# Spectral Efficient and Fair User Pairing for Full-Duplex Communication in Cellular Networks

José Mairton B. da Silva Jr.⋆, Gábor Fodor⋆†, Carlo Fischione⋆
⋆KTH, Royal Institute of Technology, Stockholm, Sweden
†Ericsson Research, Kista, Sweden

***Abstract*—A promising new transmission mode in cellular networks is the three-node full-duplex mode, which involves a base station with full-duplex capability and two half-duplex user transmissions on the same frequency channel for uplink and downlink. The three-node full-duplex mode can increase spectral efficiency, especially in the low transmit power regime, without requiring full-duplex capability at user devices. However, when a large set of users is scheduled in this mode, self-interference at the base station and user-to-user interference can substantially hinder the potential gains of full-duplex communications. This paper investigates the problem of grouping users to pairs and assigning frequency channels to each pair in a spectral efficient and fair manner. Specifically, the joint problem of user uplink/downlink frequency channel pairing and power allocation is formulated as a mixed integer nonlinear problem that is solved by a novel joint fairness assignment maximization algorithm. Realistic system level simulations indicate that the spectral efficiency of the users having the lowest spectral efficiency is increased by the proposed algorithm, while a high ratio of connected users in different loads and self-interference levels is maintained.**

## I. Introduction

Traditional cellular networks operate in half-duplex (HD) transmission mode, in which a user equipment (UE) or the base station (BS) either transmits or receives on any given frequency channel. However, the ever increasing demand to support the transmission of unprecedented data quantities has led the research community to investigate new wireless transmission technologies. Recently, in-band full-duplex (FD) has been proposed as a key enabling technology to drastically increase the spectral efficiency of conventional wireless transmission modes. In-band FD has the potential to double the spectral efficiency of traditional wireless systems operating in HD, such as time division duplex or frequency division duplex modes [1], [2]. Recent advances in antenna design, interference cancellation algorithms, self-interference (SI) suppression techniques and prototyping of FD transceivers have meant that, FD is becoming a realistic technology component of advanced wireless – including cellular – systems, especially in the low transmit power regime [3], [4].

FD transmission modes include bidirectional full-duplex and three-node full-duplex (TNFD) modes [5]. In bidirectional full-duplex, two FD-capable nodes (either a UE or the BS) transmit and receive on the same time-frequency resource. In contrast, TNFD involves three nodes, but only one of them needs to have FD capability. The FD-capable node transmits to its receiver node while receiving from another transmitter node on the same frequency channel. In cellular networks, the BS is FD-capable and transmits in the downlink (DL) and receives in the uplink (UL) from the HD UEs (Figure 1). From Figure 1, it is clear that FD operation in a cellular environment experiences new types of interference, aside from the inherently present SI.

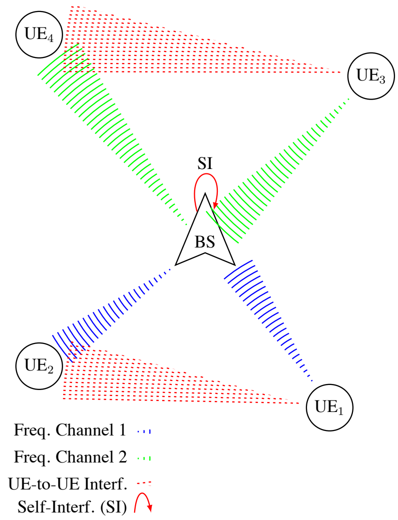


Figure 1. A full duplex cellular network employing TNFD with two UEs pairs. The base station (BS) selects pairs of UE (pairing) and jointly schedules them for TNFD transmission by allocating frequency channels in the UL and DL. As the figure illustrates, apart from SI, TNFD experiences the new UE-to-UE interference that might limit the efficiency of FD communications.

Because of the level of UE-to-UE interference depends on the UE locations and their transmission powers, coordination mechanisms are needed to mitigate the negative effect of the interference on the spectral efficiency of the system [6]. A key element of such mechanisms is UE *pairing* and frequency channel selection that determines which UEs should be scheduled for simultaneous UL and DL transmissions on specific frequency channels. As pointed out in [5], the simultaneous bidirectional communication capabilities of the nodes in bidirectional full-duplex and TNFD networks mean that the fairness among nodes may degrade by a factor of two compared with HD communications. Despite that fairness is widely recognized as important and has been well investigated in more traditional wireless communication context [7], to

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the best of our knowledge, max-min fairness has not been addressed in the full-duplex literature for TNFD. Hence, it is crucial to design efficient and fair medium access control protocols and physical layer procedures capable of supporting adequate pairing mechanisms that arise in full-duplex communications.

In this paper we focus on the problem of joint frequency channel selection and transmit power allocation assuming TNFD transmissions and a frequency selective wireless environment. Specifically, we investigate schemes that maximize the spectral efficiency of the user with the lowest achieved spectral efficiency. We formulate this problem as a mixed integer nonlinear optimization, which we call the *joint assignment and fairness maximization* (JAFM) problem, where the decision variables are the assignment matrices of UL and DL, and where each element of the matrix is a binary variable indicating the association of the user to a frequency channel. We show that the JAFM problem is a non-polynomial time (NP)-hard problem, implying that no optimal solution in polynomial time can be obtained. We derive a closed-form approximate solution by resorting to Lagrangian duality theory, and prove the optimal power allocation for the dual problem. However, the closed-form assignment cannot be solved efficiently, because this assignment problem is also NP-hard, but now with respect to the assignment variables. Therefore, we propose a greedy solution to this assignment problem and evaluate the optimal transmit powers for all UEs based on the results given by the dual solution. We also show that the duality gap between the greedy solution and the primal is bounded and diminishes as the number of frequency channels increases.

The remainder of the paper is organized as follows. Section II discusses some relevant and closely related works to our problem. Section III presents the system model and main parameters, followed by the problem formulation. In Section IV, we study the impact of the problem constraints based on the JAFM problem formulation, and show that the SINR and power constraints create an admissible area on which UL and DL users can share a frequency channel. In Section V we analyse the proposed JAFM problem and derive a closed-form approximate solution for the assignment and show the optimal power allocation based on the Lagrangian dual problem. In Section VI, we propose a greedy approximation to the primal JAFM problem based on the dual problem of Section V. Section VII presents numerical results and compares the performance of the proposed solution with different assignments and power allocation, and the impact of the users' load and SI cancelling on the fairness of FD cellular networks. The results show that our proposed solution outperforms existing methods from the literature, such as random assignment without power control, and guarantees connection to more users, and maintains a higher level of fairness. Moreover, we show that our joint assignment and power allocation solution should not be split into separated procedures, because the performance achieved by such splitting schemes is similar to a random assignment with equal power allocation.

## II. Related Works

The impact of FD radios on the design of cellular systems has been analysed only relatively recently, in [1], [6], [8]. These works provide valuable insights into the design and performance of FD cellular systems in terms of rate and energy performance. However, the problem of fair and efficient joint power and channel allocation has not been addressed.

Transmit power optimization for FD wireless networks is the topic of [9] and [10]. A dynamic power allocation scheme that maximizes the sum-rate of FD bidirectional transmissions is developed in [9], while an optimal power control scheme for FD decode-and-forward relaying systems is proposed in [10]. However, the results of these works are not directly applicable to FD networks operating in TNFD mode, since the joint channel and power allocation problem under fairness constraints has not been considered therein.

