Analyzing different quality situations within public-private partnership of infrastructure

The public sector’s point of view

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Abstract

Due to the increasing amount of private-public partnership procurements, the pros and cons of this subject area is of great concern for economists and stakeholders. Previous studies regarding the rationale of this type of procurement argued on the externalities of private-public partnership. Most arguments were upon the negative externalities, which can reduce the private-public partnership’s value. In this thesis, externalities in private-public partnership for infrastructure projects are explained by using operation and maintenance, and user’s quality in more detail and compared in different situations. This study carries that with having positively correlated qualities, the private sector will have maximum profit and by having uncorrelated qualities, and the private sector will have more profit compared to negatively correlated qualities. Furthermore the ways to design optimal contract for public sector, which gives the incentive to the private sector to enhance quality and reduce life-cycle costs, is analyzed.
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1. Introduction
According to UNECE (2012), governments around the world need to provide better and more efficient infrastructure such as transport systems, water and electrical systems, health, education etc. for public welfare. Demand for these types of infrastructures is increasing. However infrastructure projects in many countries have investment difficulties since the governments have to control the expenditures and keep the taxes low (UNECE, 2012).

One option to solve financial problem is to transfer the full responsibility of the infrastructure projects to the private sector, which might not be feasible in most cases due to market downturn or social and national security reasons. This cooperation is called private-public partnership (hereafter abbreviated as PPP). Thus, many governments use their regulatory power to protect the private sector regarding finance, management and innovative technologies (UNECE, 2012).

There are no unified definition for PPP and it differs between the countries. PPP is a long-term contract between the government and private sector to provide a public asset or service in which the private party bears the management and risk responsibilities (World Bank, 2016). The definition of PPP in the Canadian context is, where the risk is transferred between the partners,
and this relates to the provision of public services and infrastructure (The Canadian Council for Public-Private Partnership, 2016). Another definition of PPP is cooperative institutional arrangement between public and private sectors (Hodge, G. A., & Greve, C., 2007).

Under a PPP, a local authority or a central-government agency enters a long time contract (usually between 20-35 years) with a private entity for the delivery of some services. That entity takes responsibility for building, financing, managing, operating and maintaining the facility (Iossa and Martimort, 2015).

There are many pros and cons regarding the use of PPPs. One for advantage of using PPP is allowing the public sector to procure projects, which would be difficult to procure without PPP due to financial and technical difficulties (Devkar and Kalidindi, 2013). Another advantage is by having single responsibility over an asset, there is an incentive to minimize the life-cycle costs and as a result produce a more efficient asset and maintain it more efficiently (Siemiatycki and Farooqi, 2012). In addition, some studies found that by focusing on the outputs rather than inputs more incentives for the private sector to deliver higher quality service are produced (Cheung et al., 2010). Other studies found out that by transferring risks to the private sector, there is a higher incentive to innovate in the service (Eadie et al., 2013).

On the other hand there are some additional risks that imposed by PPP due to its duration, scope and complexity. These risks can be due to financial, technical, operational, sponsor, market, network and interface, industrial relations and regulatory risks (Akbiyikli and Eaton, 2004, Akintoye et al., 2000, Grimsey and Lewis, 2002, Medda, 2007, Ng and Loosemore, 2007 and Victorian Department of Treasury and Finance, 2001).

Due to discrepancy between the social demands and the public resources in many countries, the public sector is seeking to have more PPP contracts to fill up the gap (Scharle 2002). By increasing PPP projects, different situations regarding the quality of service can happen. Quality enhancement is an advantage of PPP. Therefore designing the contract in a way that prevents the private sector’s deviation from quality and effort is greatly important, since governments care about cost efficiencies and quality provision (Dewatripont & Legros, 2005). The aim of this thesis is therefore, to study ways to design a PPP contract, by using mathematical models and different quality dimensions to describe the externalities with mathematical models. By using the models, this thesis will analyze the different possible quality situations for the private sector and study ways to have better qualities from a public perspective of view.

2. Theoretical framework

There are distinct connections between financing, constructing and operating of infrastructure. Construction determines the quality of the infrastructure, which then impact either positively or negatively on operating and maintenance costs. Dewatripont and Legros (2005) argued that unless the constructor is induced to internalize potential externalities in the operating phase, inefficiencies may emerge. The possible cost of inefficiencies can be large when a bad constructed infrastructure raises the risk of costly maintenance. Private sector has incentives to enhance the operation and maintenance quality to reduce costs and also higher social benefits and quality provisions (Dewatripont & Legros, 2005). Therefore, the optimal contract for public sector is to have high operation and maintenance quality and high social benefits provisions (Holmström & Milgrom, 1991). It can be concluded that public sector’s objective is to maximize the consumer surplus (an economic term for consumer’s satisfaction, which can be calculated by the difference between the consumers’ willingness to pay for the service or good received and its market price) minus the costs, whereas the aim of the private sector is just to reduce the costs and have maximum profit.

Dewatripont and Legros (2005) argued that there can be asymmetries in information in these types of projects. Hidden action is one of the most important information asymmetries in PPP.
With a hidden action, a private sector may take actions that are not noticeable for public sector. If the price is set by private sector, they can use lower quality with higher price that gives them the same revenue compared to the higher quality and lower price. They will have the same revenue since by increasing the price, less customer will use the service and therefore, the revenue will be lower, but since they use lower quality, quality provisions will be less costly and therefore they can have same revenue. One good possible way to prevent hidden action regarding the cost monopolistic approach can be setting contract to a fixed price where the private sector will receive fixed amount fee for its service (Dewatripont & Legros, 2005). Therefore, it is recommended that public sector give fixed marginal revenue to private sector in order to prevent the monopolistic approach and increase consumer surplus.

Iossa and Martimort (2015) argued that if the project has the positive externality regarding the effort, it is highly desirable to choose bundling or PPP rather than traditional procurement. In the conditions, that project has negative externality regarding the effort, PPP and traditional procurement works the same (Iossa and Martimort, 2015). So it can be obtained that PPP is a better general procurement than traditional. However, the quality of durable assets and infrastructure may depreciate due to the contract lengths which are normally between 20-35 years. Long term contract creates an issue regarding the finding new investment for extending the contract, or new management strategies to maintain costs low in short-term (Iossa and Martimort, 2015). Another study argued that PPP is preferred when quality cannot be controlled and cost efficiencies are not the key (Hart et al. 1997). Some other, argued that rationale for PPP is internalization of positive externality due to having single responsibility over an asset, which gives incentive to reduce life-cycle costs (Blanc-Brude, Goldsmith and Välilä, 2009).

