<http://www.diva-portal.org>

Postprint

This is the accepted version of a paper published in *Journal of Sound and Vibration*. This paper has been peer-reviewed but does not include the final publisher proof-corrections or journal pagination.

Citation for the original published paper (version of record):

Pignier, N., O'Reilly, C J., Boij, S. (2017)

Identifying equivalent sound sources from aeroacoustic simulations using a numerical phased array.

Journal of Sound and Vibration, 394C: 203-219

<https://doi.org/10.1016/j.jsv.2017.01.051>

Access to the published version may require subscription.

N.B. When citing this work, cite the original published paper.

Permanent link to this version:

<http://urn.kb.se/resolve?urn=urn:nbn:se:kth:diva-192164>

Identifying equivalent sound sources from aeroacoustic simulations using a numerical phased array[☆]

Nicolas J. Pignier^{a,b,c,*}, Ciarán J. O'Reilly^{a,b}, Susann Boij^{a,b,c}

^a*KTH Royal Institute of Technology, Department of Aeronautical and Vehicle Engineering, Teknikringen 8, 100 44 Stockholm, Sweden*

^b*The Centre for ECO² Vehicle Design at KTH, Teknikringen 8, 100 44 Stockholm, Sweden*

^c*The Marcus Wallenberg Laboratory for Sound and Vibration Research (MWL), KTH Royal Institute of Technology, Teknikringen 8, 100 44 Stockholm, Sweden*

Abstract

An application of phased array methods to numerical data is presented, aimed at identifying equivalent flow sound sources from aeroacoustic simulations. Based on phased array data extracted from compressible flow simulations, sound source strengths are computed on a set of points in the source region using phased array techniques assuming monopole propagation. Two phased array techniques are used to compute the source strengths: an approach using a Moore-Penrose pseudo-inverse and a beamforming approach using dual linear programming (dual-LP) deconvolution. The first approach gives a model of correlated sources for the acoustic field generated from the flow expressed in a matrix of cross- and auto-power spectral values, whereas the second approach results in a model of uncorrelated sources expressed in a vector of auto-power spectral values. The accuracy of the equivalent source model is estimated by computing the acoustic spectrum at a far-field observer. The approach is tested first on an analytical case with known point sources. It is then applied to the example of the flow around a submerged air inlet. The far-field spectra obtained from the source models for two different flow conditions are in good agreement with the spectra obtained with a Ffowcs Williams-Hawkins integral, showing the accuracy of the source model from the observer's standpoint. Various configurations for the phased array and for the sources are used. The dual-LP beamforming approach shows better robustness to changes in the number of probes and sources than the pseudo-inverse approach. The good results obtained with this simulation case demonstrate the potential of the phased array approach as a modelling tool for aeroacoustic simulations.

Keywords: sound sources, modelling, phased array, beamforming, simulations, dual-LP beamforming

[☆]<http://dx.doi.org/10.1016/j.jsv.2017.01.051>

*Corresponding author.

Email address: pignier@kth.se (Nicolas J. Pignier)

1. Introduction

Aerodynamic noise is a significant contributor to the pass-by noise from ground vehicles. This is particularly true for high-speed trains but, as progress is being made on reducing engine and rolling noise, and as the market share of electric cars increases, it is also becoming an important issue for road vehicles travelling at lower speeds. In recent years, the awareness of the impact of traffic noise on health and economy has increased significantly and noise is now considered to be one of the major sources of pollution, alongside air pollution [1]. As a consequence, low noise emission is an important requirement for future sustainable transport systems. Developing more silent vehicles currently requires design iterations through prototyping and wind-tunnel tests. As these are costly and time-consuming, the incentive for manufacturers to rely on numerical methods has never been as high.

Currently it is still very challenging and computationally heavy to solve the acoustic field generated by the flow through direct simulation of the flow, because the acoustic field is usually several orders of magnitude smaller than the flow field [2]. When the acoustic field far from the unsteady region is of interest, the most commonly adopted strategy is to use hybrid aeroacoustic methods: the flow in the unsteady region is solved first, then the acoustic sources are computed from the flow solution and propagated away from the unsteady region using e.g. integral methods, boundary element methods or finite difference methods.

This work focuses on the source identification part of hybrid aeroacoustic methods. More precisely, the purpose of the approach presented in this paper is to obtain a simple point source model of aeroacoustic sound sources from compressible flow simulation data, for subsequent use in propagation solvers. In this paper, the terms source model and equivalent sound sources are used with the same meaning. Obtaining such a source model can be useful for many applications, where sound propagation beyond the domain of the fluid simulation is of interest, for example when the radiated sound field is scattered by objects in the far-field.

The extraction of sound sources from the flow quantities is usually performed using acoustic analogies. Acoustic analogies are reformulations of the governing equations of the flow in the form of a wave equation, with on one side the terms governing the propagation of acoustic waves in an idealized medium, and on the other side the remaining terms that are identified as sound sources. The first of the kind was developed by Lighthill [3, 4] for free flows and was later extended by Curle [5] to add the effect of solid surfaces, and by Ffowcs Williams and Hawkings [6] to account for moving solid and permeable surfaces. When only the far-field acoustics is of interest, integral formulations of the acoustic analogies are often used, assuming free-field or uniform mean flow propagation. Otherwise the source terms of the acoustic analogies can be evaluated from the flow quantities, providing a model for the sources. Although well detailed, this model is not straightforward to use as input to propagation solvers [7, 8] as it requires interpolation between the flow and the acoustic mesh, estimations of derivatives etc. In many applications, it can be useful to get a simpler model, which is the motivation behind this work.

The starting point of the method introduced in this paper is a compressible simulation of the flow field around the geometry of interest. If the flow and the acoustic fields are accurately resolved locally, it is possible to analyze the radiated acoustic field itself rather than the flow quantities in the source region, and to model this radiated field with simple sources. This approach guarantees

that the correct radiated acoustic field is retrieved from the point source model, as long as this field is accurately predicted locally in the CFD simulations. The main idea is to use a phased array of probes inside the simulation domain to identify source strengths on a selected set of point sources in the unsteady region of the flow. The array data is extracted from the flow solution, and the identification of the source strengths is performed using phased array or beamforming techniques. Beamforming is originally an experimental technique, but it is not theoretically limited to experimental measurements. Recently, a number of numerical works have focused on applying beamforming or other phased array techniques to flow simulations, to locate aeroacoustic sources. Brusniak et al. used phased array data extracted from a large eddy simulation to identify jet noise sources with a conventional beamforming approach [9]. Adam et al. applied standard beamforming techniques to lattice Boltzmann method simulations of a car side-view mirror [10]. Sandberg and Tester used phased-array techniques to break down the sources from a direct numerical simulation of a jet flow [11]. In these three studies, the phased array data was directly extracted from the flow simulations, as is done in this work. In other studies the phased array is located farther from the sources and the acoustic data has to be extrapolated from the flow solution. This extrapolation is most often performed using integral methods such as the Ffowcs Williams-Hawkins (FW-H) integral [12, 13, 14, 15, 16, 17], but other approaches can be found [18]. Extrapolation is performed when the acoustic field is not expected to be sufficiently well-resolved at the location of the array. A problem of this intermediary step is that it introduces a potential additional source of bias errors, which for the FW-H integral can for example be due to the choice of control surface of integration [19] or to the assumption of free-field propagation. In this work, the phased array is placed directly close to the sources in the sound computation domain of the flow simulations and does not rely on interpolation methods to compute the array pressure data. This work differs from previously published research by its purpose. Apart from the analysis of the sound sources — which is limited by the resolution of the beamforming method — the main purpose and novelty of this work is to use phased array techniques to construct a point source model for the radiated acoustic field, for subsequent use in a propagation solver. The locations of the sources may or may not match the locations of acoustic sources that would be found using eg. acoustic analogies: what matters most is that the radiated acoustic field is correctly represented.