Most of the recent works consider the joint subcarrier and power allocation problem [11] and the joint duplex mode selection, channel allocation, and power control problem [12] in FD networks. The cellular network model in [11] is applicable to FD mobile nodes rather than to networks operating in TNFD mode. The work reported in [12] considers the case of TNFD transmission mode in a cognitive femto-cell context with bidirectional transmissions from UEs and develops sum-rate optimal resource allocation and power control algorithms. Our objective differs from that of [12] in that we aim at maximizing the transmission rate of the low rate users and thereby taking into account the fairness of the FD system. We do not consider the proposed algorithm for rate maximization in our performance evaluation, because in [12] heterogeneous networks with bidirectional transmission from UEs were considered, which substantially changes the problem.

The aspects of fairness and quality of service (QoS) are addressed by [5], [13], and [14]. Specifically, a QoS-driven power allocation approach for bidirectional FD links is proposed by [13] and a heterogeneous statistical QoS provisioning framework is developed in [14]. Both of those papers focus on the bidirectional FD link case without considering the implications of TNFD transmissions. In contrast, [5] discusses both the bidirectional and the TNFD modes and emphasizes the importance of fairness. However, [5] does not provide a power control and channel allocation scheme that is developed with such objectives in mind.

Several deliverables of the DUPLO project are related to our work [15]. These deliverables provide methodology and numerical results on the performance of cellular and ad hoc networks employing FD radios combined with power control, channel allocation and other physical and medium access control layer algorithms. However, a fairness-optimizing joint channel and power allocation has not been investigated in that project.

In the light of this survey of related literature, we summarize our main contributions as follows:

Table I
DEFINITION OF SETS, CONSTANTS AND VARIABLES

| | |
|---|---|
| **Sets** | |
| $\mathcal{F}$ | Set of frequency channels |
| $\mathcal{I}$ | Set of UL users |
| $\mathcal{J}$ | Set of DL users |
| **Constants** | |
| $\beta$ | SI cancelling term |
| $\sigma^2$ | Thermal noise power on frequency channel $f$ |
| $\gamma^u_{\text{th}}$ | Minimum SINR required for UL users |
| $\gamma^d_{\text{th}}$ | Minimum SINR required for DL users |
| $G_{bjf}$ | Path gain between BS and DL user $j$ on frequency $f$ |
| $G_{ibf}$ | Path gain between UL user $i$ and BS on frequency $f$ |
| $P^u_{\text{max}}$ | UL maximum transmit power |
| $P^d_{\text{max}}$ | DL maximum transmit power |
| **Variables** | |
| $P^u_i$ | Transmit power of UL user $i$ |
| $P^d_j$ | Transmit power of DL user $j$ |
| $x^u_{if}$ | Assignment of UL user $i$ on frequency $f$ |
| $x^d_{jf}$ | Assignment of DL user $j$ on frequency $f$ |

- The problem formulation of JAFM and our proposed solution is new. This problem is particularly important in practical systems employing TNFD mode, where its need was first pointed out in [5].
- We propose a necessary condition for a pair of UL and DL users to share a frequency channel based on the SI cancelling level of the system, which helps us to define the admissible pairs for the assignment problem.
- Due to the intractability of JAFM, we propose a greedy joint assignment and fairness maximization algorithm (JAFMA) that is based on Lagrangian duality theory, where we show that the duality gap between JAFMA and JAFM is bounded and diminishes as the number of channels in the system increases.
- We evaluate JAFMA by a realistic system simulator and argue that the obtained numerical results and the insights help design FD cellular networks.

## III. System Model and Problem Formulation

### A. System Model

We consider a single-cell cellular system in which only the BS is FD capable, whereas the UEs operate in time division duplex mode, as illustrated in Figure 1. In the example scenario of Figure 1, the BS is subject to SI and the UEs in the UL ($\text{UE}_1$ and $\text{UE}_3$) cause UE-to-UE interference to $\text{UE}_2$ and $\text{UE}_4$ in DL. Table I shows the main sets, constants and variables used throughout the paper. We assume that within a large number of connected users in the cellular network, a smaller subset of these users are selected by an appropriate scheduler for data transmission in the UL and reception in the DL. The number of UEs in the UL and DL is denoted by $I$ and $J$, respectively. Their sum is constrained by the total number of frequency channels in the system $F$, i.e., $I \le F$ and $J \le F$. The set of UL and DL users is denoted by $\mathcal{I} = \{1, \dots, I\}$ and $\mathcal{J} = \{1, \dots, J\}$, respectively, and the set of resources by $\mathcal{F} = \{1, \dots, F\}$. In this paper we assume that at most one channel is used by a UE.

Our system model includes a frequency selective environment, such that the composite channel gains depend on the frequency. We assume that the path gains consist of small- and large-scale fading. Specifically, let $G_{ibf}$ denote the path gain between transmitter $i$ in the UL and the BS on frequency channel $f$, whereas $G_{bjf}$ denotes the path gain between BS and the receiving UE $j$ in the DL on frequency channel $f$, and $G_{ijf}$ denotes the interfering path gain between transmitter $i$ in the UL and the receiver $j$ in the DL on frequency channel $f$. The vector of transmit powers in the UL is denoted by $\mathbf{p^u} = [P^u_1 \dots P^u_I]$, whereas the vector of the DL transmit powers on the DL frequency channels is denoted by $\mathbf{p^d} = [P^d_1 \dots P^d_J]$.

To take into account the residual value of the SI power that leaks to the receiver, we define $\beta$ as the SI cancellation coefficient, such that the SI power at the receiver of the BS is $\beta P^d_j$ when the transmit power is $P^d_j$. For example, at a transmit power of $20\,\text{dBm}$ ($100\,\text{mW}$), and assuming a noise floor around $-90\,\text{dBm}$, the transmit SI has to be cancelled by $\beta = -110\,\text{dB}$ to reduce it to the same level as the noise floor [2], [4].

As illustrated in Figure 1, the UE-to-UE interference depends on the geometry of the co-scheduled UL and DL users. For example, assuming a greater path loss between $\text{UE}_1$ and $\text{UE}_4$ than between $\text{UE}_1$ and $\text{UE}_2$, it may be advantageous to pair $\text{UE}_1$ and $\text{UE}_4$ for simultaneous UL/DL transmission on the same frequency channel $f$. Therefore, the UE pairing along with the channel allocation are key functions of the system. Accordingly, we define two assignment matrices, $\mathbf{X}^u \in \{0,1\}^{I \times F}$ for the UL and $\mathbf{X}^d \in \{0,1\}^{J \times F}$ for the DL, such that

$$x^u_{if} = \begin{cases} 1, & \text{if } \text{UE}_i \text{ transmits in frequency } f \\ 0, & \text{otherwise,} \end{cases}$$

and

$$x^d_{jf} = \begin{cases} 1, & \text{if } \text{UE}_j \text{ receives in frequency } f \\ 0, & \text{otherwise.} \end{cases}$$

The signal-to-interference-plus-noise ratio (SINR) at the BS due to transmitting user $i$ in the UL on frequency channel $f$ is

$$\gamma^u_{if} = \frac{x^u_{if} P^u_i G_{ibf}}{\sigma^2 + \sum_{j=1}^{J} x^d_{jf} \beta P^d_j}, \tag{1}$$

where $x^u_{if}$ accounts for usage of frequency channel $f$ by UL user $i$, and the sum in the denominator for the possible SI created by the simultaneous transmission by the BS on frequency channel $f$. Similarly, the SINR at the receiving user $j$ due to the BS on frequency channel $f$ is given by

$$\gamma^d_{jf} = \frac{x^d_{if} P^d_j G_{bjf}}{\sigma^2 + \sum_{i=1}^{I} x^u_{if} P^u_i G_{ijf}}, \tag{2}$$

where the $x^d_{jf}$ accounts for usage of frequency channel $f$ by the BS and the sum in the denominator for the intra-cell interference between $\text{UE}_i$ and $\text{UE}_j$ on frequency channel $f$.