Although the literature focused on PPP rationale, mostly based on externalities, there is a gap in the literature to define the externalities and mathematically model it and also analyze the situations that can happen within PPP contract regarding the quality and externalities. Previous studies have used the only two externality situations that have positive and negative externalities and they did not go into the detail of externalities regarding the quality. In this thesis, it is tried to separate the quality into more detailed qualities to define the externalities based on the model created. After the model is created, it is expected that model can analyze the externalities with more detail and find ways to create optimal contract for public sector within the possible externalities situations.

### 3. Methodology

The thesis takes both a theoretical and deductive approach which process some premises and by using mathematical equations, it draws logical explanations and conclusions. The purpose is to create a general contract and to set different theories as assumptions to generate new models. It will define the key parameters and the mathematical model based on assumptions regarding the quality of operation and maintenance, social benefit, quality provision and investment. The thesis will generate mathematical models for investment stage, which leads to have a contract for public sector which gives the objectives or public sector. It will be followed by how to use the model and a numerical example based on rational assumptions. These will lead to conclusion with consideration of model limits in form of report.

The models and mathematical equations are created theoretically. Calculations are based on mathematical and economic theories. Basic related economy variables have chosen such as quality-demand function etc. which are described in detail in the model section, and they will be modeled using mathematics to explain the situations that can happen within the PPP infrastructure contracts. Therefore analyzing the equations and models will be based on mathematical and economic facts and hence it does not required data collection. It is possible that model will not work for every possible situation, which considers as limitation of the model.
4. Model

In order to have a contract which gives benefits to both, public and private sectors, different objectives of public and private sector must be analyzed. It is assumed that public sector’s objective of the service is to have a maximized consumer surplus minus operation and maintenance costs and their investment and monitoring, whereas the private sector’s objective is, just to maximize their profit. Public sector has three regulatory power to determine in the contract. First the power to set the customer’s charge and transfer it to private sector by either fixed marginal revenue or percentage of the fee (which the customers should pay to use the facility or service, for example paying a fee to use the highway), which results in variable marginal revenue for private sector. For example, give a percentage of the price charged from customers for passing the highway to the private sector, or give a fixed price per each customer that passes the highway. Second the power to determine the length of the PPP contract. Third is the initial investment, which can be either percentage of the initial investment or percentage of the investment.

4.1 Public sector’s initial investment

Before the construction of service begins, private sector needs to have an initial investment for the construction. This initial investment can be either provided by the private sector solely or by private and public sector together. Public sector can either give a fixed part of the initial investment or give a percentage of total initial investment to the private sector, as a subsidy, for some project which are not financially profitable for private sector. Hence, the private sector will not accept the project if they do not receive any subsidy to make it profitable for them. This case can happen for example for a road in a deprived area which does not absorb enough customers to pay the fee and make the project profitable, but the public sector need that project. Therefore, public sector will financially support the private sector in the initial investment phase.

Due to asymmetric information and strategic cost overruns, it is preferred to have a fixed payment from public sector to prevent the strategic cost overruns. Strategic cost overruns can be announced when the private sector have more information regarding the costs which they can use it to announce higher investment costs for the initial investment. Therefore, if the public sector has to pay the percentage of investment, they are obliged to pay more than their real payment share that was a percentage of total investment spent on the initial investment. This is due to private sector’s initial investment’s announcement will be higher than the actual and real initial investment. For example if the public sector is obliged to pay 50 percent of the total initial investment, and the total initial investment costs are 100 billion Swedish kronor, the private sector will announce 150 billion Swedish kronor spent on the initial investment. Therefore public sector is obliged to pay 75 billion Swedish kronor instead of the real share, which is 50 billion Swedish kronor.

4.2 Quality

Two issues regarding the quality effort need to be analyzed in the investment stage. First, effects of increasing quality in the future operation and maintenance costs. Second, effect of increasing welfare quality that provide customer’s attraction to use the provided service.

Thus, it is assumed that there are two kinds of quality. First is the operation and maintenance quality ($Q_{O&M}$) with illustrates the reduction of operation and maintenance costs due to increasing the quality. Second is the user’s quality, which illustrates the welfare enhancement due to increasing the quality.

4.2.1 Operation and maintenance quality

With increasing $Q_{O&M}$ to an optimal point, private sector will have better net project value. Since marginal costs will decrease. On the other hand, higher $Q_{O&M}$ demands more initial investment for operation and maintenance ($I_{O&M}$). It is assumed that $Q_{O&M}$ and operation and maintenance
costs have reversed correlation. It should be noted that the operation and maintenance costs calculated in equation 1 is based on the initial data for the first year multiplied by the total years of the contract.

\[
\text{operation and maintenance costs} = \frac{PV}{b \times Q_{O&M}}
\]

Where \( PV \) is an abbreviation for present value which is considered as a scaler and shows the effect of future costs on the present value. \( b \) defines the effect of \( Q_{O&M} \) into operation and maintenance costs. It has higher value when increasing a unit of \( Q_{O&M} \) results in lower operation and maintenance cost compared to when \( b \) has lower value. For example, if the asphalt of highway changes to a durable asphalt, the operation and maintenance cost will decrease which means \( b \) gets higher compared to normal asphalt. In this case, a same change in effort regarding \( Q_{O&M} \) of construction of the road will reduce more operation and maintenance cost compared to the reduction of operation and maintenance cost for the same change in effort regarding \( Q_{O&M} \) of construction with the normal asphalt which defines the effect of \( b \).

It is assumed in this model that, either renovation will not be needed during the contract length, or if the renovation is needed, cost of renovation is considered to be normally distributed between the operation and maintenance costs and is included in the operation and maintenance costs.

Improving the \( Q_{O&M} \) demands more initial \( I_{O&M} \). It is assumed that they have linear correlation for simplicity.

\[
I_{O&M} = a \times Q_{O&M}
\]

Where \( a \) defines the effect of \( Q_{O&M} \) on \( I_{O&M} \). It has lower value when increasing a unit of \( Q_{O&M} \) will results in lower \( I_{O&M} \) compare to a higher value for same increase in \( Q_{O&M} \). For example increasing the \( Q_{O&M} \) by using different durable material demands more initial investment than using more effort during the construction to enhance \( Q_{O&M} \) to the same amount of \( Q_{O&M} \) using the different material. Therefore \( a \) is lower for the latter situation. Figure 1 illustrates the operation and maintenance effects on investment costs and present value of operation and maintenance costs.
Therefore, the optimal $Q_{O&M}$ is where the sum of present value of operation and maintenance costs and $I_{O&M}$ is minimum.

