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Postprint

This is the accepted version of a paper published in *International Studies in Religion and Society*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Vardanyan, Y., Hesamzadeh, M R. (2016)

Coordinated production planning of risk-averse hydropower producer in sequential markets.

International Studies in Religion and Society, 26(6): 1226-1243

<http://dx.doi.org/10.1002/etep.2131>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-189943>

Coordinated production planning of risk-averse hydropower producer in sequential markets

Yelena Vardanyan^{*,†} and Mohammad Reza Hesamzadeh^{†,‡}

Electricity Market Research Group (EMReG), School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden

SUMMARY

This paper proposes a quadratic programming (QP) model for optimal coordinated production of a risk-averse hydropower producer. The day-ahead, intra-day and real-time markets are considered. A rolling planning approach is used to take advantage of sequential clearing of mentioned markets. The multi-period risk of trading in different markets is modelled as quadratic terms in the objective function. To cope with uncertain prices, three price forecasting techniques are used. The best forecasting technique is selected based on a designed Markov switch. The discrete behaviour of intra-day and real-time market prices are modelled as different Markov states. The proposed QP model is coded in GAMS (GAMS Development Corporation, Washington, DC, USA) platform and solved using the MOSEK (Mosek ApS, Copenhagen, Denmark) solver. An example of a three-reservoir system from a Swedish hydropower producer is used to examine the proposed QP model. The results show the economic gains from coordinated production planning in sequential markets. Copyright © 2015 John Wiley & Sons, Ltd.

KEY WORDS: coordinated production planning; sequential markets; quadratic programming; risk

1. INTRODUCTION

Wholesale electricity markets have been designed and implemented during the past two decades to improve the economic efficiency of the electricity industry. The power trading in deregulated markets can take place on day-ahead market, the intra-day market or the real-time market [1].

Although the electricity production and consumption is balanced in day-ahead market, imbalances in real-time market might happen. These imbalances are due to intermittent energy sources, and they are expected to increase in the power system because of continuous growth in renewable energy [2]. Accordingly, there is a significant need for the operation of intra-day and real-time markets to settle down these imbalances.

Being a flexible power source, hydropower producer can provide balancing power and earn extra profit. The issue of coordinated production is very relevant for energy-limited producers who are able to participate in all markets. Therefore, in multi-settlement markets, the hydropower producer can use a coordinated production to maximise their profits from trading in different markets. However, fewer producers are participating in real-time market compared with day-ahead market [3]. The existing hesitation could be highly correlated with extremely varying market prices, and hence the amount of risk the producer is facing. Therefore, in this context, modelling of profit risk both in a single period and between different periods is of particular importance for hydropower producers.

References on coordinated production planning of hydropower producers are very limited. References [3] and [4] discuss the stochastic coordinated bidding for a hydropower producer in day-ahead and real-time markets. Reference [5] finds the optimal bidding strategies of multiple generators

*Correspondence to: Y. Vardanyan, School of Electrical Engineering, KTH Royal Institute of Technology, Stockholm, Sweden.

[†]E-mail: yelena.vardanyan@ee.kth.se

[‡]E-mail: mrhesamzadeh@ee.kth.se

in day-ahead market. The authors in [6] develop a stochastic coordinated bidding strategy for day-ahead, intra-day and real-time markets for thermal power plants. In this paper, the market prices in three market places are considered uncertain. Reference [7] models stochastic bidding to day-ahead market taking into account price uncertainties. In [8], authors integrate strategic bids and reserve sales. The authors present stochastic bidding to day-ahead market in [9] considering price and inflow uncertainties. Reference [10] discusses an optimal bidding strategy to day-ahead and intra-day markets.

Risk management in the production planning models is discussed in the following papers. Reference [11] addresses the self-scheduling problem for a thermal power producer considering only the day-ahead market. Reference [12] introduces a risk-constrained bidding strategy to day-ahead market for a wind power producer. The authors in [13] model the risk in hydro planning model considering day-ahead market, using minimum profit and conditional value-at-risk (CVaR) models. Risk is modelled with CVaR also in reference [14], where authors develop optimal bidding strategy for generation companies.

This paper extends the previous works by proposing a framework for coordinated production planning of a profit-maximising risk-averse hydropower producer in sequential day-ahead, intra-day and real-time markets. The expected profit and the multi-period risk measure are used in the set-up model. The multi-period risk metric measures the variance of profit over different time periods and over different markets.

The prices in different market places are modelled and predicted using the mean reversion jump diffusion (MRJD), Holt Winter (HW) and autoregressive integrated moving average (ARIMA) models. These price forecasting models predict the expected value and variance of the price for the horizon day of planning. To determine the best performing forecasting technique, a Markov switch is developed based on the historical data. Using the predicted expected and variance of prices, many number of scenarios are generated and reduced in a way that the previous information about the stochastic process is maintained in a best possible way [15]. The backward reduction algorithm is used for scenario reduction. A mean-variance quadratic programming (QP) model is developed for the risk-averse hydropower producer. The QP model is solved using the generated scenarios. The whole framework models the risks in different markets with quadratic terms in the objective function. It also considers arrival of new information because of the sequence of the three mentioned markets using a rolling planning approach implemented in the framework.

This paper is organised as follows. Section 1 reviews the existing literature in the field. Section 2 explains methods to model and forecast electricity market prices and scenario generation. The three-stage stochastic production planning model is developed in Section 3. The model results are discussed and analysed in Section 4. The future work is stated in Section 5. Section 6 concludes the paper.

