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Single layer planar near-field acoustic holography for compact sources and a parallel reflector

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Abstract

We consider the problem of planar near-field acoustic holography (PNAH) and introduce a new reconstruction method that can be used to process single layer pressure measurements performed in the presence of a reflective surface that is parallel to the measurement plane. The method is specially tailored for compact sources, or for problems in which the scattered field due to the source can be neglected. The approach consists in formulating a seismic model (WRW model) in wavenumber-space and employ it for sound source reconstructions. The proposed method is validated with numerical and experimental data, and, although the most accurate results are obtained when an estimate of the surface impedance is known beforehand, we show that it can substantially improve the reconstruction performance with respect to that of free-field PNAH.

Keywords: NAH, field separation techniques, reflective environments

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1. Introduction

Near-field acoustic holography (NAH) comprises a computational method for visualizing the acoustic field of sound sources by performing near-field measurements with a certain geometry [1, 2]. Among the limitations of the NAH method, considerable attention has been paid to overcome the requirement that the sources must be located in only one of the half-spaces of the measurement plane, and that the complementary half-space must be source-free. In other words, the solution to the inhomogeneous Helmholtz equation is assumed to be a superposition of outgoing plane waves and incoming waves are neglected [2]. The incentive to overcome this limitation is the possibility of performing reconstructions in noisy or reflective environments.

Field separation techniques (FST) have therefore become a subject of remarkable interest, since they allow to filter out the contribution of incoming (as well as scattered) wave-fields from the measurements. From a data-acquisition point of view, it is a common practice to measure pressure or particle velocity in two closely spaced surfaces, or to measure Cauchy data in a single layer. Among these two categories, some studies formulate the problem with inverse boundary element and Helmholtz equation least-squares methods via the singular value decomposition [3, 4, 5, 6, 7, 8], while others use the spherical wave superposition method [9, 10, 11], the equivalent source method [12, 13, 14, 15, 16, 17] or the statistically optimized NAH method [18, 19, 20, 21, 22, 23]. There is also a recent application of the equivalent source method in which the (double-layer) measurements completely cover the source region, allowing acoustic visualization of the field in small cavities [24]. On the other hand, due to the substantial interest on measuring the angle-dependent (plane-wave) reflection coefficient of absorbing surfaces [25, 26, 27, 28], a number of Fourier-based FST

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can nowadays be used to pre-process NAH measurements in reflective environments [29, 30, 31, 32]. The strength of Fourier-based FST is their low computational complexity. Nevertheless, as of today, two common drawbacks of existing FST are that they demand twice as many measurements than standard NAH, and that in most cases the reflective surfaces are deemed acoustically rigid.

The present work introduces a new Fourier-based method suitable for measurements in the presence of a reflector that is parallel to the microphone array. The benefit of this method, hereafter WRW-based PNAH, is that it only requires knowledge of the pressure in a single layer. (The method, in principle, also allows for particle velocity predictions from particle velocity measurements, and the theory follows equivalent mathematics.) The main contribution is the application of the WRW model [33, 34] for reconstructions via Fourier-based planar NAH (PNAH), under circumstances in which a reflective surface is parallel to the measurement and prediction planes. The WRW model is a well-known method in the seismic signal processing community, and it proposes a solution to wave-field extrapolation in enclosed spaces by means of performing spatial convolutions via matrix multiplications. The main idea of WRW-based PNAH is then to calculate these spatial convolutions in wavenumber-space in order to solve the inverse problem. An additional advantage of this method is that it offers the possibility of modelling surfaces which need not be rigid, however an estimate of the surface impedance is needed such that accurate reconstructions are ensured.

2. Theory

2.1. Wave-fields in the presence of a parallel reflector

Our starting point is the seismic model WRW, and the main result we use here is the propagation model in the presence of one reflector [33, 34] (see Figure 1). The steady-state pressure field at the measurement plane $z = z_h$ is, then, the superposition of the free-field pressure and the reflected pressure due the reflector at $z = z_1$. On mathematical grounds:

$$\mathbf{p}_h = \mathbf{p}_h^{\text{ff}} + \mathbf{p}_h^{\text{rf}} = \mathbf{W}_{sh}\mathbf{p}_s + \mathbf{W}_{1h}\mathbf{R}_1\mathbf{W}_{s1}\mathbf{p}_s, \quad (1)$$

where $\mathbf{p}_s$ and $\mathbf{p}_h$ are column vectors denoting the real-space pressure at planes $z = z_s$ and $z = z_h$, respectively, any propagation matrix $\mathbf{W}_{nm}$ contains the Dirichlet Green's functions that characterize the field extrapolation from plane z_n to plane z_m , and $\mathbf{R}_1$ contains the angle-dependent reflectivity impulse responses of the surface $z = z_1$ [33]. Note that the scattered field is not included in this model, thus the derivation is applicable for cases in which the source dimensions are small compared with the acoustic wavelength, e.g. compact sources. For cases otherwise, the problem can possibly be formulated via the WRW model for two reflectors [34]. It is worth noting that the reflector size should also be large with respect to the wavelength.

Figure 1: Geometrical illustration of the wave-field extrapolation in the presence of a parallel reflector, where z_s is the source plane, z_h is the hologram (measurement) plane, and z_1 is the reflective plane.