Figure 2 illustrates the effect of operation and maintenance quality on total costs considering the investment costs and reduced present value of operation and maintenance costs.

\begin{equation}
\text{Total costs of } O&M = \text{Present Value}(\text{Operation and maintenance costs}) + I_{O&M}
\end{equation}

By using equations 1, 2, 3 total costs regarding the operation and maintenance will be:

\begin{equation}
\text{Total costs of } O&M = PV\left(\frac{1}{b \times Q_{O&M}}\right) + a \times Q_{O&M}
\end{equation}
To find the minimum total price, equation 4 is differentiated with respect of $Q_{O&M}$ and is set to zero:

\(0 = \frac{-PV}{b \times Q_{O&M}^2} + a\)

By simplifying the equation 5, the optimal $Q_{O&M}$ for minimizing the total cost is:

\(Q_{O&M} = \frac{PV}{\sqrt{a \times b}}\)

From equation 6, it can be seen that the optimal $Q_{O&M}$ is dependent on the changes on operation and maintenance costs regarding $Q_{O&M}$ and changes on $I_{O&M}$ regarding $Q_{O&M}$. Therefore, it shows that for higher $Q_{O&M}$ than the optimal calculated above, there should be extra $I_{O&M}$ which reduces the profit by increasing the total costs that is not the objective of either public or private sectors. And for lower $Q_{O&M}$ than optimal calculated above, there will be more present value of operation and maintenance costs which reduce the profit by increasing the total costs related to operation and maintenance. Hence, the optimal $Q_{O&M}$ calculated above will minimize the total costs and thus, maximize the profit regarding the total costs, which is the objective of both private and public sector.

By using equations 1 and 6, it can be written:

\(\text{Operation and maintenance costs} = \frac{PV}{b \times \sqrt{\frac{PV}{a \times b}}} = \sqrt{\frac{a \times PV}{b}}\)

By using equations 2 and 6, it can be written:

\(I_{O&M} = a \sqrt{\frac{PV}{a \times b}} = \sqrt{\frac{a \times PV}{b}}\)

From equations 7 and 8, it can be concluded that for optimal $Q_{O&M}$ calculated in equation 6:

\(I_{O&M} = \text{Operation and maintenance costs}\)

Therefore, by using equations 9 and 4 total costs of operation and maintenance regarding the optimal operation and maintenance quality can be calculated:

\(\text{Total costs of O&M} = 2 \sqrt{\frac{a \times PV}{b}}\)

### 4.2.2 User’s quality

It is assumed that user’s quality ($Q_W$) defines the welfare of the service for customers. Generalized costs for customers (GC) is defined as below:
\[ GC = \text{Total costs for customer} + \text{customers' charge} - f \times \sqrt{Q_W} \]

Where total costs for customer are the customers' costs, which are not related to the service directly. For instance the fuel and travel time etc. for passing the highway which the project’s quality enhancement cannot affect them. customers' charge is the charge that customers should pay for using the facility. For instance paying fee to pass the highway. It is assumed that by enhancing the \( Q_W \), generalized costs for customer will decrease. For example using better quality asphalt which reduce the tires or car springs damages. Therefore, it will reduce the generalized cost of the customers. Total costs for customer and customers' charge (explained in further section) are assumed to be fixed.

Figure 3 Generalized costs - Demand

Figure 3 illustrates the effect of generalized costs on demand (D). It should be noted that demand in this paper means the total demand for the PPP length. For some customers, the provided service is important to use and they are willing to pay much more than the generalized costs. They are the first units of quantity in the figure which have the most consumer surplus. There will be customers that the service is important for them, but not as much as the aforementioned group of customers. They are willing to pay less than the previous group, but still more than generalized costs. By increasing the quantity, willingness to pay will decrease for new units of quantity, since the service for new units is not as important as for the previous units, therefore consumer surplus is less for them and so on. The intersection between the demand line and generalized costs, is where the group of customers are willing to pay exactly the generalized cost, therefore consumer surplus for them is zero. After that, next group are willing to pay less than generalized costs, thus they will not use the service since they do not want to pay the generalized costs for it. Hence it can be said that demand is where the demand line above will intersect with generalized costs.

As the customers' charge decreases, demand will increase since the customers should pay less for the service with the same quality. But since it is considered that total costs for customers and customer's charge are fixed, therefore \( GC \) changes is solely relying to changes of \( \sqrt{Q_W} \). This means changes on quality is only achievable to absorb demand.
As it is illustrated on figure 3, by changing GC line position, demand and consumer surplus, which is the area above the GC and below Demand line, will change. Intersection of GC and demand line will give the total demand. Lower GC will result in higher demand since the intersection will have more quantity, and higher consumer surplus. By increasing \( Q_W \), GC will decrease, thus demand and consumer surplus will increase. Hence, \( c \) can be defined as changes in \( Q_W \) and have positive correlation with \( D \) as it is shown in equation 12.

\[
D = c \times \sqrt{Q_W}
\]

Where \( c \) defines the effect of \( Q_W \) on demand. It has higher value when increasing \( Q_W \) will result in higher demand compare to its lower value. For example increasing the same amount of user’s quality on smoothness of the asphalt may absorb fewer customers than increasing the safety of the road by using guardrails etc., hence latter has higher \( c \).

By using equations 11 and 12, demand can be written using GC:

\[
D = c \times (\text{Total costs for customer} + \text{customers' charge} - \text{GC})
\]

It is assumed that \( Q_W \) has linear correlation with investment for user’s quality \( I_W \):

\[
I_W = d \times Q_W
\]

Where \( d \) is constant, which defines the effect of \( Q_W \) on \( I_W \). It has higher value when increasing \( Q_W \) will result in higher \( I_W \) compare to lower a value. The same example for \( I_{O&M} \) and \( Q_{O&M} \) can be assumed for this equation too.

It should be noted that \( I_W \) and \( I_{O&M} \) do not have shared parts and are completely separate. Which means \( I_{O&M} \) is just the investment for enhancing \( Q_{O&M} \), while \( I_W \) is the investment for constructing the project and \( Q_W \) enhancement. Therefore, there will be no cost which is considered to be invested by both \( I_{O&M} \) and \( I_W \). Figure 4 illustrates the effect of user’s quality on the investment costs and present value of revenue.
The optimal $Q_W$ for public sector, is to have maximum consumer surplus minus the total costs. Hence, it will be the maximum difference between the revenue and $I_W$ or in mathematical words, when the slope of revenue curve is the same as slope of investment line.

For profit maximizing private sector when price is constant (it will be shown the best option is for public sector is to give constant price to private sector), the optimal $Q_W$ is the maximum difference of Demand $(D)$ times price $(P)$ (which is present value of total revenue $(PVR)$) and $I_W$, since it will lead to maximum profit. Therefore, the optimal $Q_W$ for both public and private sector is equal.

\[(15)\]

\[ \text{Profit} = PVR - I_W - I_{O&M} - \text{Operation and maintenance cost} \]

![Figure 5](image_url)  
*Figure 5 Difference between present value of total revenue and investment regarding user’s quality*

Figure 5 illustrates the effect of user’s quality on total costs regarding the present value of revenue and investments.

