Valuation of Contingent Convertible Bonds

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Abstract

Contingent convertible bonds are hybrid capital instruments, contingent on some form of indicator of financial distress of the issuing bank. Following the financial crisis, these instruments are proposed as a solution to the moral hazard issue of banks too big to fail. With the increased capital requirements of the Basel III directive, contingent capital enables banks to increase their capitalization without issuing expensive equity. Also, in times of historically low interest rates, these instruments might be interesting for investors in search of higher yields, as well as long term investors wanting to implement counter-cyclical investment strategies. However, due to the high complexity of these instruments, valuation has proven difficult. The purpose of this thesis is to value instruments contingent on the bank’s common equity tier 1 to risk-weighted assets ratio. We build our model upon the work of Glasserman & Nouri (2012), and extend it to include contingency on risk-weighted assets, instant non-continuous conversion to equity, and a combination of fixed imposed loss and fixed conversion price as terms of conversion. We use a capital structure model in continuous time to define asset dynamics, asset claims and the event of conversion and liquidation of the bank. Thereafter we use two important results from Glasserman & Nouri (2012) to value the discounted cash flows to holders of debt and contingent debt. From this, we arrive at closed form solutions for the coupon rates of these securities.

Keywords: Contingent Convertible Bonds, Hybrid Capital, Capital Structure, Capital Adequacy Regulation, Basel III, Risk-Neutral Valuation
Sammanfattning

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List of Symbols

$\alpha$ threshold level for conversion
$\beta$ threshold level for liquidation
$\gamma$ fixed imposed loss
$\eta$ modified drift of the Brownian motion $\tilde{X}_t$
$\delta$ payout rate from assets
$\theta$ corporate tax rate
$\lambda$ drift of $\tilde{X}_t$
$\mu$ expected growth rate of assets
$\rho_D$ recovery rate of debt
$\sigma$ volatility of assets
$\tau_C$ time of conversion
$\tau_L$ time of liquidation
$\phi$ lower bound for price of equity at conversion
$\psi$ price of equity used for conversion
$\omega_i$ risk-weight applied to asset $i$
$a_i$ value of asset $i$
$A_t$ total value of assets at time $t$
$c_C$ coupon rate for contingent convertible bond
$c_D$ coupon rate for ordinary bond
$C_t$ book value of contingent convertible bond at time $t$
$D_t$ book value of ordinary debt at time $t$
$P_C$ principal of contingent convertible bond
$P_D$ principal of ordinary debt
$r$ risk-free interest rate
$Q_t$ value of shareholder’s equity at time $t$
$X_t$ value of risk-weighted assets at time $t$
$Y_t$ value of risk-free assets at time $t$
$\mathbb{P}$ objective probability measure
$\mathbb{Q}$ risk-neutral probability measure
1 Introduction

The financial crisis of 2007-08 was detrimental to the world's economy as a whole, but arguably the financial sector and banks in particular received the hardest blow. Lehman Brothers collapsing and filing for bankruptcy in September 2008 prompted governments to take action in form of relief packages and bail-outs when banks all over the world entered serious financial distress. These bail-outs have been criticized as they make tax payers pay for the risk taking of banks, which in turn creates a moral hazard as banks are more likely to take excessive risks when the promise of a bail out is implied. The alternative to bail-outs in such a deep financial crisis is very probably a systemic collapse of the global financial system, which arguably would have been much worse for the tax-payers that bail-out critics want to defend. This dilemma where tax-payers seem to receive the short end of the stick no matter the actions of their governments is commonly known as the too big to fail problem.

Much of the ex post discussion about the financial crisis and in particular the bail-outs of banks have been about leverage. It is the collective consensus of the financial world that if banks had held more equity and less debt in relation to their risk the crisis could have been, if not averted, much less widespread. There are studies written post-crisis suggesting that banks who held their risk managers in high regard and payed them well had less stock price volatility before the crisis and less losses during the crisis. (Ellul & Yerramilli, 2010). The appointment of overt leverage as the main antagonist, coupled with the wish to protect tax payers from financial losses have led governments and financial regulators around the world to implement stricter capital requirements. The most important of these regulatory changes is the Third Basel Accord, or Basel III, which is an agreement between most of the larger economies of the world concerning the regulation of banks.

Basel III is an agreement on bank-regulation in general but there is special focus on hindering excessive leverage by imposing requirements on how much capital banks must hold. These requirements on capital are structured into classes of capital, where each class is distinguished by how risky the capital is. The riskier the capital the cheaper it generally is as a means of finance. Basel III then requires banks to hold a certain percentage of their Risk Weighted Assets (RWA) in every respective category. RWA is a risk weighting of the banks assets where certain weights are assigned to different types of assets. At the core of a banks capital is Common Equity Tier 1 (CET1) consisting of shareholders equity and retained earnings which is considered practically risk-free. CET1 is part of the tier 1 class of capital which consists of CET1 and Additional Tier 1 (AT1). Outside of the core capital, classified as tier 2 and upwards, are different types of debt and other financial instruments. An example of the capital requirements of Basel III is that a bank must hold at least 4.5% of their RWA as CET1 and at least 6% of their RWA as tier 1 capital, leaving a 1.5% segment that is allowed to consist of AT1. (Basel Committee on Banking Supervision, 2012)

These regulations are focused on making banks hold capital. As with anything, everyone does not agree with this solution. An objection made by Calomiris & Herring (2011) is that a large amount of capital does not equal a financially healthy bank. Instead they advocate for the importance of holding the right type of capital and understanding on how to measure risk.

Banks want to access the cheapest viable financing, tax payers want to avoid bail outs and thus regulators impose capital requirements. Contingent convertible bonds are a type of hybrid capital that offers a possible solution to this puzzle. Many academic texts, including this thesis, refer to contingent convertible bonds as CCBs but they are also often called CoCos. CCBs are a form of debt which provides some form of loss absorption mechanism when the bank shows signs of financial distress. This provides buffers for the tax payers as with converting debt the responsibility of bailing out banks that are too big
to fail falls on the investors holding the CCBs rather than on the governments. The banks incentives are to reap some of the benefits of leverage, such as tax shields, and still have the protection offered by equity buffers when needed. CCBs have some form of loss absorption mechanism that activates and reduces the leverage of the issuing bank upon some predefined event often referred to as the “trigger”. As this reduces leverage in the type of situations that leverage is considered dangerous, CCBs are often classified by regulators as less risky than other forms of debt. Depending on the specifics of the instrument a CCB can even be classified as AT1. This provides banks with a less expensive way of satisfying the capital ratio requirements of Basel III which is a major incentive to issue CCBs.

Lloyds became the first bank to issue in 2009 when they sold CCB’s to about 120 000 retail investors (The Guardian, 2009). The bank had already received substantial amounts of bail-out capital from the UK government and was in serious financial distress. The CCBs were issued to avoid the need to receive additional support from the taxpayers when regulators found a 29 billion pound shortfall in the banks core capital. This first issuance of CCBs was not very successful for Lloyds as the interest rate was set to 12% and some of the contracts were not to expire until 2020, making them very expensive in today’s financial climate with historically low interest rates.

The possibility to count CCBs towards different classes of capital, as with many regulatory impositions, causes some disparity on the emerging CCB market. There is no immediate need for a consistent pricing model as far as the issuing banks are concerned. As long as the CCBs are less expensive to issue than other members of the same class of capital, there is incentive for the banks to issue the instruments (Brodén, 2016). The investors who in this regard are unaffected by the regulatory demands will treat the CCBs as any other financial instrument and consequently be careful not to undercharge. There is limited research on how this disparity of incentives has affected the market for CCBs but intuitively, overpricing is a reasonable consequence.

There are various kinds of CCB’s in existence and a plethora of proposed instruments from the academic community. The type of loss absorption mechanism, the trigger that is employed and the terms of the conversion itself varies greatly between CCBs. This is an issue when developing pricing models for CCBs. As the instruments are contingent on the financial health of the bank they are mathematically complex and since important characteristics vary it is probably impossible to develop a “catch all” model. The solution is resorting to specific models for specific CCBs. Commonly issued are CCBs with conversion to equity as loss absorption mechanism, the ratio of CET1/RWA as trigger mechanism and a combination of fixed imposed loss and fixed conversion price as the terms of conversion. These CCBs are popular as they under certain circumstances are classified as AT1 and thus can be counted towards the regulatory demands on tier 1 capital. A consistent pricing model for such CCBs does not exists and it is the aim of this thesis to supply one.

### 1.1 Problem Formulation

The aim of this thesis is to value contingent convertible bonds using a capital structure model in continuous time. We build our work upon the model of Glasserman & Nouri (2012) and extend it to include instant conversion to equity and a mix of fixed imposed loss and fixed conversion prices. Furthermore, we aim to value instruments contingent on the common equity tier 1 to risk-weighted assets ratio.

### 1.2 Purpose

Although a reliable pricing model is of value to banks and investors, there is a broader reason for writing this thesis. CCB’s are in theory part of the solution to our too big to fail problem but are held back by a
lack of understanding of how to price them, which creates insecurity in the market. An effort to develop a consistent and comprehensible model is of potential value to taxpayers overall if it can contribute to preventing government bailouts and relief packages in times of financial distress.