2. MULTI-PREDICTOR MODEL FOR DAY-AHEAD, INTRA-DAY AND REAL-TIME MARKETS

The price forecasting in the competitive electricity markets is challenging mainly because special characteristics of commodity market are reflected in market price dynamics [16,17]. Reference [18] provides a detailed review on the price-forecasting techniques for day-ahead market. Regression (or causal) models [19,20], exponential-smoothing models [21] and stochastic models [22] are three broad classes of techniques for forecasting day-ahead market prices. In contrast, forecasting models for intra-day and real-time market prices are few [23–25]. The multi-predictor model in this paper is developed based on three individual forecasting techniques. Each individual model is, potentially, very suitable to capture some specific characteristics of price dynamic.

2.1. Mean reversion jump diffusion model

The stochastic continuous-time MRJD process for a time series $\{p_t\}_{t=1}^T$ is defined as follows:

$$dp_t = (v - \eta p_t)dt + \zeta dD_t + J_t dR_t \quad (1)$$

In (1), $\frac{v}{\eta}$ is the long-term mean. The D_t is Brownian motion, which is responsible for frequent small fluctuations around $\frac{v}{\eta}$ proportional to ζ . The Poisson process R_t is producing non-frequent big spikes of size J_t , which is normally distributed $N(\mu, \zeta)$ with arrival rate of λ [26].

2.2. Holt Winter model

The standard HW model for a time series $\{p_t\}_{t=1}^T$ with a unique seasonal pattern is as follows [21,27]:

$$\gamma_t = \alpha(p_t/I_{t-s}) + (1 - \alpha)(\gamma_{t-1} + T_{t-1}) \quad (2)$$

$$T_t = \beta(\gamma_t - \gamma_{t-1}) + (1 - \beta)T_{t-1} \quad (3)$$

$$I_t = \delta(p_t/\gamma_t) + (1 - \delta)I_{t-s} \quad (4)$$

$$\tilde{p}_t(h) = (\gamma_t + hT_t)I_{t-s+h} \quad (5)$$

where γ_t is the exponential component, T_t is the trend and I_t is the seasonal component with period s . The α , β and δ are smoothing parameters, which belong to the interval $[0, 1]$. $\tilde{p}_t(h)$ is the forecast with h hours forward.

2.3. Autoregressive integrated moving average model

The ARIMA processes incorporate a wide range of non-stationary series, which in turn, after differencing finitely many times, reduces to ARMA process [28,29]. The ARMA (a and b) process can be expressed as $\phi(B)p_t = \theta(B)n_t$, where $\{p_t\}_{t=1}^T$ is the time series, ϕ and θ are, respectively, a^{th} and b^{th} degree polynomials and B is backward shift operator defined by $B^j p_t = p_{t-j}$. In addition, ϵ_t is white noise sequence with normal distribution $\epsilon_t \sim N(0, \sigma^2)$. The ϕ and θ are expressed as follows:

$$\left. \begin{aligned} \phi(B) &= 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p, \\ \theta(B) &= 1 + \theta_1 B + \theta_2 B^2 + \dots + \theta_q B^q, \end{aligned} \right\} \quad (6)$$

2.4. Markov model for predicting the state of intra-day and real-time market prices

The real-time and intra-day market prices have discrete behaviour in the sense that in addition to price levels, we need to forecast price states. The intra-day market price at each bidding interval t belongs to one of the following four states: (i) no selling or buying price exists; (ii) only buying price exists; (iii) only selling price exists; and (iv) both buying and selling price exist. The state of intra-day market price can be modelled using a four-state Markov process. This is shown in Figure 1.

The probabilities of the transition matrix for intra-day Markov model are estimated using historical intra-day market prices. Based on the intra-day prices, the binary pair (b_t^{sell}, b_t^{buy}) is defined as follows for each bidding period t :

$$b_t^{sell(buy)} = \begin{cases} 1 & \text{if a selling(buying) price exists} \\ 0 & \text{otherwise} \end{cases} \quad (7)$$

Figure 1. Network representation of four-state Markov process for intra-day market prices.

We define o_t as the parameter that shows the state of intra-day price at time t :

$$o_t^{intra-day} = \begin{cases} 1, & \text{if } (b_t^{sell}, b_t^{buy}) = (0, 0) \\ 2, & \text{if } (b_t^{sell}, b_t^{buy}) = (0, 1) \\ 3, & \text{if } (b_t^{sell}, b_t^{buy}) = (1, 0) \\ 4, & \text{if } (b_t^{sell}, b_t^{buy}) = (1, 1) \end{cases} \quad t = 1, 2, \dots, T \quad (8)$$

Let $O_{ij} = \{o_t^{intra-day} : o_t^{intra-day} = j, o_{t-1}^{intra-day} = i, t = 1, \dots, T\}$, then element $(i \text{ and } j)$ of transition probability matrix pr_{ij} for $i, j = 1, \dots, 4$ can be calculated as

$$pr_{ij} = \frac{Card(O_{ij})}{\sum_{n=1}^4 Card(O_{i,n})}, i, j = 1, \dots, 4 \quad (9)$$

The real-time market prices have the same discrete behaviour. The following four states can be distinguished for real-time market prices: (i) no up-regulating or down-regulating price exists; (ii) only up-regulating price exists; (iii) only down-regulating price exists; and (iv) both up-regulating and down-regulating price exist. Similar to the intra-day market, the state of real-time market price can be modelled using a four-state Markov process. The states of the real-time Markov model are defined as follows:

$$o_t^{real-time} = \begin{cases} 1, & \text{if } (b_t^{up}, b_t^{down}) = (0, 0) \\ 2, & \text{if } (b_t^{up}, b_t^{down}) = (0, 1) \\ 3, & \text{if } (b_t^{up}, b_t^{down}) = (1, 0) \\ 4, & \text{if } (b_t^{up}, b_t^{down}) = (1, 1) \end{cases} \quad t = 1, 2, \dots, T \quad (10)$$

The transition probability matrix of Markov model for real-time prices is estimated using the real-time historical data. The process is similar to the one for the intra-day Markov model, and it is omitted for brevity.