Since operation and maintenance cost and $I_{O&M}$ are functions of $Q_{O&M}$, they have no effect on the maximizing the profit regarding $Q_W$. The difference of $PVR$ and $I_W$ can be calculated from equations 12 and 14 as:

\[(16)\]

\[ PVR - I_W = D \times PV(P) - I_W = c \times \sqrt{Q_W} \times PV(P) - d \times Q_W \]

In order to maximize the equation above, and get Optimal $Q_W$ for profit maximizing private sector, the equation is differentiated with respect to $Q_W$ and sets to zero:

\[(17)\]

\[ 0 = \frac{cPV(P)}{2\sqrt{Q_W}} - d \]

Therefore:

\[(18)\]

\[ Q_W = \left(\frac{cPV(P)}{2d}\right)^2 \]
Equation 18 illustrates that with increasing the price, private sector has more incentives to enhance user’s quality. It should be noted that the price used in equations above is present value of price, which means the price given in future, worth different value than the price today. It is shown in this paper that fixed price is the optimal price for the public sector (see customer’s charge section). Therefore optimal $Q_{W}$ for private sector is fixed and depends on the initial agreement fixed price from the public sector, which will be given to private sector for each customer.

### 4.3 Customer’s charge

There are two different options to transfer the customers’ charge to the private sector: giving the percentage of the fee to private sector or giving the fixed priced which means fixed marginal revenue.

By giving the percentage of the fee and the power of setting the customer’s charge to private sector, the public sector will give the monopoly power to the private sector. By reducing the customers fee (Price), more customers will use the service (higher demand). However, more demand will also increase the marginal costs due to more required maintenance and operation costs. The optimal price for private sector is where marginal revenue and marginal cost will be equal. In this case, the optimal and most profitable price is the intersection of marginal revenue and marginal cost lines. Figure 6 illustrates the choice of price regarding the demand and marginal revenue.

![Variable Marginal Revenue](image)

*Figure 6 Variable marginal revenue*

Once marginal revenue and marginal cost are equal, then for each additional customer, marginal costs will increase and will be more than marginal revenues. Therefore, the revenue gained for that additional customer is less than the costs that it produces. Thus, private sector will set the price according to the demand that they can have, when marginal costs and marginal revenues are equal. Whilst in order to maximizing consumer surplus, qualities must be enhanced to the point where the demand and marginal costs are equal. This means the price should be set lower to have more demand and therefore, lower profit. Lower profit is due to additional units of quantity after the marginal costs and revenue are equal. Since for those additional units, marginal costs are higher than marginal revenue, which produces more costs than revenue.

Revenue for the private sector is quantity or in this case demand (D) times price (P) which is the user’s charge that each user should pay to use the service.

\[
R_{D} = D \times P_{D}
\]
Marginal revenue (MR) is the derivative of revenue regarding the quantity which shows the rate of change of revenue for an extra unit of quantity which is equal to demand in this case.

\[
MR_D = D \times P_D' + P_D
\]  

As shown in equation 20, if the rate of change of price regarding the demand \((P_D')\) is set to zero, which means the price is fixed for customers to use the service, marginal revenue will be fixed and equal to price. But for variable price, marginal revenue is decreasing for the higher demand since the rate of change of price should be negative for the same quality to absorb more customer. Therefore marginal revenue for variable price will be less than fixed price and decreasing. By increasing \(Q_W\), \(P_D'\) will increase, which means by having more \(Q_W\) rate of change of prices will increase for extra unit of quantity. For simplicity, it can be assumed that \(Q_W\) and \(P_D'\) have correlation with the same demand which is shown in equation 21:

\[
P_D' = -\frac{1}{g \times Q_W}
\]  

From equation 21, it can be seen that by increasing \(Q_W\) for the same demand, \(P_D'\) will increase also and get closer to zero. It means that the price needs to get less low to absorb the next unit of quantity when \(Q_W\) is higher compared to lower \(Q_W\).

By using equation 12, 20 and 21, it can be written that:

\[
MR_D = c \times \sqrt{Q_W} \times \frac{1}{g \times Q_W} + P_D = -\frac{c}{g \times \sqrt{Q_W}} + P_D
\]

Thus, by increasing \(Q_W\), marginal revenue will increase since the negative part of the equation is reducing. By having equal marginal revenue and marginal demand, private sector will not get any profit from the last quantity unit added, since the revenues that it provide is the same as the costs that it produce. Since marginal revenue is decreasing and marginal costs are increasing, from that point and after, private sector will lose profit and have negative profit, since revenues added for the next unit of quantity is less than added costs. On the other hand, for every previous quantity added, private sector have profit, since marginal revenues were higher than marginal costs. This means the previous units of quantity, before the quantity where marginal revenue and marginal costs are equal, revenues added were more than costs added related to the units. Therefore private sector could benefit from each of the added units of quantity.

The slope of the marginal cost is the effect of \(Q_{O&M}\) on the operation and maintenance costs. It is steep with lower \(Q_{O&M}\) and it is gradual when \(Q_{O&M}\) is higher. Therefore private sector is willing to do more \(Q_{O&M}\) enhancement to an optimal calculated \(Q_{O&M}\) where they will get reducing operation and maintenance costs, since it will increase the demand for the equal marginal revenue and marginal costs, which leads to higher profit.

In the public sector’s perspective, the optimal \(Q_W\) is the intersection of Demand and Marginal cost lines to have maximum consumer surplus. This means lower price for more demand, which results in more costs and less revenue and hence, less profit for the private sector.

By giving fixed marginal revenue to private sector, private sector has the incentive to reduce the slope of marginal costs by enhancing the \(Q_{O&M}\) in order to have more demand and as a result, more revenue and more profit. Since the costs are reduced. By using equation 16, it can be seen that fixed marginal revenue means, zero changes in rate of change of price or in the other words, fixed price from public sector to private sector for each customer using the service. In
In this case, public sector will acquire more quality and consumer surplus and private sector will acquire more profit, which is favorable contract for both sectors. Therefore regardless of quality effects on each other \( Q_{O&M} \) and \( Q_W \) it is desirable for public sector to have a fixed marginal revenue contract and not monopolistic approach.

By having different qualities, the slope of Marginal Cost line will change. It is steeper with lower \( Q_{O&M} \) and it is gradual with higher \( Q_{O&M} \). It should be noted that higher \( Q_{O&M} \) needs more initial investment. Therefore, the initial point of marginal cost will be with higher price. As it is illustrated in the Figure 7, the maximum demand, which has positive correlation with \( Q_W \) that produce the maximum profit, is not for the highest quality due to highest investment. Contradictory, maximum demand or maximum \( Q_W \) (positive correlation of demand and \( Q_W \)) is for the optimal quality investment.