Thus, the spectral efficiency for each user is given by the Shannon Eq. (in bits/s/Hz) for the UL and DL as

$$C^u_i = \sum_{f=1}^{F} C^u_{if} = \sum_{f=1}^{F} \log_2(1 + \gamma^u_{if}), \tag{3a}$$

$$C^d_j = \sum_{f=1}^{F} C^d_{jf} = \sum_{f=1}^{F} \log_2(1 + \gamma^d_{jf}), \tag{3b}$$

where the sum over the frequency dimension is because at most one SINR is non-zero along the frequency dimension.

### B. Problem Formulation

Our goal is to jointly consider the problem of frequency assignment to UEs in the UL and DL (*pairing*), while maximizing the minimum spectral efficiency of all users (***max-min spectral efficiency***), thus increasing the fairness of the system. Specifically, the problem is formulated as a Joint Assignment and Fairness Maximization (JAFM) problem as follows

$$\underset{\mathbf{X}^u, \mathbf{X}^d, \mathbf{p}^u, \mathbf{p}^d}{\text{maximize}} \quad \min_{\forall i,j}\{C_i^u, C_j^d\} \tag{4a}$$

$$\text{subject to} \quad \sum_{f=1}^{F} \gamma_{if}^u \geq \gamma_{\text{th}}^u, \ \forall i, \tag{4b}$$

$$\sum_{f=1}^{F} \gamma_{jf}^d \geq \gamma_{\text{th}}^d, \ \forall j, \tag{4c}$$

$$P_i^u \leq P_{\text{max}}^u, \ \forall i, \tag{4d}$$

$$P_j^d \leq P_{\text{max}}^d, \ \forall j, \tag{4e}$$

$$\sum_{i=1}^{I} x_{if}^u \leq 1, \ \forall f, \tag{4f}$$

$$\sum_{f=1}^{F} x_{if}^u \leq 1, \ \forall i, \tag{4g}$$

$$\sum_{j=1}^{J} x_{jf}^d \leq 1, \ \forall f, \tag{4h}$$

$$\sum_{f=1}^{F} x_{jf}^d \leq 1, \ \forall j, \tag{4i}$$

$$x_{if}^u, \ x_{jf}^d \in \{0,1\}, \ \forall i,j,f. \tag{4j}$$

The optimization variables of JAFM are $\mathbf{p}^u$, $\mathbf{p}^d$, $\mathbf{X}^u$ $\mathbf{X}^d$. Constraints (4b) and (4c) ensure a minimum SINR to be achieved in the DL and UL, respectively. Constraints (4d) and (4e) limit the transmitting power and constraints (4f)-(4i) assure that only one UE in the UL and DL can use frequency channel $f$ and that any given frequency channel is used by at most one UE in the UL and DL. Let us further denote by $\mathcal{X}$ the set of matrices $\mathbf{X}^u$ and $\mathbf{X}^d$ that satisfy the constraints (4f)-(4j) and by $\mathcal{P}$ the set of vectors $\mathbf{p}^u$ and $\mathbf{p}^d$ that satisfy the constraints (4d)-(4e).

JAFM is a mixed integer nonlinear programming (MINLP) problem that belongs to the category of multi-level nonlinear bottleneck assignments, wherein the linear integer subproblem is NP-hard [16]. Therefore, to analyse JAFM, we develop an approximate approach based on Lagrangian duality in Section V. Afterwards, by using the insights given by the dual problem, we propose a greedy approximation to problem (4) in Section VI. Moreover, for simplicity we rename the primal problem (4) as P-JAFM.

Before analysing problem (4), we establish some preliminary results that will be instrumental in establishing an approximate solution to problem (4) in the subsequent sections.

## IV. Preliminary Results

In this section, we characterize the maximum number of users sharing the frequency channel, and the region of the transmit power for which users can share frequency channels. We use these two results in the following section to derive an approximate solution to problem (4).

TNFD implies that an UL and a DL transmission share a frequency channel $f$. The following lemma shows that in a TNFD system, complying with constraints (4f)-(4i), the number of users sharing a frequency channel $f$ is bounded.

**Lemma 1.** Consider optimization Problem (4). Then, for every frequency channel $f$ and feasible assignment matrices $\mathbf{X}^u$ and $\mathbf{X}^d$, the following inequality holds

$$\sum\nolimits_{i=1}^{I} x_{if}^u + \sum\nolimits_{j=1}^{J} x_{jf}^d \leq 2. \tag{5}$$

*Proof.* Since $I, J \leq F$, we have the inequality $I + J \leq 2F$. Moreover, if we sum constraints (4f) and (4h), we notice that $\sum_{i=1}^{I} x_{if}^u + \sum_{j=1}^{J} x_{jf}^d$ is at most 2. ■

Lemma 1 establishes that for feasible assignment matrices $\mathbf{X}^u$ and $\mathbf{X}^d$, there can be at most one pair of UL-DL users sharing a frequency channel $f$. We will use this result in the following subsection to derive an approximate closed-form solution to the assignment matrices of problem (4).

Now we turn our attention to characterize the admissible area of transmit powers for which a pair of UL and DL users can share a frequency channel $f$. In problem 4, constraints (4b)-(4e) require a minimum SINR level for all users, implying that we need to identify the set of corresponding transmit powers within the constraint set $\mathcal{P}$. Based on the SINR and power constraints of Problem 4, we establish Lemma 2 to determine whether a pair is admissible or not. We refer to the set of such powers as *admissible areas* (Figure 2).

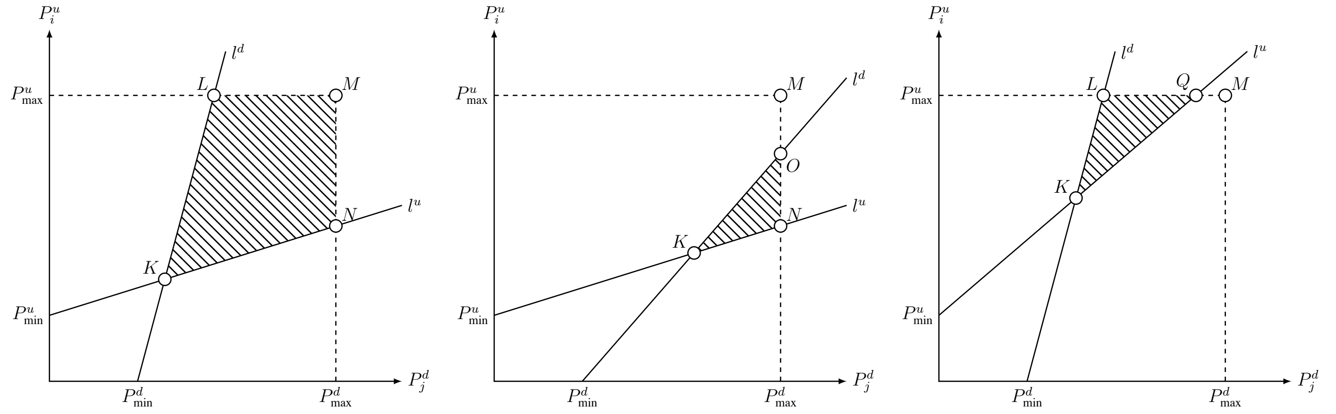
Figure 2 shows three possible areas in terms of $(P_j^d, P_i^u)$ where both the UL and DL users fulfil the minimum SINR requirement. In the figure, the lines $l^u$ and $l^d$ correspond to constraints (4b) and (4c) whereas the rectangular area drawn by the dashed line represents the constraint set $\mathcal{P}$. The points $P_{\text{min}}^u$ and $P_{\text{min}}^d$ fulfil constraints (4b) and (4c) with equality and HD transmission from both users, and can be calculated as