Provided the above geometry and a spatially homogeneous reflective surface, the convolution operators embedded in the matrices in Eq. (1) are all invariant in real-space (i.e., it reduces to a space-invariant system), and all of them can be (globally) represented with a unique spatial impulse response [35, 36]. Henceforth, the convolution theorem holds, and the wavenumber-space equivalent of Eq. (1) via the spatial Fourier transform follows

$$\tilde{\mathbf{p}}_h = \mathbf{G}_W \tilde{\mathbf{p}}_s, \quad (2)$$

where

$$\mathbf{G}_W = \mathbf{G}_{sh} + \mathbf{G}_{h1} \tilde{\mathbf{R}}_1 \mathbf{G}_{s1}. \quad (3)$$

Here $\mathbf{G}_{nm} = \text{diag}(e^{jk_z(z_m - z_n)})$ for any parallel planes z_n and z_m , j is the imaginary unit, and the wavenumber k_z is defined for propagating and evanescent waves as

$$k_z = \begin{cases} \sqrt{k^2 - k_x^2 - k_y^2}, & \text{if } k^2 \geq k_x^2 + k_y^2, \\ j\sqrt{k_x^2 + k_y^2 - k^2}, & \text{if } k^2 < k_x^2 + k_y^2, \end{cases} \quad (4)$$

where $k \in \mathbb{R}^+$ is the acoustic wavenumber.

The operator $\tilde{\mathbf{R}}_1$ is a diagonal matrix that contains the plane-wave reflection coefficient [37, 38] of the surface $z = z_1$, and it is defined as a function of the incidence angle, $\theta = \cos^{-1}(k_z/k)$, as follows

$$\tilde{\mathbf{R}}_1 = \text{diag}\left(\frac{\cos \theta - \beta}{\cos \theta + \beta}\right), \quad (5)$$

where β is the specific acoustic admittance. If the reflection coefficient is assumed to be angle-independent, it follows that $\tilde{\mathbf{R}}_1 = \text{diag}((1 - \beta)/(1 + \beta))$ [27]. The above expression is only valid for a locally reacting surface, where the pressure (and the particle velocity) at every point on the surface only depends on the pressure (particle velocity) at that point [37].

It is important to stress that the operator $\mathbf{G}_W$ can be singular: (i) at the origin of wavenumber-space for distances $\Delta_z = z_1 - z_h$ that are odd integers of quarter of the acoustic wavelength, and (ii) for propagating waves whose wavelengths are multiple integers of the measurement aperture size. An illustration of these singularities (or modes) is shown in Figure 2. It is then worth mentioning that the existence of singularities

Figure 2: Illustration of the singularities of the WRW-based propagator $\mathbf{G}_W$. (a) Modes due to the reflector distance $\Delta_z = z_1 - z_h$. (b) Modes due to the hologram (measurement) aperture size L . Here $\lambda = 2\pi/k$ and $(\lambda_x, \lambda_y) = (2\pi/k_x, 2\pi/k_y)$.

due to the reflector distance has a (temporal) frequency limit: the closer the surface is to the measurements, the higher the frequency at which these singularities appear. On the other hand, the singularities due to the aperture are determined by the propagating plane waves whose wavelengths (and their integer multiples) exactly match the aperture length, and these singularities only exist once the singularities due to reflector distance appear. A more detailed mathematical explanation can be found in Appendix A.

2.2. WRW-based PNAH

The novelty in our formulation is the application of the WRW model for single layer Fourier-based PNAH reconstructions. The formulation follows from inversion of the WRW-based propagator defined in Eq. (3), for which a regularization strategy must be chosen. Although regularization filters proposed for free-field PNAH (see [39, 40]) can be applied, we introduce the WRW-based Tikhonov filter, which is specially designed for the proposed WRW-based PNAH formulation.

Let us first assume that the specific acoustic admittance is known. Then, the Tikhonov minimization functional in general form follows

$$J(\tilde{\mathbf{p}}_s) = \frac{1}{2} \|\mathbf{G}_W \tilde{\mathbf{p}}_s - \tilde{\mathbf{p}}_h\|_2^2 + \frac{1}{2} \delta^2 \|\mathbf{D} \tilde{\mathbf{p}}_s\|_2^2, \quad (6)$$

where $\delta > 0$ is the regularization parameter, which can be chosen, for instance, with the L-curve or the generalized cross-validation (GCV) methods [41], and $\mathbf{D}$ is a wavenumber weighting matrix. The global minimum of J for known β is the regularized inverse of Eq. (2), or reconstructed source spectrum

$$\tilde{\mathbf{p}}_s = \mathbf{F} \mathbf{G}_W^{-1} \tilde{\mathbf{p}}_h, \quad (7)$$

where $\mathbf{F}$ is the WRW-based Tikhonov filter, which is defined in Section 2.3 below. The reconstructed source pressure, $\mathbf{p}_s$, follows from inverse spatial Fourier transformation of the above result. Note that Eq. (7) resembles the inverse solution of free-field PNAH if $\mathbf{G}_{sh}$ replaces $\mathbf{G}_W$, and $\mathbf{F}$ is any of the low-pass filter functions used in free-field PNAH (see [39, 40]).

2.3. WRW-based Tikhonov regularization

The WRW-based Tikhonov filter in general form is then expressed as

$$\mathbf{F} = (\mathbf{G}_W^H \mathbf{G}_W + \delta^2 \mathbf{D}^H \mathbf{D})^{-1} \mathbf{G}_W^H \mathbf{G}_W, \quad (8)$$

where superscript H denotes the Hermitian matrix transpose. Among the possible choices for the operator $\mathbf{D}$, we here take those according to the WRW-based Tikhonov filter in standard and modified forms, in a similar way to that postulated in [39]. The definition of δ can be found in Appendix B, where it is shown that it depends on the following problem parameters: frequency, propagation distances and reflector admittance.

The Tikhonov filter is in standard form when the operator $\mathbf{D}$ is the identity matrix. In such a case, the reconstructed wavenumber coefficients in Eq. (6) are weighted equally. On the other hand, the WRW-based modified Tikhonov filter follows from a similar construction to that in free-field PNAH [39]. Instead of the identity matrix, a usual choice for $\mathbf{D}$ is the complementary filter of Eq. (8), that is to say,

$$\mathbf{D} = \delta^2 (\mathbf{G}_W^H \mathbf{G}_W + \delta^2 \mathbf{I})^{-1}. \quad (9)$$

In this case, the reconstructed wavenumber coefficients are weighted with a differentiating (high-pass filter) operator. A straightforward connection to free-field Tikhonov filters [39] can be found when $\tilde{\mathbf{R}}_1$ is a null matrix. From a regularization perspective, the free-field filters do not account for the singularities introduced by $\mathbf{G}_W$, whereas the WRW-based filters are specially designed to account for such singularities.