2.5. Best performing price predictor

Suppose a set of N competing price predictors producing forecasts $p_t^{(1)}$ to $p_t^{(N)}$ of real price $\hat{p}_t$. If $\{\epsilon_t^{(1)}, \dots, \epsilon_t^{(N)}\}$ is the set of errors for price predictors 1 to N , then minimum element of this set (ϵ_t^*) determines the best performing price predictor for period t . The ϵ_t^* and its associated predictor can be used to design a Markov model that can predict the best performing predictor for period t in the multi-predictor model. Specifically, let ϵ^{ARIMA} , ϵ^{HW} and ϵ^{MRJD} be absolute errors for ARIMA, HW and MRJD forecasting techniques as compared with the real prices. The parameter e_t defines the state of Markov model at time t :

$$e_t = \begin{cases} 1, & \text{if } \epsilon^{ARIMA} < \epsilon^{HW} \quad \text{and} \quad \epsilon^{ARIMA} < \epsilon^{MRJD} \\ 2, & \text{if } \epsilon^{HW} < \epsilon^{ARIMA} \quad \text{and} \quad \epsilon^{HW} < \epsilon^{MRJD} \\ 3, & \text{if } \epsilon^{MRJD} < \epsilon^{HW} \quad \text{and} \quad \epsilon^{MRJD} < \epsilon^{ARIMA} \end{cases} \quad (11)$$

Let $E_{ij} = \{e_t : e_t = j, e_{t-1} = i, t = 1, \dots, T\}$, then the 3×3 transition probability matrix is defined as

$$pr_{ij}^{SW} = \frac{Card(E_{ij})}{\sum_{n=1}^3 Card(E_{i,n})}, i, j = 1, \dots, 3 \quad (12)$$

Using the transition matrix earlier, we can determine the best predictor for forecasting the day-ahead, intra-day and real-time market prices.

To model the probabilistic information on random prices, a large number of scenarios are initially generated. The initial number of scenarios is then reduced such that the probability

information is maintained as much as possible. The problem of optimal scenario reduction can be stated as

$$\text{Minimise } D_J = \sum_{i \in J} \omega_i \text{ Minimise } e_{j \notin J} \|p^i - p^j\| \quad (13)$$

$$\text{Subject to : } J \subset \{1, \dots, N\}, \text{Card}\{J\} = N - n \quad (14)$$

The optimisation problem (13), (14) can be approximately solved using the backward reduction technique. In this reduction technique, price scenario p^k is deleted such that $D_{J^{k-1} \cup \{p^k\}} = \text{Min}_{p \notin J^{k-1}} D_{J^{k-1} \cup \{p^k\}}$ [30].

3. THE SCENARIO-BASED QUADRATIC PROGRAMMING MODEL

The three-market coordinated production model for a risk-averse profit-maximising hydropower producer with multi-period risk measure and rolling planning is presented in the following. The uncertainties related to the prices in different market places are reflected via expected prices and covariance matrices, which are calculated based on the price scenarios. The price scenarios are generated based on the best performing forecasted technique suggested by Markov switch model presented in the previous section.

The expected profit of the hydropower producer $E[\Pi(s, t)] = E[\pi^d(s, t)] + E[\pi^{sell}(s, t)] + E[\pi^{buy}(s, t)] + E[\pi^{up}(s, t)] + E[\pi^{down}(s, t)] + E[\pi^f(s, t)]$ can be calculated as

$$E[\Pi(s, t)]T = \left[\sum_{t=1}^T \left(\hat{p}_t^d x_t^d + \hat{p}_t^{sell} x_t^{sell} - \hat{p}_t^{buy} x_t^{buy} + \hat{p}_t^{up} x_t^{up} - \hat{p}_t^{down} x_t^{down} \right) + \hat{p}^f \sum_j \sum_{r \in R_j} \gamma_r m_{j,T} \right] \quad (15)$$

where

$$\begin{aligned} \hat{p}_t^d &= \sum_{s=1}^{S_k} \omega_s p_{s,t}^d, \quad \hat{p}_t^{sell} = \sum_{s=1}^{S_k} \omega_s p_{s,t}^{sell}, \quad \hat{p}_t^{buy} = \sum_{s=1}^{S_k} \omega_s p_{s,t}^{buy} \\ \hat{p}_t^{up} &= \sum_{s=1}^{S_k} \omega_s p_{s,t}^{up}, \quad \hat{p}_t^{down} = \sum_{s=1}^{S_k} \omega_s p_{s,t}^{down}, \quad \hat{p}^f = \sum_{s=1}^{S_k} \omega_s p_s^f \end{aligned} \quad (16)$$

The multi-period variance of the profit $\text{Var}[\Pi(s, t)] = \text{Var}[\pi^d(s, t)] + \text{Var}[\pi^{sell}(s, t)] + \text{Var}[\pi^{buy}(s, t)] + \text{Var}[\pi^{up}(s, t)] + \text{Var}[\pi^{down}(s, t)] + \text{Var}[\pi^f(s, t)]$ is derived as

$$\begin{aligned} \text{Var}[\Pi] &= \sum_{i=1}^T \sum_{j=1}^T \left[x_i^d W_{i,j}^d x_j^d + x_i^{sell} W_{i,j}^{sell} x_j^{sell} + x_i^{buy} W_{i,j}^{buy} x_j^{buy} + \right. \\ &\quad \left. x_i^{up} W_{i,j}^{up} x_j^{up} + x_i^{down} W_{i,j}^{down} x_j^{down} + \text{Var}(p_s^f) \sum_j \sum_{r \in R_j} \gamma_r^2 m_{j,T}^2 \right] \end{aligned} \quad (17)$$

The positive semidefinite matrix $W \geq 0$ is the covariance matrix that is calculated based on the preserved price scenarios. Moreover, the variance of future prices is calculated based on future price scenarios. Equation (17) is derived based on the variance definition under the assumption that prices at different market places are mutually independent. This means that in the variance matrix, only diagonal elements are considered non-zeros. The multi-period variance in (17) measures the risk of profit change between different periods of horizon day of planning. The risk of profit change in one single period ($\text{Var}_s[\Pi(s, t)]$) is predicted using the historical data.