![Figure 7 Constant marginal revenue](image)

Figure 8 illustrates the equation 6, which the optimal \( Q_{O&M} \) that generates the optimal marginal costs for private sector is calculated by using the marginal revenue and marginal costs. As discussed before, slope of marginal costs is the effect of \( Q_{O&M} \). The more \( Q_{O&M} \) the gradual the slope, but the initial point starts with the higher price due to larger initial investment for the higher \( Q_{O&M} \). It can be concluded that \( Q_{O&M} \) has an optimal point which has been calculated and is not the largest possible \( Q_{O&M} \) due to the initial investment and profit maximizing behavior of private sector. Therefore, for higher \( Q_{O&M} \) than optimal \( Q_{O&M} \), initial investment is too high that even with more gradual slope, marginal cost line of them will intersect earlier with marginal revenue line that reduces the private sector’s profit.
4.4 Contract length

By having a shorter contract, private sector is willing to reduce $Q_{O&M}$ to the optimal quality of their contract length since they are not obliged to operate and maintain the service after the contracts ends. Nevertheless, they still need the higher demand which can be obtained by increasing $Q_W$. This scenario can be useful when $Q_{O&M}$ and $Q_W$ have negative correlation.

According to equation 18, future interest rates can change the present value of $P$ which will results in changing the user’s quality enhancing incentives for private sector. It can be concluded that if the future interest rates are higher, net value of $P$ is less than net value of $P$ with lower future interest rates, which will lower the user’s quality enhancing incentives for private sector. On the other hand, if future interest rates are lower, present value of $P$ is higher than net value of $P$ with higher future interest rate, which results in higher user’s quality enhancement incentives for private sector.

4.5 Private sector’s game tree

In some cases, improving the $Q_{O&M}$ in the construction stage will reduces $Q_W$. For example, using durable asphalt will delay the repairing and hence, reduce the maintenance cost but it may have worse quality for the road and cars, which has negative externality. In some cases, improving the $Q_{O&M}$ in the construction stage will increase the $Q_W$ also. For instance using high quality and durable asphalt may reduce the repair costs and it will increase $Q_W$, which has positive externality. In some other cases, $Q_{O&M}$ and $Q_W$ might not have any effects on each other. For example using durable tar for asphalt may not increase or decrease $Q_W$ of the road.

Due to profit maximization incentives of private sector, they will choose to have higher $Q_{O&M}$ to reduce their operation and maintenance costs and as a result, a lower marginal cost. Therefore, in the scenarios that user’s quality is not observable, and it has negative correlation with operation and maintenance quality, private sector will reduce $Q_W$ to have profit maximization. Hence, PPP may reduce its value.

Figure 9 illustrates the Private sector’s three different quality situations which can happen for different kinds of projects: Positively correlated $Q_{O&M}$ and $Q_W$, negatively correlated $Q_{O&M}$ and $Q_W$ and uncorrelated $Q_{O&M}$ and $Q_W$. 

\[\text{Figure 8 Quality effect with Constant Marginal Revenue}\]
It should be noted that these quality situations are not changeable and are according to specifics of each projects. Thus there is no choice for private sector to choose them or change them to other situation. Thus, studying all of them is required for the public sector to be able to design the contract for the specific situation for specific project.

By having negatively correlated $Q_{O&M}$ and $Q_W$ (Corr($Q_{O&M}$, $Q_W$)<0), two different situation for observable $Q_W$ and Unobservable $Q_W$ should be analyzed.

4.5.1 Negatively correlated $Q_{O&M}$ and $Q_W$ with unobservable $Q_W$

This situation can happen when customers cannot observe the true user’s quality and they observe the unreal user’s quality. For instance a hospital with good appearance and seems to be clean in scale of human-eye size, but in fact, it is not decontaminated from smaller scales contamination which will reduce user’s quality. In the meantime, by increasing the $Q_{O&M}$, true $Q_W$ will decrease.

If the $Q_W$ is unobservable by public sector and customers, it is preferable for public sector to have a shorter length contract. This will reduce the private sector incentives to enhance $Q_{O&M}$ and as a result, higher $Q_W$ compare to longer length contract. Monitoring is needed in this situation and it is not avoidable in order to have a balance between the different qualities. These scenarios are hard to detect for public sector (it can be detectable after using the service and check the final results) and especially not detectable during the construction. Therefore, there is a limitation of uncertainty whether monitoring is needed or not. It should be noted that after the true $Q_W$ is observed by the results, demand will decrease and this section will follow up by the consequences of the next section, observable $Q_W$. Therefore, it is unlikely that private sector chooses this solution, unless they are certain that results cannot be detectable before their contract ends.

4.5.2 Negatively correlated $Q_{O&M}$ and $Q_W$ with observable $Q_W$

If the $Q_W$ is observable for customers, profit maximizing private sector is required to increase $Q_W$ to the optimal point to have the optimal demand. It is assumed that $Q_{O&M}$ and $Q_W$ have linear negative effect on each other:

$Q_{O&M} = \text{direct } Q_{O&M} - e Q_W$

Where $e$ is a positive constant which defines the correlation between $Q_{O&M}$ and $Q_W$ and direct $Q_{O&M}$ is the direct effort of $Q_{O&M}$ for private sector to increase $Q_{O&M}$ to the optimal calculated $Q_{O&M}$ in equation 6. In this case where qualities are negatively correlated, direct $Q_{O&M}$ is bigger than $Q_{O&M}$ since the effect of $Q_W$ is negative on $Q_{O&M}$. It is assumed that private sector enhances $Q_W$ first and then, calculates the $Q_{O&M}$ required. At last, direct $Q_{O&M}$ can be calculated according to equation 23 and according to that, private sector enhances the
direct $Q_{O&M}$. Therefore, the effect of enhancing $Q_{O&M}$ on $Q_W$ can be disregarded. Profit for private sector according to equation 15 is:

$$Profit = PVR - I_W - I_{O&M} - \text{Operation and maintenance cost}$$

It is assumed that private sector first invests on $I_W$ since it is required for constructing the project, and as a result, $Q_{O&M}$ decreases due to negative correlation with $Q_W$. It should be noted that $I_{O&M}$ is the operation and maintenance investment which is just for direct operation and maintenance quality which private sector is investing to have that optimal quality. Operation and maintenance cost should be considered with $Q_{O&M}$ from equation 23, since $Q_W$ is affecting that and it is the real $Q_{O&M}$ for the service considering the effect of $Q_W$. Considering equations 1, 2, 12 and 14 profit regarding the qualities is:

\[ Profit = c \times \sqrt{Q_W} \times PVP - d \times Q_W - a \times \text{direct } Q_{O&M} - \frac{PV}{b \times Q_{O&M}} \]