1.3 Structure of the Thesis

Chapter two is meant to provide the reader with a thorough presentation of contingent capital bonds. The aim is to describe all the different factors that can differ in CCBs both real and theorized. The discussion attempts to include how different choices when formatting a CCB will affect the issuing bank, the investor buying the CCB, regulators as well as the market for the issued CCBs and the issuing banks stock. Most of these differing factors are instrumental to valuation which is why a rudimentary understanding of loss absorption mechanisms, trigger mechanisms and terms of conversion is necessary to understand the thoughts and work presented in the thesis in general and chapter four especially.

The third chapter of the thesis contains a model of the banks capital structure. Such a model is necessary for valuation by cause of the contingent qualities of CCBs. The capital structure of the issuing bank is structured into assets, debt, CCBs and equity and a mathematical model for each of these is derived using a set of assumptions that are presented continuously throughout the chapter. Much of this work is based on that of Paul Glasserman and Behzad Nouri which is possibly the most important previous literature enabling this thesis.

Valuation, or chapter four, explains the mathematical models of the components of debt and CCBs and derives an arbitrage free value for the coupon rates that is consistent with the made assumptions and model of the bank presented in chapter three. The expressions derived in this chapter are to be interpreted as the results of the thesis.

Chapter five contains a numerical example including a sensitivity analysis. Chapter six concludes and gives recommendations for further work.
2 Contingent Convertible Bonds

The main idea with CCBs is, as explained in section 1, to supply banks with financing in times of financial distress and simultaneously exploit the benefits of leverage without losing the safety of equity. This is achieved through issuing bonds with some sort of loss absorption mechanism triggered in times of financial distress. As such, the specifics on how this is to be achieved are undetermined and it is the purpose of this chapter to determine and explain the parameters that distinguish different CCBs.

The type of loss absorption mechanism is obviously important as it is the main idea behind the contract, but there are other parameters that differ between CCBs. The trigger mechanism, hereafter simply called the "trigger" is what defines more accurately what has previously been referred to as financial distress, meaning it is a predefined event that causes the loss absorption mechanism to take place. The last important parameter is called the "terms of conversion" this is mostly interesting when the loss absorption mechanism is a conversion to equity and mandates how exactly, and at what price, the conversion is to be made. The following chart based on one by Avdjiev et al. (2013) is an illustration of how these components are aligned.

![Figure 1: Main design features of contingent convertible bonds](image)

The employed trigger can vary greatly between CCBs, the most being CET1/RWA falling below a certain threshold. An instrument with a discretionary trigger has an assigned third party, most often the local regulator, who decides when the loss absorption mechanism is to take effect. A mechanical trigger is activated when a predetermined value falls below, or rises above a limit. The major difference between the two is that a discretionary trigger is an active decision, while the mechanical ones are predefined and automatic by the time of triggering.

Within the mechanical group of triggers are the two subgroups market-value and book-value. What differs between the two are the values that define the threshold. RWA/CET1 is an example of a book value trigger, that is to say that the value of the ratio is determined by accounting figures. A market value trigger is defined by values on the financial market, stock prices for example. There are two prominent types of loss absorption mechanisms, principal write-down and conversion to equity. Both of these mechanisms have been used in instruments actually issued as well as theorized by the academic community (Reuters, 2014).
A principal write-down is an absolving of debt. CCBs that employ the principal write-down will upon triggering absolve the bank of parts or all of the principal debt incurred when the instrument was issued. Conversion to equity differs from the principal write-down as debt is not written down but converted. If the CCB triggers, the bank will issue new equity that will thereafter belong to the holder of the CCB. Often the amount of issued equity will be equal to the principal, meaning a full conversion from debt to equity, but there is a plethora of ways this could be set up. These different methods of handling the conversion to equity is what this thesis, in accordance with most literature on the subject, names the terms of conversion.

For the banks issuing CCBs a very important factor is how regulators will classify the instruments. As mentioned in the introduction, Basel III imposes demands on the amount of capital banks hold. Regarding CCBs requirements on tier 1 capital specifically are very significant. There are many examples of CCBs that regulators have approved as AT1-capital and as such they can be counted towards the requirements on Tier 1. Such instruments, that is CCBs that are considered AT1, are often called convertible AT1-instruments. This gives convertible AT1-instruments the key property that they will satisfy the demands of regulators while still giving the issuing bank some of the benefits of leverage.

2.1 Loss Absorption Mechanism

The type of loss absorption mechanism employed is critical to how a CCB will be viewed by investors. From their perspective, buying a conversion to equity CCB is somewhat like buying a hybrid of bonds and stocks. As long as the bank stays financially healthy the instrument is uncomplicated and behaves just like a bond, but upon triggering the investor will instead hold equity. This makes conversion to equity preferable over a principal write-down from the investors point of view as investors receive nothing in the case of a principal write-down. Since the expected value of the investor is greater in a conversion to equity CCB, investors will require smaller coupons than they would if a principal write-down system was in place.

Buying CCBs with a principal write-down is substantially less complicated than buying one with conversion to equity. A conversion to equity instrument comes with more insecurity as there is heavy co-variance between the stock price and the trigger. This has potential to become expensive for the investor in e.g. a financial crisis as they could receive equity that is rapidly loosing value. In the worst case scenario, this could mean that the investor is unable to sell their shares before liquidation of the bank due to unresponsiveness or difficulties in finding a buyer. If the principal is very large this liquidity risk increases substantially. This general difficulty of pricing equity in a future potential state of financial distress along with the added liquidity risk is part of what makes conversion to equity CCBs difficult to price and will be further discussed later in this chapter.

The banks preference regarding conversion to equity or principal write-downs is, of course, inverted to that of the investor. Writing down debt is better for the current shareholder as issuing new equity to the holder of the CCB would dilute value for current shareholders. As such, the bank is willing to pay a larger coupon for CCBs with a principal write-down than one with conversion to equity.

The choice between the two loss absorption mechanisms presented here generally doesn’t affect capital classification as much as what trigger is chosen. So long as the bank is not under excessive leverage in times of financial distress it is not crucial to regulators whether they issue new equity or not (Avdjiev et al., 2013).

A principal write-down triggered in a CCB does not have to be permanent. An example of this is when the Swedish retail bank Handelsbanken issued CCBs in February 2015 (Globalnewswire, 2015). If
these instruments trigger the principal write-down is temporal and if the bank emerges from financial
distress, which in Handelsbankens case means CET1/RWA returning to above 8%, the bank are once
again indebted to the holder of the CCB. This makes these instruments similar to deeply subordinated
debt, the difference being that the bank would not technically have defaulted when canceling coupons
in times of financial distress and the investor having no claim whatsoever in the event of a bankruptcy.
The similarity between these types of CCBs and subordinated debt makes for a nice illustration of the
previously mentioned possibilities of CCBs as a countermeasure to the too big to fail problem.

2.2 Trigger Mechanism

What trigger mechanism is employed is perhaps the most defining feature of a CCB. The trigger is what
supplies the instrument with the key trait of injecting capital into the issuing bank when it needs it the
most.

2.2.1 The Bank, Investors and Regulators

A bank concerned with staying financially healthy in the long run will be very interested in what kind of
trigger is employed in their CCB contracts, as the trigger determines what specifically is to be considered
financial distress. The trigger is therefore a tool for calibrating a CCB to fit a scenario that the bank, or
its regulators, believe the bank is ill prepared for. This makes the choice of trigger mechanism influential
in satisfying regulator demands and insuring against financial distress.

Pricing and complexity are the dominant effects the choice of trigger has on investors buying the
CCB. The investors are not as concerned about exactly under what circumstances the loss absorption
mechanism kicks in as long as they can make reasonable estimations of the probability of triggering. If
modeling this probability is too difficult, the instrument becomes very hard to price correctly making
it an unpredictable investment. As a CCB triggering is generally negative for the investor holding the
CCB, it is most often the case that a trigger that is more likely to take effect makes investors demand
larger coupons. So long as there is reasonably accurate ways to account for the investors expected cost
of the contract triggering, what trigger is employed mostly affects the investors risk and therefore the
size of the coupons.

The choice of trigger is probably the most important consideration in a CCB for regulators. As
their objective is financial stability the definition of financial distress and when investors are to provide
banks with financial relief is a crucial component. As previously mentioned, the regulatory demands on
what capital the banks are to hold play a central role for CCBs and it is regarding the choice of trigger
that this really comes in to play. The trigger decides when the issuing bank receives a bail-in from the
investors and as such affects the risk of the bank failing. Regulators will strongly prefer a trigger that
can be trusted to activate the loss absorption mechanism before any costs of the banks hypothetical
default befall the tax payers. As such, a CCB with a very dependable trigger that is "strict" in the sense
that it activates at early signs of financial distress will be appreciated by regulators as it is not much
more risky than equity for the bank. A CCB with a less strict trigger or one that the regulator is not
convinced will surely trigger in times of financial distress because of its structure will be frowned upon.
How regulators perceive the soundness of the trigger will naturally affect the capital classification of the
CCB.