$$P_{\text{min}}^u = \frac{\gamma_{\text{th}}^u \sigma^2}{G_{ibf}}, \qquad P_{\text{min}}^d = \frac{\gamma_{\text{th}}^d \sigma^2}{G_{bjf}}. \tag{6}$$

The point $K$ is the intersection point between the lines $l^u$ and $l^d$, whereas points $L, M, N, O$ and $Q$ help to define the admissible (dashed) area.

Figure 2(a) shows the first area, where $\gamma_{iMf}^u > \gamma_{\text{th}}^u$ and $\gamma_{jMf}^d \geq \gamma_{\text{th}}^d$, with corner points $L$, $M$ and $N$. At corner point $L$, the UL user is at the maximum power $P_{\text{max}}^u$ while the DL user fulfils constraint (4c) with equality, whose coordinates are $(P_{jLf}^d, P_{\text{max}}^u)$ where $P_{jLf}^d$ is given by

$$P_{jLf}^d = \frac{\gamma_{\text{th}}^d(\sigma^2 + P_{\text{max}}^u G_{iLjLf})}{G_{bjLf}}. \tag{7}$$




(a) Area 1 given by $\gamma_{iMf}^u > \gamma_{\text{th}}^u$ and $\gamma_{jMf}^d \geq \gamma_{\text{th}}^d$ (b) Area 2 given by $\gamma_{iMf}^u > \gamma_{\text{th}}^u$ and $\gamma_{jMf}^d < \gamma_{\text{th}}^d$ (c) Area 3 given by $\gamma_{iMf}^u \leq \gamma_{\text{th}}^u$ and $\gamma_{jMf}^d \geq \gamma_{\text{th}}^d$

Figure 2. The 3 admissible areas for a user $i$ in the UL and a user $j$ in the DL to share a frequency channel $f$ that fulfil constraints (4b)-(4e).

Similarly, at corner point $N$ the DL user is at maximum power $P_{\max}^d$ while the UL user fulfils constraint (4b) with equality, whose coordinates are $(P_{\max}^d, P_{iNf})^u$ where $P_{iNf}^u$ is given by

$$P_{iNf}^u = \frac{\gamma_{\text{th}}^u(\sigma^2 + P_{\max}^d \beta)}{G_{iNbf}}. \tag{8}$$

The second area is shown in Figure 2(b), where $\gamma_{iMf}^u > \gamma_{\text{th}}^u$ and $\gamma_{jMf}^d < \gamma_{\text{th}}^d$, with corner points $O$ and $N$. The corner point $N$ is the same as in the first area, but at corner point $O$, the DL user is at maximum power $P_{\max}^d$ and also fulfils constraint (4c) with equality. Thus, the coordinates of point $O$ are $(P_{\max}^d, P_{iOf}^u)$, where $P_{iOf}^u$ is given by

$$P_{iOf}^u = \frac{P_{\max}^d G_{bjOf} - \gamma_{\text{th}}^d \sigma^2}{G_{iOjOf}\gamma_{\text{th}}^d}. \tag{9}$$

Finally, Figure 2(c) shows the third area, where $\gamma_{iMf}^u \leq \gamma_{\text{th}}^u$ and $\gamma_{jMf}^d \geq \gamma_{\text{th}}^d$, with corner points $L$ and $Q$. The corner point $L$ is the same as in the first area, but at corner point $Q$, the UL user is at maximum power $P_{\max}^u$ and fulfils constraint (4b) with equality. Similarly to corner point $O$, the coordinates of point $Q$ are $(P_{jQf}^d, P_{\max}^u)$, where $P_{jQf}^d$ is given by

$$P_{jQf}^d = \frac{P_{\max}^u G_{iQbf} - \gamma_{\text{th}}^u \sigma^2}{\beta \gamma_{\text{th}}^u}. \tag{10}$$

From constraints (4b)-(4e), we can derive the following Lemma that will help us to determine if an admissible area exists for a given pair of UL and DL users. We say that a pair of users is an admissible pair if an associated admissible area exists.

**Lemma 2.** Consider optimization Problem (4). A pair of an UL user $i$ and a DL user $j$ sharing frequency channel $f$ is an *admissible pair* if the residual SI term is such that $\beta \leq \beta_{ijf}^{\max}$, where $\beta_{ijf}^{\max}$ is

$$\beta_{ijf}^{\max} = \begin{cases} \dfrac{G_{bjf}(P_{\max}^u G_{ibf} - \gamma_{\text{th}}^u \sigma^2)}{\gamma_{\text{th}}^u \gamma_{\text{th}}^d (P_{\max}^u G_{ijf} + \sigma^2)}, \text{if } \gamma_{jMf}^d > \gamma_{\text{th}}^d, \\ \dfrac{P_{\max}^d G_{bjf} G_{ibf} - \sigma^2 \gamma_{\text{th}}^d (G_{ijf}\gamma_{\text{th}}^u + G_{ibf})}{\gamma_{\text{th}}^u \gamma_{\text{th}}^d P_{\max}^d G_{ijf}}, \\ \text{if } \gamma_{jMf}^d \leq \gamma_{\text{th}}^d. \end{cases} \tag{11}$$

*Proof.* See Appendix C. ■

Based on the previous result, a pair of UL and DL users is admissible if that pair fulfils Lemma 2. As we will see in the next section, once this is guaranteed, the optimal transmit powers lie in the admissible area to which the pair belongs.

## V. Solution via Lagrange Dual Problem

In this section we analyse JAFM through duality theory, where we first derive an equivalent version of problem (4) in Section V-A, which will help us to mathematically treat the objective function (4a) as a linear function. In Section V-B we form the partial Lagrangian function and derive a closed-form solution for the assignments $\mathbf{X}^u, \mathbf{X}^d$, and in Section V-C we show that the optimal power allocation, given a pair of users reusing frequency channel $f$, is within the admissible areas shown in Section IV. Finally, Section V-D summarizes the insights given by the solution of the dual problem, which are important to develop the greedy approximation of problem (4) on Section VI, named JAFMA.