In this study, the acoustic power of the sources are identified using two different phased array techniques. The first approach is a direct method based on generalized inverse techniques, using a Moore-Penrose pseudo-inverse to compute the source strengths from the cross-spectral matrix of the phased array. The second approach is a beamforming technique using linear programming (LP) deconvolution, adapted from Dougherty’s work [20] and denoted dual-LP beamforming. The first approach is theoretically able to resolve coherent/incoherent sources, whereas the second approach can only resolve incoherent sources. The pseudo-inverse approach provides an L_2 solution of the inverse problem. The L_1 generalized inverse beamforming proposed by Suzuki [21] and developed by Zavala et al. [22, 23] could also be used, to provide more sparsity in the resulting source distribution, if needed. In this work, monopole sources are used, but the method can be extended to other source types, such as dipoles. The resulting source model is expressed in terms of source locations, source types and source auto-power and cross-power spectral values. From this model, simple formulae can be derived, which allow to compute the acoustic power spectral density (PSD) at a far-field observer, assuming free-field radiation. In this study, the accuracy of the source model

Figure 1: Overview of the method in two steps: 1) sound source modelling using phased array techniques and 2) validation of the source model using analytical propagation to a far-field observer.

to radiate the correct acoustic field is assessed by comparing the far-field spectrum to reference spectra, from analytical or from other numerical approaches.

The paper is structured as follows. In the first section, the phased array source modelling approach is presented, with theoretical details on the pseudo-inverse and the dual-LP beamforming approaches. Parallels between both methods are drawn. The formulae to compute the far-field spectrum from the source model are also introduced. The following section deals with an application of the method to model the acoustic sources generated by the flow around a submerged air inlet, previously studied by Pignier et al. [19]. Before applying them to the flow case, the source and array configurations are tested on an analytical case. The source models are computed, and then assessed by comparing the far-field spectra they radiate with analytically predicted spectra for the test case, and FW-H spectra for the flow case. In the next section, the results and limitations of the modelling approach are discussed. Finally, conclusions are made.

2. Source modelling approach

An overview of the approach used in this study is shown in Fig. 1. An array of probes, or numerical “microphones”, is placed in the direct sound computation region, outside of the unsteady region of the flow. In the region of the array, it is assumed that the acoustic field is the dominating fluctuating field. Pressure data is recorded at the probes. In a first step, the source model is computed on a set of source points in the unsteady flow region, assuming known transfer functions from all source positions to all probe positions. To validate the source model, the PSD spectrum at a far-field observer is computed in a second step. Details about each of these steps are given in this section.

2.1. Formulation of the problem

The analysis is performed in the frequency domain. N probes are placed at the positions $\mathbf{x}_{1..n..N}$ and M test sources at positions $\mathbf{y}_{1..m..M}$ as described in Fig. 1. The transfer function between a

source at $\mathbf{y}_m$ and the probe at $\mathbf{x}_n$ is noted $g_{m,n}$. In this study a simple free-field monopole Green's function is used ($g_{m,n} = \exp(-ikr_{m,n})/r_{m,n}$) but other models, such as a dipole or a monopole in moving medium, could be used. The pressure at probe n is modelled as

$$p_n = \sum_{i=1}^M s_i g_{n,i} \quad (1)$$

or in vector form for the phased array

$$\mathbf{P} = \mathbf{G}\mathbf{S} \quad (2)$$

where $\mathbf{P} = (p_n)$ is a $N \times 1$ vector, $\mathbf{G} = (g_{n,m})$ is a $N \times M$ matrix and $\mathbf{S} = (s_m)$ is a $M \times 1$ vector. $|s_m|^2$ represents the auto-spectrum of the source m per Hz at the considered frequency. Assuming that $\mathbf{G}$ is known, the problem to solve consists in finding $\mathbf{S}$.

2.2. Moore-Penrose pseudo-inverse solution

It is possible to find an approximate solution to Eq. (2) directly, using generalized inverse techniques [24]. A minimum L_2 -norm solution to Eq. (2) is given by $\mathbf{S} = \mathbf{G}^+ \mathbf{P}$ where $\mathbf{G}^+$ is the Moore-Penrose pseudo-inverse. Depending on whether the system is underdetermined or overdetermined, and in particular cases on the rank of $\mathbf{G}$, this solution can be either exact or approximate. In this work the solutions for both underdetermined ($N < M$) and overdetermined ($N > M$) systems are used. This solution provides the complex source strengths, including amplitude and phase. However, if the signals recorded at the probes contain not only acoustic waves, but also hydrodynamic waves or noise, the results are contaminated. To use this method the probes have to be placed in a region of the flow where the fluctuating field consists solely of the acoustic field. This condition is hard to rigorously satisfy in practice. A less biased and therefore less restrictive expression can be obtained using complex conjugates as

$$\mathbf{S}\mathbf{S}^\dagger = \mathbf{G}^+ \mathbf{P}\mathbf{P}^\dagger (\mathbf{G}^+)^{\dagger}. \quad (3)$$

The elements of the array cross spectral matrix (CSM) $\mathbf{P}\mathbf{P}^\dagger$ are computed through Welch averaging over the time sample, as

$$(\mathbf{P}\mathbf{P}^\dagger)_{nn'} = p_n \bar{p}_{n'} = \frac{2}{wKB_e} \sum_{k=1}^K X_n \bar{X}_{n'} \quad (4)$$

where X_n denotes the Fourier transform of the acoustic pressure at probe n , K the number of averages, B_e the frequency resolution bandwidth of the Fourier transform and w a correction factor for the time window.

Except for the diagonal elements which contain self-noise, the elements of $\mathbf{P}\mathbf{P}^\dagger$ are free from bias errors due to uncorrelated signals between the probes [25] such as uncorrelated hydrodynamic fluctuations — at least the fluctuations for which the correlation length of the flow structures is smaller than the distance between the probes — or uncorrelated numerical noise, provided that a sufficient number of averages is performed. $\mathbf{S}\mathbf{S}^\dagger$ is a square matrix made of the elements $(s_m \bar{s}_{m'})$. The solution of Eq. (3) is referred to in this paper as the *pseudo-inverse solution*. This approach presents some similarities with the generalized inverse beamforming introduced by Suzuki [21].

The main difference between the methods is that Suzuki uses L_1 generalized inverse techniques to improve the resolution of the source map, instead of the L_2 solution given by the Moore-Penrose pseudo-inverse. Using the L_1 solution may increase the sparsity of the source distribution. As resolution is not the focus of this study, only the L_2 solution is used.

In this work, the Moore-Penrose inverse $\mathbf{G}^+$ is evaluated using singular value decomposition (SVD) [26]. Regularization is used, where any singular value less than $\max(M, N)\|\mathbf{G}\|_2\epsilon$ is treated as zero, with ϵ being the machine epsilon. The fast computation method based on full rank Cholesky factorization introduced by Courrieu [27] was also tested, but led to inaccurate predictions of the source strengths, probably because of poor matrix conditioning. Indeed, for some configurations of sources and phased array, $\mathbf{G}$ can be ill-conditioned and the estimation of $\mathbf{G}^+$ can be subject to large errors. This effect can be partly prevented by using regularization techniques, absent in the fast computation method. A detail study of conditioning issues for this type of problem, and associated solutions, can be found in the papers by Nelson and Yoon [26, 28].

2.3. Linear programming beamforming

The main idea behind beamforming methods consists in getting information on the source strengths by steering the microphone data at the source locations using an appropriate choice of transformation. Doing so formally avoids the use of a Moore-Penrose inverse and the conditioning issues that may be associated. The resulting output of this operation is commonly called the standard beamforming output. As is, the standard beamforming output is however not a solution of the inverse problem. A so-called deconvolution equation can be derived relating the standard beamforming output to the actual modelled source strengths. In this study, this equation is solved using linear programming [20].

