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Potential games for subcarrier allocation in multi-cell networks with D2D communications

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Abstract—This paper investigates the subcarrier allocation problem for uplink transmissions in a multi-cell network, where device-to-device communications are enabled. We focus on maximizing the aggregate transmission rate in the system accounting for both inter- and intra-cell interference. This problem is computationally hard due to its nonconvex and combinatorial nature. However, we show that it can be described by a potential game, and thus a Nash equilibrium can be found using iterative algorithms based on best/better response dynamics. In particular, we propose a simple iterative algorithm with limited signaling that is guaranteed to converge to an equilibrium point, corresponding to a local maximum of the potential function. Using extensive simulations, we show that the algorithm converges quickly also for dense networks, and that the distance to the true optimum is often small, at least for the small-sized networks for which we were able to compute the true optimum.

Index Terms—Subcarrier allocation, Device-to-Device, potential games, Nash equilibrium.

I. INTRODUCTION

Device-to-Device (D2D) communication as an underlay to cellular networks is a promising technology for enhancing the performance of next generation (5G) systems [1]. The basic idea of D2D communication is to allow mobile devices in close proximity to communicate directly, bypassing the base station (BS). This leads to several advantages, such as reducing the load on the cellular system and improving coverage, throughput, transmission latency, energy, and spectral efficiency [2].

Due to the expenses and scarcity of licensed spectrum, it is preferred to implement D2D communication in *underlay in-band mode*, where D2D transmitters opportunistically access the frequency s occupied by cellular users under the network control. Although this mode can improve both the spectral efficiency and the system capacity, it leads to new interference situations and signaling limitations for which efficient resource allocation (RA) methods are needed.

Considering the spectrum reuse from an optimization perspective usually leads to nonconvex and mixed integer formulations, where the optimal solution is in general very hard to compute, even for small-sized networks. Therefore, most existing works propose heuristic or suboptimal methods [3]–[6]. Optimal results are only obtained in very special situations. For example, the authors of [7] provide optimal solution to the resource sharing problem, but for a greatly simplified model of only one cellular user and one D2D pair. Other works limit

the RA analysis to a single cell case, considering different objectives (e.g., energy efficiency and sum-rate) with different system constraints [4,8]–[10].

In this work, we consider the subcarrier allocation problem for a multi-cell network, where D2D communications share the uplink (UL) resources with traditional cellular users¹. Our objective is to maximize the total rate of the system by taking advantage of frequency diversity among channels and by properly managing the interference level on each subcarrier. In this hybrid network, there is not only inter-cell multiple-access interference due to frequency reuse between cells, but also the intra-cell interference due to the presence of D2D connections (see Fig. 1). Moreover, in classical UL cellular systems, interfering users are located at a distance that amounts to at least the cell radius. Due to the D2D links, interfering transmissions can operate at any distance, potentially jeopardizing the system performance.

Given the nonconvex and combinatorial nature of this problem, we leverage on the theory of potential games to guarantee convergence to a Nash equilibrium via best response dynamics. Indeed, potential games have been already shown to be a valid approach for resource allocation in multi-cell wireless systems when trying to maximize user SINRs and energy efficiency [12] or striving to minimize interference [13].

Our main contribution in this paper is to apply the theory of potential games to sum-rate maximization in the challenging multi-cell D2D scenario. We propose a low-complexity iterative algorithm that is guaranteed to converge to a Nash equilibrium of the potential game. By numerical examples, we show that the algorithm converges quickly, also for dense networks, and that the solution obtained is not too far from the true optimum (at least not for the small-sized networks where we are able to find the truly optimal resource allocation).