### A. Problem Transformation

As a first step of solving problem (4), we consider the standard equivalent hypograph [17, Sec. 3.1.7] form of problem (4), where one new variable and two more constraints are included:

$$\begin{aligned} \underset{\mathbf{X}^u, \mathbf{X}^d, \mathbf{p}^u, \mathbf{p}^d, t}{\text{maximize}} \quad & t && \text{(12a)} \\ \text{subject to} \quad & C_i^u \geq t, \forall i, && \text{(12b)} \\ & C_j^d \geq t, \forall j && \text{(12c)} \\ & \text{Constraints (4b)-(4j)} && \text{(12d)} \end{aligned}$$

where $t > 0$ is an additional variable with respect to (4). Notice that problem (12), similarly to problem (4a), is a MINLP. It is possible to relax its integer constraints, but the resulting problem is not convex as established by the following result.

**Result 1.** The continuous relaxation of problem (12), given by letting $\mathbf{X}^u \in [0,1]^{I\times F}$ and $\mathbf{X}^d \in [0,1]^{J\times F}$, is not convex.

*Proof.* See Appendix A. ■

It is important to know that the relaxed problem is not convex, because it shows that relaxing the integer constraint does not lead to a problem that can be solved by conventional solvers and methods, which motivates us to develop an alternative solution approach.

### B. Solution for Assignment Matrices $\mathbf{X}^u, \mathbf{X}^d$

We can now form the *partial* Lagrangian function by considering constraints (12b)-(12d) of problem (12) and ignoring the integer constraints (4f)-(4j). To this end, we introduce Lagrange multipliers $\boldsymbol{\lambda}^u$, $\boldsymbol{\lambda}^d$, $\boldsymbol{\delta}^u$, $\boldsymbol{\delta}^d$ , where the superscript $u$ denotes the dimension of $I$ and $d$ of $J$, respectively. The partial Lagrangian is a function of the Lagrange multipliers and the optimization variables $\mathbf{X}^u, \mathbf{X}^d, \mathbf{p}^u, \mathbf{p}^d, t$ as follows:

$$
\begin{aligned}
L(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}, \mathbf{X}^{u,d}, \mathbf{p}^{u,d}, t) \triangleq\, & t\left(\sum_{i=1}^{I}\lambda_i^u + \sum_{j=1}^{J}\lambda_j^d - 1\right) \\
& + \sum_{i=1}^{I}\left(\delta_i^u\left(\gamma_{\text{th}}^u - \sum_{f=1}^{F}\gamma_{if}^u\right) - \lambda_i^u C_i^u\right) \\
& + \sum_{j=1}^{J}\left(\delta_j^d\left(\gamma_{\text{th}}^d - \sum_{f=1}^{F}\gamma_{jf}^d\right) - \lambda_j^d C_j^d\right), \quad (13)
\end{aligned}
$$

where by a slight abuse of notation we introduce $\boldsymbol{\lambda}^{u,d}$ to denote the two vectors $\boldsymbol{\lambda}^u$ and $\boldsymbol{\lambda}^d$, and we assume $\mathbf{X}^u, \mathbf{X}^d \in \mathcal{X}$, and $\mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}$.

Let $g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d})$ denote the dual function obtained by minimizing the partial Lagrangian (13) with respect to the Lagrange multipliers and variable $t$. Thus, the dual function is

$$
g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}) = \inf_{t\in\mathbb{R}^+\ \mathbf{X}^{u,d}\in\mathcal{X}\ \mathbf{p}^{u,d}\in\mathcal{P}} L(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}, \mathbf{X}^{u,d}, \mathbf{p}^{u,d}, t) \quad (14a)
$$

$$
g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d}) = \begin{cases} \inf_{\mathbf{X}^{u,d}\in\mathcal{X}\ \mathbf{p}^{u,d}\in\mathcal{P}}\Big[\sum_i q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) + \\ \sum_j q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})\Big], & \text{if } \sum_i \lambda_i^u + \sum_j \lambda_j^d = 1 \\ -\infty, & \text{otherwise,} \end{cases} \quad (14b)
$$

where from equality (14b) follows that the linear function $t\big(\sum_{i=1}^{I}\lambda_i^u + \sum_{j=1}^{J}\lambda_j^d - 1\big)$ is lower bounded when it is identically zero, and $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ are

$$
q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) \triangleq \delta_i^u\left(\gamma_{\text{th}}^u - \sum\nolimits_{f=1}^{F}\gamma_{if}^u\right) - \lambda_i^u C_i^u, \quad (15a)
$$

$$
q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) \triangleq \delta_j^d\left(\gamma_{\text{th}}^d - \sum\nolimits_{f=1}^{F}\gamma_{jf}^d\right) - \lambda_j^d C_j^d. \quad (15b)
$$

Notice that we can find the infimum of $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ as

$$
\inf_{\substack{\mathbf{X}^{u,d}\in\mathcal{X}\\ \mathbf{p}^{u,d}\in\mathcal{P}}} q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) = \delta_i^u\left(\gamma_{\text{th}}^u - \gamma_{if^{(i)}}^{u,\max}\right) - \lambda_i^u \log_2\left(1 + \gamma_{if^{(i)}}^{u,\max}\right), \quad (16)
$$

where $f^{(i)} = \arg\max_f \gamma_{if}^{u,\max}$ and $\gamma_{if}^{u,\max} = \max_{\mathbf{p^{u,d}}\in\mathcal{P}} \gamma_{if}^u$. We define $\gamma_{if}^{u,\max}$ as the maximum SINR that UL user $i$ can achieve on frequency resource $f$, where the maximization is done over the power variables $\mathbf{p}^{u,d} \in \mathcal{P}$. Note that $\gamma_{if}^{u,\max}$ is a function of the given assignment matrices $\mathbf{X}^{u,d}$, and it should be evaluated for the DL user that is receiving in frequency resource $f$. Due to constraints (4f)-(4j) on the partial Lagrangian and using Lemma 1, the summations in (15) include a single frequency channel that maximizes the $\gamma_{if}^{u,\max}$ in (16). Then, the chosen $f^{(i)}$ is unique, because a UL user cannot be connected to more than one frequency channel. Based on this, we propose a solution for $x_{if}^u$ and $x_{jf}^d$ given by

$$
x_{if^{(i)}}^{u\star} = \begin{cases} 1, & \text{if } f^{(i)} = \arg\max_f \gamma_{if}^{u,\max} \\ 0, & \text{otherwise,} \end{cases} \quad (17a)
$$

$$
x_{jf^{(j)}}^{d\star} = \begin{cases} 1, & \text{if } f^{(j)} = \arg\max_f \gamma_{jf}^{d,\max} \\ 0, & \text{otherwise.} \end{cases} \quad (17b)
$$

**Remark 1.** Recall we need to find the dual function (14b) of problem (4), which includes the infimum of the partial Lagrangian function (14a). This requires finding the infimum of $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ as in (15), and (16). These infima are obtained at $x_{if}^{u\star}$ and $x_{jf}^{d\star}$, given by Eqs (17), because Eqs. (17) assign to UL and DL users frequency channels $f^{(i)}$and $f^{(j)}$ that maximizes $\gamma_{if}^{u,\max}$ and $\gamma_{jf}^{d,\max}$, thus minimizing $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$.

Note that from Eqs. (17) and constraints (4f)-(4h), we can uniquely associate a user $i$ in the UL with a frequency channel $f$ and do the same with a user $j$ in the DL. However, the solutions are still intertwined through the SINRs $\gamma_{if}^u$ and $\gamma_{jf}^d$. This implies that if an UL user changes its allocation, this modification will impact the DL users, whose modifications are then reflected in the SINRs $\gamma_{if}^u$ and $\gamma_{jf}^d$, which makes the problem of finding the assignment matrix complex. Due to the high complexity of this solution, we use the results from Section V-C and Section V-D to reformulate Eqs. (17) as an NP-Hard problem in Section VI, where we propose a greedy approximation solution whose solution is the assignment matrices searched for in Eqs. (17).

