Design of slender steel members

A comparison between the reduced stress method and the effective width method

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Abstract

As of now, the most common way in Sweden, to address the issue of local buckling of steel structures is through the procedure called the effective width method. A less common procedure for dealing with local buckling is the reduced stress method. The benefit of the latter method is that, when combined with finite element analysis, results in a less tedious design process. However, this method is often labelled as a method that results in an overconservative design. Therefore, the purpose of this report is to compare and evaluate the reduced stress method against the effective width method and nonlinear finite element method. The nonlinear FE-analyses are performed with intention of simulating the real behaviour of the structure and serve as a reference for the other two methods. The comparison is conducted through a series of analyses, on different steel members with various load configurations and slenderness in order to include the most common cases in the construction industry. This report resulted in recommendations for when the reduced stress method could be a relevant design procedure, with emphasis on providing reliable and accurate results compared to FE-analyses. Furthermore, the report resulted in proposed further studies, both regarding the improvement of the reduced stress method and other structural elements that should be studied. The result from the report indicates that the reduced stress method can be used when the effect of patch loading is small. Furthermore, it is recommended to obtain the critical stresses from a linear finite element analysis rather than from hand calculations, as to not end up with over-conservative results.

Keywords: Steel structures, finite element method, reduced stress method, effective width method, nonlinear finite element
Sammanfattning


Nyckelord: Stålkonstruktion, finita element metoden, reducerad spänningsmetoden, effektiva bredd metoden, olinjär finita element analys
Preface

The topic of this master thesis was initiated by the consultant company ELU in Stockholm in cooperation with the Department of Civil and Architectural Engineering, the division of Structural Engineering and Bridges at the Royal Institute of Technology.

First and foremost, we would like to thank our supervisors Christoffer Svedholm and Professor Costin Pacoste who has contributed immensely with their guidance and support throughout the work. Special acknowledgement goes to ELU for presenting us with the opportunity to perform our thesis at their office, and to the engineers at ELU for providing insight and expertise.

We would also like to thank our examiner Professor Raid Karoumi who has been a good mentor through the whole the master program.

Finally, we would also like to express our gratitude to David Samvin from Jönköping University for his great enthusiasm and inspirational lectures on nonlinear finite element behaviour.

Stockholm, June 2016

Daniel Samvin

Oskar Skoglund
# Nomenclature

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Unit</th>
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<tbody>
<tr>
<td>LVL</td>
<td>Level</td>
<td>-</td>
</tr>
<tr>
<td>C.B</td>
<td>Column buckling</td>
<td>-</td>
</tr>
<tr>
<td>LTB</td>
<td>Lateral torsional buckling</td>
<td>-</td>
</tr>
<tr>
<td>RSM</td>
<td>Reduced stress method</td>
<td>-</td>
</tr>
<tr>
<td>EWM</td>
<td>Effective width method</td>
<td>-</td>
</tr>
<tr>
<td>FEM</td>
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<td>-</td>
</tr>
<tr>
<td>FE</td>
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<td>-</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite element analysis</td>
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</tr>
<tr>
<td>$L$</td>
<td>Length</td>
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</tr>
<tr>
<td>$t_f$</td>
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</tr>
<tr>
<td>$t_w$</td>
<td>Web thickness</td>
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</tr>
<tr>
<td>$h$</td>
<td>Height</td>
<td>m</td>
</tr>
<tr>
<td>$c$</td>
<td>Width of a part of a cross-section</td>
<td>m</td>
</tr>
<tr>
<td>$A_{\text{gross}}$</td>
<td>Gross area</td>
<td>m$^2$</td>
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<tr>
<td>$A_{\text{eff}}$</td>
<td>Effective area</td>
<td>m$^2$</td>
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<td>$a_{\text{eff}}$</td>
<td>Effective length</td>
<td>m</td>
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<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
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<tr>
<td>$\alpha$</td>
<td>Imperfection factor</td>
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</tr>
<tr>
<td>$\alpha_{cr}$</td>
<td>Load amplification factor</td>
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<tr>
<td>$\alpha_{ult,k}$</td>
<td>Load amplification factor</td>
<td>-</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Manufacturing process factor</td>
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<td>$\rho$</td>
<td>Reduction factor</td>
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<tr>
<td>$\lambda$</td>
<td>non dimension slenderness</td>
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</table>
Φ  Value to determine χ -
χ  Reduction factor -
\(χ_{LTmod}\)  Modified reduction factor -
f  Modification factor -
E  Modulus of elasticity  Pa
I  Moment of inertia  m^4
I_{eff}  Effective moment of inertia  m^4
W_{eff}  Effective section modulus  m^3
W_{pl}  Plastic section modulus  m^3
W_{el}  Elastic section modulus  m^3
N_{Ed}  Design normal force  N
N_{cr}  Critical buckling load  N
N_{Ed}  Design resistance normal force  N
M_{Ed}  Design bending moment  Nm
M_{Rd}  Unreduced bending resistance  Nm
M_{Rd,i}  Unreduced bending resistance  Nm
M_{cr}  Elastic critical moment  Nm
V_{Ed}  Design shear force  N
\(\gamma_{M1}\)  Partial factor -
\(\gamma_{M2}\)  Partial factor -
η  Conversion factor -
ε  Strain -
ψ  Stress ratio -
f_y  Yield strength  Pa
f_u  Ultimate strength  Pa
σ  Stress  Pa
\(\sigma_{x,Ed}\)  Local longitudinal stress  Pa
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<td>$\sigma_{z,Ed}$</td>
<td>Local transverse stress</td>
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<tr>
<td>$\tau_{Ed}$</td>
<td>Local shear stress</td>
<td>Pa</td>
</tr>
<tr>
<td>$k_\sigma$</td>
<td>Plate buckling factor</td>
<td></td>
</tr>
<tr>
<td>$k_{yy}$</td>
<td>Interaction factor</td>
<td></td>
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<tr>
<td>$k_{zz}$</td>
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<tr>
<td>$k_{zy}$</td>
<td>Interaction factor</td>
<td></td>
</tr>
<tr>
<td>$a_{ult,k}$</td>
<td>Minimum load amplifier</td>
<td></td>
</tr>
<tr>
<td>$C_{my}$</td>
<td>Equivalent uniform moment factor</td>
<td></td>
</tr>
<tr>
<td>$C_{mz}$</td>
<td>Equivalent uniform moment factor</td>
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Chapter 1

Introduction

1.1 Background

Steel structures, especially steel bridges, are often composed by slender plates. If these plates are sufficiently slender, the section will buckle locally before the yield stress of the material is reached in any fibre of the cross-section; a cross-section that is susceptible to local buckling is denoted as a class 4 cross-section. Eurocode EN 1993-1-5, which is the design standard used for plated structural elements, describes three approaches for designing structural members belonging to cross-sectional class 4 with regard to post-critical behavior. The three approaches are as follows; the effective width method, the reduced stress method and numerical simulation with finite element method, which are covered in section 4, 10 and annex C of Eurocode EN 1993-1-5.

In the effective width method, the individual compressed parts in the cross-section are reduced to account for the influence of local buckling. In the reduced stress method, the stresses are instead adjusted and can be obtained directly from the FE-model and thus the number of steps in the analysis shortened. However, the method of reduced stress is rather briefly addressed in the Eurocode, compared to the effective width method. Furthermore, the procedure of the reduced stress method that is described in the Eurocode often give an over conservative result. The lack of information in the Eurocode and that the method of reduced stress is labeled as over conservative is a contributing factor that it is not as widely used among designers as the effective width method.

The third method mentioned is a computer simulation based on a nonlinear approach of the finite element method. This method is considered to reflect the true capacity of the structure, however it is more time consuming and in many cases a lot more complex.

According to Christoffer Svedholm and Professor Costin Pacoste the reduced stress method is common among German designers, (Braun & Kuhlmann, 2012), but not among Swedish. Furthermore, they believe that this method will be less tedious when combined with a detailed FE model, compared to effective width method.
CHAPTER 1. INTRODUCTION

1.2 Aim and purpose

The purpose of this master thesis is to provide the engineers at the consultant company ELU AB with guidelines in how to address the issue of local buckling of thin plated structural members and when to use the different methods that are recommended in the Eurocode as to not end up with an over conservative result.

1.3 Limitations

This master thesis encompasses a rather extensive parametric study of an I-girder and a box-section column. It includes several load combinations to evoke different failure modes and a varying degree of slenderness of the plates constituting the cross-section. The failure modes consist of patch loading, shear buckling, column buckling and lateral torsional buckling.

1.4 Method

This section presents the procedure that has been used throughout this report. The initial step of the report is a step of validation, which aims at confirming known experimental results of steel members with a nonlinear FE-analysis; this is to ensure that the nonlinear FE-analysis is able to capture the true capacity of the structure. Once the experimental results are confirmed, a parametric study is performed. The FE-models used in the parametric study are obtained by modifying the models from the experimental verification by altering cross-sectional properties, loads and boundary conditions, which will give rise to different stress distributions. All studied cross-sections will be of cross-sectional class 4. These structural members are analyzed with a linear and nonlinear approach of the FE-method. A nonlinear representation is needed in order to capture the “real” behaviour of the loaded member and to account for the stress redistribution associated with local buckling. The nonlinear FE-method will therefore serve as a reference for both of the analytical methods. For the same structural members, the load bearing capacity is calculated by using the effective width method and the reduced stress method, this is performed by using Mathead and Matlab. The linear FE-analysis is used to obtain buckling modes and corresponding critical stresses; the buckling modes are in turn used in the nonlinear analyses to seed the geometrical imperfections.
Chapter 2

Literature review

This chapter presents the theory of the effective width method (EWM), the reduced stress method (RSM) and nonlinear FE-analysis (FEA). The EWM is very commonly used by designers and therefore only a brief description is presented. Meanwhile a more detailed description of the RSM is given. Germany proposed the introduction of the reduced stress method, based on their experience with such an approach, to the Eurocode, (1993-1-5, 2008). Some of the advantages with the reduced stress method are that the resistance is based on the gross cross-section, it also relates to any loading condition and to non-regular geometry (Stranghöner, et al., 2012). The method is also suitable and effective when verifying the resistance based on the stresses obtained from FEM. Unlike the reduced stress method, the effective width method has certain limitations.

The following requirements that need to be fulfilled in order for the effective width method to be applied are, (D. Beg, 2012):

- The shape of the panels should be rectangular with a maximum deviation from the horizontal line of 10°.
- Unstiffened openings and cut-outs should not exceed 5 % of the width of the plate element.
- The thickness of the panel should be constant, if not, the smallest one should be used in the calculation.

2.1 Reduced stress method

The reduced stress method accounts for local buckling by limiting the yield stress of the plated cross-section and assuming a linear, but reduced, stress distribution. The method does not allow any load-shedding between the plate elements that constitutes the cross-section and the resistance is thus governed by the plate element that buckles first. In the reduction of the yield stress all stress components are taken into account according to von Mises yield criterion. Therefore, a verification of the combination of the different load components is not necessary, which leads to a resistance verification in a single step, (D. Beg, 2012) page 161.
The verification of the resistance is calculated according to section 10 in EN 1993-1-5 as:

\[
\left( \frac{\gamma_M \sigma_{x,Ed}}{\rho_x f_y} \right)^2 + \left( \frac{\gamma_M \sigma_{z,Ed}}{\rho_z f_y} \right)^2 - \left( \frac{\gamma_M \sigma_{x,Ed}}{\rho_x f_y} \right) + 3 \left( \frac{\gamma_M \tau_{Ed}}{\chi_w f_y} \right)^2 \leq 1
\]

\begin{align*}
\rho_x &= \frac{\lambda_p - 0.055(3 + \psi)}{\lambda_p^2} \leq 1.0 \\
\rho_z &= \frac{\lambda_p - 0.188}{\lambda_p^2} \leq 1.0 \\
\rho_z &= \frac{1}{\varphi_p + \sqrt{\varphi_p^2 - \lambda_p}} \leq 1.0
\end{align*}

- \( \rho_x \) is the reduction factor for longitudinal stresses, calculated according to equation (2.2) or (2.3) for internal and outstand compression elements respectively.
- \( \rho_z \) is the reduction factor for transverse stresses, calculated according to equation (2.4).
- \( \chi_w \) is the reduction factor for shear stresses, calculated according to section 5.3(1) in Eurocode, (1993-1-5, 2008).

The reduction factor \( \rho \) can either be determined according to section 4, 5 or annex B in EN 1993-1-5. The factors calculated according to section 4 and 5 utilise as much of the post-critical resistance for buckling as possible, (D. Beg, 2012) page 161, and is calculated in accordance with equation (2.2) and (2.3). Meanwhile the reduction factor presented in annex B does not fully utilize that reserve and is therefore more suitable for problems were column like buckling behaviour prevails, calculated according to equation (2.4), (D. Beg, 2012).

The reduced stress method is based on Von Mises yield criterion and takes thus into account the whole stress field when determining the slenderness factor and also the reduction of the yield strength in the material, when considering local buckling.
2.1. REDUCED STRESS METHOD

The plate slenderness is taken according to Eurocode, (1993-1-5, 2008), as:

\[
\lambda_p = \sqrt{\frac{\alpha_{ult,K}}{\alpha_{cr}}} \tag{2.5}
\]

- \( \alpha_{cr} \) is the minimum load amplification factor for which the complete stress state has to be increased in order to reach the critical elastic resistance.
- \( \alpha_{ult,K} \) is the minimum load amplification factor for which the complete stress state has to be increased in order to reach the resistance in the most critical point of the plate.

The factor \( \alpha_{cr} \) can either be determined by appropriate software, such as EBPlate\(^1\), or according to equation (2.6), the latter is more suitable for simple load cases, (D. Beg, 2012). Perhaps the most suitable method and the most accurate is to determine \( \alpha_{cr} \) and \( \alpha_{ult,K} \) directly from the FE-model.

\[
\frac{1}{\alpha_{cr}} = \frac{1+\psi_x}{4\alpha_{cr,x}} + \frac{1+\psi_z}{4\alpha_{cr,z}} + \left( \frac{1+\psi_x}{4\alpha_{cr,x}} + \frac{1+\psi_z}{4\alpha_{cr,z}} \right)^2 + \frac{1-\psi_x}{2\alpha_{cr,x}} + \frac{1-\psi_z}{2\alpha_{cr,z}} + \frac{1}{\alpha_{cr,z}} \tag{2.6}
\]

\[
\alpha_{cr,x} = \frac{\sigma_{cr,x}}{\sigma_{x,Ed}} \tag{2.7}
\]

\[
\alpha_{cr,z} = \frac{\sigma_{cr,z}}{\sigma_{z,Ed}} \tag{2.8}
\]

\[
\alpha_{cr,t} = \frac{\tau_{cr,t}}{\tau_{t,Ed}} \tag{2.9}
\]

The factor \( \alpha_{ult,K} \) is based on the assumption that the resistance is reached when yielding of the most critical point in the plate occurs, without plate buckling, and is calculated as (1993-1-5, 2008):

\[
\frac{1}{\alpha_{ult,K}} = \frac{\left( \frac{\sigma_{x,Ed}}{f_y} \right)^2 + \left( \frac{\sigma_{z,Ed}}{f_y} \right)^2 - \left( \frac{\sigma_{x,Ed}}{f_y} \right) \left( \frac{\sigma_{z,Ed}}{f_y} \right) + 3 \left( \frac{\tau_{Ed}}{f_y} \right)^2}{2} \tag{2.10}
\]

\(^1\) EBPlate version 2.01 is a free software developed by CTICM which is used to determine the critical stresses associated with elastic plate type buckling, (Naumes, et al., 2009) and can be downloaded from (Anon., 2016).
2.1.1 Improvements of RSM

As mentioned before and specified in the design documentations, the reduced stress method does not account for any load shedding between the plate elements of the cross-section. However, there are approaches that are not yet specified in, (1993-1-5, 2008) that consider further straining of the cross-section after the first plate buckles which are presented in, (Johansson, et al., 2007) and (Naumes, et al., 2009).

Three different levels of the reduced stress method are to be considered, see also Figure 2.1:

- **Level 1**: corresponding to the approach previously presented, where the plate buckling strength is limited by the weakest plated element.
- **Level 2**: utilizes the straining capacities of the weakest plates until the plate buckling strength of the strongest plate is reached.
- **Level 3**: utilizes the straining capacities of both the weakest plates and the strongest plates.

![Figure 2.1: Development of limiting stresses for the three levels, reproduced from (Johansson, et al., 2007).](image)

\[
\sigma_{\text{Limit,h}} \quad \sigma_{\text{Limit,b1}} \quad \varepsilon_{\text{Limit,h}} \quad \varepsilon_{\text{Limit,b1}} \quad \varepsilon_y
\]

Figure 2.1: Development of limiting stresses for the three levels, reproduced from (Johansson, et al., 2007). \( \sigma_{\text{Limit,h}} \) and \( \sigma_{\text{Limit,b1}} \) is the limiting stress for the weaker and stronger plate respectively.

2.1.2 Proposed implementation

Figure 2.1 depicts the development of strength for the three different levels. As can be seen for the second and the third level the stress distribution deviates from the linear behaviour and thus becomes hard to implement in a comparison of allowed stresses in a FE-model. As a means to overcome this problem, an interpolation between the two limiting stresses (for the weakest plates and the strongest respectively). This interpolation considers the contribution of
the bending resistance from the different plates composing the cross-section. As a simplification, the stresses for the cross-sectional part subjected to tensile forces are also reduced. This procedure has been proven to be an adequate approximation for the cases covered in chapter 4.

The interpolated reduction factor $\varrho$ can be calculated as follows and is applicable for axial stresses:

$$\rho = \rho_1 \frac{M_{Rd,1}}{M_{Rd}} + \rho_2 \frac{M_{Rd,2}}{M_{Rd}}$$

- $\rho_1$ is the reduction factor for the weakest plate.
- $\rho_2$ is the reduction factor for the strongest plate.
- $M_{Rd,1}$ is the unreduced bending resistance of the weakest plate.
- $M_{Rd,2}$ is the unreduced bending resistance of the strongest plate.
- $M_{Rd}$ is the unreduced bending resistance of the entire cross-section.

Equation (2.11) is suitable for when the axial stresses are induced by a bending moment.

For the case of uniform compressive stresses the reduction factor $\rho$ can be interpolated as follows:

$$\rho = \rho_1 \frac{A_1}{A} + \rho_2 \frac{A_2}{A}$$

- $A_1$ is the area of the weakest plate.
- $A_2$ is the area of the strongest plate.
- $A$ is the total area.

### 2.2 Effective width method

In the effective width method, the influence of local buckling is taken into account by calculating an effective cross-section with reduced dimensions and by assuming a linear stress distribution over the reduced cross-section. The method assumes that certain regions of the cross-section remain effective while others are ineffective in resisting the load; this is to reflect the nonlinear stress distribution over the cross-section when a steel plate buckles and thus also to reflect the capacity of the plated structure.

If the cross-section parts have lower slenderness than the limits specified in Eurocode, the influence of local buckling is negligible. Figure 2.2 describes each part of the computation model of the members used in this thesis, according to EWM.
Figure 2.2: Computational model for the cross-sections used in this report.

- $C.G$ is the centre of gravity for the gross cross-section.
- $C.G_e$ is the centre of gravity for the effective cross-section.
- $N.A$ is the neutral axis coinciding with the centre of gravity for the gross cross-section.
- $B$ is the ineffective region.
- $N.A_e$ is the neutral axis coinciding with the centre of gravity for the effective cross-section.

The effective width is calculated through a reduction factor, $\rho$, according to equation (2.2) or (2.3). The reduction factor depends on the slenderness of the plate, $\lambda_{pl}$, and is calculated as:

$$\lambda_{p} = \frac{f_y}{\sigma_{cy}} = \frac{b}{t} \sqrt{\frac{28.4\varepsilon}{k_\sigma}}$$

- $b$ is the width of the considered cross-sectional part.
- $t$ is the thickness of the considered cross-sectional part.
- $\varepsilon$ is a factor considering the steel grade and calculated according to table 5.2 in (1993-1-5, 2008).
- $k_\sigma$ is the buckling coefficient and can be obtained from table 4.1 and 4.2 in (1993-1-5, 2008).

The distribution of the effective width over the cross-section can be obtained from EN 1993-1-5 table 4.1 and 4.2. Finally, the effective area and section modulus can be calculated for the total effective cross-section. This procedure should only be applied for cross-sectional parts under compression. Further calculations need to be made in order to determine the bearing capacity of the structure, which are presented in (1993-1-1, 2008).

### 2.3 Finite element analysis

In a nonlinear analysis the structures stiffness is no longer constant but varies as the structure deforms so that the equilibrium equations must be based on the deformed geometry, (Cook, et al., 2001). The nonlinear behaviour also covers material yielding, local buckling, initial imperfections and boundary conditions that changes with increasing load. In chapter 2.3.1 the material model to account for yielding and local buckling is described, and in chapter 2.3.2 the initial imperfections are described.
2.3.1 Material model

The stress-strain relation used for all the experiments and the parametric studies is computed by using equations (2.14) to (2.19), which will result in the material model depicted in Figure 2.3, (BSK07, 2007). The general properties used for the steel material are the same for all test specimens; where the young modulus, $E = 210$ GPa, Poisson’s ratio, $\nu = 0.3$, and the density, $\rho = 7800$ kg/m$^3$.

Figure 2.3: Simplified stress/strain diagram for steel material.

\[ \varepsilon_1 = \varepsilon_{\text{nom}} = \frac{f_y}{E} \]  
(2.14)

\[ \varepsilon_2 = 0.025 - 5\times\frac{f_u}{E} \]  
(2.15)

\[ \varepsilon_3 = 0.02 + 50\times\frac{f_u - f_y}{E} \]  
(2.16)

\[ \varepsilon_{\text{true}} = \ln(1 + \varepsilon_{\text{nom}}) \]  
(2.17)

\[ \sigma_{\text{true}} = \sigma_{\text{nom}}(1 + \varepsilon_{\text{nom}}) \]  
(2.18)

\[ \varepsilon_{\text{plastic}} = \varepsilon_{\text{true}} - \frac{\sigma_{\text{true}}}{E} \]  
(2.19)
2.3.2 Imperfections

Initial imperfections and conditions are implemented in the finite element model to account for what the real structural element exhibits in its unloaded state. The initial imperfections consist of both structural and geometric imperfections.

**Structural imperfections:**

The structural imperfections, such as residual stresses, are obtained from (BSK07, 2007), and are illustrated in Figure 2.4. Where $f_{yk}$ is the characteristic yield strength and $\sigma_c$ is the balancing compression stresses.

![Figure 2.4: Residual stress patterns according to (BSK99, 1999).](image)

As a means to implement the residual stresses, presented in Figure 2.4, in the FE-model a simplification of the stress field is made, which is illustrated in Figure 2.5 and taken from (Gozzi, 2007).

![Figure 2.5: Simplified residual stress pattern used in the FE-models (Gozzi, 2007).](image)

**Geometric imperfections:**

The geometrical imperfections are seeded from eigenmodes obtained from a linear buckling analysis. The magnitudes of the imperfections are taken as the essential manufacturing
tolerances depicted in table D.1.1 and D.1.3 in (EN1090-2, 2012), a summarization of the tolerances are presented in Table 2.1.

The number and the combination of eigenmodes are such that predicts the lowest buckling load. However, the amplitude of the combination of the eigenmodes should not exceed any of the essential manufacturing tolerances depicted in, (EN1090-2, 2012) page 119.

When seeding the imperfections, the essential manufacturing tolerances are reduced to 80 %, (1993-1-5, 2008) page 46. A further reduction is carried out when combining the imperfection, where upon a leading imperfection is chosen and the accompanying imperfections are reduced to 70 %, (1993-1-5, 2008) page 48.

Table 2.1: Summarization of the essential manufacturing tolerances.

<table>
<thead>
<tr>
<th>Criterion:</th>
<th>Parameter</th>
<th>Permitted deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate curvature</td>
<td>Deviation $\Delta$ as a function of the plate height $b$</td>
<td>$\Delta = \pm \frac{b}{200}$ if $\frac{b}{t} \leq 80$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta = \pm \frac{b^2}{16000t}$ if $80 \leq \frac{b}{t} \leq 200$</td>
</tr>
<tr>
<td>Flange distortion of I-section</td>
<td>Deviation $\Delta$ as a function of the flange width $b$.</td>
<td>$\Delta = \pm \frac{b}{150}$ if $\frac{b}{t} \leq 20$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\Delta = \pm \frac{b^2}{3000t}$ if $\frac{b}{t} &gt; 20$</td>
</tr>
<tr>
<td>Global straightness of the component</td>
<td>Deviation from straightness $\Delta$ as a function of the member length $L$.</td>
<td>$\Delta = \pm \frac{L}{750}$</td>
</tr>
</tbody>
</table>
Chapter 3

Experimental verification

Two different experiments are chosen to be verified by nonlinear FE-analysis. The intention of the experimental verification is to be able to justify the results obtained from the coming parametric study. The chosen experiments include two common structural elements, an I-girder and a box-section column. Details regarding the experiment of the I-girder and box-section column are presented in chapter 3.1 and 3.2 respectively.

3.1 Lateral torsional buckling, I-girder

3.1.1 Experiment description

The experiment is carried out to investigate distortional buckling of a double symmetric I-section girder. The experimental setup is depicted in Figure 3.1. Two types of lateral bracing systems are applied, one at the supports and one at mid-span. The lateral bracing system at the supports prevent lateral deflection and twist. The effective bracing at mid-point is applied over a distance of 100 mm and this configuration prevent, to some degree, the lateral deflection and the rotation of the top flange, meanwhile the twist of the top flange is completely prevented. The test specimen is fabricated by welding a web plate together with the flanges from an IPE-profile, illustrated in Figure 3.1. The material and geometrical properties are presented in Table 3.1 and Table 3.2 respectively, with denotations according to Figure 3.1. Several strain gauges are placed along the beam according to Figure 3.2. A more detailed description of the experimental setup is presented by the authors of the experiment, (Zirakian & Showkati, 2007).

Table 3.1: Material properties of the test specimen.

<table>
<thead>
<tr>
<th>Structural part</th>
<th>( f_y ) [MPa]</th>
<th>( f_u ) [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web plate</td>
<td>239.80</td>
<td>1086.35</td>
</tr>
<tr>
<td>Flange</td>
<td>279.31</td>
<td>894.35</td>
</tr>
</tbody>
</table>
3.1. LATERAL TORSIONAL BUCKLING, I-GIRDER

Table 3.2: Geometrical properties, [mm].

<table>
<thead>
<tr>
<th>Specimen</th>
<th>h</th>
<th>b</th>
<th>t</th>
<th>s</th>
<th>r</th>
<th>d</th>
<th>a</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>S180-3600</td>
<td>180</td>
<td>64</td>
<td>6.3</td>
<td>4.85</td>
<td>7</td>
<td>140.5</td>
<td>20.1</td>
<td>3600</td>
</tr>
</tbody>
</table>

Figure 3.1: Experimental setup.

Figure 3.2: Strain gauge at quarter point and strain gauge 0.54 m from mid-span.

3.1.2 FE-model

A full-length model of the beam is used to account for the possibility of asymmetric buckling modes. The mesh is constructed of S8 shell element and the mesh density is 15 mm. The nonlinear analysis is performed with the Riks method, with a multilinear material model, presented in chapter 2.3.1. The imperfections used in the experiment are taken according to Table 2.1.

The boundary conditions are locked in the vertical direction (y-axis) in the first support and locked in the horizontal and vertical direction (x-axis and y-axis) in the second support. The lateral bracing placed in the vicinity of the support prevents lateral deflection (y-direction) and twist (rotation around x-axis).
Owing to the uncertainty of the lateral bracing at mid-span, two cases are tried. First case (case 1) prevents lateral deflection (y-direction), twist (rotation around z-axis), and rotation (rotation around y-axis) of the top flange. The second case (case 2) is by only preventing the top flange to twist.

Figure 3.3: A full length model of the experimental case 2.

3.1.3 Convergence analysis

A mesh refinement is done to assure that convergence is obtained for the current mesh density of 10 mm. In Figure 3.4 a force strain plot for sensor S3 is illustrated for the current mesh density of 10 mm and a refined mesh of density 0.5 mm.

Figure 3.4: Convergence analyses and mesh refinement.
3.1.4 Results

The critical buckling load, \( P_{cr} \), along with the maximum load associated with failure, \( P_{Rd} \), for case 1 and case 2 and the experimentally obtained value is presented in Table 3.3. Hand-calculation is presented in Appendix A.1.

Table 3.3: Critical and ultimate load.

<table>
<thead>
<tr>
<th>Test</th>
<th>( P_{cr} ) [kN]</th>
<th>( P_{Rd} ) [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>101.90</td>
<td>32.13</td>
</tr>
<tr>
<td>Case 2</td>
<td>29.72</td>
<td>22.50</td>
</tr>
<tr>
<td>Experiment</td>
<td>---</td>
<td>24.13</td>
</tr>
<tr>
<td>Hand-calculation</td>
<td>29.72</td>
<td>19.05</td>
</tr>
</tbody>
</table>

In Figure 3.5 and Figure 3.6, the force strain relationship for case 2 is shown together with the experimental result for the different strain gauges, placed according to Figure 3.2. The deformed shape associated with failure is illustrated in Figure 3.7 for case 2.

Figure 3.5: Force strain curve for gauge S2 (left) and S3 (right) compared to the results obtained from nonlinear FE-analysis.

Figure 3.6: Force strain curve for gauge S6 (left) and S7 (right) compared to the results obtained from nonlinear FE-analysis.
The difficulties of the experiment laid in the interpretation of the boundary conditions, the residual stresses and the crookedness of the member. The first case with a completely fixed upper flange at mid-span proved to be wrong, not only by looking at the bearing capacity but also the failure mode did not fully agree, compared with the mode presented in the experimental report, (Zirakian & Showkati, 2007). The second case reflected the bearing capacity and the failure mode in a much more accurate way. However, further improvements could be made by studying the residual stresses and the bracing at mid-span even more carefully. Nevertheless, this is neither necessary nor fruitful since there are uncertainties in the geometrical imperfections that can give rise to similar deviations in the result.

The relative difference between the hand calculation and FEM is approximately 15 %. The deviation might be in the choice of crookedness of the member, especially since no local imperfections were seeded.

### 3.2 Column buckling, box-section

#### 3.2.1 Experiment description

The experiment is carried out to study the post-failure behaviour of a box-section column. The experimental setup and the cross-section are shown in Figure 3.8. The column is subjected to a uniform compression load through a hydraulic actuator. Acting, as boundary conditions are two cylindrical hinges that are fitted to each end of the column at a distance of 250 mm at both ends, with a rotational stiffness spanning from 8.69 kNm to 14.6 kNm.

The material and geometrical properties are presented in Table 3.4 and Table 3.5 respectively, with denotations according to Figure 3.8. Several strain gauges are placed along the beam according to Figure 3.8. A more detailed description of the experiment is presented by the authors of the experiment, (Ban, et al., 2012).
3.2. COLUMN BUCKLING, BOX-SECTION

Table 3.4: Material properties of the test specimen.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$f_y$ [MPa]</th>
<th>$f_u$ [MPa]</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B5 – 460</td>
<td>531.9</td>
<td>657</td>
<td>0</td>
<td>0.028</td>
<td>0.140</td>
</tr>
</tbody>
</table>

Table 3.5: Geometrical properties of the test specimen.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$B$ [mm]</th>
<th>$t$ [mm]</th>
<th>$h_0$ [mm]</th>
<th>$L$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>B5 – 460</td>
<td>102.2</td>
<td>10.81</td>
<td>91.39</td>
<td>4019.9</td>
</tr>
</tbody>
</table>

Figure 3.8: Experimental set up with gauge applied and hinged at both ends, (Ban, et al., 2012).

The material properties in Table 3.4 are the nominal values of the strains and they are used together with Equations (2.14) to (2.19) to describe the material curve shown in Figure 2.3. The nominal values are obtained from the measurement presented in the experimental article, (Ban, et al., 2012).

3.2.2 FE-model

A full-length model of the column is used to account for the possibility of asymmetric buckling modes. The mesh is constructed of S8 shell element and the mesh density is 15 mm. The nonlinear analysis is performed with the Riks method, with a material model according to Figure 2.3.
Chapter 3. Experimental Verification

Figure 3.9: A full length model of the experimental case with residual stresses and boundary conditions.

The boundary condition at the bottom end are locked in all directions except the rotation in the transversal direction. The second end is locked in all directions except in longitudinal direction and rotation around the transversal direction. A spring rotation stiffness is applied at both ends of the column in order to idealize the rational restraint at the hinges, spanning from 8.69 kNm to 14.62 kNm. Two analyses are performed and presented with the upper and lower value of the restraint.

3.2.3 Imperfection

The initial geometrical imperfection is accounted for by adding a global crookedness of the column corresponding to three discrete locations $v_2 = 6.85$ mm, see Figure 3.10. The initial residual stresses are introduced to account for the butt-welds in the box-section and are taken according to Figure 2.4.

Figure 3.10: Initial bending measurements obtained from the experiment, (Ban, et al., 2012).
3.2.4 Convergence analysis

A mesh refinement is done to assure that convergence is obtained for the current mesh density of 15 mm. The force-displacement plot is illustrated in Figure 3.11.

![Force-displacement plot with mesh refinement.](Image)

**Figure 3.11:** The force-displacement plot with mesh refinement.

3.2.5 Results

Two different results are obtained considering the lower boundary and upper boundary of the rotational stiffness. The critical buckling load $N_{cr}$ and the design buckling resistance $N_{b,Rd}$ for both boundaries are presented in Table 3.6. The hand calculation is presented in Appendix A.2.

<table>
<thead>
<tr>
<th>Test</th>
<th>$N_{cr}$ [kN]</th>
<th>$N_{b,Rd}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper boundary</td>
<td>2527.1</td>
<td>1123.6</td>
</tr>
<tr>
<td>Lower boundary</td>
<td>2496.6</td>
<td>944.9</td>
</tr>
<tr>
<td>Experiment</td>
<td>---</td>
<td>930.6</td>
</tr>
<tr>
<td>Hand-calculation upper</td>
<td>---</td>
<td>1202.0</td>
</tr>
<tr>
<td>Hand-calculation lower</td>
<td>---</td>
<td>1195.0</td>
</tr>
</tbody>
</table>
CHAPTER 3. EXPERIMENTAL VERIFICATION

Figure 3.12: Force versus displacement obtained from the experiment and FE-analysis.

Figure 3.13: Deformed shape at the point of failure, the left figure displays Von Mises stresses and the right displays deformation.

The bearing capacity obtained from the FE-analysis and the experimental test is almost identical. The relative difference between the FE-analysis and the experimental are 1.5 % and 21 % for lower and upper boundary respectively.

The deformation in the elastic region are quite similar, but the finite element model is not able to fully reflect the behaviour once it starts to plasticise. However, the aim of the analysis is to capture the ultimate resistance, rather than the behaviour at failure. Furthermore, the span of the rotational stiffness results in an interval for the bearing capacity. Unfortunately, the experimental bearing capacity is not fully covered in this interval, see Figure 3.12. The discrepancy in the bearing capacity and that the plastic behaviour is not fully reflected is most likely due to that no local imperfections are seeded in the analysis.
Chapter 4

Parametric study

For the purpose of creating a parametric study to compare the three design methods, MATLAB, Mathcad and the FEM software Brigade/plus are used. Two conceptually different cases are studied. The first case presented in chapter 4.1 is lateral torsional buckling of an I-girder and the second case is column buckling of a box-section presented in chapter 4.2. For each case a hand calculation is performed in accordance with the reduced stress method and effective width method, see Appendix A.3 to A.10.

In the parametric study the I-girder and the column are subjected to two different sets of loadings. Furthermore, the slenderness of the plates constituting the various cross-sections are varied by altering the plate thickness. A dimensionless slenderness parameter is used for this purpose. The slenderness parameter of the plate, $\lambda_{plate}$, is related to the width-to-thickness ratio that is the limit for a class 4 cross-section, presented in Table 4.1. The limits are obtained from Table 5.2 in (1993-1-1, 2008).

<table>
<thead>
<tr>
<th>Cross-section part</th>
<th>Part subjected to bending</th>
<th>Part subjected to compression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal compression parts</td>
<td>$\lambda_{plate} = \frac{c/t}{124\varepsilon}$</td>
<td>$\lambda_{plate} = \frac{c/t}{42\varepsilon}$</td>
</tr>
<tr>
<td>Outstand flanges</td>
<td>-</td>
<td>$\lambda_{plate} = \frac{c/t}{14\varepsilon}$</td>
</tr>
</tbody>
</table>

All the models are constructed of S8 shell element with a mesh density confirmed through a mesh refinement, the mesh density is specified in respective chapter. The nonlinear analysis is performed in accordance with the Riks method, with a multilinear hardening model; see the material curve in Figure 2.3. The combination of mode shapes is determined by running several different analyses with different combinations of mode shapes. The most detrimental combination is chosen. The magnitudes of the structural and geometric imperfections are taken in accordance with chapter 2.3.2 and the combined effect of the geometric imperfections are made sure not to exceed the magnitudes in Table 2.1.
4.1 Lateral torsional buckling, I-girder

4.1.1 Description

In this chapter an I-girder subjected to two different types of loading is studied, see Figure 4.1. A parametric analysis is performed by varying the slenderness of the web and the flanges one at the time. The length of the girder and the steel grade is kept constant during the parametric study; more details are presented in Appendix B.1.

![Case 1 and Case 2](image)

Figure 4.1: I-girders loaded under pure bending and loaded by two point loads at quarter point respectively.

For each case, two different analyses are performed. The first is to vary the slenderness of the web plate and the second is to vary the slenderness of the flange.

**Case 1 - Plate girder subjected to pure bending**

A welded I-girder is subjected to two equal end moments with fixed-fixed boundary conditions, illustrated in Figure 4.2. The end moment is represented in the FE-model by applying a point moment in a reference point which is in turn connected with a kinematic coupling to the end of both the flanges and the web. The reference point is placed at the neutral axis in both ends of the girder. The boundary condition is modelled as fixed in both ends, by applying the boundary conditions in the reference point. The mesh density is set to 30 mm.

![Case 1 and Case 2](image)

Figure 4.2: The left figure illustrates the boundary conditions at the reference point and the right figure illustrates the kinematic coupling.
Case 2 - Plate girder subjected to two point loads

A welded I-girder is subjected to two equal concentrated loads, with a load length of 100 mm, at quarter point. The support conditions are of type end-forked boundary conditions, where the beam is restrained from vertical and lateral movement in both ends and longitudinal in one end, shown in Figure 4.3. At the location of the support the girder is provided with stiffeners to prevent local crushing of the web. In the FE-model: the vertical and longitudinal restrains are applied along the edges of the end of the girder meanwhile the lateral restrains are applied to the edge of the stiffeners. The thickness of the stiffeners is kept constant and set to 12 mm. The mesh size is set to 30 mm.

![Illustration of the boundary conditions at the each sides of the I-girder.](image)

4.1.2 Results – Lateral torsional buckling

In this chapter the result from the lateral torsional buckling for the I-girder is presented. Each structural case and load case is divided into a separate section. The result contains FEM analyses, hand calculation with respect to the effective width method and the reduced stress method. The reduced stress method is calculated in three different ways, according to level 1, level 3 and the proposed implementation presented in chapter 2.1.2. The level 1 calculation is the approach recommended in the Eurocode.

The following denotation is used: Finite element method – FEM, Effective width method – EWM, Reduced stress method level 1 – RSM-1, Reduced stress method level 3 – RSM-3, Reduced stress method implementation – RSM-IMP.

Case 1 - Plate girder subjected to pure bending

In Figure 4.4 and Figure 4.5 the bearing capacity is plotted as a function of the plate slenderness \( \lambda_{plate} \) of the web plate. Meanwhile the slenderness for the flange plate is set constant to \( \lambda_{plate} = 0.96 \). The slenderness concept is described in the section accompanying Table 4.1.

The results of FEM, EWM and RSM are plotted against each other in Figure 4.4; the calculations are based on critical stresses obtained through simple hand calculation methods. Furthermore, FEM, EWM and RSM are plotted against each other in Figure 4.5. However,
unlike the previous case, the critical stresses, used for RSM calculations, are obtained from the finite element model corresponding to each slenderness value. The critical stresses are obtained through a linear buckling analysis without any residual stresses.

Figure 4.4: Moment bearing capacity, $M_{Rd}$, as a function of the plate slenderness for the web plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.

Figure 4.5: Moment bearing capacity, $M_{Rd}$, as a function of the plate slenderness for the web plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.

The two analytical methods (EWM and RSM), presented in Figure 4.4 and Figure 4.5, are in good agreement with the results obtained in the nonlinear FE-analyses, with an exception for
the RSM-1 which for high slenderness ratios deviates substantially. The reason for this deviation is that the slender web plate unintentionally reduces the capacity of the flange plate.

Furthermore, the flanges carry the majority of the bending moment, which amplifies the differences additionally. The critical stresses obtained through hand calculations assume that the edge rotational restraints are hinged. Meanwhile when the critical stresses are obtained from FE-analyses the rotational restraints from the flange plates are utilized and thus the bearing capacity is increased, and is approaching the values obtained from the nonlinear FE-analyses.

In Figure 4.6 and Figure 4.7 the bearing capacity is plotted as a function of the plate slenderness $\lambda_{plate}$ of the flange plate. Meanwhile the slenderness for the web plate is set constant to $\lambda_{plate} = 0.96$.

![Figure 4.6: Moment bearing capacity, $M_{Rd}$, as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.](image-url)
Figure 4.7: Moment bearing capacity, $M_{Rd}$, as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.

The effect of that the majority of the external bending moment is allocation to the flange plate is visualized when comparing the difference between EWM and RSM-1 for Figure 4.4 and Figure 4.6. This allocation of forces results in a smaller deviation between EWM and RSM when the flange plate is being reduced.

There is a noteworthy difference between Figure 4.6 and Figure 4.7 when comparing the relative difference of the RSM and the EWM, which highlights one of the difficulties of obtaining the critical stress from a linear FE-analysis. In Figure 4.6 the relative difference between RSM-1 and EWM is increasing with increasing slenderness of the web plate, as is intuitive when studying how the method is formulated with the weakest plate governing the resistance. In contrast to the behaviour in Figure 4.6 the relative difference of RSM-1 and EWM is decreasing with increasing plate slenderness in Figure 4.7. The reason for this is that the first local buckling mode that excited flange buckling behaviour is chosen, which also, inevitably, include buckling of the web plate. This buckling mode is chosen since there are no realistic buckling modes corresponding to a dominant flange buckling behaviour. Furthermore, the flange buckling behaviour becomes more pronounced, compared to the buckling of the web plate, when the relative difference of the slenderness increases. This means that for lower slenderness values of the flange plate the critical stress obtained in FE-analysis is underestimated leading to a lower prediction of the final resistance.
Case 2 - Plate girder subjected to two point loads

In Figure 4.8 and Figure 4.9 the bearing capacity is plotted as a function of the plate slenderness $\lambda_{\text{plate}}$ of the web plate. Meanwhile the slenderness for the flange plate is set constant to $\lambda_{\text{plate}} = 0.96$.