The objective function of the QP model for production planning is defined as the convex combination of the expected profit and the multi-period variance of profit. This is formulated in (18).

$$\text{Maximise } E[\Pi] - \chi \text{Var}[\Pi] \quad (18)$$

χ in (18) is interpreted as a risk aversion level, and it belongs to $[0, \infty]$. The value of χ gives trade-off between expected profit and risk.

The risk-averse hydropower producer solves the following quadratic programme to find its optimal coordinated production planning.

$$\begin{aligned} & \text{Maximise } \sum_{t=1}^T \left(\hat{p}_t^d x_t^d + \hat{p}_t^{\text{sell}} x_t^{\text{sell}} - \hat{p}_t^{\text{buy}} x_t^{\text{buy}} + \hat{p}_t^{\text{up}} x_t^{\text{up}} - \hat{p}_t^{\text{down}} x_t^{\text{down}} \right) \\ & + \hat{p}^f \sum_{j=1}^J \sum_{r \in R_j} \gamma_r m_{j,T} - \chi \left(\sum_{i=1}^T \sum_{j=1}^T W_{i,j} x_i^d x_j^d + \sum_{i=1}^T \sum_{j=1}^T W_{i,j} x_i^{\text{sell}} x_j^{\text{sell}} \right. \\ & + \sum_{i=1}^T \sum_{j=1}^T W_{i,j} x_i^{\text{buy}} x_j^{\text{buy}} + \sum_{i=1}^T \sum_{j=1}^T W_{i,j} x_i^{\text{up}} x_j^{\text{up}} + \sum_{i=1}^T \sum_{j=1}^T W_{i,j} x_i^{\text{down}} x_j^{\text{down}} \\ & \left. + \text{Var}(p_s^f) \sum_j \sum_{r \in R_j} \gamma_r^2 m_{j,T}^2 \right) \end{aligned} \quad (19)$$

subject to

$$m_{j,t} = m_{j,t-1} - \sum_{n=1}^N Q_{j,t,n} - S_{j,t} + \sum_{n=1}^N Q_{j-1,t-\tau_j,n} + S_{j-1,t-\tau_j} + I_{j,t} \quad (20)$$

$$G_{j,t} \leq \sum_{n=1}^N \mu_{j,n} Q_{j,t,n} \quad (21)$$

$$x_t^d + x_t^{\text{sell}} + x_t^{\text{up}} = x_t^{\text{buy}} + x_t^{\text{down}} + \sum_j G_{j,t} \quad (22)$$

$$x_t^{\text{buy}} + x_t^{\text{down}} \leq x_t^d \quad (23)$$

$$Q_{j,t,n} \leq \bar{Q}_{j,n} \quad (24)$$

$$m_{j,t} \leq \bar{m}_j \quad (25)$$

The constraint (20) sets balance in the reservoirs, which means new content of reservoir is equal to old content of reservoir plus water inflow minus water outflow. The generation and discharge relation in each power plant is stated in constraint (21). The constraint (22) guarantees that dispatched quantity on day-ahead market together with offered selling and upregulation production volume are equal to the total generation quantity plus offered buying and downregulation production. In addition, the buying and downregulation volumes should not exceed the dispatched volume to day-ahead market, which is achieved by constraint (23). Maximum discharge capacity and maximum reservoir content is set by the constraints (24) and (25).

The QP model (19–25) is non-linear but convex. The convexity guarantees the global optimum solution.

On the other hand, the information about intra-day and real-time markets is revealed continuously. In order to benefit from the information releasing over time, the scenario tree is updated with arrival of new information. Ideally, we can update the scenario tree once a new set of information on intra-day and real-time prices exists. However, to keep the model computationally tractable, we update the scenario tree every few hours period t , which we call ‘iteration’. For each iteration, new scenario tree is used, which contains the updated forecasts for intra-day and real-time market prices. The implementation of the rolling planning to update the intra-day and real-time prices every 4-h period is depicted in Figure 2. Note that each scenario contains information for both intra-day and real-time market prices.

The whole framework of coordinated production planning is presented in Figure 3.

4. RESULTS AND DISCUSSIONS

4.1. Three-reservoir system

An example three-reservoir system from Swedish hydropower network is studied. Reservoir 1 is larger, which then is followed by smaller reservoirs 2 and 3. Every reservoir has local inflow and a

local generator. Table I sets out the maximum storage capacity ($\bar{m}_j$), the maximum flow ($\bar{Q}_j$), the maximum production capacity for each power plant ($\bar{G}_j$) and water delay time (τ_j). For predicting day-ahead, intra-day and real-time market prices, the historical data for these prices from 10 March 2012 to 10 March 2013 are taken from the Nordic electricity market website [31].

Figure 2. Illustration of the rolling planning approach implemented in the quadratic programming model.

Figure 3. Flowchart presenting steps in the whole framework.

The future electricity price is estimated by taking the average value of forward contracts. The average of the forward market prices is calculated as 40€/MWh. The initial reservoir content is considered 60% of its maximum storage capacity.