### C. Solution for $\mathbf{p}^u, \mathbf{p}^d$

Since the SINR from the UL is not separable from that of the DL, they must be analyzed jointly. We therefore need to find the powers that jointly minimize expression (14b) $g(\boldsymbol{\lambda}^{u,d}, \boldsymbol{\delta}^{u,d})$, thus we can formulate the subproblem of power setting as

$$
\begin{aligned}
\underset{\mathbf{p}^u,\mathbf{p}^d}{\text{minimize}} \quad & -\sum_{i=1}^{I}\left(\delta_i^u\gamma_{if(i)}^u + \lambda_i^u\log_2(1+\gamma_{if(i)}^u)\right) \\
& -\sum_{j=1}^{J}\left(\delta_j^d\gamma_{jf(j)}^d + \lambda_j^d\log_2(1+\gamma_{jf(j)}^d)\right) \quad (18a) \\
\text{subject to} \quad & \mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}, \quad (18b)
\end{aligned}
$$

where it is assumed that all the frequencies $f(i)$ and $f(j)$ have been assigned without the terms related to $\gamma_{\text{th}}^u$ and $\gamma_{\text{th}}^d$, since these terms are constant and do not affect the problem in (18).

The following lemma reveals an important property of the optimal transmit powers that will be useful in finding the solutions to (18).

**Lemma 3.** Consider optimization Problem 18. The optimal transmit power pair $(P_i^{u\star}, P_j^{d\star})$ of a user pair sharing the same frequency channel has at least one component equal to either $P_{\max}^u$ or $P_{\max}^d$.

*Proof.* Suppose that there is a user $i'$ or $j'$ that do not share the same frequency channel. Then the interference term is zero and the SINR is equal to the signal to noise ratio (SNR) and the maximum power minimizes the objective sum of problem (18) for these users. Alternatively, suppose user $i$ in the UL shares the frequency channel $f$ with user $j$ in the DL.

User $i$ is only associated with user $j$ and vice-versa, since these users use the same frequency channel, i.e., $f(i) = f(j)$. Therefore, the optimization for these two users can be taken independently of all the other users.

The objective function can be redefined as

$$O(P_i^u, P_j^d) \triangleq \frac{\delta_i^u P_i^u G_{ibf}}{\sigma^2 + P_j^d \beta} + \lambda_i^u \log_2\left(1 + \frac{P_i^u G_{ibf}}{\sigma^2 + P_j^d \beta}\right) + \frac{\delta_j^d P_j^d G_{bjf}}{\sigma^2 + P_i^u G_{ijf}} + \lambda_j^d \log_2\left(1 + \frac{P_j^d G_{bjf}}{\sigma^2 + P_i^u G_{ijf}}\right) \quad (19)$$

Thus, we have that for $\alpha \in \mathbb{R}, \alpha > 1$ and $\mathbf{p}^u, \mathbf{p}^d \in \mathcal{P}$:

$$O(\alpha P_i^u, \alpha P_j^d) = \frac{\delta_i^u P_i^u G_{ibf}}{\frac{\sigma^2}{\alpha} + P_j^d \beta} + \lambda_i^u \log_2\left(1 + \frac{P_i^u G_{ibf}}{\frac{\sigma^2}{\alpha} + P_j^d \beta}\right) + \frac{\delta_j^d P_j^d G_{bjf}}{\frac{\sigma^2}{\alpha} + P_i^u G_{ijf}} + \lambda_j^d \log_2\left(1 + \frac{P_j^d G_{bjf}}{\frac{\sigma^2}{\alpha} + P_i^u G_{ijf}}\right)$$

Note that $O(\alpha P_i^u, \alpha P_j^d) > O(P_i^u, P_j^d)$. Therefore, similarly to [18], the optimal power pair $(P_i^{u\star}, P_j^{d\star})$ is bounded by the maximum power constraints $(P_{\max}^u, P_{\max}^d)$, which concludes the proof. ■

From this lemma it follows that we must look for solutions into the two functions of the form $O(P_i^u, P_{\max}^d)$ and $O(P_{\max}^u, P_j^d)$. Next, the following result establishes that the points that maximize $O(P_i^u, P_j^d)$ are in the *corner points* of the constraint set $\mathcal{P}$.

**Lemma 4.** The function $O(P_i^u, P_{\max}^d)$ given in Eq. (19) is convex for positive $P_i^u$ and is maximized at the corner points of the constraint set $\mathcal{P}$.

*Proof.* See Appendix B. ■

Lemma 4 implies that to maximize $O(P_i^u, P_j^d)$ we must look in the corner points of the constraint set $\mathcal{P}$. Moreover, as shown in Section IV, we should look in a subset of $\mathcal{P}$ in order to fulfil the SINR constraints (4b)-(4c), the admissible area of pair $i$ and $j$. Based on the optimal transmit powers and the closed-form solution for the assignment, we will now analyse the Lagrange dual problem and use its implications to develop the greedy approximation of P-JAFM on Section VI.

### D. Insights from the Dual

Once the transmit powers are determined looking for the corner points in the admissible area of the UL-DL pairs as shown in Section IV, we can formulate the Lagrange dual problem to problem (12) as

$$\underset{\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d}}{\text{maximize}} \quad g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d}) \quad (20a)$$

$$\text{subject to} \quad \sum\nolimits_{i=1}^{I} \lambda_i^u + \sum\nolimits_{j=1}^{J} \lambda_j^d = 1, \quad (20b)$$

$$\lambda_i^u,\ \lambda_j^d,\ \delta_i^u,\ \delta_j^d \geq 0, \forall i, j, \quad (20c)$$

where recall that $g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d})$ is the solution of problem (14b). Notice that we can interpret the dual problem above as a weighted sum spectral efficiency maximization of both UL and DL users, where the weights are chosen based on a convex combination of both users' spectral efficiencies.

For every pair, it is necessary to decide which corner point within the admissible area will be used. Since the values of $\boldsymbol{\lambda}^{u,d}$ are tied through Eq. (17) and the SINRs, we need to check all possible combinations of the admissible areas of different pairs. However, by analyzing the structure of the dual function, we note that $\boldsymbol{\delta}^{u,d}$ are lower bounded by 0 and the terms $\delta_i^u\left(\gamma_{\text{th}}^u - \gamma_{if}^u\right)$ and $\delta_j^d\left(\gamma_{\text{th}}^d - \gamma_{jf}^d\right)$ are negative for a pair $(i, j)$ associated with frequency $f$.

Since the dual maximizes $g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d})$ and the terms above are negative, either the SINRs are equal to the minimum ($\gamma_{\text{th}}^u$ or $\gamma_{\text{th}}^d$), or $\boldsymbol{\delta}^{u,d}$ is 0. Thus, the terms with $\boldsymbol{\delta}^{u,d}$ do not impact $g(\boldsymbol{\lambda}^{u,d},\ \boldsymbol{\delta}^{u,d})$. The dual maximizes the negative convex combination of the spectral efficiencies of UL and DL, which results in a value close to 1 for the $\lambda^{u,d}$ related to the user with minimum spectral efficiency in the system, implying that the optimal value of the Lagrange dual problem is close to the negative of the minimum spectral efficiency in the system. Moreover, recall that the closed-form solutions for $\mathbf{X}^{u,d}$ on (17) are still tied through the SINRs $\gamma_{if}^u$ and $\gamma_{jf}^d$, meaning that the solution for the assignment is still complex.