Furthermore, FEM, EWM and RSM are plotted against each other in Figure 4.8; the calculations are based on critical stresses obtained through simple hand calculation methods. However, unlike the previous case FEM, EWM and RSM are plotted against each other in Figure 4.9 with the critical stresses obtained from the finite element model corresponding to each slenderness value.

For the result in Figure 4.8 to Figure 4.11 the effective width method and the reduced stress method is calculated based on the assumption that the second order effects of lateral torsional buckling have no influence on the interaction between the transverse forces and axial forces. This results in a cross-sectional resistance verification and a separate verification against lateral torsional buckling, only the decisive resistance is presented. The cross-sectional verification is performed according to chapter 7 in EN 1993-1-5 for EWM and according to the verification format, Equation (2.1), for RSM. The reasoning behind this assumption can be found in the discussion section of the report.

Figure 4.8: Design resistance, $P_{Rd}$, as a function of the plate slenderness for the web plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.
CHAPTER 4. PARAMETRIC STUDY

Figure 4.9: Design resistance, $P_{Rd}$, as a function of the plate slenderness for the web plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.

The governing resistance in Figure 4.8 for the effective width method is the moment bearing capacity with respect to lateral torsional buckling for a slenderness ratio between 1.0 and 1.07 and for higher slenderness values it is the patch loading resistance that is decisive. Meanwhile for the reduced stress method, in Figure 4.8 and Figure 4.9, the verification format is governing the resistance for all values of the slenderness.

Unlike the previous results, Figure 4.4 to Figure 4.7, the FEM does not follow a continuous pattern. This bilinear pattern can be entirely explained by the eigenmodes that are seeded (different set of eigenmodes can substantially alter the bearing capacity).

In Figure 4.10 and Figure 4.11 the bearing capacity is plotted as a function of the plate slenderness $\lambda_{plate}$ of the flange plate. Meanwhile the slenderness for the web plate is set constant to $\lambda_{plate} = 0.96$. 

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4.1. LATERAL TORSIONAL BUCKLING, I-GIRDER

Figure 4.10: Design resistance, $P_{Rd}$, as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.

Figure 4.11: Design resistance, $P_{Rd}$, as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.
The governing resistance in Figure 4.10 for the effective width method is with respect to lateral torsional buckling for all values of the slenderness. Meanwhile for the reduced stress method the verification format regarding the web plate is governing the resistance for all values of the slenderness.

The resistance according to the verification format is almost constant for all slenderness values in Figure 4.10; the reason for this is that the flange plates have marginal effect on the bearing capacity of the web plate.

Furthermore, the calculated capacity of EWM is greater than the capacity in FEM. There could be several reasons for this. The first and simplest explanation is that EWM is unconservative in this case. However, a more plausible explanation might be that the assumption made concerning whether the second order effects of lateral torsional buckling have any influence on the interaction verification or not, is false; if the second order effects would have been included the resistance in EWM would match the resistance in FEM much better. A third explanation could be that the amplitude of the imperfections seeded are conservative and therefore that the true capacity is not covered by a nonlinear FE-analysis. Nevertheless, the capacity of EWM and RSM are calculated with the same assumption and the capacity is substantially higher in EWM, due to the conservative verification format of RSM.

The decisive resistance in Figure 4.11 is for RSM-1 the capacity due to lateral torsional buckling, for all slenderness values. For RSM-3 and RSM-IMP lateral torsional buckling is governing for slenderness values between 1-1.2 and 1-1.1 respectively. In all other cases the cross-sectional resistance is decisive. Lateral torsional buckling is governing the resistance because the critical stresses obtained from FEM are underestimated for the flange plate, which gives a lower resistance. This underestimation is decreasing with increasing slenderness and for higher values of the slenderness the cross-sectional verification is governing the resistance for RSM-3 and RSM-IMP. The fact that RSM-1 corresponds so well with FEM seems to be a mere coincidence, considering all the uncertainties with the second order effects of lateral torsional buckling and the critical stresses.
4.2 Column buckling

4.2.1 Description

In this chapter a simply supported column with a butt-welded box-section subjected to two different types of loading is studied, see Figure 4.12. A parametric analysis is performed by varying the slenderness of the two sets of plates that constitutes the cross-section (flange and web plates). All cross-sectional data are held constant except for the plate thickness; more details are presented in Appendix B.2.

![Figure 4.12: To the left a column buckling by centric axial load and to the right the combination of centric axial load and transverse load.](image)

For each studied load case, two different analyses are performed. The first is to vary the slenderness of the flange and web plates simultaneously. The second analysis is to keep the web plates at a constant slenderness and increase the slenderness of the flange plates.

**Column buckling induced by centric axial load**

A but-welded box-section is subjected to centric axial load. The line of action coincides with the centre of gravity of the cross-section.

In the FE-model two reference points are created, one at the centre of the top end respectively bottom end of the column. At the top end, the load is applied together with a restriction of lateral translation in both directions, allowing only for rotation and axial translation. At the bottom end the column is restricted from lateral translation in both directions as well as axial translation, allowing only for rotation. The reference point is in turn connected through a kinematic coupling to the end of the column, see Figure 4.13. The mesh density is set to 40 mm.
4.2.2 Results – Column buckling

In this chapter, the results from the column buckling for the box-section are presented. Each structural case and load case is divided into a separate section. The result contains FEM analyses, hand calculation with respect to the effective width method and the reduced stress method. The reduced stress method is calculated in three different ways, according to level 1, level 3 and the proposed implementation presented in chapter 2.1.2. The level 1 calculation is the approach recommended in the Eurocode.

Column buckling induced by centric axial load

The bearing capacity is plotted in Figure 4.14 and Figure 4.15 as a function of the plate slenderness $\lambda_{plate}$ of the web and flange plate.
4.2. COLUMN BUCKLING

Figure 4.14: Design resistance, $N_{Rd}$, as a function of the plate slenderness for the flange and web plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.

Figure 4.15: Design resistance, $N_{Rd}$, as a function of the plate slenderness for the flange and web plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.
In both of the cases, presented in Figure 4.14 and Figure 4.15, the resistance calculated with the reduce stress method, RSM-1, RSM-3 and RSM-IMP give the same result. This is due to the fact that each plate constituting the cross-section have the same slenderness and are subjected to full compression. This is also the reason why the resistance is almost the same for RSM and EWM in Figure 4.14. The slight discrepancy between the two methods is due that the in RSM even the junction between the plates are reduced, in contrary to EWM. Furthermore, the two methods, RSM and EWM are in good agreement with the finite element analysis.

For a uniaxial fully compressed plate element the critical stress obtained through hand calculations corresponds well with the critical stress obtained from FEM. However, in this case the critical stress is slightly lower in FEM than in the hand calculation, which gives a marginally lower bearing capacity of the RSM based on critical stresses from FEA. The reason for this is not easy to pin-point but it is most likely due to the discretization of the model and that the critical stress would increase with a denser mesh. However, a denser mesh would severely punish the computational time of the nonlinear analysis and the small discrepancy can therefore be neglected.

The bearing capacity is plotted in Figure 4.16 and Figure 4.17 as a function of the plate slenderness $\lambda_{plate}$ of the flange plate. Meanwhile the slenderness of the web plate is set constant to $\lambda_{plate} = 1.1$.

![Plot of $N_{Rd}$ vs. $\lambda_{Plate}$](image)

**Figure 4.16:** Design resistance $N_{Rd}$, as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.
4.2. COLUMN BUCKLING

The reason why there is a discontinuity in Figure 4.16 at the slenderness value of $\lambda_{plate} = 1.1$ and that the relative difference between RSM-1 and the other methods increases with increasing slenderness is that in the RSM-1 the weakest plate in the cross-section governs the resistance.

Two critical stresses are obtained from FEA, one for the flange plate and web plate respectively. The critical stress of the slenderer flange plate is corresponding to the first local buckling mode and for the stockier web plate, an eigenmode corresponding to dominant buckling behaviour for the web plate is used. In RSM-1 the weakest plate governs the resistance and give a joint reduction factor for the cross-section. For RSM-3 and RSM-IMP two reduction factors are instead calculated. Intuitively the approach in RSM-3 and RSM-IMP seems reasonable and should correspond well with FEA. However, this is not the case and the bearing capacity deviates with increasing slenderness for RSM-3 and RSM-IMP, meanwhile for RSM-1 the bearing capacity is more or less precise. This indicates that the critical stresses from FEM might not be able to be allocated to a certain plate element but is considering the cross-section as a whole.

**Column buckling induced by centric axial load and transverse load**

In the case of column buckling induced by a combination of centric axial design load, $N_{Rd}$, and transverse design load, $P_{Rd}$, a relation between them is set to a fixed value for each analysis, as follows:

$$P_{Rd} = 0.22 \times N_{Rd}$$
The column is loaded in such a way that bending occurs around its minor axis, which means that the concentrated forces are acting on the web plate. The reduced stress method is only calculated according to a level 1 calculation.

The design resistance, $N_{R_d}$, is plotted in Figure 4.18 and Figure 4.19 as a function of the plate slenderness $\lambda_{plate}$ of the web and flange plate. Furthermore, FEM, EWM and RSM are plotted against each other in Figure 4.18 and the calculations are based on critical stresses obtained through simple hand calculation methods.

In Figure 4.19, FEM, EWM and RSM are plotted against each other with the critical stresses obtained from the finite element model corresponding to each slenderness value.

![Figure 4.18: Design resistance $N_{R_d}$ as a function of the plate slenderness for the flange and web plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.](image-url)
The governing design verification for the EWM is the interaction of bending and axial compression, for all values of the slenderness in Figure 4.18 and Figure 4.19. For RSM two design verifications are performed; one with respect to the flange plate, which is only subjected to uniform compression from bending moment and axial force; and one with respect to the web plate, which is subjected to transversal stresses and a linearly varying axial stress. The governing verification is with respect to the flange plate, both for Figure 4.18 and Figure 4.19, for all values of the slenderness.

The bearing capacity is plotted in Figure 4.20 and Figure 4.21 as a function of the plate slenderness $\lambda_{plate}$ of the flange plate. Meanwhile the slenderness of the web plate is set constant to $\lambda_{plate} = 1.1$. 

Figure 4.19: Design resistance $N_{Rd}$, as a function of the plate slenderness for the flange and web plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.


Figure 4.20:  Design resistance \(N_{Rd}\), as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained through hand calculations.

Figure 4.21:  Design resistance \(N_{Rd}\), as a function of the plate slenderness for the flange plate, comparing FEM, EWM and RSM. Critical stresses obtained from FE-analysis.
In Figure 4.20 and Figure 4.21 the same type of design verifications are performed, as in the previous case (illustrated in Figure 4.18 and Figure 4.19). For EWM, the governing design verification is the interaction of bending and axial compression; for RSM the governing verification is with respect to the flange plate.

Picking geometric imperfections to be seeded have imposed great problems for the analysis of the combined action of axial load and transversal load. The problem consisted of finding a suitable global buckling mode to be seeded; this lead to that only the axial force is used to seed imperfections for the nonlinear finite element analysis. When the critical stresses are to be obtained only local buckling modes are required, therefore allowing the transverse force to be added. The different way of handling the eigenmodes could have led to the dissimilarities between the FEM analysis and RSM.
Chapter 5

Conclusions

5.1 Reduced stress method

5.1.1 Verification format

The verification format in section 10 of EN 1993-1-5 has certain shortcomings commented upon in (Johansson, et al., 2009) and (Braun & Kuhlmann, 2012). A concern raised by (Johansson, et al., 2009) and also noted in this thesis is the inconsistency of the results provided by the verification format. For instance, an increase of a reduction factor in either the axial direction or an increase in the transverse direction due to choice of another buckling curve (Johansson, et al., 2009) might lead to a lower prediction of the resistance. The problem of inconsistency is caused by the fact that the reduction factors are introduced in the denominator of the von Mises yield criterion. This is best visualized if the reduction factors are viewed as a cause of change of the stress state in either the axial or transversal direction which inevitably alters the stress resultant and thus the point on the yield surface, that might lead to a higher respectively lower resistance. Whether the resistance is increasing or decreasing by a change of the reduction factors depends on the current stress state and the relation between the transverse and axial stress.

By studying the verification format the following correlation can be concluded:

- If \( \sigma_{x,Ed} > \sigma_{z,Ed} \), an increase of the reduction factor for axial stresses, \( \rho_x \), will increase the bearing capacity.
- If \( \sigma_{x,Ed} < \sigma_{z,Ed} \), an increase of the reduction factor for axial stresses, \( \rho_x \), will decrease the bearing capacity.
- If \( \sigma_{x,Ed} > \sigma_{z,Ed} \), an increase of the reduction factor for axial stresses, \( \rho_z \), will decrease the bearing capacity.
- If \( \sigma_{x,Ed} < \sigma_{z,Ed} \), an increase of the reduction factor for axial stresses, \( \rho_z \), will increase the bearing capacity.

This inconsistency is something that should be revised, leading to that an increase of the reduction factor should always lead to a higher resistance and wise versa. In (Braun & Kuhlmann, 2012) the verification format has been revised to better describe the biaxial stress.
state, this revision is not implemented in this report. Their modification would bring consistency but also a lower prediction of the bearing capacity.

The interaction between bending and shear force is shown to be quite weak which is reflected by the interaction verification in section 7 of EN (1993-1-5, 2008) for the effective width method, (Johansson, et al., 2009). This is because that the interaction verification in section 7 allows for the different internal forces to be allocated to different elements in the cross-section. For instance, most of the shear force is carried by the web plate meanwhile, most of the moment is carried by the flange plates, for an I-girder. Unlike the effective width method, the verification format for the reduced stress method does not allow for this distribution of internal forces, which leads to unfavourable results. Therefore, EN-1993-1-5 recommends that this verification should only be performed where other cross-sectional verification methods are not applicable.

5.1.2 Reduction factors

It should be noted that it is not only the verification format that is the reason for the conservative results. The reduction factors are often underestimated, due to an all too high prediction of the slenderness parameters, compared to the EWM, when the whole stress field is being considered. The overestimation of the slenderness factor can be traced back to how the critical load amplification factor, $\alpha_{cr}$, is calculated with the hand calculation methods. If the slenderness factor is obtained with critical stress from FEA instead, the slenderness factor is often lower than in the EWM. This is a natural consequence of the favourable effect of the boundary conditions of the plate is being considered.

5.1.3 Reduced stress method LVL 1

It is reasonable to say that the calculation procedure presented and recommended in EN 1993-1-5 for the reduced stress method will provide results on the safe side. How conservative these results are, depends mainly on the difference in slenderness between the plates that constitutes the cross-section. However, a substantial difference in individual slenderness can still give acceptable results depending on what type of resistance that is sought and how much each individual plate is contributing to the bearing capacity, compare for instance Figure 4.4 and Figure 4.6; where the relative difference in slenderness between the flange plate and the web plate are the same. However, the plots are quite different in terms of relative difference between EWM and RSM. The reason is that most of the moment is carried by the flanges and rather small portion by the web plate. Therefore, the fact that the web plate is being “unfairly” reduced due to the slender flange has a rather small impact on the total bearing capacity.

Furthermore, it should be noted that the method and the results are still conservative compared to the EWM even for plates of equal slenderness. This is due to the fact that even the parts of the cross-section that is subjected to tensile forces are reduced; compared to EWM where only the compressed parts are being reduced.

When RSM-1 is combined with critical stresses from FEM the agreement with nonlinear FEA have been good throughout the report; except for one instance, when all stress components interact in the verification format, see Figure 4.9.
5.1.4 Reduced stress method LVL 3

The reduced stress method with the level 3 approach is equivalent to the EWM and manage to consider the load shedding between plates with different slenderness in a satisfying way. However, the method becomes complicated and requires an iterative procedure when, for instance, bending moment and axial compression is combined. The tedious task of locating the neutral axis makes this approach less attractive.

5.1.5 Reduced stress method proposed implementation

The proposed implementation is aimed to take the load shedding into consideration in a simple manner and to give a joint reduction factor for the entire cross-section, which is meant to be equivalent to a calculation according to RSM-3. For the cases covered in this report the proposed implementation is in good agreement with RSM-3. However, this implementation, in its current state, cannot give a joint reduction factor when for instance axial compression and bending moment is combined. This, unfortunately, limits the usefulness substantially.

5.1.6 Reduced stress method based on FE-analysis

The reduced stress method is very suitable in combination with a linear elastic FE analysis and provides accurate results. The slenderness of the structure and thus the reduction factor is easy to obtain. The factor $\alpha_{ult,k}$ is calculated as the minimum load amplification factor to reach yielding in the most critical point ($\alpha_{ult,k} = F_y/F_{Ed}$) which is readily obtained from the contour plot of a linear analysis. The factor $\alpha_{cr}$ is obtained from a linear buckling analysis which should correspond to the lowest eigenmode that resembles local buckling of the desired plate.

However, it might be problematic to choose a relevant point for where to calculate $\alpha_{ult,k}$; as points of singularities and stress concentrations will appear in the contour plot. Therefore, it will become the job of the designer to be able to determine what stress levels that are realistic and to be used in the design. Furthermore, the relevant eigenmode could be difficult to identify if the structure is complex and give rise to many different local buckling modes.

5.2 Finite element analysis

Nonlinear finite element is a far more demanding design method compared to EWM and RSM in terms of both the analyst’s time and expertise. Even trivial structural problems can impose difficulties. The many sources of the nonlinear behaviour, especially the geometric imperfections, have proven to be challenging. Seeding the geometric imperfections have been a trial and error procedure and has led to sorting through numerous amount of eigenmodes, in order to find the most detrimental combination without exceeding any of the essential manufacturing tolerances. This tedious task of going through eigenmodes can be avoided if the analyst feels comfortable with prescribing her own imperfections, which however requires more experience.
Furthermore, there are several material models available, which are suitable for different purposes. The material model used in this paper, presented in Figure 2.3, is modelled with a continuous yield plateau. The problem of having a continuous yield plateau is that the structure loses too much of its bending stiffness as soon as yielding starts. This will cause a lower prediction of the local buckling strength, leading to conservative results. A way to overcome this is to model the material without a yield plateau and have strain hardening directly, (Johansson, et al., 2007). However, this was tested and resulted in an insignificant change of the results in the cases studied in this report, but might be relevant for other structural problems.

5.3 Assumptions

An assumption that has greatly affected the results in Figure 4.8 to Figure 4.11 is whether the influence of lateral torsional buckling has an effect on the interaction equations in chapter 7 in EN-1993-1-5. In the calculation of the axial stresses EN-1993-1-5 says:

“Action effects $M_{Ed}$ and $N_{Ed}$ should include global second order effects where relevant”.

For lateral torsional buckling the second order effect was, in this report, deemed to be of no relevance; and motivated as follows: The second order effects that appear when the compressed top flange buckles and deviates from its original axis is a bending moment around its weaker axis, produced by the compressive stresses and the eccentricity in the top flange. This will lead to a linear stress distribution across the top flange with a point of zero stress at the junction between the web and the flange plate. Since patch loading is mainly depending on the stress field in the web plate, the effect of second order moment is not taken into account.

Furthermore, the local increase of axial compression due to the concentrated force will not affect the bearing capacity with respect to lateral torsional buckling. This is because lateral torsional buckling is a global phenomenon, which takes the entire stress field of the flange plate into account and local stress concentrations are not considered.

5.4 Orthotropic deck

The analysis of orthotropic decks is not comprised in this master thesis, nevertheless conclusions can be drawn from the results of the parametric study of ordinary beams and also from the result of other reports.

For orthotropic plates there are two types of buckling behaviour; a local buckling phenomenon that occurs in subpanels between the longitudinal stiffeners and a global buckling phenomenon that involves the whole stiffened plate. In the effective width method, the area of the member is reduced in two steps, first considering local buckling and then further by considering global buckling. In the reduced stress method, the interaction between local and global buckling is not considered, noted upon in (Stranghöner, et al., 2012). In the RSM the smallest value of the local and global reduction factor will prevail. This will, for aspect ratios of $\rho_{loc}/\rho_{glob}$ close to 1.0, lead to un-conservative results. However, the reduced stress method becomes conservative especially for larger aspect ratios when the weak
stiffeners unavoidably reduce the resistance of the relatively stronger subpanels, (Stranğhöner, et al., 2012).

5.5 Future work

For the reduced stress method to be fully equivalent with the effective width method and to be able to predict accurate and reliable results compared to nonlinear finite element analysis, certain aspects of the method has to be revised; the main aspect is the verification format. Future work suitable for another master thesis could be to compare RSM, EWM and FEM when considering orthotropic decks and also to perform a case study on a real bridge and compare the material use when applying different design methods.
Chapter 6

Recommendations

In this chapter recommendations for when the reduced stress method and effective width method should be applied are outlined. The recommendations are based on the parametric study performed in this report. Two main aspects have been taken into consideration; the difference in individual slenderness amongst the plated elements in the cross-section and the influence of the entire stress field.

Cross-section constituted by plates of different slenderness

When the cross-section is composed of plates with different slenderness extra precaution is advised as to not end up with an overconservative design. Even for relatively small differences in slenderness the reduced stress method with the approach presented in Eurocode (RSM-1) yields overconservative results, if the critical stresses are obtained through hand calculations; RSM-1 is in this case not to be recommended. However, if the critical stresses are obtained from a linear finite element analysis RSM-1 will most often provide reliable results.

If critical stresses from a finite element model is not available the improved modification of the reduced stress method (RSM-3) can be applied, and for most problems this approach provides results that are in agreement with the effective width method (EWM) and nonlinear FEM.

Cross-section subjected to the complete stress field

For more or less uniaxial stress problems the recommendations presented above are fully applicable. Once all the stress components are considered, the verification format of RSM becomes unreliable and provides results that may be conservative, compared to EWM and nonlinear FEM. If the designer feels comfortable with using RSM even for a more complicated stress field, critical stresses from FEM are recommended to be used. In sections where patch loading can be decisive for the resistance RSM is not recommended and the designer should use nonlinear FEM or EWM.
Bibliography


Stranghöner, N., Kuhn, B. & Beg, D., 2012. Design of plated structures according to EN 1993-1-5 with emphasis on longitudinal compression, Germany: Ernst & Sohn Verlag für Architektur und technische Wissenschaften GmbH & Co.

Appendix A.1
Lateral torsional buckling - I-girder - verification

A double symmetric welded I-girder is subjected to a central loading at mid-span. The I-girder has a lateral bracing system at the supports in order to prevent lateral deflection and twist. Furthermore, the twist of the top flange is completely prevented.

Inputs:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{y,el}$</td>
<td>$9.8698 \times 10^{-5} \text{m}^3$</td>
</tr>
<tr>
<td>$f_y$</td>
<td>239.8 MPa</td>
</tr>
<tr>
<td>$L_1$</td>
<td>3.6 m</td>
</tr>
<tr>
<td>$\lambda_{M1}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$P_{cr,FEM}$</td>
<td>29.72 kN</td>
</tr>
</tbody>
</table>

Value obtained from buckling analysis in BRIGADE.

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

$$\lambda_{LT} := \sqrt{\frac{W_{y,el} f_y}{M_{cr}}} = 0.941$$

Imperfection factors for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.5 - rolled I-section

$h/b > 2$ Gives curve C

$\alpha_{LT} := 0.49$

The factor $\theta_{LT}$ is calculated according to section 6.3.2.3

$$\beta := 0.75$$

$$\lambda_{LT,0} := 0.4$$

$$\phi_{LT} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT} - \lambda_{LT,0} \right) + \beta \lambda_{LT}^2 \right] = 0.964$$

The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.676$$
The correction factor $\kappa_c$ is obtained according to table 6.6

$$\kappa_c := 0.86$$

The factor $f$ is calculated according to section 6.3.2.3

$$f := 1 - 0.5 \left(1 - \kappa_c\right) \left[1 - 2 \left(\chi_{LT} - 0.8\right)^2\right] = 0.933$$

The modified reduction factor $\chi_{LT,\text{mod}}$ is calculated according to equation 6.58

$$\chi_{LT,\text{mod}} := \frac{\chi_{LT}}{f} = 0.724 \quad < 1.0 \quad \text{ok!}$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd} := \chi_{LT,\text{mod}} \frac{W_{y,\text{el}} f_y}{\gamma_M 1} = 17.142 \text{kN} \cdot \text{m}$$

$$P_{Ed} := \frac{4 \cdot M_{b,Rd}}{L_1} = 19.046 \text{kN} \quad \text{Design load}$$
Appendix A.2
Column buckling - verification

A simply supported column with a butt-welded box-section subjected to a monotonic centric axial load through a hydraulic actuator. The column has a pined-end support and the line of action coincides with the centre of gravity of the cross-section.

The following calculations have been made:
- Effective width method, considering column buckling.

![Diagram of a column with dimensions and properties](image)

**Inputs:**

\[ I_y := 5.578 \times 10^6 \text{mm}^4 \]

\[ I_z := I_y \]

\[ f_y := 531.9 \text{MPa} \]

\[ b := 102.2 \text{mm} \]

\[ t := 10.81 \text{mm} \]

\[ \mu := 0.5 \]

\[ E := 210 \text{GPa} \]

\[ L_1 := 4519.9 \text{mm} \]

\[ A_1 := 3952.1 \text{mm}^2 \]

\[ \gamma_{M1} := 1.0 \]

\[ E_s := 210 \text{GPa} \]

\[ h_0 := 80.81 \text{mm} \]

**Classify the cross-section**

\[ \epsilon_1 := \sqrt{\frac{235 \text{MPa}}{f_y}} = 0.665 \]

\[ 33 \cdot \epsilon_1 = 21.935 \]

\[ \frac{h_0}{t} = 7.475 \quad \text{The cross-section belongs to class (1)} \]

**Critical buckling load for upper- and lower boundary obtained from FE-analysis**

\[ N_{\text{cr.upper}} := 2527.11 \text{kN} \]

\[ N_{\text{cr.lower}} := 2496.6 \text{kN} \]

\[ \lambda_{\text{upper}} := \sqrt{\frac{A_1 \cdot f_y}{N_{\text{cr.upper}}}} = 0.912 \]

A.2.1
\[ \lambda_{\text{lower}} = \frac{A_1 f_y}{\sqrt{N_{\text{cr.lower}}}} = 0.918 \quad \text{EN 1993-1-1 2005 section 6.3.1.2} \]

According to EN 1993-1-1 2005 table 6.2 - choice of buckling curve

\[ a > 0.5 t_f \quad b/t_f < 30 \quad h/t_w < 30 \quad \text{Gives curve (c)} \]

Using EN 1993-1-1 2005 table 6.1 to obtain the imperfection factor

\[ \alpha := 0.49 \quad \text{Imperfection factor from curve (c)} \]

\[ \phi_{\text{upper}} := 0.5 \left[ 1 + \alpha \left( \lambda_{\text{upper}} - 0.2 \right) + \lambda_{\text{upper}}^2 \right] \]

\[ \phi_{\text{lower}} := 0.5 \left[ 1 + \alpha \left( \lambda_{\text{lower}} - 0.2 \right) + \lambda_{\text{lower}}^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi_{\text{upper}} := \frac{1}{\phi_{\text{upper}} + \sqrt{\phi_{\text{upper}}^2 - \lambda_{\text{upper}}^2}} = 0.592 \quad < 1.0 \text{ ok!} \]

\[ \chi_{\text{lower}} := \frac{1}{\phi_{\text{lower}} + \sqrt{\phi_{\text{lower}}^2 - \lambda_{\text{lower}}^2}} = 0.589 \quad < 1.0 \text{ ok!} \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{b.Rd.\text{upper}} := \frac{\chi_{\text{upper}} A_1 f_y}{\gamma M_1} = 1245.394 \text{kN} \]

\[ N_{b.Rd.\text{lower}} := \frac{\chi_{\text{lower}} A_1 f_y}{\gamma M_1} = 1238.261 \text{kN} \]

Second order effects need to be taken into account, caused by the load eccentricity in which give rise to end moments. Members which are subjected to combined bending and axial compression should satisfy the criterion presented in EN 1993-1-1:2005, (6.61)

\[ \left( \frac{X_y N_{\text{Rk}}}{\gamma M_1} \right) + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\gamma M_{y,Rk}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\gamma M_{z,Rk}} \leq 1.0 \quad (6.61) \]
\[ \chi_{LT} := 1.0 \quad \Delta M_{y,Ed} := 0 \quad e_{upper} := 1.28 \times 10^{-3} \cdot m \quad e_{lower} := 8.8 \times 10^{-3} \cdot m \]

Finding at which point the maximum deflection occur and determine the stress at that point.

Given

\[ P_{Ed} := 1 \cdot kN \]

\[ M_{y,Ed,upper} := e_{upper} P_{Ed} = 1.28 \times 10^{-3} \cdot kN \cdot m \]

\[ M_{y,Ed,lower} := e_{lower} P_{Ed} = 8.8 \times 10^{-3} \cdot kN \cdot m \]

The deflection function is obtained from beam cases - table A.3

\[ f(x) := \frac{M_{y,Ed,lower} L_1^2}{6 E_s I_y} \left[ \left( 1 - \frac{x}{L_1} \right) \left( 1 - \left( 1 - \frac{x}{L_1} \right)^2 \right) + \frac{M_{y,Ed,upper} x}{M_{y,Ed,lower} L_1} \left( 1 - \left( \frac{x}{L_1} \right)^2 \right) \right] \]

First plot the function and then finding the maximum x-value, in which give the deflection magnitude.

\[ x := 0 \]

\[ x_{\text{star}} := \text{Maximize}(f, x) \]

\[ x_{\text{star}} = 1.991 \text{m} \quad \text{The distance where the maximum deflection occur} \]

\[ f(x_{\text{star}}) = 0.011 \cdot \text{mm} \quad \text{The maximum deflection} \]

Linear interpolation of the moments, where \( M_{\text{ed}} = k \cdot x + m_1 \)

\[ m_1 := M_{y,Ed,lower} \]

\[ k := \frac{M_{y,Ed,upper} - M_{y,Ed,lower}}{L_1} \]

A.2.3
$M_{Ed.1}(x) := k \cdot x + m_1$ \hspace{1cm} \text{Final expression for the moment at maximum deflection}

$M_{Ed.1}(\bar{x}_{\text{star}}) = 5.487 \times 10^{-3} \text{kN-m}$

$M_{Ed}(N_{Ed}) := 5.487 \times 10^{-3} \text{m} \cdot N_{Ed}$

Centre of gravity

$$y_{GC} := \frac{b \cdot t \cdot \left[ \frac{(b \cdot t)^2}{2} \right]}{2} + \frac{A_1}{2} = 34.428 \text{mm}$$

Plastic section modulus

$$W_{pl.y} := 2 \cdot A_1 \cdot y_{GC} = 2.721 \times 10^5 \text{mm}^3$$

$$M_{y,Rk} := W_{pl.y} \cdot f_y$$

Determine the value of the interaction factor $k_{yy}$ for plastic cross-sectional properties class 1

$$W_{el,y} := \frac{I_y}{b} = 1.092 \times 10^5 \text{mm}^3$$

$$W_{y,1} := \frac{W_{pl.y}}{W_{el,y}} = 2.493 \quad > 1.5 \text{ not ok!} \quad \text{Gives} \quad W_y := 1.5$$

Two interations are needed for both the upper- and lower boundary in order to determine the maximum design resistance load $N_{Ed}$. Using EN 1993-1-1 2005

$$\psi := \frac{M_{y,Ed,upperpart}}{M_{y,Ed,lowerpart}} = 0.145 \quad -1 < \psi > 1 \quad \text{Gives}$$

$$C_{my,upper}(N_{Ed}) := 0.79 + 0.21 \cdot \psi + 0.36 \cdot (\psi - 0.33) \cdot \frac{N_{Ed}}{N_{cr,upper}}$$

$$C_{my,lower}(N_{Ed}) := 0.79 + 0.21 \cdot \psi + 0.36 \cdot (\psi - 0.33) \cdot \frac{N_{Ed}}{N_{cr,lower}}$$
\[ C_{yy.upper}(N_{Ed}) = 1 + (W_y - 1) \left[ 2 - \frac{1.6}{W_y} C_{my.upper}(N_{Ed})^2 \lambda_{upper} (1 + \lambda_{upper}) \right] \]

\[ C_{yy.lower}(N_{Ed}) = 1 + (W_y - 1) \left[ 2 - \frac{1.6}{W_y} C_{my.lower}(N_{Ed})^2 \lambda_{lower} (1 + \lambda_{lower}) \right] \]

\[ k_{yy.upper}(N_{Ed}) := \frac{C_{my.upper}(N_{Ed})}{C_{yy.upper}(N_{Ed}) \left( 1 - \chi_{upper} \frac{N_{Ed}}{N_{cr.upper}} \right)} \]

\[ k_{yy.lower}(N_{Ed}) := \frac{C_{my.lower}(N_{Ed})}{C_{yy.lower}(N_{Ed}) \left( 1 - \chi_{lower} \frac{N_{Ed}}{N_{cr.lower}} \right)} \]

Given

\[ N_{Ed} := 1 \cdot kN \]

\[ \frac{N_{Ed}}{N_{b.Rd.upper}} + k_{yy.upper}(N_{Ed}) \frac{M_{Ed}(N_{Ed}) + \Delta M_{y.Ed}}{M_{y.Rk}} = 1 \]

\[ \chi_{LT} \frac{\gamma_{M1}}{\gamma_{M1}} = 1 \]

\[ N_{Ed.upper} := \text{Find}(N_{Ed}) = 1.202 \times 10^3 \cdot kN \quad \text{Design buckling resistance upper value} \]

Given

\[ N_{Ed} := 1 \cdot kN \]

\[ \frac{N_{Ed}}{N_{b.Rd.lower}} + k_{yy.lower}(N_{Ed}) \frac{M_{Ed}(N_{Ed}) + \Delta M_{y.Ed}}{M_{y.Rk}} = 1 \]

\[ \chi_{LT} \frac{\gamma_{M1}}{\gamma_{M1}} = 1 \]

\[ N_{Ed.lower} := \text{Find}(N_{Ed}) = 1.195 \times 10^3 \cdot kN \quad \text{Design buckling resistance lower value} \]
Appendix A.3
Lateral torsional buckling - Endmoments - web class 4

A welded I-girder is subjected to two equal end moments with fixed-fixed boundary conditions.

The following calculations have been made:
- Effective width method, considering lateral torsional buckling.
- Reduced stress method: According to level 1, level 3, proposed implementation by obtaining the critical stress from hand-calculations and from linear FE-analysis.

![Diagram of I-girder]

Flange in class 3
Web in class 4
Slenderness ratio 1.1

Inputs:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_f$</td>
<td>200 mm</td>
</tr>
<tr>
<td>$t_f$</td>
<td>9 mm</td>
</tr>
<tr>
<td>$h_w$</td>
<td>600 mm</td>
</tr>
<tr>
<td>$t_w$</td>
<td>3.964775 mm</td>
</tr>
<tr>
<td>$L$</td>
<td>4 m</td>
</tr>
<tr>
<td>$f_y$</td>
<td>355 MPa</td>
</tr>
<tr>
<td>$\gamma_{M1}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$E$</td>
<td>210 GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>$z$</td>
<td>304.5 mm</td>
</tr>
</tbody>
</table>

Classification: weld sizes are omitted

Web:

$$\epsilon := \sqrt{\frac{235 \text{ MPa}}{f_y}} \quad \frac{h_w}{t_w} = 151.333$$

Limit for class 4 part subjected to bending

$$124 \cdot \epsilon = 100.888 \quad < 120 \quad \text{The web belongs to class 4}$$

Flange: outstand full compression

$$\frac{b_f - t_w}{2t_f} = 10.891$$

A.3.1
Limit for class 4

\[ 14 \cdot \varepsilon = 11.391 \]  
\[ \geq 10.83 \quad \text{The flange belongs to class 3} \]

Effective area: internal compression - table 4.1 EN 1993-1-5

\[ \psi := -1 \quad \text{Gives} \quad k_\sigma := 23.9 \]

Determine the reduction factor (\( \rho \))

\[
\lambda_p := \frac{h_w}{t_w \sqrt{28.4 \cdot \varepsilon \cdot k_\sigma}} = 1.34
\]

Larger than 0.673 - using equation (4.2)

\[
\rho := \frac{\lambda_p - 0.055(3 + \psi)}{\lambda_p^2} = 0.685
\]

< 1.0 ok!

\[ b_{\text{eff}} := \frac{h_w}{2} = 205.549 \text{·mm} \]

Effective length

\[ b_{c1} := 0.4 \cdot b_{\text{eff}} = 82.22 \text{·mm} \]

\[ b_{c2} := 0.6 \cdot b_{\text{eff}} = 123.33 \text{·mm} \]

Calculation of a new centre of gravity (\( y_{\text{GC,red}} \))

\[
y_{\text{GC,red}} := \frac{\frac{h_w \cdot t_w}{2} \left( \frac{h_w + t_f}{2} \right) + b_f \cdot t_f \left( h_w + t_f \right) - C \cdot t_w \left( \frac{t_f + h_w}{2} + b_{c2} + \frac{C}{2} \right)}{A_{\text{eff}}} = 293.104 \text{·mm}
\]
Effective moment of inertia

\[
I_{y.1} := \frac{2b_{c1}t_{f}^{3}}{12} + b_{f}t_{f}(y_{G.C.red})^{2} + b_{f}t_{f}(h_{w} + t_{f} - y_{G.C.red})^{2}
\]

\[
I_{y.2} := b_{c1}t_{w}\left(h_{w} + \frac{t_{f}}{2} - y_{G.C.red} - \frac{b_{c1}}{2}\right)^{2} + \left(b_{c2} + h_{w} / 2\right)t_{w}\left(y_{G.C.red} - \frac{t_{f}}{2} - \frac{b_{c2} + h_{w}}{2}\right)^{2}
\]

\[
I_{y.3} := \frac{t_{w}b_{c1}^{3}}{12} + t_{w}\left(b_{c2} + \frac{h_{w}}{2}\right)^{3}
\]

\[
I_{y.eff} := I_{y.1} + I_{y.2} + I_{y.3} = 3.933 \times 10^{8}\text{mm}^{4}
\]

\[
W_{y.eff} := \frac{I_{y.eff}}{h_{w} + t_{f} - y_{G.C.red}} = 1.244978 \times 10^{6}\text{mm}^{3}
\]

\[
I_{y.gross} := 2\left[\frac{b_{f}t_{f}^{3}}{12} + b_{f}t_{f}\left(h_{w} / 2 + \frac{t_{f}}{2}\right)^{2}\right] + \frac{t_{w}h_{w}^{3}}{12} = 4.052 \times 10^{8}\text{mm}^{4}
\]

\[
M_{cr} := 1907.8\text{kN} \cdot \text{m}
\]

The critical moment was obtained from LTBeam

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[
\lambda_{LT} := \sqrt{\frac{W_{y.eff}f_{y}}{M_{cr}}} = 0.4813
\]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

h/b > 2  Gives  curve d

\[
\alpha_{LT} := 0.76
\]

The factor \(\theta_{LT}\) is calculated according to section 6.3.2.3

\[
\beta := 0.75 \quad \lambda_{LT.0} := 0.4
\]

\[
\phi_{LT} := 0.5\left[1 + \alpha_{LT}\left(\lambda_{LT} - \lambda_{LT.0}\right) + \beta \lambda_{LT}^{2}\right] = 0.618
\]

A.3.3
The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$\chi_{LT} := \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.931$$

The correction factor $\kappa_c$ is obtained according to table 6.6

$$\kappa_c := 1$$

The factor $f$ is calculated according to section 6.3.2.3

$$f := 1 - 0.5(1 - \kappa_c)\left[1 - 2\left(\lambda_{LT} - 0.8\right)^2\right] = 1$$

The modified reduction factor $\chi_{LT,mod}$ is calculated according to equation 6.58

$$\chi_{LT,mod} := \frac{\chi_{LT}}{f} = 0.931 \quad \text{< 1.0 ok!}$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd} := \chi_{LT,mod} \frac{W_{y,eff}}{\gamma_1} = 411.619\text{kN}\cdot\text{m}$$

Reduced stress method level 1

$$\sigma_E := \frac{\pi \cdot E \cdot t_w^2}{12 (1 - \nu^2) \cdot h_w^2} = 8.288\text{MPa}$$

$$\sigma_{cr,x} := \kappa \cdot \sigma_E = 198.075\text{MPa}$$

$$\alpha_{cr,x} := \frac{\sigma_{cr,x}}{\sigma_{x,Ed}} \quad \text{and} \quad \alpha_{ult,k} := \frac{f_y}{\sigma_{x,Ed}} \quad \text{gives} \quad \lambda_p := \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,x}}}$$

$$\lambda_p := \sqrt{\frac{f_y}{\sigma_{cr,x}}} = 1.339$$

$\lambda_p$ is larger than 0.673: using equation 4.2 EN 1993-1-5 (internal compression)

$$\rho := \frac{\lambda_p - 0.055 \cdot (3 + \psi)}{\lambda_p} = 0.686 \quad \text{< 1.0 ok!}$$

A.3.4
\[ I_{y, \text{gross.}} = 2 \left[ \frac{b_f t_f^3}{12} + b_f t_f \left( \frac{h_w}{2} + \frac{t_f}{2} \right)^2 \right] + \frac{t_w h_w^3}{12} = 4.052 \times 10^8 \cdot \text{mm}^4 \]

\[ W_{y, \text{el}} := \frac{I_{y, \text{gross.}}}{z} = 1.331 \times 10^6 \cdot \text{mm}^3 \]

The critical moment was obtained from LTBeam

\[ M_{\text{cr.}} := 1907.8 \text{ kN} \cdot \text{m} \]

Non-dimensional slenderness with a reduction factor \( \rho \) included in order to achieve the same principle as the effective width method.

\[ \lambda_{LT.} := \frac{W_{y, \text{el}} \rho f_y}{M_{\text{cr}}} = 0.412 \]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\( h/b > 2 \) \quad \text{Gives curve d}

\[ \alpha_{LT.} := 0.76 \]