The multi-predictor model for price forecasting is used to forecast the electricity prices in day-ahead, intra-day and real-time markets. The parameters of MRJD, ARIMA and HW models are derived using Matlab software. For day-ahead market, HW method parameters are taken α , β and $\delta=0.1$. For MRJD method, the corresponding parameters for day-ahead market are estimated as follows: long term mean parameters $\nu=1.04$ and $\eta=0.04$, the parameter corresponding to frequent small fluctuations $\zeta=2.86$, the mean and the standard deviation of non-frequent spikes are $\mu=31.5$ and $\gamma=2.06$ and the corresponding arrival rate $\lambda=0.0034$. ARIMA model parameters for day-ahead market prices are set out in Table II.

The estimated transition matrices $P_{intra-day}$, $P_{real-time}$ and P_{switch} for intra-day market, real-time market and switch are presented in (26), (27) and (28), respectively. In these transition matrices, the symbol “*” means that the system has never been in the state associated to the asterisk row or column.

$$P_{intra-day} = \begin{pmatrix} * & * & * & * \\ * & 0.4020 & 0.1843 & 0.4132 \\ * & 0.1655 & 0.4501 & 0.3845 \\ * & 0.2157 & 0.2232 & 0.5614 \end{pmatrix} \quad (26)$$

$$P_{real-time} = \begin{pmatrix} 0.6526 & 0.1877 & 0.1582 & 0.0015 \\ 0.0798 & 0.8724 & 0.0302 & 0.0176 \\ 0.1143 & 0.0466 & 0.8211 & 0.0178 \\ 0.0291 & 0.3964 & 0.5564 & 0.0182 \end{pmatrix} \quad (27)$$

$$P_{switch} = \begin{pmatrix} 0.2619 & * & 0.7381 \\ * & * & * \\ 0.2222 & * & 0.7786 \end{pmatrix} \quad (28)$$

Using estimated parameters, 1000 scenarios are initially generated and reduced to 30 scenarios. The daily absolute error is calculated using the real prices and the simulated prices using all three forecasting techniques for the whole year. Then, diurnal absolute error using all three forecasting tools is depicted in Figure 4. According to Figure 4, 23% of cases ARIMA method outperforms, and 77% cases HW method beats its competitors. The MRJD happened to be the worst predictor for our price data. This behaviour is also consistent with (28).

The QP model is coded in GAMS 24.2 platform and solved using MOSEK solver. The solver is using interior point method and is suitable for solving large-size linear, convex quadratic and conic quadratic programmes [32]. Because quadratic programmes are convex, the solver guarantees the global optimum. The whole simulation is run on a computer with Intel(R) Core(TM)2 Quad CPU Q 9400 at 2.66 GHz and 4 GB RAM. The objective function value together with execution time for all iterations are summarised in Table III. The model statistics for the three-reservoir hydropower system is stated in Table IV. According to Table III, the total computation time is 0.359 s, which clearly shows that the model is applicable to bigger hydropower systems. Moreover, fast execution time will allow updating the information in the model in every hour using rolling planning approach.

The optimal coordinated production planning of the hydropower producer in three sequential markets with risk factor $\chi=0.001$ is set out in Tables V and VI. The production volumes to day-ahead market remain the same for all iterations (the first column in the Table V). In contrast, intra-day production volumes (selling and buying) and real-time production volumes (upregulation and

Table I. Data for three-reservoir system.

Reservoir	$\bar{m}_j$ (HE)	$\bar{Q}_j$ (m ³ /s)	$\bar{G}_j$ (MW)	τ_j (h)
1	305 856	340	95	0
2	1392	310	50	0.5
3	4008	330	90	2
Total	311 256	980	235	2.5

Table II. Estimated parameters for autoregressive integrated moving average model, day-ahead market.

Parameter	Estimate	Parameter	Estimate
φ_1	0.868	ϕ_{48}	-0.02
φ_2	-0.25	ϕ_{96}	0.07
φ_{20}	-0.1	ϕ_{168}	-0.06
φ_{21}	0.15	θ_1	0.022
φ_{22}	-0.05	θ_2	-0.004
φ_{23}	0.12	θ_3	0.04
φ_{24}	0.23	θ_4	-0.01
φ_{25}	-0.28	θ_5	-0.02

Figure 4. The daily absolute error for autoregressive integrated moving average (ARIMA), mean reversion jump diffusion (MRJD) and Holt Winter (HW) methods.

downregulation) are changing when time evolves and new price information reveals. Changes in production volumes throughout the iterations can be seen from Tables V and VI.

Results in Tables V and VI show that in the planning period, the hydropower producer allocates the production differently in different markets.

4.1.1. Discussion in terms of risk factor. Simulation results have shown that the QP model is very sensitive to the risk factor. Table VII illustrates the impact of the inclusion of the risk factor in the discharge plan. For that purpose, the model is simulated with $\chi=0$ (no risk) and $\chi=0.001$ (small weight to risk factor). The discharge plans for both cases are set out in Table VII. For the no-risk case, the model allocates the maximum production to day-ahead market nearly for all hours and withdraws it via down-regulation. These types of strategies are expected in the multi-settlement markets. When we include a small risk more realistic, productions are achieved. The model allocates base production to day-ahead market and saves some amount for intra-day and real-time markets depending on the expected prices.

Table III. Computation time of quadratic programming model with respect to rolling planning, $\chi=0.001$.

	Iteration 1	Iteration 2	Iteration 3	Iteration 4
$E[II] - \chi \text{Var}[II]$ (€)	195 135	193 740	180 326	179 750
Computation time (seconds)	0.156	0.063	0.078	0.062

Table IV. Model statistics for the three-reservoir hydropower system.

	Iteration 1	Iteration 2	Iteration 3	Iteration 4
Single variables	483	483	483	483
Single equations	435	870	1305	1740

Table V. The optimal coordinated production allocation in day-ahead and intra-day markets with risk factor $\chi = 0.001$ and four iterations of rolling planning.