3. Numerical study

3.1. Simulation setup

The pressure field radiated by a point-like source [2] is reconstructed via the free-field and WRW-based PNAH methods. The source is located 1 cm away from the source plane in the z -axis and at $(-1.3, 2.1)$ cm in the (x, y) plane, and it has 1 cm radius and oscillates with radial velocity 1 cm/s. The distance between the reflector and the hologram plane is 5 cm, and the reflected field is computed via numerical integration (see Appendix C). The distance from the hologram to the source plane is 3 cm, the aperture contains 32×32 measurements positions inter-spaced 6 cm. The frequencies of interest span from 200 Hz to 2500 Hz in steps of 100 Hz. Complex Gaussian noise is added to the hologram measurements such that the signal-to-noise ratio (SNR) is 35 dB, and the aperture is extrapolated via first-order linear predictive border padding to 128×128 samples [42] in order to minimize wraparound.

The reconstruction performance of the free-field PNAH [2] and WRW-based PNAH methods is quantified with the relative reconstruction error in decibels:

$$\varepsilon = 20 \log_{10} \left(\frac{\|\mathbf{p}_s^{\text{rec}} - \mathbf{p}_s^{\text{ref}}\|_2}{\|\mathbf{p}_s^{\text{ref}}\|_2} \right), \quad (10)$$

where superscripts “rec” and “ref” denote, respectively, reconstructed and reference source pressure. The relative spatial reconstruction error is also computed as follows:

$$e(x, y) = 20 \log_{10} \left(\frac{|\mathbf{p}_s^{\text{rec}}(x, y) - \mathbf{p}_s^{\text{ref}}(x, y)|}{|\mathbf{p}_s^{\text{ref}}(x, y)|} \right), \quad (11)$$

In addition, the idea of studying sound power ratio is taken from [32] and rewritten as the reflected-to-direct sound power ratio:

$$\text{RDR} = 20 \log_{10} \left(\frac{\|\mathbf{p}_h^{\text{rf}}\|_2}{\|\mathbf{p}_h^{\text{ff}}\|_2} \right). \quad (12)$$

3.2. Influence of angle-dependent reflectivity spectra and regularization filters

The purpose of this study is two-fold: to evaluate the reconstruction performance with angle-independent and angle-dependent reflectivity spectra, and with the use of various regularization filters. The specific acoustic admittance is assumed to be known *a priori*, and two materials are considered: a non-porous, highly reflective material ($\beta = 0.05$), and a porous material ($\beta = 0.52 + j0.26$). As a parenthesis here, at present we shall focus on how accurate the reconstruction method is when the material properties are known, yet a reasonable step to follow in the future is to evaluate the method when these properties are unknown. The regularization filters used in WRW-based PNAH (the choices of $\mathbf{F}$ in Eq. (7)), are summarized in Table 1.

Table 1: Summary of WRW-based and free-field regularization filters.

	Filter	Expression $\mathbf{F}$	Norm weighting $\mathbf{D}$	Reflectivity $\mathbf{R}_1$
WRW-based	WT	$(\mathbf{G}_W^H \mathbf{G}_W + \delta^2 \mathbf{D}^H \mathbf{D})^{-1} \mathbf{G}_W^H \mathbf{G}_W$	$\mathbf{I}$	$\frac{1-\beta}{1+\beta}$
	θ WT			$\frac{k_z-k\beta}{k_z+k\beta}$
	WMT		$\delta^2 (\mathbf{G}_W^H \mathbf{G}_W + \delta^2)^{-1}$	$\frac{1-\beta}{1+\beta}$
	θ WMT			$\frac{k_z-k\beta}{k_z+k\beta}$
Free-field	T	$(\mathbf{G}_{sh}^H \mathbf{G}_{sh} + \delta^2 \mathbf{D}^H \mathbf{D})^{-1} \mathbf{G}_{sh}^H \mathbf{G}_{sh}$	$\mathbf{I}$	$\frac{k_z-k\beta}{k_z+k\beta}$
	MT		$\delta^2 (\mathbf{G}_{sh}^H \mathbf{G}_{sh} + \delta^2)^{-1}$	
	E	See Appendix D		
	ME			

The notation used in the table is clarified in the following list:

- WRW-based filters: angle-independent and angle-dependent standard Tikhonov forms (WT and θ WT), and angle-independent and angle-dependent modified Tikhonov forms (WMT and θ WMT)
- Free-field filters: standard Tikhonov (T), modified Tikhonov (MT), exponential (E), and modified exponential (ME).

The GCV method is used to select the relevant regularization filter cut-off, and its expression follows [41]

$$\text{GCV} = \frac{\|(\mathbf{I} - \mathbf{F}) \tilde{\mathbf{p}}_h\|_2^2}{[\text{tr}(\mathbf{I} - \mathbf{F})]^2}, \quad (13)$$

where the cut-off is found via $k_{co} = \arg \min_{k_{co}} \text{GCV}$. The GCV method is used here since it does not require knowledge of the SNR, and, unlike the L-curve, it does not need to compute reconstructions.

In principle, free-field regularization filters can be used in combination with a propagator $\mathbf{G}_W$ irrespective of the angle-dependency, however the errors with angle-independent reflectivity are in practice much higher than with angle-dependent reflectivity. Free-field PNAH [2] is regularized with the free-field Tikhonov filter (T). Unlike WRW-based PNAH, free-field PNAH is based on the use of the free-field propagator $\mathbf{G}_{sh}$ instead of $\mathbf{G}_W$.