Considering equations 23 and 24, profit can be written as:

\[ Profit = c \times \sqrt{Q_W} \times PVP - d \times Q_W - a \times \text{direct } Q_{O&M} - \frac{PV}{b \times (\text{direct } Q_{O&M} - e Q_W)} \]

Profit regarding $Q_W$ can be written as:

\[ Profit (Q_W) = c \times \sqrt{Q_W} \times PVP - d \times Q_W + \frac{PV}{be \times Q_W} \]

In order to maximize the profit regarding $Q_W$, equation 25 is differentiated regarding $Q_W$ and sets to zero:

\[ 0 = \frac{cPVP}{2\sqrt{Q_W}} - d - \frac{PV}{be \times Q_W^2} \]

By simplifying equation above, the optimal $Q_W$ for observable $Q_W$ when $Q_{O&M}$ and $Q_W$ are negatively correlated can be calculated (see appendices). With having optimal $Q_W$, $Q_{O&M}$ and as a results, direct $Q_{O&M}$ can be calculated according to equations 6 and 23.

Equation 27 demonstrates that private sector will increase the both qualities to their optimal point without monitoring, which increases the value of PPP compared to traditional procurement. By using equation 6, 23 and 27, optimal $Q_{O&M}$ can be calculated. By using equation 24, maximum profit for the private sector can be calculated.

\[4.5.3\] Positively correlated $Q_{O&M}$ and $Q_W$

By having positively correlated $Q_{O&M}$ and $Q_W$ (Corr($Q_{O&M}$, $Q_W$) > 0), private sector will increase $Q_{O&M}$ to an optimal point and therefore, $Q_W$ will increase also. The favorable situation for public sector is to have higher $Q_W$ and $Q_{O&M}$ to have an optimal point of maximizing the consumer surplus while minimizing the costs, hence it is better to have a longer contract, which will increase the private sector’s incentive to enhance $Q_{O&M}$ and as a result, enhanced $Q_W$. Due
to monopoly power which was showed that reduces the quality, which in this case is both $Q_{O&M}$ and $Q_W$, it is better for public sector to choose constant marginal revenue to gain maximum $Q_{O&M}$ and $Q_W$. In this situation, observable or unobservable qualities does not matter, since private sector is willing to enhance $Q_{O&M}$ to reduce operation and maintenance costs, and as a result, $Q_W$ increases.

It should be noted that direct $Q_{O&M}$, $Q_W$ and maximized profit can be calculated using equation 23 by using $-e$ instead of $e$. Therefore, it can be written:

$$Q_{O&M} = \text{direct } Q_{O&M} + e Q_W$$

It is assumed that private sector will invest on $I_W$ since it is required for constructing the project, and as a result, they can benefit from increasing $Q_{O&M}$ due to its positively correlation with $Q_W$. It should be noted that $I_{O&M}$ is the operation and maintenance investment which is just for direct operation and maintenance quality which private sector is investing to have that optimal quality. Operation and maintenance cost should be considered with $Q_{O&M}$ from equation 23, since $Q_W$ is affecting that. It should be noted that for large $Q_W$ and small $Q_{O&M}$, direct $Q_{O&M}$ can be negative. It can be interpreted that private sector does not require to enhance direct $Q_{O&M}$ and can disregard investing on operation and maintenance quality, but since the effect of $Q_W$ on $Q_{O&M}$ is large enough, operation and maintenance cost will optimize and reduced significantly. Therefore, for the situations that direct $Q_{O&M}$ calculated to be negative, it can be set to zero and continue the calculation by setting the direct $Q_{O&M}$ to zero. Considering equations 1, 2, 12, 14 and 28 profit regarding the qualities can be written as:

$$\begin{align*}
\text{(29)} \\
\text{Profit} &= c \times \sqrt{Q_W} \times PV - d \times Q_W - a \times \text{direct } Q_{O&M} - \frac{PV}{b(\text{direct } Q_{O&M} + e Q_W)}
\end{align*}$$

Profit regarding $Q_W$ can be written as:

$$\begin{align*}
\text{(30)} \\
\text{Profit} &= c \times \sqrt{Q_W} \times PV - d \times Q_W - \frac{PV}{be \times Q_W}
\end{align*}$$

In order to maximize the profit, equation 30 is differentiated regarding $Q_W$ and sets to zero:

$$\begin{align*}
\text{(31)} \\
0 &= \frac{cPV}{2\sqrt{Q_W}} - d + \frac{PV}{be \times Q_W^2}
\end{align*}$$

By simplifying equation above, the optimal $Q_W$ for observable $Q_W$ when $Q_{O&M}$ and $Q_W$ are negatively correlated can be calculated (see appendices). With having optimal $Q_W$, $Q_{O&M}$ and as a results, direct $Q_{O&M}$ can be calculated according to equations 6 and 28.

It can be concluded that in this situation, private sector will increase the qualities to an optimal point without monitoring, which increases the value of PPP in comparison with traditional procurement. It should be noted that for positively correlated qualities, unobservable user’s quality and observable user’s quality works the same, since private sector is willing to enhance operation and maintenance quality and as a result, user’s quality will be enhanced.
4.5.4 Uncorrelated $Q_{O&M}$ and $Q_W$ with observable $Q_W$

By having uncorrelated $Q_{O&M}$ and $Q_W$ and observable $Q_W$, private sector will choose to increase $Q_{O&M}$ to the optimal calculated ($Q_{O&M} = \sqrt{\frac{PV}{ab}}$) point to have maximized profit. They will also increase $Q_W$ to the optimal calculated ($Q_W = \left(\frac{c_{PV}}{2d}\right)^2$) point to have maximized profit. In this situation, public sector will get maximized consumer surplus regarding the minimized costs possible without monitoring.

4.5.5 Uncorrelated $Q_{O&M}$ and $Q_W$ with unobservable $Q_W$

By having uncorrelated $Q_{O&M}$ and $Q_W$ and unobservable $Q_W$ for customers and public sector, private sector will choose to increase $Q_{O&M}$ to the optimal calculated ($Q_{O&M} = \sqrt{\frac{PV}{ab}}$) point to have maximized profit. However, they will not increase $Q_W$ at all. Public sector cannot force the private sector to enhance $Q_W$ due to asymmetric information, unless they have monitoring on the quality performance. As it is described in negatively correlated qualities with unobservable user’s quality section, after the results are detected, the true $Q_W$ and demand and hence, profit will decrease. Therefore, it is unlikely that private sector chooses this situation unless they are certain that results cannot be detected within their contract period.

In conclusion, it can be seen that in many different situations, monitoring is not required for quality enhancement in PPP. By comparing quality enhancement for PPP without monitoring and traditional procurement, which requires monitoring in order for quality enhancement, it can be concluded that PPP works better for public sector. In the situations described that monitoring was needed, PPP works the same as traditional procurement.