Hours	Day-ahead market		Selling intra-day market				Buying intra-day market			
	All Iteration		Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 1	Iteration 2	Iteration 3	Iteration 4
1	0	121	121	121	121	121	0	0	0	0
2	0	121	121	121	121	121	0	0	0	0
3	0	56	56	56	56	56	0	0	0	0
4	0	0	0	0	0	0	0	0	0	0
5	108	92	92	92	92	92	0	0	0	0
6	37	54	54	54	54	54	0	0	0	0
7	70	109	110	110	110	110	0	0	0	0
8	55	70	31	31	31	31	0	0	0	0
9	20	0	41	41	41	41	0	0	0	0
10	0	124	113	113	113	113	0	0	0	0
11	101	7	0	0	0	0	0	0	0	0
12	131	82	76	76	76	76	0	0	0	0
13	66	235	118	118	37	37	30	0	0	0
14	142	50	86	86	114	114	0	0	21	21
15	56	147	116	116	117	117	0	0	0	0
16	41	163	96	96	143	143	0	0	19	19
17	23	63	73	73	66	66	0	0	0	0
18	97	44	95	95	55	55	0	0	0	0
19	0	200	185	185	106	91	0	0	0	0
20	34	81	10	10	48	79	0	0	0	0
21	5	136	90	90	96	53	0	0	0	14
22	65	52	48	48	0	0	0	0	1	17
23	36	88	84	84	22	19	0	0	0	3
24	27	127	121	121	101	78	0	0	0	3

Table VI. The optimal coordinated production allocation in real-time market with risk factor $\chi=0.001$ and four iterations of rolling planning.

Hours	Up-regulation real-time market				Down-regulation real-time market			
	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 1	Iteration 2	Iteration 3	Iteration 4
1	114	114	114	114	0	0	0	0
2	7	7	7	7	0	0	0	0
3	35	35	35	35	0	0	0	0
4	194	194	194	194	0	0	0	0
5	0	0	0	0	5	5	5	5
6	9	9	9	9	0	0	0	0
7	8	21	21	21	11	0	0	0
8	110	122	122	122	0	0	0	0
9	215	174	174	174	0	0	0	0
10	111	121	121	121	0	0	0	0
11	105	105	105	105	0	0	0	0
12	7	0	0	0	0	0	0	0
13	0	36	102	102	37	3	0	0
14	20	0	0	0	0	17	0	0
15	0	0	62	62	0	0	0	0
16	20	49	70	70	12	0	0	0
17	149	135	145	145	0	0	0	0
18	81	32	83	83	0	0	0	0
19	35	46	129	144	0	0	0	0
20	107	162	0	0	0	0	0	0
21	94	144	133	143	0	4	0	0
22	63	85	151	170	5	22	0	0
23	0	0	0	0	0	0	0	0
24	0	0	0	0	27	24	0	0

In Table VIII, $\chi=0$ models a risk-neutral hydropower producer, and $\chi=0.004$ models a risk-averse hydropower producer.

Table VIII shows that the continuous increase of risk factor brings continuous decrease in the expected profit ($E[\pi]$) and continuous decrease in risk ($Var[\pi]$). The expected profit with maximum risk (standard deviation of 17 320€) is $E[\pi]=223\ 511\text{€}$. However, for a risk-averse producer with minimum risk (standard deviation of 3742€), the expected profit is only $E[\pi]=181\ 962\text{€}$. Therefore, there is no optimal value for χ . The level of weight assigned to the risk factor highly depends on the producer risk attitude. The expected profit versus profit standard deviation is depicted in Figure 5.

4.1.2. The economic gain from coordinated production. To show the economic gain from coordinated production, the expected profit and variance under the following five cases are studied: (Case 1) coordinated production under three sequential markets, (Case 2) coordinated production under two sequential markets, (Case 3) separated production in day-ahead market, (Case 4) separated production in intra-day market and (Case 5) separated production in real-time market. Case 1 is the suggested three-market coordinated production model. The results from Case 1 are compared with the results of four models Cases 2–5. Case 2 considers day-ahead and real-time markets to allocate production. Cases 3–5 consider only one market for trading the energy. The results for risk factor $\chi=0.01$ is tabulated in Table IX.

Table IX clearly shows that under coordinated production, hydropower producer has much richer strategies to maximise its profit. The coordinated production planning under three-sequential market has the highest profit for the hydropower producer.

The coordinated and separated production planning for hour 13 of day-ahead market is drawn in Figure 6. In the coordinated case, the hydropower producer allocates smaller volume at 41.62€ price, because there is still an opportunity to trade on intra-day or on the real-time markets. In contrast, the separated model allocates the higher volume under the same price, because in this case, the only alternative is to sell the water with the expected future price of 40€.

Table VII. The optimal coordinated production allocation in day-ahead, intra-day and real-time markets with maximum risk $\chi=0$ and risk level $\chi=0.001$.