Figures 3 and 4 respectively show the reconstruction errors for WRW-based PNAH as a function of frequency for WRW-based and free-field regularization filters. Both figures have two sub-figures corresponding to the materials: $\beta = 0.05$ and $\beta = 0.52 + j0.26$. The errors in Figures 4(a) and (b) are normalized with respect to the lowest error obtained with WRW-based filters in Figures 3(a) and (b), respectively.

The results reveal that the most adequate filter for a highly reflective surface is the angle-dependent

Figure 3: Simulated relative reconstruction errors with WRW-based PNAH via WRW-based regularization filters for the two materials (a) $\beta = 0.05$ and (b) $\beta = 0.52 + j0.26$.

Figure 4: Simulated relative reconstruction errors with WRW-based PNAH via free-field regularization filters for the two materials (a) $\beta = 0.05$ and (b) $\beta = 0.52 + j0.26$. The errors in (a) are relative to the error via the θ WT filter in Fig. 3(a), while the errors in (b) are relative to the error via the θ WMT filter in Fig. 3(b).

WRW-based Tikhonov filter (θ WT), while for a more absorptive surface the angle-dependent WRW-based modified Tikhonov filter (θ WMT) gives better results. It is worth underlining that the highly-reflective material is not fully rigid, however, its associated spectrum is nearly constant (or angle-independent) thus the angle-independence assumption holds.

Surprisingly, the reconstruction errors obtained with the free-field modified Tikhonov filter (MT) are comparable to (and often lower than) the errors obtained with the WRW-based Tikhonov filters. In particular, as seen for $f > 2100$ Hz in Fig. 4(a), all free-field regularization filters outperform the θ WT filter, yet their performance appears to be comparable to that of the θ WMT filter in this frequency range. In a similar manner, as seen for $f > 2100$ Hz in Fig. 4(b), all free-field regularization filters except the T filter outperform the θ WMT, WT and WMT filters.

In Figure 5, the reconstruction error with free-field PNAH [2] is shown, together with the reflected-to-direct sound ratio (RDR) from Eq. (12). As expected, the reconstruction error (circles) appears to follow a rather similar curve as the RDR (triangles), that is to say, a smaller sound power ratio implies a stronger free-field, hence a lower reconstruction error with free-field PNAH.

On the other hand, the peaks in the error curves at 700, 1400 and 2100 Hz, particularly Figure 4(b), are caused by the fact that the propagator $\mathbf{G}_W$ is singular at the radiation circle (see Appendix A). As a consequence, the free-field regularization filters are incapable of dealing with the singularity, and this

Figure 5: Simulated reconstruction error in dB employing free-field PNAH ($-o-$), and reflected-to-direct sound power ratio ($-\Delta-$) in dB for (a) $\beta = 0.05$ and (b) $\beta = 0.52 + j0.26$.

severely contaminates the reconstructions with wraparound errors. This phenomenon is clearly illustrated in Figure 6, where the relative spatial reconstruction error for both materials at 700 Hz is shown, with the use of WRW-based PNAH and various regularization filters, as well as free-field PNAH. Overall, the lowest

Figure 6: Simulated relative spatial reconstruction error at $f = 700$ Hz for the highly reflective material (top row) using the filters (a) WT, (b) θ WT, (c) T and (d) free-field PNAH, and for the porous material (bottom row) using the filters (e) WMT, (f) θ WMT, (g) MT and (h) free-field PNAH. The source location is illustrated with an encircled cross.

relative spatial reconstruction errors are obtained with the angle-dependent WRW-based Tikhonov filters, in Figures 6(b) and (f). In particular, since the highly reflective material is nearly angle-independent (specially for real incidence angles), the angle-independent WRW-based (as well as the free-field) Tikhonov filters also give reasonable results for the highly reflective material.

The cause of the geometrical patterns (or wraparound errors), particularly in Figures 6(f) and (g), is that the penalty imposed by the modified Tikhonov filter, due to its high-pass filter nature, does not account

for the cost introduced by singular propagating waves in Eq. (6). This also explains why the solutions in Figures 6(b) and (c) with the WRW-based and free-field standard Tikhonov filters, do not suffer as much from wraparound errors since they penalize the entire wavenumber spectrum equally. In addition, the results for angle-independent WRW-based filters, i.e., Figs. 6(a) and (e), reveal that no wraparound errors are introduced, and this can be confirmed since the angle-independent propagator $\mathbf{G}_W$ is non-singular unless $f \geq 1700$ Hz and the admittance is sufficiently close to zero (for details see Appendix A). Free-field PNAH, depicted in Figures 6(e) and (h), appears to be accurate for source localization, yet the reconstructed field is severely distorted in the remainder of the aperture. Figure 7 shows the reconstructed sound fields along the diagonal of the microphone array, providing a complementary view of the results discussed above. Wraparound is specially noticeable with the free-field Tikhonov (T) and modified Tikhonov (MT) filters, while it is less noticeable for the WRW-based filters. It can also be seen that, for the case of the porous material, the wraparound is much more efficiently reduced when the angle-dependency is not taken into account, however the overall result appears to be much closer to the reference pressure when this dependency is considered.

Figure 7: Simulated reconstructed and reference sound fields at $f = 700$ Hz, with WRW-based PNAH and various regularization filters, as well as with free-field PNAH. (a) $\beta = 0.05$ and (b) $\beta = 0.52 + j0.26$.

Lastly, it is worth mentioning that when the source is not in the center of the measurement aperture, edge artifacts are introduced because of leakage in wavenumber-space, much alike free-field PNAH [2]. These artifacts can be minimized with the application of extrapolation techniques.