The main steps and equations of the dual-LP beamforming approach are presented thereafter. Most of the theory is based on Dougherty's work [20], but the notations, the choice of normalization and the final step are changed. The key steps in the theory are therefore shown here. To start, a quantity b_m is defined as

$$b_m = \mathbf{W}_m^\dagger \mathbf{P} \quad (5)$$

where $\mathbf{W}_m = (w_{m,n})$ is a $N \times 1$ vector, commonly called a *steering vector*. Taking the amplitude-squared of b_m and using Eq. (2), under the assumption that the sources are incoherent, gives after a few steps

$$\tilde{\mathbf{B}} = \mathbf{A} \tilde{\mathbf{S}} \quad (6)$$

where $\tilde{\mathbf{B}} = (|b_m|^2)$ and $\tilde{\mathbf{S}} = (|s_m|^2)$ are $M \times 1$ vectors, and $\mathbf{A} = (|\mathbf{W}_i^\dagger \mathbf{G}_j|^2)$ is a $M \times M$ matrix. The coefficients $|b_m|^2$ are calculated from Eq. (5), giving

$$|b_m|^2 = |\mathbf{W}_m^\dagger \mathbf{P}|^2 = \mathbf{W}_m^\dagger \mathbf{P} \mathbf{P}^\dagger \mathbf{W}_m. \quad (7)$$

$|b_m|^2$ is the standard beamforming output for the source m . In this study, the steering vectors $\mathbf{W}_i$ are chosen as being parallel to $\mathbf{G}_i$, $\mathbf{W}_i = \alpha_i \mathbf{G}_i$, leading to

$$A_{i,j} = |\alpha_i|^2 |\mathbf{G}_i^\dagger \mathbf{G}_j|^2. \quad (8)$$

α_i is chosen so that for a single source located at $\mathbf{y}_{m_0}$, $|b_{m_0}|^2 = |s_{m_0}|^2$, giving $|\alpha_i|^2 = 1/|\mathbf{G}_i^\dagger \mathbf{G}_i|^2$.

With this choice of α_i , $\mathbf{A}$ is a square matrix of positive elements, with unitary diagonal. $\mathbf{A}$ is the array point spread function (PSF), sometimes called array beamform function [29]. The beamform map $\tilde{\mathbf{B}}$ represents the distribution of source strenghts squared convolved with the PSF. $\tilde{\mathbf{S}}$ represents the actual distribution of source strenghts squared in the model. Eq. (6) is solved using a deconvolution algorithm. Several deconvolution algorithms are proposed in the literature, among which the Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS) [29, 30, 31]. The DAMAS algorithm attempts at approaching the solution of Eq. (6) by minimizing the 2-norm of $\mathbf{A}\tilde{\mathbf{S}} - \tilde{\mathbf{B}}$ using a Gauss-Seidel iteration. It has shown evidence of robustness, and has been widely applied to various acoustic experiments. However it was shown to have a high computational cost and a slow convergence [20]. Dougherty recently proposed an approach based on linear programming [20]. This method consists in formulating a linear problem from Eq. (6) that can be solved using standard linear programming methods. In this approach, Dougherty proposes to maximize the product $\mathbf{A}\tilde{\mathbf{S}}$ under the constraint $\mathbf{A}\tilde{\mathbf{S}} < \tilde{\mathbf{B}}$. Maximizing $\mathbf{A}\tilde{\mathbf{S}}$ is equivalent to maximizing the sum of the components of $\mathbf{A}\tilde{\mathbf{S}}$. Defining a column vector $\mathbf{C}$ by $C_{m'} = \sum_{m=1}^M A_{mm'}$ for $m' = 1, \dots, M$ gives the following linear problem

$$\begin{aligned} \max_{\tilde{\mathbf{S}}} \quad & \mathbf{C}^T \tilde{\mathbf{S}} \\ \text{such that} \quad & \mathbf{A}\tilde{\mathbf{S}} \leq \tilde{\mathbf{B}}, \tilde{\mathbf{S}} \geq 0. \end{aligned} \quad (9)$$

Eq. (9) is solved in Matlab using a dual-simplex algorithm [32]. As it consists in maximizing the product $\mathbf{A}\tilde{\mathbf{S}}$ while keeping it below $\tilde{\mathbf{B}}$, the solution obtained with this approach is referred to as the solution *from below*. In this study, preliminary tests have shown that this approach can sometimes struggle to find an accurate solution to the targeted equality $\mathbf{A}\tilde{\mathbf{S}} = \tilde{\mathbf{B}}$, and that a better least-square estimate can be obtained for some frequencies by approaching the solution *from above*, solving the following problem

$$\begin{aligned} \min_{\tilde{\mathbf{S}}} \quad & \mathbf{C}^T \tilde{\mathbf{S}} \\ \text{such that} \quad & \mathbf{A}\tilde{\mathbf{S}} \geq \tilde{\mathbf{B}}, \tilde{\mathbf{S}} \geq 0. \end{aligned} \quad (10)$$

Dougherty claims that the approach *from below* is physically more realistic. As the focus of this work is to get an accurate source model and not necessarily a realistic one, both approaches *from below* and *from above* are systematically used and the solution $\tilde{\mathbf{S}}$ is selected as the one that minimizes the 2-norm error $\|\mathbf{A}\tilde{\mathbf{S}} - \tilde{\mathbf{B}}\|_2$. Because of the use of the dual-simplex algorithm and because of the dual approach *from below* and *from above*, this method is referred to as dual-LP beamforming.

2.4. Comparison between the methods

The Moore-Penrose inverse solution and the dual-LP beamforming solution are two approximate solutions to the initial problem given by Eq. (2). To compare both methods, an extended standard beamforming output $\tilde{\mathbf{B}}_{\text{ext}}$ may be defined as

$$\tilde{\mathbf{B}}_{\text{ext}} = \Delta^\dagger \mathbf{G}^\dagger \mathbf{P} \mathbf{P}^\dagger \mathbf{G} \Delta. \quad (11)$$

If Λ is a diagonal matrix whose diagonal elements are given by α_m , then $\tilde{\mathbf{B}}_{\text{ext}}$ is a diagonal matrix whose diagonal elements correspond to the standard beamforming outputs of Section 2.3. Using the expression $\mathbf{G}^+ = (\mathbf{G}^\dagger \mathbf{G})^{-1} \mathbf{G}^\dagger$ for an over-determined system [26] (provided that $\mathbf{G}^\dagger \mathbf{G}$ is invertible) in Eq. (3) leads to

$$\mathbf{S}\mathbf{S}^\dagger = [(\mathbf{G}^\dagger \mathbf{G})^{-1}]^\dagger \mathbf{G}^\dagger \mathbf{P} \mathbf{P}^\dagger \mathbf{G} (\mathbf{G}^\dagger \mathbf{G})^{-1}. \quad (12)$$

If Λ was chosen as $(\mathbf{G}^\dagger \mathbf{G})^{-1}$, the extended standard beamforming output $\tilde{\mathbf{B}}_{\text{ext}}$ would be equal to the Moore-Penrose inverse solution $\mathbf{S}\mathbf{S}^\dagger$. In the standard beamforming approach, with the chosen steering vector weights, the diagonal elements of Λ are given by $\alpha_i = 1/\mathbf{G}_i^\dagger \mathbf{G}_i$, highlighting the resemblance between the standard beamforming output and the pseudo-inverse solution. The deconvolution approach can be seen as a correction of the standard beamforming output, performed under the assumption that the sources are incoherent.

The main advantage of the dual-LP beamforming approach is that it formally avoids matrix inversion, and transforms the problem into a linear problem that can be solved using linear programming techniques. It is therefore less prone to errors related to poor matrix conditioning, which may occur when the system given by Eq. (2) is highly under-determined. This is studied in more details in Section 3.3. The main advantage of the pseudo-inverse solution is that it identifies both coherent and incoherent sources, whereas the dual-LP beamforming solution only models incoherent sources. One consequence is that the predicted directivity of the modelled sources from the dual-LP beamforming only depends on the choice of source type and not on the interaction between sources. In particular, choosing a model of monopole radiation would not correctly describe a strongly directive flow generated sound.