The factor \( \theta_{LT.} \) is calculated according to section 6.3.2.3

\[ \lambda_{LT.0.} := 0.4 \]

\[ \phi_{LT.} := 0.5 \left[ 1 + \alpha_{LT.} \left( \lambda_{LT.} - \lambda_{LT.0.} \right) + \beta \lambda_{LT.}^2 \right] = 0.568 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.} := \frac{1}{\phi_{LT.} + \sqrt{\phi_{LT.}^2 - \beta \lambda_{LT.}^2}} = 0.99 \]

The modified reduction factor \( \chi_{LT, \text{mod}} \) is calculated according to equation 6.58

\[ \chi_{LT, \text{mod.}} := \chi_{LT.} = 0.99 \quad < 1.0 \text{ ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b, \text{Rd.LVL1}} := \frac{W_{y, \text{el}} \rho f_y}{\gamma_M1} = 320.312 \text{ kN} \cdot \text{m} \]

A.3.5
Reduced stress method level 3

The calculation with RSM is performed according to the theory of level 3, more information are presented in the report.

\[ \sigma_1(x) := \rho f_y \]
\[ \sigma_2(x) := \frac{h_w - x}{x + \frac{t_f}{2}} f_y \]
\[ \sigma_t(x) := \frac{(h_w - x) + t_f}{x + \frac{t_f}{2}} f_y \]

Force equilibrium:

\[ \text{Tension}(x) := b_f t_f \left( \frac{\sigma_t(x) + \sigma_2(x)}{2} \right) + (h_w - x) t_w \frac{\sigma_2(x)}{2} \]

Given

\[ x := 1 \cdot m \]

\[ \text{Compression}(x) := b_f t_f f_y + x t_w \frac{\sigma_1(x)}{2} \]

\[ \text{Tension}(x) \neq \text{Compression}(x) \]

\[ X := \text{Find}(x) = 309.695 \cdot \text{mm} \]

Tension side:

\[ \sigma_{\text{tension}} := \frac{(h_w - X) + t_f}{X + \frac{t_f}{2}} f_y = 338.175 \cdot \text{MPa} \]

A.3.6
Web contribution:

\[
\sigma_{2, \text{web}} := \frac{h_w - X}{X + \frac{t_f}{2}} f_y = 328.006\text{·MPa}
\]

\[
R_{\text{web}} := \frac{\left(h_w - X\right) t_w \sigma_{2, \text{web}}}{2} = 188.766\text{·kN}
\]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[
\sigma_{\text{AVG}} := \frac{\sigma_{\text{tension}} + \sigma_{2, \text{flange}}}{2} = 333.091\text{·MPa}
\]

\[
R_{\text{flange}} := b_f t_f \sigma_{\text{AVG}} = 599.563\text{·kN}
\]

Compression side:

Web contribution:

\[
\sigma_{1, \text{c}} := \rho f_y = 243.233\text{·MPa}
\]

\[
R_{\text{c,web}} := \frac{X t_w \sigma_{1, \text{c}}}{2} = 149.33\text{·kN}
\]

Flange contribution:

\[
R_{\text{c,flange}} := b_f t_f f_y = 639\text{·kN}
\]

\[LA := X + \frac{t_f}{2}\] Lever arm

Moment capacity:

\[
M_{\text{Rd}} := R_{\text{c,web}} \frac{2}{3} X + R_{\text{c,flange}} LA + R_{\text{web}} \frac{2}{3} \left(h_w - X\right) + R_{\text{flange}} \left(h_w - X + \frac{t_f}{2}\right)
\]

\[
M_{\text{Rd}} = 444.889\text{·kN·m}
\]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[
\lambda_{LT..} := \sqrt{\frac{M_{\text{Rd}}}{M_{\text{cr}}}} = 0.4829
\]

A.3.7
Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\( h/b > 2 \) gives curve d

\[ \alpha_{LT} := 0.76 \]

The factor \( \theta_{LT} \) is calculated according to section 6.3.2.3

\[ \beta := 0.75 \quad \lambda_{LT.0} := 0.4 \]

\[ \phi_{LT} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT} - \lambda_{LT.0} \right) + \beta \cdot \lambda_{LT}^2 \right] = 0.619 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \cdot \lambda_{LT}^2}} = 0.93 \]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[ \chi_{LT,mod} := \chi_{LT} = 0.93 \quad \text{< 1.0 ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b.Rd.LVL3} := \chi_{LT,mod} \cdot \frac{M_{Rd}}{\gamma_{M1}} = 413.756 \text{ kNm} \]

The design moment capacity according to RSM level 3 is 413.756 kNm
Proposed implementation

The proposed implementation allows stress redistribution between the weaker and stronger plate. A linear stress distribution is assumed by solving for a reduction factor $\rho$ for both the weaker and stronger plate.

\[
M_{Rd,flange} := b_{f}t_{f}f_{y}(h_{w} + t_{f})
\]

\[
M_{Rd,web} := \frac{h_{w}^{2}t_{w}}{6}f_{y}
\]

\[
\rho_{web} := \rho \quad \rho_{flange} := 1
\]

\[
\rho_{proposed} := \rho_{web} \frac{M_{Rd,web}}{M_{Rd,flange} + M_{Rd,web}} + \rho_{flange} \frac{M_{Rd,flange}}{M_{Rd,flange} + M_{Rd,web}} = 0.944
\]

\[
\lambda_{LT,proposed} := \sqrt{\frac{W_{y,el}\rho_{proposed}f_{y}}{M_{cr}}} = 0.483
\]

\[
\Phi_{LT,proposed} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT,proposed} - \lambda_{LT,0} \right) + \beta \lambda_{LT,proposed} \right] = 0.619
\]

The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

\[
\chi_{LT,proposed} := \frac{1}{\Phi_{LT,proposed} + \sqrt{\Phi_{LT,proposed}^{2} - \beta \lambda_{LT,proposed}^{2}}} = 0.9296
\]

The modified reduction factor $\chi_{LT,mod}$ is calculated according to equation 6.58

\[
\chi_{LT,mod,proposed} := \chi_{LT,proposed} = 0.9296 < 1.0 \text{ ok!}
\]

The design resistance moment is calculated according to equation 6.55

\[
M_{b,Rd,proposed} := \chi_{LT,mod,proposed}M_{Rd} = 413.562 \text{ kN}\cdot\text{m}
\]

The design moment capacity according to RSM proposed is 413.526 kNm
Reduced stress method level 1 based on FEM results

The critical load is obtained from the first local buckling mode of the web plate. The critical load for which the web plate buckles is denoted with $M_{cr,FEM}$. 

$$M_{cr,FEM} := 364.581 \text{kN} \cdot \text{m}$$

$$\lambda_{p,FEM} := \frac{f_y W_{y,el}}{M_{cr,FEM}} = 1.138$$

$\lambda_{pR}$ is larger than 0.748: using equation 4.3 EN 1993-1-5 (outstand compression)

$$\rho_{FEM} := \frac{\lambda_{p,FEM} - 0.055 (3 + \psi)}{\lambda_{p,FEM}^2} = 0.794 \quad < 1.0 \text{ ok!}$$

$$\lambda_{LT,R,FEM} := \sqrt{\frac{W_{y,el} \rho_{FEM} f_y}{M_{cr}}} = 0.4433$$

$$\phi_{LT,R,FEM} := 0.5 \left[ 1 + \alpha_{LT,R,FEM} (\lambda_{LT,R,FEM} - \lambda_{LT,0..}) + \beta \lambda_{LT,R,FEM}^2 \right] = 0.59$$

The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$\chi_{LT,R,FEM} := \frac{1}{\phi_{LT,R,FEM} + \sqrt{\phi_{LT,R,FEM}^2 - \beta \lambda_{LT,R,FEM}^2}} = 0.963 \quad < 1.0 \text{ ok!}$$

The modified reduction factor $\chi_{LT,mod}$ is calculated according to equation 6.58

$$\chi_{LT,mod,R,FEM} := \chi_{LT,R,FEM} = 0.963$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd,LVL1,FEM} := \chi_{LT,mod,R,FEM} \frac{W_{y,el} \rho_{FEM} f_y}{\gamma_{M1}} = 361.048 \text{kN} \cdot \text{m}$$

Moment capacity

The design moment capacity according to RSM level 1 is 361.048 kNm
Reduced stress method level 3 based on FEM results

Lateral torsional buckling:

The calculation with RSM is performed according to the theory of level 3, more information are presented in the report.

Force equilibrium:

\[\sigma_{1,FEM}^{(x)} := \rho_{FEM} \cdot f_y\]
\[\sigma_{2,FEM}^{(x)} := \frac{h_w - x}{x + \frac{t_f}{2}} \cdot f_y\]
\[\sigma_{t,FEM}^{(x)} := \frac{h_w - x}{x + \frac{t_f}{2}} \cdot f_y\]

Tension_{FEM}^{(x)} := b_f \cdot t_f \left( \frac{\sigma_{t,FEM}^{(x)} + \sigma_{2,FEM}^{(x)}}{2} \right) + (h_w - x) \cdot t_w \cdot \frac{\sigma_{2,FEM}^{(x)}}{2}

Given
\[x := 1 \cdot m\]

Compression_{FEM}^{(x)} := b_f \cdot t_f \cdot f_y + x \cdot t_w \cdot \frac{\sigma_{1,FEM}^{(x)}}{2}

Tension_{FEM}^{(x)} = Compression_{FEM}^{(x)}

\[X := \text{Find}(x) = 306.047 \cdot \text{mm}\]

Tension side:

\[\sigma_{\text{tension,FEM}} := \frac{(h_w - X) + t_f}{X + \frac{t_f}{2}} \cdot f_y = 346.319 \cdot \text{MPa}\]
Web contribution:

\[ \sigma_{2\text{.FEM.}} := \frac{h_w - X}{X + \frac{t_f}{2}} \cdot f_y = 336.031\cdot\text{MPa} \]

\[ R_{\text{web.FEM}} := \frac{(h_w - X) \cdot t_w \cdot \sigma_{2\text{.FEM.}}}{2} = 195.815\cdot\text{kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{\text{AVG.FEM}} := \frac{\sigma_{\text{tension.FEM}} + \sigma_{2\text{.FEM.}}}{2} = 341.175\cdot\text{MPa} \]

\[ R_{\text{flange.FEM}} := b_f t_f f_y = 614.115\cdot\text{kN} \]

Compression side:

Web contribution:

\[ \sigma_{1\text{.c.FEM.}} := \rho_{\text{FEM}} \cdot f_y = 281.735\cdot\text{MPa} \]

\[ R_{c\text{.web.FEM}} := \frac{X \cdot t_w \cdot \sigma_{1\text{.c.FEM.}}}{2} = 170.93\cdot\text{kN} \]

Flange contribution:

\[ R_{c\text{.flange.FEM}} := b_f t_f f_y = 639\cdot\text{kN} \]

\[ L_{\text{FEM}} := X + \frac{t_f}{2} \quad \text{Lever arm} \]

Moment capacity:

\[ M_{R\text{d.FEM.1}} := R_{c\text{.web.FEM}} \left(\frac{2}{3}X + R_{c\text{.flange.FEM}} L_{\text{FEM}}\right) \]

\[ M_{R\text{d.FEM.2}} := R_{\text{web.FEM}} \left(\frac{2}{3}(h_w - X) + R_{\text{flange.FEM}} \left(h_w - X + \frac{t_f}{2}\right)\right) \]

\[ M_{R\text{d.FEM.}} := M_{R\text{d.FEM.1}} + M_{R\text{d.FEM.2}} = 454.973 \text{ kN m} \]

A.3.12
Non-dimensional slenderness with a reduction factor \( \rho \) included in order to achieve the same principle as the effective width method.

\[
\lambda_{LT,FEM} := \sqrt{\frac{M_{Rd}}{M_{cr}}} = 0.4829
\]

\[
\phi_{LT,FEM} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT,FEM} - \lambda_{LT,0} \right) + \beta \lambda_{LT,FEM}^2 \right]
\]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[
\chi_{LT,FEM} := \frac{1}{\phi_{LT,FEM} + \sqrt{\phi_{LT,FEM}^2 - \beta \lambda_{LT,FEM}^2}} = 0.93
\]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[
\chi_{LT,mod,FEM} := \chi_{LT,FEM} = 0.93 \quad \text{< 1.0 ok!}
\]

The design resistance moment is calculated according to equation 6.55

\[
M_{b,Rd,LVL3,FEM} := \chi_{LT,mod,FEM} \frac{M_{Rd}}{\gamma_{M1}} = 413.756 \text{ kNm}
\]

The design moment capacity according to RSM level 3 is 413.756 kNm
Proposed implementation based on FEM result

\[
\rho_{p,FEM} := \rho_{FEM} \frac{M_{Rd.web}}{M_{Rd.web} + M_{Rd.flange}} + \rho_{flange} \frac{M_{Rd.flange}}{M_{Rd.web} + M_{Rd.flange}} = 0.963
\]

**Lateral torsional buckling:**

\[
\lambda_{LT,p,FEM} := \sqrt{\frac{W_{y,el} \rho_{p,FEM} f_y}{M_{cr}}} = 0.4884
\]

\[
\phi_{LT,p,FEM} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT,p,FEM} - \lambda_{LT,0} \right) + \beta \lambda_{LT,p,FEM}^2 \right]
\]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[
\chi_{LT,p,FEM} := \frac{1}{\phi_{LT,p,FEM} + \sqrt{\phi_{LT,p,FEM}^2 - \beta \lambda_{LT,p,FEM}^2}} = 0.926
\]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[
\chi_{LT,mod,p,FEM} := \chi_{LT,p,FEM} = 0.926 \quad \text{< 1.0 ok!}
\]

The design resistance moment is calculated according to equation 6.55

\[
M_{b,Rd.LVL1,p,FEM} := \chi_{LT,mod,p,FEM} \frac{W_{y,el} \rho_{p,FEM} f_y}{\gamma_M 1} = 421.109 \text{ kNm}
\]

The design moment capacity according to RSM proposed is 421.109 kNm
Appendix A.4
Lateral torsional buckling - End moments - flange class 4

Flange in class 4
Web in class 3
Slenderness ratio 1.1

Inputs:
\[ b_f := 200 \text{ mm} \quad t_f := 7.731 \text{ mm} \quad h_w := 600 \text{ mm} \quad t_w := 6.2661 \text{ mm} \]
\[ L := 4 \text{ m} \quad f_y := 355 \text{ MPa} \quad \gamma_{M1} := 1.0 \quad E := 210 \text{ GPa} \]
\[ v := 0.3 \]

Classification: weld sizes are omitted

Web:
\[ \varepsilon = \sqrt{\frac{235 \text{ MPa}}{f_y}} \quad \frac{h_w}{t_w} = 95.753 \]
Limit for class 4 part subjected to bending

\[ 124 \cdot \varepsilon = 100.888 \quad > 95.753 \quad \text{The web belongs to class 3} \]

Flange: outstand full compression

\[ \frac{b_f - t_w}{2t_f} = 12.53 \]
Limit for class 4

\[ 14 \cdot \varepsilon = 11.391 \quad < 12.53 \quad \text{The flange belongs to class 4} \]
Effective area: outstanding compression elements - table 4.1 EN 1993-1-5

ψ := −1 \quad \text{Gives} \quad k_\sigma := 0.43

Determine the reduction factor (ρ)

\[ \lambda_p := \frac{b_f - t_w}{2 t_f} = 0.827 \quad \text{Larger than 0.748 - using equation (4.3)} \]

\[ \rho := \frac{\lambda_p - 0.188}{\lambda_p} = 0.934 \quad < 1.0 \text{ ok!} \]

\[ b_{\text{eff}} := \rho \frac{b_f - t_w}{2} = 90.509 \text{ mm} \quad \text{Effective length} \]

\[ b_{\text{red}} := (1 - \rho) \frac{b_f - t_w}{2} = 6.358 \text{ mm} \]

\[ A_{\text{gross}} := h_w t_w + 2 b_f t_f = 6.852 \times 10^3 \text{ mm}^2 \]

\[ A_{\text{red}} := 2 b_{\text{red}} t_f = 98.305 \text{ mm}^2 \]

\[ A_{\text{eff}} := A_{\text{gross}} - A_{\text{red}} = 6.754 \times 10^3 \text{ mm}^2 \]

Effective cross-section area

\[ I_{y, \text{gross}} := 2 \left[ \frac{b_f t_f^3}{12} + b_f t_f \left( \frac{h_w}{2} + \frac{t_f}{2} \right)^2 \right] + \frac{t_w h_w^3}{12} = 3.983 \times 10^8 \text{ mm}^4 \]

\[ z_{\text{eff}} := \frac{A_{\text{gross}} \left( \frac{h_w}{2} + \frac{t_f}{2} \right)}{A_{\text{eff}}} = 308.288 \text{ mm} \]

\[ I_{y, \text{eff}} := I_{y, \text{gross}} + A_{\text{gross}} \left[ z_{\text{eff}} - \left( \frac{h_w}{2} + \frac{t_f}{2} \right)^2 \right] - 2 \left( \frac{b_{\text{red}} t_f^3}{12} + t_f b_{\text{red}} z_{\text{eff}}^2 \right) = 3.891 \times 10^8 \text{ mm}^4 \]

A.4.2
\[ W_{y,\text{eff}} := \frac{I_{y,\text{eff}}}{z_{\text{eff}}} = 1.262 \times 10^6 \text{mm}^3 \]

The critical moment obtained from LTBeam

\[ M_{cr} := 1639.4 \text{kN}\cdot\text{m} \]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[ \lambda_{LT} := \sqrt{\frac{W_{y,\text{eff}} f_y}{M_{cr}}} = 0.5228 \]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\[ h/b > 2 \quad \text{Gives} \quad \text{curve d} \]

\[ \alpha_{LT} := 0.76 \]

The factor \( \theta_{LT} \) is calculated according to section 6.3.2.3

\[ \beta := 0.75 \quad \lambda_{LT,0} := 0.4 \]

\[ \phi_{LT} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT} - \lambda_{LT,0} \right) + \beta \lambda_{LT}^2 \right] = 0.649 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.897 \]

The correction factor \( \kappa_c \) is obtained according to table 6.6

\[ \kappa_c := 1 \]

The factor \( f \) is calculated according to section 6.3.2.3

\[ f := 1 - 0.5 \left( 1 - \kappa_c \right) \left[ 1 - 2 \left( \lambda_{LT} - 0.8 \right)^2 \right] = 1 \]

The modified reduction factor \( \chi_{LT,\text{mod}} \) is calculated according to equation 6.58

\[ \chi_{LT,\text{mod}} := \frac{\chi_{LT}}{f} = 0.897 < 1.0 \quad \text{ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,Rd} := \chi_{LT,\text{mod}} \frac{W_{y,\text{eff}} f_y}{\gamma_{M1}} = 402.102 \text{kN}\cdot\text{m} \]
Reduced stress method level 1

The calculation with RSM is performed according to the theory of level 1, more information are presented in the report and in Eurocode.

### Plate buckling: using EN 1993-1-5 chapter 10 page 35

\[ k_{\sigma, R} := 0.43 \] Buckling factor according to EN 1993-1-5 table 4.1

\[ \sigma_E := \frac{\pi^2 \cdot E \cdot t_f^2}{12 \left(1 - \nu^2\right) \left(\frac{b - t_w}{2}\right)^2} = 1.209 \times 10^3 \cdot \text{MPa} \]

\[ \sigma_{cr,x} := k_{\sigma, R} \cdot \sigma_E = 519.858 \cdot \text{MPa} \]

\[ \lambda_{p,R} := \sqrt{\frac{f_y}{\sigma_{cr,x}}} = 0.826 \]

\( \lambda_{pR} \) is larger than 0.748: using equation 4.3 EN 1993-1-5 (outstand compression)

\[ \rho_R := \frac{\lambda_p - 0.188}{\lambda_p^2} = 0.934 \quad < 1.0 \text{ ok!} \]

\[ W_{y,el} := \frac{I_{y, gross}}{\left(\frac{h_w + t_f}{2}\right)} = 1.311 \times 10^6 \cdot \text{mm}^3 \]

\[ \lambda_{LT,R} := \sqrt{\frac{W_{y,el} \cdot f_y}{M_{cr}}} = 0.515 \]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\( h/b > 2 \) Gives curve d

\[ \alpha_{LT,R} := 0.76 \]

The factor \( \theta_{LT} \) is calculated according to section 6.3.2.3

\[ \beta_R := 0.75 \quad \lambda_{LT,0.R} := 0.4 \]

\[ \phi_{LT,R} := 0.5 \left[ 1 + \alpha_{LT,R} \left( \lambda_{LT,R} - \lambda_{LT,0.R} \right) + \beta \cdot \lambda_{LT,R}^2 \right] = 0.643 \]
The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$
\chi_{LT,R} := \frac{1}{\phi_{LT,R} + \sqrt{\phi_{LT,R}^2 - \beta \lambda_{LT,R}^2}} = 0.904
$$

The modified reduction factor $\chi_{LT,\text{mod}}$ is calculated according to equation 6.58

$$
\chi_{LT,\text{mod},R} := \chi_{LT,R} = 0.904 < 1.0 \text{ ok!}
$$

The design resistance moment is calculated according to equation 6.55

$$
M_{b,Rd,LVL1} := \chi_{LT,\text{mod},R} \frac{W_{y,el'} \rho' f_y}{\gamma_{M1}} = 392.952 \text{ kN}\cdot\text{m}
$$
Reduced stress method level 3

The calculation with RSM is performed according to the theory of level 3, more information are presented in the report.

\[
\sigma_1(x) := \frac{x}{x + t_f} \cdot f_y \\
\sigma_2(x) := \frac{h_w - x}{x + t_f} \cdot f_y \\
\sigma_f(x) := \frac{(h_w - x) + t_f}{x + t_f} \cdot f_y \\
\text{Tension}(x) := b_f \cdot t_f \left( \frac{\sigma_f(x) + \sigma_2(x)}{2} \right) + \left( h_w - x \right) \cdot t_w \frac{\sigma_2(x)}{2} \\
\text{Compression}(x) := b_f \cdot t_f \cdot f_y + x \cdot t_w \frac{\sigma_1(x)}{2} \\
\text{Given} \\
x := 1 \cdot m \\
\text{Tension}(x) \neq \text{Compression}(x) \\
X := \text{Find}(x) = 303.741 \cdot \text{mm} \\
\text{Tension side:} \\
\sigma_{\text{tension}} := \frac{(h_w - X) + t_f}{X + t_f} \cdot f_y = 346.473 \cdot \text{MPa}
\]
Web contribution:

\[
\sigma_2 := \frac{h_w - X}{X + t_f} \cdot f_y = 337.661 \text{ MPa}
\]

\[
R_{\text{web}} := \frac{(h_w - X) \cdot t_w \cdot \sigma_2}{2} = 313.415 \text{ kN}
\]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[
\sigma_{\text{AVG}} := \frac{\sigma_{\text{tension}} + \sigma_2}{2} = 342.067 \text{ MPa}
\]

\[R_{\text{flange}} := b_f \cdot t_f \cdot \sigma_{\text{AVG}} = 528.904 \text{ kN}\]

Compression side:

Web contribution:

\[
\sigma_{1,c} := \frac{X}{X + t_f} \cdot f_y = 346.189 \text{ MPa}
\]

\[R_{\text{c.web}} := \frac{X \cdot t_w \cdot \sigma_{1,c}}{2} = 329.445 \text{ kN}\]

Flange contribution:

\[R_{\text{c.flange}} := b_f \cdot t_f \cdot \rho \cdot f_y = 512.874 \text{ kN}\]

\[LA := X + \frac{t_f}{2} \quad \text{Lever arm}\]

Moment capacity:

\[
M_{\text{Rd}} := R_{\text{c.web}} \cdot \frac{2}{3} \cdot X + R_{\text{c.flange}} \cdot LA + R_{\text{web}} \cdot \frac{2}{3} \left( h_w - X \right) + R_{\text{flange}} \left( h_w - X + \frac{t_f}{2} \right)
\]

\[M_{\text{Rd}} = 445.112 \text{ kN} \cdot \text{m}\]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[
\lambda_{LT} := \sqrt{\frac{M_{\text{Rd}}}{M_{\text{cr}}}} = 0.5211
\]

A.4.7
Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\(\frac{h}{b} > 2 \quad \text{Gives curve d}\)

\(\alpha_{LT} := 0.76\)

The factor \(\theta_{LT}\) is calculated according to section 6.3.2.3

\(\beta := 0.75 \quad \lambda_{LT.0} := 0.4\)

\(\phi_{LT} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT} - \lambda_{LT.0} \right) + \beta \lambda_{LT}^2 \right] = 0.648\)

The reduction factor \(\chi_{LT}\) is calculated according to equation 6.57

\[\chi_{LT} := \frac{1}{\phi_{LT} \pm \sqrt{\phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.899\]

The modified reduction factor \(\chi_{LT,\text{mod}}\) is calculated according to equation 6.58

\(\chi_{LT,\text{mod}} := \chi_{LT} = 0.899 < 1.0 \quad \text{ok!}\)

The design resistance moment is calculated according to equation 6.55

\[M_{b.Rd.LVL3} := \chi_{LT,\text{mod}} M_{Rd} = 400.057 \text{kN.m}\]

The design moment capacity according to RSM level 3 is 400.057 kNm
Proposed implementation

The proposed implementation allows stress redistribution between the weaker and stronger plate. A linear stress distribution is assumed by solving for a reduction factor $\rho$ for both the weaker and stronger plate.

$$\rho \cdot f_y$$

$$M_{Rd.\text{flange}} := b_t \cdot t_f \cdot f_y (h_w + t_f)$$

$$M_{Rd.\text{web}} := \frac{h_w \cdot t_w}{6} \cdot f_y$$

$$\rho_{\text{web}} := 1 \quad \rho_{\text{flange}} := \rho$$

$$\rho_{\text{proposed}} := \rho_{\text{web}} \cdot \frac{M_{Rd.\text{web}}}{M_{Rd.\text{flange}} + M_{Rd.\text{web}}} + \rho_{\text{flange}} \frac{M_{Rd.\text{flange}}}{M_{Rd.\text{flange}} + M_{Rd.\text{web}}} \approx 0.953$$

$$\chi_{\text{LT,proposed}} := \frac{W_{Y,el} \cdot \rho_{\text{proposed}} \cdot f_y}{M_{cr}} = 0.52$$

$$\Phi_{\text{LT,proposed}} := 0.5 \left[ 1 + \alpha_{\text{LT}} \cdot (\chi_{\text{LT,proposed}} - \chi_{\text{LT,0}}) + \beta \chi_{\text{LT,proposed}}^2 \right] = 0.647$$

The reduction factor $\chi_{\text{LT}}$ is calculated according to equation 6.57.

$$\chi_{\text{LT,proposed}} := \frac{1}{\Phi_{\text{LT,proposed}} + \sqrt{\Phi_{\text{LT,proposed}}^2 - \beta \cdot \chi_{\text{LT,proposed}}^2}} = 0.8995$$

The modified reduction factor $\chi_{\text{LT,mod}}$ is calculated according to equation 6.58.

$$\chi_{\text{LT,mod,proposed}} := \chi_{\text{LT,proposed}} = 0.8995 \leq 1.0 \text{ ok!}$$

A.4.9
The design resistance moment is calculated according to equation 6.55

\[ M_{b,Rd,\text{proposed}} = \chi_{LT,\text{mod,proposed}} \frac{W_{y,el,\text{proposed}} f_y}{\gamma_{M1}} = 398.987 \text{kN} \cdot \text{m} \]

The design moment capacity according to RSM proposed is 398.987 kNm
Reduced stress method level 1 based on FEM results

The critical load is obtained from the first local buckling mode of the web plate. The critical load for which the web plate buckles is denoted with $M_{cr,FEM}$.

$$M_{cr,FEM} := 701.17 \text{ kN}\cdot\text{m}$$

$$\lambda_{p,FEM} := \frac{f_y W_y,el}{\sqrt{M_{cr,FEM}}} = 0.815$$

$\lambda_{pR}$ is larger than 0.748: using equation 4.3 EN 1993-1-5 (outstand compression)

$$\rho_{FEM} := \frac{\lambda_{p,FEM} - 0.188}{\lambda_{p,FEM}^2} = 0.944 \quad < 1.0 \text{ ok!}$$

$$\lambda_{LT,R,FEM} := \frac{W_y,el \rho_{FEM} f_y}{\sqrt{M_{cr}}} = 0.5177$$

$$\phi_{LT,R,FEM} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT,R,FEM} - \lambda_{LT,0} \right) + \beta \lambda_{LT,R,FEM}^2 \right] = 0.645$$

The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$\chi_{LT,FEM} := \frac{1}{\phi_{LT,R,FEM} + \sqrt{\phi_{LT,R,FEM}^2 - \beta \lambda_{LT,R,FEM}^2}} = 0.901 \quad < 1.0 \text{ ok!}$$

The modified reduction factor $\chi_{LT,mod}$ is calculated according to equation 6.58

$$\chi_{LT,mod,R,FEM} := \chi_{LT,R,FEM} = 0.901$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd,LVL1,FEM} := \frac{W_y,el \rho_{FEM} f_y}{\gamma M1} = 396.127 \text{ kN}\cdot\text{m}$$

The design moment capacity according to RSM level 1 FEM is 396.127 kNm
Reduced stress method level 3 based on FEM results

The cross-sectional verification is performed as for the calculation of RSM level 1 and will therefore not be repeated here. Furthermore the reduction factor for the flange plate is calculated as for RSM level 1 based on FEM results.

\[ \rho_{\text{FEM}} = 0.944 \]

\[ \text{Force equilibrium:} \]

\[ \sigma_{1,\text{FEM}}(x) := \frac{x}{x + t_f} f_y \]

\[ \sigma_{2,\text{FEM}}(x) := \frac{h_w - x}{x + t_f} f_y \]

\[ \sigma_{t,\text{FEM}}(x) := \frac{(h_w - x) + t_f}{x + t_f} f_y \]

\[ \text{Tension}_{\text{FEM}}(x) := b_f t_f f_y \left( \frac{\sigma_{t,\text{FEM}}(x) + \sigma_{2,\text{FEM}}(x)}{2} \right) + \left( h_w - x \right) t_w \frac{\sigma_{2,\text{FEM}}(x)}{2} \]

\[ \text{Compression}_{\text{FEM}}(x) := b_f t_f \rho_{\text{FEM}} f_y + x t_w \frac{\sigma_{1,\text{FEM}}(x)}{2} \]

Given

\[ x = 1 \cdot \text{m} \]

\[ \text{Tension}_{\text{FEM}}(x) = \text{Compression}_{\text{FEM}}(x) \]

\[ X := \text{Find}(x) = 303.04 \cdot \text{mm} \]

\[ \text{Tension side:} \]

\[ \sigma_{\text{tension,FEM}} := \frac{(h_w - X) + t_f}{X + t_f} f_y = 348.055 \cdot \text{MPa} \]
Web contribution:

\[ \sigma_{2,FEM.} := \frac{h_w - X}{X + t_f} f_y = 339.224 \text{ MPa} \]

\[ R_{\text{web,FEM}} := \frac{(h_w - X) \cdot t_w \sigma_{2,FEM.}}{2} = 315.611 \text{ kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{\text{AVG,FEM}} := \frac{\sigma_{\text{tension,FEM}} + \sigma_{2,FEM.}}{2} = 343.64 \text{ MPa} \]

\[ R_{\text{flange,FEM}} := b_f t_f \sigma_{\text{AVG,FEM}} = 531.335 \text{ kN} \]

Compression side:

Web contribution:

\[ \sigma_{1.c,FEM} := \frac{X}{X + t_f} f_y = 346.169 \text{ MPa} \]

\[ R_{\text{c,web,FEM}} := \frac{X t_w \sigma_{1.c,FEM}}{2} = 328.666 \text{ kN} \]

Flange contribution:

\[ R_{\text{c,flange,FEM}} := b_f t_f \rho_{FEM} f_y = 518.28 \text{ kN} \]

\[ L_{\text{FEM}} := X + \frac{t_f}{2} \quad \text{Lever arm} \]

Moment capacity:

\[ M_{\text{Rd,FEM.1}} := R_{\text{c,web,FEM}} \frac{2}{3} X + R_{\text{c,flange,FEM}} L_{\text{FEM}} \]

\[ M_{\text{Rd,FEM.2}} := R_{\text{web,FEM}} \frac{2}{3} (h_w - X) + R_{\text{flange,FEM}} \left( h_w - X + \frac{t_f}{2} \right) \]

\[ M_{\text{Rd,FEM}} := M_{\text{Rd,FEM.1}} + M_{\text{Rd,FEM.2}} \]

\[ M_{\text{Rd,FEM}} = 447.784 \text{ kN} \cdot \text{m} \]

A.4.13
Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[ \lambda_{LT,FEM} := \frac{M_{Rd}}{M_{cr}} = 0.5211 \]

\[ \phi_{LT,FEM} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT,FEM} - \lambda_{LT,0} \right) + \beta \lambda_{LT,FEM}^2 \right] = 0.648 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT,FEM} := \frac{1}{\phi_{LT,FEM} + \sqrt{\phi_{LT,FEM}^2 - \beta \lambda_{LT,FEM}^2}} = 0.899 \]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[ \chi_{LT,mod,FEM} := \chi_{LT,FEM} = 0.899 \quad < 1.0 \text{ ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,Rd,LVL3,FEM} := \chi_{LT,mod,FEM} \frac{M_{Rd}}{\gamma_{M1}} = 400.057 \text{ kN m} \]

The design moment capacity according to RSM level 3 FEM is 400.057 kNm
Proposed implementation based on FEM result

\[ \rho_{\text{web.}} := 1 \quad \rho_{\text{flange.}} := \rho_{\text{FEM}} \]

\[ \rho_{\text{proposed.FEM}} := \rho_{\text{web.}} \frac{M_{\text{Rd.web}}}{M_{\text{Rd.flange}} + M_{\text{Rd.web}}} + \rho_{\text{flange.}} \frac{M_{\text{Rd.flange}}}{M_{\text{Rd.flange}} + M_{\text{Rd.web}}} = 0.96 \]

\[ \lambda_{\text{LT,proposed.FEM}} := \frac{W_{y,\text{el}} \rho_{\text{proposed.FEM}} f_y}{M_{\text{cr}}} = 0.522 \]

\[ \Phi_{\text{LT,proposed.FEM}} := 0.5 \left[ 1 + \alpha_{\text{LT}} \left( \lambda_{\text{LT,proposed.FEM}} - \lambda_{\text{LT},0} \right) + \beta_{\lambda} \lambda_{\text{LT,proposed.FEM}}^2 \right] = 0.649 \]

The reduction factor \( \chi_{\text{LT}} \) is calculated according to equation 6.57

\[ \chi_{\text{LT,proposed.FEM}} := \frac{1}{\Phi_{\text{LT,proposed.FEM}} + \sqrt{\Phi_{\text{LT,proposed.FEM}}^2 - \beta_{\lambda} \lambda_{\text{LT,proposed.FEM}}^2}} = 0.898 \]

The modified reduction factor \( \chi_{\text{LT,mod}} \) is calculated according to equation 6.58

\[ \chi_{\text{LT,mod,proposed.FEM}} := \chi_{\text{LT,proposed.FEM}} = 0.898 \quad < 1.0 \text{ ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{\text{b.Rd,proposed.FEM}} := \chi_{\text{LT,mod,proposed.FEM}} \frac{W_{y,\text{el}} \rho_{\text{proposed.FEM}} f_y}{\gamma_{M1}} = 401.237 \text{ kNm} \]

The design moment capacity according to RSM proposed FEM is 401.237 kNm
Appendix A.5
Lateral torsional buckling - point load - web class 4

An I-girder is subjected to two equal point loads at quarter point. The supports are of forked end boundary conditions. It should be noted that the girder is not restrained at the point loads. Furthermore it has been assumed that the local effects of the concentrated force does not contribute to the global stability of the girder. Therefore in the interaction between transverse force and axial force the effect of lateral torsional buckling does not have to be considered.

The following calculations have been made:
- Effective width method, considering patch load, shear buckling and lateral torsional buckling.
- Reduced stress method (RSM): According to level 1, level 3 and proposed implementation by obtaining the critical stress from hand-calculations and from linear FE-analysis.

Inputs:
- \( b_f := 200 \text{ mm} \)
- \( t_f := 9 \text{ mm} \)
- \( h_w := 600 \text{ mm} \)
- \( t_w := 3.964775 \text{ mm} \)
- \( L := 4 \text{ m} \)
- \( f_y := 355 \text{ MPa} \)
- \( \gamma_M1 := 1.0 \)
- \( E := 210 \text{ GPa} \)
- \( S_s := 100 \text{ mm} \)
- \( \gamma_M0 := 1.0 \)
- \( f_{yw} := f_y \)
- \( \eta := 1.2 \)
- \( v := 0.3 \)
- \( z := 304.5 \text{ mm} \)

Classification: weld sizes are omitted

Web:

\[
\varepsilon_w := \sqrt{\frac{235 \text{ MPa}}{f_y}} \times \frac{h_w}{t_w} = 151.333
\]

Limit for class 4 part subjected to bending

\( 124 \cdot \varepsilon = 100.888 < 120 \) The web belongs to class 4

Flange: outstand full compression

\[
\frac{b_f - t_w}{2} \times \frac{1}{t_f} = 10.891
\]
Limit for class 4

\[ 14 \varepsilon = 11.391 \quad > 10.83 \quad \text{The flange belongs to class 3} \]

Effective area: internal compression - table 4.1 EN 1993-1-5

\[ \psi := -1 \quad \text{Gives} \quad k_\sigma := 23.9 \]

Determine the reduction factor (\( \rho \))

\[ \lambda_p := \frac{h_w}{28.4 \varepsilon \sqrt{k_\sigma}} = 1.34 \]

Larger than 0.673 - using equation (4.2)

\[ \rho := \frac{\lambda_p - 0.055 \cdot (3 + \psi)}{\lambda_p^2} = 0.685 \quad \text{< 1.0 ok!} \]

\[ b_{\text{eff}} := \frac{h_w}{2} = 205.549 \text{mm} \quad \text{Effective length} \]

\[ b_{c1} := 0.4 \cdot b_{\text{eff}} = 0.082 \text{m} \]

\[ b_{c2} := 0.6 \cdot b_{\text{eff}} = 0.123 \text{m} \]

Calculation of a new centre of gravity (\( y_{GC,\text{red}} \))
Effective moment of inertia

\[
I_{y.1} := \frac{2 \cdot b_f \cdot t_f^3}{12} + b_f \cdot t_f \cdot y_{GC.red}^2 + b_f \cdot t_f \cdot (h_w + t_f - y_{GC.red})^2
\]

\[
I_{y.2} := b_c1 \cdot t_w \left( h_w + \frac{t_f}{2} - y_{GC.red} - \frac{b_c1}{2} \right)^2 + \left( b_c2 + \frac{h_w}{2} \right)^2 \cdot t_w \left( y_{GC.red} - \frac{t_f}{2} - \frac{b_c2 + h_w}{2} \right)
\]

\[
I_{y.3} := \frac{t_w \cdot b_c1^3}{12} + t_w \left( \frac{b_c2 + h_w}{2} \right)^3
\]

\[
I_{y.eff} := I_{y.1} + I_{y.2} + I_{y.3} = 3.933 \times 10^8 \cdot \text{mm}^4
\]

\[
W_{y.eff} := \frac{I_{y.eff}}{(h_w + t_f - y_{GC.red})} = 1.245 \times 10^6 \cdot \text{mm}^3
\]

\[
M_{cr} := 337.43 \times 10^3 \cdot \text{N} \cdot \text{m}
\]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[
\lambda_{LT} := \sqrt{\frac{W_{y.eff} f_y}{M_{cr}}} = 1.144
\]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

h/b > 2 \quad \text{Gives} \quad \text{curve d}

\[
\alpha_{LT} := 0.76
\]

The factor \( \alpha_{LT} \) is calculated according to section 6.3.2.3

\[
\beta := 0.75 \quad \lambda_{LT.0} := 0.4
\]

\[
\phi_{LT} := 0.5 \left[ 1 + \alpha_{LT} (\lambda_{LT} - \lambda_{LT.0}) + \beta \cdot \lambda_{LT}^2 \right] = 1.274
\]
The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$\chi_{LT} := \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.482$$

The correction factor $\kappa_c$ is obtained according to table 6.6

$$\kappa_c := 1.0$$

The actual moment distribution is not covered in table 6.6. Therefore the more conservative value, corresponding to uniform moment distribution along the entire length of the beam, is used.

The factor $f$ is calculated according to section 6.3.2.3

$$f := 1 - 0.5 \cdot (1 - \kappa_c) \left[ 1 - 2 \cdot (\lambda_{LT} - 0.8)^2 \right] = 1$$

The modified reduction factor $\chi_{LT,mod}$ is calculated according to equation 6.58

$$\chi_{LT,mod} := \frac{\chi_{LT}}{f} = 0.482 \quad \leq 1.0 \text{ ok!}$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd} := \frac{\chi_{LT,mod} \cdot W_{y,eff} \cdot f_y}{\gamma M_1} = 213.033 \cdot \text{kN} \cdot \text{m}$$

$$P_{Rd} := \frac{M_{b,Rd}}{\frac{L}{4}} = 213.033 \cdot \text{kN}$$ Design resistance with respect to lateral torsional buckling

**Patch loading**

Effective load length according to section 6.5 EN 1993-1-5

$$m_1 := \frac{f_y \cdot b_f}{f_{yw} \cdot t_w} = 50.444 \quad \text{Equation 6.8}$$

$$m_2 := 0.02 \cdot \left( \frac{h_w}{t_f} \right)^2 = 88.889 \quad \text{Assuming } \lambda_{f} > 0.5 \text{ using equation 6.9}$$

Load application type (a), see the figure below from figure 6.1 EN 1993-1-5
\[ k_F := 6 + 2 \cdot \left( \frac{h_w}{a} \right)^2 = 6.05 \quad \text{Type (a) according to figure 6.1} \]

\[ l_y := S_s + 2 \cdot k_F \left[ 1 + \sqrt{m_1 + m_2} \right] = 330.471 \cdot \text{mm} \quad \text{Equation 6.10} \]

\[ F_{cr} := 0.9 \cdot k_F \cdot E \cdot \frac{t_w^3}{h_w} = 118.771 \cdot \text{kN} \quad \text{Equation 6.5} \]

\[ \lambda_F := \frac{l_y \cdot t_w \cdot f_{yw}}{F_{cr}} = 1.979 \quad \text{Equation 6.4} \]

\[ \chi_F := \frac{0.5}{\lambda_F} = 0.253 \quad < 1.0 \quad \text{ok!} \]

\[ L_{eff} := \chi_F \cdot l_y = 83.497 \cdot \text{mm} \quad \text{Equation 6.2} \]

\[ F_{Rd} := \frac{f_{yw} \cdot L_{eff} \cdot t_w}{\gamma_{M1}} = 117.521 \cdot \text{kN} \quad \text{Design buckling resistance under transversal forces} \]

**Shear resistance**

No stiffeners gives according to chapter 5.1 (2), EN 1993-1-5: 2006

\[ \frac{h_w}{t_w} = 151.333 > \frac{72 \cdot \varepsilon}{\eta} = 48.817 \quad \text{Shear buckling needs to be checked} \]

Web with transverse stiffeners at supports. The slenderness parameter \( \lambda_w \) is calculated according to equation 5.5 EN 1993-1-5

\[ \lambda_w := \frac{h_w}{86.4 \cdot t_w \cdot \varepsilon} = 2.153 \]

Non-rigid end post, the factor \( \chi_w \) is taken from table 5.1, where \( \lambda_w > 1.08 \)

\[ \chi_w := \frac{0.83}{\lambda_w} = 0.386 \]

We have no contribution from the flanges since there are no transverse

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stiffeners except at the supports.

