software I-DEAS. Simulation results from the 2D FE model with wear features were compared with the wear simulated by Flodin [17]. The results from the 3D FE model representing a spur gear pair with interference were compared with the experimental and simulated results from Spiegelberg [18]. In both cases, the results showed good agreement. The global-local FE meshing method that is used in papers B and D was applied to two long cylinders in contact (see Figs. 6 and 7), and the results from those simulations agreed very well with an analytical solution based on Hertzian theory [19]. In order to verify the 3D model representing a conical involute gear pair, a test rig has been designed (see Fig. 8), but unfortunately there was no time within this PhD project to finalise the building of the test rig and to carry out the necessary experiments.
Fig. 7. Ratio of approach distance (calculated using FE and analytically) to size of contact region. Notice that a contact region is defined as a region in the solid where the outer boundary will (entirely or partly) be in contact.

Fig. 8. Drawing and block diagram of test rig.
4 Summary of appended papers
This chapter summarises the four appended papers and lists the contributions.

4.1 Paper A: Analytical geometry of straight conical involute gears
Although conical involute gears are frequently used in anti-backlash schemes, no detailed mathematical geometry description of conical involute gear teeth hitherto exists. In this paper we derive the parametric equations for a straight conical involute gear tooth surface and its offset surface. These formulas are then used to create a finite element model with a specific surface layer. Such a surface layer enables meshing control or modelling of surface properties such as case hardening and surface roughness. In addition, we derive an expression for the minimum value of the inner transverse addendum modification coefficient that avoids undercutting of the whole gear tooth.

Contributions:

- The parametric equations for a straight conical involute gear tooth surface (composed of flank and fillet surface) and its offset surface have been derived. These formulas have then been used to create an FE model with a specific surface layer of a straight conical involute gear tooth (and a spur gear tooth as well by setting the cone angle equal to zero). Such a surface layer can be used for meshing control or to model surface properties such as case hardening and surface roughness.
- The boundary point between the involute and fillet curve has been investigated and, if the tooth profile is not undercut, the boundary point can be determined analytically with the presented formula.
- An expression that enables checking whether a tooth profile is undercut or not has been developed. In addition, an expression for the minimum value of the inner transverse addendum modification coefficient that avoids undercutting of the whole gear tooth has been derived.

4.2 Paper B: Simulation of wear in gears with flank interference – A mixed FE and analytical approach
Gears in precision mechanisms, such as industrial robots, are often designed to have no backlash. Deviations from the ideal gear geometries and unfavourable deformations during operation may cause interference between interacting tooth flanks, resulting in high and strongly varying friction. Mild wear of the tooth flanks may improve such conditions. Therefore, this theoretical study has been conducted of wear in spur gears with interference. A mixed finite element (FE) and analytical approach is used. The FE method is used to determine contact loads between the interacting gear teeth. The main drawback with FE analyses of this type of problem is normally the computation time needed. Therefore, a novel FE meshing method is used, giving a dense FE mesh in the contact regions and a coarse mesh in the rest of the teeth. Based on the FE determined loads between the interacting teeth, the contact pressures and the contact widths are then easily determined using analytical expressions based on Hertz theory. The wear of a point on a tooth flank is determined by integrating the product of sliding distance and contact pressure during the time it is in contact with its mating flank.
Contributions:

- The parametric equations for a wear modified involute spur gear have been derived based on the parametric equations that define an involute spur gear.
- A mixed FE and analytical method to simulate wear in gears has been presented and the analytical expressions that enable such an approach have been derived. In addition, a flowchart for the simulation method has been presented.
- It has been shown that wear may eliminate the interference of gears subjected to double-flank action and, as a result, reduce the non-linear friction torque.

4.3 Paper C: A general finite element model of involute gears

Involute gears comprise primarily spur gears, helical gears, straight conical involute gears and conical involute gears. Robust and effective parameterised finite element models of involute gears should be based on analytically derived mathematical representations of their shape. In this paper we derive a mathematical description of conical involute gears that is also capable of representing three other types of involute gear. The equations and the intervals for the surface parameters are then used to create a general finite element model of involute gears.