This game between banks, regulators and investors is a recurring theme regarding most aspects of
CCBs but it is perhaps most influential when it comes to the selection of a trigger. The banks want cheap
financing and a trigger that is less likely to activate will make investors require smaller coupons. If the
CCB doesn’t pass the regulators demands for it to be considered for example tier 1 capital, on the other hand, the bank will have to achieve the regulators demand on tier 1 capital with more expensive equity or other CCBs. The bank thus misses an opportunity to achieve the cheaper financing they wanted in the first place. This causes the banks to sometimes have incentive to issue CCBs that just barely meets regulators criteria for a specific class of capital. If banks issue that kind of CCB, they can replace something considered more expensive, e.g. equity, in their capital structure with the CCB. This way the banks satisfy the capital adequacy regulation on tier 1 capital while also lowering their weighted average cost of capital. When performing this maneuver several banks, in particular Swedish ones, have issued a CCB with the CET1/RWA trigger discussed above, although with a so called ”dual trigger” where either one of two scenarios will trigger the contract. (nordic-fi, 2014), (Reuters, 2015), (Moody’s, 2015). In all of the cited cases the two triggers have both been based on CET1/RWA, one at group level triggering on 8% and one at bank level on 5.125%. 5.125% is coincidentally also the limit for how low a trigger of CET1/RWA can be set if the CCB in question is to be considered AT1-capital according to the Basel III framework. (Basel Committee on Banking Supervision, 2012)

2.2.2 Mechanical Triggers

Many of the banks that have issued CCBs have opted for using some form of CET1/RWA trigger, but there are many other types that have been issued and even more that have been suggested by the academic community. CET1/RWA is an example of a mechanical, book value trigger. Another trigger belonging to that category is one proposed by Glasserman & Nouri (2012), based on the book value of total assets. They support Sundaresan & Wang’s (2010) theory that using the market value on a trigger concerning total assets, e.g. using the stock price, leads to multiple possible triggering events and that the price of the CCB becomes impossible to determine. Mechanical book value triggers have the advantage that there are clear rules from the Basel III directive as to what levels the triggers should be set to in order to be considered a specific class of capital. A valid concern regarding book value triggers raised by Avdjiev et al. (2013) is that book values are only calculated and disclosed to the public every so often. As even large banks and other financial institutions can fail very fast, Lehman brothers being an excellent example, there are possible scenarios where a book value trigger fails to react and activate the loss absorption mechanism before it is too late. The discontinuous nature of updates on book values also affects the liquidity of the CCB as an asset. Because the information is the most reliable when it is updated, there are possible scenarios where the information will not be able to do so, or be forced to sell at a disadvantageous price. Another potential weakness of book value triggers occurs when the issuing bank would benefit from the CCB triggering. In an adequately defined CCB the bank would always stand to loose from the CCB triggering, but if the CCB is not set up with care the bank could value the gains from the loss absorption mechanism higher than the costs and stigma of triggering. This, coupled with the fact that Basel III allows certain banks some degree of control over how their RWA is to be calculated, gives incentive and possibility of accounting manipulation. A bank could potentially change the way they calculate their RWA to make the CCB trigger earlier then it initially would have.

The other half of the mechanical family of triggers are the market-value ones. These types of triggers are never, or at least very rarely issued by banks, but there are several such CCBs theorized by the academic community, see for example Albul et al. (2010) or Pennacchi (2010). However, the most influential of these is perhaps Flannery (2009), who propose a capital ratio trigger similar to the previously
explained CET1/RWA trigger but based on the contemporaneous market value. Flannery argues that a book value trigger is much too easy for the issuer to affect, and that such accounting measures are common in times of financial crisis. An analogy to the crisis of 2008 when banks were "adequately and well capitalized" according to their books just before failing or receiving government relief packages is an excellent example on the potential accounting manipulation of book value triggered CCBs in a crisis. Flannery discusses some of the ways a market value trigger could cause market manipulations such as share dilution effects and death spirals, but since the reasoning is heavily dependent on what the terms of conversion are, those discussions are left to that segment of this thesis.

2.2.3 Discretionary triggers

Looking at the illustration at the beginning of this chapter, the discretionary triggers are still left to be discussed. The illustration shows a distinction within the discretionary category, namely at whose discretion the CCB should be triggered. There are, at least in academic theory, CCBs where the issuer of the CCB is the one to decide when to trigger the contract (Bolton & Samama, 2012). These types of contracts come very close to being a simple combination of a bond and a put option where the bank is short. As such, arguments for whether these instruments are indeed CCBs or not are mostly dependent on who does the arguing. The instruments can be understood as either a CCB with a "issuers discretion trigger", or with equal legitimacy as the mentioned combination of bond and option.

The far more common type of discretionary trigger and the most often referred to when using the term is the regulatory discretionary trigger. These kind of triggers employ the local regulators as a third party who decides when the CCB triggers. Generally, there will be some kind of agreement as to what scenarios will cause the regulator to trigger the contract but these scenarios will be considerably less well-defined than the ones in the mechanical triggers. Regulatory triggers comes with the advantage of the obvious agreement of regulators. It is from the regulators point of view the most reliable type of trigger because it provides them with some direct control over banks capital ratios in tough financial climates. As long as regulators can be trusted to be reactive enough regulatory triggers will activate in time to give a struggling bank relief before it fails. Because of this reliability, regulatory triggers are arguably superior at making sure taxpayers are punished as little as possible by the too big to fail phenomenon.

There are however some major disadvantages to regulatory triggers. Perhaps the most important one being that they are very unreliable as far as the investor is concerned. It is difficult to price a CCB that does not have a clear and predefined scenario where it triggers since calculating the probability of triggering and all the expected values that go along with that becomes virtually impossible. This, ironically, has potential to cause something that places high on most financial regulators list of things to avoid; financial instability (Albul et al., 2010). Since the CCBs pay a high yield in a low interest environment they are enticing to investors, but the amount of uncertainty that comes with a regulatory trigger will cause volatility of both stock and CCB price to be high and increase the chances of investors panicking and disrupting the market. A way of responding to this issue is for regulators to be very clear regarding when they are going to trigger a CCB. However the more regulators move towards predefined scenarios where they will trigger the instruments the less sense the regulatory trigger make as it simply moves towards being a mechanical trigger in all but formality.

The last potential problem with discretionary triggers to be discussed here is the influence the triggering of a CCB could have on the stock price of the bank. The relevance of this issue largely depends the terms of conversion and only arise if the loss absorption mechanism in place is a conversion to equity.
There are potential problems when a regulator triggers a contract - making the investor essentially buy stock for the principal of the instrument at a predefined price. If this happens at a fixed imposed loss, e.g. the CCB is set up in such a way that the investors will receive stocks to a value of $X$, where $X$ is predefined, the investors will have calculated the price of the contract thinking that they will retain at least $X$ in the case of the CCB triggering. The investors potential losses are much greater using discretionary triggers as the fact that regulators triggered the contract will probably affect the stock price of the bank negatively. The investor in the CCB will then own equity in a rapidly failing bank, that just had to announce the absence of faith from regulators in the banks solvency. Investors could therefore have a very hard time to liquidate the equity quickly. This is a scenario possible with most kinds of CCBs, but it is more significant regarding regulatory discretionary triggers because these triggers do not supply the stock market with continuous flows of information as to how close the CCBs are to triggering. If the trigger had been mechanical, the stock market would have been continuously aware of how close the banks CCBs were to triggering and adjusted accordingly before the trigger went off - considerably dampening the potential losses of the investor in the CCB.

Last among the triggers and completely omitted from the illustration in the beginning of the chapter are the dual triggers. These are commonly theorized and even more commonly issued by financial institutions. One already discussed example is many Swedish banks issuing CCBs with one CET1/RWA trigger on group level, and one on bank level. Dual triggers work exactly as one would think, instead of one trigger there are two. Dual triggers can be set up so that both of the employed triggers have to trigger, or so that one or the other triggering will activate the loss absorption mechanism. The individual triggers within a dual trigger setup function as the already described single triggers do and can in theory belong to any of the explained categories. McDonald (2010) is among the most cited working with dual triggers, but there has been significant research done by Sundaresan & Wang (2010) as well.

2.3 Terms of Conversion

In the case of a CCB with a conversion to equity as loss absorption mechanism the discussion of how the conversion is to take place becomes relevant. Perhaps the most intuitive and simple solution is to simply trade the principal of the CCB for equity in a 1:1 ratio. Such a setup theoretically implies that investors can with certainty say that their direct and immediate losses upon conversion are zero as in a perfect market, they simply sell the equity immediately and collect the principal in cash.

There are however more advanced ways of handling the terms of conversion. The two most prominent ones are called fixed imposed loss and fixed conversion price. This chapter will examine these two, along with some hybrid versions denoted other terms of conversion, and end in a discussion of how these different terms of conversion affect the issuing bank and investors as well as the market for CCBs and the banks stock.

2.3.1 Fixed Imposed loss

If the price of equity used for conversion is predefined as a ratio of the stock price at the time of conversion, it implies a loss incurred by the investor depending on the set ratio. If the ratio is 1:1, as in the example above, the loss of the investor is zero and the conversion if fair, but in other cases an immediate loss is imposed on the investor since one dollar of principal buys less than a dollar of equity. These terms of conversion are called a fixed imposed loss as the losses of the investor is constant while the price of conversion varies.
Pennacchi (2010), Glasserman & Nouri (2012) as well as Albul et al. (2010) are prominent authors of research papers on CCBs with terms of conversion within the fixed imposed loss category.

2.3.2 Fixed Conversion Price

The near opposite of a fixed imposed loss is called a fixed conversion price. A fixed conversion price implies that the imposed loss of the investor varies with the stock price at the time of conversion while the actual price of equity upon conversion remains constant. This makes CCBs with a fixed conversion price somewhat more complicated to price than their counterpart as the imposed losses are contingent on the market value of equity and as such stochastic in their nature.

In general, nothing definite can be said as to which of the two introduced terms of conversion generates the higher coupon. The coupon is, of course, related to the potential losses of the investor. Regarding CCBs with fixed conversion prices these potential losses vary with stock price and if a fixed imposed loss is used, the losses vary with the parameter set in the CCB. As such the size of coupons will be affected by the terms of conversion, but which of the here presented categories is being used does not in itself imply anything about the size of the coupon.