Hours	$\chi=0$					$\chi=0.001$				
	Day-ahead volume	Selling volume	Buying volume	Up-regulation volume	Down-regulation volume	Day-ahead volume	Selling volume	Buying volume	Up-regulation volume	Down-regulation volume
1	235	178	0	0	235	0	121	0	114	0
2	0	0	0	178	0	0	121	0	7	0
3	235	0	0	0	235	0	56	0	35	0
4	235	0	0	0	235	0	0	0	194	0
5	235	178	0	0	235	108	92	0	0	5
6	235	106	0	0	235	37	54	0	9	0
7	235	0	0	0	235	70	110	0	21	0
8	235	0	0	0	235	55	31	0	122	0
9	235	222	0	0	235	20	41	0	174	0
10	235	178	0	0	235	0	113	0	121	0
11	235	235	235	0	0	101	0	0	105	0
12	235	235	0	0	235	131	76	0	0	0
13	235	235	0	0	235	66	37	0	102	0
14	235	178	0	0	235	142	114	21	0	0
15	235	235	0	0	235	56	117	0	62	0
16	235	235	0	0	235	41	143	19	70	0
17	0	235	0	0	0	23	66	0	145	0
18	235	178	0	0	235	97	55	0	83	0
19	0	0	0	89	0	0	91	0	144	0
20	0	235	0	0	0	34	79	0	0	0
21	235	0	190	0	46	5	53	0	143	0
22	235	0	0	0	235	65	0	0	170	0
23	235	0	0	0	235	36	19	0	0	0
24	235	0	64	0	102	27	78	0	0	0

Table VIII. The change of expected profit and profit variance of the hydropower producer with respect to risk factor.

χ	0	0.001	0.002	0.003	0.004
$E[\pi^d]$	186 936	44 845	45 857	47 482	48 095
$E[\pi^{sell}]$	128 934	73 760	69 151	63 132	57 346
$E[\pi^{buy}]$	20 675	1666	898	686	226
$E[\pi^{up}]$	11 529	75 836	78 780	77 853	71 187
$E[\pi^{down}]$	152 096	164	1.1	0	0
$E[\pi^f]$	68 883	14 400	11 007	7407	5560
Total	223 511	207 011	203 896	195 188	181 962
$Var[\pi^d]$	8×10^7	2×10^6	2×10^6	2×10^6	2×10^6
$Var[\pi^{sell}]$	6×10^7	1×10^7	1×10^7	9×10^6	7×10^6
$Var[\pi^{buy}]$	7×10^6	56251	36965	21 687	0
$Var[\pi^{up}]$	2×10^6	7×10^6	6×10^6	5×10^6	4×10^6
$Var[\pi^{dn}]$	6×10^7	1547	0	0	0
$Var[\pi^f]$	1×10^8	5×10^6	3×10^6	1×10^6	1×10^6
Total Variance	30×10^7	3×10^7	2×10^7	17×10^6	14×10^6
Total standard deviation	17 320	5477	4472	4123	3742

Figure 5. Profit frontier.

4.2. Eighteen-reservoir system

A numerical example is provided studying real 18-reservoir cascaded hydropower system from a Swedish river. The layout of the system is depicted in Figure AI, stated in I. The physical characteristics of the system are summarised in Table X.

The optimization problem for 18-reservoir system again is coded in GAMS and is solved with MOSEK solver using the same computer. The total computation time in this case is 0.748 s instead. This proves the claim stated earlier that the model is applicable for a big system. The discharge plan for ‘Iteration 4’ is summarised in the Table XI. According to the results, the model suggests to allocate electric power to day-ahead market and keeps some volume for correcting actions via intra-day and real-time markets. Hence, the results are consistent independent of the size of the hydro system the model is applied.

5. FURTHER DISCUSSIONS AND FUTURE WORKS

Possible extension of the current work will be to build multi-stage stochastic coordinated bidding to day-ahead, intra-day and real-time markets for a risk-averse hydropower producer. In this case, the objective function will be modelled as a convex combination of the expected profit and time-consistence risk measure (29).

Table IX. The change of expected profit and profit variance of the hydropower producer with respect to different cases and risk factor 0.01.

	Case 1	Case 2	Case 3	Case 4	Case 5
$E[\pi^d]$	47 729	60 140	70 439	0	0
$E[\pi^{sell}]$	75 863	0	0	110 608	0
$E[\pi^{buy}]$	793	0	0	0	0
$E[\pi^{up}]$	63 778	79 059	0	0	81 185
$E[\pi^{down}]$	415	0	0	0	0
$E[\pi^f]$	2226	2222	2222	2222	2222
Total	188 388	141 421	72 661	112 830	83 407
$Var[\pi^d]$	2×10^6	3×10^6	3×10^6	0	0
$Var[\pi^{sell}]$	3×10^6	0	0	5×10^6	0
$Var[\pi^{buy}]$	8353	0	0	0	0
$Var[\pi^{up}]$	2×10^6	3×10^6	0	0	3×10^6
$Var[\pi^{down}]$	5459	0	0	0	0
$Var[\pi^f]$	1×10^5	1×10^5	1×10^5	1×10^5	1×10^5
Total	7×10^6	6×10^6	3×10^6	5×10^6	3×10^6

Figure 6. Day-ahead market production for hour 13 using coordinated production planning (solid curve) and separate production planning (dashed curve).

Table X. Data for 18-reservoir system.

Reservoir	$\bar{m}_j$ (HE)	$\bar{Q}_j$ (m^3/s)	$\bar{G}_j$ (MW)	τ_j (min.)
1	2253	108	65	60
2	1921	50	27	480
3	1798	165	75	15
4	4220	160	60	45
5	2602	50	25	45
6	8008	340	95	30
7	1392	310	50	120
8	4008	330	90	360
9	2083	450	45	60
10	1000	315	83	20
11	900	320	25	20
12	0	320	21	30
13	3000	480	110	210
14	0	450	42	10
15	0	450	78	30
16	0	450	203	30
17	795	450	52	120
18	2808	1040	590	0

Table XI. The optimal coordinated production allocation in day-ahead, intra-day and real-time markets applied on the big 18-reservoir system $\chi=0.0001$.