4. Experimental study

4.1. Experimental setup

A photograph and a schematic of the experimental setup can be found in Figure 8. All measurements are performed in the anechoic chamber of the Marcus Wallenberg Laboratory for Sound and Vibration Research at KTH. A line array of eight B&K 4260 microphones is used and the data is acquired with B&K PULSE (via a VXI E1432A system) with temporal sampling frequency of 5 kHz, FFT block size of 2048 samples, and Hanning windowing with 50% overlap. The microphones are calibrated in an impedance tube excited with plane waves within a 4.5 kHz bandwidth. The line array is manually shifted such as to have 24×16

measurement positions inter-spaced with 1 cm. The distance from the source plane to the hologram plane is 2 cm, and the distance from the hologram plane to the reflector is 8.5 cm. The reflective surface is a steel sheet

Figure 8: (a) Photograph and (b) schematic (side view) of the experimental setup.

of dimensions $0.5 \times 0.5 \text{ m}^2$ and 2 mm thickness, and it is mounted above a wood layer of 8 mm thickness. The acoustic source consists of an omnidirectional point-like source, and its bottom lid is positioned 5 cm away from the source plane in the z -axis and is centered in the measurement aperture. The frequencies of interest are $f = 400, 800, 1000, 1250, 1600, 2000, \text{ and } 2500 \text{ Hz}$. The phase information at the measurement points is obtained with a reference microphone placed in the vicinity of the source. The reconstructions are performed via the free-field and the WRW-based PNAH methods, and the source plane pressure is obtained in the absence of the reflector. Similarly to the numerical study, the hologram aperture is extrapolated via basic border padding to 64×64 samples [42]. The preference of basic over linear predictive border padding for experimental measurements can be attributed to the (substantially lower) noise level that is measured in the anechoic chamber.

If no knowledge of the surface impedance is given beforehand, the system of equations described by Eq. (2) becomes nonlinear and under-determined, adding a layer of complexity which is out of the scope of this paper. However, knowledge of the reference source pressure can be used in order to find a value for the specific acoustic admittance that minimizes the reconstruction error. In mathematical terms, the following combinatorial search is performed: $\beta = \arg \min_{\beta_0} \varepsilon(\beta_0)$, for $\beta_0 \in \mathbb{C}^n$, with $n = 2601$ values in the complex unit circle. This is equivalent to model the sound field with $\mathbf{G}_W$ for each value of β_0 , and compute the corresponding reconstruction errors. For WRW-based regularization filters, the value of δ is changed accordingly for each value of β_0 (see Appendix B), whereas free-field regularization filters remain the same irrespective of β_0 . Both WRW-based and free-field filters are designed with various cut-off wavenumbers. The minimum of Eq. (13) is then sought to find the optimal filter cut-off, and such filter is used to calculate the error $\varepsilon(\beta_0)$. This procedure is iterated for all values in β_0 , and the smallest error corresponds with the optimal value β . An instance of the reconstruction error for the set β_0 is shown at two frequencies for the θ WT filter in Figure 9 below.

The significance of estimating these admittance values in such a way, is that there exists a solution space in which a given admittance provides minimum reconstruction error. The admittance associated with such optimal solution is not necessarily a (fully) physically meaningful quantity since it accounts for modelling effects such as aperture extrapolation and regularization, as well as possible scattering and diffraction.

Figure 9: Experimental reconstruction error in dB with WRW-based PNAH and the θ WT filter versus complex specific acoustic admittance at (a) $f = 400$ Hz and (b) $f = 1250$ Hz. Knowledge of the reference source pressure is used to compute the errors. The admittances that provide minimum error are: (a) $\beta = 0.82 - j0.27$ and (b) $\beta = 0$.

Moreover, the admittance values might also include effects from the wood layer on which the steel is mounted. Nevertheless these values are relevant because they provide the most accurate reconstructions within the (physically defined) set of complex-valued admittances, β_0 ; henceforth indicating that reconstruction can potentially be accurate if the admittance (or a close estimate of it) is known.

4.2. Influence of angle-dependent reflectivity spectra and regularization filters

Figure 10 shows the reconstruction error in dB versus frequency, for the regularization strategies used in the numerical study. The reconstruction via free-field PNAH [2] is also included in the right-most figure. The

Figure 10: Experimental relative reconstruction errors in dB versus frequency, for (a) WRW-based PNAH via WRW-based filters and (b) WRW-based PNAH via free-field filters as well as free-field PNAH. Knowledge of the reference source pressure is used to compute the errors.

results reveal that the WRW-based filters do not provide as good reconstructions as in the numerical study. The reconstruction errors with these filters are bounded within ± 10 dB, compared with their performance in the numerical study with errors bounded within -20 dB and -10 dB. On the other hand, most free-field regularization filters, as well as the free-field PNAH method, perform as good as the WRW-based filters,

with the exponential filter (E) being the best performer on average among all filters. The admittances that produce the error values in Figure 10 are shown in Figure 11. Since the combinatorial search accounts for

Figure 11: Real part or conductance (top row) and imaginary part or susceptance (bottom row) of the specific acoustic admittance, β , that gives minimum reconstruction error with experimental data for varying frequency. The regularization filters in (a) and (c) are WRW-based filters, while in (b) and (d) are free-field filters. Knowledge of the reference source pressure is used to compute the admittance.

potential modelling effects (including regularization), one would expect the admittance estimates to be close to each other. Therefore the large discrepancies among admittances, specially at 1600 and 2000 Hz with WRW-based filters, are in a way unjustifiable.