Readers interested in papers on CCBs with a fixed conversion price are referred to the work of McDonald (2010) and to some extent Li (2016) who’s pricing model is capable of handling a fixed imposed loss as well as a fixed conversion price.

2.3.3 Other Terms of Conversion

Combinations of fixed imposed loss and a fixed conversion price are in this thesis titled other terms of conversion. Such combinations consist of a fixed imposed loss structure with a floor for the stock price at which the terms of conversion are changed to instead be a fixed conversion price. As long as the stock price at the time of conversion is above a specific ratio of the original stock price the terms of conversion are some variant of a fixed imposed loss. If, however, the stock price drops below the predetermined threshold there is a fixed conversion price agreed upon that becomes the new terms of conversion. Other than the obvious difference that the terms of conversion themselves are dependent on the stock price, these hybrid terms of conversion work as explained in the previous subsections of this chapter.

This thesis assumes that conversion to equity is complete in the sense that upon triggering, all the conversion to equity that is contracted to take place converts instantly and fully. Glasserman & Nouri (2012) propose a CCB which converts gradually to continuously keep the bank just capitalized enough to satisfy the capital requirements. While efficient and reasonable in theory, the instrument would rely on a continuous stream of equity ownership passing to the investor which in practice would be very hard to implement.

2.3.4 The Bank, Investors and the Market

The owners of a bank issuing a CCB with conversion to equity as loss absorption mechanism in conjunction with fixed imposed loss as terms of conversion will be concerned about possible dilution of the market value of equity in the event of the CCB triggering (Brodén, 2016). Specifically, the shareholders that together own the bank prior to the CCB triggering can be put in a situation where a substantial amount of the ownership of the bank falls to the CCB holder upon triggering of the instrument. In a realistically constructed CCB the banks stock price would have to have fallen dramatically from the time of issuing for this event to take place, so the ownership the CCB holder receive would indeed be worth as much as agreed upon when the CCB was issued. As such, these dilution effects of certain CCBs are
not primarily a problem regarding value but one of control. A shareholder who has ownership interests that are not strictly financial have potential incentives to retain their control of the bank and could therefore view the CCBs in question as possible means for hostile takeovers. Long term investors holding original shares could also dislike the possibility of dilution effects if they as investors are less reactive than the general market. In the event of a financial crisis or similar, such shareholders could believe in that the struggling bank will eventually recover from financial distress and therefore retain their shares in hopes of recouping their losses. Such a strategy is more likely if the bank is partially financed with CCBs as the instruments are constructed as a support in times of financial distress. The dilution effect consequentially has the potential to severely hamper this long term investor strategy.

The dilution effects discussed above are related to the perhaps most voiced concern regarding CCBs as a whole, that of possible incentives for dishonest market manipulation. Dishonest in this context refers to market manipulation with the objective of driving down the stock price of the issuing bank. An investor holding the CCB who recognizes the above described potential problems for the owners of the issuing bank could also recognize the potential for their own gain stemming from the exact same phenomenon. The holder of the CCB could have incentive to spread negative information or short-sell stock of the issuing banks to reduce the stock price enough to trigger the contract. If the CCB holder has simply spread false information, the stock price will recover once the misconceptions are cleared up but at that point the CCB will already have triggered and provided the CCB holder with an opportunity to buy stock at an advantageous price. An other perhaps somewhat more likely scenario is that the holder of the CCB believes with relative certainty that the CCB will trigger at some point in the future, which leads to an incentive to decrease the stock price since it will give the CCB holder a better price of conversion.

Incentives for market manipulation for the CCB holders are, as already argued, most often given rise from relying completely on fixed imposed losses as the terms of conversion. The prerequisites for creating incentives for original stockholders to commit dishonest market manipulation are somewhat more general. Simply put, if the original stockholders judge that the advantages of the loss absorption mechanism outweigh the disadvantages of triggering the CCB they have incentive to intentionally devise a triggering of the CCB. This prospect is arguably more troublesome than CCB holders having incentives for dishonesty as the original stockholders have substantially more influence over the stock price of the bank. Triggering the CCB seems advantageous for stock holders if the price of conversion is set too high. That generally only occurs when applying a fixed conversion price as the conversion price of a CCB with fixed imposed loss follows the stock price itself, making it pointless to deliberately decrease the stock price in hopes of a high price of conversion.

Principal write-downs as loss absorption mechanisms are the worst possible example of a high price of conversion. Since the CCB holder doesn’t receive anything at all upon triggering of such CCBs this, left unchecked, has major potential to cause market manipulations. The widely used solution to this in issued principal write-down CCBs has been to use the temporal write-downs explained at the end of section 2.1. Temporal write-downs restricts the possibilities of causing dishonest incentives as the write-downs are reverted as soon as the bank is out of financial distress. As such, almost all potential gains for the bank and its owners are nullified in the long run and the incentives for market manipulation are void.

There are a few proposed solutions to the problems of dilution effects and market manipulation. Coffee (2010) proposes that instead of converting the principal of the CCB to shares, it should be converted to preferred stock as that is generally less volatile than common stock, severely hampering the rise of manipulation incentives. He further argues that a higher cost of capital on the preferred stock than
on the bond is a must to prevent incentives for the original stock holders to cause a triggering. This increased cost of capital in times of financial distress is not, according to Coffee, worrying from a public point of view as unpaid dividends does not lead to the bank defaulting. The argument has merit, but has been met with the notion that added complexity in structure and pricing is the last thing CCBs need in order to be properly implemented in banks capital structures and investors portfolios.

A solution that has been more widely accepted by the academic community as well as implemented in CCBs actually issued are the combinations of fixed imposed loss and fixed conversion prices discussed in section 2.3.3. These constellations of terms of conversion attempt to make the price of equity at conversion consistently high enough to dissuade the CCB holders from performing market manipulations while also keeping them low enough to make sure that the original shareholders have strictly honest incentives as well. For a more detailed and in depth analysis of this equilibrium Albul et al. (2010) can be consulted where the limits for these conversion prices are mathematically derived in a model applying fixed imposed losses with a stock price floor.

The last important consequence of the terms of conversion is an effect on investors and ends this chapter on a more positive note. A CCB with fixed imposed losses, or at the very least a hybrid containing only some aspects of fixed conversion prices has counter-cyclical effects on the investors portfolio (Bolton & Samama, 2012). As the CCB yields a relatively high coupon in periods of financial stability when the banks is doing well there are few arguments to make against their value for investors in such financial climates. In recessions and times of financial stress for the issuing bank the holder of a CCB often stands before very possible immediate losses, but there is potential long term gain in this as well. The argument comes very close to the one warning about dilution effects and the two phenomenons are indeed correlated. Given that the bank eventually recovers, investors do however stand to gain in the long term from buying stock in bank when the stock price is low. If the investor as a result from this gains ownership of an extraordinarily large part of the bank, the dilution effects are prominent. However, if this is not the case and the bank eventually emerges from their state of financial distress, the original stock owners will be satisfied with receiving the loss absorption mechanism that saved the bank and the previous CCB holder who bought shares at a low price will stand to gain from the stock price increases to come.
3 Model of the Bank

To value the contingent convertible bonds we first need to define a model of its cash flows. This chapter presents a complete model of the bank’s capital structure, starting from assets and continuing with defining the bank’s liabilities as claims on the cash flows from the bank’s assets.

3.1 Assets

Assets are hereafter assumed to follow geometric Brownian motion, which to the best of our knowledge is the approach of all previous researchers, except for Penniachi (2010) and Chen et al. (2013), who both model asset dynamics as a jump diffusion process. This assumption captures the non-continuous behavior of financial assets in time of financial distress and is thought to model the event of financial crisis. However, this assumption drastically increases complexity, why we continue with the assumption of a mere diffusion process. Furthermore, we assume that the bank’s assets generate a continuous cash flow to its investors proportional to total assets, as is common practice. If we assume that the bank’s assets have a constant drift rate of $\mu$, we can describe the dynamics of the assets under the objective measure $P$ on the probability space $(\Omega, F, P)$ as

$$dA_t = (\mu - \delta)A_t dt + \sigma A_t dW_t,$$  \hspace{1cm} (1)

where $\delta$ is the constant continuous payout rate to investors, $\sigma$ is the constant volatility of the bank’s assets, $W_t$ is a $P$ - Wiener process, and $A_t$ is adapted to the $\sigma$-algebra generated by $W_t$, i.e. $\{F_t\}_{t \geq 0} = \sigma\{W_s : 0 \leq s \leq t\}$. This representation of $A_t$ is clearly dependent on individual expectations on the expected growth rate $\mu$ of assets. To develop a valuation model independent of individual risk preferences, we change from the objective measure $P$ to the equivalent martingale measure $Q$, the risk neutral measure, under which the discounted gains processes of financial securities are martingales. We therefore would require that the process be dependent on the risk free interest rate, and not the expected rate of return. To change from measure $P$ to $Q$, we use the Girsanov theorem, which answers the question about what happens to a $P$ - Wiener process when we change measure to $Q$. Since we change measure to the risk neutral one, the new drift becomes $(r - \delta)$, where $r$ is the risk free interest rate, with a term structure assumed constant and flat. The change of measure is done by $dQ = L_t dP$, where $L_t$ is the likelihood process, which satisfies

$$\begin{cases} 
    dL_t = \frac{\mu - r}{\sigma}L_t dW_t \\
    L_0 = 1 
\end{cases}$$  \hspace{1cm} (2)