2.5. Far-field acoustic spectrum from modelled sources

The power spectral density at a given observer away from the sources is computed for each frequency as [25]

$$\text{PSD}_{\text{obs}} = \frac{2}{wKB_e} \sum_{k=1}^K |X_{\text{obs},k}|^2. \quad (13)$$

From this expression, using in turn Eqs. (4) and (1) leads to

$$\text{PSD}_{\text{obs}} = \sum_{i,j=1}^M s_i g_{\text{obs},i} \overline{s_j g_{\text{obs},j}}. \quad (14)$$

For the Moore-Penrose inverse approach, Eq. (14) can be written as

$$\text{PSD}_{\text{obs}} = \mathbf{G}_{\text{obs}}^\dagger [\mathbf{S}\mathbf{S}^\dagger]^T \mathbf{G}_{\text{obs}} \quad (15)$$

where $\mathbf{G}_{\text{obs}} = (g_{\text{obs},i})$ is a $M \times 1$ vector. For the dual-LP beamforming approach, Eq. (14) can be expressed as

$$\text{PSD}_{\text{obs}} = \mathbf{G}_{\text{obs}}^\dagger \text{diag}(\tilde{\mathbf{S}}) \mathbf{G}_{\text{obs}} \quad (16)$$

since $s_i \overline{s_j} = 0$ for $i \neq j$. In Eq. (16), $\text{diag}(\tilde{\mathbf{S}})$ is the diagonal matrix whose diagonal is the vector $\tilde{\mathbf{S}}$.

3. Application: sound source modelling on a submerged air inlet

The phased array source modelling method is applied to compressible detached eddy simulation (DES) solutions of the flow around a submerged air inlet, to model the aeroacoustic sources and to estimate the far-field acoustic spectrum from these sources. The far-field spectrum obtained through the use of a FW-H integral is used as a reference. The analysis is performed at a low Mach number. The setup of the simulations and some relevant flow results are briefly presented here. More details about the flow and the FW-H analysis can be found in the paper by Pignier et al. [19]. The results of the phased array analysis are shown at the end of this section.

3.1. Flow solution

Figure 2: Model of NACA inlet used in the numerical simulations, and terms used to characterize the inlet. The duct is truncated in the figure.

The submerged air inlet studied in this paper is a generic NACA inlet, as shown in Fig. 2. Its dimensions are based on a submerged inlet studied experimentally by Mossman et al. in a wind tunnel [33], and are given in Pignier et al. [19]. Results for the simulations are presented in terms of non dimensional variables, defined in Table 1. The subscript 0 corresponds to quantities related to the free-stream, while the subscript 1 denotes quantities averaged over the duct entrance plane. f_1 is used to scale the acoustic results in this study. Simulations are performed with a flow speed of 60 m s^{-1} in the free-stream corresponding to a Mach number of 0.1727 and for a duct depth of $D = 5.079 \times 10^{-2} \text{ m}$, giving $f_0 = 1181 \text{ Hz}$. The source modelling method is applied to two flow cases corresponding to velocity ratios ν of 0.4 and 0.8.

Name	Inlet velocity ratio	Charact. freq.	Charact. freq.
Symbol	ν	f_0	f_1
Definition	$\frac{V_1}{V_0}$	V_0/D	V_1/D

Table 1: Non dimensional parameters used in this study.

The flow simulations are run on the commercial software STAR-CCM+ [34] in its release 10.02.010. Steady-state results of the flow are first obtained through RANS simulations, and from

there time-dependent DESs are run. Turbulence is modelled using a SST $k - \omega$ model [35]. Improved Delayed DES (IDDES) [36] is used for the time-dependent computations. The solution is advanced in time using a second-order implicit temporal discretization scheme. Space derivatives are obtained with a hybrid second-order upwind/bounded central-differencing convection scheme, which behaves as second order within its bounds [37]. A time-step of 4×10^{-5} s is used, meaning that the convection of a flow particle over the length of the inlet is solved in about 240 time-steps. This time-step gives 21 samples per period at f_0 and corresponds to a Nyquist frequency of 12500 Hz, which is one order of magnitude above the frequencies of interest. For the time-dependent studies, simulations are run until at least two convection cycles over the inlet are completed, before values are monitored and data extracted for the acoustic post-processing.

Figure 3: Domain and boundary conditions.

The NACA inlet described in Fig. 2 is located in a flat plate under a free stream as shown in Fig. 3. A so-called free stream boundary condition is used on the free boundaries of the domain, with a prescribed Mach number of 0.1727, a pressure of 101325 Pa and a temperature of 300 K. The free-stream boundary condition is based on the method of characteristics and treats the flow as one-dimensional near the boundary [38, 39, 40], with a spatial coordinate taken along the normal to the boundary. In this setup, this boundary condition was shown to exhibit low reflections of the acoustic waves [19]. The mesh is shown in Fig. 4. The grid used in this study has a total of 13.3 M cells. A finer grid of 47.8 M cells was shown to give consistent results [19] for the flow and for the acoustic spectrum at a far-field observer.

Results for the DESs of the flow are presented at two velocity ratios: 0.4 and 0.8. Visualization of the instantaneous vorticity is shown in Fig. 5 for both velocity ratios. Streamwise vortices are generated from the edges of the ramp side-walls and are convected with the flow. The vortex cores on both sides are split by the inlet: part of the vortical field is sucked inside the duct and part is forced over the lip, downstream of the inlet. For high velocity ratios, most of the vortical region is sucked inside the duct, whereas for low velocity ratio, most of it is forced over the lip. The flow around the lip is rather turbulent for a velocity ratio of 0.4 and rather laminar for a velocity ratio of 0.8.

3.2. Acoustic analysis setup

The phased array approach is used to get a source model corresponding to the flow generated sound. The positions of the phased array and of the source test grid are shown in Fig. 6. In a previous numerical study, it was shown that the sound sources are mainly located on the surface

Figure 4: Visualization of the grid for the DESs.

of the inlet, on both sides of the lip and a short distance downstream along the vortex wakes [19]. Accordingly, the source grid is chosen so that it covers a sufficiently large area centred around the lip. Pressure data is extracted on a phased array of width $11.4D$, over the source grid, at a distance of $h = 5.51D$ from the inlet.

To assess the source model, a reference is needed. This reference is taken as the solution of the FW-H integral [6] at an observer in the far-field. The far-field spectra from the source models obtained with the phased array approach are computed according to the formulae given earlier in Section 2.5, and compared with the FW-H reference. The position of the permeable control surface on which the FW-H integral is computed is represented in Fig. 6. Farassat’s formulation 1A [41] of the FW-H integral is used, as implemented in Star-CCM+, using a source-time-dominant approach [42]. The observer position is represented in red in Fig. 6, at a distance of $100D$ from the lip and a polar angle of 90° relative to the free stream.

The phased array is designed as a subset of the FW-H permeable surface, allowing for a meaningful comparison of the far-field spectra, free from numerical dissipation and dispersion effects. Data is also extracted on two other phased arrays, at vertical distances $h = 2D$ and $h = 3D$ from

Figure 5: Instantaneous vorticity of the flow around the inlet, for velocity ratios of 0.4 and 0.8.

Figure 6: Positions of the numerical phased array, of the source grid and of the FW-H control surface used for comparison. The phased array and the source grid are represented here with 49 points.

the source grid. The source grid and the phased array are chosen to be of the same width, for simplicity purpose. In practice, the phased array is defined on a regular grid and then matched to the nearest cell centres of the mesh, adding some degree of pseudo-randomness in the phased array distribution. Several combinations of probe and source numbers are used, up to 289 sources and 100 probes.