The design shear resistance is taken according equation 5.2

\[
V_{bw,Rd} := \frac{X_w f_y h_w t_w}{\gamma M1 \cdot \sqrt{3}} = 187.982 \cdot kN
\]

**Interaction:**

Interaction between transverse force and bending moment

\[
M_{Ed}(P_{Ed}) := P_{Ed} \cdot \frac{L}{4}
\]

\[
\eta_1(P_{Ed}) := \frac{M_{Ed}(P_{Ed})}{W_{y,eff} f_y}
\]

\[
\eta_2(P_{Ed}) := \frac{P_{Ed}}{F_{Rd}}
\]

Given

\[P_{Ed} := 1 \cdot kN\]

\[
\eta_2(P_{Ed}) + 0.8 \cdot \eta_1(P_{Ed}) = 1.4
\]

\[P_{Ed.1} := \text{Find}(P_{Ed}) = 135.669 \cdot kN\]

Interaction between shear force and bending moment

\[M_{f,Rd} := (h_w + t_f) \cdot t_f \cdot b_f \cdot f_y = 389.151 \cdot kN \cdot m\]

\[M_{pl,Rd} := M_{f,Rd} + \left(\frac{h_w}{2}\right)^2 \cdot t_w \cdot f_y = 515.826 \cdot kN \cdot m\]

\[M_{Ed.(P_{Ed.2})} := P_{Ed.2} \cdot \frac{L}{4}\]

\[
\eta_1.(P_{Ed.2}) := \frac{M_{Ed.(P_{Ed.2})}}{W_{y,eff} f_y}
\]

\[
\eta_3(P_{Ed.2}) := \frac{P_{Ed.2}}{V_{bw,Rd}}
\]

\[P_{Ed.2} := 174.577 \cdot kN\]
\[ \eta_1(P_{Ed.2}) + \left(1 - \frac{M_{f,Rd}}{M_{pl,Rd}}\right) \cdot (2 \cdot \eta_3(P_{Ed.2}) - 1)^2 = 0.576 \]

**Answer:**

\[ P_{Ed} = 117.521 \text{ kN} \]

The decisive load obtained from the interaction between transverse force and bending moment

**Reduced stress method level 1**

A cross sectional verification is performed according to the verification format in section 10 in EN-1993-1-5. Furthermore a verification considering lateral torsional buckling is also performed considering only axial stresses.

**Cross sectional verification:**

**Design stresses:**

\[ I_{y,gross} = 2 \left[ \frac{b_f t_f^3}{12} + b_f t_f \left( \frac{h_w}{2} + \frac{t_f}{2} \right)^2 \right] + \frac{t_w h_w^3}{12} = 4.052 \times 10^8 \text{ mm}^4 \]

\[ W_{el} \text{ is determined at a point located at the junction between the web and the flange.} \]

\[ V_{Ed} := 49.80 \text{ kN} \]

\[ M_{Ed,R} := V_{Ed} \frac{L}{4} \]

\[ A_{web} := h_w \cdot t_w \]

\[ W_{el} := \frac{I_{y,gross}}{h_w} \]

\[ \tau_{Ed} := \frac{V_{Ed}}{A_{web}} = 20.934 \cdot \text{MPa} \]

\[ \sigma_{Ed,x} := \frac{M_{Ed,R}}{W_{el}} = 36.872 \cdot \text{MPa} \]

\[ \sigma_{Ed,z} := \frac{V_{Ed}}{\chi F \cdot I_{y} \cdot t_w} = 150.433 \cdot \text{MPa} \]

**Critical stresses:**

\[ k_T := 5.34 + 4 \left( \frac{h_w}{a} \right)^2 = 5.44 \quad \text{EN 1993-1-5 equation (A.5)} \]

\[ \sigma_E := \frac{\pi^2 \cdot E \cdot t_w^2}{12 \left(1 - \nu^2\right) h_w^2} = 8.288 \cdot \text{MPa} \]

\[ \sigma_{cr,x} := k_T \cdot \sigma_E = 198.075 \cdot \text{MPa} \]

\[ \tau_{cr,T} := k_T \cdot \sigma_E = 45.082 \cdot \text{MPa} \]
\[ \sigma_{cr.c.z} := 1.881 \cdot \sigma_E = 15.589 \text{MPa} \]

\[ \sigma_{cr.p.z} := 124.91 \text{MPa} \quad \text{Obtained from EB-plate} \]

The stress from EB-plate was obtained under the assumption that the two point loads were separated enough to not influence each other. This was a problem since only one concentrated force could be applied at the time.

Due to the fact that the concentrated load is only applied to a certain part of the flange, plate like buckling will prevail.

\[ \frac{\sigma_{cr.p.z}}{\sigma_{cr.c.z}} = 8.013 \gg 2.0 \]

Thus:

\[ \sigma_{cr.z} := 124.91 \text{MPa} \]

Load amplification factors:

\[ \alpha_{cr.x} := \frac{\sigma_{cr.x}}{\sigma_{Ed.x}} = 5.372 \]

\[ \alpha_{cr.z} := \frac{\sigma_{cr.z}}{\sigma_{Ed.z}} = 0.83 \]

\[ \alpha_{cr.\tau} := \frac{\tau_{cr.\tau}}{\tau_{Ed}} = 2.154 \]

\[ \psi_x := -1.0 \quad \psi_z := 1.0 \]

\[ \alpha_{cr} := \left[ \frac{1 + \psi_x}{4 \cdot \alpha_{cr,x}} + \frac{1 + \psi_z}{4 \cdot \alpha_{cr,z}} + \left( \frac{1 + \psi_x}{4 \cdot \alpha_{cr,x}} + \frac{1 + \psi_z}{4 \cdot \alpha_{cr.z}} \right)^2 + \frac{1 - \psi_x}{2 \cdot \alpha_{cr.x}} + \frac{1 - \psi_z}{2 \cdot \alpha_{cr.z}} + \frac{1}{\alpha_{cr.\tau}} \right]^{1/2} \]

\[ \alpha_{cr} = 0.722 \]

\[ \alpha_{ult.k} := \sqrt{\left( \frac{\sigma_{Ed.x}}{f_y} \right)^2 + \left( \frac{\sigma_{Ed.z}}{f_y} \right)^2 - \left( \frac{\sigma_{Ed.x}}{f_y} \right) \left( \frac{\sigma_{Ed.z}}{f_y} \right) + 3 \cdot \left( \frac{\tau_{Ed}}{f_y} \right)^2} \]

\[ \alpha_{ult.k} = 2.526 \]

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\[ \lambda_{p.R} := \frac{\alpha_{\text{ult.k}}}{\alpha_{\text{cr}}} = 1.87 \quad \text{Slenderness parameter} \]

Reduction factors:

\[ \rho_x := \frac{\lambda_{p.R} - 0.055 \cdot (3 + \psi_x)}{\lambda_{p,R}^2} = 0.503 \quad < 1.0 \text{ ok!} \]

From table B.1 EN 1993-1-5

\[ \lambda_{p,0} := 0.8 \quad \alpha_p := 0.34 \]

\[ \phi_p := 0.5 \left[ 1 + \alpha_p \left( \lambda_{p,R} - \lambda_{p,0} \right) + \lambda_{p,R} \right] = 1.617 \]

\[ \rho_z := \frac{1}{\phi_p + \sqrt{\phi_p^2 - \lambda_{p,R}}} = 0.403 \]

Verification:

\[ \left( \frac{\sigma_{\text{Ed},x}}{\rho_x f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed},z}}{\rho_z f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed},x}}{\rho_x f_y} \right) \left( \frac{\sigma_{\text{Ed},z}}{\rho_z f_y} \right) + 3 \left( \frac{\tau_{\text{Ed}}}{X_w f_y} \right)^2 = 1 \]

The cross sectional bearing capacity is \( P_{rd} = 49.80 \text{ kN} \).

**Lateral torsional buckling:**

The bearing capacity of the I-girder is the lowest value of obtained from either the cross sectional verification or the verification considering lateral torsional buckling. The interaction between the different stresses are not needed in this verification since the transverse stresses are not assumed to influence the bearing capacity of the I-girder with respect to lateral torsional buckling.

\[ \lambda_p := \frac{f_y}{\sigma_{\text{cr},x}} = 1.339 \]

\( \lambda_p \) is larger than 0.673: using equation 4.2 EN 1993-1-5 (internal compression)

\[ \rho := \frac{\lambda_p - 0.055 \cdot (3 + \psi)}{\lambda_p^2} = 0.686 \quad < 1.0 \text{ ok!} \]

\[ I_{y,\text{gross.}} := 2 \left[ \frac{b f t_f^3}{12} + b f t_f \left( \frac{h_w}{2} + \frac{t_f}{2} \right)^2 \right] + \frac{t_w h_w^3}{12} = 4.052 \times 10^8 \text{ mm}^4 \]
Non-dimensional slenderness with a reduction factor \( \rho \) included in order to achieve the same principle as the effective width method.

\[
\lambda_{LT} := \sqrt{\frac{W_{y,el} \rho f_y}{M_{cr}}} = 0.9797
\]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\( h/b > 2 \) gives curve d

\( \alpha_{LT} := 0.76 \)

The factor \( \theta_{LT} \) is calculated according to section 6.3.2.3

\[
\beta := 0.75 \quad \lambda_{LT,0} := 0.4
\]

\[
\phi_{LT} := 0.5 \left[ 1 + \alpha_{LT} \cdot \left( \lambda_{LT} - \lambda_{LT,0} \right) + \beta \lambda_{LT}^2 \right]
\]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[
\chi_{LT} := \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^2 - \beta \lambda_{LT}^2}} = 0.572
\]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[
\chi_{LT,mod} := \chi_{LT} = 0.572 < 1.0 \text{ ok!}
\]

The design resistance moment is calculated according to equation 6.55

\[
M_{b,Rd.LVL1} := \chi_{LT,mod} \cdot \frac{W_{y,el} \rho f_y}{\gamma M1} = 185.077 \text{ kN} \cdot \text{m}
\]

A moment capacity of 185.077 kNm is equivalent to \( \text{Prd} = 185.077 \text{kN} \). Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM level 1 is 49.8 kN.
Reduced stress method level 3

The stresses, and therefore also the reduction, is calculated for a point located at the junction between the web and the flange plate.

\[ V_{Ed.} := 48.20 \text{kN} \]
\[ M_{Ed.R.} := V_{Ed.} \cdot \frac{L}{4} \]
\[ \tau_{Ed.} := \frac{V_{Ed.}}{A_{web}} \]
\[ \sigma_{Ed.z} := \frac{V_{Ed.}}{x_{F.r} \cdot l \cdot t_{w}} \]
\[ \sigma_{Ed.x} := \frac{M_{Ed.R.}}{W_{el}} \]

Load amplification factors:
\[ \alpha_{cr.x} := \frac{\sigma_{cr.x}}{\sigma_{Ed.x}} = 5.55 \]
\[ \alpha_{cr.z} := \frac{\sigma_{cr.z}}{\sigma_{Ed.z}} = 0.858 \]
\[ \alpha_{cr.\tau} := \frac{\tau_{cr.\tau}}{\tau_{Ed}} = 2.225 \]

\[ \alpha_{cr} := \left[ \frac{1 + \psi_{x}}{4 \cdot \alpha_{cr.x}} + \frac{1 + \psi_{z}}{4 \cdot \alpha_{cr.z}} + \left( \frac{1 + \psi_{x}}{4 \cdot \alpha_{cr.x}} + \frac{1 + \psi_{z}}{4 \cdot \alpha_{cr.z}} \right)^{2} + \frac{1 - \psi_{x}}{2 \cdot \alpha_{cr.x}} + \frac{1 - \psi_{z}}{2 \cdot \alpha_{cr.z}} + \frac{1}{2} \right]^{-1} \]
\[ \alpha_{cr} = 0.746 \]

\[ \alpha_{ult.k} := \sqrt{\left( \frac{\sigma_{Ed.x}}{f_{y}} \right)^{2} + \left( \frac{\sigma_{Ed.z}}{f_{y}} \right)^{2} - \left( \frac{\sigma_{Ed.x}}{f_{y}} \right) \cdot \left( \frac{\sigma_{Ed.z}}{f_{y}} \right) + 3 \cdot \left( \frac{\tau_{Ed}}{f_{y}} \right)^{2}} \]
\[ \alpha_{ult.k} = 2.609 \]

\[ \lambda_{p.R.} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} = 1.87 \quad \text{Slenderness parameter} \]

Reduction factors:
\[ \rho_{x} := \frac{\lambda_{p.R.} - 0.055 \cdot (3 + \psi_{x})}{\lambda_{p.R.}^{2}} = 0.503 \quad < 1.0 \text{ ok!} \]
\( \phi_p := 0.5 \left[ 1 + \alpha_p \left( \lambda_{p.R} - \lambda_{p.0} \right) + \lambda_{p.R} \right] = 1.617 \)

\( \rho_Z := \frac{1}{\phi_p + \sqrt{\phi_p^2 - \lambda_{p.R}}} = 0.403 \)

The calculation with RSM is performed according to the theory of level 3, more information are presented in the report.

\[
\begin{align*}
\sigma_1(x) &:= \rho_x \cdot f_y \\
\sigma_2(x) &:= \frac{h_w - x}{t_f} \cdot f_y \\
\sigma_4(x) &:= \frac{(h_w - x) + t_f}{x + \frac{t_f}{2}} \cdot f_y \\
\text{Tension}(x) &:= b_f t_f \left( \frac{\sigma_1(x) + \sigma_2(x)}{2} \right) + (h_w - x) t_w \cdot \frac{\sigma_2(x)}{2}
\end{align*}
\]

Given

\( x := 1 \cdot m \)

\( \text{Compression}(x) := b_f t_f f_y + x \cdot t_w \cdot \frac{\sigma_1(x)}{2} \)

\( \text{Tension}(x) = \text{Compression}(x) \)

\( X := \text{Find}(x) = 316.235 \cdot \text{mm} \)
Tension side:

\[ \sigma_{\text{tension}} := \frac{(h_w - X) + t_f}{X + \frac{t_f}{2}} \cdot f_y = 324.043 \cdot \text{MPa} \]

Web contribution:

\[ \sigma_2 := \frac{h_w - X}{X + \frac{t_f}{2}} \cdot f_y = 314.081 \cdot \text{MPa} \]

\[ R_{\text{web}} := \frac{(h_w - X) \cdot t_w \cdot \sigma_2}{2} = 176.681 \cdot \text{kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{\text{AVG}} := \frac{\sigma_{\text{tension}} + \sigma_2}{2} = 319.062 \cdot \text{MPa} \]

\[ R_{\text{flange}} := b_f \cdot t_f \cdot \sigma_{\text{AVG}} = 574.312 \cdot \text{kN} \]

Compression side:

Web contribution:

\[ \sigma_{1,c} := \rho_x \cdot f_y = 178.646 \cdot \text{MPa} \]

\[ R_{\text{c.web}} := \frac{X \cdot t_w \cdot \sigma_{1,c}}{2} = 111.993 \cdot \text{kN} \]

Flange contribution:

\[ R_{\text{c.flange}} := b_f \cdot t_f \cdot f_y = 639 \cdot \text{kN} \]

\[ \text{LA} := X + \frac{t_f}{2} \quad \text{Lever arm} \]

Moment capacity:

\[ M_{\text{Rd}} := R_{\text{c.web}} \cdot \frac{2}{3} \cdot X + R_{\text{c.flange}} \cdot \text{LA} + R_{\text{web}} \cdot \frac{2}{3} \cdot (h_w - X) + R_{\text{flange}} \cdot \left( h_w - X + \frac{t_f}{2} \right) \]

\[ M_{\text{Rd}} = 427.538 \cdot \text{kN} \cdot \text{m} \]
Reduction factor in the longitudinal direction

\[ W_{el.} := \frac{t_y \cdot \text{guss}}{h_w + t_f} \]

\[ \rho_{x,3} := \frac{M_{Rd}}{f_y \cdot W_{el.}} = 0.905 \]

\( \rho_{x,3} \) is a reduction factor, used for the entire cross-section, that is equivalent to a level 3 calculation.

Verification:

\[ \left( \frac{\sigma_{\text{Ed}.x.}}{\rho_{x,3} \cdot f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed}.z.}}{\rho_{x,3} \cdot f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed}.x.}}{\rho_{x,3} \cdot f_y} \right) \left( \frac{\sigma_{\text{Ed}.z.}}{\rho_{x,3} \cdot f_y} \right) + 3 \left( \frac{\tau_{\text{Ed.}}}{x_w \cdot f_y} \right)^2 = 1 \]

The cross sectional bearing capacity is \( P_{rd} = 48.20 \text{ kN} \).

Lateral torsional buckling:

When determining the bearing capacity with respect to LTB the reduction factor will only be calculated for the longitudinal stress component. This reduction factor was already calculated for RSM level 1 LTB. The moment bearing capacity \( M_{Rd} \) is calculated with the same procedure as above, but with a reduction factor only considering the longitudinal stress component.

\( \rho = 0.686 \)  

Same as before

Force equilibrium:

\[ \sigma_{1.LVL3}(x) := \rho \cdot f_y \]

\[ \sigma_{2.LVL3}(x) := \frac{h_w - x}{x + \frac{t_f}{2}} \cdot f_y \]

\[ \sigma_{\tau.LVL3}(x) := \frac{h_w - x + t_f}{x + \frac{t_f}{2}} \cdot f_y \]

Tension\(_{\text{LVL3}}(x) := b_f \cdot t_f \left( \frac{\sigma_{\tau.LVL3}(x) + \sigma_{2.LVL3}(x)}{2} \right) + \left( h_w - x \right) \cdot t_w \cdot \frac{\sigma_{2.LVL3}(x)}{2} \]

Given

\[ x_w := 1 \cdot m \]

Compression\(_{\text{LVL3}}(x) := b_f \cdot t_f \cdot f_y + x \cdot t_w \cdot \frac{\sigma_{1.LVL3}(x)}{2} \]

Tension\(_{\text{LVL3}}(x) = \text{Compression\(_{\text{LVL3}}(x) \)}

\[ X := \text{Find}(x) = 309.695 \cdot \text{mm} \]
Tension side:

\[ \sigma_{\text{tension.LVL3}} := \frac{(h_w - X) + t_f}{t_f} \cdot f_y = 338.175 \cdot \text{MPa} \]

Web contribution:

\[ \sigma_{2.\text{LVL3}.} := \frac{h_w - X}{t_f} \cdot f_y = 328.006 \cdot \text{MPa} \]

\[ R_{\text{web.LVL3}} := \frac{(h_w - X) \cdot t_w \cdot \sigma_{2.\text{LVL3}.}}{2} = 188.766 \cdot \text{kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{\text{AVG.LVL3}} := \frac{\sigma_{\text{tension.LVL3}} + \sigma_{2.\text{LVL3}.}}{2} = 333.091 \cdot \text{MPa} \]

\[ R_{\text{flange.LVL3}} := b_f \cdot t_f \cdot \sigma_{\text{AVG.LVL3}} = 599.563 \cdot \text{kN} \]

Compression side:

Web contribution:

\[ \sigma_{1.c.\text{LVL3}} := \rho \cdot f_y = 243.233 \cdot \text{MPa} \]

\[ R_{\text{c.web.LVL3}} := \frac{X \cdot t_w \cdot \sigma_{1.c.\text{LVL3}}}{2} = 149.33 \cdot \text{kN} \]

Flange contribution:

\[ R_{\text{c.flange.LVL3}} := b_f \cdot t_f \cdot f_y = 639 \cdot \text{kN} \]

\[ LA_{\text{LVL3}} := X + \frac{t_f}{2} \quad \text{Lever arm} \]

Moment capacity:

\[ M_{\text{Rd.LVL3.1}} := R_{\text{c.web.LVL3}} \cdot \frac{2}{3} \cdot X + R_{\text{c.flange.LVL3}} \cdot LA_{\text{LVL3}} \]

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\[ M_{\text{Rd.LVL3.2}} := R_{\text{web.LVL3}} \cdot \frac{2}{3} \left( h_w - X \right) + R_{\text{flange.LVL3}} \left( h_w - X + \frac{t_f}{2} \right) \]

\[ M_{\text{Rd.LVL3}} := M_{\text{Rd.LVL3.1}} + M_{\text{Rd.LVL3.2}} = 444.889 \text{ kN-m} \]

\[ W_{y,el} = 1.331 \times 10^6 \text{ mm}^3 \text{ \quad Same as before} \]

Non-dimensional slenderness with a reduction factor \( \rho \) included in order to achieve the same principle as the effective width method.

\[ \lambda_{\text{LT.LVL3}} := \sqrt{\frac{M_{\text{Rd}}}{{M_{\text{cr}}}}} = 1.1256 \]

\[ \phi_{\text{LT.LVL3}} := 0.5 \left[ 1 + \alpha_{\text{LT}} \left( \lambda_{\text{LT.LVL3}} - \lambda_{\text{LT.0}} \right) + \beta \lambda_{\text{LT.LVL3}}^2 \right] \]

The reduction factor \( \chi_{\text{LT}} \) is calculated according to equation 6.57

\[ \chi_{\text{LT.LVL3}} := \frac{1}{\phi_{\text{LT.LVL3}} + \sqrt{\phi_{\text{LT.LVL3}}^2 - \beta \lambda_{\text{LT.LVL3}}^2}} = 0.491 \]

The modified reduction factor \( \chi_{\text{LT,mod}} \) is calculated according to equation 6.58

\[ \chi_{\text{LT.mod.LVL3}} := \chi_{\text{LT.LVL3}} = 0.491 < 1.0 \text{ ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,\text{Rd.LVL3}} := \chi_{\text{LT.mod.LVL3}} \cdot \frac{M_{\text{Rd.LVL3}}}{\gamma_{M1}} = 218.648 \text{ kN-m} \]

A moment capacity of 218.648 kNm is equivalent to \( \text{Prd} = 218.648 \text{ kN} \). Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM level 3 is 48.2 kN.
Proposed implementation

The proposed implementation allows stress redistribution between the weaker and stronger plate. A linear stress distribution is assumed by solving for a reduction factor $\rho$ for both the weaker and stronger plate.

Cross sectional verification:

The proposed implementation does only allow load shedding between the weaker and stronger plate for axial stresses. For the transverse stresses the flanges are assumed to have no influence in the load bearing capacity. However, for the cross sectional verification the full stress field must be considered, even in the determination of the reduction factor in the axial direction.

\[
V_{\text{proposed}} := 48.2 \text{ kN} \quad M_{\text{proposed}} := V_{\text{proposed}} \cdot \frac{L}{4}
\]

\[
\tau_{\text{Ed.p}} := \frac{V_{\text{proposed}}}{A_{\text{web}}} = 20.262 \text{ MPa} \quad \sigma_{\text{Ed.x.p}} := \frac{M_{\text{proposed}}}{W_{\text{el}}} = 35.688 \text{ MPa}
\]

\[
\sigma_{\text{Ed.z.p}} := \frac{V_{\text{proposed}}}{X_F \cdot t_y \cdot t_w} = 145.599 \text{ MPa}
\]

Critical stresses are the same as before.

Load amplification factors:

\[
\alpha_{\text{cr.x.p}} := \frac{\sigma_{\text{cr.x}}}{\sigma_{\text{Ed.x.p}}} = 5.55 \quad \alpha_{\text{cr.z.p}} := \frac{\sigma_{\text{cr.z}}}{\sigma_{\text{Ed.z.p}}} = 0.858 \quad \alpha_{\text{cr.} \tau} := \frac{\tau_{\text{cr.} \tau}}{\tau_{\text{Ed.p}}} = 2.225
\]

\[
a_1 := \frac{1 + \psi_x}{4 \cdot \alpha_{\text{cr.x.p}}} \quad a_2 := \frac{1 + \psi_z}{4 \cdot \alpha_{\text{cr.z.p}}}
\]

A.5.17
\[
\alpha_{\text{cr.p}} := \left[ a_1 + a_2 + \left( \frac{a_1 + a_2}{2} \right)^2 + \frac{1 - \psi_x}{2 \cdot \alpha_{\text{cr.x.p}}} + \frac{1 - \psi_z}{2 \cdot \alpha_{\text{cr.z.p}}} + \frac{1}{\alpha_{\text{cr.\tau.p}}} \right]^2 = 0.746
\]

\[
\alpha_{\text{ult.k.p}} := \sqrt{\left[ \left( \frac{\sigma_{\text{Ed.x.p}}}{f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.p}}}{f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.p}}}{f_y} \right) \left( \frac{\sigma_{\text{Ed.z.p}}}{f_y} \right) + 3 \left( \frac{\tau_{\text{Ed.p}}}{f_y} \right)^2 \right]} = 1.167
\]

\[
\lambda_{p.p} := \sqrt{\frac{\alpha_{\text{ult.k.p}}}{\alpha_{\text{cr.p}}}} = 1.87 \quad \text{Slenderness parameter}
\]

\[
\rho_{x.web} := \rho_{x.p} - 0.055 \cdot (3 + \psi_x) = 0.503 < 1.0 \text{ ok!}
\]

\[
\phi_{p.p} := 0.5 \left[ 1 + \alpha_p \left( \lambda_{p,R} - \lambda_{p,0} \right) + \lambda_{p,R} \right] = 1.617
\]

\[
\rho_{z.p} := \frac{1}{\phi_{p.p} + \sqrt{\phi_{p.p} - \lambda_{p.p}}} = 0.403 \quad \rho_{\text{flange}} := 1
\]

\[
M_{\text{Rd.flange}} := f_y \cdot b_t \cdot t_f \cdot \left( h_w + t_f \right) = 389.151 \cdot \text{kN}\cdot\text{m}
\]

\[
M_{\text{Rd.web}} := \frac{h_w \cdot h_w^3}{12} \cdot \frac{f_y}{h_w} = 84.45 \cdot \text{kN}\cdot\text{m}
\]

\[
\rho_{x.p} := \rho_{x.web} \cdot \frac{M_{\text{Rd.web}}}{M_{\text{Rd.web}} + M_{\text{Rd.flange}}} + \rho_{\text{flange}} \cdot \frac{M_{\text{Rd.flange}}}{M_{\text{Rd.web}} + M_{\text{Rd.flange}}} = 0.911
\]

**Verification:**

\[
\frac{\left( \frac{\sigma_{\text{Ed.x.p}}}{\rho_{x,p} \cdot f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.p}}}{\rho_{z,p} \cdot f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.p}}}{\rho_{x,p} \cdot f_y} \right) \left( \frac{\sigma_{\text{Ed.z.p}}}{\rho_{z,p} \cdot f_y} \right) + 3 \left( \frac{\tau_{\text{Ed.p}}}{f_y} \right)^2}{\left( \frac{\psi_{x,p}}{\rho_{x,p} \cdot f_y} \right)^2} = 1
\]

The cross sectional bearing capacity is \(\text{Prd} = 48.20 \text{ kN}\).
Lateral torsional buckling:

\[ \rho = 0.685 \quad \text{Same as level 1} \]

\[ \rho_{\text{imp}} := \rho \cdot \frac{M_{\text{Rd.web}}}{M_{\text{Rd.web}} + M_{\text{Rd.flange}}} + \rho_{\text{flange}} \cdot \frac{M_{\text{Rd.flange}}}{M_{\text{Rd.web}} + M_{\text{Rd.flange}}} = 0.944 \]

Non-dimensional slenderness with a reduction factor \( \rho \) included in order to achieve the same principle as the effective width method.

\[ \lambda_{LT.p} := \sqrt{\frac{W_{y, el} \rho_{\text{imp}} \cdot f_y}{M_{cr}}} = 1.1495 \]

\[ \phi_{LT.p} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT.p} - \lambda_{LT,0} \right) + \beta \cdot \lambda_{LT.p}^2 \right] \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.p} := \frac{1}{\phi_{LT.p} + \sqrt{\phi_{LT.p}^2 - \beta \cdot \lambda_{LT.p}^2}} = 0.479 \]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[ \chi_{LT,mod.p} := \chi_{LT.p} = 0.479 \quad \leq 1.0 \text{ ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b, Rd, proposed} := \chi_{LT,mod.p} \cdot \frac{W_{y, el} \rho_{\text{imp}} \cdot f_y}{\gamma M1} = 213.807 \text{ kN.m} \]

A moment capacity of 213.807 kNm is equivalent to \( Prd = 213.807 \) kN. Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM\textsubscript{proposed} is 48.20 kN.

Reduced stress method level 1 based on FEM results

The critical buckling load \( P_{cr} \) is obtained from the buckling analysis and it correspond to the first local buckling mode. The bearing capacity of the I-girder is the lowest value of obtained from either the cross sectional verification or the verification considering lateral torsional buckling. The interaction between the different stresses are not needed in this verification since the transverse stresses are not assumed to influence the bearing capacity of the I-girder with respect to lateral Cross sectional verification:

\[ P_{\text{cr}} := 82.915 \text{ kN} \]

\[ P_{Ed, \text{FEM}} := 73.179 \text{ kN} \]
\[ M_{Ed.FEM} := \frac{P_{Ed.FEM}}{4} \]

\[ \alpha_{cr.FEM} := \frac{P_{cr}}{P_{Ed.FEM}} \]

\[ \sigma_{Rk.1} := \frac{M_{Ed.FEM}}{W_{cl}} \]

\[ \sigma_{Rk.2} := \frac{P_{Ed.FEM}}{\chi F^{-1} y_{tw}} \]

\[ \tau_{Rk.3} := \frac{P_{Ed.FEM}}{h_{w} t_{w}} \]

\[ \alpha_{ult.FEM} := \sqrt{\left[ \left( \frac{\sigma_{Rk.1}}{f_{y}} \right)^2 + \left( \frac{\sigma_{Rk.2}}{f_{y}} \right)^2 - \left( \frac{\sigma_{Rk.1}}{f_{y}} \right) \left( \frac{\sigma_{Rk.2}}{f_{y}} \right) + 3 \cdot \left( \frac{\tau_{Rk.3}}{f_{y}} \right)^2 \right]^{-1}} = 1.719 \]

\[ \lambda_{p.FEM} := \sqrt{\frac{\alpha_{ult.FEM}}{\alpha_{cr.FEM}}} = 1.232 \]

Reduction factors:

\[ \rho_{x.FEM} := \frac{\lambda_{p.FEM} - 0.055 \left( 3 + \psi_{x} \right)}{\lambda_{p.FEM}^2} = 0.739 \quad < 1.0 \text{ ok!} \]

\[ \phi_{p.FEM} := 0.5 \left[ 1 + \alpha_{p} \left( \lambda_{p.FEM} - \lambda_{p.0} \right) + \lambda_{p.FEM} \right] \]

\[ \rho_{z.FEM} := \frac{1}{\phi_{p.FEM} + \sqrt{\phi_{p.FEM}^2 - \lambda_{p.FEM}}} = 0.619 \]

Verification:

\[ \left( \frac{M_{Ed.FEM}}{W_{cl}} \right)^2 + \left( \frac{P_{Ed.FEM}}{\chi F^{-1} y_{tw}} \right)^2 - \left( \frac{M_{Ed.FEM}}{W_{cl}} \right) \cdot \left( \frac{P_{Ed.FEM}}{\chi F^{-1} y_{tw}} \right) + 3 \cdot \left( \frac{P_{Ed.FEM}}{h_{w} t_{w}} \right)^2 = 1 \]

Lateral torsional buckling:

The load amplification factor, \( \alpha_{cr} \), is obtained from FEM and will inevitably include the entire stress field and not just the axial stresses. For this reason the entire stress field has to be considered when calculating the load amplification factor, \( \alpha_{ult,k} \), otherwise an overconservative value of the slenderness will be obtained.

\[ P_{Ed.FEM.1} := 192.05 \text{kN} \]

\[ M_{Ed.FEM.1} := \frac{P_{Ed.FEM.1} L}{4} \]

\[ \alpha_{cr.FEM.1} := \frac{P_{cr}}{P_{Ed.FEM.1}} \]
\[ \sigma_{Rk.1.1} := \frac{M_{Ed.FEM.1}}{W_{el}} \quad \sigma_{Rk.1.2} := \frac{P_{Ed.FEM.1}}{\chi_{Fy}^1 t_w} \quad \tau_{Rk.1.3} := \frac{P_{Ed.FEM.1}}{h_{w}^1 t_w} \]

\[ \alpha_{ult.FEM.1} := \sqrt{\left[ \left( \frac{\sigma_{Rk.1.1}}{f_y} \right)^2 + \left( \frac{\sigma_{Rk.1.2}}{f_y} \right)^2 - \left( \frac{\sigma_{Rk.1.1}}{f_y} \right) \left( \frac{\sigma_{Rk.1.2}}{f_y} \right) + 3 \left( \frac{\tau_{Rk.1.3}}{f_y} \right)^2 \right]} } \]

\[ \alpha_{ult.FEM.1} = 0.655 \]

\[ \lambda_{p,FEM.1} := \frac{\alpha_{ult.FEM.1}}{\alpha_{cr,FEM.1}} = 1.232 \]

Reduction factors:

\[ \rho_{x,FEM.1} := \frac{\lambda_{p,FEM.1} - 0.055 \left( 3 + \psi_{x} \right)}{\lambda_{p,FEM.1}^2} = 0.739 \quad \text{< 1.0 ok!} \]

\[ \chi_{LT} := \sqrt{\frac{W_{y,el} \rho_{x,FEM.1} f_y}{M_{cr}}} = 1.0174 \]

\[ \phi_{LT.1} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT.1} - \lambda_{LT.0} \right) + \beta \lambda_{LT.1}^2 \right] \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.1} := \frac{1}{\phi_{LT.1} + \sqrt{\phi_{LT.1}^2 - \beta \lambda_{LT.1}^2}} = 0.55 \]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[ \chi_{LT,mod.1} := \chi_{LT.1} = 0.55 \quad \text{< 1.0 ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,Rd.LVL1.p} := \chi_{LT,mod.1} \cdot \frac{W_{y,el} \rho_{x,FEM.1} f_y}{\gamma_{M1}} = 192.05 \text{ kN\cdot m} \]

A moment capacity of 192.05 kNm is equivalent to \( Prd = 192.05 \text{ kN} \). Therefore the interaction format will be decisive and the bearing capacity of the girder according to the cross sectional verification is 73.179 kN.

A.5.21
Reduced stress method level 3 based on FEM results

Cross sectional verification:

\[ P_{\text{cr.3}} := 82.915 \cdot \text{kN} \]
\[ P_{\text{Ed.FEM.3}} := 72.123 \cdot \text{kN} \]

\[ \alpha_{\text{cr.FEM.3}} := \frac{P_{\text{cr.3}}}{P_{\text{Ed.FEM.3}}} \]
\[ M_{\text{Ed.FEM.3}} := P_{\text{Ed.FEM.3}} \cdot \frac{L}{4} \]

\[ \sigma_{Rk.3.1} := \frac{M_{\text{Ed.FEM.3}}}{W_{\text{el}}} \]
\[ \sigma_{Rl.3.2} := \frac{P_{\text{Ed.FEM.3}}}{\chi_{F} \cdot f_{y} \cdot t_{w}} \]
\[ \tau_{R.k.3.3} := \frac{P_{\text{Ed.FEM.3}}}{h_{w} \cdot t_{w}} \]

\[ \alpha_{\text{ult.FEM.3}} := \sqrt{\left[ \left( \frac{\sigma_{Rk.3.1}}{f_{y}} \right)^{2} + \left( \frac{\sigma_{Rl.3.2}}{f_{y}} \right)^{2} - \left( \frac{\sigma_{Rk.3.1}}{f_{y}} \right) \left( \frac{\sigma_{Rl.3.2}}{f_{y}} \right) + 3 \cdot \left( \frac{\tau_{R.k.3.3}}{f_{y}} \right)^{2} \right]^{-1}} \]

\[ \alpha_{\text{ult.FEM.3}} = 1.744 \]

\[ \lambda_{p.3} := \sqrt{\frac{\alpha_{\text{ult.FEM.3}}}{\alpha_{\text{cr.FEM.3}}}} = 1.232 \]

Reduction factors:

\[ \rho_{x.3} := \frac{\lambda_{p.3} - 0.055 \left( 3 + \psi_{x} \right)}{2 \lambda_{p.3}} = 0.739 \quad \text{< 1.0 ok!} \]

\[ \phi_{p.3} := 0.5 \left[ 1 + \alpha_{p} \left( \lambda_{p.3} - \lambda_{p.0} \right) + \lambda_{p.3} \right] \]

\[ \rho_{z.3} := \frac{1}{\phi_{p.3} + \sqrt{\phi_{p.3}^{2} - \lambda_{p.3}}} = 0.619 \]

The calculation with RSM is performed according to the theory of level 3, more information are presented in the report.
Force equilibrium:

\[ \sigma_{3,1}(x) := \rho_{x,FEM} \cdot 3 \cdot f_y \]

\[ \sigma_{3,2}(x) := \frac{h_w - x}{t_f} \cdot f_y \]

\[ \sigma_{3,4}(x) := \frac{(h_w - x) + t_f}{x + \frac{t_f}{2}} \cdot f_y \]

Tension \[ T(x) := b_f \cdot t_f \left( \frac{\sigma_{3,4}(x) + \sigma_{3,2}(x)}{2} \right) + (h_w - x) \cdot t_w \cdot \frac{\sigma_{3,2}(x)}{2} \]

Given

\[ x := 1 \cdot m \]

Compression \[ C(x) := b_f \cdot t_f \cdot f_y + x \cdot t_w \cdot \frac{\sigma_{3,1}(x)}{2} \]

Tension \[ T(x) = Compression(x) \]

\[ X := \text{Find}(x) = \text{307.849 mm} \]

Tension side:

\[ \sigma_{\text{tension}} := \frac{(h_w - X) + t_f}{X + \frac{t_f}{2}} \cdot f_y = 342.274 \cdot \text{MPa} \]
Web contribution:

\[ \sigma_{2.3} := \frac{h_{w} - X}{t_{f}} \cdot f_{y} = \frac{332.045 \cdot \text{MPa}}{X + \frac{t_{f}}{2}} \]
\[ R_{\text{web.3}} := \frac{(h_{w} - X) \cdot t_{w} \cdot \sigma_{2.3}}{2} = 192.306 \cdot \text{kN} \]

Flange contribution:

Simplification of the lever arm: \( (X + t_{f})/2 \)

\[ \sigma_{\text{AVG.3}} := \frac{\sigma_{\text{tension.3}} + \sigma_{2.3}}{2} = 337.159 \cdot \text{MPa} \]
\[ R_{\text{flange.3}} := b_{f} \cdot t_{f} \cdot \sigma_{\text{AVG.3}} = 606.887 \cdot \text{kN} \]

Compression side:

Web contribution:

\[ \sigma_{1.c.3} := \rho_{x,FEM.3} \cdot f_{y} = 262.493 \cdot \text{MPa} \]
\[ R_{\text{c.web.3}} := \frac{X \cdot t_{w} \cdot \sigma_{1.c.3}}{2} = 160.193 \cdot \text{kN} \]

Flange contribution:

\[ R_{\text{c.flange.3}} := b_{f} \cdot t_{f} \cdot f_{y} = 639 \cdot \text{kN} \]
\[ \text{LA}_{3} := X + \frac{t_{f}}{2} \quad \text{Lever arm} \]

Moment capacity:

\[ M_{\text{Rd.3}} := R_{\text{c.web.3}} \cdot \frac{2}{3} \cdot X + R_{\text{c.flange.3}} \cdot \text{LA}_{3} + R_{\text{web.3}} \cdot \frac{2}{3} \left( h_{w} - X \right) + R_{\text{flange.3}} \left( h_{w} - X + \frac{t_{f}}{2} \right) \]
\[ M_{\text{Rd.3}} = 451.137 \cdot \text{kN} \cdot \text{m} \]

Reduction factor in the longitudinal direction

A.5.24
### Verification:

\[
\left( \frac{\sigma_{Rk.3.1}}{\rho_{x.3.FEM}} f_y \right)^2 + \left( \frac{\sigma_{Rl.3.2}}{\rho_{z.3.FEM}} f_y \right)^2 - \left( \frac{\sigma_{Rk.3.1}}{\rho_{x.3.FEM}} f_y \right) \left( \frac{\sigma_{Rl.3.2}}{\rho_{z.3.FEM}} f_y \right) + 3 \left( \frac{\tau_{R.k.3.3}}{\chi_w f_y} \right)^2 = 1
\]

### Lateral torsional buckling:

The minimum load amplification factor, \( \alpha_{cr} \), for lateral torsional buckling will inevitably include the full stress field and not just the longitudinal stress component. The minimum load amplification factor, \( \alpha_{ult} \), considering the critical point for yielding will be calculated for the entire stress field.