The paper is organized as follows. In § II, we present the system model together with a general formulation of the subcarrier allocation problem. §III gives some background material on potential games. §IV develops our proposed strategic game formulation for the multi-cell D2D resource allocation problem, and the dynamic process which is guaranteed to converge to an equilibrium. This section also discusses the

¹The choice of sharing uplink instead of downlink resources is motivated by the asymmetric traffic loads in the two directions, and by the BS's ability to better handle the interference, compared to the mobile devices [11].

possible practical implementation of the proposed method. Numerical results are presented in §V. Finally, in §VI we summarize the paper and outline possible future extensions.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. System model

We consider UL transmissions in an LTE-A-like multi-cell system, where network-assisted D2D communication is enabled. We assume that mode selection to determine if each user should communicate in cellular or D2D mode has already been performed [14], and focus on the allocation of time-frequency resources to users.

The network is populated by a set $\mathcal{B}$ of BSs, each which serves users in its own cell. The set of D2D receivers, denoted by $\mathcal{D}$, can be seen as a set of virtual base stations, each serving a single user. We introduce the notation $\mathcal{K} = \mathcal{B} \cup \mathcal{D}$ for the set of receivers in the system. For each receiver $k \in \mathcal{K}$, we let $\mathcal{C}_k$ be the set of users served by k . Thus, $\mathcal{C}_k$ is a singleton if $k \in \mathcal{D}$, while it may contain many users if $k \in \mathcal{B}$.

The system has a set $\mathcal{F}$ of F orthogonal time-frequency physical resource blocks (RBs). Each BS is responsible for assigning RBs to the users in its own cell. For communications in cellular mode, we consider the channel allocation policy of legacy LTE systems, i.e., RBs are assigned orthogonally within the cell. For D2D communications, on the other hand, we assume *underlay in-band D2D communication*, where D2D transmitters opportunistically access the UL frequency resources occupied by cellular users; see Fig. 1 for an illustration.

Let G_{nk}^f denote the channel gain (which includes path loss and shadowing) between transmitter- n and receiver- k on RB- f and let P_n^f be the transmit power level used by transmitter- n on RB- f . We assume that transmit powers are allocated on a slower time-scale than RBs (as in the LTE UL power control [15]), and consider the transmit power matrix $\mathbf{P}$ to be constant and known. For each $k \in \mathcal{K}$, $\mathbf{X}_k$ represents the RB assignment to all users that transmit to receiver k . If $k \in \mathcal{B}$, $\mathbf{X}_k$ is the RB allocations for all UL transmissions in cell- k and if $k \in \mathcal{D}$, $\mathbf{X}_k$ is the RB allocation for the transmission of D2D pair- k . We label each element of $\mathbf{X}_k$ with x_n^f , which is 1 if user- $n \in \mathcal{C}_k$ is transmitting on RB- f , and 0 otherwise. We let $\mathbf{X}_{-k}$ be the set of allocation decisions taken by all receivers except receiver- k , and $\mathbf{X} = (\mathbf{X}_k, \mathbf{X}_{-k})$, $\forall k \in \mathcal{K}$ be the overall resource allocation in the system.

A Shannon-like capacity is considered as a measure of the instantaneous achievable rate on the links in the system. Specifically, the normalized rate (with respect to the bandwidth) in bps/Hz that user- $n \in \mathcal{C}_k$ can achieve on RB- f is

$$R_n^f(\mathbf{X}_{-k}) = \log_2 \left(1 + \frac{P_n^f G_{nk}^f}{\sigma^2 + I_k^f(\mathbf{X}_{-k})} \right), \quad (1)$$

where σ^2 is the noise power (assumed equal for all RBs), and $I_k^f(\mathbf{X}_{-k}) = \sum_{q \in \mathcal{K} \setminus \{k\}} \sum_{m \in \mathcal{C}_q} P_m^f G_{mk}^f x_m^f$ is the interference perceived at receiver- k on RB- f , which depends on the

resource allocation in all cells and D2D pairs, except the one which receiver- k belongs to.

Finally, we denote by F_n the number of RBs to be assigned to transmitter- n . We assume that $\sum_{n \in \mathcal{C}_k} F_n \leq F$, $\forall k \in \mathcal{K}$, so that the number of available RBs is sufficient to accommodate all communication requests.