For both the phased array and FW-H analyses, 2500 samples are recorded from the simulations, corresponding to 0.1 s, or 10 convection cycles of the flow particles over the inlet. By recording 10 convection cycles, it is ensured that the potential low frequency phenomena related to the largest length scale of the inlet are not missed. Power spectral densities (PSD) are calculated with a Welch method [43] by averaging over 21 sample series of 500 samples with an overlap of 400 samples, using a Hanning window on each sample series. The number of averages is limited by the simulation time and gives a relative random error in the spectra of about ± 1 dB/Hz. The resulting spectra have a frequency resolution of 50 Hz, up to 12.500 Hz. The same Fourier processing is

used throughout the next sections.

3.3. Assessment of the phased array approach on an analytical case

Before applying the phased array approach to the flow simulation of the submerged air inlet, the method is first tested on an analytical source to assess its accuracy for various numbers of probes and source points. For this validation case, an artificial set of monopoles is defined in the source model plane of Fig. 6, following a spiral shape. The phased array is placed at $z = 5.51D$. The positions of the artificial monopoles are symbolized by triangles in the plots of Fig. 9. These monopoles are set to radiate incoherent white noise, that is analytically propagated to the probes, for a total time of 0.1 s. This total time corresponds to the duration of the flow simulations around the inlet. The acoustic pressure is also evaluated at a far-field observer at $z = 100D$. To simulate external noise at the probes, white noise with a signal-to-noise ratio of 10 dB is added to the signal at each probe, in addition to the white noise radiated from the sources. The sources are defined as uncorrelated, meaning that an infinite number of averages in the spectral estimation of the source cross-powers would lead to $s_i \overline{s_j} = 0$ for $i \neq j$. Uncorrelated sources are chosen as the flow sound sources are expected to be overall uncorrelated — local correlated sources may exist, but are not captured by the phased array approach when the distance between two sources in the model is larger than the correlation length. The sources are modelled using both pseudo-inverse and dual-LP beamforming methods, and the acoustic spectrum obtained from these sources is computed at the far-field observer using Eqs. (15) and (16). Various number of sources and probes are tested. The average absolute errors in dB in the PSD spectrum estimation compared to the analytical reference are shown in Figs. 7 and 8 for both methods.

The first observation that can be made is that the dual-LP beamforming approach performs better in this test than the pseudo-inverse approach for a large range of values of (M, N) . The pseudo-inverse approach only provides a good model for the radiated sound for some nearly equal values of M and N (for nearly squared G), with a best solution at (49, 81) giving an average error in the spectrum of about 2.1 dB. When M is much greater than N , or the other way around, the far-field spectrum predicted from the source model has more than 10 dB difference in average with the analytical spectrum. The large errors observed when $N < 10$ could be explained by spatial aliasing which occurs at frequencies as low as 2 kHz with this probes spacing. When M and N are greater than 100, it seems that the solution is inaccurate even with squared G. Finally, local error maxima are also found along the line $M = N$. The dual-LP beamforming approach, on the other end, performs well as long as $M \geq N$. The average error decreases with increasing M , which may be explained by the fact that an improved resolution of the source grid allows for a better match with the real sources. For fixed M , the graph suggests that there is an optimal corresponding value of N . This can for example be seen for $M = 100$, when increasing the value of N : the average error decreases at first and then increases. Intuitively, it would seem rational that a larger number of probes should improve the solution as more information is brought into the system. However, this is only the case for low values of N : further increase of N causes a degradation of the solution. This may be due to the conditioning of the matrix A in Eq. (6) which is degraded when N becomes too large. The minimum error in the spectrum is found for $(M, N) = (289, 25)$, and is of around 1.2 dB.

Figure 7: Error in the prediction of the overall SPL (dB) at the observer in the far-field using the pseudo-inverse method to compute the source model. The colorbar represents the average absolute error with the reference spectrum.

This test shows that the dual-LP beamforming approach performs better and is more robust to changes in the number of probes and sources than the pseudo-inverse method. Although the pseudo-inverse provides a correlated source model, as it finds sources with non-zero cross-power values, using it as implemented in this work can lead to large errors. The pseudo-inverse method could certainly be improved by using stronger conditioning of the matrix G , as discussed in section 2.2. Finally, the error in the spectrum can also be reduced by increasing the number of averages in the CSM computation. A total time of 0.2 s instead of 0.1 s was observed to reduce the error for $(M, N) = (289, 25)$ from 1.2 dB down to 0.9 dB. As it is computationally heavy to get longer times in the flow simulations, only 0.1 s was used.

Results for the source models for the pseudo-inverse approach and the dual-LP beamforming approach are shown in Figs. 9. The source models are shown for values of M and N giving the lowest absolute errors in dB, meaning $(M, N) = (49, 81)$ for the pseudo-inverse approach and $(M, N) = (289, 25)$ for the dual-LP beamforming approach. The pseudo-inverse solution gives a model where the energy is distributed over the sources and do not locally match the analytical sources. This was expected, because the pseudo-inverse solution is a least-square solution of the inverse problem. In the least-square sense, a distributed source strength representation is better than a sparse representation, as explained for example in [21]. For the pseudo-inverse solution, the rms values of the modelled sources is much larger than the ones of the analytical sources. It is because the modelled sources are correlated, and the non-zero cross-power spectral values will cause sources to interfere with each other when propagated to the far-field. For the dual-LP beamforming model, a nearly sparse representation of the sources is found, that follows the spiral shape of the analytical sources. When there is a modelled source close to an analytical source, the modelled source SPL matches well the one of the analytical source, as can be seen for example for the source located at about $(2.1D, 4.2D)$. If there is not any modelled source at the position of the analytical source, the power of the source is distributed to the closest modelled sources, as can be seen for example at about $(3.9D, 2.1D)$.

Figure 8: Error in the prediction of the overall SPL (dB) at the observer in the far-field using the dual-LP beamforming approach to compute the source model. The colorbar represents the average absolute error with the reference spectrum.

From these results it is clear that the dual-LP beamforming gives a more realistic model of the sources than the pseudo-inverse approach, at least for uncorrelated sources. The pseudo-inverse method does not allow a good analysis of the source locations. In terms of predicted radiated acoustic field from the source models, both approaches can give a good solution, but the dual-LP beamforming approach is more stable for a wider range of values of (M, N) .

3.4. Acoustic source modelling on the inlet

Based on the values of (M, N) that were identified as optimal for the pseudo-inverse and the dual-LP beamforming approach, source modelling using phased array analysis is performed on the flow case of the NACA inlet. Figs. 10 and 11 show the source models respectively found with the pseudo-inverse and the dual-LP beamforming methods, for a velocity ratio of 0.4. Figs. 12 and 13 show the results for a velocity ratio of 0.8. In these four figures, the models are represented in six plots: the top left represents the overall auto-power of the sources, integrated over all frequencies, whereas the other plots show the auto-power levels integrated over various frequency bands.

The results for a velocity ratio of 0.4 are analysed first. As previously observed in Section 3.3, the pseudo-inverse method gives a distributed source model over the whole source region, for all frequency bands. This distributed model is however centred around the lip region. The dual-LP beamforming approach gives a rather sparse model. At low frequencies, a few point sources are identified over the centre of the entrance of the inlet. At higher frequencies over $2 f_1$, the sources are more distributed and cover the lip region. For high frequencies over $5 f_1$, two clear source regions appear, on both sides of the lip, in the regions of strong vorticity (see Fig. 5). Interestingly, a few point sources are also identified on the side wall edges of the inlet, at around 50-60 percent of the ramp length, and may correspond to physical sound sources caused by the generation of vortices along the sharp edges. Spurious sources are also seen locally on the edge of the source model, which may be attributed to spurious effects from the flow simulation: over $5 f_1$, parts of the mesh between the inlet and the phased array contains less than 15 cells per wavelength, which

Figure 9: Point source model for the analytical spiral case, using pseudo-inverse (left) and dual-LP beamforming (right). Sound-pressure levels of the sources in dB. The triangles represent the positions of the analytical monopoles. The coordinates are normalized by D .

is not sufficient to accurately propagate the sound waves [19]. These sources are anyway of lower amplitude than the more centralized sources.