\( P_{cr.3} = 82.915 \text{kN} \)
\( P_{Ed.FEM.3.LT} := 213.239 \text{kN} \)

\( \alpha_{cr.FEM.3.LT} := \frac{P_{cr.3}}{P_{Ed.FEM.3.LT}} \)
\( M_{Ed.FEM.3.LT} := \frac{P_{Ed.FEM.3.LT} L}{4} \)

\( \sigma_{Rk.LT1} := \frac{M_{Ed.FEM.3.LT}}{W_{el}} \)
\( \sigma_{Rl.LT2} := \frac{P_{Ed.FEM.3.LT}}{X F^{-1} y t_w} \)
\( \tau_{R.k.LT} := \frac{P_{Ed.FEM.3.LT}}{h_w t_w} \)

\( \alpha_{ult.FEM.3} := \sqrt[3]{\left( \frac{\sigma_{Rk.LT1}}{f_y} \right)^2 + \left( \frac{\sigma_{Rl.LT2}}{f_y} \right)^2 - \left( \frac{\sigma_{Rk.LT1}}{f_y} \right) \left( \frac{\sigma_{Rl.LT2}}{f_y} \right) + 3 \left( \frac{\tau_{R.k.LT}}{f_y} \right)^2} \)

\( \alpha_{ult.FEM.3} = 0.59 \)

\( \lambda_{p.FEM.3.LT} := \frac{\alpha_{ult.FEM.3}}{\alpha_{cr.FEM.3.LT}} = 1.232 \)

### Reduction factors:

\( \rho_{x.FEM.3.LT} := \frac{\lambda_{p.FEM.3.LT} - 0.055 \left( 3 + \psi_x \right)}{\lambda_{p.FEM.3.LT}^2} \)

\( \rho_{x.FEM.3.LT} = 0.739 \leq 1.0 \text{ ok!} \)
Force equilibrium:

\[ \sigma_{1,LT}(x) := \rho_{x,FEM,LT} f_y \]
\[ \sigma_{2,LT}(x) := \frac{(h_w - x)}{t_f} \cdot f_y \]
\[ \sigma_{LT}(x) := \frac{(h_w - x) + t_f}{X + \frac{t_f}{2}} \cdot f_y \]

Tension_{LT}(x) := b_f t_f \left( \frac{\sigma_{LT}(x) + \sigma_{2,LT}(x)}{2} \right) + \left( h_w - x \right) \cdot t_w \cdot \frac{\sigma_{2,LT}(x)}{2}

Given

\( x := 1 \cdot m \)

Compression_{LT}(x) := b_f t_f f_y + x \cdot t_w \cdot \frac{\sigma_{1,LT}(x)}{2}

Tension_{LT}(x) = Compression_{LT}(x)

\( X_{LT} := \text{Find}(x) = 307.849 \cdot \text{mm} \)

Tension side:

\[ \sigma_{\text{tension,LT}} := \frac{(h_w - X_{LT}) + t_f}{X_{LT} + \frac{t_f}{2}} \cdot f_y = 342.274 \cdot \text{MPa} \]

Web contribution:

\[ \sigma_{2,LT,3} := \frac{h_w - X}{X + \frac{t_f}{2}} \cdot f_y = 314.081 \cdot \text{MPa} \]

\[ R_{\text{web,LT}} := \frac{(h_w - X) \cdot t_w \cdot \sigma_{2,LT,3}}{2} = 176.681 \cdot \text{kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{\text{AVG,LT}} := \frac{\sigma_{\text{tension,LT}} + \sigma_{2,LT,3}}{2} = 328.178 \cdot \text{MPa} \]
\[ R_{\text{flange/LT}} := b_f \cdot t_f \cdot \sigma_{AVG/LT} = 590.72 \cdot \text{kN} \]

**Compression side:**

**Web contribution:**

\[ \sigma_{1.c/LT} := \rho \cdot f_{y/LT} = 262.493 \cdot \text{MPa} \]

\[ R_{c.web/LT} := \frac{X \cdot t_w \cdot \sigma_{1.c/LT}}{2} = 164.557 \cdot \text{kN} \]

**Flange contribution:**

\[ R_{c.flange/LT} := b_f \cdot t_f \cdot f_y = 639 \cdot \text{kN} \]

\[ L_{ALT} := X_{LT} + \frac{t_f}{2} \quad \text{Lever arm} \]

**Moment capacity:**

\[ M_{Rd/LT.1} := R_{c.web/LT} \cdot \frac{2}{3} \cdot X_{LT} + R_{c.flange/LT} \cdot L_{ALT} = 233.363 \cdot \text{kN} \cdot \text{m} \]

\[ M_{Rd/LT.2} := R_{web/LT} \cdot \frac{2}{3} \left( h_w - X_{LT} \right) + R_{flange/LT} \left( h_w - X_{LT} + \frac{t_f}{2} \right) = 209.65 \cdot \text{kN} \cdot \text{m} \]

\[ M_{Rd/LT} := M_{Rd/LT.1} + M_{Rd/LT.2} \]

\[ W_y.el = 1.331 \times 10^6 \cdot \text{mm}^3 \quad \text{Same as before} \]

Non-dimensional slenderness with a reduction factor \( \rho \) included in order to achieve the same principle as the effective width method.

\[ \lambda_{LT.3} := \sqrt{\frac{M_{Rd/LT}}{M_{cr}}} = 1.1458 \]

\[ \phi_{LT.3} := 0.5 \left[ 1 + \alpha_{LT.3} \left( \lambda_{LT.3} - \lambda_{LT.0} \right) + \beta \cdot \lambda_{LT.3}^2 \right] = 1.276 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.3} := \frac{1}{\phi_{LT.3} + \sqrt{\phi_{LT.3}^2 - \beta \cdot \lambda_{LT.3}^2}} = 0.481 \]

A.5.27
The modified reduction factor $\chi_{LT,\text{mod}}$ is calculated according to equation 6.58

$$\chi_{LT,\text{mod}} := \chi_{LT} = 0.481 < 1.0 \text{ ok!}$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd,LVL3,FEM} := \chi_{LT,\text{mod}} \frac{M_{Rd,LT}}{\gamma_1} = 213.239 \text{ kN} \cdot \text{m}$$

A moment capacity of 213.239 kNm is equivalent to $P_{rd} = 213.239$ kN. Therefore the interaction format will be decisive and the bearing capacity of the girder according to the cross sectional verification is 72.123 kN.

**Proposed implementation based on FEM result**

**Cross sectional verification:**

$$P_{cr,p} := 82.915 \text{ kN}$$

$$P_{Ed,FEM,p} := 71.3 \text{ kN}$$

$$M_{Ed,FEM,p} := P_{Ed,FEM} \frac{L}{4}$$

$$\alpha_{cr,FEM,p} := \frac{P_{cr}}{P_{Ed,FEM}}$$

$$\sigma_{Rk,p,1} := \frac{M_{Ed,FEM,p}}{W_{el}}$$

$$\sigma_{Rk,p,2} := \frac{P_{Ed,FEM,p}}{\chi_{F} \cdot l_{y} \cdot t_{w}}$$

$$\tau_{Rk,p,3} := \frac{P_{Ed,FEM,p}}{h_{w} \cdot t_{w}}$$

$$\alpha_{ult,FEM,p} := \sqrt{\frac{\left[ \left( \frac{\sigma_{Rk,p,1}}{f_{y}} \right)^2 + \left( \frac{\sigma_{Rk,p,2}}{f_{y}} \right)^2 - \left( \frac{\sigma_{Rk,p,1}}{f_{y}} \right) \left( \frac{\sigma_{Rk,p,2}}{f_{y}} \right) + 3 \cdot \left( \frac{\tau_{Rk,p,3}}{f_{y}} \right)^2 \right]}{\left( \frac{\sigma_{Rk,p,1}}{f_{y}} \right)^2}}$$

$$\lambda_{p,FEM,p} := \sqrt{\frac{\alpha_{ult,FEM,p}}{\alpha_{cr,FEM}}} = 1.249$$

**Reduction factors:**

$$\rho_{x,FEM} := \frac{\lambda_{p,FEM,p} - 0.055 \cdot (3 + \psi_{x})}{\lambda_{p,FEM,p}} = 0.73 \text{ < 1.0 ok!}$$

$$\phi_{p,FEM,p} := 0.5 \left[ 1 + \alpha_p (\lambda_{p,FEM,p} - \lambda_{p,0}) + \lambda_{p,FEM,p} \right]$$
\[ \rho_{z, \text{FEM}} := \frac{1}{\phi_{p, \text{FEM}, p} + \sqrt{\phi_{p, \text{FEM}, p}^2 - \lambda_{p, \text{FEM}, p}}} = 0.61 \]

\[ \rho_{x, \text{p, FEM}} := \rho_{x, \text{FEM}} \cdot \frac{M_{\text{Rd, web}}}{M_{\text{Rd, web}} + M_{\text{Rd, flange}}} + \rho_{\text{flange}} \cdot \frac{M_{\text{Rd, flange}}}{M_{\text{Rd, web}} + M_{\text{Rd, flange}}} = 0.952 \]

**Verification:**

\[ \left( \frac{\sigma_{Rk, p, 1}}{\rho_{x, \text{p, FEM}, f_y}} \right)^2 + \left( \frac{\sigma_{Rk, p, 2}}{\rho_{z, \text{FEM}, f_y}} \right)^2 - \left( \frac{\sigma_{Rk, p, 1}}{\rho_{x, \text{p, FEM}, f_y}} \right) \left( \frac{\sigma_{Rk, p, 2}}{\rho_{z, \text{FEM}, f_y}} \right) + 3 \cdot \left( \frac{\tau_{Rk, p, 3}}{\chi_{W, f_y}} \right)^2 = 1 \]

**Lateral torsional buckling:**

The reduction factor for the axial stress component, \( \rho_{x, \text{p, FEM}} \), will be the same as in the calculation above, this is because we still consider the entire stress field in both of the load amplification factors.

\[ \lambda_{\text{LT, p, FEM}} := \sqrt{\frac{W_{y, \text{el}}}{\phi_{x, \text{t, FEM}, f_y}}} = 1.1544 \]

\[ \phi_{\text{LT, p, FEM}} := 0.5 \left[ 1 + \alpha_{\text{LT}} \cdot \left( \lambda_{\text{LT, p, FEM}} - \lambda_{\text{LT}, 0} \right) + \beta \cdot \lambda_{\text{LT, p, FEM}}^2 \right] \]

The reduction factor \( \chi_{\text{LT}} \) is calculated according to equation 6.57

\[ \chi_{\text{LT, p, FEM}} := \frac{1}{\phi_{\text{LT, p, FEM}} + \sqrt{\phi_{\text{LT, p, FEM}}^2 - \beta \cdot \lambda_{\text{LT, p, FEM}}^2}} = 0.477 \]

The modified reduction factor \( \chi_{\text{LT, mod}} \) is calculated according to equation 6.58

\[ \chi_{\text{LT, mod, p, FEM}} := \chi_{\text{LT, p, FEM}} = 0.477 \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b, \text{Rd, p, FEM}} := \chi_{\text{LT, mod, p, FEM}} \cdot \frac{W_{y, \text{el}} \rho_{x, \text{p, FEM}, f_y}}{\gamma M_1} = 214.538 \text{ kN} \cdot \text{m} \]

A moment capacity of 214.538 kNm is equivalent to \( P_{\text{rd}} = 214.538 \) kN. Therefore the interaction format will be decisive and the bearing capacity of the girder according to the cross sectional verification is 71.3 kN.

A.5.29
Appendix A.6
Lateral torsional buckling - point load - flange class 4

An I-girder is subjected to two equal point loads at quarter point. The supports are of forked end boundary conditions. It should be noted that the girder is not restrained at the point loads. Furthermore it has been assumed that the local effects of the concentrated force does not contribute to the global stability of the girder. Therefore in the interaction between transverse force and axial force the effect of lateral torsional buckling does not have to be considered.

The following calculations have been made:

- Effective width method, considering patch load, shear buckling and lateral torsional buckling.
- Reduced stress method (RSM): According to level 1, level 3 and proposed implementation by obtaining the critical stress from hand-calculations and from linear FE-analysis.

Inputs:

\[
\begin{align*}
  b_f & := 200\text{-mm} & t_f & := 7.731\text{-mm} & h_w & := 600\text{-mm} & t_w & := 6.2661\text{-mm} \\
  L & := 4\text{-m} & f_y & := 355\text{-MPa} & \gamma_{M1} & := 1.0 & E & := 210\text{-GPa} \\
  S_s & := 100\text{-mm} & \gamma_{M0} & := 1.0 & f_{yw} & := f_y & \eta & := 1.2 \\
  \nu & := 0.3
\end{align*}
\]

Classification: weld sizes are omitted

Web:

\[

\varepsilon := \frac{235\text{-MPa}}{\sqrt{f_y}} \quad \frac{h_w}{t_w} = 95.753
\]

Limit for class 4 part subjected to bending

\[
124 \cdot \varepsilon = 100.888 < 120 \quad \text{The web belongs to class 3}
\]

A.6.1
Flange: outstand full compression

\[
\frac{b_f - t_w}{2t_f} = 12.53
\]

Limit for class 4

\[
14 - \varepsilon = 11.391 > 10.83 \quad \text{The flange belongs to class 4}
\]

Effective area: outstand compression elements - table 4.1 EN 1993-1-5

\[\psi := -1 \quad \text{Gives} \quad \kappa \sigma := 0.43\]

Determine the reduction factor (\(\rho\))

\[
\lambda_p := \frac{2t_f}{28 \cdot \varepsilon \cdot \sqrt{k \sigma}} = 0.827 \quad \text{Larger than 0.748 - using equation (4.3)}
\]

\[
\rho := \frac{\lambda_p - 0.188}{\lambda_p^2} = 0.934 < 1.0 \text{ ok!}
\]

\[b_{\text{eff}} := \rho \frac{b_f - t_w}{2} = 90.509\,\text{mm} \quad \text{Effective length}\]

\[b_{\text{red}} := (1 - \rho) \frac{b_f - t_w}{2} = 6.358\,\text{mm}\]

\[A_{\text{gross}} := h_w t_w + 2 \cdot b_f t_f = 6.852 \times 10^3 \cdot \text{mm}^2\]

\[A_{\text{red}} := 2 \cdot b_{\text{red}} t_f = 98.305 \cdot \text{mm}^2\]

Effective cross-section area

\[A_{\text{eff}} := A_{\text{gross}} - A_{\text{red}} = 6.754 \times 10^3 \cdot \text{mm}^2\]

Effective moment of inertia

\[
I_{y,\text{gross}} := 2 \left[ \frac{b_f t_f^3}{12} + b_f t_f \left( \frac{h_w}{2} + \frac{t_f}{2} \right)^2 \right] + \frac{t_w h_w^3}{12} = 3.983 \times 10^8 \cdot \text{mm}^4
\]

A.6.2
\[ z_{\text{eff}} := \frac{A_{\text{gross}} \left( \frac{h_w}{2} + \frac{t_f}{2} \right)}{A_{\text{eff}}} = 308.288 \text{ mm} \]

\[ I_{y,\text{eff}} := I_{y,\text{gross}} + A_{\text{gross}} \left[ z_{\text{eff}} - \left( \frac{h_w}{2} + \frac{t_f}{2} \right) \right]^2 - 2 \left( \frac{b_{\text{red}} t_f^3}{12} + t_f b_{\text{red}} z_{\text{eff}}^2 \right) = 3.891 \times 10^8 \text{ mm}^4 \]

\[ W_{y,\text{eff}} := \frac{I_{y,\text{eff}}}{z_{\text{eff}}} = 1.262 \times 10^6 \text{ mm}^3 \]

The critical moment obtained from LTBeam

\[ M_{\text{cr}} := 437.19 \text{ kN} \cdot \text{m} \]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[ \lambda_{\text{LT}} := \sqrt{\frac{W_{y,\text{eff}} f_y}{M_{\text{cr}}}} = 1.0124 \]

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\[ h/b > 2 \quad \text{Gives} \quad \text{curve d} \]

\[ \alpha_{\text{LT}} := 0.76 \]

The factor \( \theta_{\text{LT}} \) is calculated according to section 6.3.2.3

\[ \beta := 0.75 \quad \lambda_{\text{LT},0} := 0.4 \]

\[ \phi_{\text{LT}} := 0.5 \left[ 1 + \alpha_{\text{LT}} \left( \lambda_{\text{LT}} - \lambda_{\text{LT},0} \right) + \beta \lambda_{\text{LT}}^2 \right] = 1.117 \]

The reduction factor \( \chi_{\text{LT}} \) is calculated according to equation 6.57

\[ \chi_{\text{LT}} := \frac{1}{\phi_{\text{LT}} + \sqrt{\phi_{\text{LT}}^2 - \beta \lambda_{\text{LT}}^2}} = 0.553 \]

\[ \kappa_c := 1 \quad \text{The correction factor } \kappa_c \text{ is obtained according to table 6.6} \]

The factor \( f \) is calculated according to section 6.3.2.3

\[ f := 1 - 0.5 \left( 1 - \kappa_c \right) \left[ 1 - 2 \left( \lambda_{\text{LT}} - 0.8 \right)^2 \right] = 1 \]
The modified reduction factor $\chi_{LT,\text{mod}}$ is calculated according to equation 6.58

$$\chi_{LT,\text{mod}} := \frac{\chi_{LT}}{\gamma} = 0.553 \quad < 1.0 \text{ ok!}$$

The design resistance moment is calculated according to equation 6.55

$$M_{b,Rd} := \chi_{LT,\text{mod}} \frac{W_{y,\text{eff}} f_y}{\gamma M_1} = 247.668 \text{ kN} \cdot \text{m}$$

$$P_{Rd} := \frac{M_{b,Rd}}{\frac{L}{4}} = 247.668 \text{ kN}$$ Design resistance

**Patch loading**

Effective load length according to section 6.5 EN 1993-1-5

$$m_1 := \frac{f_y b_f}{f_y w t_w} = 31.918 \quad \text{Equation 6.8}$$

$$m_2 := 0.02 \left( \frac{h_w}{t_f} \right)^2 = 120.465 \quad \text{Assuming } \lambda_F > 0.5 \text{ using equation 6.9}$$

Load application type (a), see the figure below from figure 6.1 EN 1993-1-5

$$a := L - 2 \cdot 100 \text{ mm}$$

$$k_F := 6 + 2 \left( \frac{h_w}{a} \right)^2 = 6.05 \quad \text{Type (a) according to figure 6.1}$$

$$l_y := S_s + 2 \cdot t_f \left( 1 + \sqrt{m_1 + m_2} \right) = 306.33 \text{ mm} \quad \text{Equation 6.10}$$

$$F_{cr} := 0.9 k_F \frac{t_w^3}{h_w} = 468.865 \text{ kN} \quad \text{Equation 6.5}$$
Shear resistance

No stiffeners gives according to chapter 5.1 (2), EN 1993-1-5: 2006

\[
\frac{h_w}{t_w} = 95.753 \quad > \quad \frac{72 \cdot \varepsilon}{\eta} = 48.817 \quad \text{Shear buckling needs to be checked}
\]

Web with transverse stiffeners at supports. The slenderness parameter \( \lambda_w \) is calculated according to equation 5.5 EN 1993-1-5

\[
\lambda_w := \frac{h_w}{86.4 \cdot t_w \cdot \varepsilon} = 1.362
\]

Non-rigid end post, the factor \( \chi_w \) is taken from table 5.1, where \( \lambda_w > 1.08 \)

\[
\chi_w := \frac{0.83}{\lambda_w} = 0.609
\]

We have no contribution from the flanges since there are no transverse stiffeners except at the supports.

The design shear resistance is taken according equation 5.2

\[
V_{bw,Rd} := \frac{X_w \cdot f_y \cdot h_w \cdot t_w}{\gamma M_1 \sqrt{3}} = 469.541 \cdot \text{kN}
\]

Interaction:

Interaction between transverse force and bending moment

\[
M_{Rd} := \frac{W_{y,eff} \cdot f_y}{\gamma M_1}
\]
\[ M_{Ed}(P_{Ed}) := \frac{P_{Ed} \cdot L}{4} \]
\[ \eta_1(P_{Ed}) := \frac{M_{Ed}(P_{Ed})}{M_{Rd}} \]
\[ \eta_2(P_{Ed}) := \frac{P_{Ed}}{F_{Rd}} \]

Given

\[ P_{Ed} := 1 \text{kN} \]

\[ \eta_2(P_{Ed}) + 0.8 \eta_1(P_{Ed}) = 1.4 \]

\[ P_{Ed} := \text{Find}(P_{Ed}) = 262.976 \text{kN} \]

Interaction between shear force and bending moment

\[ M_{f,Rd} := (h_w + t_f) \cdot t_f \cdot b_f \cdot f_y = 333.584 \text{kN} \cdot \text{m} \]

\[ M_{pl,Rd} := M_{f,Rd} + \left( \frac{h_w}{2} \right)^2 \cdot t_w \cdot f_y = 533.786 \text{kN} \cdot \text{m} \]

\[ M_{Ed}(P_{Ed}) := \frac{P_{Ed} \cdot L}{4} \]
\[ \eta_1(P_{Ed}) := \frac{M_{Ed}(P_{Ed})}{M_{Rd}} \]
\[ \eta_3(P_{Ed}) := \frac{P_{Ed}}{V_{bw,Rd}} \]

\[ P_{Ed} := 381.9 \text{kN} \]

\[ \eta_1(P_{Ed}) + \left( 1 - \frac{M_{f,Rd}}{M_{pl,Rd}} \right) \left( 2 \cdot \eta_3(P_{Ed}) - 1 \right)^2 = 1 \quad \text{Ok!} \]

**Answer:**

\[ P_{Ed} = 247.67 \text{kN} \]

The decisive load obtained from the lateral torsional buckling.
Reduced stress method level 1

A cross sectional verification is performed according to the verification format in section 10 in EN-1993-1-5. Furthermore a verification considering lateral torsional buckling is also performed considering only axial stresses.

Cross sectional verification:

Two plate elements need to be verified. The slender flange plate will only be subjected to longitudinal stresses (as an approximation) and therefore the verification format is not needed. Meanwhile the stockier web plate (cross sectional class 3) will be verified with the verification format considering the entire stress field. It should be noted that there could still be a need for a reduction in the web plate when considering the entire stress field, even though the plate is in class 3 (for axial stresses).

Design stresses for the web plate:

The stresses are determined at a point located at the junction between the web and the flange.

\[ V_{Ed,w} := 127.10 \text{kN} \]
\[ M_{Ed,R.w} := \frac{V_{Ed,w} \cdot L}{4} \]
\[ A_{web} := h_w \cdot t_w \]
\[ W_{el} := \frac{I_{y,gross}}{h_w} \]
\[ \tau_{Ed,w} := \frac{V_{Ed,w}}{A_{web}} = 33.806 \text{MPa} \]
\[ \sigma_{Ed,x.w} := \frac{M_{Ed,R.w}}{W_{el}} = 95.722 \text{MPa} \]
\[ \sigma_{Ed,z.w} := \frac{V_{Ed.w}}{\chi F_{ly,t_w}} = 159.651 \text{MPa} \]

Critical stresses:

\[ k_\tau := 5.34 + 4 \left( \frac{h_w}{a} \right)^2 = 5.44 \quad \text{EN 1993-1-5 equation (A.5)} \]
\[ \sigma_E := \frac{\pi^2 E \cdot t_w^2}{12 \left(1 - \nu^2\right) h_w^2} = 20.701 \text{MPa} \]
\[ k_{\sigma,1} := 4.0 \]
\[ \sigma_{cr,x} := k_{\sigma,1} \cdot \sigma_E = 82.803 \text{MPa} \]
\[ \tau_{cr,\tau} := k_\tau \cdot \sigma_E = 112.607 \text{MPa} \]
\[ \sigma_{cr,c.z} := 1.881 \cdot \sigma_E = 38.938 \text{MPa} \quad \text{Hand calculated approximation, column type buckling.} \]
\[ \sigma_{cr,p.z} := 311.99 \text{MPa} \quad \text{Obtained from EB-plate, plate type buckling} \]
The stress from EB-plate was obtained under the assumption that the two point loads were separated enough to not influence each other. This was a problem since only one concentrated force could be applied at the time.

Due to the fact that the concentrated load is only applied to a certain part of the flange, plate-like buckling will prevail.

\[ \frac{\sigma_{cr.p.z}}{\sigma_{cr.c.z}} = 8.012 \quad >> \quad 2.0 \]

Thus:

\[ \sigma_{cr.z} := 311.99 \text{ MPa} \]

Load amplification factors:

\[ \alpha_{cr.x} := \frac{\sigma_{cr.x}}{\sigma_{Ed.x,w}} = 0.865 \]

\[ \alpha_{cr.z} := \frac{\sigma_{cr.z}}{\sigma_{Ed.z,w}} = 1.954 \]

\[ \alpha_{cr.\tau} := \frac{\tau_{cr.\tau}}{\tau_{Ed.w}} = 3.331 \]

\[ \psi_x := -1.0 \quad \psi_z := 1.0 \]

\[ \alpha_{cr} := \sqrt{\left[ \frac{1 + \psi_x}{4 \cdot \alpha_{cr.x}} + \frac{1 + \psi_z}{4 \cdot \alpha_{cr.z}} + \left( \frac{1 + \psi_x}{4 \cdot \alpha_{cr.x}} + \frac{1 + \psi_z}{4 \cdot \alpha_{cr.z}} \right)^2 + \frac{1 - \psi_x}{2 \cdot \alpha_{cr.x}^2} + \frac{1 - \psi_z}{2 \cdot \alpha_{cr.z}^2} + \frac{1}{\alpha_{cr.\tau}^2} \right]^{-1}} \]

\[ \alpha_{cr} = 0.677 \]

\[ \alpha_{ult.k} := \sqrt{\frac{\sigma_{Ed.x,w}}{f_y}^2 + \frac{\sigma_{Ed.z,w}}{f_y}^2 - \left( \frac{\sigma_{Ed.x,w}}{f_y} \right) \left( \frac{\sigma_{Ed.z,w}}{f_y} \right) + 3 \left( \frac{\tau_{Ed.w}}{f_y} \right)^2} \]

\[ \alpha_{ult.k} = 2.351 \]

\[ \chi_{p.w} := \sqrt{\frac{\alpha_{ult.k}}{\alpha_{cr}}} = 1.864 \quad \text{Slenderness parameter} \]
Reduction factors:

\[ \rho_{x,w} := \frac{\lambda_{p,w} - 0.055 \left( 3 + \psi \right)}{\lambda_{p,w}^{2}} = 0.505 \quad < 1.0 \text{ ok!} \]

From table B.1 EN 1993-1-5

\[ \lambda_{p,0} := 0.8 \quad \alpha_{p} := 0.34 \]

\[ \phi_{p,w} := 0.5 \left[ 1 + \alpha_{p} \left( \lambda_{p,w} - \lambda_{p,0} \right) + \lambda_{p,w} \right] = 1.613 \]

\[ \rho_{z,w} := \frac{1}{\phi_{p,w} + \sqrt{\phi_{p,w}^{2} - \lambda_{p,w}}} = 0.405 \]

Verification:

\[ \left( \frac{\sigma_{Ed,x,w}}{\rho_{x,w} f_{y}} \right)^{2} + \left( \frac{\sigma_{Ed,z,w}}{\rho_{z,w} f_{y}} \right)^{2} - \left( \frac{\sigma_{Ed,x,w}}{\rho_{x,w} f_{y}} \right) \left( \frac{\sigma_{Ed,z,w}}{\rho_{z,w} f_{y}} \right) + 3 \left( \frac{\tau_{Ed,w}}{\chi_{w} f_{y}} \right)^{2} = 1 \]

The cross sectional bearing capacity is \( P_{rd} = 127.1 \text{ kN} \), with respect to the web.

Lateral torsional buckling:

The flange belongs to class 4 with respect to axial stresses, which is assumed to be the only stress component present for the flange plate that contributes to LTB (there is a small shear stress component). For the capacity with respect to lateral torsional buckling the axial stresses will only be considered and therefore no reduction factor for the web needs to be calculated (since it is a level 3 cross section) as in the case above where the reduction factor for the web was \( \rho_{x,w} = 0.505 \), this reduction factor is considered to be a result of the local effect and should therefore not effect the bearing capacity with respect to lateral torsional buckling.

\[ V_{Ed,f} := 266.85 \text{ kN} \quad M_{Ed,R,f} := V_{Ed,f} \frac{L}{4} \quad W_{el,f} := \frac{l_{y, gross}}{h_{w} + t_{f}} \frac{h_{w} + t_{f}}{2} \]

\[ \tau_{Ed,f} := 0 \text{ MPa} \quad \sigma_{Ed,z} := 0 \text{ MPa} \]

\[ \sigma_{Ed,x} := \frac{M_{Ed,R,f}}{W_{el,f}} = 203.561 \text{ MPa} \]

Critical stress:

\[ \sigma_{E,1} := \frac{\pi^{2} \cdot E \cdot t_{f}^{2}}{12 \left( 1 - \nu^{2} \right) \left( \frac{b_{f} - t_{w}}{2} \right)^{2}} = 1.209 \times 10^{3} \text{ MPa} \]
\( \sigma_{cr,x.1} := k_\sigma \cdot \sigma_{E,1} = 519.858 \cdot \text{MPa} \quad \quad \quad k_\sigma = 0.43 \)

\( \lambda_{p,R} \cdot \sqrt{\frac{f_y}{\sigma_{cr,x.1}}} = 0.826 \)

\( \lambda_{p,R} \) is larger than 0.748: using equation 4.3 EN 1993-1-5 (outstand compression)

\( \rho_{R} := \frac{\lambda_{p,R} - 0.188}{\lambda_{p,R}} = 0.935 \quad \quad < 1.0 \text{ ok!} \)

\( \lambda_{LT,R} := \sqrt{\frac{W_{el,f} \rho_{R} f_y}{M_{cr}}} = 0.9975 \)

Imperfection factor for lateral torsional buckling curves, according to table 6.3, using buckling curves from table 6.4 - welded I-section

\( h/b > 2 \quad \text{Gives} \quad \text{curve d} \)

\( \alpha_{LT,R} := 0.76 \)

The factor \( \theta_{LT} \) is calculated according to section 6.3.2.3

\( \beta_R := 0.75 \quad \lambda_{LT,0,R} := 0.4 \)

\( \phi_{LT,R} := 0.5 \left[ 1 + \alpha_{LT,R} \left( \lambda_{LT,R} - \lambda_{LT,0,R} \right) + \beta \cdot \lambda_{LT,R}^2 \right] = 1.1 \)

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\( \chi_{LT,R} := \frac{1}{\phi_{LT,R} + \sqrt{\phi_{LT,R}^2 - \beta \lambda_{LT,R}^2}} = 0.561 \)

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\( \chi_{LT,mod,R} := \chi_{LT,R} = 0.561 \quad \quad < 1.0 \text{ ok!} \)

The design resistance moment is calculated according to equation 6.55

\( M_{b,Rd,LVL1} := \chi_{LT,mod,R} \frac{W_{el,f} \rho_{R} f_y}{\gamma_{M1}} = 244.195 \cdot \text{kNm} \)

A moment capacity of 244.195 kNm is equivalent to Prd = 244.195 kN. Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM level 1 is 127.1 kN.

A.6.10
Reduced stress method level 3

The calculation with RSM is performed according to the theory of level 3, more information are presented in the report. First a cross sectional verification will be performed considering the entire stress field. As we have seen before the entire stress field will result in a reduction factor for the web plate as well. For the global stability verification (with respect to lateral torsional buckling) the axial stress component is considered and therefore only a reduction of the flange plate is needed.

Cross sectional verification:

Design stresses for the web plate:

Stresses are determined at a point located at the junction between the web and the flange.

\[ V_{Ed.w.3} := 124.18 \text{kN} \quad M_{Ed.R.w.3} := V_{Ed.w.3} \frac{L}{4} \]

\[ \tau_{Ed.w.3} := \frac{V_{Ed.w.3}}{A_{web}} = 33.03 \text{MPa} \]

\[ \sigma_{Ed.x.w.3} := \frac{M_{Ed.R.w.3}}{W_{el}} = 93.523 \text{MPa} \]

\[ \sigma_{Ed.z.w.3} := \frac{V_{Ed.w.3}}{\chi F^1 y^1 w} = 155.983 \text{MPa} \]

Load amplification factors:

\[ \alpha_{cr.x.3} := \frac{\sigma_{cr.x}}{\sigma_{Ed.x.w.3}} = 0.885 \]

\[ \alpha_{cr.z.3} := \frac{\sigma_{cr.z}}{\sigma_{Ed.z.w.3}} = 2 \]

\[ \alpha_{cr.\tau.3} := \frac{\tau_{cr.\tau}}{\tau_{Ed.w.3}} = 3.409 \]

\[ a_1 := \frac{1 + \psi_x}{4 \alpha_{cr.x.3}} \quad a_2 := \frac{1 + \psi_z}{4 \alpha_{cr.z.3}} \]

\[ \alpha_{cr.3} := a_1 + a_2 + \left( a_1 + a_2 \right)^2 + \frac{1 - \psi_x}{2 \alpha_{cr.x.3}^2} + \frac{1 - \psi_z}{2 \alpha_{cr.z.3}^2} + \frac{1}{\alpha_{cr.\tau.3}^2} \left( \frac{1}{2} \right)^{-1} \]

\[ = 0.693 \]

A.6.11
\[ \alpha_{\text{ult.k.3}} := \left[ \left( \frac{\sigma_{\text{Ed.x.w.3}}}{f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.w.3}}}{f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.w.3}}}{f_y} \right) \left( \frac{\sigma_{\text{Ed.z.w.3}}}{f_y} \right) + 3 \left( \frac{\tau_{\text{Ed.w.3}}}{f_y} \right)^2 \right]^{-\frac{1}{2}} \]

\[ \alpha_{\text{ult.k.3}} = 2.406 \]

\[ \lambda_{p.3} := \sqrt{\frac{\alpha_{\text{ult.k.3}}}{\alpha_{\text{cr.3}}}} = 1.864 \quad \text{Slenderness parameter} \]

**Reduction factors:**

\[ \rho_{x.3} := \frac{\lambda_{p.3} - 0.055 (3 + \psi_x)}{\lambda_{p.3}^2} = 0.505 \quad < 1.0 \text{ ok!} \]

\[ \phi_{p.3} := 0.5 \left[ 1 + \alpha_p \left( \lambda_{p.3} - \lambda_p.0 \right) + \lambda_{p.3} \right] = 1.613 \]

\[ \rho_{z.3} := \frac{1}{\phi_{p.3} + \sqrt{\phi_{p.3}^2 - \lambda_{p.3}}} = 0.405 \]

**Force equilibrium:**

\[ \sigma_1(x) := \frac{x}{x + t_f} f_y \rho_{x.3} \]

\[ \sigma_2(x) := \frac{h_w - x}{x + t_f} f_y \]

\[ \sigma_f(x) := \frac{(h_w - x) + t_f}{x + t_f} f_y \]

\[ \text{Tension}(x) := b_f \cdot t_f \left( \frac{\sigma_1(x) + \sigma_2(x)}{2} \right) + (h_w - x) \cdot t_w \cdot \frac{\sigma_2(x)}{2} \]

A.6.12
Compression(x) := \frac{b_f t_f \rho_R f_y + x t_w \sigma_1(x)}{2}

Given

x := 1 \cdot m

Tension(x) \equiv Compression(x)

X := Find(x) = 328.504 \text{ mm}

Tension side:

\[ \sigma_{\text{tension}} := \frac{(h_w - X) + t_f}{X + t_f} f_y = 294.811 \text{ MPa} \]

Web contribution:

\[ \sigma_2 := \frac{h_w - X}{X + t_f} f_y = 286.649 \text{ MPa} \]

\[ R_{\text{web}} := \frac{(h_w - X) t_w \sigma_2}{2} = 243.827 \text{ kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{\text{AVG}} := \frac{\sigma_{\text{tension}} + \sigma_2}{2} = 290.73 \text{ MPa} \]

\[ R_{\text{flange}} := b_f t_f \sigma_{\text{AVG}} = 449.527 \text{ kN} \]

Compression side:

Web contribution:

\[ \sigma_{1.c} := \frac{X}{X + t_f} f_y \rho_{x,w,3} = 175.116 \text{ MPa} \]

\[ R_{c,\text{web}} := \frac{X t_w \sigma_{1.c}}{2} = 180.233 \text{ kN} \]
Flange contribution:

\[ R_{c.flange} := b_f \cdot t_f \cdot \rho R \cdot f_y = 513.121 \text{kN} \]

\[ LA := X + \frac{t_f}{2} \quad \text{Lever arm} \]

Moment capacity:

\[ M_{Rd.3} := R_{c.web} \left( \frac{2}{3} \cdot X + R_{c.flange} \cdot LA + R_{web} \left( \frac{2}{3} \cdot (h_w - X) + R_{flange} \left( h_w - X + \frac{t_f}{2} \right) \right) \right) \]

\[ M_{Rd.3} = 377.931 \text{kN} \cdot \text{m} \]

\[ \rho_{x,3} := \frac{M_{Rd.3}}{f_y \cdot W_{el}} = 0.802 \quad \text{An equivalent reduction factor for the cross-section corresponding to a level 3 calculation.} \]

Verification:

\[ \left( \frac{\sigma_{Ed,x,w,3}}{\rho_{x,3} \cdot f_y} \right)^2 + \left( \frac{\sigma_{Ed,z,w,3}}{\rho_{z,w,3} \cdot f_y} \right)^2 - \left( \frac{\sigma_{Ed,x,w,3}}{\rho_{x,3} \cdot f_y} \right) \left( \frac{\sigma_{Ed,z,w,3}}{\rho_{z,w,3} \cdot f_y} \right) + 3 \cdot \left( \frac{\tau_{Ed,w,3}}{\chi_w \cdot f_y} \right)^2 = 1 \]

The cross sectional bearing capacity is \( Prd = 124.18 \text{kN} \).

Lateral torsional buckling:

The global stability verification will only consider the axial stress component, resulting in only a reduction factor for the flange plate. This reduction factor for the flange plate is already for RSM level 1 and now used to calculate the moment capacity for the cross section with respect to RSM level 3.

Force equilibrium:

\[ \sigma_{3.1}(x) := \frac{x}{x + t_f} \cdot f_y \]

\[ \sigma_{3.2}(x) := \frac{h_w - x}{x + t_f} \cdot f_y \]

\[ \sigma_{t,3}(x) := \frac{(h_w - x) + t_f}{x + t_f} \cdot f_y \]

\[ \text{Tension}_{3}(x) := b_f \cdot t_f \left( \frac{\sigma_{t,3}(x) + \sigma_{3.2}(x)}{2} \right) + (h_w - x) \cdot t_w \cdot \frac{\sigma_{3.2}(x)}{2} \]

\[ \text{Compression}_{3}(x) := b_f \cdot t_f \cdot \rho R \cdot f_y + x \cdot t_w \cdot \frac{\sigma_{3.1}(x)}{2} \]

A.6.14
Given

\( x := 1 \cdot \text{m} \)

\( \text{Tension}_3(x) = \text{Compression}_3(x) \)

\( X_3 := \text{Find}(x) = 303.709 \cdot \text{mm} \)

**Tension side:**

\[
\sigma_{\text{tension.3}} := \frac{(h_w - X_3) + t_f}{X_3 + t_f} \cdot f_y = 346.545 \cdot \text{MPa}
\]

**Web contribution:**

\[
\sigma_{2.3} := \frac{h_w - X_3}{X_3 + t_f} \cdot f_y = 337.733 \cdot \text{MPa}
\]

\[
R_{\text{web.3}} := \frac{(h_w - X_3) \cdot t_w \cdot \sigma_{2.3}}{2} = 313.515 \cdot \text{kN}
\]

**Flange contribution:**

**Simplification of the lever arm:** \((X_3 + t_f)/2\)

\[
\sigma_{\text{AVG.3}} := \frac{\sigma_{\text{tension.3}} + \sigma_{2.3}}{2} = 342.139 \cdot \text{MPa}
\]

\[
R_{\text{flange.3}} := b_f \cdot t_f \cdot \sigma_{\text{AVG.3}} = 529.015 \cdot \text{kN}
\]

**Compression side:**

**Web contribution:**

\[
\sigma_{1.3} := \frac{X_3}{X_3 + t_f} \cdot f_y = 346.188 \cdot \text{MPa}
\]

\[
R_{\text{c.web.3}} := \frac{X_3 \cdot t_w \cdot \sigma_{1.3}}{2} = 329.41 \cdot \text{kN}
\]

**Flange contribution:**

\[
R_{\text{c.flange.3}} := b_f \cdot t_f \cdot pR \cdot f_y = 513.121 \cdot \text{kN}
\]
Lever arm

\[ \text{LA}_3 := X_3 + \frac{t_f}{2} \]

Moment capacity:

\[ M_{\text{Rd.3.}} := R_{\text{c.web.3}} \cdot \frac{2}{3} \cdot X_3 + R_{\text{c.flange.3}} \cdot \text{LA}_3 + R_{\text{web.3}} \cdot \frac{2}{3} \left( h_w - X_3 \right) + R_{\text{flange.3}} \left( h_w - X_3 + \frac{t_f}{2} \right) \]

\[ M_{\text{Rd.3.}} = 445.234 \text{kN m} \]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[ \lambda_{LT.} := \sqrt{\frac{M_{\text{Rd.3.}}}{M_{\text{cr}}}} = 1.0092 \]

\[ \phi_{LT.} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT.} - \lambda_{LT.0} \right) + \beta \lambda_{LT.}^2 \right] = 1.113 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.} := \frac{1}{\phi_{LT.} + \sqrt{\phi_{LT.}^2 - \beta \lambda_{LT.}^2}} = 0.555 \]

The modified reduction factor \( \chi_{LT,\text{mod}} \) is calculated according to equation 6.58

\[ \chi_{LT,\text{mod.}} := \chi_{LT.} = 0.555 \quad < 1.0 \text{ ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,\text{Rd.LVL3}} := \chi_{LT,\text{mod.}} \cdot \frac{M_{\text{Rd.3.}}}{\gamma_{M1}} = 246.917 \text{kN m} \]

A moment capacity of 246.917 kNm is equivalent to \( \text{Prd} = 246.917 \text{ kN} \). Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM level 3 is 124.18 kN.
**Proposed implementation**

The proposed implementation allows stress redistribution between the weaker and stronger plate. A linear stress distribution is assumed by solving for a reduction factor $\rho$ for both the weaker and stronger plate.