Fig. 1. Example of both inter-cell and intra-cell interference in the UL transmissions of cellular network with D2D communications. For illustration purposes, we show only the interference caused by transmissions in Cell-1.

B. Problem formulation

We consider the problem of allocating RBs to users, aiming at maximizing the aggregate system rate while assigning a given number of RBs to each link and ensuring orthogonality among cellular transmissions within the same cell. This problem can be formally stated as the following integer programming problem

$$\begin{aligned} & \underset{\{x_n^f\}}{\text{maximize}} && \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{C}_k} \sum_{f \in \mathcal{F}} R_n^f(\mathbf{X}_{-k}) x_n^f \\ & \text{subject to} && \sum_{n \in \mathcal{C}_k} x_n^f \leq 1, \quad \forall f \in \mathcal{F}, \forall k \in \mathcal{K}, \end{aligned} \quad (2a)$$

$$\begin{aligned} & \sum_{f \in \mathcal{F}} x_n^f = F_n, \quad \forall n \in \mathcal{C}_k, \forall k \in \mathcal{K}, \\ & x_n^f \in \{0, 1\}, \quad \forall f \in \mathcal{F}, \forall n \in \mathcal{C}_k, \forall k \in \mathcal{K}, \end{aligned} \quad (2b)$$

where constraints (2a) are the orthogonality constraints, which are active only for $k \in \mathcal{B}$ because $|\mathcal{C}_k| = 1$ for $k \in \mathcal{D}$, while constraints (2b) ensure that each link is assigned the required number of RBs.

III. PRELIMINARIES ON POTENTIAL GAMES

In this section we briefly introduce the theory of potential games [16], which we will use to design a solution for the multi-cell D2D resource allocation problem formulated in (2).

A strategic game can be described by a triplet $\mathcal{G} = [\mathcal{K}, \{\mathcal{X}_k\}, \{U_k\}]$, where $\mathcal{K}$ is the set of players, $\mathcal{X}_k$ is the set of all possible strategies for the k th player, each strategy is represented by $\mathbf{X}_k$. $U_k(\mathbf{X}_k, \mathbf{X}_{-k})$ denotes the payoff for player- k ; a scalar function which depends on the strategy taken by all players of the game. Since any change in strategy from one player affects all other players, this triggers a dynamic process where players iteratively update their own strategies

as a reaction to the changes in the strategy of other players. Let us recall some useful definitions and results:

Definition 1 (Best- and better-response dynamics). *The best-response dynamic is a strategy updating rule where each player updates its strategy by selecting the one that produces the highest utility, assuming that the other players do not change their current strategies. That is, given a strategy profile $\bar{\mathbf{X}} = (\bar{\mathbf{X}}_k, \bar{\mathbf{X}}_{-k})$, player- k chooses its new strategy $\mathbf{X}_k \in \mathcal{X}_k$ such that*

$$\mathbf{X}_k \in \{\tilde{\mathbf{X}}_k \in \mathcal{X}_k : U_k(\tilde{\mathbf{X}}_k, \bar{\mathbf{X}}_{-k}) \geq U_k(\mathbf{X}_k, \bar{\mathbf{X}}_{-k}), \forall \mathbf{X}_k \in \mathcal{X}_k\}. \quad (3)$$

In the less demanding better-response dynamics, instead, the strategy update of player- k is defined by replacing condition (3) by

$$\mathbf{X}_k \in \{\tilde{\mathbf{X}}_k \in \mathcal{X}_k : U_k(\tilde{\mathbf{X}}_k, \bar{\mathbf{X}}_{-k}) \geq U_k(\bar{\mathbf{X}}_k, \bar{\mathbf{X}}_{-k})\}. \quad (4)$$