At a velocity ratio of 0.8, the results are relatively similar. Before commenting on the results, it should be noted that f_1 is two times higher for this velocity ratio than for the velocity ratio of 0.4. Therefore the analysis of the results over $2.5 f_1$ is subject to precautions, because of excessive grid-induced dissipation [19]. For the same reason, over $5 f_1$, the sources are found with a negligible and meaningless amplitude. For the pseudo-inverse method, the source power is distributed over the model, as for the lower velocity ratio. For the dual-LP beamforming approach, the distribution is sparser and closer to the source distribution obtained using the FW-H analogy [19]. At low frequencies, sources are found around the centre of the lip, as for the lower velocity ratio. In the frequency band $[0.7-1.3] f_1$, a few sources with a low but non negligible amplitude are found towards the $+x$ - y edges of the model. In this region, the mesh transitions from a fine to a coarser mesh, see Fig. 4, and it is possible that spurious sound sources may be generated by the flow at these positions. In the frequency band $[1.3-2] f_1$, sources are mainly found on the outer parts of the strong vorticity regions at around $(1D, \pm 3D)$, over the lip, and along the side wall edges of the inlet. Sound sources are also identified at $(-6D, -3D)$ and $(5D, -1D)$, but are attributed to the beamforming algorithm: they were not observed with a different source grid. For both the pseudo-inverse and the dual-LP beamforming methods, the source model is not fully symmetrical with respect to the $y = 0$ plane, as would be expected. A reason for this may be the weakness of the radiated sound field, which is about 40 dB lower in amplitude than for $\nu = 0.4$. The phased array data may be contaminated by numerical effects or non-physical sound sources related for example to the mesh.

3.5. Far-field acoustic spectra

Using the source models found with the pseudo-inverse and dual-LP beamforming approaches, the acoustic spectrum is computed at the far-field observer location following the formulae derived

Figure 10: Point source model obtained using pseudo-inverse, for the NACA inlet case with a velocity ratio of 0.4. Overall and per frequency bands values of the sound-pressure levels of the sources in dB, using $(M, N) = (49, 81)$. The coordinates are normalized by D .

Figure 11: Point source model obtained using dual-LP beamforming, for the NACA inlet case with a velocity ratio of 0.4. Overall and per frequency bands values of the sound-pressure levels of the sources in dB, using $(M, N) = (289, 25)$. The coordinates are normalized by D .

Figure 12: Point source model obtained using pseudo-inverse, for the NACA inlet case with a velocity ratio of 0.8. Overall and per frequency bands values of the sound-pressure levels of the sources in dB, using $(M, N) = (49, 81)$. The coordinates are normalized by D .

Figure 13: Point source model obtained using dual-LP beamforming, for the NACA inlet case with a velocity ratio of 0.8. Overall and per frequency bands values of the sound-pressure levels of the sources in dB, using $(M, N) = (289, 25)$. The coordinates are normalized by D .

Figure 14: Far-field acoustic spectrum for velocity ratios of 0.4, left, and 0.8, right, using $(M, N) = (49, 81)$ for the pseudo-inverse approach and $(M, N) = (289, 25)$ for the dual-LP beamforming approach.

in Section 2.5. Free-field monopole radiation is used, to be consistent with the nature of the source models identified previously. This choice is consistent with the FW-H integral used in this study which also assumes free-field radiation. Fig. 14 shows the PSD spectra for the velocity ratios of 0.4 and 0.8, obtained with the source models from the two phased array methods, compared with the spectra obtained with use of the FW-H integral. Overall, the pseudo-inverse and the beamforming methods give similar spectra, which was expected since both source models are designed to radiate the same sound field at the phased array. What is more interesting is that the spectra obtained with the phased array methods show a good agreement with the spectra obtained with the FW-H integral using the permeable control surface. For a velocity ratio of 0.4, in the frequency range corresponding to maximum sound levels, the differences are of maximum 2-3 dB. For a velocity ratio of 0.8, the tones are well predicted with both source models, with a good agreement in amplitude and width for the first two tones at 1.8 and $2.6 f_1$. The third tone at $3.5 f_1$ is over-predicted by the source models, compared with the FW-H spectrum. After the fourth tone, the levels drop faster with the source models than with the FW-H integral, perhaps because the phased array approach uses less data from the simulations than the FW-H integral, capturing less information on the low amplitude high frequency sound waves that are distorted by the coarse mesh.

Fig. 15 shows the effect of the number of probes and point sources on the far-field spectrum obtained with the phased array method, for a velocity ratio of 0.4. Convergence in the spectrum is reached for $(M, N) = (289, 25)$. Using 9, 25, or 100 probes in the phased array does not affect the results, except at very high and very low frequencies where the results are anyway meaningless. The limiting case of 1 point source is also tested. Using a single point source and more than 25 probes gives good results for frequencies below $2 f_1$. The interest of using a single or a few point sources is to get a simple frequency-domain point source model of the flow generated sound for further use in propagation codes: the source map itself would not give much information.

Computing times for the source models with the pseudo-inverse and beamforming approaches are shown in Tables 2 and 3 for various number of probes and sources. Computations are per-

Figure 15: Far-field acoustic spectrum for velocity ratios of 0.4, with $N = 25$ and varying M , left, and $M = 289$ and varying N , right.

formed in Matlab on a single Intel CORE i7 cpu. The general trend for the computing time is logically to increase with increasing number of probes and sources. Computational times for the pseudo-inverse method are much lower than for the dual-LP beamforming approach. For the pseudo-inverse method, the increase in computing time is much larger with N than with M because the CSM estimate is the longest step. For the dual-LP beamforming approach, both the CSM estimate and the deconvolution algorithm take most of the computing time, explaining a strong increase of the computing time with respect to both N and M . For both methods, computing times are relatively low and allow for quick tests of various phased array and source model configurations.

$N \downarrow M \rightarrow$	1	25	100	289
9	0.46 s	0.51 s	1.10 s	2.46 s
25	1.76 s	1.85 s	3.65 s	5.28 s
100	25.02 s	29.43 s	34.16 s	35.83 s

Table 2: Computational time required for the Moore-Penrose inverse approach (estimation of the cross-spectral matrix from pressure time data included), on a single Intel CORE i7 cpu, for a velocity ratio of 0.4.

$N \downarrow M \rightarrow$	1	25	100	289
9	8.59 s	15.11 s	36.70 s	59.67 s
25	14.04 s	14.64 s	32.35 s	117.20 s
100	38.58 s	46.14 s	75.94 s	192.94 s

Table 3: Computational time required for the dual-LP beamforming approach (estimation of the cross-spectral matrix from pressure time data included), on a single Intel CORE i7 cpu, for a velocity ratio of 0.4.

Finally, Fig. 16 shows the effect of a change in the distance from the inlet to the phased array on the estimation of the far-field spectrum. For varying h , the results appear unchanged up to

Figure 16: Far-field acoustic spectrum for velocity ratios of 0.4, with $(N, M) = (25, 289)$ and varying distance h between the inlet and the array.

about $4f_1$, above which they differ increasingly. At higher frequencies, the levels are lower when the array is placed further to the inlet and to the unsteady flow. This is probably due to numerical dissipation of the acoustic waves by the grid.