Figure 12 shows the relative spatial reconstruction errors for various frequencies and regularization filters via WRW-based PNAH, as well as via free-field PNAH. No wraparound errors can be observed, which supports the fact that solutions with wraparound cannot yield minimum reconstruction errors. Edge artifacts are found, and this is due to the spatial windowing required prior to the fast Fourier transform. The spatial errors via the WRW-based PNAH method and Tikhonov filter (fourth column) are seemingly identical to those found via the free-field PNAH method (fifth column), and this is so because the regularization filter used in free-field PNAH is the Tikhonov filter as well. Furthermore, the errors at 1000 Hz for the WT and θ WT filters are much alike the sound field of a point source, and the admittance found in these cases is zero (see Figure 11(a)). Despite the accurate localization of the source, the relative spatial reconstruction error only consists of the phase difference due to the direct and reflected propagation distances (i.e., the reflective surface introduces no phase differences). In addition, possible scattering and/or diffraction can be observed at 2000 Hz (bottom row of Figure 12). Figure 13 further shows the reconstructed sound fields along a diagonal of the microphone array. Overall, the main conclusion is that WRW-based PNAH performs substantially worse than expected from the numerical results. In order to explain such poor reconstructions, a more thorough analysis is presented in the subsequent discussion section, with emphasis on WRW-based filters.

4.3. Discussion on the GCV method

To seek for explanations, an approach is to further use the reference source pressure and determine whether the GCV method is appropriate to choose the regularization parameter under the given experimental conditions. In other words, the authors put under test the hypothesis that the GCV method fails at obtaining

Figure 12: Experimental relative spatial reconstruction errors for three frequencies (rows). Top row: reconstruction error at $f = 400$ Hz via WRW-based PNAH and filters (a) WT, (b) θ WT, (c) E, (d) T, and (e) via free-field PNAH. Middle row: reconstruction error at $f = 1000$ Hz via WRW-based PNAH and filters (f) WT, (g) θ WT, (h) E, (i) T, and (j) via free-field PNAH. Bottom row: reconstruction error at $f = 2000$ Hz via WRW-based PNAH and filters (k) WT, (l) θ WT, (m) E, (n) T, and (o) via free-field PNAH. The source location is illustrated with an encircled cross.

Figure 13: Experimental reconstructed and reference sound fields with WRW-based PNAH and various regularization filters, as well as with free-field PNAH. (a) $f = 400$ Hz and (b) $f = 2000$ Hz.

the appropriate regularization parameter (or filter cut-off), and that there might be an optimal δ which further minimizes the reconstruction errors in the experimental study.

4.3.1. A combinatorial search for (δ, β)

In order to verify if there is at all an optimal δ different from that found via the GCV, the GCV minimization is avoided and the following combinatorial search is instead performed: $(\delta, \beta) = \arg \min_{\delta_0, \beta_0} \varepsilon(\delta_0, \beta_0)$, for $\delta_0 \in [0.05, 0.5]$, and $\beta_0 \in \mathbb{C}^{2601}$ as in Section 4.2. The set δ_0 has 51 elements. The results of running such an algorithm show that there is an optimal pair (δ, β) which indeed produces a much smaller error. Figure 14 shows the relative reconstruction errors versus admittance (compare with Figure 9), whereas Figure 15 shows the reconstruction errors versus frequency (compare with Figure 10). The optimal admittances

Figure 14: Experimental reconstruction error in dB with WRW-based PNAH and the θ WT filter versus complex specific acoustic admittance at (a) $f = 400$ Hz and (b) $f = 1250$ Hz. The errors are obtained through combinatorial search (δ, β) and knowledge of the reference source pressure. The admittances that provide minimum error are: (a) $\beta = 0.74 - j0.05$ and (b) $\beta = 0.11 + j0.19$.

Figure 15: Experimental relative reconstruction errors in dB versus frequency for WRW-based PNAH and WRW-based regularization filters. The errors are obtained through combinatorial search (δ, β) and knowledge of the reference source pressure.

found with the use of WRW-based filters are shown in Figure 16 (compare with Figure 11). These estimated admittances are much closer to each other than found via the GCV minimization algorithm. Relative spatial reconstruction errors are shown in Figure 17 and the reconstructed sound fields along a diagonal of the array are shown in Figure 18 (compare with Figures 12 and 13, respectively). The relative reconstruction errors show no significant influence of the angle-dependency of the regularization filter; and no wraparound patterns are found (which, again, is expected since no solution with wraparound can yield minimum reconstruction error). In addition, potential diffraction and/or scattering effects are noticeable at 1000 and 2000

Hz (middle and bottom rows in Figure 17).

Figure 16: Real part or conductance (left) and imaginary part or susceptance (right) of the specific acoustic admittance, β , that gives minimum reconstruction error with experimental data, for WRW-based filters and varying frequency. Knowledge of the reference source pressure is used to compute the optimal pair (δ, β) .

Figure 17: Experimental relative spatial reconstruction errors for three frequencies (rows) via the combinatorial search (δ, β) . Top row: reconstruction error at $f = 400$ Hz via WRW-based PNAH and filters (a) WT, (b) θ WT, (c) WMT and (d) θ WMT. Middle row: reconstruction error at $f = 1000$ Hz via WRW-based PNAH and filters (e) WT, (f) θ WT, (g) WMT and (h) θ WMT. Bottom row: reconstruction error at $f = 2000$ Hz via WRW-based PNAH and filters (i) WT, (j) θ WT, (k) WMT and (l) θ WMT. The source location is illustrated with an encircled cross.

Summarizing, the experimental results indicate that WRW-based PNAH critically depends on the choice of the regularization parameter, and, in particular, that the GCV appears to fail under the experimental conditions. In spite of these findings, arguably speaking, the results do not fully prove that the GCV method

Figure 18: Reconstructed and reference sound fields (with experimental data) for various WRW-based filters via combinatorial search (δ, β) at (a) $f = 400$ Hz and (b) $f = 2000$ Hz.

is inappropriate since the lack of knowledge of the admittance can be a questionable concern. Therefore a complementary numerical simulation, reproducing the experimental conditions, is also performed, in which the admittance is known and its estimation is avoided.