With the Girsanov theorem, $W_t^Q = W_t - \frac{\mu - r}{\sigma}t$ is a $Q$ - Wiener process. An important insight of this change of measure is that it alters the drift, but leaves the diffusion unaffected, i.e. the bank’s assets have the same volatility in the objective and the risk neutral world. On the probability space $(\Omega, F^Q, Q)$, equipped with the filtration $\{F_t^Q\}_{t \geq 0} = \sigma\{W_s^Q : 0 \leq s \leq t\}$, the dynamics of the value of the bank’s assets can be described by the stochastic differential equation

$$dA_t = (r - \delta)A_t dt + \sigma A_t dW_t^Q.$$  \hspace{1cm} (3)

With a use of Itô’s formula (on $\ln(A_t)$), an analytic solution to (3) can be derived as

$$A_t = A_0 \exp((r - \delta - \frac{\sigma^2}{2})t + \sigma W_t^Q).$$  \hspace{1cm} (4)
Capital adequacy regulation, and this thesis in particular, focus on the ratio between shareholder’s equity and risk-weighted assets, and not the ratio between equity and total assets. Previous work on contingent capital bonds have also focused the equity-assets ratio, e.g. Glasserman & Nouri (2012), or the asset-liability ratio like Li (2016). To extend this previous research, and develop a valuation model aimed at the instruments currently traded in the markets, we divide total assets into risk free and risk weighted assets. Let $X_t$ denote the value of risk-weighted assets at time $t$, and $Y_t$ the value of risk free assets. In practice, this fragmentation of assets would be done by either the Basel II framework, or by the bank’s internal ratings based (IRB) approach. Let $\omega_i$ denote the risk-weight assigned to asset $a_i(t)$.

The partition of the bank’s assets can be written as

$$X_t = \sum_i \omega_i a_i(t), \quad Y_t = \sum_i (1 - \omega_i) a_i(t) = A_t - X_t. \quad (5)$$

The danger of assuming that assets can fully be partitioned into two parts is that this does not account for the fact that RWA also includes some off-balance sheet exposures, and thus the sum $X_t + Y_t$ could be larger than $A_t$ in practice. Given this partition of total assets, we know that the risk free assets are indeed risk free and thus only earn the risk free rate $r$. The risk-weighted assets will have the same volatility as total assets, $\sigma$, since $Y_t$ has zero volatility, and zero covariance with $X_t$. With a payout rate from the risk-free assets of $r$, the value of these will remain constant, i.e. $Y_t = Y_0$.

With the goal of arriving at the dynamics of the risk-weighted assets, we note that since $X_t = A_t - Y_0$, and $Y_0$ is constant, we must have $dX_t = dA_t$. This could also be shown by a trivial use of Itô’s lemma. A better way to describe the risk-weighted assets is by the integral equation

$$X_t = X_0 + (r - \delta) \int_0^t A_s ds + \sigma \int_0^t A_s dW_s, \quad X_0 = A_0 - Y_0. \quad (6)$$

As with (4), $X_t$ can also be written as

$$X_t = X_0 \exp((r - \delta - \frac{\sigma^2}{2})t + \sigma W_t). \quad (7)$$

### 3.2 Liabilities

Assuming a capital structure consisting of debt, equity and contingent capital, we use that these are all claims on the cash flows of the bank’s assets. At some point in time $t \in (0, T]$ of the maturity of the debt, the losses of the bank may leave it insolvent, defaulting on the payments to its debt holders, resulting in bankruptcy or liquidation of the firm. This event can be defined in a number of ways. Merton (1973) defines the event of bankruptcy as value of assets being less than principal of debt at maturity, assuming only zero-coupon bonds in the capital structure. Both Glasserman & Nouri (2012) and Li (2016) assumes that regulatory intervention will occur before the event of actual bankruptcy, which is a reasonable assumption for large financial institutions. Glasserman & Nouri (2012) define the event of liquidation as when the value of assets falls below the current book value of debt. Li (2016) on the other hand, bases liquidation on the event that the asset-liability ratio drops below a predetermined limit.

Continuing on our path of studying instruments contingent on the shareholder’s equity to risk-weighted assets ratio, we define the event of liquidation as the first time this ratio falls below the limit $\beta \in (0, 1)$, which under the Basel III framework would be $\beta = 0.045$. From the balance sheet identity we have

$$Q_t = A_t - D_t \iff Q_t = X_t + Y_0 - D_t - C_t, \quad (8)$$
where \( Q_t \) denotes the value of shareholder’s equity at time \( t \), \( D_t \) the book value of debt, and \( C_t \) denotes the book value of contingent convertible debt. The capital requirement at time \( t \) can thus be written as

\[
Q_t \geq \beta X_t \iff (1 - \beta)X_t \geq D_t + C_t - Y_0.
\] (9)

The time of liquidation of the bank is the first time that the bank fails to maintain the capital requirement (9). From this we can define the time of liquidation as the stopping time

\[
\tau_L = \inf\{ t > 0 : Q_t \leq \beta X_t \}. \tag{10}
\]

### 3.2.1 Debt

We assume that the bank at time \( t = 0 \) issues debt with principal value \( P_D \), and maturity \( T \). Through this transaction, the bank raises a cash amount of \( D_0 \). The debt pays a continuous coupon of \( c_D P_D \) to its investors, where \( c_D \) is the coupon rate, paid until the time \( t = \min\{T, \tau_L\} \). Should it happen that \( \tau_L < T \), i.e. that the bank is liquidated by regulators, the debt holders receive cash at the recovery rate \( \rho_D \in [0, 1] \), of the principal \( P_D \) at time \( \tau_L \), where a \( \rho_D = 1 \) would indicate that the bond is risk free. An extension of this model could be made by dividing debt into different tranches, each of which with a certain coupon rate, \( c_i \), recovery rate, \( \rho_i \), and principal, \( P_i \), but with the same maturity \( T \). This could be modeled to include both senior/junior debt, as well as deposits, for example. To summarize, the future cash flows to the debt holders are continuous coupon payments, the principal value of debt if the bank is not liquidated, and cash at the recovery rate of the principal if the bank is liquidated. The present values of these cash flows are as follows:

- **Coupon payments until maturity or liquidation**
  \[
  \int_t^{\min\{T, \tau_L\}} c_D P_D e^{-r(s-t)} ds. \tag{11}
  \]

- **Principal at maturity**
  \[
  e^{-r(T-t)} P_D \mathbb{1}_{\{T < \tau_L\}} + \mathbb{1}_{\{T \geq \tau_L\}} = \begin{cases} 1, & \text{if } T < \tau_L \\ 0, & \text{otherwise.} \end{cases} \tag{12}
  \]

- **Recovered principal in the event of liquidation**
  \[
  e^{-r(\tau_L-t)} P_D \mathbb{1}_{\{T \geq \tau_L\}}. \tag{13}
  \]

The cash flows defined above denotes the market value of debt. Like Glasserman & Nouri (2012), we define the book value of debt as the total amount that the bank owes its creditors, discounted by the yield, \( y_D \), at which the debt was issued

\[
D_t = e^{-y_D(T-t)} P_D + \int_t^T c_D P_D e^{-y_D s} ds = P_D \left( e^{-y_D(T-t)} \left( 1 - \frac{c_D}{y_D} \right) + \frac{c_D}{y_D} \right). \tag{14}
\]

If we assume that the debt was issued on par, i.e. with \( D_0 = P_D \), and that the yield of the debt equals the coupon rate, \( c_D = y_D \), we note that the debt will have a constant book value. For valuation purposes, we continue with this assumption.
3.2.2 Contingent Convertible Bonds

We assume that the bank issues contingent convertible bonds at time $t = 0$, simultaneously as the ordinary debt, with the same maturity $T$. We do not make any assumptions regarding whether the CCB is issued on top of existing debt in a recapitalization replacing equity, or if it replaces part of the bank’s ordinary debt. The CCB is issued with a principal value of $P_C$. Upon issuance, the CCB pays a continuous coupon to its investors, $c_C P_C$, where $c_C$ denotes the coupon rate. Since the instruments are contingent on the CET1/RWA ratio, and convertible to equity upon the event that the CET1/RWA ratio for the first time falls below the contract parameter $\alpha \in (\beta, 1)$. This is equivalent to $Q_t \leq \alpha X_t$, and the time of conversion can thus be defined as the stopping time

$$\tau_C = \inf\{t > 0 : Q_t \leq \alpha X_t\}. \quad (15)$$

A natural constraint to impose on $\alpha$ would be $\alpha < \beta$, hence $\tau_C < \tau_L$, since conversion has to occur prior to liquidation for the instrument to be a true CCB, otherwise it would just posses the characteristics of a subordinated bond. Upon the event of conversion, the principal of the bond is converted into equity of the bank. Conversion is instant and permanent, meaning that even if the bank’s CET1/RWA ratio increases beyond $\alpha$, the CCB holders still hold equity. An alternative to this would be to model conversion as continuous process like Glasserman & Nouri (2012), where conversion occurs every time the critical capital ratio is reached, until the contingent capital is depleted. This component has the advantage that it reduces the risks of the investors, however, the instruments in practice, to the best of our knowledge, all converts discontinuously and instantly, why we continue with that assumption.