5 How to use the model and numerical examples

To show how the model works, a numerical example is provided based on assumed rational numbers for variables and constants in the model. First it is considered that qualities are uncorrelated.

Figure 10 illustrates the changes in optimal calculated $Q_{O&M}$ regarding $a$ and by having constant $b = 5$ and $PV = 0.8$ using equation 6.

![Figure 10 Operation and maintenance quality changes regarding a](image)
It can be seen in figure 10 that by increasing $a$, optimal $Q_{O&M}$ decreases for constants $b$ and $PV$. Therefore, if $a$ is bigger, more operation and maintenance investment is required to reach the optimal calculated quality. It can be concluded that for larger $a$, profit decreases due to investment increasing.

Figure 11 illustrates the changes in optimal calculated $Q_{O&M}$ regarding $b$ and by having constant $a = 0.05$ and $PV = 0.8$ using equation 6.

![Figure 11 Operation and maintenance quality changes regarding b](image)

It can be seen in figure 11 that by increasing $b$, optimal $Q_{O&M}$ decreases for constants $a$ and $PV$. Therefore, if $b$ is bigger, less operation and maintenance costs is required, and it reduces the optimal calculated $Q_{O&M}$. It can be concluded that for larger $b$, profit increases due to reducing the optimal $Q_{O&M}$ which reduces the operation and maintenance investment considering constant $a$ and equation 2.

By using equation 10 and the aforementioned assumed values for $a = 0.05$, $b = 5$ and $PV = 0.8$, $I_{O&M} = Operation and maintenance costs = 0.09$. Therefore, it can be written that costs of $O&M = 0.18$. It is considered that costs are in billion Swedish Kronor. $Q_{O&M}$ can be calculated using the assumed values, using equation 6. $Q_{O&M} = 1.8$.

By using equations 8, 10, 12 and 14, it can be written that:

\[
(32) \quad PVR - I_w = D \times PV(P) - I_w = \frac{(cPVP)^2}{2d} - \frac{(cPVP)^2}{4d} = \frac{(cPVP)^2}{4d}
\]

It can be seen that the positive part of the equation above or the income of private sector is \(\frac{(cPVP)^2}{2d}\) which is twice as \(\frac{(cPVP)^2}{4d}\) = $I_w$.

Figure 12 illustrates the changes in optimal calculated $Q_w$ regarding $c$ and by having constant $P = 0.000,000,05$ billion Swedish Kronor (50 SEK), $d = 0.0002$ and $PV = 0.8$ using equation 18.
It can be seen in figure 12 that by increasing \(c\), optimal \(Q_{w}\) increases for constants \(P\), \(d\) and \(PV\). Therefore, if \(c\) is bigger, more customers absorbs and increases the demand, and it increases the optimal calculated \(Q_{w}\). Therefore, private sector requires to enhance \(Q_{w}\) more when \(c\) is bigger.

Figure 13 illustrates the changes in optimal calculated \(Q_{w}\) regarding \(p\) and by having constant \(c = 500,000\), \(d = 0.0002\) and \(PV = 0.8\) using equation 18.

It can be seen in figure 13 that by increasing \(P\), optimal \(Q_{w}\) increases for constants \(c\), \(d\) and \(PV\). Therefore, if \(P\) is bigger, generalized costs increases, and it increases the optimal calculated \(Q_{w}\). Therefore, private sector requires to enhance \(Q_{w}\) more when \(P\) is bigger.

By using equations 24, 31 and 32, it can be written that:

\[
Profit = \frac{(cPV)^2}{4d} - 2 \sqrt{\frac{a \times PV}{b}}
\]
By using aforementioned assumptions, profit can be calculated as:

\[
Profit = \left( \frac{500000 \times 0.8 \times 0.00000005}{4 \times 0.0002} \right)^2 - 2 \sqrt{\frac{0.05 \times 0.8}{5}} = 0.5 - 0.18 = 0.32 \text{ BSEK}
\]

In order to find optimal qualities and profit, when qualities are negatively correlated, it is assumed that \( e = 0.5 \). By using previous assumptions and equation 27, it can be written that:

\[
Q_W = 10.54
\]

According to equations 6, 23 and 27-a, it can be written:

\[
direct Q_{O&M} = 7
\]

Equation 23-a illustrates that in order to have maximum profit in this example, private sector increases the direct \( Q_{O&M} \) to 7. Profit can be calculated using equation 25.

\[
Profit = -0.24 \text{ BSEK}
\]

From equation 25-a, it can be seen that for negatively correlated qualities with correlation coefficient of 0.5, profit is negative. It means that even with the optimal profit maximizing behavior, profit is negative, which means this project is not profitable for private sector, unless they get part of initial investment from public sector to make it profitable for them.

In order to find optimal qualities and profit, when qualities are positively correlated, it is assumed that \( e = 0.5 \) using equation 28. By using previous assumptions and equation 31, it can be written that:

\[
Q_W = 2500
\]

According to equations 6, 28 and 31-a, it can be written:

\[
direct Q_{O&M} = -1248.2
\]

According to equation 28-a, Optimal \( Q_{O&M} \) is negative, which means private sector does not have to enhance \( Q_{O&M} \) at all. Therefore, it can be considered \( Q_{O&M} = 0 \).

Therefore, by using equations 31-a,28-a and 29, profit can be calculated:

\[
Profit = 0.5 \text{ BSEK}
\]

As it is shown, it can be seen that private sector’s profit is maximized when qualities are positively correlated. Since they can maximize user’s quality and it affects the operation and maintenance quality and reduce the operation and maintenance investment and costs. Profit is bigger when qualities are uncorrelated compared to negatively correlated qualities.
6 Conclusions and discussion

With increasing the need for infrastructure and lack of public financing, governments decided to create a solution to use private sector capital for these types of facilities. By using public-private partnership, governments transfer the finance, construction, operation and maintenance of the infrastructure projects to the private sector. By doing so, public sector will receive the infrastructure needed without using its capital and private sector will have a full control over the asset and will get profit out of the project. Therefore, private capital is used for public good. Public will pay the taxes or customer’s charge as revenue to the private sector during the PPP contract. Customer’s charge can be the bill that customers have to pay for passing the road in road projects or paying the hospital fixed fee or etc. Therefore, the initial investment for public is very low since private sector is responsible for financing the project and operation of the project will be costly for public sector since they have to pay customer’s charge or taxes.

This study, has explained the externalities with more detail by using two different quality dimensions and studied the different possible situation for private sector regarding the quality choices. It has suggested some ways for public sector to design the contract in a way to have maximum possible quality enhancement by the private sector in different quality situations.