Contributions:

- Mathematical descriptions of conical involute gear tooth flanks, fillets and roots have been derived. The descriptions include both the parametric equations for the surfaces and the intervals for the surface parameters. Straight conical involute gears, helical gears and spur gears can be represented by the equations by letting the helix angle, the cone angle, or both be equal to zero. All gear types may have addendum modification, which is specified by an own design parameter.
- The boundary point between the involute and fillet curve in a transverse gear profile has been investigated. If the tooth profile is not undercut, the boundary point can be determined analytically with the presented formula. Otherwise, the boundary point can be determined using the numerical algorithm presented here.
- A method for creating a general FE model of involute gears.

4.4 Paper D: Transmission error and friction in anti-backlash conical involute gear transmissions: A global-local FE approach

This paper reports on a theoretical study of transmission errors and friction in anti-backlash conical involute gear transmissions. Such transmissions are used to reduce or eliminate the backlash in mechatronic products such as industrial robots and thereby improving system stability. A global-local finite element approach reduced the computation time needed to solve the non-linear contact problems. This approach yields a dense FE mesh in the contact regions and a coarse mesh in the rest of the teeth.

Contributions:
The meshing of conical involute gears mounted on parallel axes has been analysed and analytical expressions that determine the flank interference, relative velocity and flank forces between the mating tooth flanks derived.

An anti-backlash conical involute gear transmission has been investigated both analytically by derived formulas and by finite element contact simulations. A global-local FE approach has been used and a method of determining the dense-meshed contact regions has been presented. In addition an analytical investigation of appropriate sizes of the contact regions has been conducted.

The FE simulations show that the transmission error changes abruptly between the characteristic states for the gear transmission. A derived analytical expression shows that this phenomenon is explained by a rigid component of the transmission error, which is caused by axial movements of one of the gears.

The influence of friction has been investigated and it is shown by derived analytical expressions that when one of the gears moves in its axial direction, the axial friction forces will vary in size and reach their maximum at rotation reversals or other brief stops, or when the points of tangency of the gear flanks are at or near the instantaneous axis of rotation. In addition, it is shown by FE simulations that self-locking effects may arise if the coefficient of friction is larger or in the neighbourhood of a derived critical value. It is also shown that even if none of the gears move in their axial direction, the friction forces between the meshing gear flanks cause an oscillating axial force. All this indicates that the behaviour of the gear transmission during the state of axial freedom is unstable and as result may be very complex or chaotic. Finally, it is shown that during the state of interference, the friction torque varies strongly and is relatively large for small load torques.
5 Discussion and future work

The results in appended paper B and D show that the friction torque is high and varies strongly if the gears are subjected to flank interference. The simulated wear in paper B reduced the interference between the spur gear flanks that in turn reduced the non-linear friction torque. The same results should apply to other types of involute gears since every transverse tooth profile approximately represents a spur gear. On the other hand, the derived sliding velocity in paper D indicates that axial movements of one of the gears (due to an extra degree of freedom) in a conical involute gear transmission introduces additional sliding that changes the direction and magnitude of the sliding velocity. This in turn will affect the wear of the gear flanks and might give rise to an irregular reduction of interference.

The simulation and a derived analytical expression in paper D reveal that the axial motion of a spring-loaded conical involute gear has a significant influence on the transmission error since it introduces a rigid component into the error. Such a component should also exist in other types of spring-loaded anti-backlash gear transmissions.

Although, anti-backlash involute gear transmissions are commonly used in mechatronic products, little is known about their transmission properties, and particularly their dynamic transmission properties. Thus dynamic models need to be developed and test rigs need to be built to measure the dynamic transmission error and friction torque. It is also important to investigate both static and dynamic behaviour using more sophisticated models for the coefficient of friction than the Coulomb friction model (see [20]). The high friction forces caused by the flank interference mean that there is also a need to examine thermal behaviour (see [21]) and its effects on transmission properties. Furthermore, a gear transmission is currently usually represented in the control system as a single linear spring with some damping, but the introduction of more precise and faster precision mechatronic products is creating an increasing demand for more accurate dynamic modelling of the gear transmission in the control system. Therefore, an ultimate goal must be to develop one or several models that capture essential dynamic behaviour, but are not too complex for real-time control.
References