Hours	Day-ahead volume	Selling volume	Buying volume	Up-regulation volume	Down-regulation volume
1	0	976	0	220	0
2	11	276	0	190	0
3	143	288	0	0	143
4	0	0	0	624	0
5	292	727	77	0	120
6	169	418	0	0	0
7	299	533	0	0	215
8	139	199	0	477	0
9	97	388	0	848	0
10	92	675	0	714	92
11	375	0	0	626	0
12	535	459	0	0	0
13	381	124	0	477	0
14	529	797	0	0	0
15	144	510	0	359	0
16	370	1046	0	153	36
17	21	470	0	516	0
18	634	279	0	762	0
19	0	347	0	1014	0
20	148	421	0	36	0
21	99	196	0	866	0
22	474	0	0	773	0
23	69	0	0	0	36
24	290	307	0	0	116

$$\text{Maximise}(1 - \sigma)E[\Pi] + \chi\Phi(E[\Pi]) \quad (29)$$

In (29), $\Phi(\cdot)$ is a real-valued function for the risk measure [33].

As another avenue for future work, the multi-predictor model developed in this paper can be augmented with more predictors and combined forecasts. The combined forecast based on M predictors producing forecasts $p_t^{(1)}$ to $p_t^{(M)}$ of real price P_t for period t can be defined as

$$\hat{p}_t = \sum_{k=1}^M \alpha_k p_t^{(k)} \quad (30)$$

where α_k ($\alpha_1 + \dots + \alpha_M = 1$) is the weight of predictor k in the combined price forecast $\hat{p}_t$ [16].

In general, hydropower plants are characterised by very low variable generation and start-up cost. In the current work, we assumed that the variable generation and start-up cost are negligible. However, introducing binary variables, which allow modelling the variable generation and start-up cost, will provide more detailed hydropower planning model. Hence, considering unit commitment in the model could be another extension of the work.

Another important thing to consider in the future is the head effect while modelling hydropower plants [34, 35]. When reservoirs are big enough, the head effect for short-term planning is negligible. However, modelling head dependency for small reservoirs will bring additional value.

6. CONCLUSION

This paper proposes a QP model to generate optimal coordinated production planning of a risk-averse profit-maximising hydropower producer. The model handles uncertainties in an elegant way while keeping the model deterministic, hence, fast to solve. The day-ahead, intra-day and real-time markets are considered in the model. The risk is defined as the multi-period variance over different market

places. The multi-period risks over different markets are modelled as quadratic terms in the objective function. The model provides a framework to analyse the tradeoff between maximum profit and the multi-period risk that a hydropower producer faces. The MRJD, HW and ARIMA techniques are employed in the multi-predictor price model. The best performing predictor is selected using a designed Markov switch. The discrete behaviour of the intra-day and real-time markets are modelled as different Markov states. The simulation results show the economic gains from optimal coordinated production planning. The fast execution time and convexity of the model guarantee the usefulness of model for big hydrosystems.

7. LIST OF SYMBOLS AND ABBREVIATIONS

7.1. Symbols

j	index for power plants $j = 1, \dots, J$
n	index for discharging segments $n = 1, \dots, N$
t	index for planning periods $t = 1, \dots, T$
k	index for iterations in rolling planning $k = 1, \dots, K$
s	index for market price scenarios $s = 1, \dots, S_k$
R_j	set for power plants downstream hydropower plant j
ω_s	probabilities associated with the price scenarios
$I_{j,t}$	inflow level to each power plant time (HE)
$\bar{G}_j$	maximum power production at plant j (MW)
$\bar{m}_j$	maximum reservoir content (HE)
m_j^0	initial reservoir content (HE)
$\mu_{j,n}$	marginal production equivalent at plant j segment n (MWh/HE)
γ_j	expected future production equivalent for plant j (MWh/HE)
$\bar{Q}_{j,n}$	maximum discharge level in plant j at segment n (HE)
p_s^f	realised future electricity price scenarios (€/MWh)
$\hat{p}^f$	expected future electricity price (€/MWh)
$p_{s,t}^d$	realised day-ahead market price scenarios (€/MWh)
$\hat{p}_t^d$	hourly expected day-ahead market prices (€/MWh)
$p_{s,t}^{sell/buy}$	intra-day market prices for each hour and scenario (€/MWh)
$\hat{p}_t^{sell/buy}$	hourly expected intra-day market prices (€/MWh)
$p_{s,t}^{up/down}$	real-time market prices for each hour and scenario (€/MWh)
$\hat{p}_t^{up/down}$	hourly expected real-time market prices (€/MWh)
τ_j	delay time for the water between power plants (h)
$W_{i,j}^{d}$	covariance matrix for day-ahead market price scenarios
$W_{i,j}^{sell/buy}$	covariance matrix for intra-day market price scenarios
$W_{i,j}^{up/down}$	covariance matrix for real-time market price scenarios
$\pi_{s,t}^d$	day-ahead market profit (€)
$\pi_{s,t}^{sell/buy}$	intra-day market profit (€)
$\pi_{s,t}^{up/down}$	real-time market profit (€)
χ	risk aversion level
$G_{j,t}$	generation level at each power plant and hour (MWh)
$m_{j,t}$	content of reservoir j at the end of hour t (HE)
$m_{j,T}$	reservoir content at the end of the planning period (HE)
$Q_{j,t,n}$	discharged volume for hourly bids, for each power plant, segment and hour (HE)
$S_{j,t}$	spillage from reservoir j during hour t (HE)
x_t^d	hourly bid volumes to day-ahead market (MWh)
$x_t^{sell/buy}$	hourly selling and buying bid volumes to intra-day market (MWh)
$x_t^{up/down}$	hourly up and down bid volumes to real-time market (MWh)
z	objective function value (€).

7.2. Abbreviations

QP	Quadratic Programming
CVaR	Conditional value-at-risk
MRJD	Mean Reversion Jump Diffusion
HW	Holt Winter
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average

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Appendix I

Figure A1. Eighteen-reservoir cascaded hydro system.