4. Discussion

A good match is observed between the spectra obtained with the phased array source modelling approach and the FW-H approach, at this specific observer located at a polar angle of 90° from the inlet. The two methods have a radically different approach and it is interesting to see that the far-field results are so similar. This good match is particularly interesting because the phased array source modelling method has a lower computational cost than the FW-H method: it does not require any time nor space interpolation, nor numerical integration. But still the main interest of the phased array approach resides in the equivalent point source model that is computed. This source model can be directly implemented in a propagation solver, whereas using an acoustic analogy would require several additional steps to reduce the complex distributed source output to only a few simple point sources. Another advantage of the phased array approach is that hydrodynamic fluctuations at the location of the array can be partially filtered out through ensemble averaging, an operation that is not performed in the FW-H approach. The plots of the source models for the dual-LP beamforming approach are consistent with the locations of the sources according to the FW-H acoustic analogy [19] and are the next point of discussion.

The source models obtained with the dual-LP beamforming approach, see Figs. 11 and 13, show acoustic sources on both sides of the lip at high frequencies, on the surface adjacent to the turbulent vortex regions. This is consistent with previous results [19]. At low frequencies, the sources reduce to a few points centred over the entrance of the inlet, showing the limits of resolution of the beamforming approach when the phased array is close to the sources. However, resolution issues are not a problem in this study, because the sources identified through the phased array approaches are used as a source model, regardless of whether they accurately correspond to real source positions or not. For $\nu = 0.4$, the fact that the sources are overall found in the lip region

enforces the idea that there are not any spurious sound sources in the simulation. For $\nu = 0.8$, see Fig. 13, the beamforming analysis reveals what may be spurious sound sources, at frequencies of $0.7 f_1$ and higher. However, their amplitudes are lower than the sources in the lip region and they should not significantly affect the far-field acoustic results. These sources may also exist at $\nu = 0.4$, but would be masked by the greater amplitude “physical” sources. If spurious non-physical sources are suspected to deteriorate the far-field results, manually removing them from the source model could be a solution.

As it was stated earlier, the results for the far-field pressure compare very well with the results from the FW-H integral, showing that the source models represent well the radiated acoustic field at least from the observer’s standpoint. The position of the observer is the next point of discussion.

In this study, the observer is placed vertically over the inlet and the microphone array is expected to capture most of the acoustic energy radiated in this direction. It is not certain whether the source model would work as well to predict the sound radiated in other directions. To obtain the spectrum at shallow angles or downstream, it may be needed to change the phased array position accordingly. A possibility would be to use a partially cylindrical or spherical array to cover a larger surface over the inlet and therefore cover all directions. This is left to future work.

One important requirement of the phased array method is that the flow simulations should accurately resolve sound propagation up to the phased array. The FW-H integral, on the other hand, can in some cases be used with solid control surfaces in incompressible simulations. The main interest of using the phased array modelling method is that it decouples sound source modelling and sound propagation, giving at a relatively low computational cost a point source model of the flow generated sound, and making possible the use of more complex sound propagation methods, including scattering by near-by surfaces, or convection effects.

The sources are modelled with monopoles in this study, despite the fact that the sound sources created by the unsteady flow on the surface of the inlet are of dipole type. Theoretically, it should be possible to use any source type in the model, such as dipole or even tailored Green’s functions based on the mean flow quantities. This is only a matter of computing a different matrix $\mathbf{G}$ in Eq. 2. It should also be possible to extend the matrix $\mathbf{G}$ to include dipoles or higher-order multipoles as was done for example by Suzuki [21]: this could extend the applicability of the method to directional aeroacoustic sources. This study was restricted to monopoles to keep the source model as simple as possible.

Finally, the phased array method is applicable to the flow simulations of this study because the free stream flow is essentially laminar. If it was turbulent, turbulent pressure fluctuations would be captured in the phased array data and would degrade the results of the source modelling. The cross-spectra in the CSM would reduce the effect of non-correlated turbulent content between the various probes, but locally in the array these effects may still be the cause of biased results. A possible improvement to the method would be to use two phased arrays at a distance larger than the turbulence correlation scale and to perform cross-correlations between these arrays to attenuate the hydrodynamic pressure fluctuations due to the turbulent eddies.

5. Conclusion

A source modelling approach based on numerical phased array data was presented. This approach uses phased array techniques to compute an equivalent point source model from phased array data extracted from a flow simulation. Two phased array techniques were used: a direct approach based on pseudo-inverse and cross-spectral computations, and a beamforming approach using linear programming deconvolution denoted dual-LP beamforming. A test on analytical uncorrelated sources showed that both approaches perform well for some phased array and source model configurations, but that the dual-LP beamforming approach was more robust than the pseudo-inverse approach, the latter being very sensitive to changes in the number of probes in the array or point sources in the model. The dual-LP beamforming approach provides an accurate and relatively sparse model of uncorrelated sources, whereas the pseudo-inverse approach provides a more distributed model of correlated sources.

The method was applied on the example of the flow around a submerged air inlet at a low Mach number. It was shown to give consistent results compared to the FW-H acoustic analogy, both in terms of source locations and far-field spectrum. The source model obtained through the dual-LP beamforming approach gives interesting information about the source locations at high frequencies, but the resolution is not sufficient to give detailed information about the source locations at low frequencies: the main advantage of the phased array approach is that it provides a point source model for the flow generated sound, with a relative freedom in the number of chosen point sources. This model can be used as input to propagation codes, e.g. to study the scattering by near-by surfaces or to simulate the pass-by noise from a moving vehicle. The approach using dual-LP beamforming was demonstrated to be robust to changes in the number of probes in the phased array, in the number of sources in the model and in the distance of the phased array to the source model. Future work may include additional studies on the phased array shape, scanning of three-dimensional source regions, extension to dipole or tailored transfer functions, or use of other beamforming algorithms.

To sum up, the source modelling approach presented in this paper was shown to perform well on the submerged air inlet case, and proved to be an efficient and useful tool to extract equivalent sound sources from compressible flow simulations for further use in propagation codes.

Acknowledgements

The work presented here is part of the project: “Noise Propagation from Sustainable Vehicle Concepts” conducted at the Centre for ECO² Vehicle Design, a VinnExcellence Centre performing multi-vehicular multidisciplinary research to support a sustainable vehicle design development. The partners of the centre are gratefully acknowledged. The DES computations were performed on resources provided by the Swedish National Infrastructure for Computing (SNIC) at PDC Centre for High Performance Computing (PDC-HPC).

References

- [1] N. J. Pignier, “The impact of traffic noise on economy and environment: a short literature study: Performed within the scope of the ECO² project noise propagation from sustainable ground vehicles.” KTH Royal Institute of Technology, 2015.