This thesis has illustrated that, by separating the qualities into user’s quality and operation and maintenance quality, three situations can happen. They can be positively correlated, negatively correlated or uncorrelated. After calculating different situations’ incentive for quality enhancement, it is concluded that there is an optimal point for qualities in the situations that they are uncorrelated, positively or negatively correlated. After calculating qualities, profit can be calculated for private sector. It is shown that private sector is willing to increase both qualities to an optimal calculated point. By comparing the qualities with traditional procurement, it can be easily seen that in PPP, qualities will increase without monitoring, whereas in traditional procurement, qualities will not increase without monitoring. In the situation that qualities are uncorrelated, it is shown that private sector is willing to optimize the qualities according to separated optimal qualities calculated to have maximum profit. It is shown that private sector will have maximum profit when qualities are positively correlated, and they have higher profit when qualities are uncorrelated compared to negatively correlated qualities.

In conclusion, for every situation possible with considering the model limitations, private sector has incentive to increase both qualities and as a result, increasing consumer surplus and reducing the total costs to a balanced optimal point, which is assumed to be the public sector’s objective.

According to the calculations in this thesis, that public sector should take away the price setting power from private sector, since private sector will choose monopolistic approach which will reduce the consumer surplus and user’s quality. In that case they have the same profit but with less quality. Therefore, the maximum consumer surplus can be reached by public sector setting the customer’s charge and paying fixed fee per each customer to the private sector.

Considering paying fixed fee to the private sector per each customer, private sector has incentive to enhance user’s quality to absorb more customer and as a result, more revenue. On the other hand increasing user’s quality demands more initial investment. Thus, there is an optimal point where private sector can have maximum profit by having maximum difference between revenue and costs related to user’s quality. Moreover, by increasing the operation and maintenance quality, operation and maintenance costs will decrease and therefore less costs for private sector. However, increasing operation and maintenance quality demands more initial investment. Hence, there is an optimal point for operation and maintenance quality which gives the maximum profit for the private sector. It is shown that private sector is willing to increase the qualities and reduce the costs, which is in line with public sector objectives. Thus, it can be a win-win situation for both parties.
Public and private sector can use the model that has been created and described in this study to calculate the externalities by dividing it to two different quality dimensions. Private sector’s plausible behavior is different quality situations and their profit can also be calculated.

7 Model Limitations
These models created based on some key assumptions. First is that the objective of public sector is to have maximized consumer surplus minus the costs. This can cause some limitations, for instance the case of prisons where with increasing consumer surplus, demand will increase, which is not governments’ objective. Therefore it can be assumed government objective in this scenario is to have correlated qualities with unobservable \( Q_W \). In this special case, government wants to have high true \( Q_W \), which is unobservable and unreal \( Q_W \) which is observable and low to decrease the demand.

Another limitation is the revenue for private sector, which is considered to be fixed price per each customer. This can be the scenarios which low quality and minimized costs is the final objective for public sector. In this case, revenue function must be changed since profit maximizing private sector has incentives to increase \( Q_W \) to increase the demand and thus their profit, which is contradicted with public sector’s objective. In these scenarios, revenues for private sector must be changed and not consider fixed fee per customer, which incentivize the private sector to increase the customers for fixed fee.

Another limitation is renovation costs. It may be possible that the service requires renovation within the PPP contract length. This model did not include the renovation which might occur and change the profit and quality enhancements. For simplicity, it can be assumed that renovation costs are included in the initial investment costs, but the effect of operation and maintenance quality should be considered for the renovation costs and time.

Last and most important limitation is the series of assumptions that has been used. Although it is tried to be realistic as much as possible, but there are not based on empirical data and they are not precise. Therefore for using the model in real-life situation, all of the assumptions must be calculated based on empirical data and follow the steps of the model based on new empirical equations.

8 Suggestions and recommendations for further studies
There are limitations to this study, which are mentioned on the limitation section. One of the important situations that this paper could not cover is that prison facility which is believed that public sector’s objective is different from the assumed objective. It is worth considering that specific situation by defining the user’s quality and operation and maintenance quality.

Another important suggestion for further studies can be testing the model and assumptions in real-life with empirical data. It is believed that some of the assumptions may change in with empirical data. Therefore model must be created with the new equations replacing the unprecise assumptions.

Lastly, the most important suggestion for further study can be measuring quality. In this paper, it is assumed that quality can be measurable and one unit difference of quality can be observed, whereas it is hard to create measuring system for quality.
References


Infrastructure Australia, National Public Private Partnership Guideline: Overview, 2008


Appendices

Calculation of equation 27: Equation 27 can be simplified as:

\[
0 = \frac{c_{PV}}{2\sqrt{Q_w}} - d - \frac{PV}{be \times Q_w^2} \Rightarrow \\
0 = \frac{c_{PV} \times be \times Q_w^{3/2}}{2be \times Q_w^2} - \frac{2dbe \times Q_w^2}{2be \times Q_w^2} - \frac{2PV}{2be \times Q_w^2} \Rightarrow \\
0 = \frac{c_{PV} \times be \times Q_w^{3/2} - 2dbe \times Q_w^2 - 2PV}{2be \times Q_w^2} \Rightarrow \\
0 = -2dbe \times Q_w^2 + c_{PV} \times be \times Q_w^{3/2} - 2PV \Rightarrow
\]

To solve equation above, a general solution is provided in figure 14.

\[
\begin{align*}
& a < 0, \quad b > 0, \quad 0 < c < -\frac{0.10549 b^4}{a^2}, \quad x = \frac{0.25 b^2}{a^2} - 0.144338 \\
& \left(0.86602 \left(\frac{b^6}{a^6} + \frac{8b^2c}{a^2}\right)\right) + \\
& \sqrt{\frac{3b^4}{a^2} + \frac{80.6349c^2}{a^2}} + \\
& 3.1748 \sqrt{\frac{5.19615 \sqrt{256a^3b^4c^5 + 27b^4c^4 + 128a^3c^3 + 27b^4c^2}}{a^2}} + 16c \left(\frac{a^2}{a}\right) - 0.5
\end{align*}
\]

General solution for:

\[
0 = ax^2 + bx^{3/2} - c
\]

Calculation of equation 31: Equation 31 can be simplified as

\[
0 = \frac{c_{PV}}{2\sqrt{Q_w}} - d + \frac{PV}{be \times Q_w^2} \Rightarrow
\]
\[ 0 = \frac{cPVP \times be \times Q^3}{2be \times Q^2} - \frac{2dbe \times Q^2}{2be \times Q^2} + \frac{2PV}{2be \times Q^2} \Rightarrow \]

\[ 0 = -2\frac{Q^2}{2be \times Q^2} + \frac{cPVP \times be \times Q^3}{2be \times Q^2} + 2PV \Rightarrow \]

To solve equation above, a general solution is provided in figure 14.