To shield the issuer from potentially massive dilution effects, resulting from the fact that a relatively small principal can buy a large portion of equity in times of crisis, the price of equity used for conversion has a built in floor of a portion $\phi$ of the stock price at issuance ($t = 0$). In the case where this fixed conversion price is not used, the price for conversion is a portion $\gamma$ of the stock price at conversion. This is a combination of fixed imposed loss and fixed conversion price as terms of conversion. Using that the value of equity at $t$ is $Q_t$, the conversion price is therefore $\max\{\gamma Q_{\tau_c}, \phi Q_0\}$. Hence, at conversion, CCB holders receive a portion $\psi = P_C / \max\{\gamma Q_{\tau_c}, \phi Q_0\}$ of the bank’s equity.

As an investor in the CCB, the future cash flows will consist of continuous coupon payments on the bond until conversion or maturity, the principal of the bond in the event that conversion does not occur, and the value future value of equity in the event of conversion, which will consist of dividend payments until maturity or liquidation, and the remaining equity value at that time. The present values of these cash flows are as follows:

- **Coupon payments until conversion or maturity**
  \[
  \int_t^{\min\{T, \tau_C\}} c_C P_C e^{-r(s-t)} ds. \quad (16)
  \]

- **Principal at maturity**
  \[
  e^{-r(T-t)} P_C \mathbb{1}_{\{T < \tau_C\}}. \quad (17)
  \]

- **Equity at conversion**
  \[
  e^{-r(\tau_C - t)} \psi Q_{\tau_C} \mathbb{1}_{\{T \geq \tau_C\}}, \quad \psi = \frac{P_C}{\max\{\gamma Q_{\tau_c}, \phi Q_0\}}. \quad (18)
  \]
As with the ordinary debt, the cash flows stated above are what constitutes the market value of the contingent debt. To measure the book value we once again discount all future promised cash flows, and discount them with the yield, $y_C$, at which the contingent debt was originally issued

$$\begin{align*}
C_t &= \mathbf{1}_{\{t<\tau_C\}} \left( e^{-y_C(T-t)} P_C + \int_t^T c_C P_C e^{-y_C s} ds \right) = \mathbf{1}_{\{t<\tau_C\}} P_C \left( e^{-y_C(T-t)} \left( 1 - \frac{c_C}{y_C} \right) + \frac{c_C}{y_C} \right). \quad (19)
\end{align*}$$

Again, we can see that the book value of contingent debt will remain constant with the assumption the the yield equals the coupon rate, $c_C = y_C$, and that that it was issued on par, i.e. that the principal equals the cash amount raised from issuance, $C_0 = P_C$. With these constant book values we can determine that conversion occurs when the risk-weighted assets hit the threshold $X_{\tau_C} = P_D + P_C - Y_0 1_{\{T<\tau_C\}} c_C P_C$, and that liquidation of the bank occurs at $X_{\tau_L} = \frac{P_D - Y_0}{1-\beta}$.

### 3.2.3 Equity

As stated, the bank’s assets pay a continuous yield of $\delta$ to its investors, until the event of liquidation of the bank. This cash flow is proportional to the value of the bank’s assets. Since the payments to both holders of debt and contingent convertible debt are proportional to the principal value of respective debt, the dividends paid to equity holders are the remains of $\delta$, dependent of $A_t$. In analogy with Glasserman & Nouri (2012), building upon Leland (1994) and Leland & Toft (1996), assuming a corporate tax rate of $\theta \in (0, 1)$ we can define the cash flow to shareholders as

$$\begin{align*}
\delta A_t - (1-\theta)(c_D P_D + \mathbf{1}_{\{T<\tau_C\}} c_C P_C).
\end{align*}$$

This definition of dividends allows for a negative cash flow to shareholders, when assets value is sufficiently small. In analogy with previous research (e.g. Glasserman & Nouri (2012)) we interpret negative dividends as the issuance of new equity to existing shareholders, i.e. a rights offering, the proceeds from which is used to pay coupons to debt holders, to avoid default.

We define the value of shareholder’s equity after the introduction of CCBs to the capital structure by extending (8) to also include contingent debt, and to use the constant book values of debt

$$\begin{align*}
Q_t &= A_t - D_t - C_t = X_t + Y_0 - P_D - P_C.
\end{align*}$$

This measure of shareholder’s equity will in our valuation be used as the market value of equity. However, this is not fully accurate, since book and market values of equity tend to differ. For most firms, this assumption is simply unreasonable, but for banks who hold a large portion of market traded assets and marking their values to market, this simplification is somewhat plausible. As mentioned by Metzler & Reesor (2014), this is a common mix-up in the capital structure literature, when market and book values are treated as interchangeable. An easy fix for this problem is scaling the book value of equity by the bank’s P/B-ratio, and using it as the market value of equity, assuming a constant P/B-ratio.

At time $t = \tau_C$, when the losses of the bank has eroded the shareholder’s equity and the CET1/RWA ratio has fallen below the critical level $\alpha$, the bank retires the contingent convertible bonds, and new equity is issued to the CCB holders. The amount of equity issued is such that the new equity holders, i.e. the CCB holders, own a portion of $\psi = P_C / \max\{\gamma Q_{\tau_C}, \phi Q_0\}$ of the bank’s equity, and the original equity holders now own a portion of $(1 - \psi)$ of equity.
4 Valuation

Our goal with this chapter is to calculate the expected values of the discounted cash flows defined in (11)-(13) and (16)-(18). In analogy with Glasserman & Nouri (2012), we introduce a variable for the logarithm of the ratio of asset value at time $t$ and at 0, although we use risk-weighted assets. We also introduce a variable $\tilde{m}_t$ for the running mean of $\tilde{X}_t$ over the interval $[0, t]$.

$$\tilde{X}_t = \ln \left( \frac{X_t}{X_0} \right), \quad \tilde{m}_t = \min_{0 \leq s \leq t} \tilde{X}_s. \quad (22)$$

If we recall that $X_t$ can be written on the form $X_t = X_0 \exp(\lambda t + \sigma W^Q_t)$, where $\lambda = r - \delta - \sigma^2/2$, it is clear that $\tilde{X}_t = \lambda t + \sigma W^Q_t$, which is Brownian motion (and not geometric Brownian motion) with drift $\lambda$ and drift $\sigma$. Clearly $\tilde{X}_t$ has a $N(\lambda t, \sigma \sqrt{t})$-distribution, since $W^Q_t$ has expectation 0 and standard deviation $\sigma \sqrt{t}$, and $\lambda t$ is deterministic. From Glasserman & Nouri (2012) we have the density of the running minimum

$$\mathbb{Q}(\tilde{m}_t \leq m) = \Phi \left( \frac{m - \lambda t}{\sigma \sqrt{t}} \right) + e^{2\lambda m/\sigma^2} \Phi \left( \frac{m + \lambda t}{\sigma \sqrt{t}} \right), \quad (24)$$

where $\Phi$ denotes the cumulative distribution function of the standard normal distribution.

Judging by the cash flows that we set out to value, it is clear that an important step is taking the expected value of the discount factor at a stopping time together with the indicator function of the same stopping time, i.e.

$$\mathbb{E}_Q \left[ e^{-r_{\tau_i}} \mathbb{1}_{\{\tau_i \leq T\}} \right], \quad i = L, C. \quad (25)$$

With a change of drift using the Girsanov theorem, together with a use of the distribution of the running minimum with the new drift, Glasserman & Nouri (2012) calculates this expectation as

$$\left( \frac{X_{\tau_i}}{X_0} \right) \frac{\lambda t}{\sigma^2} \Phi \left( \frac{\ln \left( \frac{X_{\tau_i}}{X_0} \right) - \eta T}{\sigma \sqrt{T}} \right) + \left( \frac{X_{\tau_i}}{X_0} \right) \frac{\lambda t}{\sigma^2} \Phi \left( \frac{\ln \left( \frac{X_{\tau_i}}{X_0} \right) + \eta T}{\sigma \sqrt{T}} \right), \quad (26)$$

where $\eta = \sqrt{2r\sigma^2 + \lambda^2}$ is the new drift, and we have used from section 2.3.2 the values of $X_{\tau_i}$. Again, $X_{\tauC} = \frac{P_0 + P_2 - Y_0}{1 - \alpha}$ is the level of the risk-weighted assets at conversion, and $X_{\tauL} = \frac{P_D - Y_0}{1 - \beta}$ is the level at liquidation.