## VI. Approximate Solution via Greedy Method

Given the results on the closed-form solution to the assignment problem (17) the optimal power allocation problem (18) from Section V, we can solve the dual problem by checking exhaustively which pair of UL and DL users jointly solve Eq. (17), where the power allocation for each pair is given by the solution of the admissible areas (see Section V-C). Furthermore, Section V-D showed that the dual problem (20) maximizes the user with minimum spectral efficiency in the system. Therefore, we can solve the dual problem by selecting the pair of UL and DL users that maximize the spectral efficiency of the user with minimum spectral efficiency of the pair.

However, such exhaustive solution demands an extremely high number of iterations that depend on the number of frequency channels, which in practical cellular systems is essentially impossible due to the high number of frequency channels. Thus, in order to use the results from Section V and solve the dual problem (20) in an efficient manner, we reformulate the dual problem (20) on Section VI-A to solve the assignment and propose a greedy approximation on Section VI-B that aims at maximizing the sum of the minimum spectral efficiency of the UL-DL pairs, named JAFMA. Finally, in Section VI-C we discuss the efficiency and complexity of JAFMA when compared to the P-JAFM (4) and dual (20) problems.

### *A. The Dual as a 3-Dimensional Assignment Problem*

As seen in Lemma 2, the SI cancellation term $\beta$ must fulfil $\beta \leq \beta_{ijf}^{\max}$ for every assigned pair of UL user $i$ and DL user $j$ on frequency channel $f$. Thus, an algorithm that aims to solve the JAFM problem needs to identify the UL-DL user *candidates* for every frequency $f$. Notice that due to the minimum SINR constraints (4b) and (4c) in JAFM, a given UL user $i$ or DL user $j$ may not have a pair with which it would form an admissible pair. On the other hand, once a pair fulfils Lemma 2, there is an admissible area where both users fulfil constraints (4b) and (4c).

From the reasoning in Section V-D and using Lemma 4, it follows that we can select the corner point that maximizes the minimum spectral efficiency of the two users in the pair, and thereby maximizes the sum of the minimum spectral efficiencies over all pairs. Note that the power allocation problem (18) formulation comes from minimizing $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d}) + q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ for the admissible pair $(i, j)$. Thus, we can use the observations on maximizing the sum of the minimum spectral efficiency to reformulate the solution for the assignment (17) as an Axial 3-Dimensional Assignment Problem (3-DAP) given by:

$$\begin{aligned}
\underset{\mathbf{X}}{\text{maximize}} \quad & \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{f=1}^{F} s_{ijf}x_{ijf} && \text{(21a)}\\
\text{subject to} \quad & \sum_{j=1}^{J}\sum_{f=1}^{F} x_{ijf} = 1,\ \forall i, && \text{(21b)}\\
& \sum_{i=1}^{I}\sum_{f=1}^{F} x_{ijf} = 1,\ \forall j, && \text{(21c)}\\
& \sum_{i=1}^{I}\sum_{j=1}^{J} x_{ijf} = 1,\ \forall f, && \text{(21d)}\\
& x_{ijf} \in \{0,1\},\ \forall i,j,f. && \text{(21e)}
\end{aligned}$$

The matrix $\mathbf{S} = [s_{ijf}] \in \mathbb{R}^{I\times J\times F}$ represents the minimum spectral efficiency of the pair of users $i$ and $j$ on frequency channel $f$, which is explicitly defined as $s_{ijf} = \min\{C_{if}^u, C_{jf}^d\}$. We define $\mathbf{X} = [x_{ijf}] \in \{0,1\}^{I\times J\times F}$ as the binary assignment matrix. Recall that $C_{if}^u$ and $C_{jf}^d$ depend on the powers, thus we select the power vector from the corner point $(P_j^d, P_i^u)$ that maximizes $s_{ijf}$. The selection of the corner point that maximizes $s_{ijf}$ is according to the solution of problem (18), which uses Lemma 4 to assure that the optimal solution is indeed one of the corner points.

**Remark 2.** From Remark 1 it follows that we can assign UL and DL users frequency channels $f(i)$ and $f(j)$ that maximize $\gamma_{if}^{u,\max}$ and $\gamma_{jf}^{d,\max}$ from the minimization of $q_i^u(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$ and $q_j^d(\mathbf{X}^{u,d}, \mathbf{p}^{u,d})$, and this is precisely the formulation of the Axial 3-DAP (21). Therefore, the solutions to problem (21) are the assignment matrices searched for in equation (17) and together with the optimal power allocation, solves the dual problem (20). For clarity, we rename the Axial 3-DAP (21) problem as the D-JAFM problem to indicate its connection with the dual problem.

The D-JAFM problem has $(n!)^2$ possible assignments and is a well known NP-hard problem [16, Section 8.2]. Therefore, by using the insights in the preceding section, we develop a greedy algorithm to solve D-JAFM and approximate the P-JAFM problem.

### *B. A Greedy Approximation Solution to the JAFM Problem*

Differently from other works that reduce the assignment problem (21) to two-dimensions [12], we aim at jointly assigning the pairs to frequencies. We propose a greedy algorithm with reduced complexity to solve the D-JAFM problem, where using the power allocation from Section V-C we approximate P-JAFM problem. Algorithm 1 shows the steps of the proposed greedy solution, called Joint Fairness Assignment Maximization Algorithm (JAFMA). JAFMA evaluates $\beta_{ijf}^{\max}$ for every user and frequency (see line 5). If the assumptions of Lemma 2 are fulfilled for the pair $(i, j)$ on frequency channel $f$, JAFMA evaluates the powers that maximize the minimum spectral efficiency $s_{ijf}$ based on the admissible area for the pair (see Figure 2 and line 7 of Algorithm 1).

Once we have the $s_{ijf}$ for all the admissible pairs, JAFMA sorts the matrix $\mathbf{S}$ in descending order of the minimum spectral efficiency of users in the pair (see line 14). If the maximum value is not unique, JAFMA selects the pairs that attain this maximum and within these pairs, JAFMA selects the pair with maximum sum of spectral efficiency (see line 17). If the maximum is unique, select the pair and frequency that achieves that maximum (see line 19).

If there is a pair that has not been assigned to a frequency channel, JAFMA checks which user (either UL or DL) has higher spectral efficiency and assigns the frequency channel to it (see lines 22-23). As for the non-selected user, JAFMA accounts it for the disconnected users statistics (see line 24), which will help us to later define the ratio of connected users. At the end, JAFMA removes the assigned users and frequency channels from the matrix $\mathbf{S}$ and continues the loop over all users until all users have been assigned to a frequency channel (see lines 29-30).