4.3.2. A numerically reproduced experiment

The variables used to render this simulation are $f = 1000$ Hz, $\beta = 0.2$ and $\text{SNR} = 50$ dB. The remaining sampling parameters, propagation distances and extrapolation strategy are the same as in the experiments. The results found with such synthetic experiment are strikingly similar to those found in Section 4.2. As a matter of fact, the main reason appears to be the choice of spatial sampling rate, and this is illustrated in Figure 19 in more detail, where the GCV functional and the reconstruction error are plotted versus filter cut-off wavenumber, for the θWT filter. It is then clear that the minimum of the GCV functional is found at the highest cut-off wavenumber, which differs from the cut-off that provides minimum reconstruction error. On the other hand, if the microphone spacing is coarser, like in the numerical validation, the highest cut-off wavenumber is lower, and the minimum of the GCV progressively approaches the minimum of the error. These findings indeed indicate that the GCV fails at selecting the appropriate regularization parameter given the microphone spacing used in the experiments. Nevertheless, there is a value of δ that provides optimal reconstruction error, meaning that other parameter choice methods than the GCV can potentially improve the reconstructions with WRW-based PNAH.

5. Conclusions

Reconstruction methods used to process near-field acoustic holography (NAH) measurements in reflective environments require nowadays two measurement layers, such that the outgoing and incoming wave-fields can be separated. We introduce in this paper a novel single layer NAH method, named WRW-based PNAH, that accounts for the presence of a reflective surface that is parallel to the measurement plane. The main contributions of this work are: (i) the application of the WRW model, a numerical method used in seismic exploration and room acoustics, for reconstructions with Fourier-based PNAH in the presence of a parallel

Figure 19: Illustration of the use of the GCV method (with numerically reproduced experimental conditions) to find the optimal regularization parameter. (a) GCV functional in dB and (b) reconstruction error versus cut-off wavenumber of the θ WT filter.

reflector, and (ii) the introduction of the WRW-based Tikhonov regularization filters. The WRW-based PNAH method is validated numerically and experimentally, and particular attention is paid to the angle-dependency of the reflective surface and the choice of regularization strategy. The performance of the free-field PNAH method is also investigated, in order to assess the improvement of employing WRW-based PNAH method in the presence of the reflector. One of the findings is that the free-field PNAH method may provide accurate reconstructions, sometimes comparable with those of the WRW-based PNAH method, when the reflective material tends to be of an absorptive kind. For a more highly reflective material the use of the WRW-based PNAH method can provide much better reconstructions than the free-field PNAH method. It is worth stating that these results can also vary depending on the distance between the array and the reflector: a larger (smaller) distance implies a weaker (stronger) reflection. Furthermore, it is important to stress here that the purpose of the WRW-based PNAH method is to filter out the reflected field and only visualize the free-space pressure field radiated by the source; however it is at present not a purpose to estimate the acoustic admittance of the reflector, as it was performed in the experimental analysis. In order to provide a valid (or accurate) admittance input to the WRW-based PNAH method, other estimation methods can be used, such as Tamura's method. Another significant conclusion is that the regularization parameter choice method becomes increasingly critical for accurate reconstructions as the spacing between microphones is smaller: a tradeoff between reconstruction accuracy and sound field resolution. An important limitation so far is the requirement of the sources being compact, however the WRW model for two reflectors may be applicable to characterize the vibrations of extended planar sources, such as baffled plates, and still only require one measurement layer. Lastly, another major constraint of the WRW-based PNAH method is the orientation of the reflective surface with respect to the microphone array, which is at present under investigation, for most geometries of practical interest might be other than the parallel case.

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Appendix A. The singularities of $\mathbf{G}_W$

For angle-independent plane-wave reflection coefficients, if the reflector is considered acoustically rigid then $\mathbf{G}_W$ is singular when any discrete wavenumber pair (k_x, k_y) satisfies

$$e^{2jk_z\Delta_z} = -1, \quad (\text{A.1})$$

where $\Delta_z = z_1 - z_h$. This equation is equivalent to

$$2k_z\Delta_z = (2\mu + 1)\pi, \quad (\text{A.2})$$

where $\mu \in \mathbb{N}$. Clearly in this equation only propagating waves lead to a singular $\mathbf{G}_W$ since k_z must be real (and positive), thus the singularity condition becomes after Eq. (4)

$$k^2 = k_x^2 + k_y^2 + \frac{(2\mu + 1)^2\pi^2}{4\Delta_z^2}, \quad (\text{A.3})$$

and the lower bound for k follows $k \geq \pi/(2\Delta_z)$. In few words, $\mathbf{G}_W$ is singular at the origin of wavenumber-space when $k_z = k$ is an odd integer of $\pi/(2\Delta_z)$. For acoustic wavenumbers k in the interval between two consecutive odd integers, the measurement aperture size L can also introduce n_L singular propagating waves $(k_x, k_y) \neq (0, 0)$, where $n_L = k \bmod 2\pi/L$. Here the modulo operation gives the amount of propagating waves that can be observed in (or that can fit) the measurement aperture. In practical terms, the aperture should not be equal to any integer multiple of the acoustic wavelength if minimum wraparound is desired.

For angle-dependent plane-wave reflection coefficients there are also chances to find wavenumbers which make $\mathbf{G}_W$ singular. Eq. (A.1) must then be rewritten as

$$e^{2jk_z\Delta_z} = -\frac{k_z + k\beta}{k_z - k\beta}. \quad (\text{A.4})$$

The exponential term in this equation can be expanded into a power series, i.e., $e^x \approx \sum_{n=0}^N x^n/n!$ where $N \gg 1$, in order to obtain a polynomial approximation of the singular wavenumbers:

$$1 + \sum_{n=1}^N a_n k_z^n \approx -\frac{k_z + k\beta}{k_z - k\beta}, \quad \text{where} \quad a_n = \frac{(2j\Delta_z)^n}{n!}. \quad (\text{A.5})$$

The $N + 1$ roots of k_z are categorized into: N complex roots (which convey no physical meaning since the medium is not deemed viscous in this study), and one real root. The latter, regardless of the polynomial degree $N + 1$, corresponds to $(k_x^2 + k_y^2)^{1/2} = k$, which means that the spectrum of $\mathbf{G}_W$ is singular at the radiation circle.