- [2] C. K. Tam, "Computational aeroacoustics: an overview of computational challenges and applications," *International Journal of Computational Fluid Dynamics*, vol. 18, no. 6, pp. 547–567, 2004.
- [3] M. J. Lighthill, "On sound generated aerodynamically. I. General theory," *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, vol. 211, no. 1107, pp. 564–587, 1952.
- [4] M. J. Lighthill, "On sound generated aerodynamically. II. Turbulence as a source of sound," *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, vol. 222, no. 1148, pp. 1–32, 1954.
- [5] N. Curle, "The influence of solid boundaries upon aerodynamic sound," *Proceedings of the Royal Society of London. Series A. Mathematical and Physical Sciences*, vol. 231, no. 1187, pp. 505–514, 1955.
- [6] J. F. Williams and D. L. Hawkings, "Sound generation by turbulence and surfaces in arbitrary motion," *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences*, vol. 264, no. 1151, pp. 321–342, 1969.
- [7] J. Hammar, C. J. O'Reilly, and G. Efraimsson, "Simulation of aerodynamically generated noise using the wave expansion method," in *22nd AIAA/CEAS Aeroacoustics Conference, Lyon, France*, 2016.
- [8] C. J. O'Reilly, "A wave expansion method for acoustic propagation in lined flow ducts," *Applied Acoustics*, vol. 90, pp. 54 – 63, 2015.
- [9] L. Brusniak, M. L. Shur, and P. R. Spalart, "Phased array imaging of jet noise sources in a large-eddy simulation," *AIAA Paper*, vol. 2444, 2006.
- [10] J. Adam, D. Ricot, and A. Menoret, "Direct aeroacoustic source identification based on lattice boltzmann simulation and beamforming technique," in *15th AIAA/CEAS Aeroacoustic Conference, AIAA paper*, vol. 3182, 2009.
- [11] R. Sandberg and B. Tester, "Mach-number scaling of individual azimuthal modes of subsonic co-flowing jets," *Journal of Fluid Mechanics*, vol. 793, pp. 209–228, 2016.
- [12] Y. Du and P. J. Morris, "Numerical investigation of the noise source locations of supersonic jets using the beamformed method," in *Proc. 50th AIAA Aerospace Science Meeting, AIAA*, vol. 1169, p. 2012, 2012.
- [13] D. B. Armstrong, A. Najafi-Yazdi, L. Mongeau, and V. Raymond, "Numerical simulations of flow over a landing gear with noise reduction devices using lattice boltzmann method," in *19th AIAA/CEAS Aeroacoustics Conference, AIAA Paper*, vol. 2114, pp. 27–29, 2013.
- [14] C. Nelson, A. B. Cain, R. Dougherty, K. S. Brentner, and P. J. Morris, "Application of synthetic array techniques for improved simulations of hot supersonic jet noise," in *53rd AIAA Aerospace Sciences Meeting*, p. 0507, 2015.
- [15] Y. Du and P. J. Morris, "The separation of radiating and non-radiating near-field pressure fluctuations in supersonic jets," *Journal of Sound and Vibration*, vol. 355, pp. 172–187, 2015.
- [16] D. P. Lockard, W. M. Humphreys, M. R. Khorrami, E. Fares, D. Casalino, and P. A. Ravetta, "Comparison of computational and experimental microphone array results for an 18%-scale aircraft model," in *21st AIAA/CEAS Aeroacoustics Conference, Dallas, Texas*, 2015.
- [17] V. Fleury and R. Davy, "Analysis of jet–airfoil interaction noise sources by using a microphone array technique," *Journal of Sound and Vibration*, vol. 364, pp. 44–66, 2016.
- [18] S. Redonnet and J. Bulté, "Landing gear noise sources identification through an application of array methods to experimental and computational data," in *22nd AIAA/CEAS Aeroacoustics Conference, Lyon, France*, 2016.
- [19] N. J. Pignier, C. J. O'Reilly, and S. Boij, "Aerodynamic and aeroacoustic analyses of a submerged air inlet in a low-mach-number flow," *Computers & Fluids*, vol. 133, pp. 15 – 31, 2016.
- [20] R. P. Dougherty, R. C. Ramachandran, and G. Raman, "Deconvolution of sources in aeroacoustic images from phased microphone arrays using linear programming," *International Journal of Aeroacoustics*, vol. 12, no. 7-8, pp. 699–717, 2013.
- [21] T. Suzuki, "L1 generalized inverse beam-forming algorithm resolving coherent/incoherent, distributed and multipole sources," *Journal of Sound and Vibration*, vol. 330, no. 24, pp. 5835 – 5851, 2011.
- [22] P. A. Zavala, W. De Roeck, K. Janssens, J. R. Arruda, P. Sas, and W. Desmet, "Monopole and dipole identification using generalized inverse beamforming," in *16th AIAA/CEAS Aeroacoustics Conference, Stockholm, Sweden*, 2010.
- [23] P. Zavala, W. D. Roeck, K. Janssens, J. Arruda, P. Sas, and W. Desmet, "Generalized inverse beamforming with optimized regularization strategy," *Mechanical Systems and Signal Processing*, vol. 25, no. 3, pp. 928 – 939, 2011.

- [24] S. Campbell and C. Meyer, *Generalized Inverses of Linear Transformations*. Society for Industrial and Applied Mathematics, 2009.
- [25] J. S. Bendat and A. G. Piersol, "Engineering applications of correlation and spectral analysis," *New York, Wiley-Interscience*, 1980. 315 p., vol. 1, 1980.
- [26] P. A. Nelson and S.-H. Yoon, "Estimation of acoustic source strength by inverse methods: Part i, conditioning of the inverse problem," *Journal of Sound and Vibration*, vol. 233, no. 4, pp. 639–664, 2000.
- [27] P. Courrieu, "Fast computation of moore-penrose inverse matrices," *Neural Information Processing-Letters and Reviews*, vol. 8, no. 2, pp. 25–29, 2005.
- [28] S.-H. Yoon and P. A. Nelson, "Estimation of acoustic source strength by inverse methods: Part ii, experimental investigation of methods for choosing regularization parameters," *Journal of Sound and Vibration*, vol. 233, no. 4, pp. 665–701, 2000.
- [29] T. F. Brooks and W. M. Humphreys, "A deconvolution approach for the mapping of acoustic sources (DAMAS) determined from phased microphone arrays," *Journal of Sound and Vibration*, vol. 294, no. 4, pp. 856–879, 2006.
- [30] R. P. Dougherty, "Extensions of DAMAS and benefits and limitations of deconvolution in beamforming," *AIAA paper*, vol. 2961, no. 11, 2005.
- [31] T. F. Brooks and W. M. Humphreys, "Extension of DAMAS phased array processing for spatial coherence determination (DAMAS-C)," *AIAA paper*, vol. 2654, p. 2006, 2006.
- [32] J. Nocedal and S. Wright, *Numerical optimization*, 2nd ed. SpringerVerlag, New York, 2006.
- [33] E. A. Mossman and L. M. Randall, "An experimental investigation of the design variables for NACA submerged duct entrances." NACA-RM-A7I30, 1948.
- [34] "STAR-CCM+." <http://www.cd-adapco.com/products/star-ccm>. Accessed: 2016-02-16.
- [35] F. R. Menter, "Two-equation eddy-viscosity turbulence models for engineering applications," *AIAA journal*, vol. 32, no. 8, pp. 1598–1605, 1994.
- [36] M. L. Shur, P. R. Spalart, M. K. Strelets, and A. K. Travin, "A hybrid rans-les approach with delayed-des and wall-modelled les capabilities," *International Journal of Heat and Fluid Flow*, vol. 29, no. 6, pp. 1638–1649, 2008.
- [37] B. Leonard, "The ULTIMATE conservative difference scheme applied to unsteady one-dimensional advection," *Computer Methods in Applied Mechanics and Engineering*, vol. 88, no. 1, pp. 17 – 74, 1991.
- [38] K. W. Thompson, "Time dependent boundary conditions for hyperbolic systems," *Journal of Computational Physics*, vol. 68, no. 1, pp. 1–24, 1987.
- [39] K. W. Thompson, "Time-dependent boundary conditions for hyperbolic systems, ii," *Journal of Computational Physics*, vol. 89, no. 2, pp. 439–461, 1990.
- [40] T. J. Poinsot and S. Lele, "Boundary conditions for direct simulations of compressible viscous flows," *Journal of Computational Physics*, vol. 101, no. 1, pp. 104–129, 1992.
- [41] K. S. Brentner and F. Farassat, "Analytical comparison of the acoustic analogy and kirchhoff formulation for moving surfaces," *AIAA journal*, vol. 36, no. 8, pp. 1379–1386, 1998.
- [42] K. Brentner, "A new algorithm for computing acoustic integrals," in *Proceedings of the IMACS 14th World Congress on Computational and Applied Mathematics*, vol. 2, pp. 592–595, 1994.
- [43] P. D. Welch, "The use of fast Fourier transform for the estimation of power spectra: A method based on time averaging over short, modified periodograms," *IEEE Transactions on Audio and Electroacoustics*, vol. 15, no. 2, pp. 70–73, 1967.