Definition 2 (Potential game). *A strategic game $\mathcal{G} = [\mathcal{K}, \{\mathcal{X}_k\}, \{U_k\}]$ is an exact potential game if there exists a function $\Phi : \mathcal{X}_1 \times \mathcal{X}_2 \times \dots \times \mathcal{X}_{|\mathcal{K}|} \rightarrow \mathcal{R}$ such that for any $k \in \mathcal{K}$*

$$U_k(\mathbf{X}_k, \mathbf{X}_{-k}) - U_k(\mathbf{X}'_k, \mathbf{X}_{-k}) = \Phi(\mathbf{X}_k, \mathbf{X}_{-k}) - \Phi(\mathbf{X}'_k, \mathbf{X}_{-k}), \quad (5)$$

where $\mathbf{X}_k$ and $\mathbf{X}'_k$ are two different strategies of player- k . Any such function Φ is called the exact potential function of $\mathcal{G}$.

Definition 3 (Nash equilibrium (NE)). *Given a strategic game $\mathcal{G} = [\mathcal{K}, \{\mathcal{X}_k\}, \{U_k\}]$, the K -tuple $(\mathbf{X}_1^*, \mathbf{X}_2^*, \dots, \mathbf{X}_{|\mathcal{K}|}^*) \in \mathcal{X}_1 \times \mathcal{X}_2 \times \dots \times \mathcal{X}_{|\mathcal{K}|}$ is a NE if $U_k(\mathbf{X}_k^*, \mathbf{X}_{-k}^*) \geq U_k(\mathbf{X}_k, \mathbf{X}_{-k}^*), \forall \mathbf{X}_k \neq \mathbf{X}_k^*, \forall k = 1, \dots, |\mathcal{K}|$. Put in words: at the NE no player has an incentive to unilaterally change its strategy.*

Potential games possess important properties that relate the optimizers (local optima) of the potential function to pure-strategy NE points of the game.

Lemma 1 ([17], Theorem 15). *If Φ is a potential function of the game $\mathcal{G}$ and $\mathbf{X}^* \in \operatorname{argmax}_{\mathbf{X}} \Phi(\mathbf{X})$ is a maximizer of the potential function, then $\mathbf{X}^*$ is a NE of the game.*

Another attractive property of finite potential games whose potential function is bounded from above, is that iterative processes based on one-side better response dynamic converge to the equilibrium set, as established by the following result:

Lemma 2 ([17], Theorem 19). *Let $\mathcal{G}$ be a potential game. Then both the best response dynamic and the better response dynamic will converge to a NE in a finite number of steps.*

IV. GAME FORMULATION FOR THE MULTI-CELL D2D RA

In this section we are interested in applying the results from Section III to the resource allocation problem in (2).

A. Solution based on best response dynamic

We propose a strategic game between all receivers in the set $\mathcal{K}$. The game is described by $\mathcal{G} = [\mathcal{K}, \{\mathcal{X}_k\}, \{U_k\}]$, where $\mathcal{X}_k$ is the set of all possible allocation decisions for all transmissions to receiver- $k \in \mathcal{K}$, while U_k is the utility

function of the k th player, given by the sum of all the achievable bit rates in the system, that is

$$U_k(\mathbf{X}_k, \mathbf{X}_{-k}) = \sum_{f \in \mathcal{F}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{C}_k} R_n^f(\mathbf{X}_{-k}) x_n^f. \quad (6)$$

U_k is a scalar function corresponding to the objective in Problem (2). Note that, despite the fact that each player- k aims at maximizing U_k with respect to its own strategy only, all players' utilities are chosen to be the same, which means that we can define

$$\Phi(\mathbf{X}) = U_k(\mathbf{X}_k, \mathbf{X}_{-k}), \quad \forall k \in \mathcal{K}. \quad (7)$$

Proposition 1. *The game $\mathcal{G} = [\mathcal{K}, \{\mathbf{X}_k\}, \{U_k\}]$ is an exact potential game with potential function $\Phi(\mathbf{X})$ given in Eq. (7).*

Proof. $\mathcal{G}$ is an identical interest game [18], for which condition (5) can be easily verified. $\square$

Since $\mathcal{G}$ is an exact potential game, the best response dynamic will converge to a NE of the game (Lemma 2). Thus, given any initial resource allocation for all transmissions in the system, the players take turn to play the game in sequential manner, choosing their best response strategy. This iterative process terminates when no player is willing to change its strategy, that is, when a NE is achieved.