**Cross sectional verification:**

**Design stresses for the web plate:**

Stresses are determined at a point located at the junction between the web and the flange.

\[
V_{Ed.w.p} := 124.1 \text{kN} \\
M_{Ed.R.w.p} := V_{Ed.w.p} \frac{L}{4}
\]

\[
\tau_{Ed.w.p} := \frac{V_{Ed.w.p}}{A_{web}} = 33.008 \text{MPa}
\]

\[
\sigma_{Ed.x.w.p} := \frac{M_{Ed.R.w.p}}{W_{el}} = 93.463 \text{MPa}
\]

\[
\sigma_{Ed.z.w.p} := \frac{V_{Ed.w.p}}{X_{F} \cdot \gamma_{w} \cdot Y_{w}} = 155.883 \text{MPa}
\]

**Load amplification factors:**

\[
\alpha_{cr.x.p} := \frac{\sigma_{cr.x}}{\sigma_{Ed.x.w.p}} = 0.886
\]

\[
\alpha_{cr.z.p} := \frac{\sigma_{cr.z}}{\sigma_{Ed.z.w.p}} = 2.001
\]

\[
\alpha_{cr.\tau.p} := \frac{\tau_{cr.\tau}}{\tau_{Ed.w.p}} = 3.411
\]

\[
a_{1.p} := \frac{1 + \psi_{x}}{4 \cdot \alpha_{cr.x.p}} \\
a_{2.p} := \frac{1 + \psi_{z}}{4 \cdot \alpha_{cr.z.p}}
\]

\[
\alpha_{cr.p} := \left[ a_{1.p} + a_{2.p} + \left( a_{1.p} + a_{2.p} \right)^{2} + \frac{1 - \psi_{x}}{2 \cdot \alpha_{cr.x.p}^{2}} + \frac{1 - \psi_{z}}{2 \cdot \alpha_{cr.z.p}^{2}} + \frac{1}{\alpha_{cr.\tau.p}^{2}} \right]^{1/2} = 0.693
\]
\[
\alpha_{\text{ult.k.p}} := \sqrt{\left(\frac{\sigma_{\text{Ed.x.w.p}}}{f_y}\right)^2 + \left(\frac{\sigma_{\text{Ed.z.w.p}}}{f_y}\right)^2 - \left(\frac{\sigma_{\text{Ed.x.w.p}}}{f_y}\right)\left(\frac{\sigma_{\text{Ed.z.w.p}}}{f_y}\right) + 3\left(\frac{\tau_{\text{Ed.w.p}}}{f_y}\right)^2}\]^{-1}

\[
\alpha_{\text{ult.k.p}} = 2.408
\]

\[
\lambda_{\text{p.w.p}} := \frac{\alpha_{\text{ult.k.p}}}{\alpha_{\text{cr.p}}} = 1.864 \quad \text{Slenderness parameter}
\]

**Reduction factors:**

\[
\rho_{\text{x.w.p}} := \frac{\lambda_{\text{p.w.p}} - 0.055(3 + \psi_x)}{\lambda_{\text{p.w.p}}} = 0.505 \quad \text{< 1.0 ok!}
\]

\[
\phi_{\text{p.w.p}} := 0.5\left[1 + \alpha_p\left(\lambda_{\text{p.w.p}} - \lambda_{\text{p,0}}\right) + \lambda_{\text{p.w.p}}\right] = 1.613
\]

\[
\rho_{\text{z.w.p}} := \frac{1}{\phi_{\text{p.w.p}} + \sqrt{\phi_{\text{p.w.p}}^2 - \lambda_{\text{p.w.p}}}} = 0.405
\]

\[
M_{\text{Rd.flange}} := b_ft_fy(h_w + t_f)
\]

\[
M_{\text{Rd.web}} := \frac{h_w^2t_w}{6}fy
\]

\[
\rho_{\text{web}} := \rho_{\text{x.w.p}} \quad \rho_{\text{flange}} := \rho_R
\]

\[
\rho_{\text{proposed}} := \rho_{\text{web}}\frac{M_{\text{Rd.web}}}{M_{\text{Rd.flange}} + M_{\text{Rd.web}}} + \rho_{\text{flange}}\frac{M_{\text{Rd.flange}}}{M_{\text{Rd.flange}} + M_{\text{Rd.web}}} = 0.812
\]

**Verification:**

\[
\left(\frac{\sigma_{\text{Ed.x.w.p}}}{\rho_{\text{proposed}}fy}\right)^2 + \left(\frac{\sigma_{\text{Ed.z.w.p}}}{\rho_{\text{z.w.p}}fy}\right)^2 - \left(\frac{\sigma_{\text{Ed.x.w.p}}}{\rho_{\text{proposed}}fy}\right)\left(\frac{\sigma_{\text{Ed.z.w.p}}}{\rho_{\text{z.w.p}}fy}\right) + 3\left(\frac{\tau_{\text{Ed.w.p}}}{\chi_wfy}\right)^2 = 1
\]

The cross sectional bearing capacity is \(Prd = 124.1 \text{ kN}\).
Lateral torsional buckling:

\[ M_{Rd.flange.p} := b_f t_f f_y \left( h_w + t_f \right) \]

\[ M_{Rd.web.p} := \frac{h_w^2 t_w}{6} f_y \]

\[ \rho_{web.p} := 1 \quad \rho_{flange.p} := \rho_R \]

\[ \rho_{proposed.p} := \rho_{web.p} \frac{M_{Rd.web.p}}{M_{Rd.flange.p} + M_{Rd.web.p}} + \rho_{flange.p} \frac{M_{Rd.flange.p}}{M_{Rd.flange.p} + M_{Rd.web.p}} = 0.953 \]

\[ \lambda_{LT.proposed} := \sqrt{\frac{\text{W}_{\text{el.f}} \rho_{proposed.p} f_y}{M_{cr}}} = 1.007 \]

\[ \Phi_{LT.proposed} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT.proposed} - \lambda_{LT.0} \right) + \beta \lambda_{LT.proposed} \right] = 1.111 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.proposed} := \frac{1}{\Phi_{LT.proposed} + \sqrt{\Phi_{LT.proposed}^2 - \beta^2 \lambda_{LT.proposed}^2}} = 0.5556 \]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[ \chi_{LT.mod.proposed} := \chi_{LT.proposed} = 0.5556 \quad \text{< 1.0 ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,Rd.proposed} := \chi_{LT.mod.proposed} \frac{\text{W}_{\text{el.f}} \rho_{proposed.p} f_y}{\gamma M_1} = 246.513 \text{ kN\cdotm} \]

A moment capacity of 246.513 kNm is equivalent to \( \text{Prd} = 246.513 \text{ kN} \). Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM level 1 is 124.1 kN.
Reduced stress method level 1 based on FEM results

The critical buckling load \( P_{cr} \) is obtained from the buckling analysis and it correspond to the first local buckling mode. The cross sectional verification will be performed with the same procedure as RSM level 1.

A cross sectional verification is performed according to the verification format in section 10 in EN-1993-1-5. Furthermore a verification considering lateral torsional buckling is also performed. Therefore the complete stress field will have to be used when calculating, \( \alpha_{ult,k} \).

Cross sectional verification:

Design stresses for the web plate:

Stresses are determined at a point located at the junction between the web and the flange.

\[
V_{Ed,w,F} := 204.4 \text{kN} \quad P_{cr} := 245.609 \text{kN}
\]

\[
\tau_{Ed,w,F} := \frac{V_{Ed,w,F}}{A_{web}} = 54.367 \text{MPa}
\]

\[
M_{Ed,R.w,F} := \frac{V_{Ed,w,F} L}{4}
\]

\[
\sigma_{Ed,x.w,F} := \frac{M_{Ed,R.w,F}}{W_{el}} = 153.939 \text{MPa}
\]

\[
\sigma_{Ed,z.w,F} := \frac{V_{Ed,w,F}}{\chi_f l_{y,w}} = 256.748 \text{MPa}
\]

Load amplification factors:

\[
\alpha_{cr,F} := \frac{P_{cr}}{V_{Ed,w,F}} = 1.202
\]

\[
\alpha_{ult,k,F} := \sqrt{\left( \frac{\sigma_{Ed,x.w,F}}{f_{y}} \right)^2 + \left( \frac{\sigma_{Ed,z.w,F}}{f_{y}} \right)^2 - \left( \frac{\sigma_{Ed,x.w,F}}{f_{y}} \right) \left( \frac{\sigma_{Ed,z.w,F}}{f_{y}} \right) + 3 \cdot \left( \frac{\tau_{Ed,w,F}}{f_{y}} \right)^2}^{-1}
\]

\[
\alpha_{ult.k,F} = 1.462
\]

\[
\chi_{p,w,F} := \sqrt{\frac{\alpha_{ult,k,F}}{\alpha_{cr,F}}} = 1.103 \quad \text{Slenderness parameter}
\]

A.6.20
Reduction factors:
\[ \rho_{\text{x,w.F}} := \frac{\lambda_{\text{p,w.F}} - 0.055 \cdot (3 + \psi_x)}{\lambda_{\text{p,w.F}}} = 0.816 \quad \text{< 1.0 ok!} \]
\[ \phi_{\text{p,w.F}} := 0.5 \left[ 1 + \alpha_p \left( \lambda_{\text{p,w.F}} - \lambda_{\text{p,0}} \right) + \lambda_{\text{p,w.F}} \right] = 1.103 \]
\[ \rho_{\text{z,w.F}} := \frac{1}{\phi_{\text{p,w.F}} + \sqrt{\phi_{\text{p,w.F}}^2 - \lambda_{\text{p,w.F}}}} = 0.694 \]

Verification:
\[ \left( \frac{\sigma_{\text{Ed,x,w.F}}}{\rho_{\text{x,w.F}} f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed,z,w.F}}}{\rho_{\text{z,w.F}} f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed,x,w.F}}}{\rho_{\text{x,w.F}} f_y} \right) \left( \frac{\sigma_{\text{Ed,z,w.F}}}{\rho_{\text{z,w.F}} f_y} \right) + 3 \left( \frac{\tau_{\text{Ed,w.F}}}{\chi_{w} f_y} \right)^2 = 1 \]

The cross sectional bearing capacity is \( Prd = 204.4 \) kN, with respect to the web.

Lateral torsional buckling:
The load amplification factor, \( \alpha_{\text{cr}} \), is based on the critical stress obtained from FEM. This value considers the whole stress field. However, as an assumption the transverse stresses are set to zero for the web plate.
\[ V_{\text{Ed.f.F}} := 198.053 \text{ kN} \quad M_{\text{Ed.R.f.F}} := V_{\text{Ed.f.F}} \frac{L}{4} \quad \sigma_{\text{Ed.x.f.F}} := 0 \quad \alpha_{\text{cr.f.F}} := \frac{P_{\text{cr}}}{V_{\text{Ed.f.F}}} \]
\[ \tau_{\text{Ed.f.F}} := \frac{V_{\text{Ed.f.F}}}{2} \left( \frac{h_w + 1 f}{2} \right) = 15.108 \text{ MPa} \]

The shear stresses are calculated for a section in the middle of the top flange.
\[ \sigma_{\text{Ed.x.f.F}} := \frac{M_{\text{Ed.R.f.F}}}{W_{\text{el.f}}} = 151.081 \text{ MPa} \]
\[ \alpha_{\text{ult.k.f.F}} := \sqrt{\left( \frac{\sigma_{\text{Ed.x.f.F}}}{f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.f.F}}}{f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.f.F}}}{f_y} \right) \left( \frac{\sigma_{\text{Ed.z.f.F}}}{f_y} \right) + 3 \left( \frac{\tau_{\text{Ed.f.F}}}{f_y} \right)^2}^{-1} \]
\[ \alpha_{\text{ult.k.f.F}} = 2.315 \]
\[ \lambda_{\text{p.f.F}} := \sqrt{\frac{\alpha_{\text{ult.k.f.F}}}{\alpha_{\text{cr.f.F}}}} = 1.366 \quad \text{Slenderness parameter} \]
$\lambda_{pR}$ is larger than 0.748: using equation 4.3 EN 1993-1-5 (outstand compression)

$$\rho_{R.F} := \frac{\lambda_{p.f.F} - 0.188}{\lambda_{p.f.F}} = 0.631 < 1.0 \text{ ok!}$$

$$\lambda_{LT.R.FEM} := \sqrt{\frac{W_{el.f} \cdot \rho_{R.F} \cdot f_y}{M_{cr}}} = 0.8197$$

$$\phi_{LT.R.FEM} := 0.5 \left[ 1 + \alpha_{LT.R} \left( \lambda_{LT.R.FEM} - \lambda_{LT.0.R} \right) + \beta \lambda_{LT.R.FEM}^2 \right] = 0.911$$

The reduction factor $\chi_{LT}$ is calculated according to equation 6.57

$$\chi_{LT.R.FEM} := \frac{1}{\phi_{LT.R.FEM} + \sqrt{\phi_{LT.R.FEM}^2 - \beta \lambda_{LT.R.FEM}^2}} = 0.674$$

The modified reduction factor $\chi_{LT,mod}$ is calculated according to equation 6.58

$$\chi_{LT.mod.R.FEM} := \chi_{LT.R.FEM} = 0.674 < 1.0 \text{ ok!}$$

The design resistance moment is calculated according to equation 6.55

$$M_{b.Rd.LVL1.FEM} := \chi_{LT.mod.R.FEM} \frac{W_{el.f} \cdot \rho_{R.F} \cdot f_y}{\gamma_{M1}} = 198.053 \text{ kN} \cdot \text{m}$$

A moment capacity of 198.053 kNm is equivalent to the design bearing capacity $Prd = 198.053$ kN.
Reduced stress method level 3 based on FEM results

The critical buckling load $P_{cr}$ is obtained from the buckling analysis and it corresponds to the first local buckling mode. The cross sectional verification will be performed with the same procedure as RSM level 3.

Cross sectional verification:

Design stresses for the web plate:

Stresses are determined at a point located at the junction between the web and the flange.

$$V_{Ed.F.3} := 202.6 \text{ kN} \quad \quad \quad \quad \quad \quad \quad \quad P_{cr} = 245.609 \text{ kN}$$

$$\tau_{Ed.F.3} := \frac{V_{Ed.F.3}}{A_{web}} = 53.888 \text{ MPa} \quad \quad \quad \quad \quad \quad \quad M_{Ed.R.F.3} := V_{Ed.F.3} \frac{L}{4}$$

$$\sigma_{Ed.x.F.3} := \frac{M_{Ed.R.F.3}}{W_{el}} = 152.583 \text{ MPa}$$

$$\sigma_{Ed.z.F.3} := \frac{V_{Ed.F.3}}{\chi_F t_y t_w} = 254.487 \text{ MPa}$$

Load amplification factors:

$$\alpha_{cr.F.3} := \frac{P_{cr}}{V_{Ed.w.F}} = 1.202$$

$$\alpha_{ult.k.F.3} := \sqrt{\left(\frac{\sigma_{Ed.x.F.3}}{f_y}\right)^2 + \left(\frac{\sigma_{Ed.z.F.3}}{f_y}\right)^2 - \left(\frac{\sigma_{Ed.x.F.3}}{f_y}\right)\left(\frac{\sigma_{Ed.z.F.3}}{f_y}\right) + 3\left(\frac{\tau_{Ed.F.3}}{f_y}\right)^2}^{-1}$$

$$\alpha_{ult.k.F.3} = 1.475$$

$$\lambda_{p.F.3} := \sqrt{\frac{\alpha_{ult.k.F.3}}{\alpha_{cr.F.3}}} = 1.108 \quad \text{Slenderness parameter}$$

Reduction factors:

$$\rho_{x.F.3} := \frac{\lambda_{p.F.3} - 0.055\left(3 + \psi_{x}\right)}{\lambda_{p.F.3}^2} = 0.813 \quad < 1.0 \text{ ok!}$$

$$\phi_{p.F.3} := 0.5\left[1 + \alpha_p\left(\lambda_{p.F.3} - \lambda_{p.0}\right) + \lambda_{p.F.3}\right] = 1.106$$

A.6.23
\[
\rho_{z,F.3} := \frac{1}{\phi_{p,F.3} + \sqrt{\phi_{p,F.3}^2 - \lambda_{p,F.3}}} = 0.691
\]

Force equilibrium:

\[
\sigma_{1,F}(x) := \frac{x}{x + t_f} \cdot f_y \cdot \rho_{x,F.3}
\]

\[
\sigma_{2,F}(x) := \frac{h_w - x}{x + t_f} \cdot f_y
\]

\[
\sigma_{t,F}(x) := \frac{(h_w - x) + t_f}{x + t_f} \cdot f_y
\]

Tension \( F(x) := b_f \cdot t_f \cdot \left( \frac{\sigma_{t,F}(x) + \sigma_{2,F}(x)}{2} \right) + (h_w - x) t_w \cdot \frac{\sigma_{2,F}(x)}{2} \)

Compression \( F(x) := b_f \cdot t_f \cdot \rho_{R,F} \cdot f_y + x t_w \cdot \frac{\sigma_{1,F}(x)}{2} \)

Given

\[ x := 1 \text{ m} \]

Tension \( F(x) = \text{Compression} \( F(x) \)

\[ X_F := \text{Find}(x) = 337.619 \text{ mm} \]

Tension side:

\[
\sigma_{\text{tension},F} := \frac{(h_w - X_F) + t_f}{X_F + t_f} \cdot f_y = 277.66 \text{ MPa}
\]
Web contribution:

\[
\sigma_{2,F.3} := \frac{h_w - X_F}{X_F + t_f} f_y = 269.713 \text{ MPa}
\]

\[
R_{\text{web,F}} := \frac{(h_w - X_F) t_w \sigma_{2,F.3}}{2} = 221.719 \text{ kN}
\]

Flange contribution:

Simplification of the lever arm: \((X_F + t_f)/2\)

\[
\sigma_{\text{AVG,F}} := \frac{\sigma_{\text{tension,F}} + \sigma_{2,F.3}}{2} = 273.687 \text{ MPa}
\]

\[
R_{\text{flange,F}} := b_f \cdot t_f \cdot \sigma_{\text{AVG,F}} = 423.175 \text{ kN}
\]

Compression side:

Web contribution:

\[
\sigma_{1,c,F} := \frac{X_F}{X_F + t_f} f_y \rho_{x,F.3} = 282.144 \text{ MPa}
\]

\[
R_{c,\text{web,F}} := \frac{X_F \cdot t_w \cdot \sigma_{1,c,F}}{2} = 298.445 \text{ kN}
\]

Flange contribution:

\[
R_{c,\text{flange,F}} := b_f \cdot t_f \cdot \rho_{R,F} \cdot f_y = 346.448 \text{ kN}
\]

\[
L_{\text{A,F}} := X_F + \frac{t_f}{2} \quad \text{Lever arm}
\]

Moment capacity:

\[
M_{\text{Rd,3,F}} := R_{c,\text{web,F}} \cdot \frac{2}{3} X_F + R_{c,\text{flange,F}} \cdot L_{\text{A,F}} + R_{\text{web,F}} \cdot \frac{2}{3} (h_w - X_F) + R_{\text{flange,F}} \left( h_w - X_F + \frac{t_f}{2} \right)
\]

\[
M_{\text{Rd,3,F}} = 336.933 \text{ kN-m}
\]

\[
\rho_{x,3,F} := \frac{M_{\text{Rd,3,F}}}{f_y \cdot W_{\text{el}}} = 0.715 \quad \text{An equivalent reduction factor for the cross-section corresponding to a level 3 calculation.}
\]

A.6.25
Verification:

\[ \left( \frac{\sigma_{Ed.x.F.3}}{\rho_{x.3.F} f_y} \right)^2 + \left( \frac{\sigma_{Ed.z.F.3}}{\rho_{z.3.F} f_y} \right)^2 - \frac{3}{2} \left( \frac{\tau_{Ed.F.3} \rho_{x.3.F} f_y}{X_{w} f_y} \right)^2 = 1 \]

The cross sectional bearing capacity is \( Prd = 202.6 \text{ kN} \).

Lateral torsional buckling:

The load amplification factor, \( \alpha_{cr} \) is based on the critical stress obtained from FEM. This value considers the whole stress field. However, as an assumption the transverse stresses are set to zero for the flange plate. The reduction factor for the web plate is set to 1.0 this is because that for the axial stresses (which govern lateral torsional buckling) the web is in class 3. The reduction factor for the flange plate is calculated as in level 1 for FEM and will give the exact same reduction factor as before

\[ \rho_{R.F} = 0.631 \]

The reduction factor for the axial direction for the flange plate obtained from RSM level 1 with FEM.

\[ \text{Force equilibrium:} \]

\[ \sigma_{1.3.F}(x) := \frac{x}{x + t_f} f_y \quad \sigma_{2.3.F}(x) := \frac{h_w - x}{x + t_f} f_y \quad \sigma_{t.3.F}(x) := \frac{(h_w - x) + t_f}{x + t_f} f_y \]

Tension\(_{3,F}(x) := b_f t_f \left( \frac{\sigma_{1.3.F}(x) + \sigma_{2.3.F}(x)}{2} \right) + (h_w - x) t_w \frac{\sigma_{2.3.F}(x)}{2} \]

Compression\(_{3,F}(x) := b_f t_f \rho_{R.F} f_y + x t_w \frac{\sigma_{1.3.F}(x)}{2} \]

Given

\[ x := 1 \cdot \text{m} \]

\[ \text{Tension}_{3,F}(x) = \text{Compression}_{3,F}(x) \]

A.6.26
X := Find(x) = 326.986·mm

**Tension side:**

\[ \sigma_{tension.3.F} := \frac{(h_w - X) + t_f}{X + t_f} f_y = 297.758\text{·MPa} \]

Web contribution:

\[ \sigma_{2.3.} := \frac{h_w - X}{X + t_f} f_y = 289.558\text{·MPa} \]

\[ R_{web.3.F} := \frac{(h_w - X) \cdot t_w \cdot \sigma_{2.3.}}{2} = 247.679\text{·kN} \]

Flange contribution:

Simplification of the lever arm: \((X + t_f)/2\)

\[ \sigma_{AVG.3.F} := \frac{\sigma_{tension.3.F} + \sigma_{2.3.}}{2} = 293.658\text{·MPa} \]

\[ R_{flange.3.F} := b_f \cdot t_f \cdot \sigma_{AVG.3.F} = 454.054\text{·kN} \]

**Compression side:**

Web contribution:

\[ \sigma_{1.c.3.F} := \frac{X}{X + t_f} f_y = 346.801\text{·MPa} \]

\[ R_{c.w.F} := \frac{X \cdot t_w \cdot \sigma_{1.c.3.F}}{2} = 355.284\text{·kN} \]

Flange contribution:

\[ R_{c.f.F} := b_f \cdot t_f \cdot \rho R_F f_y = 346.448\text{·kN} \]

\[ LA_{3.F} := X + \frac{t_f}{2} \quad \text{Lever arm} \]

A.6.27
Moment capacity:

\[ M_{\text{Rd.F.3}} := R_{c.w.F} \cdot \frac{2}{3} X + R_{c.f.F} \cdot LA_{3,F} + R_{\text{web.3.F}} \cdot \frac{2}{3} \left( h_w - X \right) + R_{\text{flange.3.F}} \cdot \left( h_w - X + \frac{t_f}{2} \right) \]

\[ M_{\text{Rd.F.3}} = 362.87 \text{kN}\cdot\text{m} \]

Non-dimensional slenderness EN 1993-1-1 section 6.3.2.2

\[ \lambda_{LT.3} := \sqrt{\frac{M_{\text{Rd.F.3}}}{M_{cr}}} = 0.911 \]

\[ \phi_{LT.3} := 0.5 \left[ 1 + \alpha_{LT} \left( \lambda_{LT.3} - \lambda_{LT.0} \right) + \beta \lambda_{LT.3} \right]^2 = 1.005 \]

The reduction factor \( \chi_{LT} \) is calculated according to equation 6.57

\[ \chi_{LT.3} := \frac{1}{\phi_{LT.3} + \sqrt{\phi_{LT.3}^2 - \beta \lambda_{LT.3}^2}} = 0.614 \]

The modified reduction factor \( \chi_{LT,mod} \) is calculated according to equation 6.58

\[ \chi_{LT,mod.3} := \chi_{LT.3} = 0.614 \quad \text{< 1.0 ok!} \]

The design resistance moment is calculated according to equation 6.55

\[ M_{b,\text{Rd.LVL3.FEM}} := \chi_{LT,mod.3} \frac{M_{\text{Rd.F.3}}}{\gamma_{M1}} = 222.799 \text{kN}\cdot\text{m} \]

A moment capacity of 222.8 kNm is equivalent to the design bearing capacity \( P_{\text{Rd}} = 222.8 \text{kN} \). The design bearing capacity is according to the interaction, where \( P_{\text{Rd}} = 202.6 \text{kN} \).
Proposed implementation based on FEM result

Cross sectional verification:

Design stresses for web plate:

\[
V_{\text{Ed.w.p.F}} := 202.9 \text{ kN} \\
M_{\text{Ed.R.w.p.F}} := V_{\text{Ed.w.p.F}} \frac{L}{4}
\]

\[
\tau_{\text{Ed.p.F}} := \frac{V_{\text{Ed.w.p.F}}}{A_{\text{web}}} = 53.968 \text{ MPa}
\]

\[
\sigma_{\text{Ed.x.p.F}} := \frac{M_{\text{Ed.R.w.p.F}}}{W_{\text{el}}} = 152.809 \text{ MPa}
\]

\[
\sigma_{\text{Ed.z.p.F}} := \frac{V_{\text{Ed.w.p.F}}}{X_{F}t_y t_w} = 254.864 \text{ MPa}
\]

Load amplification factors:

\[
\alpha_{\text{cr.p.F}} := \frac{P_{\text{cr}}}{V_{\text{Ed.w.p.F}}} = 1.21
\]

\[
\alpha_{\text{ult.k.p.F}} := \sqrt{\left(\frac{\sigma_{\text{Ed.x.p.F}}}{f_y}\right)^2 + \left(\frac{\sigma_{\text{Ed.z.p.F}}}{f_y}\right)^2 - \left(\frac{\sigma_{\text{Ed.x.p.F}}}{f_y}\right)\left(\frac{\sigma_{\text{Ed.z.p.F}}}{f_y}\right) + 3\left(\frac{\tau_{\text{Ed.p.F}}}{f_y}\right)^2}
\]

\[
\alpha_{\text{ult.k.p.F}} = 1.473
\]
\[ \lambda_{p.w.p.F} := \frac{\alpha_{\text{ult.k.p.F}}}{\alpha_{\text{cr.p.F}}} = 1.103 \quad \text{Slenderness parameter} \]

Reduction factors:

\[ \rho_{x.w.p.F} := \frac{\lambda_{p.w.p.F} - 0.055(3 + \psi_x)}{\lambda_{p.w.p.F}} = 0.816 \quad \text{< 1.0 ok!} \]

\[ \phi_{p.w.p.F} := 0.5 \left[ 1 + \alpha_p \left( \lambda_{p.w.p.F} - \lambda_{p.0} \right) + \lambda_{p.w.p.F} \right] = 1.103 \]

\[ \rho_{z.w.p.F} := \frac{1}{\phi_{p.w.p.F} + \sqrt{\phi_{p.w.p.F}^2 - \lambda_{p.w.p.F}}} = 0.694 \]

\[ M_{\text{Rd.flange.F}} := b_f t_f f_y \left( h_w + t_f \right) \]

\[ M_{\text{Rd.web.F}} := \frac{b_w t_w t_f f_y}{6} \]

\[ \rho_{\text{web.F}} := \rho_{x.w.p.F} \quad \rho_{\text{flange.F}} := \rho_{R.F} \]

\[ \rho_{-p.F} := \rho_{\text{web.F}} \frac{M_{\text{Rd.web.F}}}{M_{\text{Rd.flange.F}} + M_{\text{Rd.web.F}}} + \frac{\rho_{\text{flange.F}} M_{\text{Rd.flange.F}}}{M_{\text{Rd.flange.F}} + M_{\text{Rd.web.F}}} = 0.684 \]

Verification:

\[ \left( \frac{\sigma_{\text{Ed.x.p.F}}}{\rho_{p.F} f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.p.F}}}{\rho_{z.w.p.F} f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.p.F}}}{\rho_{p.F} f_y} \right) \left( \frac{\sigma_{\text{Ed.z.p.F}}}{\rho_{z.w.p.F} f_y} \right) + 3 \left( \frac{\tau_{\text{Ed.p.F}}}{\chi_w f_y} \right)^2 = 1 \]

The cross sectional bearing capacity is \( P_{rd} = 202.9 \) kN.

Lateral torsional buckling:

The reduction factor for the web plate is set to 1.0, same argument as before. The reduction factor for the flange plate is taken from RSM level 1.

\[ \rho_{\text{web.F.LT}} := 1.0 \quad \rho_{\text{flange.F.LT}} := \rho_{R.F} \]

\[ \rho_{p.F.LT} := \rho_{\text{web.F.LT}} \frac{M_{\text{Rd.web.F}}}{M_{\text{Rd.flange.F}} + M_{\text{Rd.web.F}}} + \rho_{\text{flange.F.LT}} \frac{M_{\text{Rd.flange.F}}}{M_{\text{Rd.flange.F}} + M_{\text{Rd.web.F}}} = 0.737 \]
\[
\chi_{\text{LT},\text{p.F}} := \sqrt[\varepsilon]{\frac{W_{\text{el.f}} \rho_{\text{p.F,LT}} f_y}{M_{\text{cr}}}} = 0.885
\]

\[
\Phi_{\text{LT},\text{p.F}} := 0.5 \left[ 1 + \alpha_{\text{LT}} \left( \lambda_{\text{LT},\text{p.F}} - \lambda_{\text{LT},0} \right) + \beta \lambda_{\text{LT},\text{p.F}}^2 \right] = 0.978
\]

The reduction factor \( \chi_{\text{LT}} \) is calculated according to equation 6.57

\[
\chi_{\text{LT},\text{p.F}} := \frac{1}{\Phi_{\text{LT},\text{p.F}} + \sqrt{\Phi_{\text{LT},\text{p.F}}^2 - \beta \lambda_{\text{LT},\text{p.F}}^2}} = 0.6304
\]

The modified reduction factor \( \chi_{\text{LT},\text{mod}} \) is calculated according to equation 6.58

\[
\chi_{\text{LT},\text{mod},\text{p.F}} := \chi_{\text{LT},\text{p.F}} = 0.6304 \quad < 1.0 \text{ ok!}
\]

The design resistance moment is calculated according to equation 6.55

\[
M_{\text{b.Rd},\text{p.F}} := \chi_{\text{LT}.\text{mod},\text{p.F}} W_{\text{el.f}} \rho_{\text{p.F,LT}} f_y = 216.088 \text{ kN.m}
\]

A moment capacity of 216.088 kNm is equivalent to \( \text{Prd} = 216.088 \text{ kN} \). Therefore the interaction format will be decisive and the bearing capacity of the girder according to RSM is 202.9 kN.
Appendix A.7
Column buckling - Centric axial load - plate slenderness 1.1

A simply supported column with a butt-welded box-section subjected to a centric axial loading. The line of action coincides with the centre of gravity of the cross-section. The plate slenderness of both the web and the flange plate is set to 1.1.

The following calculations have been made:
- Effective width method, considering column buckling.
- Reduced stress method (RSM), considering column buckling: According to level 1, level 3 and proposed implementation by obtaining the critical stress from hand-calculations and from linear FE-analysis.

Inputs:
\[
\begin{align*}
    a & := 300\text{ mm} \\
    t & := 7.7742\text{ mm} \\
    f_y & := 355\text{ MPa} \\
    E & := 210\text{ GPa} \\
    \mu & := 1.0 \\
    L & := 4\text{ m} \\
    h & := 300\text{ mm} - t \\
    v & := 0.3 \\
    \gamma_{M1} & := 1.0 \\
    c & := a - t \\
    I_y & := 2\left[\frac{a\cdot t^3}{12} + a\cdot t\left(\frac{a}{2}\right)^2 + \frac{t\cdot a^3}{12}\right] = 1.4 \times 10^8\text{ mm}^4
\end{align*}
\]

Classify the cross-section
\[
\varepsilon := \frac{235\text{ MPa}}{f_y} = 0.814
\]
\[
\begin{align*}
    & 33\cdot \varepsilon = 26.849 \\
    & 38\cdot \varepsilon = 30.917 \\
    & 37.589 \\
    & 42\cdot \varepsilon = 34.172
\end{align*}
\]
The cross-section belongs to class (4)
\[ N_{cr} := \frac{\pi^2 \cdot E \cdot I_y}{(\mu \cdot L)^2} = 1.813 \times 10^7 \cdot N \]

Calculation of effective area for the cross-section,

EN 1993-1-5: 2006 (4.4), table 4.1

\[ k_\sigma := 4 \quad \psi := 1 \]

Determine the reduction factor (\( \rho \))

\[ \lambda_p := \frac{\frac{a-t}{t}}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = 0.813 \quad \text{Larger than 0.673 - using equation (4.2)} \]

\[ \rho := \frac{\lambda_p - 0.055 \cdot (3 + \psi)}{\lambda_p^2} = 0.897 \quad < 1.0 \text{ ok!} \]

\( a_{eff} := 0.5 \rho \cdot (a - t) = 131.049 \cdot \text{mm} \)

Effective length

A\(_{\text{gross}} := 4 \cdot a \cdot t = 9.329 \times 10^{-3} \cdot \text{m}^2 \)

A\(_{\text{red}} := 4 \cdot (1 - \rho) \cdot (a - t) \cdot t = 9.369 \times 10^{-4} \cdot \text{m}^2 \)

\[ A_{\text{eff}} := A_{\text{gross}} - A_{\text{red}} = 8.392 \times 10^3 \cdot \text{mm}^2 \]

Effective cross-section area

According to EN 1993-1-1 2005 table 6.2 - choice of buckling curve

\( a/t_r > 30 \quad a/t_\psi > 30 \quad \text{Gives curve (b)} \)

Using EN 1993-1-1 2005 table 6.1 to obtain the imperfection factor

\[ \alpha := 0.34 \quad \text{Imperfection factor from curve (b)} \]

\[ \lambda := \frac{\sqrt{A_{\text{eff}} \cdot f_y}}{N_{cr}} = 0.405 \quad \text{EN 1993-1-1 2005 section 6.3.1.2} \]
\[ \Phi := 0.5 \left[ 1 + \alpha (\lambda - 0.2) + \lambda^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi := \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} = 0.924 \quad \text{< 1.0 ok!} \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{b,Rd} := \frac{\chi \cdot A_{eff} \cdot f_y}{\gamma_{M1}} = 2752.61 \text{ kN} \]

Design buckling resistance
Reduced stress method

The reduced stress method will give the same results for a calculation according to level 1, level 3 and proposed implementation, therefore only one calculation will be presented. The calculation according to level 1 is performed with stresses obtained from hand-calculation and from FE-analysis.

\[ \sigma_E := \frac{\pi^2 \cdot E \cdot t^2}{12 \left(1 - \nu^2\right) \cdot h^2} = 134.329 \text{ MPa} \]

\[ \psi_R := 1 \text{ gives } k_{\sigma,p} := 4 \text{ Buckling factor according to EN 1993-1-5 table 4.1} \]

\[ \sigma_{cr,x} := k_{\sigma,p} \cdot \sigma_E = 537.317 \text{ MPa} \]

\[ \alpha_{cr,x} := \frac{\sigma_{cr,x}}{\sigma_{x,Ed}} \text{ and } \alpha_{ult,k} := \frac{f_y}{\sigma_{x,Ed}} \text{ gives } \lambda_p := \sqrt{\frac{\alpha_{ult,k}}{\sigma_{cr,x}}} \]

Finally: \[ \lambda_p := \sqrt{\frac{f_y}{\sigma_{cr,x}}} \] Where \( \sigma_{cr,x} = \sigma_{cr} \)

\( \lambda_p \) is larger than 0.673: using equation 4.2 EN 1993-1-5 (internal compression)

\[ \rho_R := \frac{\lambda_p - 0.055 \cdot (3 + \psi)}{\lambda_p^2} = 0.897 \leq 1.0 \text{ ok!} \]

\[ \sigma_{cr} := \frac{N_{cr}}{A_{gross}} \]

\[ \lambda_R := \sqrt{\frac{\rho_R f_y}{\sigma_{cr}}} = 0.405 \]

\[ \Phi_R := 0.5 \left[ 1 + \alpha \left( \lambda_R - 0.2 \right) + \lambda_R^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi_R := \frac{1}{\Phi_R + \sqrt{\Phi_R^2 - \lambda_R^2}} = 0.9242 \leq 1.0 \text{ ok!} \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{b,Rd,R} := \frac{\chi_R A_{gross} \rho_R f_y}{\gamma_{M1}} = 2745.148 \text{ kN} \text{ Design buckling resistance} \]
Reduced stress method based on FEM results

The critical buckling load $N_{cr,FEM}$ is obtained from the buckling analysis and it correspond to the first local buckling mode.

$$N_{cr,FEM} := 4416.96 \text{kN}$$

$$\alpha_{ult.k,FEM} := \frac{A_{\text{gross}}f_y}{N_{Ed}} \quad \text{and} \quad \alpha_{cr,FEM} := \frac{N_{cr,FEM}}{N_{Ed}} \quad \text{eliminating } N_{Ed}$$

Finally:

$$\lambda_{p,FEM} := \sqrt{\frac{A_{\text{gross}}f_y}{N_{cr,FEM}}}$$

$\lambda_p$ is larger than 0.673: using equation 4.2 EN 1993-1-5 (internal compression)

$$\rho_{R,FEM} := \frac{\lambda_{p,FEM} - 0.055(3 + \psi)}{\lambda_{p,FEM}^2} = 0.861 \quad < 1.0 \text{ ok!}$$

$$\lambda_{R,FEM} := \sqrt{\frac{\rho_{R,FEM}f_yA_{\text{gross}}}{N_{cr}}} = 0.397$$

$$\Phi_{R,FEM} := 0.5\left[1 + \alpha\left(\lambda_{R,FEM} - 0.2\right) + \lambda_{R,FEM}^2\right]$$

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

$$\chi_{R,FEM} := \frac{1}{\Phi_{R,FEM} + \sqrt{\Phi_{R,FEM}^2 - \lambda_{R,FEM}^2}} = 0.9274 \quad < 1.0 \text{ ok!}$$

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

$$N_{b,Rd,R,FEM} := \frac{\chi_{R,FEM}A_{\text{gross}}\rho_{R,FEM}f_y}{\gamma_{M1}} = 2645.779 \text{kN} \quad \text{Design buckling resistance}$$
Appendix. A.8
Column buckling with different plate slenderness

A simply supported column with a butt-welded box-section subjected to a centric axial loading. The line of action coincides with the centre of gravity of the cross-section. The plate slenderness of the web is set to 1.1 and the flange plate is set to 1.2.

The following calculations have been made:
- Effective width method, considering column buckling.
- Reduced stress method (RSM), considering column buckling: According to level 1, level 3 and proposed implementation by obtaining the critical stress from hand-calculations and from linear FE-analysis.