Appendix B. The Tikhonov regularization parameter in WRW-based PNAH

The Tikhonov regularization parameter in WRW-based PNAH can be related to some of the physical parameters of the problem. In order to show this, the Tikhonov filter can be interpreted as a smoothing operator (or low-pass filter function), thus, at the cut-off point, $(k_x^2 + k_y^2)^{1/2} = k_{co}$, it follows that the filter has magnitude $1/2$. In other words, from Eq. (8), the value of δ can be found via the expression

$$\frac{|\mathbf{G}_{W\{k_{co}\}}|^2}{|\mathbf{G}_{W\{k_{co}\}}|^2 + \delta^2} = \frac{1}{2}, \quad (\text{B.1})$$

where the subscript $\{k_{co}\}$ denotes that the operator $\mathbf{G}_W$ is evaluated at the cut-off wavenumber k_{co} , by means of applying the following relationships

$$\tilde{\mathbf{R}}_{1\{k_{co}\}} = \frac{k_{ev} - k\beta}{k_{ev} + k\beta}, \quad (\text{B.2a})$$

$$\mathbf{G}_{nm\{k_{co}\}} = e^{jk_{ev}(z_m - z_n)}, \quad (\text{B.2b})$$

to Eq. (3), where $k_{ev} = j\sqrt{k_{co}^2 - k^2}$. Finally, Eq. (B.1) yields

$$\delta = |\mathbf{G}_{W\{k_{co}\}}| = \left| e^{jk_{ev}(z_h - z_s)} + \frac{k_{ev} - k\beta}{k_{ev} + k\beta} e^{jk_{ev}(2z_1 - z_h - z_s)} \right|. \quad (\text{B.3})$$

An analogous procedure can be followed for angle-independent reflectivity spectra, and the result follows

$$\delta = \left| e^{jk_{ev}(z_h - z_s)} + \frac{1 - \beta}{1 + \beta} e^{jk_{ev}(2z_1 - z_h - z_s)} \right|. \quad (\text{B.4})$$

Lastly, it can be further said here that the free-field Tikhonov filter families [39] have a regularization parameter which only depends on the free-field term, that is,

$$\delta = |\mathbf{G}_{sh\{k_{co}\}}| = \left| e^{jk_{ev}(z_h - z_s)} \right|. \quad (\text{B.5})$$

Appendix C. Calculation of the reflected field

Let us first define the (steady-state) pressure field radiated by the point-like source in free space [2]

$$p^{\text{ff}}(r) = -au_r Z_0 \frac{e^{jkr}}{r}, \quad (\text{C.1})$$

where a and u_r are the radius and radial velocity of the source, respectively, Z_0 is the characteristic impedance of air, and r is the distance from the source center to the receiver position. Then, the reflected pressure field propagating away from a locally reacting reflective surface follows [26]

$$p^{\text{rev}}(r) = au_r Z_0 \int_0^{+\infty} \frac{\sqrt{k^2 - k_r^2} - k\beta}{\sqrt{k^2 - k_r^2} + k\beta} \frac{k_r}{\sqrt{k^2 - k_r^2}} e^{j\sqrt{k^2 - k_r^2}(z_h + z_0)} J_0(rk_r) dk_r, \quad (\text{C.2})$$

where $k_r = (k_x^2 + k_y^2)^{1/2}$, z_0 is the source position along the z -axis, and J_0 is the zero-order Bessel function of the second kind. This integration is numerically evaluated at every point within the measurement aperture, i.e., $r \in z = z_h$, and the hologram pressure field is constructed by field superposition (see Section 2.1). The scattered field due to source dimensions is neglected.

Appendix D. Exponential regularization filters

This appendix includes the mathematical expressions for the exponential regularization filters. It is important to stress that these filters have been originally designed for application to free-field PNAH [2, 40], however, given their flexible design, it is interesting to investigate their application to WRW-based PNAH.

Unlike Tikhonov filters, the exponential filter is not designed upon a minimization functional. Rather, the exponential filter is designed to minimize ringing effects due to the finiteness of the measurement aperture and steep filter decay rates. The exponential filter reads in wavenumber-space [2]

$$F = \begin{cases} 1 - \frac{1}{2}e^{-(1-k_r/k_{co})/\phi}, & k_r \leq k_{co}, \\ \frac{1}{2}e^{(1-k_r/k_{co})/\phi}, & k_r > k_{co}, \end{cases} \quad (\text{D.1})$$

where $\phi \in (0, 1)$ is a decay rate parameter and $k_r = (k_x^2 + k_y^2)^{1/2}$. At cut-off, this filter has magnitude 1/2 as well. Thus, alike Tikhonov filters, some propagating content can potentially be attenuated by the exponential filter as the cut-off point approaches the radiation circle (e.g. measurements with poor signal-to-noise ratios). The so-called modified exponential filter (or cosine-tapered window), due to its parametrization

of cut-off and slope, is then designed to control such attenuation of propagating waves. The mathematical expression is [40]

$$F = \begin{cases} 1, & k_r < k_{co}(1 - \phi) + k\phi, \\ \frac{1}{2} + \frac{1}{2} \cos \left[\frac{k_r - k_{co}(1 - \phi) + k\phi}{2(k_{co} - k)\phi} \pi \right], & k_{co}(1 - \phi) + k\phi \leq k_r \leq k_{co}(1 + \phi) - k\phi, \\ 0, & k_r > k_{co}(1 + \phi) - k\phi, \end{cases} \quad (\text{D.2})$$

where k is the acoustic wavenumber. Then the resulting filter matrix for either filter follows $\mathbf{F} = \text{diag}(F)$.

It is important to highlight here that, for both the exponential filters, the GCV functional becomes two-dimensional since it depends on cut-off and slope parameters.

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