We now turn our attention to deriving the best response of player- k . Given $\mathbf{X}_{-k}$, player- k updates its strategy by solving the following optimization problem

$$\begin{aligned} & \underset{\mathbf{X}_k}{\text{maximize}} && U_k(\mathbf{X}_k, \mathbf{X}_{-k}), \\ & \text{subject to} && \sum_{n \in \mathcal{C}_k} x_n^f \leq 1, \quad \forall f \in \mathcal{F} \\ & && \sum_{f \in \mathcal{F}} x_n^f = F_n, \quad \forall n \in \mathcal{C}_k, \\ & && x_n^f \in \{0, 1\}, \quad \forall n \in \mathcal{C}_k, \forall f \in \mathcal{F}. \end{aligned} \quad (8)$$

The objective function in (8) is given in Eq. (6) and it can be expressed explicitly as function of $\mathbf{X}_k$ as

$$\begin{aligned} U_k(\mathbf{X}_k, \mathbf{X}_{-k}) = & \sum_{f \in \mathcal{F}} \left[\sum_{n \in \mathcal{C}_k} \log_2 \left(1 + \frac{P_n^f G_{nk}^f}{\sigma^2 + I_k^f(\mathbf{X}_{-k})} \right) x_n^f \right. \\ & \left. + \sum_{q \in \mathcal{K}, m \in \mathcal{C}_q} \log_2 \left(1 + \frac{P_m^f G_{mq}^f}{\sigma^2 + I_q^f(\mathbf{X}_{-\{q,k\}}) + \sum_{n \in \mathcal{C}_k} P_n^f G_{nq}^f x_n^f} \right) x_m^f \right], \end{aligned} \quad (9)$$

where $I_q^f(\mathbf{X}_{-\{q,k\}}) = \sum_{c \in \mathcal{K} \setminus \{q,k\}} \sum_{l \in \mathcal{C}_c} P_l^f G_{lq}^f x_l^f$ is the interference levels at the q th receiver that does not include the effect from the use of RB- f in cell- k , which is given by $\sum_{n \in \mathcal{C}_k} P_n^f G_{nq}^f x_n^f$ instead.

Problem (8) belongs to the family of nonlinear combinatorial optimization problems, hence achieving its optimal solution quickly becomes prohibitive when the number of users and/or RBs increases.

It is worth noting that to compute $U_k(\mathbf{X}_k, \mathbf{X}_{-k})$ in (9), player- k needs to retrieve from every other player at most

$2F$ scalar values, representing the useful power and the interference level measured on each assigned RB. Then, player- k must be able to remove the contribution of its cell to the global interference at the other receivers, thus obtaining $I_q^f(\mathbf{X}_{-\{q,k\}}), \forall q, \forall f$.

B. Solution based on better response dynamic

The nonlinearity of Problem (8), together with the amount of information required to be exchanged between the players, encouraged us to redefine the utility function of the players.