Slenderness ratio a(1.1) b(1.2)

Inputs:
\[
\begin{align*}
\alpha &:= 300\text{-mm} \\
\beta_1 &= 7.7742\text{-mm} \\
\beta_2 &= 7.1418\text{-mm} \\
E &= 210\text{-GPa} \\
\mu &= 1.0 \\
L &= 4\text{-m} \\
h_1 &= 300\text{-mm} - t_1 \\
h_2 &= 300\text{-mm} \\
\gamma_{M1} &= 1.0 \\
c_1 &= a - t_1 \\
v &= 0.3 \\
f_y &= 355\text{-MPa} \\
c_2 &= a - t_2 \\
I_z &= 2 \left( \frac{t_1 \cdot a^3}{12} + \frac{a \cdot t_2^3}{12} + \frac{a \cdot t_2^3}{4} \right) = 1.314 \times 10^{-4} \text{-m}^4
\end{align*}
\]

Classify the cross-section

\[
\varepsilon := \sqrt{\frac{235\text{-MPa}}{f_y}} = 0.814
\]

\[
\frac{c_1}{t_1} = 37.589 \\
33 \cdot \varepsilon = 26.849
\]

\[
\frac{c_2}{t_2} = 41.006 \\
42 \cdot \varepsilon = 34.172
\]

The cross-section belongs to class (4)
\[ N_{cr} := \frac{\pi^2 \cdot E \cdot I_z}{(\mu \cdot L)^2} = 1.702 \times 10^7 \cdot N \]

Calculation of effective area for the cross-section,

EN 1993-1-5: 2006 (4.4), table 4.1

\[ k_\sigma := 4 \quad \psi := 1 \]

Determine the reduction factor \((\rho)\)

\[ \lambda_{p.1} := \frac{t_1}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = 0.815 \]

\[ \lambda_{p.2} := \frac{t_2}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = 0.885 \]

\[ \rho_1 := \frac{\lambda_{p.1} - 0.055 \cdot (3 + \psi)}{\lambda_{p.1}^2} = 0.896 \quad \text{< 1.0 ok!} \]

\[ \rho_2 := \frac{\lambda_{p.2} - 0.055 \cdot (3 + \psi)}{\lambda_{p.2}^2} = 0.849 \quad \text{< 1.0 ok!} \]

\[ a_{eff.1} := 0.5 \rho_1 \cdot (a - t_1) = 130.871 \cdot \text{mm} \quad \text{Effective length} \]

\[ a_{eff.2} := 0.5 \rho_2 \cdot (a - t_2) = 124.288 \cdot \text{mm} \]

\[ A_{gross} := 2 \left(a \cdot t_1 + a \cdot t_2\right) = 8.95 \times 10^3 \cdot \text{mm}^2 \]

\[ A_{red.1} := 2 \left(1 - \rho_1\right) \left(a - t_1\right) t_1 = 473.984 \cdot \text{mm}^2 \]

\[ A_{red.2} := 2 \left(1 - \rho_2\right) \left(a - t_2\right) t_2 = 632.511 \cdot \text{mm}^2 \]

\[ A_{eff} := A_{gross} - A_{red.1} - A_{red.2} = 7.843 \times 10^3 \cdot \text{mm}^2 \quad \text{Effective cross-section area} \]

According to EN 1993-1-1 2005 table 6.2 - choice of buckling curve

\[ a/t_\gamma > 30 \quad a/t_\psi > 30 \quad \text{Gives curve (b)} \]

A.8.2
Using EN 1993-1-1 2005 table 6.1 to obtain the imperfection factor

\[ \alpha := 0.34 \quad \text{Imperfection factor from curve (b)} \]

\[ \lambda := \frac{A_{\text{eff}} f_y}{N_{\text{cr}}} = 0.404 \quad \text{EN 1993-1-1 2005 section 6.3.1.2} \]

\[ \Phi := 0.5 \left[ 1 + \alpha (\lambda - 0.2) + \lambda^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi := \frac{1}{\Phi + \sqrt{\Phi^2 - \lambda^2}} = 0.924 \quad < 1.0 \text{ ok!} \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{b,Rd} := \frac{\chi \cdot A_{\text{eff}} f_y}{\gamma_{M1}} = 2573.573 \text{ kN} \quad \text{Design buckling resistance} \]
Reduced stress method level 1

\[ \sigma_E := \frac{\pi^2 \cdot E \cdot t_2^2}{12 \left(1 - \nu^2\right) h_1^2} = 113.364 \cdot \text{MPa} \]

\[ \psi_R := 1 \quad \text{gives} \quad k_{\sigma,p} := 4 \quad \text{Buckling factor according to EN 1993-1-5 table 4.1} \]

\[ \sigma_{\text{cr,x}} := k_{\sigma,p} \sigma_E = 453.455 \cdot \text{MPa} \]

\[ \alpha_{\text{cr,x}} := \frac{\sigma_{\text{cr,x}}}{\sigma_{x,Ed}} \quad \text{and} \quad \alpha_{\text{ult,k}} := \frac{f_y}{\sigma_{x,Ed}} \quad \text{gives} \quad \lambda_p := \frac{\alpha_{\text{ult,k}}}{\alpha_{\text{cr,x}}} \]

Finally: \[ \lambda_p := \frac{f_y}{\sigma_{x,Ed}} \quad \text{Where} \quad \sigma_{\text{cr,x}} = \sigma_{\text{cr}} \]

\( \lambda_p \) is larger than 0.673: using equation 4.2 EN 1993-1-5 (internal compression)

\[ \rho_R := \frac{\lambda_{p,2} - 0.055 \cdot (3 + \psi)}{\lambda_{p,2}^2} = 0.849 \quad < 1.0 \ \text{ok!} \]

\[ \sigma_{\text{cr}} := \frac{N_{\text{cr}}}{A_{\text{gross}}} \]

\[ \lambda_R := \sqrt{\frac{\rho_R \cdot f_y}{\sigma_{\text{cr}}}} = 0.398 \]

\[ \Phi_R := 0.5 \left[ 1 + \alpha \left( \lambda_R - 0.2 \right) + \lambda_R^2 \right] = 0.613 \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi_R := \frac{1}{\Phi_R + \sqrt{\Phi_R^2 - \lambda_R^2}} = 0.9269 \quad < 1.0 \ \text{ok!} \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{\text{b,Rd,R}} := \frac{\chi_R \cdot A_{\text{gross}} \cdot \rho_R \cdot f_y}{\gamma M_1} = 2499.475 \cdot \text{kN} \quad \text{Design buckling resistance} \]

A.8.4
Reduced stress method level 3

In the case of centric axial load the resistance according to level 2 will be equal to the resistance of level 3.

\[ A_a := 2a \cdot t_2 = 4.285 \times 10^3 \cdot \text{mm}^2 \]

\[ A_b := 2a \cdot t_1 = 4.665 \times 10^3 \cdot \text{mm}^2 \]

\[ \sigma_{E,b} := \frac{\pi^2 \cdot E \cdot t_1^2}{12 \cdot (1 - \nu^2) \cdot (a - t_2)^2} = 133.75 \cdot \text{MPa} \]

\[ \sigma_{cr,x} := 4 \sigma_{E,b} = 534.999 \cdot \text{MPa} \]

\[ \lambda_{p,b} := \sqrt{\frac{f_y}{\sigma_{cr,x}}} = 0.815 \]

\[ \rho_{R,b} := \frac{\lambda_{p,b} - 0.055 \cdot (3 + \psi)}{\lambda_{p,b}^2} = 0.896 \quad \text{< 1.0 ok!} \]

\[ \rho_{R,a} := 0.848 \]

\[ \lambda_{R} := \sqrt{\frac{(A_a \rho_{R,a} + A_b \rho_{R,b}) f_y}{N_{cr}}} = 0.404 \quad \text{< 1.0 ok!} \]

\[ \Phi_{R} := 0.5 \left[ 1 + \alpha \left( \lambda_{R} - 0.2 \right) + \lambda_{R}^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi_{R} := \frac{1}{\Phi_{R} + \sqrt{\Phi_{R}^2 - \lambda_{R}^2}} = 0.9246 \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{b,Rd,R} := \frac{\chi_{R} \cdot (A_a \rho_{R,a} + A_b \rho_{R,b}) f_y}{\gamma_{M1}} = 2564.694 \text{ kN} \]

A.8.5
Proposed implementation

\[ \rho_{\text{interpolation}} := \rho_{R.a} \frac{\Lambda_a}{A_{\text{gross}}} + \rho_{R.b} \frac{\Lambda_b}{A_{\text{gross}}} = 0.873 \]

\[ \lambda_{R.\text{Implementation}} := \sqrt{\frac{A_{\text{gross}} \rho_{\text{interpolation}} f_y}{N_{cr}}} = 0.404 \quad \text{Fully equivalent} \]

\[ \Phi_{R.\text{Implementation}} := 0.5 \left[ 1 + \alpha \left( \lambda_{R.\text{Implementation}} - 0.2 \right) + \lambda_{R.\text{Implementation}}^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \lambda_{R.\text{Implementation}} := \frac{1}{\Phi_{R.\text{Implementation}} + \sqrt{\Phi_{R.\text{Implementation}}^2 - \lambda_{R.\text{Implementation}}^2}} = 0.9246 \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

\[ N_{b.\text{Rd.\text{Implementation}}} := \frac{\lambda_{R.} \left( \Lambda_a \rho_{R.a} + \Lambda_b \rho_{R.b} \right) f_y}{\gamma_{M1}} = 2564.694 \text{kN} \quad \text{Design buckling resistance} \]
Reduced stress method level 1 based on FEM results

The critical buckling load $N_{cr,FEM}$ is obtained from the buckling analysis and it correspond to the first local buckling mode.

$$N_{cr,FEM} := 3795.33 \text{kN}$$

$$\alpha_{ult,FEM} := \frac{A_{\text{gross}} f_y}{N_{Ed}}$$

$$\alpha_{cr,FEM} := \frac{N_{cr,FEM}}{N_{Ed}}$$

eliminating $N_{Ed}$ and finally:

$$\lambda_{p,FEM} := \frac{A_{\text{gross}} f_y}{N_{cr,FEM}}$$

$\lambda_p$ is larger than 0.673: using equation 4.2 EN 1993-1-5 (internal compression)

$$\rho_{R,FEM} := \frac{\lambda_{p,FEM} - 0.055(3 + \psi)}{\lambda_{p,FEM}^2} = 0.83 \quad \text{< 1.0 ok!}$$

$$\lambda_{R,FEM} := \sqrt{\frac{\rho_{R,FEM} f_y A_{\text{gross}}}{N_{cr}}} = 0.394$$

$$\Phi_{R,FEM} := 0.5 \left[ 1 + \alpha \left( \lambda_{R,FEM} - 0.2 \right) + \lambda_{R,FEM}^2 \right]$$

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

$$\chi_{R,FEM} := \frac{1}{\Phi_{R,FEM} + \sqrt{\Phi_{R,FEM}^2 - \lambda_{R,FEM}^2}} = 0.9286 \quad \text{< 1.0 ok!}$$

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

$$N_{b,Rd,FEM} := \frac{\chi_{R,FEM} A_{\text{gross}} \rho_{R,FEM} f_y}{\gamma_{M1}} = 2449.192 \text{kN} \quad \text{Design buckling resistance}$$
Reduced stress method level 3 based on FEM results

In the first local buckling mode the buckles for the flange plate are a lot more pronounced than the buckles in the web plate. Indicating that the critical load for the web plate should, as expected, have a higher critical load. Therefore a buckling mode corresponding to plate buckles in the web and almost no buckles in the flange, gave a critical load of: 5884.2 kN.

\[ N_{cr.FEM.2} := 5884.2 \text{ kN} \]

\[ \rho_{R,FEM.1} := \rho_{R,FEM} \quad \text{same as before, calculated for the slender plate (1.2)} \]

\[ \lambda_{p,FEM.2} := \sqrt{\frac{A_{\text{gross}} f_y}{N_{cr,FEM.2}}} = 0.7348 \]

\[ \rho_{R,FEM.2} := \frac{\lambda_{p,FEM.2} - 0.055(3 + \psi)}{\lambda_{p,FEM.2}^2} = 0.953 \quad \text{calculated for the stockier plate (1.1)} \]

\[ \lambda_{R,FEM.1.2} := \sqrt{\frac{(A_a \rho_{R,FEM.1} + A_b \rho_{R,FEM.2}) f_y}{N_{cr}}} = 0.4086 \quad < 1.0 \text{ ok!} \]

\[ \Phi_{R,FEM.1.2} := 0.5 \left[ 1 + \alpha \left( \lambda_{R,FEM.1.2} - 0.2 \right) + \lambda_{R,FEM.1.2}^2 \right] = 0.619 \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi_{R,FEM.1.2} := \frac{1}{\Phi_{R,FEM.1.2} + \sqrt{\Phi_{R,FEM.1.2}^2 - \lambda_{R,FEM.1.2}^2}} = 0.9227 \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

Design buckling resistance:

\[ N_{b,Rd,R,FEM.1.2} := \frac{\chi_{R,FEM.1.2} (A_a \rho_{R,FEM.1} + A_b \rho_{R,FEM.2}) f_y}{\gamma_{M1}} = 2621.896 \text{ kN} \]

The calculated bearing capacity with the proposed implementation is fully equivalent with a level 3 calculation, as has been proved in previous calculations, RSM level 3 and proposed implementation.
Appendix A.9
C.B. induced by centric axial load and transverse load

A pined-end butt-welded box-section is subjected to a centric axial load, the line of action coincides with the centre of gravity of the cross-section. Furthermore, the column is also subjected to a concentrated transverse load applied at mid-span allowing for rotation around the minor principal axis of bending.

Column buckling - slenderness ratio of 1.1

Inputs:

\[ a := 300\, \text{mm} \quad t := 7.7742\, \text{mm} \quad f_y := 355\, \text{MPa} \quad E := 210\, \text{GPa} \]

\[ \mu := 1.0 \quad L := 4\, \text{m} \quad h := 300\, \text{mm} - t \quad t_w := t \]

\[ \gamma_{M1} := 1.0 \quad c := a - t \quad v := 0.3 \quad h_w := a - t \]

\[ S_s := 100\, \text{mm} \quad f_yf := f_y \quad f_{yw} := f_{yf} \quad b_f := a - t \]

\[ t_f := t \quad P := 201.27\, \text{kN} \]

Bending moment resistance

\[
I_{z,\text{gross}} := 2 \left[ \frac{a \cdot t^3}{12} + a \cdot t \left( \frac{a}{2} \right)^2 + \frac{t \cdot a^3}{12} \right] = 1.4 \times 10^8 \, \text{mm}^4
\]

Classify the cross-section

Flange:

\[
\epsilon := \sqrt{\frac{235\, \text{MPa}}{f_y}} = 0.814
\]

\[
\frac{c}{t} = 37.589
\]

The flange belongs to class 4
Web:

\[ 72 \cdot \varepsilon = 58.58 \]
\[ \frac{c}{t} = 37.589 \]
\[ 83 \cdot \varepsilon = 67.53 \]

The web belongs to class 1

\[ 124 \cdot \varepsilon = 100.888 \]

The cross-section belongs to class (4)

\[ N_{cr,z} := \frac{\pi^2 \cdot E \cdot I_{z, gross}}{(\mu \cdot L)^2} = 1.813 \times 10^7 \cdot N \]

Calculation of effective area for the cross-section with respect to bending moment.

EN 1993-1-5: 2006 (4.4), table 4.1

\[ k_{\sigma,1} := 4 \quad \psi := 1 \quad k_{\sigma,2} := 23.9 \]

Determine the reduction factor (\( \rho \))

\[ \lambda_{p.1} := \frac{a-t}{t} \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma,1}}} = 0.813 \quad \text{Larger than 0.673 - using equation (4.2)} \]

\[ \lambda_{p.2} := \frac{a-t}{t} \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma,2}}} = 0.333 \quad \text{Lower than 0.673} \quad \rho = 1.0 \quad \text{no reduction needed} \]

\[ \rho := \frac{\lambda_{p.1} - 0.055 \cdot (3 + \psi)}{\lambda_{p.1}^2} = 0.897 \quad < 1.0 \text{ ok!} \]

Effective cross-section area

\[ a_{eff} := 0.5 \cdot \rho \cdot (a - t) + \frac{t}{2} = 0.135 \text{ m} \]

\[ A_{gross} := 4 \cdot a \cdot t = 9.329 \times 10^{-3} \text{ m}^2 \quad \text{Gross area} \]

\[ A_{red} := (1 - \rho) \cdot (a - t) \cdot t = 234.22 \text{ mm}^2 \]

\[ A_{eff} := A_{gross} - A_{red} = 9.095 \times 10^3 \text{ mm}^2 \quad \text{Effective area} \]

\[ y_{GC} := \frac{t \cdot a^2 + (a - t \cdot A_{red}) \cdot a}{A_{eff}} = 146.137 \text{ mm} \]
\[ I_{z,\text{eff.1}} := 2 \left[ \frac{t \cdot a^3}{12} + a \cdot t \left( \frac{a}{2} - y_{GC} \right)^2 \right] \]

\[ I_{z,\text{eff.2}} := 2 \left[ \frac{a_{\text{eff}} \cdot t^3}{12} + a_{\text{eff}} \cdot t \left( a - y_{GC} \right)^2 \right] \]

\[ I_{z,\text{eff.3}} := \frac{a \cdot t^3}{12} + a \cdot t \cdot y_{GC}^2 \]

\[ I_{z,\text{eff}} := I_{z,\text{eff.1}} + I_{z,\text{eff.2}} + I_{z,\text{eff.3}} = 1.346 \times 10^8 \cdot \text{mm}^4 \quad \text{Second area moment of inertia} \]

\[ W_{z,\text{el.eff}} := \frac{I_{z,\text{eff}}}{a - y_{GC}} = 8.745 \times 10^5 \cdot \text{mm}^3 \quad \text{Effective elastic section modulus} \]

\( \chi_{LT} := 1.0 \quad \text{Where} \quad h \leq 2b \)

\[ M_{z,Rk} := W_{z,\text{el.eff}} \cdot f_y = 310.445 \cdot \text{kN} \cdot \text{m} \quad \text{Moment resistance} \]

\[ M_{z,Ed} := \frac{P \cdot L}{4} = 201.27 \cdot \text{kN} \cdot \text{m} \quad \text{External first order bending} \]

\[ \frac{M_{z,Ed}}{M_{z,Rk}} = 0.648 \quad \text{< 1.0 ok!} \]

Axial resistance:

Classify the cross-section with regard to axial resistance

\[ \varepsilon := \sqrt{\frac{235 \cdot \text{MPa}}{f_{y}}} = 0.814 \]

\[ 33 \cdot \varepsilon = 26.849 \]

\[ 38 \cdot \varepsilon = 30.917 \]

\[ \frac{\varepsilon}{t} = 37.589 \quad 42 \cdot \varepsilon = 34.172 \]

The cross-section belongs to class (4)

Calculation of effective area for the cross-section,

EN 1993-1-5: 2006 (4.4), table 4.1

\[ k_{\sigma} := 4 \quad \psi := 1 \]

Determine the reduction factor (\( \rho \))

A.9.3
 detal1 100x664
\[
\lambda_{p,N} := \frac{a-t}{t} = 0.813 \quad \text{Larger than 0.673 - using equation (4.2)}
\]
\[
\rho_N := \frac{\lambda_{p,N} - 0.055 \cdot (3 + \psi)}{\lambda_{p,N}^2} = 0.897 \quad < 1.0 \text{ ok!}
\]
\[
a_{\text{eff,N}} := 0.5 \cdot (a-t) = 131.049 \cdot \text{mm} \quad \text{Effective length}
\]
\[
A_{\text{gross}} = 9.329 \times 10^{-3} \text{ m}^2
\]
\[
A_{\text{red,N}} := 4 \cdot (1 - \rho) \cdot (a-t) \cdot t = 9.369 \times 10^{-4} \text{ m}^2
\]
\[
A_{\text{eff,N}} := A_{\text{gross}} - A_{\text{red,N}} = 8.392 \times 10^3 \text{ mm}^2 \quad \text{Effective cross-section area}
\]
According to EN 1993-1-1 2005 table 6.2 - choice of buckling curve
\[
a/t_r > 30 \quad a/t_w > 30 \quad \text{Gives curve (b)}
\]
Using EN 1993-1-1 2005 table 6.1 to obtain the imperfection factor
\[
\alpha := 0.34 \quad \text{Imperfection factor from curve (b)}
\]
\[
\lambda_z := \sqrt{\frac{A_{\text{eff,N}} f_y}{N_{\text{cr,z}}}} = 0.405 \quad \text{EN 1993-1-1 2005 section 6.3.1.2}
\]
\[
\Phi_z := 0.5 \left[ 1 + \alpha \left( \lambda_z - 0.2 \right) + \lambda_z^2 \right]
\]
Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2
\[
\chi_z := \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \lambda_z^2}} = 0.924 \quad < 1.0 \text{ ok!}
\]
Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1
\[
N_{b,Rd} := \frac{\chi_z \cdot A_{\text{eff,N}} f_y}{\gamma M1} = 2752.61 \text{ kN} \quad \text{Design buckling resistance with regard to axial resistance.}
\]
Second order effects need to be taken into account. Members which are subjected to combined bending and axial compression should satisfy the criterion presented in EN 1993-1-1:2005, (6.62)

\[
\frac{N_{Ed}}{\left(\chi \frac{N_{Rk}}{\gamma M1}\right)} + k_{zz} \frac{M_{Z,Ed} + \Delta M_{Z,Ed}}{M_{Z,Rk} \gamma M1} \leq 1.0
\]  

(6.62)

\[\Delta M_{Z,Ed} := 0 \quad \text{No eccentricity}\]

Interations are needed in order to determine the maximum design resistance load \(N_{Ed}\). Using EN 1993-1-1 2005

\[N_{Rk} := A_{eff} N_{f_y}\]

\[C_{mi.0}(N_{Ed.1}) := 1 - 0.18 \frac{N_{Ed.1}}{N_{cr.z}}\]

\[k_{zz}(N_{Ed.1}) := \frac{C_{mi.0}(N_{Ed.1})}{1 - \chi \frac{N_{Ed.1}}{N_{cr.z}}}\]

Given

\[N_{Ed.1} := 1 \text{-kN}\]

\[
\frac{N_{Ed.1}}{\left(\chi \frac{N_{Rk}}{\gamma M1}\right)} + k_{zz}(N_{Ed.1}) \frac{M_{Z,Ed} + \Delta M_{Z,Ed}}{M_{Z,Rk} \gamma M1} = 1
\]

\[N_{Ed} := \text{Find}(N_{Ed.1}) = 899.025 \text{-kN} \quad \text{Design buckling resistance}\]
Patch loading

Effective load length according to section 6.5 EN 1993-1-5

The cross section is of type box-section and therefore the factor $m_1$ will be calculated with half of the flange width.

$$m_1 := \frac{b_f}{f_{yw} t_w} = 18.795 \quad \text{Equation 6.8}$$

$$m_2 := 0.02 \left( \frac{h_w}{t_f} \right)^2 = 28.259 \quad \text{Assuming } \lambda_F > 0.5 \text{ using equation 6.9}$$

Load application type (a), see the figure below from figure 6.1 EN 1993-1-5

![Diagram of load application](image)

$$k_F := 6 + 2 \left( \frac{h_w}{L} \right)^2 = 6.011 \quad \text{Type (a) according to figure 6.1}$$

$$l_y := s_S + 2 \cdot t_f \left( 1 + \sqrt{m_1 + m_2} \right) = 222.204\text{-mm} \quad \text{Equation 6.10}$$

$$F_{cr} := 0.9 \cdot k_F \cdot E \cdot \frac{t_w^3}{h_w} = 1.827 \times 10^3 \cdot \text{kN} \quad \text{Equation 6.5}$$

$$\lambda_F := \sqrt{\frac{l_y t_w f_{yw}}{F_{cr}}} = 0.579 \quad \text{Equation 6.4}$$

$$\chi_F := \frac{0.5}{\lambda_F} = 0.863 \quad < 1.0 \ \text{ok!}$$

$$L_{eff} := \chi_F l_y = 191.743\text{-mm} \quad \text{Equation 6.2}$$

$$F_{Rd} := 2 \cdot \frac{f_{yw} L_{eff} t_w}{\gamma M_1} = 1.058 \times 10^3 \cdot \text{kN} \quad \text{Design buckling resistance under transversal forces}$$
Interaction between transverse force, bending moment and axial force:

The interaction is performed according to subchapter 7.2 (1), EN 1993-1-5. The action effects \( N_{\text{Ed}} \) and \( M_{\text{Ed}} \), in the interaction verification, need to include second order effects from the normal force. Therefore the reduction factors for column buckling need to be included in \( \eta_1 \).

\[
\eta_1 := 1 \quad \text{\( \eta_1 \) is the usage ratio of the bending moment and axial force and it is calculated to 1.0 for an axial force \( N_{\text{Ed}} = 891.526 \) kN.}
\]
\[
\eta_2 := \frac{P}{F_{\text{Rd}}} = 0.19
\]
\[
\eta := \eta_2 + 0.8 \cdot \eta_1
\]
\[
\eta = 0.99 \quad \text{< 1.4  ok!}
\]

The design buckling resistance is 899.025 kN, where the interaction between the centric load and bending moment is decisive.
Reduced stress method level 1

A cross sectional verification is performed according to the verification format in section 10 in EN-1993-1-5. The action effects $N_{Ed}$ and $M_{Ed}$, in the interaction verification, need to include second order effects. The cross-section verification is performed in the mid point of the column, where the second order moment has its biggest amplitude and where the point load is located. In the mid point of the column, there are however no shear forces, but the effect of them will be overshadowed by the interaction of the bending moment and the concentrated force. Only a cross sectional verification is performed because it includes the second order effects from column buckling in the worst cross section.

Cross sectional verification - Local buckling:

$I_{z,\text{gross}} = 1.4 \times 10^8 \text{mm}^4$ \hspace{1cm} \text{same as before}

Second order moment:

$N_{Ed,R} := 848.51 \text{kN}$

$W_{z,\text{el}} := \frac{I_{z,\text{gross}}}{a}$

$P_{Ed,R} := 0.22433449 \cdot N_{Ed,R}$

The constant part in the equation is the relation between $P_{Ed}$ and $N_{Ed,R}$, in which are used in the performed analyses.

$k_R := \frac{N_{Ed,R}}{\sqrt{I_{z,\text{gross}} \cdot E}}$ \hspace{1cm} $e_{0,R} := \frac{L}{250}$

The deflection at mid-point is evaluated for a fully effective cross-section, this is because the reduction factor due to plate buckling is lower than 0.5 and thus can be neglected from the global analysis.

The solution to the differential equation gives the deflection at mid-point to:

$$w_{\text{max},R} := \frac{P_{Ed,R} \tan \left( k_R \cdot \frac{L}{2} \right)}{2 \cdot k_R \cdot E \cdot I_{z,\text{gross}}} - \frac{P_{Ed,R}}{2 \cdot k_R \cdot E \cdot I_{z,\text{gross}}} \cdot \frac{L}{2} + \frac{e_{0,R}}{\left( \frac{\pi}{k_R \cdot L} \right)^2} - 1 + e_{0,R}$$

$$w_{\text{max},R} = 25.839 \text{mm}$$
The total moment including the second order effect

\[ M_{2,\text{Ed.R}} := \frac{P_{\text{Ed.R}} \cdot L}{4} + N_{\text{Ed.R}} \cdot w_{\text{max,R}} = 212.275 \cdot \text{kN} \cdot \text{m} \]

**Design stresses:**

Both the flange plate and the web plate needs to be verified. In the flange plate the transverse stresses from the point load will be neglected. In the web plate the full stress field is considered, however the distribution of stresses are more favourable and therefore might lead to a higher capacity than the flange plate so both plates need to be verified.

\[ W_{z,\text{el}} = 9.331 \times 10^5 \cdot \text{mm}^3 \]

\[ \sigma_M := \frac{M_{2,\text{Ed.R}}}{W_{z,\text{el}}} = 227.504 \cdot \text{MPa} \quad \text{Stress from the bending moment in the junction between the flange and the web plate} \]

\[ \sigma_N := \frac{N_{\text{Ed.R}}}{A_{\text{gross}}} = 90.954 \cdot \text{MPa} \]

\[ \psi_x := \frac{\sigma_N - \sigma_M}{\sigma_N + \sigma_M} = -0.429 \]

\[ k_{\sigma,R,w} := 7.81 - 6.29 \cdot \psi_x + 9.78 \cdot \psi_x^2 = 12.305 \]

\[ k_{\sigma,R,f} := 4.0 \]

\[ \sigma_{\text{Ed,x}} := \frac{M_{2,\text{Ed.R}}}{W_{z,\text{el}}} + \frac{N_{\text{Ed.R}}}{A_{\text{gross}}} = 318.457 \cdot \text{MPa} \quad \text{Total stress in the axial direction used for both flange plate and web plate} \]

\[ \sigma_{\text{Ed,z}} := \frac{P_{\text{Ed.R}}}{\chi_F \cdot t_y \cdot t_w^2} = 63.848 \cdot \text{MPa} \quad \text{Total stress in the transversal direction used for web plate} \]

**Critical stresses:**

\[ \sigma_E := \frac{\pi^2 \cdot E \cdot t^2}{12 \left(1 - \nu^2\right) \cdot h_w^2} = 134.329 \cdot \text{MPa} \]

\[ \sigma_{\text{cr,x,f}} := k_{\sigma,R,f} \cdot \sigma_E = 537.317 \cdot \text{MPa} \]

\[ \sigma_{\text{cr,x,w}} := k_{\sigma,R,w} \cdot \sigma_E = 1.653 \times 10^3 \cdot \text{MPa} \]

\[ \sigma_{\text{cr,c,z}} := 1.881 \cdot \sigma_E = 252.673 \cdot \text{MPa} \]

\[ \sigma_{\text{cr,p,z}} := 988.35 \cdot \text{MPa} \quad \text{Obtained from EB-plate} \]
Due to the fact that the concentrated load is only applied to a certain part of the flange, plate like buckling will prevail.

\[
\frac{\sigma_{cr.p.z}}{\sigma_{cr.c.z}} = 3.912 > 2.0
\]

Thus:

\[
\sigma_{cr.z} = \sigma_{cr.p.z}
\]

Load amplification factors:

\[
\alpha_{cr.x.f} := \frac{\sigma_{cr.x.f}}{\sigma_{Ed.x}} = 1.687 \quad \alpha_{cr.x.w} := \frac{\sigma_{cr.x.w}}{\sigma_{Ed.x}} = 5.19
\]

\[
\frac{\sigma_{cr.z}}{\sigma_{Ed.z}} = 15.48 \quad \text{Uses only for web plate}
\]

\(\tau_{Ed}\) is equal to zero, which means the term \((1/\alpha_{cr.\tau}^2)\) in the equation for the load amplification factor will be disregarded, (zero).

\[
\psi_{z} := 1.0
\]

\[
\alpha_{cr.w} := \left[ \frac{1 + \psi_{x}}{4\alpha_{cr.x.w}} + \frac{1 + \psi_{z}}{4\alpha_{cr.z}} \right] \left[ \frac{\left( 1 + \psi_{x} \right)^2}{4\alpha_{cr.x.w}} + \frac{\left( 1 + \psi_{z} \right)^2}{4\alpha_{cr.z}} + \frac{1 - \psi_{x}}{2\alpha_{cr.x.w}} + \frac{1 - \psi_{z}}{2\alpha_{cr.z}} \right]^{1/2}\]

\[
\alpha_{cr.w} = 4.287
\]

\[
\alpha_{ult.k.w} := \left[ \frac{\left( \frac{\sigma_{Ed.x}}{f_y} \right)^2 + \left( \frac{\sigma_{Ed.z}}{f_y} \right)^2 - \left( \frac{\sigma_{Ed.x}}{f_y} \right) \left( \frac{\sigma_{Ed.z}}{f_y} \right)}{\alpha_{cr.w}} \right]^{1/2}
\]

\[
\alpha_{ult.k.w} = 1.217
\]

\[
\lambda_{p.R.w} := \frac{\alpha_{ult.k.w}}{\alpha_{cr.w}} = 0.533 \quad \lambda_{p.R.f} := \frac{f_y}{\sqrt{\sigma_{cr.x.f}}} = 0.813 \quad \text{Slenderness parameter}
\]

Reduction factors:

\[
\rho_{x.w} := \frac{\lambda_{p.R.w} - 0.055\left( 3 + \psi_{x} \right)}{\lambda_{p.R.w}^2} = 1.379 > 1.0 \text{ indicates that the web is not in class 4 and therefore, no need to reduce the web.}
\]

\[
\psi_{x.f} := 1.0
\]
\[ \rho_{x,f} := \frac{\lambda_{p,R,f} - 0.055(3 + \psi_{x,f})}{\lambda_{p,R,f}^2} = 0.897 \leq 1.0 \text{ ok!} \]

The reduction factors in the transverse direction will not be calculated. This is because in the web plate the full stress field gives a cross-sectional class lower than class 4. And for the flange plate the transverse stresses are zero and therefore there is no need to calculate a reduction factor.

**Verification of the web:**

\[ \left( \frac{\sigma_{Ed,x}}{f_y} \right)^2 + \left( \frac{\sigma_{Ed,z}}{f_y} \right)^2 - \left( \frac{\sigma_{Ed,x}}{f_y} \right) \left( \frac{\sigma_{Ed,z}}{f_y} \right) = 0.67573 \]

**Verification of the flange:**

\[ \left( \frac{\sigma_{Ed,x}}{\rho_{x,f} \cdot f_y} \right) = 1 \]

The cross sectional verification shows that the flange plate is more prone to buckling than the web plate and therefore governs the resistance of the cross-section.

The cross sectional bearing capacity is \( P_{rd} = 190.4 \) kN and \( N_{Ed} = 848.5 \) kN with respect to the flange.
Reduced stress method level I based on FEM result

The critical load is taken from the first local buckling mode from the linear FE-analysis. The first local buckling mode excited both the buckling behaviour in the flange plate and the web plate. However, the buckles were much more pronounced in the flange plate, and therefore the critical stress in the web plate might have been slightly underestimated.

Cross sectional verification - Local buckling:

\[ I_{z,\text{gross}} = 1.4 \times 10^8 \text{mm}^4 \quad W_{z,\text{el}} = 9.331 \times 10^5 \text{mm}^3 \quad \text{same as before} \]

Design stresses:

\[ N_{Ed,F} := 922.14 \text{kN} \]

\[ P_{Ed,F} := 0.22433449 N_{Ed,F} \]

The constant part in the equation is the relation between \( P_{Ed} \) and \( N_{Ed,R} \), which are used in the performed analyses.

\[ k_F := \sqrt{\frac{N_{Ed,F}}{I_{z,\text{gross}}E}} \quad e_0,F := \frac{L}{250} \]

The deflection at mid-point is evaluated for a fully effective cross-section, this is because the reduction factor due to plate buckling is lower than 0.5 and thus can be neglected from the global analysis.

\[ w_{\text{max},F} := \frac{P_{Ed,F}}{2 \cdot k_F^3 \cdot E \cdot I_{z,\text{gross}}} \left[ \tan \left( \frac{k_F \cdot L}{2} \right) + \frac{-P_{Ed,F}}{2 \cdot k_F^2 \cdot E \cdot I_{z,\text{gross}}} \right] + \frac{e_0,F}{ \left( \frac{\pi}{k_F \cdot L} \right)^2 - 1} + e_0,F \]

\[ w_{\text{max},F} = 26.738 \text{mm} \]

The total moment including second order effect

\[ M_{2,Ed,F} := \frac{P_{Ed,F} \cdot L}{4} + N_{Ed,F} w_{\text{max},F} = 231.524 \text{kN} \cdot \text{m} \]
Design stresses:

Both the flange plate and the web plate needs to be verified. In the flange plate the transverse stresses from the point load will be neglected. In the web plate the full stress field is considered, however the distribution of stresses are more favourable and therefore might lead to a higher capacity than the flange plate so both plates need to be verified.

\[
W_{z,el} = 9.331 \times 10^5 \cdot \text{mm}^3
\]

\[
\sigma_{M,F} := \frac{M_{2,Ed,F}}{W_{z,el}} = 248.134 \cdot \text{MPa}
\]

Stress from the bending moment in the junction between the flange and the web plate

\[
\sigma_{N,F} := \frac{N_{Ed,F}}{A_{gross}} = 98.846 \cdot \text{MPa}
\]

\[
\psi_{x,F} := \frac{\sigma_{N,F} - \sigma_{M,F}}{\sigma_{N,F} + \sigma_{M,F}} = -0.43
\]

\[
k_{\sigma,F,w} := 7.81 - 6.29 \cdot \psi_{x,F} + 9.78 \cdot \psi_{x,F}^2 = 12.305
\]

\[
k_{\sigma,F,f} := 4.0
\]

\[
\sigma_{Ed,x,F} := \frac{M_{2,Ed,F}}{W_{z,el}} + \frac{N_{Ed,F}}{A_{gross}} = 346.98 \cdot \text{MPa}
\]

Total stress in the axial direction used for both flange plate and web plate

\[
\sigma_{Ed,z,F} := \frac{P_{Ed,F}}{X_F f_y t_w^2} = 69.388 \cdot \text{MPa}
\]

Total stress in the transversal direction used for web plate

Critical stresses:

\[
P_{cr,F} := 2063.69 \cdot \text{kN}
\]

Load amplification factors:

\[
\alpha_{cr,F} := \frac{P_{cr,F}}{P_{Ed,F}}
\]

\[
\alpha_{ult,k.w,F} := \left[ \left( \frac{\sigma_{Ed,x,F}}{f_y} \right)^2 + \left( \frac{\sigma_{Ed,z,F}}{f_y} \right)^2 - \left( \frac{\sigma_{Ed,x,F}}{f_y} \right) \left( \frac{\sigma_{Ed,z,F}}{f_y} \right) \right]^{-\frac{1}{2}}
\]

\[
\alpha_{ult,k.w,F} = 1.116
\]
\( \alpha_{\text{ult.k.f.F}} := \sqrt{\left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right)^2} \)

\( \alpha_{\text{ult.k.f.F}} = 1.023 \)

\( \lambda_{p.f.w} := \sqrt{\frac{\alpha_{\text{ult.k.w.F}}}{\alpha_{\text{cr.F}}}} = 0.335 \quad \lambda_{p.f.f} := \sqrt{\frac{\alpha_{\text{ult.k.f.F}}}{\alpha_{\text{cr.F}}}} = 0.32 \) Slenderness parameter

Reduction factors:

\( \rho_{x.w.F} := \frac{\lambda_{p.f.w} - 0.055(3 + \psi_x)}{\lambda_{p.f.w}^2} = 1.726 \) \( > 1.0 \) indicates that the web is not in class 4 and therefore, no need to reduce the web.

\( \rho_{x.f.f} := \frac{\lambda_{p.f.f} - 0.055(3 + \psi_x)}{\lambda_{p.f.f}^2} = 0.977 \) \( < 1.0 \) ok!

The reduction factors in the transverse direction will not be calculated. This is because in the web plate the full stress field gives a cross-sectional class lower than class 4. And for the flange plate the transverse stresses are zero and therefore there is no need to calculated a reduction factor.

Verification of the web:

\( \left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.F}}}{f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right) \left( \frac{\sigma_{\text{Ed.z.F}}}{f_y} \right) = 0.80249 \)

Verification of the flange:

\( \frac{\sigma_{\text{Ed.x.F}}}{\rho_{x.f.f} f_y} = 1 \)

The cross sectional verification shows that the flange plate is more prone to buckling than the web plate and therefore governs the resistance of the cross-section.

The cross sectional bearing capacity is \( P_{rd} = 206.87 \) kN and \( N_{Ed} = 922.14 \) kN with respect to the flange.
Appendix A.10
C.B. induced by centric axial load and transverse load

A pined-end butt-welded box-section is subjected to a centric axial load, the line of action coincides with the centre of gravity of the cross-section. Furthermore, the column is also subjected to a concentrated transverse load applied at mid-span allowing for rotation around the minor principal axis of bending.

Column buckling - slenderness ratio of \( a(1.1), b(1.2) \)

Inputs:

\[
\begin{align*}
\text{a} & := 300 \text{ mm} & t_w & := 7.7742 \text{ mm} & t_f & := 7.1418 \text{ mm} & L & := 4 \text{ m} \\
h_w & := a - t_f & b_f & := a - t_w & c_w & := a - t_f & c_f & := a - t_w \\
\gamma_{M1} & := 1.0 & f_y & := 355 \text{ MPa} & E & := 210 \text{ GPa} & \mu & := 1.0 \\
S_s & := 100 \text{ mm} & f_{yw} & := f_y & f_{yw} & := f_y & \nu & := 0.3 \\
P & := 185.29 \text{ kN} \\
\end{align*}
\]

Bending moment resistance

\[
I_{z,\text{gross}} := \frac{2}{12} \left[ \frac{a \cdot t_f^3}{12} + a \cdot t_f \left( \frac{a}{2} \right)^2 + \frac{t_w \cdot a^3}{12} \right] = 1.314 \times 10^8 \text{ mm}^4
\]

Classify the cross-section

Flange:

\[
\varepsilon := \sqrt{\frac{235 \text{ MPa}}{f_y}} = 0.814
\]

\[
\frac{c_f}{t_f} = 40.918
\]

The flange belongs to class 4
Web:

\[
\begin{align*}
\frac{c_w}{t_w} &= 37.671 \quad 83\varepsilon = 67.53 \\
The web belongs to class 1 & \quad 124\varepsilon = 100.888
\end{align*}
\]

The cross-section belongs to class (4)

\[
N_{cr,z} := \frac{\pi^2 \cdot E \cdot I_z \text{,gross}}{(\mu \cdot L)^2} = 1.702 \times 10^7 \text{N}
\]

Calculation of effective area for the cross-section with respect to bending moment.

EN 1993-1-5: 2006 (4.4), table 4.1

\[
k_{\sigma,1} := 4 \quad \psi := 1 \quad k_{\sigma,2} := 23.9
\]

Determine the reduction factor (\(\rho\))

\[
\lambda_{p,1} := \frac{a-t_w}{\frac{t_f}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma,1}}}} = 0.885 \quad \text{Larger than 0.673 - using equation (4.2)}
\]

\[
\lambda_{p,2} := \frac{a-t_f}{\frac{t_w}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma,2}}}} = 0.333 \quad \text{Lower than 0.673} \quad \rho = 1.0 \quad \text{no reduction needed}
\]

\[
\rho := \frac{\lambda_{p,1} - 0.055 \cdot (3 + \psi)}{\lambda_{p,1}^2} = 0.849 \quad < 1.0 \text{ ok!}
\]

Effective cross-section area

\[
a_{\text{eff}} := 0.5 \cdot \rho \cdot (a - t_w) = 0.124 \text{ m}
\]

\[
A_{\text{gross}} := 2 \cdot a \cdot t_f + 2 \cdot a \cdot t_w = 8.95 \times 10^3 \cdot \text{mm}^2 \quad \text{Gross area}
\]

\[
A_{\text{red}} := (1 - \rho) \cdot (a - t_w) \cdot t_f = 315.573 \cdot \text{mm}^2
\]

\[
A_{\text{eff}} := A_{\text{gross}} - A_{\text{red}} = 8.634 \times 10^3 \cdot \text{mm}^2 \quad \text{Effective area}
\]
\[ y_{GC} := \frac{t_w a^2 + (a t_f - A_{red}) a}{A_{eff}} = 144.518 \text{ mm} \]

\[ I_{z.eff.1} := 2 \left[ \frac{t_w a^3}{12} + a t_w \left( \frac{a}{2} - y_{GC} \right)^2 \right] \]

\[ I_{z.eff.2} := 2 \left[ \left( \frac{a_{eff} + \frac{t_w}{2}}{2} \right)^2 t_f^3 \right] \left( a_{eff} + \frac{t_w}{2} \right) t_f \left( a - y_{GC} \right)^2 \]

\[ I_{z.eff.3} := \frac{a t_f^3}{12} + a t_f y_{GC}^2 \]

\[ I_{z.eff} := I_{z.eff.1} + I_{z.eff.2} + I_{z.eff.3} = 1.241 \times 10^8 \text{ mm}^4 \quad \text{Second area moment of inertia} \]

\[ W_{z.el.eff} := \frac{I_{z.eff}}{a - y_{GC}} = 7.979 \times 10^5 \text{ mm}^3 \quad \text{Effective elastic section modulus} \]

\[ \chi_{LT} := 1.0 \quad \text{Where} \quad h \leq 2b \]

\[ M_{z.Rk} := W_{z.el.eff} f_y = 283.245 \text{ kN} \cdot \text{m} \quad \text{Moment resistance} \]

\[ M_{z.Ed} := \frac{P \cdot L}{4} = 185.29 \text{ kN} \cdot \text{m} \quad \text{External first order bending} \]

\[ \frac{M_{z.Ed}}{M_{z.Rk}} = 0.654 \quad < 1.0 \text{ ok!} \]

**Axial resistance:**

Classify the cross-section with regard to axial resistance

\[ \varepsilon := \sqrt{\frac{235 \text{ MPa}}{f_y}} = 0.814 \]

\[ \varepsilon = \frac{33 \cdot \varepsilon}{26.849} = 38 \cdot \varepsilon = 30.917 \]

\[ \frac{c_w}{t_w} = 37.671 \quad \frac{c_f}{t_f} = 40.918 \quad 42 \cdot \varepsilon = 34.172 \]

**The cross-section belongs to class (4)**
Calculation of effective area for the cross-section, 
EN 1993-1-5: 2006 (4.4), table 4.1

$k_\sigma := 4 \quad \psi := 1$

Determine the reduction factor ($\rho$)

$$\lambda_{p,N,w} := \frac{a-t_f}{t_w} = 0.815 \quad \text{Larger than 0.673 - using equation (4.2)}$$

$$\lambda_{p,N,f} := \frac{t_f}{28.4 \cdot \varepsilon \cdot k_\sigma} \quad \text{Larger than 0.673 - using equation (4.2)}$$

$$\rho_{N,w} := \frac{\lambda_{p,N,w} - 0.055 \cdot (3 + \psi)}{\lambda_{p,N,w}^2} = 0.896 \quad < 1.0 \text{ ok!}$$

$$\rho_{N,f} := \frac{\lambda_{p,N,f} - 0.055 \cdot (3 + \psi)}{\lambda_{p,N,f}^2} = 0.849 \quad < 1.0 \text{ ok!}$$

$$a_{\text{eff,}N,w} := 0.5 \rho_{N,w} \left(a - t_f\right) = 131.154 \text{- mm} \quad \text{Effective length - web}$$

$$a_{\text{eff,}N,f} := 0.5 \rho_{N,f} \left(a - t_w\right) = 124.02 \text{- mm} \quad \text{Effective length - flange}$$

$$A_{\text{gross}} = 8.95 \times 10^{-3} \text{ m}^2$$

$$A_{\text{red,}N,w} := 2 \left(1 - \rho_{N,w}\right) \left(a - t_f\right) t_w = 475.009 \text{ mm}^2$$

$$A_{\text{red,}N,f} := 2 \left(1 - \rho_{N,f}\right) \left(a - t_w\right) t_f = 631.145 \text{ mm}^2$$

$$A_{\text{eff}} := A_{\text{gross}} - A_{\text{red,}N,w} - A_{\text{red,}N,f} = 7.843 \times 10^3 \text{ mm}^2 \quad \text{Effective cross-section area}$$
According to EN 1993-1-1 2005 table 6.2 - choice of buckling curve

\[ \frac{a}{t_y} > 30 \quad \frac{a}{t_y} > 30 \quad \text{Gives curve (b)} \]

Using EN 1993-1-1 2005 table 6.1 to obtain the imperfection factor

\[ \alpha := 0.34 \quad \text{Imperfection factor from curve (b)} \]

\[ \lambda_z := \sqrt{\frac{A_{\text{eff}N}}{N_{c.r.z}}} = 0.404 \quad \text{EN 1993-1-1 2005 section 6.3.1.2} \]

\[ \Phi_z := 0.5 \left[ 1 + \alpha \left( \lambda_z - 0.2 \right) + \lambda_z^2 \right] \]

Using equation 6.49 obtained from EN 1993-1-1 2005 section 6.3.1.2

\[ \chi_z := \frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \lambda_z^2}} = 0.924 \quad < 1.0 \quad \text{ok!} \]

Using equation 6.47 obtained from EN 1993-1-1 2005 section 6.3.1.1

Design buckling resistance with regard to axial resistance.

\[ N_{b.Rd} := \frac{\chi_z A_{\text{eff}N} f_y}{\gamma M_1} = 2573.675 \text{kN} \]

Second order effects need to be taken into account. Members which are subjected to combined bending and axial compression should satisfy the criterion presented in EN 1993-1-1:2005, (6.62)

\[ \frac{N_{Ed}}{\chi_z \frac{N_{Rk}}{\gamma M_1}} + k_{zz} \frac{M_{z.Ed} + \Delta M_{z.Ed}}{M_{z.Rk}} \leq 1.0 \quad (6.62) \]

\[ \Delta M_{z.Ed} := 0 \quad \text{No eccentricity} \]

Iterations are needed in order to determine the maximum design resistance load \( N_{Ed} \).