4.1 Debt

4.1.1 Coupon Payments

The market value at issuance of coupon payments from the ordinary debt is the expected value of (11) under the measure $\mathbb{Q}$, with respect to the trivial $\sigma$-algebra $\mathcal{F}_0^Q$ (i.e. non-conditional expectation)

$$\mathbb{E}_Q \left[ \int_0^{\min(T, \tauL)} c_D P_D e^{-rs} ds \right]. \quad (27)$$

Evaluating the integral, taking the deterministic terms out of the expectation and finally splitting the expectation in two by the law of total expectation, we arrive at
\[
\frac{c_D P_D}{r} \left( 1 - \mathbb{E}_Q \left[ e^{-r \min(\tau_L, T)} \right] \right) = \frac{c_D P_D}{r} \left( 1 - e^{-rT} \mathbb{Q}(\tau_L > T) - \mathbb{E}_Q \left[ e^{-r\tau_L} I_{\{\tau_L \leq T\}} \right] \right). \tag{28}
\]

The remaining expectation can be evaluated directly with (26). The probability that liquidation does not occur during before maturity is the compliment to the event that liquidation does occur, i.e. \( \mathbb{Q}(\tau_L > T) = 1 - \mathbb{Q}(\tau_L \leq T) \). Using the running minimum \( \tilde{\mu}_T \), we note that \( \mathbb{Q}(\tau_L \leq T) = \mathbb{Q}(\tilde{\mu}_T \leq \ln(X_{\tau_L}/X_0)) \), since the probability that liquidation occurs before time \( T \) is equivalent to the probability that the running minimum of \( \tilde{X}_t \) falls below \( \ln(X_{\tau_L}/X_0) \) on the interval \([0, T]\), which means that the risk-weighted assets have decreased to the liquidation boundary \( \beta \). Thanks to this fact, we are able to use the density (24), and the expression (28) becomes

\[
\frac{c_D P_D}{r} \left( 1 - e^{-rT} \left( 1 - \left( \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right)}{\sigma \sqrt{T}} - \frac{\lambda T}{\sigma \sqrt{T}} \right) + e^{2\lambda \ln(X_{\tau_L}/X_0)/\sigma^2} \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) + \lambda T}{\sigma \sqrt{T}} \right) \right) \right) - \left( \frac{X_{\tau_L}}{X_0} \right)^{\frac{\lambda + \eta}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) - \eta T}{\sigma \sqrt{T}} \right) - \left( \frac{X_{\tau_L}}{X_0} \right)^{\frac{\lambda + \eta}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) + \eta T}{\sigma \sqrt{T}} \right) \right). \tag{29}
\]

### 4.1.2 Principal at Maturity

The value of the principal at maturity, in the case that the bank is not liquidated during the maturity of debt, is the expected value of (12), i.e.

\[
\mathbb{E}_Q \left[ e^{-rT} P_D I_{\{T \leq \tau_L\}} \right] = e^{-rT} P_D \mathbb{Q}(T < \tau_L) = e^{-rT} P_D (1 - \mathbb{Q}(\tau_L \leq T)). \tag{30}
\]

Again, using the fact that \( \mathbb{Q}(\tau_L \leq T) = \mathbb{Q}(\tilde{\mu}_T \leq \ln(X_{\tau_L}/X_0)) \), we can use the distribution of the running minimum (24). With this, the expression (30) can be evaluated to

\[
e^{-rT} P_D \left( 1 - \left( \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) - \lambda T}{\sigma \sqrt{T}} \right) + e^{2\lambda \ln(X_{\tau_L}/X_0)/\sigma^2} \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) + \lambda T}{\sigma \sqrt{T}} \right) \right) \right). \tag{31}
\]

### 4.1.3 Recovered Principal at Liquidation

The value of the principal of debt, if the bank is liquidated by regulators before time \( T \) is the expected value of the cash flows (13)

\[
\mathbb{E}_Q \left[ e^{-r\tau_L} \rho_D P_D I_{\{\tau_L \leq T\}} \right] = \rho_D P_D \mathbb{E}_Q \left[ e^{-r\tau_L} I_{\{\tau_L \leq T\}} \right]. \tag{32}
\]

All that remains is to apply (26) to the expectation, and we arrive at

\[
\rho_D P_D \left( \left( \frac{X_{\tau_L}}{X_0} \right)^{\frac{\lambda + \eta}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) - \eta T}{\sigma \sqrt{T}} \right) + \left( \frac{X_{\tau_L}}{X_0} \right)^{\frac{\lambda + \eta}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_{\tau_L}}{X_0} \right) + \eta T}{\sigma \sqrt{T}} \right) \right). \tag{33}
\]

### 4.1.4 Coupon Rate

To keep the book value of debt constant, we assumed that the debt was issued with a yield equal to the coupon rate, and on par, i.e. with \( P_D = D_0 \). Thus, we have imposed a constraint on the values of future cash flow calculated in this chapter, and to align these to this initial constraint is our final step of valuation. In other words, we have to solve for the coupon rate \( c_D \) that equates \( P_D \) to the cash flows of debt. From this, we arrive at the coupon rate
\[ c_D = r \left( \frac{1 - e^{-rT}Q(T < \tau_D)}{1 - e^{-rT}Q(T < \tau_D) - \rho_B \mathbb{E}_Q \left[ e^{-r\tau_D}1_{(\tau_D \leq T)} \right]} \right). \tag{34} \]

To evaluate this remaining expression, we can simply apply (24) and (26). An important insight from (34) is that when \( \rho_B = 1 \), and the bond is indeed risk free, we have \( c_D = r \), as was also pointed out by Glasserman & Nouri (2012).

### 4.2 Contingent Convertible Bonds

#### 4.2.1 Coupon Payments

Analogous to debt, the market value at \( t = 0 \) of coupon payments on the contingent capital is given by

\[
\mathbb{E}_Q \left[ \int_0^{\min(T, \tau_C)} c_CP_C e^{-rs}ds \right] = \frac{c_CP_C}{r} \left( 1 - e^{-rT}Q(\tau_C > T) - \mathbb{E}_Q \left[ e^{-r\tau_C}1_{(\tau_C \leq T)} \right] \right). \tag{35} \]

Again, what remains is applying (24) and (26) to (35), which yields

\[
\frac{c_CP_C}{r} \left( 1 - e^{-rT} \left( 1 - \left( \Phi \left( \frac{\ln \left( \frac{X_C}{X_0} \right) - \lambda T}{\sigma \sqrt{T}} \right) + e^{2\lambda \ln(X_C/X_0)/\sigma^2} \Phi \left( \frac{\ln \left( \frac{X_C}{X_0} \right) + \lambda T}{\sigma \sqrt{T}} \right) \right) - \left( \frac{X_C}{X_0} \right)^{\frac{\lambda + \eta}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_C}{X_0} \right) - \eta T}{\sigma \sqrt{T}} \right) - \left( \frac{X_C}{X_0} \right)^{\frac{\lambda + \eta}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_C}{X_0} \right) + \eta T}{\sigma \sqrt{T}} \right) \right) \right). \tag{36} \]

#### 4.2.2 Principal at Maturity

The expected value of the principal at maturity as defined in (12) can be calculated precisely as the principal of ordinary debt, i.e. with a use of the distribution of the running minimum of the logarithm of the evolution of risk-weighed assets. This yields

\[
\mathbb{E}_Q \left[ e^{-rT}P_C1_{(T < \tau_C)} \right] = e^{-rT}P_CQ(T < \tau_C) = e^{-rT}P_C(1 - Q(\tau_C \leq T)). \tag{37} \]

Once again, we use the fact that \( Q(\tau_C \leq T) = Q(\tilde{m}_T \leq \ln(X_C/X_0)) \). Thus, (37) becomes

\[
e^{-rT}P_C \left( 1 - \Phi \left( \frac{\ln \left( \frac{X_C}{X_0} \right) - \lambda T}{\sigma \sqrt{T}} \right) + e^{2\lambda \ln(X_C/X_0)/\sigma^2} \Phi \left( \frac{\ln \left( \frac{X_C}{X_0} \right) + \lambda T}{\sigma \sqrt{T}} \right) \right). \tag{38} \]

#### 4.2.3 Equity at Conversion

In the case where the bank’s risk-weighted assets hit the boundary \( X_{\tau_C} \), the principal of the CCB convert to newly issued equity at a value of \( \psi Q_{\tau_C} \), where \( \psi = \frac{P_C}{\max \{ \psi Q_{\tau_C}, \phi Q_B \}} \), as defined in section 3.2.2, and \( Q_{\tau_C} = \phi(1 - \alpha) \). As was previously mentioned, this is a book value of equity being used as the market value, which can be somewhat corrected by scaling with the bank’s P/B-ratio. The expected present value of this cash flow is

\[
\mathbb{E}_Q \left[ e^{-r\tau_C} \psi Q_{\tau_C}1_{(T \geq \tau_C)} \right] = \psi Q_{\tau_C} \mathbb{E}_Q \left[ e^{-r\tau_C}1_{(T \geq \tau_C)} \right]. \tag{39} \]

To this remaining expectation, we apply (26) a final time to arrive at
\[
\psi Q_{\tau C}\left( \left( \frac{X_{\tau C}}{X_0} \right)^{\frac{\lambda}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_{\tau C}}{X_0} \right) - \eta T}{\sigma \sqrt{T}} \right) + \left( \frac{X_{\tau C}}{X_0} \right)^{\frac{\lambda}{\sigma^2}} \Phi \left( \frac{\ln \left( \frac{X_{\tau C}}{X_0} \right) + \eta T}{\sigma \sqrt{T}} \right) \right). \tag{40}
\]

4.2.4 Coupon Rate

We have assumed that the contingent convertible debt was issued on par, with \( P_C = C_0 \), as with debt. This, together with the assumption that the yield at which the CCB was issued equaled the coupon rate, keeps the book value of the contingent debt constant over time, and simplifies valuation. Our only remaining step is thus matching the cash raised at issuance \( C_0 = P_C \) with the expected present values of the CCB, and in particular, finding the coupon rate \( c_C \) that solves this equation. This yields

\[
c_C = r \left( \frac{1 - e^{-rT}Q(T < \tau_C) - \psi P_C Q_{\tau C} \mathbb{E}_Q \left[ e^{-r\tau_C} \mathbbm{1}_{\{\tau_C \leq T\}} \right]}{1 - e^{-rT}Q(T < \tau_C) - \mathbb{E}_Q \left[ e^{-r\tau_C} \mathbbm{1}_{\{\tau_C \leq T\}} \right]} \right). \tag{41}
\]