We consider a reference overall allocation strategy $\bar{\mathbf{X}}$. Then, the new utility function for player- k is defined as follows

$$\begin{aligned} \tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}}) = & \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{C}_k} \left[R_n^f(\bar{\mathbf{X}}_{-k}) x_n^f \right. \\ & + \sum_{\substack{q \in \mathcal{K}, m \in \mathcal{C}_q \\ q \neq k}} \left(R_m^f(\bar{\mathbf{X}}_{-q}) \bar{x}_m^f \right. \\ & \left. \left. + \frac{\partial R_m^f(\bar{\mathbf{X}}_{-q})}{\partial I_q^f(\bar{\mathbf{X}}_{-q})} \left(P_n^f G_{nq}^f x_n^f - P_n^f G_{nq}^f \bar{x}_n^f \right) \right) \right], \end{aligned} \quad (10)$$

where

$$\frac{\partial R_m^f(\bar{\mathbf{X}}_{-q})}{\partial I_q^f(\bar{\mathbf{X}}_{-q})} = - \frac{P_m^f G_{mq}^f}{(\ln 2)(P_m^f G_{mq}^f + I_q^f(\bar{\mathbf{X}}_{-q}) + \sigma^2)(I_q^f(\bar{\mathbf{X}}_{-q}) + \sigma^2)} \quad (11)$$

represents the rate sensitivity to the interference variations, which can be computed at receiver- q .

$\tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}})$ represents the first-order Taylor approximation of the users' rate around the interference level given by the reference allocation $\bar{\mathbf{X}}$. Since the rate is a convex function of the interference, $\tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}})$ is a lower bound of $U_k(\mathbf{X}_k, \bar{\mathbf{X}}_{-k})$. This bound represents a tight approximation of the original utility function under the reasonable assumption that the cumulative interference experienced at each receiver is much higher than the potential interference contribution of a single user.

We can further simplify Eq. (10) by subtracting constant terms that do not depend on the allocation decision of player- k , obtaining

$$\begin{aligned} \tilde{U}_k'(\mathbf{X}_k, \bar{\mathbf{X}}) = & \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{C}_k} \left[R_n^f(\bar{\mathbf{X}}_{-k}) \right. \\ & + \sum_{\substack{q \in \mathcal{K}, m \in \mathcal{C}_q \\ q \neq k}} \left(\frac{\partial R_m^f(\bar{\mathbf{X}}_{-q})}{\partial I_q^f(\bar{\mathbf{X}}_{-q})} \left(P_n^f G_{nq}^f \right) \right] x_n^f \quad (12) \\ = & \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{C}_k} \tilde{E}_n^f(\bar{\mathbf{X}}) x_n^f. \end{aligned}$$

Finally, we reformulate Problem (8) as follows. Given the current strategy profile $\bar{\mathbf{X}}$, player- k updates its strategy by

solving

$$\begin{aligned} & \underset{\mathbf{X}_k}{\text{maximize}} \quad \sum_{f \in \mathcal{F}} \sum_{n \in \mathcal{C}_k} \tilde{E}_n^f(\bar{\mathbf{X}}) x_n^f \\ & \text{subject to} \quad \sum_{n \in \mathcal{C}_k} x_n^f \leq 1, \quad \forall f \in \mathcal{F}, \\ & \quad \sum_{f=1}^F x_n^f = F_n, \quad \forall n \in \mathcal{C}_k, \\ & \quad x_n^f \in \{0, 1\}, \quad \forall f \in \mathcal{F}, \forall n \in \mathcal{C}_k, k \in \mathcal{B}, \end{aligned} \quad (13)$$

which is now an integer linear formulation, and thus easier to solve than Problem (8) [19]. Indeed, here we consider a given benefit for each user-resource assignment, removing the optimization variables from the argument of the logarithm function as in (9). Moreover, differently from (8), here the objective function depends on at most F scalar values to be retrieved from every other player of the game (i.e., the rate sensitivity in (11) for all assigned RBs).