Using EN 1993-1-1 2005

\[ N_{Rk} := A_{\text{eff}N} f_y \]

\[ C_{mi.0} (N_{Ed.1}) := 1 - 0.18 \frac{N_{Ed.1}}{N_{c.r.z}} \]
Given

\[
N_{\text{Ed.1}} := 1 \text{ kN}
\]

\[
\frac{N_{\text{Ed.1}}}{(\chi_{z_N} N_{Rk})^{\gamma_{M1}}} + k_{zz}(N_{\text{Ed.1}}) \frac{M_{z,\text{Ed}} + \Delta M_{z,\text{Ed}}}{M_{z,Rk}} = 1
\]

\[N_{\text{Ed}} := \text{Find}(N_{\text{Ed.1}}) = 826.367 \text{ kN} \quad \text{Design buckling resistance}\]
Patch loading

Effective load length according to section 6.5 EN 1993-1-5

The cross section is of type box-section and therefore the factor \( m_1 \) will be calculated with half of the flange width.

\[
m_1 := \frac{f_{yf} b_f}{2 f_{yw} t_w} = 18.795 \quad \text{Equation 6.8}
\]

\[
m_2 := 0.02 \left( \frac{h_w}{t_f} \right)^2 = 33.63 \quad \text{Assuming } \lambda_F > 0.5 \text{ using equation 6.9}
\]

Load application type (a), see the figure below from figure 6.1 EN 1993-1-5

\[
k_F := 6 + 2 \left( \frac{h_w}{L} \right)^2 = 6.011 \quad \text{Type (a) according to figure 6.1}
\]

\[
l_y := S_s + 2 \cdot t_f \left( 1 + \sqrt{m_1 + m_2} \right) = 217.704\text{-mm} \quad \text{Equation 6.10}
\]

\[
F_{cr} := 0.9 \cdot k_F \cdot E \frac{t_w^3}{h_w} = 1.823 \times 10^3 \text{-kN} \quad \text{Equation 6.5}
\]

\[
\lambda_F := \frac{t_f}{h_w} \frac{f_{yw}}{F_{cr}} = 0.574 \quad \text{Equation 6.4}
\]

\[
\chi_F := \frac{0.5}{\lambda_F} = 0.871 < 1.0 \quad \text{ok!}
\]

\[
L_{eff} := \chi_F l_y = 189.588\text{-mm} \quad \text{Equation 6.2}
\]

\[
F_{Rd} := 2 \cdot \frac{f_{yw} L_{eff} t_w}{\gamma M_1} = 1.046 \times 10^3 \text{-kN} \quad \text{Design buckling resistance under transversal forces}
\]
Interaction between transverse force, bending moment and axial force:

The interaction is performed according to subchapter 7.2 (1), EN 1993-1-5. The action effects \( N_{Ed} \) and \( M_{Ed} \), in the interaction verification, need to include second order effects from the normal force. Therefore the reduction factors for column buckling need to be included in \( \eta_1 \).

\[
\eta_1 := 1 \\
\eta_2 := \frac{P}{F_{Rd}} = 0.177 \\
\eta := \eta_2 + 0.8 \cdot \eta_1 \\
\eta = 0.977 < 1.4 \text{ ok!}
\]

The design buckling resistance is 826.367 kN, where the interaction between the centric load and bending moment is decisive.
Reduced stress method level 1

A cross sectional verification is performed according to the verification format in section 10 in EN-1993-1-5. The action effects $N_{Ed}$ and $M_{Ed}$, in the interaction verification, need to include second order effects. The cross-section verification is performed in the mid point of the column, where the second order moment has its biggest amplitude and where the point load is located. In the mid point of the column, there are however no shear forces, but the effect of them will be overshadowed by the interaction of the bending moment and the concentrated force. Only a cross sectional verification is performed because it includes the second order effects from column buckling in the worst cross section.

Cross sectional verification - Local buckling:

$$I_{z.\text{gross}} = 1.314 \times 10^8 \text{mm}^4$$

same as before

Second order moment:

$$N_{Ed.R} := 759.24 \text{kN} \quad W_{z.\text{el}} := I_{z.\text{gross}} \frac{2}{a}$$

$$P_{Ed.R} := 0.22433449 N_{Ed.R}$$

The constant part in the equation is the relation between $P_{Ed}$ and $N_{Ed.R}$, in which are used in the performed analyses.

The deflection at mid-point is evaluated for a fully effective cross-section, this is because the reduction factor due to plate buckling is lower than 0.5 and thus can be neglected from the global analysis.

The solution to the differential equation gives the deflection at mid-point to:

$$w_{max.R} := \frac{P_{Ed.R} \tan \left( k_R \frac{L}{2} \right)}{2 \cdot k_R^3 \cdot E \cdot I_{z.\text{gross}}} - \frac{P_{Ed.R}}{2 \cdot k_R^2 \cdot E \cdot I_{z.\text{gross}}} \cdot \frac{L}{2} + \frac{e_{0.R}}{2 k_R^2 - \left( \frac{\pi}{k_R \cdot L} \right)^2} + e_{0.R}$$

$$w_{max.R} = 25.355 \text{mm}$$

A.10.9
The total moment including the second order moment
\[
M_{2,\text{Ed.R}} := \frac{P_{\text{Ed.R}} \cdot L}{4} + N_{\text{Ed.R}} \cdot w_{\text{max,R}} = 189.574 \, \text{kN} \cdot \text{m}
\]

Design stresses:

Both the flange plate and the web plate needs to be verified. In the flange plate the transverse stresses from the point load will be neglected. In the web plate the full stress field is considered, however the distribution of stresses are more favourable and therefore might lead to a higher capacity than the flange plate so both plates need to be verified.

\[ W_{z,\text{el}} = 8.761 \times 10^5 \, \text{mm}^3 \]

\[
\sigma_M := \frac{M_{2,\text{Ed.R}}}{W_{z,\text{el}}} = 216.382 \, \text{MPa}
\]

Stress from the bending moment in the junction between the flange and the web plate

\[
\sigma_N := \frac{N_{\text{Ed.R}}}{A_{\text{gross}}} = 84.835 \, \text{MPa}
\]

\[
\psi_x := \frac{\sigma_N - \sigma_M}{\sigma_N + \sigma_M} = -0.437
\]

\[
k_{\sigma,R.w} := 7.81 - 6.29 \cdot \psi_x + 9.78 \cdot \psi_x^2 = 12.422
\]

\[
k_{\sigma,R.f} := 4.0
\]

\[
\sigma_{\text{Ed,x}} := \frac{M_{2,\text{Ed.R}}}{W_{z,\text{el}}} + \frac{N_{\text{Ed.R}}}{A_{\text{gross}}} = 301.217 \, \text{MPa}
\]

Total stress in the axial direction used for both flange plate and web plate

\[
\sigma_{\text{Ed,z}} := \frac{P_{\text{Ed.R}}}{\chi_F l_y t_w^2} = 57.78 \, \text{MPa}
\]

Total stress in the transversal direction used for web plate

Critical stresses:

\[
\sigma_{E,f} := \frac{\pi^2 \cdot E \cdot t_f^2}{12 \left(1 - \nu^2\right) \cdot b_f^2} = 113.364 \, \text{MPa}
\]

\[
\sigma_{E,w} := \frac{\pi^2 \cdot E \cdot t_w^2}{12 \left(1 - \nu^2\right) \cdot h_w^2} = 133.75 \, \text{MPa}
\]

\[
\sigma_{\text{cr,x,f}} := k_{\sigma,R,f} \cdot \sigma_{E,f} = 453.455 \, \text{MPa}
\]

\[
\sigma_{\text{cr,x,w}} := k_{\sigma,R,w} \cdot \sigma_{E,w} = 1.661 \times 10^3 \, \text{MPa}
\]

\[
\sigma_{\text{cr,c,z}} := 1.881 \cdot \sigma_{E,w} = 251.583 \, \text{MPa}
\]

A.10.10
The critical stress in the transverse direction for plate type behaviour is calculated for the web plate only.
\[ \sigma_{cr.p.z} := 988.35 \text{ MPa} \quad \text{Obtained from EB-plate} \]

Due to the fact that the concentrated load is only applied to a certain part of the flange, plate like buckling will prevail.

Thus:
\[ \frac{\sigma_{cr.p.z}}{\sigma_{cr.c.z}} = 3.929 > 2.0 \]

\[ \sigma_{cr.z} := \sigma_{cr.p.z} \]

Load amplification factors:
\[ \alpha_{cr.x.f} := \frac{\sigma_{cr.x.f}}{\sigma_{Ed.x}} = 1.505 \quad \alpha_{cr.x.w} := \frac{\sigma_{cr.x.w}}{\sigma_{Ed.x}} = 5.516 \]

\[ \alpha_{cr.z} := \frac{\sigma_{cr.z}}{\sigma_{Ed.z}} = 17.105 \quad \text{Uses only for web plate} \]

\( \tau_{Ed} \) is equal to zero, which means the term \( (1/\alpha_{cr,t}^2) \) in the equation for the load amplification factor will be disregarded, (zero).

\[ \psi_z := 1.0 \]

\[ \alpha_{cr.w} := \left[ \frac{1 + \psi_x}{4\alpha_{cr.x.w}} + \frac{1 + \psi_z}{4\alpha_{cr.z}} + \left( \frac{1 + \psi_x}{4\alpha_{cr.x.w}} + \frac{1 + \psi_z}{4\alpha_{cr.z}} \right)^2 + \frac{1 - \psi_x}{2\alpha_{cr.x.w}^2} + \frac{1 - \psi_z}{2\alpha_{cr.z}^2} \right]^{-1} \]

\[ \alpha_{cr.w} = 4.59 \]

\[ \alpha_{ult.k.w} := \sqrt{\left( \frac{\sigma_{Ed.x}}{f_y} \right)^2 + \left( \frac{\sigma_{Ed.z}}{f_y} \right)^2 - \left( \frac{\sigma_{Ed.x}}{f_y} \right) \left( \frac{\sigma_{Ed.z}}{f_y} \right)} \]

\[ \alpha_{ult.k.w} = 1.282 \]

\[ \lambda_{p.R.f} := \frac{f_y}{\sigma_{cr.x.f}} = 0.885 \quad \lambda_{p.R.w} := \sqrt{\frac{\alpha_{ult.k.w}}{\alpha_{cr.w}}} = 0.529 \quad \text{Slenderness parameter} \]
Reduction factors:

\[
\rho_{x,w} := \frac{\lambda_{p,R,w} - 0.055(3 + \psi_x)}{\lambda_{p,R,w}^2} = 1.387
\]

> 1,0 indicates that the web is not in class 4 and therefore, no need to reduce the web.

\[
\psi_{x,f} := 1.0
\]

\[
\rho_{x,f} := \frac{\lambda_{p,R,f} - 0.055(3 + \psi_{x,f})}{\lambda_{p,R,f}^2} = 0.849
\]

< 1.0 ok!

The reduction factors in the transverse direction will not be calculated. This is because in the web plate the full stress field gives a cross-sectional class lower than class 4. And for the flange plate the transverse stresses are zero and therefore there is no need to calculated a reduction factor.

Verification of the web:

\[
\left(\frac{\sigma_{Ed,x}}{f_y}\right)^2 + \left(\frac{\sigma_{Ed,z}}{f_y}\right)^2 - \left(\frac{\sigma_{Ed,x}}{f_y}\right)\left(\frac{\sigma_{Ed,z}}{f_y}\right) = 0.60834
\]

Verification of the flange:

\[
\left(\frac{\sigma_{Ed,x}}{\rho_{x,f} f_y}\right) = 1
\]

The cross-sectional verification shows that the flange plate is more prone to buckling than the web plate and therefore governs the resistance of the cross-section.

The cross-sectional bearing capacity is \( Prd = 170.3\) kN and \( N_{Ed} = 759.2\) kN with respect to the flange.
Reduced stress method level 1 based on FEM result

The critical load is taken from the first local buckling mode from the linear FE-analysis. The first local buckling mode excited both the buckling behaviour in the flange plate and the web plate. However, the buckles were much more pronounced in the flange plate, and therefore the critical stress in the web plate might have been slightly underestimated.

Cross sectional verification - Local buckling:

\[ I_{z,\text{gross}} = 1.314 \times 10^8 \text{mm}^4 \quad W_{z,\text{el}} = 8.761 \times 10^5 \text{mm}^3 \]

Design stresses:

\[ N_{\text{Ed.F}} := 890.14 \text{kN} \]

\[ P_{\text{Ed.F}} := 0.22433449 N_{\text{Ed.F}} \]

The constant part in the equation is the relation between \( P_{\text{Ed}} \) and \( N_{\text{Ed.R}} \), in which are used in the performed analyses.

The deflection at mid-point is evaluated for a fully effective cross-section, this is because the reduction factor due to plate buckling is lower than 0.5 and thus can be neglected from the global analysis.

The solution to the differential equation gives the deflection at mid-point to:

\[ w_{\text{max,F}} = \frac{P_{\text{Ed,F}} \tan \left( \frac{k_F \frac{L}{2}}{2} \right)}{2 \cdot k_F \cdot E \cdot I_{z,\text{gross}}} + \frac{-P_{\text{Ed,F}}}{2 \cdot k_F \cdot E \cdot I_{z,\text{gross}}} + \frac{e_{0,F}}{\left( \frac{\pi}{k_F \cdot L} \right)^2 - 1} + e_{0,F} \]

\[ w_{\text{max,F}} = 27.056 \text{mm} \]

Second order moment

\[ M_{2,\text{Ed.F}} := \frac{P_{\text{Ed,F}} L}{4} + N_{\text{Ed,F}} w_{\text{max,F}} = 223.773 \text{kN} \cdot \text{m} \]

A.10.13
Design stresses:
Both the flange plate and the web plate need to be verified. In the flange plate the transverse stresses from the point load will be neglected. In the web plate the full stress field is considered, however the distribution of stresses are more favourable and therefore might lead to a higher capacity than the flange plate so both plates need to be verified.

\[ W_{z,\text{el}} = 8.761 \times 10^5 \text{mm}^3 \]

\[ \sigma_{M,F} := \frac{M_{2,\text{Ed,F}}}{W_{z,\text{el}}} = 255.416 \text{MPa} \]
Stress from the bending moment in the junction between the flange and the web plate

\[ \sigma_{N,F} := \frac{N_{\text{Ed,F}}}{A_{\text{gross}}} = 99.461 \text{MPa} \]

\[ \psi_{x,F} := \frac{\sigma_{N,F} - \sigma_{M,F}}{\sigma_{N,F} + \sigma_{M,F}} = -0.439 \]

\[ k_{\sigma,F,w} := 7.81 - 6.29 \psi_x + 9.78 \psi_x^2 = 12.422 \]

\[ k_{\sigma,F,f} := 4.0 \]

\[ \sigma_{\text{Ed,x,F}} := \frac{M_{2,\text{Ed,F}}}{W_{z,\text{el}}} + \frac{N_{\text{Ed,F}}}{A_{\text{gross}}} = 354.878 \text{MPa} \]
Total stress in the axial direction used for both flange plate and web plate

\[ \sigma_{\text{Ed,z,F}} := \frac{P_{\text{Ed,F}}}{X_F l_y t_w/2} = 67.742 \text{MPa} \]
Total stress in the transversal direction used for web plate

Critical stresses:

\[ P_{\text{cr,F}} := 1728.89 \text{kN} \]
\[ P_{\text{cr,w}} := 4392.06 \text{kN} \]

Load amplification factors:

\[ \alpha_{\text{cr,F}} := \frac{P_{\text{cr,F}}}{P_{\text{Ed,F}}} \]
\[ \alpha_{\text{cr,w}} := \frac{P_{\text{cr,w}}}{P_{\text{Ed,F}}} \]

\[ \alpha_{\text{ult.k.w,F}} := \left( \frac{\sigma_{\text{Ed,x,F}}}{f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed,z,F}}}{f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed,x,F}}}{f_y} \right) \left( \frac{\sigma_{\text{Ed,z,F}}}{f_y} \right) \right)^{-1} \]

\[ \alpha_{\text{ult.k.w,F}} = 1.088 \]

A.10.14
\[ \alpha_{\text{ult.k.f.F}} := \left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right)^2 \]  

\[ \alpha_{\text{ult.k.f.F}} = 1 \]

\[ \lambda_{p.F.f} := \sqrt{\frac{\alpha_{\text{ult.k.f.F}}}{\alpha_{\text{cr.F}}}} = 0.34 \quad \lambda_{p.F.w} := \sqrt{\frac{\alpha_{\text{ult.k.w.F}}}{\alpha_{\text{cr.w}}}} = 0.487 \quad \text{Slenderness parameter} \]

Reduction factors:

\[ \rho_{x,w,F} := \frac{\lambda_{p,F.w} - 0.055(3 + \psi_x)}{\lambda_{p,F.w}^2} = 1.459 \quad > 1,0 \quad \text{indicates that the web is not in class 4 and therefore, no need to reduce the web.} \]

\[ \rho_{x,f,F} := \frac{\lambda_{p,F.f} - 0.055(3 + \psi_{x,f})}{\lambda_{p,F.f}^2} = 1.038 \quad > 1,0 \quad \text{No need to reduce the flange.} \]

The reduction factors in the transverse direction will not be calculated. This is because in the web plate the full stress field gives a cross-sectional class lower than class 4. And for the flange plate the transverse stresses are zero and therefore there is no need to calculated a reduction factor.

**Verification of the web:**

\[ \left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right)^2 + \left( \frac{\sigma_{\text{Ed.z.F}}}{f_y} \right)^2 - \left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right) \left( \frac{\sigma_{\text{Ed.z.F}}}{f_y} \right) = 0.84497 \]

**Verification of the flange:**

\[ \left( \frac{\sigma_{\text{Ed.x.F}}}{f_y} \right) = 1 \]

The cross sectional verification shows that the flange plate is more prone to buckling than the web plate and therefore governs the resistance of the cross-section.

The cross sectional bearing capacity is \( P_{\text{rd}} = 199.7 \) kN and \( N_{\text{Ed}} = 890.14 \) kN with respect to the flange.
Appendix B.1 I-girder

Details regarding the FE-models corresponding to the case of lateral torsional buckling induced by two equal end moments respectively two concentrated loads at quarter point are presented here. The length of the I-girder was set to 4 meters with a steel grade of S355. In Figure B.1.1 the two models are illustrated for the two different load cases.

In the left figure the I-girder is subjected to equal end moments and in the right figure the I-girder is subjected to two concentrated loads at quarter point.

Slender web plate

The geometric properties and the imperfections used for the case when varying the thickness of the web plate is presented in Table B.1.1 respectively B.1.2. The denotations in the tables are as depicted in Figure B.1.2.
The thickness of the flange plate for each slenderness ratio, $\lambda_{plate}$, is solved with the equations presented in Table 4.1 in subchapter (4.1) as:

$$\lambda_{plate} = \frac{c/t}{124\varepsilon} = \frac{h_w/t_w}{124 * \sqrt{235/f_y}}$$

For a plate slenderness ratio of 1.1 this gives a plate thickness of:

$$t_w = \frac{h_w}{\lambda_{plate} * 124 * \sqrt{235/f_y}} = \frac{600}{1.1 * 124 * \sqrt{235/355}} = 5.41 \text{ mm}$$

The thickness of the flange is set to a constant value, which results in a slight change of slenderness ratio depending on the thickness of the web. The slenderness of the flange is calculated as:

$$\lambda_{plate} = \frac{c/t}{14\varepsilon} = \frac{(b_f - t_w)/2}{t_f * 14 * \sqrt{235/f_y}} = \frac{(200 - 3.96)/2}{9 * 14 * \sqrt{235/355}} = 0.96$$

Table B.1.1: Geometric properties for when varying the slenderness, $\lambda_{plate}$, of the web plate.

<table>
<thead>
<tr>
<th>$\lambda_{plate}$</th>
<th>$t_w$ [mm]</th>
<th>$t_f$ [mm]</th>
<th>$b_f$ [mm]</th>
<th>$h_w$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>5.41</td>
<td>9</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>1.2</td>
<td>4.96</td>
<td>9</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>1.3</td>
<td>4.57</td>
<td>9</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>1.4</td>
<td>4.25</td>
<td>9</td>
<td>200</td>
<td>600</td>
</tr>
<tr>
<td>1.5</td>
<td>3.96</td>
<td>9</td>
<td>200</td>
<td>600</td>
</tr>
</tbody>
</table>

Table B.1.2: Structural and geometric imperfection for when varying the slenderness, $\lambda_{plate}$, of the web plate.

<table>
<thead>
<tr>
<th>$\lambda_{plate}$</th>
<th>$b_1$ [mm]</th>
<th>$h_1$ [mm]</th>
<th>$\sigma_{b1}$ [MPa]</th>
<th>$\sigma_{h1}$ [MPa]</th>
<th>$\Delta_L$ [mm]</th>
<th>$\Delta_H$ [mm]</th>
<th>$\Delta_D$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>20.25</td>
<td>12.16</td>
<td>40.0</td>
<td>15.0</td>
<td>4.27</td>
<td>2.33</td>
<td>0.82</td>
</tr>
<tr>
<td>1.2</td>
<td>20.25</td>
<td>11.15</td>
<td>40.0</td>
<td>13.7</td>
<td>4.27</td>
<td>2.54</td>
<td>0.82</td>
</tr>
<tr>
<td>1.3</td>
<td>20.25</td>
<td>10.29</td>
<td>40.0</td>
<td>12.6</td>
<td>4.27</td>
<td>2.75</td>
<td>0.82</td>
</tr>
<tr>
<td>1.4</td>
<td>20.25</td>
<td>9.56</td>
<td>40.0</td>
<td>11.7</td>
<td>4.27</td>
<td>2.97</td>
<td>0.82</td>
</tr>
<tr>
<td>1.5</td>
<td>20.25</td>
<td>8.92</td>
<td>40.0</td>
<td>10.9</td>
<td>4.27</td>
<td>3.18</td>
<td>0.82</td>
</tr>
</tbody>
</table>

B.1.2
The global slenderness of the I-girder is calculated according to the equation presented in Table 2.1, and when set as the leading imperfection the magnitude is:

\[ \Delta_e = \pm \frac{L}{750} \times 0.8 = \pm \frac{4000}{750} \times 0.8 = 4.27 \text{ mm} \]

The curvature of the plate is set as an accompanying imperfection and calculated as:

\[ \Delta_h = \pm \frac{h_w^2}{16000 t_w} \times 0.8 \times 0.7 = \pm \frac{600^2}{16000 \times 5.41} \times 0.8 \times 0.7 = 2.33 \text{ mm} \]

The distortion of the flange is set as an accompanying imperfection and calculated as:

\[ \Delta_b = \pm \frac{b_f^2}{3000 t_f} \times 0.8 \times 0.7 = \pm \frac{200^2}{3000 \times 9} \times 0.8 \times 0.7 = 0.82 \text{ mm} \]

The width of the tensile zone of the residual stresses in the web is calculated as:

\[ h_1 = 2.25 \times t_w = 2.25 \times 5.41 = 12.16 \text{ mm} \]

The width of the tensile zone of the residual stresses in the flange is calculated as:

\[ b_1 = 2.25 \times t_f = 2.25 \times 9 = 20.25 \text{ mm} \]

The compression stresses for the web plate and the flange are solved through force equilibrium:

\[ \sigma_{h1} = \frac{f_y \times 2h_1}{(h_w - 2h_1)} = \frac{355 \times 2 \times 12.16}{(600 - 2 \times 12.16)} = 15.0 \text{ MPa} \]

\[ \sigma_{b1} = \frac{f_y \times b_1}{(b_f - b_1)} = \frac{355 \times 20.25}{(200 - 20.25)} = 40.0 \text{ MPa} \]

The eigenmodes used to seed the geometric imperfections for a plate slenderness of 1.1 loaded with two equal end moments is presented in Figure B.1.3.

Figure B.1.3: Eigenmodes used to seed the geometric imperfections for when varying the slenderness of the web plate, loaded with two equal end moments.
The eigenmodes used to seed the geometric imperfections for a plate slenderness of 1.1 loaded with two equal concentrated loads at quarter point is presented in Figure B.1.4.

Figure B.1.4: Eigenmodes used to seed the geometric imperfections for when varying the slenderness of the web plate, loaded with two concentrated loads at quarter point.

The moment bearing capacity and the design resistance regarding the point load from the finite element analyses are depicted in Table B.1.3 for each value of the plate slenderness and for the case of two equal end moments.

Table B.1.3: Moment bearing capacity and point load bearing capacity for the different web plate slenderness ratios.

<table>
<thead>
<tr>
<th>$\lambda_{\text{plate}}$</th>
<th>$M_{\text{bra}}$ [kNm]</th>
<th>$P_{\text{rd}}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>477.38</td>
<td>191.51</td>
</tr>
<tr>
<td>1.2</td>
<td>465.44</td>
<td>183.54</td>
</tr>
<tr>
<td>1.3</td>
<td>454.67</td>
<td>182.39</td>
</tr>
<tr>
<td>1.4</td>
<td>446.95</td>
<td>161.55</td>
</tr>
<tr>
<td>1.5</td>
<td>444.44</td>
<td>143.35</td>
</tr>
</tbody>
</table>

**Slender flange plate**

The geometric properties and the imperfections used for the case when varying the thickness of the flange plate is presented in Table B.1.4 respectively B.1.5. The denotations in the tables are as depicted in Figure B.1.2.

The thickness of the flange plate for each slenderness ratio, $\lambda_{\text{plate}}$, is solved with the equations presented in Table B.1.1 as:

$$\lambda_{\text{plate}} = \frac{c/t}{14c} = \frac{(b_f - t_w)/2}{t_f * 14 * \sqrt{235/f_y}}$$
For a plate slenderness ratio of 1.1 this gives a plate thickness of:

\[
t_f = \frac{(b_f - t_w)/2}{\lambda_{plate} \times 14 \times \sqrt{235/f_y}} = \frac{(200 - 6.27)/2}{1.1 \times 14 \times \sqrt{235/355}} = 7.73 \text{mm}
\]

The thickness of the web was set to a constant value with a set plate slenderness of 0.96, to have the same plate slenderness as the flange plate in the previous case.

\[
t_w = \frac{600}{0.96 \times 124 \times \sqrt{235/355}} = 6.27 \text{mm}
\]

Table B.1.4: Geometric properties for when varying the slenderness, \(\lambda_{plate}\), of the flange plate.

<table>
<thead>
<tr>
<th>(\lambda_{plate})</th>
<th>(t_w) [mm]</th>
<th>(t_f) [mm]</th>
<th>(b_f) [mm]</th>
<th>(h_w) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>6.27</td>
<td>7.73</td>
<td>200</td>
<td>601.27</td>
</tr>
<tr>
<td>1.2</td>
<td>6.27</td>
<td>7.09</td>
<td>200</td>
<td>601.91</td>
</tr>
<tr>
<td>1.3</td>
<td>6.27</td>
<td>6.54</td>
<td>200</td>
<td>603.46</td>
</tr>
<tr>
<td>1.4</td>
<td>6.27</td>
<td>6.07</td>
<td>200</td>
<td>602.93</td>
</tr>
<tr>
<td>1.5</td>
<td>6.27</td>
<td>5.67</td>
<td>200</td>
<td>603.33</td>
</tr>
</tbody>
</table>

Table B.1.5 Structural and geometric imperfection for when varying the slenderness, \(\lambda_{plate}\), of the flange plate.

<table>
<thead>
<tr>
<th>(\lambda_{plate})</th>
<th>(b_1) [mm]</th>
<th>(h_1) [mm]</th>
<th>(\sigma_{b1}) [MPa]</th>
<th>(\sigma_{h1}) [MPa]</th>
<th>(\Delta_L) [mm]</th>
<th>(\Delta_h) [mm]</th>
<th>(\Delta_b) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>17.39</td>
<td>14.11</td>
<td>33.8</td>
<td>17.5</td>
<td>4.27</td>
<td>2.02</td>
<td>0.97</td>
</tr>
<tr>
<td>1.2</td>
<td>15.95</td>
<td>14.11</td>
<td>30.8</td>
<td>17.5</td>
<td>4.27</td>
<td>2.02</td>
<td>1.05</td>
</tr>
<tr>
<td>1.3</td>
<td>14.72</td>
<td>14.11</td>
<td>28.2</td>
<td>17.5</td>
<td>4.27</td>
<td>2.02</td>
<td>1.14</td>
</tr>
<tr>
<td>1.4</td>
<td>13.66</td>
<td>14.11</td>
<td>26.0</td>
<td>17.5</td>
<td>4.27</td>
<td>2.03</td>
<td>1.23</td>
</tr>
<tr>
<td>1.5</td>
<td>12.76</td>
<td>14.11</td>
<td>24.2</td>
<td>17.5</td>
<td>4.27</td>
<td>2.03</td>
<td>1.32</td>
</tr>
</tbody>
</table>

The global slenderness of the I-girder is calculated according to the equation presented in Table 2.1 in Chapter 2, and when set as the leading imperfection the magnitude is:

\[
\Delta_L = \pm \frac{L}{750} \times 0.8 = \frac{4000}{750} \times 0.8 = 4.27 \text{mm}
\]
The curvature of the web plate is set as an accompanying imperfection and calculated as:

\[ \Delta_h = \pm \frac{h_w^2}{16000 t_w} \times 0.8 \times 0.7 = \frac{601.27^2}{16000 \times 6.27} \times 0.8 \times 0.7 = 2.02 \text{ mm} \]

The distortion of the flange is set as an accompanying imperfection and calculated as:

\[ \Delta_p = \pm \frac{b_f^2}{3000 t_f} \times 0.8 \times 0.7 = \frac{200^2}{3000 \times 7.73} \times 0.8 \times 0.7 = 0.97 \text{ mm} \]

The width of the tensile zone of the residual stresses in the web is calculated as:

\[ h_1 = 2.25 \times t_w = 2.25 \times 6.27 = 14.11 \text{ mm} \]

The width of the tensile zone of the residual stresses in the flange is calculated as:

\[ b_1 = 2.25 \times t_f = 2.25 \times 7.73 = 17.39 \text{ mm} \]

The compression stresses for the web plate and the flange are solved through force equilibrium:

\[ \sigma_{h1} = \frac{f_y \times 2h_1}{(h_w - 2h_1)} = \frac{355 \times 2 \times 14.11}{(601.27 - 2 \times 14.11)} = 17.5 \text{ MPa} \]

\[ \sigma_{b1} = \frac{f_y \times b_1}{(b_f - b_1)} = \frac{355 \times 17.39}{(200 - 17.39)} = 33.8 \text{ MPa} \]

The eigenmodes used to seed the geometric imperfections for a plate slenderness of 1.1 loaded with two equal end moments is presented in Figure B.1.5.

![Figure B.1.5: Eigenmodes used to seed the geometric imperfections for when varying the slenderness of the flange plate, loaded with two equal end moments.](image)

The eigenmodes used to seed the geometric imperfections for a plate slenderness of 1.1 loaded with two equal concentrated loads at quarter point is presented in Figure B.1.6.
Figure B.1.6: Eigenmodes used to seed the geometric imperfections for when varying the slenderness of the flange plate, loaded with two concentrated loads at quarter point.

The moment bearing capacity and the design resistance from the FE-analyses is depicted in Table B.1.6 for each value of the plate slenderness regarding to the case of two equal end moments and for the point loads.

Table B.1.6: Moment bearing capacity and the design resistance obtained from FE-analyses for the different flange plate slenderness ratios.

<table>
<thead>
<tr>
<th>$\lambda_{plate}$</th>
<th>$M_{brd}$ [kNm]</th>
<th>$P_{rd}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>446.87</td>
<td>201.29</td>
</tr>
<tr>
<td>1.2</td>
<td>414.25</td>
<td>187.04</td>
</tr>
<tr>
<td>1.3</td>
<td>493.10</td>
<td>164.97</td>
</tr>
<tr>
<td>1.4</td>
<td>370.68</td>
<td>161.55</td>
</tr>
<tr>
<td>1.5</td>
<td>349.10</td>
<td>155.17</td>
</tr>
</tbody>
</table>
Appendix B.2 Box-section column

Details regarding the FE-models corresponding to the case of column buckling induced by centric axial load and the combination of centric axial load and transverse load are presented here. The length of the column was set to 4 meters with a steel grade of S355. In Figure B.2.1 the two models are illustrated, one for each load case.

Figure B.2.1: In the left figure the column is subjected to the centric axial load and in the right figure the column is subjected to the combined load effect of centric axial load and transverse load.

The geometric properties and the imperfections used when varying the thickness of the plate is presented in Table B.2.1. The denotations in the tables are as depicted in Figure B.2.2. The width of the plates $a$ and $b$ are set equal and in Table B.2.1 the width is denoted with $a$.

Figure B.2.2: But-welded box cross-section
The thickness of the plate for each slenderness ratio, $\lambda_{plate}$, is solved with the equations presented in Table 2.1, Chapter 2 as:

$$\lambda_{plate} = \frac{c}{t} = \frac{(b - t_f)/t_w}{42 \sqrt{235/f_y}}$$

When solving for the thickness of the plates a simplification has been done by setting the thickness $t_w$ and $t_f$ equal. For a plate slenderness ratio of 1.1 this gives a plate thickness of:

$$t = \frac{a}{1 + \lambda_{plate} \cdot 42 \cdot \sqrt{235/f_y}} = \frac{300}{1 + 1.1 \cdot 42 \cdot \sqrt{235/355}} = 7.77 \text{ mm}$$

Table B.2.1: Geometric properties and imperfections for when varying the slenderness, $\lambda_{plate}$, of the plate.

<table>
<thead>
<tr>
<th>$\lambda_{plate}$</th>
<th>$t$ [mm]</th>
<th>$a$ [mm]</th>
<th>$b_c$ [mm]</th>
<th>$\sigma_c$ [MPa]</th>
<th>$\Delta_\lambda$ [mm]</th>
<th>$\Delta_a$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>7.77</td>
<td>300</td>
<td>265.02</td>
<td>46.68</td>
<td>4.27</td>
<td>0.82</td>
</tr>
<tr>
<td>1.2</td>
<td>7.14</td>
<td>300</td>
<td>267.86</td>
<td>42.59</td>
<td>4.27</td>
<td>0.82</td>
</tr>
<tr>
<td>1.3</td>
<td>6.60</td>
<td>300</td>
<td>270.28</td>
<td>39.04</td>
<td>4.27</td>
<td>0.82</td>
</tr>
<tr>
<td>1.4</td>
<td>6.14</td>
<td>300</td>
<td>272.36</td>
<td>36.03</td>
<td>4.27</td>
<td>0.82</td>
</tr>
<tr>
<td>1.5</td>
<td>5.74</td>
<td>300</td>
<td>274.17</td>
<td>31.04</td>
<td>4.27</td>
<td>0.82</td>
</tr>
</tbody>
</table>

The global slenderness of the column is calculated according to the equation presented in Table 2.1, and when set as the leading imperfection the magnitude is:

$$\Delta_\lambda = \pm \frac{L}{750} \cdot 0.8 = \frac{4000}{750} \cdot 0.8 = 4.27 \text{ mm}$$

The curvature of the plate is set as an accompanying imperfection and calculated as:

$$\Delta_a = \pm \frac{a - t_w}{200} \cdot 0.8 \cdot 0.7 = \frac{300 - 7.77}{200} \cdot 0.8 \cdot 0.7 = 0.82 \text{ mm}$$

The width of the compression zone of the residual stresses in the web is calculated as:

$$b_c = a - 2 \cdot 2.25 \cdot t_w = 300 - 2 \cdot 2.25 \cdot 7.77 = 265.02 \text{ mm}$$

The compression stresses for the plate is solved through force equilibrium:

$$\sigma_c = \frac{f_y \cdot 2 \cdot 2.25 \cdot t_w}{b_c} = \frac{355 \cdot 2 \cdot 2.25 \cdot 7.77}{265.04} = 46.8 \text{ MPa}$$

B.2.2
Equal slenderness

The slenderness of both the web plate and the flange plate is varied simultaneously and the geometric properties and imperfections are taken from Table B.2.1. The eigenmodes used to seed the geometric imperfections for a plate slenderness of 1.1 loaded with centric axial load is presented in Figure B.2.3.

![Eigenmodes](image1)

Figure B.2.3: Eigenmodes used to seed the geometric imperfections for when varying the slenderness of the plates simultaneously under centric axial load.

The eigenmodes used to seed the geometric imperfections for a plate slenderness of 1.1 loaded with a combination of centric axial load and transverse load is presented in Figure B.2.4. The eigenmodes are produced for the load case of only centric axial compression through a linear buckling analysis.

![Eigenmodes](image2)

Figure B.2.4 Eigenmodes used to seed the geometric imperfections for when varying the slenderness of the plate under a combined load of a centric axial load and transverse load.
The bearing capacity regarding centric axial load, $N_{rd,1}$, and the design resistance regarding the combined effect of centric axial load, $N_{rd,2}$, and transverse load, $P_{rd,2}$, from the finite element analyses are depicted in table B.2.2 for each value of the plate slenderness.

Table B.2.2: Centric axial load bearing capacity, $N_{rd,1}$, and bearing capacity regarding the combined effect of centric axial load, $N_{rd,2}$, and transverse load, $P_{rd,2}$, for the different plate slenderness ratios.

<table>
<thead>
<tr>
<th>$\lambda_{plate}$</th>
<th>$N_{rd,1}$ [kN]</th>
<th>$N_{rd,2}$ [kN]</th>
<th>$P_{rd,2}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>2696.6</td>
<td>977.8</td>
<td>219.3</td>
</tr>
<tr>
<td>1.2</td>
<td>2297.0</td>
<td>888.4</td>
<td>199.3</td>
</tr>
<tr>
<td>1.3</td>
<td>2009.6</td>
<td>799.2</td>
<td>179.3</td>
</tr>
<tr>
<td>1.4</td>
<td>1976.7</td>
<td>712.9</td>
<td>159.9</td>
</tr>
<tr>
<td>1.5</td>
<td>1546.3</td>
<td>640.0</td>
<td>143.6</td>
</tr>
</tbody>
</table>

**Different slenderness**

The slenderness of the web is set constant, $\lambda_{plate} = 1.1$, meanwhile the slenderness of the flange plate is varied. The geometric properties and imperfections are taken from Table B.2.1. The transverse load is applied such that induces bending about the minor principal axis.

The eigenmodes used to seed the geometric imperfections for a web slenderness of 1.1 and a flange plate slenderness of 1.2 loaded with centric axial load is presented in Figure B.2.5.

![Figure B.2.5: Eigenmodes used to seed the geometric imperfections for a web slenderness of 1.1 and a flange slenderness of 1.2 under centric axial load.](image)

The eigenmodes used to seed the geometric imperfections for a web slenderness of 1.1 and a flange plate slenderness of 1.2 loaded with a combination of centric axial load and transverse load is presented in figure B.2.6. The eigenmodes are produced for the load case of only centric axial compression through a linear buckling analysis.
Figure B.2.6: Eigenmodes used to seed the geometric imperfections for a web slenderness of 1.1 and a flange slenderness of 1.2 under a combined load of a centric axial load and transverse load.

The bearing capacity regarding centric axial load, $N_{Rd,1}$, and the design resistance regarding the combined effect of centric axial load, $N_{Rd,2}$, and transverse load, $P_{Rd,2}$, from the finite element analyses are depicted in Table B.2.5 for each value of the plate slenderness.

Table B.2.5  Centric axial load bearing capacity, $N_{Rd,1}$, and bearing capacity regarding the combined effect of centric axial load, $N_{Rd,2}$, and transverse load, $P_{Rd,2}$, for the different slenderness ratios of the flange plate.

<table>
<thead>
<tr>
<th>$\lambda_{plate,flange}$</th>
<th>$N_{Rd,1}$ [kN]</th>
<th>$N_{Rd,2}$ [kN]</th>
<th>$P_{Rd,2}$ [kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2</td>
<td>2474.3</td>
<td>946.9</td>
<td>212.5</td>
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<tr>
<td>1.3</td>
<td>2353.0</td>
<td>919.2</td>
<td>206.2</td>
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<tr>
<td>1.4</td>
<td>2337.3</td>
<td>879.4</td>
<td>197.3</td>
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<tr>
<td>1.5</td>
<td>2120.9</td>
<td>819.9</td>
<td>183.9</td>
</tr>
</tbody>
</table>

B.2.5