Another use of the distribution of the running minimum (24) and the expectation (26), gives the coupon rate. An inspection of (41) reveals that if \( \psi P_C Q_{\tau C} = 1 \), which is equivalent to a fair conversion, the coupon rate will equal the risk-free interest rate \( r \). This is a result that might seem unrealistic at first, but with the assumption of perfect capital markets, in the event of conversion the investor of a CCB will be able to liquidate its entire equity stake in the bank and receive the full principal. Analogous to the case where the ordinary debt has a recovery rate of \( \rho_D = 1 \), this implies an effective loss of 0, equivalent to a risk-free investment, thus earning the risk-free interest rate.
5 Numerical Example

To evaluate our proposed model for valuation of contingent convertible bonds, we apply the results from the previous chapter to a hypothetical scenario. In this scenario, interest rates are close to zero, similar to the current market conditions. We use a high-level trigger at 8% for conversion, and a liquidation threshold at 4.5%, as determined by Basel III. As for risk-weighted assets, a ratio of 40% of total assets might be high for traditional retail banks with low risk exposures. However, for investment banks, a 40% ratio might be low. For example, as of Q1 2015, the RWA-to-assets ratio of Goldman Sachs was roughly 65%. Furthermore, in our example the bank is well capitalized at the initial CET1/RWA ratio of 30%, which is a bit above the current capital ratios of the major Swedish banks. The initial debt-to-assets ratio is fairly low at 88%, where contingent debt constitutes 1.14% of total debt. This level of contingent debt is thought to capture current conditions, where contingent debt is a growing but still decimal part of bank’s overall financing. The recovery rate of debt is quite low at 85%, and for bank’s with a high level of deposits, which are risk free in most cases, the recovery rate might well be much higher. We use a fixed conversion price of 67% of the stock price as issuance, combined with a fixed imposed loss of 1, i.e. a fair conversion.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total assets</td>
<td>$A_0$</td>
</tr>
<tr>
<td>Risk-free assets</td>
<td>$Y_0$</td>
</tr>
<tr>
<td>Risk-weighted assets</td>
<td>$X_0$</td>
</tr>
<tr>
<td>Principal of debt</td>
<td>$P_D$</td>
</tr>
<tr>
<td>Principal of contingent debt</td>
<td>$P_C$</td>
</tr>
<tr>
<td>Recovery rate of debt</td>
<td>$\rho_D$</td>
</tr>
<tr>
<td>Weighted maturity of debt</td>
<td>$T$</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>$r$</td>
</tr>
<tr>
<td>Payout rate from assets</td>
<td>$\delta$</td>
</tr>
<tr>
<td>Asset volatility</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Lower boundary for equity price at conversion</td>
<td>$\phi$</td>
</tr>
<tr>
<td>Fixed imposed loss</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>Conversion boundary</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Liquidation boundary</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Coupon rate of debt</td>
<td>$c_D$</td>
</tr>
<tr>
<td>Coupon rate of contingent debt</td>
<td>$c_C$</td>
</tr>
</tbody>
</table>

Table 1: parameters for the numerical example, including results

From these results, the large spread on coupons between debt and contingent debt stands out. However, these results are quite close to current market yield spreads on debt versus contingent debt. A trivial observation is the coupon rate of 5.5% on the contingent convertible bonds issued by Swedbank in February 2015. However, since we have not fully matched our parameters to Swedbank’s, this result only serves as anecdotal evidence.

5.1 Sensitivity Analysis

As a way of model evaluation, we use the base case presented in the previous section and study the coupon rates dependence on volatility, debt structure, fixed imposed loss/fixed conversion price, and maturity of debt.
5.1.1 Volatility

![Volatility graph]

Figure 2: Coupon rates of debt and CCB as a function of volatility

All else equal, Figure 2 shows the coupon rates dependence of volatility, with a volatility in the interval [0, 0.15]. As expected, both debt and CCB incur higher coupon rates with higher volatility. Although, with a volatility below 4%, both debt and CCB are close to risk free, since the probability of conversion/liquidation comes close to zero for the given maturity.

5.1.2 Debt Structure

![Debt structure graph]

(a) Debt

![CCB structure graph]

(b) CCB

Figure 3: Coupon rates of debt (a) and CCB (b) when the ratio of CCB to total debt varies

Here we see a clear example of one of the most important features of contingent convertible bonds. With higher a higher level of contingent debt in the debt structure the probability of liquidation decreases. This results in a reduced expected loss imposed on debt holders, which decreases the coupon rate of debt. As for the contingent convertible bonds, the result is the converse. A greater portion of contingent debt in the capital structure does indeed lower the probability of liquidation, however this has no effect on the CCBs, since conversion always occur prior to liquidation. Instead, with higher levels of contingent debt,
more of the total losses of the bank will be borne by the holders of CCBs, thus increasing their expected losses and increasing the coupon rate. If we compare these results to those of Glasserman & Nouri (2012), we see a clear difference. In their model, coupon rates on both debt and CCB are decreasing with an increasing portion of contingent debt. The rationale behind this is since CCB holders in their model earn equity continuously without selling it, they too bear some of the losses of liquidation. Then, with more contingent debt in the capital structure reducing the probability of liquidation, their expected loss will decrease, and so will the coupon rate. In some cases, the coupon rate on CCBs even decreases below the rate for debt. To minimize coupon payments, the bank should therefore always maximize their portion of CCBs in the capital structure. In our model, since the coupon rate of debt is decreasing and convex, while the coupon rate of contingent debt is linearly increasing, a minimum on (0, 1) can be found. Figure 4 shows the weighted average coupon rate on the bank’s total debt. In this case, an optimal capital structure is reached at roughly 5%. Worth noticing is that with a level of CCBs below 2%, the coupon payments on debt exceeds the total payout rate from assets.

Figure 4: Weighted average coupon payment of both debt and CCB as a function of debt structure

5.1.3 Fixed Imposed Loss and Fixed Conversion Price

Figure 5: Coupon rate of CCB as a function of both fixed imposed loss and fixed conversion price
The price of equity for conversion is dependent on a max function of the fixed imposed loss and fixed conversion price, Figure 5 shows this dependence with $\phi \in [0,1]$ and $\gamma \in [1,2]$. The result is intuitive, the coupon rate surface is simply the maximum of two surfaces, both increasing in one axis and constant in the other.

5.1.4 Maturity

![Diagram](a)

![Diagram](b)

Figure 6: Coupon rates when the maturity varies, with $T \in [0,10]$ in (a) and $T \in [0,100]$ in (b)

The maturity of total debt, assumed equal for all kinds of debt, has an enormous impact on coupon rates, as shows by Figure 6. With a short maturity, assuming an initially well capitalized bank, the probability of conversion/liquidation decreases, and with longer maturities financial distress becomes more likely. What stands out from (b) is that with maturities above 10 years, the coupon rates on both debt and CCB decreases. The explanation for this is that we hold debt on a constant level, while the bank’s assets over time evolve. In the long run, the leverage ratio will almost certainly have decreased, the probability of conversion/liquidation will tend to zero and the coupon rates will converge. In this case they stabilize at $c_D = 0.99\%$ and $c_C = 3.45\%$. 
6 Conclusions and Further Work

Building upon the work of Glasserman & Nouri (2012), we have extended their model to include risk-weighted assets, instant conversion, and a combination of fixed imposed loss and fixed conversion price, and from this we have derived closed form solutions for the coupon rates of debt and contingent convertible bonds. We have tested our model on a hypothetical scenario, and from that base case, performed a sensitivity analysis on the coupon rates dependence on volatility, debt structure, terms of conversion, and maturity. From this we have shown that our model enables the issuer to determine an optimal capital structure, i.e. the level of contingent convertible bonds that minimizes the total payout to debt, and thus maximizes shareholder value. Overall, the results from the sensitivity analysis were as expected. However, with long maturities, coupon rates decrease as a consequence of the assumption of constant debt and evolving assets.

As for further work, a solution to the problem of decreasing coupon rates for longer maturities might be to assume constantly evolving values of debt, so that the bank’s leverage ratio over time is expected to remain constant. For contingent convertible bonds to be classified as additional tier 1 capital, a key requirement is perpetual maturity. With evolving debt values, valuation of perpetual instruments is enabled. However, these instruments are typically callable by the issuer after 5 to 10 years. A possible solution to this might be to include a call option on the underlying CCB. Another important regulatory requirement on AT1-instruments is that they include discretionary non-cumulative coupon payments. This thesis puts stress on the risk of conversion the investor incurs. There is however also a considerable risk of cancellation of coupons that has not been addressed. As such, an important step towards precise valuation is to include this feature.

Our model assumes that the book value of equity equals the market value. As mentioned, this is not an entirely inaccurate assumption for banks. We extended this assumption to assume a constant P/B-ratio, and use that ratio to scale up the book value. For further work, effort should be put to fully include the market value of equity. Another point of improvement regarding equity is to relax the assumption that common equity tier 1 equals shareholder’s equity. A remedy might be to deduct the principal value of contingent debt from the shareholder’s equity, and use that as a measure of common equity tier 1 capital.

In times of financial distress, volatility tends to rise, why the assumption of constant asset volatility might be unreasonable. To improve the model, it would be an important advancement to remove the assumption of constant volatility in favor of a local volatility model, where the volatility is a function of time and assets. As for the risk-free interest rate, the same logic applies. The term structure is assumed constant and flat, and a first extension would be to use a non-flat term structure, i.e. with different interest rates for different time periods. A final step would be to use stochastic interest rates. This would however increase complexity more than using a non-flat term structure. An ad hoc approach is to use an interest rate dependent on time and assets. This is clearly a false dependence, but it might capture the interdependence between asset value and interest rates.
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