We now show that although we have changed the utility function of the game, we can still guarantee convergence to a NE, resorting Lemma 2 and considering the following result:

Proposition 2. *Given any allocation profile $\bar{\mathbf{X}}$, solution $\mathbf{X}_k^*$ to Problem (13) is such that $U_k(\mathbf{X}_k^*, \bar{\mathbf{X}}_{-k}) \geq U_k(\bar{\mathbf{X}})$, that is $\mathbf{X}_k^*$ is a better response of player- k in game $\mathcal{G} = [\mathcal{K}, \{\mathcal{X}_k\}, \{\tilde{U}_k\}]$.*

Proof. Since $\mathbf{X}_k^* = \underset{\mathbf{X}_k}{\text{argmax}} \tilde{U}_k'(\mathbf{X}_k, \bar{\mathbf{X}}) = \underset{\mathbf{X}_k}{\text{argmax}} \tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}})$, then $\tilde{U}_k(\mathbf{X}_k^*, \bar{\mathbf{X}}) \geq \tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}}), \forall \mathbf{X}_k \in \mathcal{X}_k$. Moreover, $\tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}}) \leq U_k(\mathbf{X}_k, \bar{\mathbf{X}}_{-k}), \forall \mathbf{X}_k \in \mathcal{X}_k$ because of the linear approximation. By combining the two inequalities above we have $U_k(\mathbf{X}_k^*, \bar{\mathbf{X}}_{-k}) \geq \tilde{U}_k(\mathbf{X}_k^*, \bar{\mathbf{X}}) \geq \tilde{U}_k(\mathbf{X}_k, \bar{\mathbf{X}}) = U_k(\mathbf{X}_k, \bar{\mathbf{X}}_{-k})$. $\square$

C. Implementation guidelines for the better response dynamic

The iterative better response dynamic requires the players to follow a predetermined order. In this work, we assume that the BSs play in a sequential order, agreed upon before the gameplay. D2D receivers located within a given cell play right after their serving BS in an order decided by the BS itself, and sent to the users as a control command. All players start with a random and feasible RB allocation profile. When player- k plays, before solving Problem (13), it retrieves from each other player- q in the game the vector $\Delta_q = \left[\frac{\partial R_m^f(\bar{\mathbf{X}}_{-q})}{\partial I_q^f(\bar{\mathbf{X}}_{-q})}, f \in \mathcal{F}, m \in \mathcal{C}_q \right]$, whose elements represent the rate sensitivity on each RB, estimated after the latest strategy profile's update. Differently from ad-hoc networks, network-assisted D2D communications are coordinated by the BS, therefore we assume that each BS- $b \in \mathcal{B}$ collects the information related to the rate sensitivity of the D2D pairs under its coverage, and sends it, together with its vector Δ_b , to the next player (BS). In an LTE network, this communication of the rate sensitivity between the BSs can be naturally mapped onto the X2-Interface [15]; see Fig. 2 for illustration. When a

D2D transmitter has to play, then its serving BS is in charge of forwarding (on a control channel) the necessary information collected from the other BSs.

Fig. 2. Example of the messages needed to be exchanged between the players when player-5 updates its strategy in the better response dynamic.

It is worth mentioning that the implementation of the better response dynamic requires that each receiver knows the channel gains between its intended transmitters and the neighbouring receivers. Channel gains between mobile users and neighbouring BSs are already estimated in cellular networks for handover purposes. However, the presence of D2D communications requires a modification of the existing protocols to include measurements of the channel gains also between mobile users and neighbouring D2D receivers.

V. NUMERICAL RESULTS AND DISCUSSIONS

To study the behaviour of the proposed resource allocation algorithms based on better response dynamic, we simulate networks consisting of either 7 or 3 hexagonal cells with omnidirectional BSs and randomly placed mobile users. The number of transmissions (UL transmissions and D2D communications) is assumed to be the same in all cells. All transmitters use the same power level and the cellular users are assigned to all RBs, i.e., $\sum_{n \in C_k} F_n = F, \forall k \in \mathcal{B}$. Table I summarizes the main simulation parameters.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
Carrier frequency	1 GHz
Cell radius	500 m
Noise power	-174 dBm/Hz
Path loss coefficient	3.5
Lognormal shadow fading	6 dB
No. of RBs assigned to users	1
Users' transmit power	0.25 W
Distance between D2D pairs	50-150 m

Fig. 3 illustrates the convergence of the better response dynamic for a 7-cell network for different number of users. Each curve shows how the total bit rate evolves in a single simulation run. In this figure, one iteration corresponds to the strategy update of one player. The simulations are

initialized from a random feasible allocation. As expected, the potential function increases monotonically towards an equilibrium value. The algorithm converges quickly: the most dense network considered has 36 transmissions per cell (24 UL transmissions and 12 direct communications) and converges in less than 4 rounds (that is, in less than 364 player updates).

Fig. 3. Convergence behaviour of the better response dynamic for different realizations of a 7-cell network. Each iteration corresponds to the strategy update of one player.

To assess the validity of the proposed scheme, we consider 500 independent simulations, each with the same number of users but placed in different locations and with different channel conditions. We compare the performance of four resource allocation schemes: random resource allocation (R-RA), the proposed game with better response dynamic (Better-RA), the proposed game with best response dynamic (Best-RA), and the globally optimal resource allocation (Opt-RA). The optimal allocation is obtained by exhaustive search, for which the run-time becomes prohibitive for large-size networks. For this reason, we considered a small system with 3 cells, each of them serving 3 cellular users and 2 D2D pairs.

We characterize the efficiency of the NE points achieved using Best-RA and Better-RA by the relative performance degradation compared with Opt-RA. Fig. 4 shows that the Best-RA is within 10% of the optimal solution for almost all network configurations. However, the computation of each best response requires the solution of a combinatorial problem, which leads to runtime limitations for practical network sizes. The Better-RA performs slightly worse, but has faster convergence for all network sizes and is still within 20% of the optimal solution for almost all configurations.

Table II summarizes the results from the 500 random configurations. As we can see, both the Best- and Better-RA are significantly superior to R-RA, and not so far from the optimum. Moreover, since the set of pure-strategy Nash equilibria achieved with the better and best response dynamics is the set of local maxima of the potential function, playing the game cannot only lead to a performance improvement compared with any initial allocation, but in some cases it can achieve the global optimum (see Table II), which belongs to the set of NE.

TABLE II
SIMULATION RESULTS (3-CELL NETWORK WITH 15 LINKS)

Algorithms	Average Utility (bps/Hz)	Percentage of incidences in which optimal solution is obtained	Number of incidences in which the percentage rate loss is within 15%
Opt-RA	73.5205	100%	500
Best-RA	70.5514	19%	491
Better-RA	65.2594	2%	357
R-RA	49.8062	0%	11

Fig. 4. Histogram of the rate loss using the Best-RA and Better-RA with respect to the optimum. Results are obtained considering 500 random independent simulations of a 3-cell network with a total of 15 links.

VI. CONCLUSIONS

In this paper we investigated the problem of RBs allocation in a multi-cell wireless network, where underlay D2D connections are also enabled. Using a game-theoretic approach, we showed that the challenging nonconvex and combinatorial problem of sum-rate maximization can be described by a potential game, for which convergence to a Nash equilibrium can be ensured using best/better response dynamics. We derived a low-complexity iterative algorithm with reduced signaling that allocates the RBs to users, exploiting frequency diversity and efficiently managing both the inter- and intra-cell interference caused by the D2D connections. Numerical results showed that the proposed RA mechanism converges quickly to an equilibrium point, even for large networks. Comparisons with alternatives for small networks (for which it is possible to compute the optimal allocation by exhaustive search) showed that the achieved performance at the NEs are not too far from the optimum (in most of the cases it is less than 20% from it), and always improved compared to the initial allocation.

In this work we used fixed transmit powers. Combining subcarrier allocation with power control could potentially lead to even better performance, but is left for future work. Another interesting future extension is to design solutions which are not only distributed among the players (receivers), but that involve also the transmitters in each cell.

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