When $I + J < 2F$, i.e., when there are more resources than pairs, JAFMA selects the pair with minimum spectral efficiency and *reassigns* the newly available resources to the users in the selected pair starting from the user with minimum spectral efficiency (see line 32). The users will select the frequency channel that gives the highest path gain within the available resources (see line 35 and 38). This loop will continue until all the initially available resources have been

**Algorithm 1** Joint Fairness Assignment Maximization Algorithm (JAFMA)

1: Input: $G_{ibf},\ G_{bjf},\ G_{ijf},\ \beta,\ P^u_{\max},\ P^d_{\max}$
2: **for** $i = 1$ **to** $I$ **do**
3: **for** $j = 1$ **to** $J$ **do**
4: **for** $f = 1$ **to** $F$ **do**
5: Evaluate $\beta^{\max}_{ijf}$ according to Eq. (11)
6: **if** $\beta \leq \beta^{\max}_{ijf}$ (see Lemma 2) **then**
7: Evaluate the corner point $(P^d_j, P^u_i)$ that maximizes $s_{ijf} = \min\{C^u_{if}, C^d_{jf}\}$ (see Section IV )
8: **end if**
9: **end for**
10: **end for**
11: **end for**
12: $\mathcal{I}_a = \mathcal{J}_a = \mathcal{F}_a = \emptyset \leftarrow$ Sets of assigned users and resources are initially empty
13: **while** $\mid \mathcal{I}_a \mid \neq I$ **and** $\mid \mathcal{J}_a \mid \neq J$ **do**
14: $s_{\max} = \max s_{ijf}, \forall i, j, f \leftarrow$ Sort in descending order $s_{ijf}$
15: **if** $s_{\max}$ is not unique **then**
16: $\mathcal{S}_{\max} \leftarrow$ Set of users with minimum spectral efficiency of the pair equals to $s_{\max}$
17: $(i^\star, j^\star, f^\star) = \arg\max_{\mathcal{S}_{\max}} C^u_{if} + C^d_{jf} \leftarrow$ Select the pair and frequency with maximum sum of the spectral efficiency within the set of users $\mathcal{S}_{\max}$
18: **else**
19: $(i^\star, j^\star, f^\star) = \arg\max s_{\max} \leftarrow$ Select the pair and frequency with maximum minimum spectral efficiency
20: **end if**
21: **if** $s_{\max} = 0 \leftarrow$ If this pair is not admissible **then**
22: Assign the frequency channel to the user with maximum spectral efficiency using maximum power (either $P^u_{\max}$ or $P^d_{\max}$)
23: Assign $P^u_{\max}$ or $P^d_{\max}$ for the chosen user
24: Assign 1 in the assignment matrix (either $x^u_{i^\star f^\star}$ or $x^d_{j^\star f^\star}$) to the selected user and 0 to the non-selected
25: **else**
26: $x^u_{i^\star f^\star} = 1$ and $x^d_{j^\star f^\star} = 1$
27: Evaluate $(P^d_{j^\star}, P^u_{i^\star})$ using the admissible areas and the SINRs and spectral efficiencies using Eqs. (1)-(3)
28: **end if**
29: $\mathcal{I}_a = \mathcal{I}_a \cup \{i^\star\},\ \mathcal{J}_a = \mathcal{J}_a \cup \{j^\star\},\ \mathcal{F}_a = \mathcal{F}_a \cup \{f^\star\} \leftarrow$ Update the set of assigned resources
30: Remove the already assigned users and frequency $(i^\star, j^\star, f^\star)$ from $s_{ijf}$
31: **end while**
32: **while** $\mid \mathcal{F}_a \mid \neq F$ **do**
33: $(i^\star, j^\star, f^\star) = \arg\min_{\mathcal{I}_a, \mathcal{J}_a, \mathcal{F}_a}\{C^u_{if}, C^d_{jf}\} \leftarrow$ Get the users and frequency with minimum spectral efficiency within the already assigned
34: $\mathcal{F}_{av} = (\mathcal{F} \setminus \mathcal{F}_a) \cup \{f^\star\} \leftarrow$ New set of available resources
35: Select the frequency channel $f^\dagger$ within $\mathcal{F}_{av}$ that gives the highest desired path gain for the user with minimum spectral efficiency selected ($G_{ibf}$ if UL or $G_{bjf}$ if DL)
36: $\mathcal{F}_a = \mathcal{F}_a \cup \{f^\dagger\} \leftarrow$ Update the set of assigned resources
37: $\mathcal{F}_{av} = (\mathcal{F}_{av} \setminus \{f^\dagger\}) \leftarrow$ Update the set of available resources
38: Select the frequency channel $\hat{f}$ within $\mathcal{F}_{av}$ that gives the highest desired path gain now for the other user that was reusing frequency $f^\star$
39: $\mathcal{F}_a = \mathcal{F}_a \cup \{\hat{f}\} \leftarrow$ Update the set of assigned resources
40: **end while**
41: **Output**: $\mathbf{X}^u, \mathbf{X}^d, \mathbf{p}^u, \mathbf{p}^d$

used. At the end, JAFMA selects the pairs that maximize the minimum spectral efficiency within each pair, which means that JAFMA maximizes the sum of the pairwise minimum spectral efficiencies of the system.

With JAFMA in hand, we can compare its performance with existing algorithms to assign and to allocate power that current system designers use. For the frequency assignment, the simplest solution is a random allocation of frequency channel to UL and DL users [19]. For the power allocation, the common solution is Equal Power Allocation (EPA) with maximum power transmission for UL users and for the BS [20], [21]. In Section VII we will compare the performance of JAFMA with different variations of a random assignment and EPA.

### *C. Discussion*

The complexity of JAFMA relies on sorting in descending order the values of minimum spectral efficiency of every pair on the frequency channel, which can be interpreted as sorting a matrix with dimensions $I \times J \times F$ (see the matrix element $s_{ijf}$ on line 7). With the heap sorting algorithm, the worst-case solution has complexity of $O(n \log n)$ [22], where $n$ is the size of the vector. Since the matrix to be sorted has size $I \times J \times F$, we represent it as a vector of size $IJF$, and assuming that $I = J = F$, the worst-case complexity of the greedy solution is $O(F^3 \log F)$. If compared with the exhaustive solution of D-JAFM that has worst-case complexity of $O(F!^2)$ and considering $F = 10$, JAFMA requires $O(10^3)$ while D-JAFM requires $O(10^{13})$. Although problem (21) has been studied before, to the best of our knowledge, our proposed greedy solution has never been proposed. Other heuristics were developed for this problem [23, Section 10.2.5], but they rely on metaheuristics, graph theory, and on some simplifications of the cost $s_{ijf}$. These other solutions cannot be used in our case, because they either require unacceptable simplifications or are computationally too complex.

**Remark 3.** To analyse the performance of the proposed JAFMA, let us assume that $\mathbf{X}^\star$ and $\mathbf{X}_G$ are the optimal solution of D-JAFM and the one given by JAFMA, respectively, and let the objective function (21a) be $c(\mathbf{X})$. In Hausmann et al. [24, Corollary 4], it is proved in a more general context of problem (21) (intersection of matroids) that any greedy solution, without specifying any particular solution, yields a performance guarantee of $1/3$, i.e., $c(\mathbf{X_G})/c(\mathbf{X}^\star) \geq 1/3$. Therefore, our greedy solution is an *approximated* solution of D-JAFM. Moreover, as we will see in the numerical results on Section VII, the proposed JAFMA has a tighter performance than the guaranteed for any greedy algorithm.

Note that problem (21) aims at solving the closed-form solution of the assignment (17) taking into account the optimal power allocation (see Section V-C) and insights from the dual problem (see Section V-D), which shows that D-JAFM has its basis on the dual problem (20). Therefore, it is necessary to take into account the duality gap between D-JAFM and P-JAFM problems. Recall that the objective function of P-JAFM (4) is the minimum spectral efficiency of UL and DL users.

Based on the works of Weeraddana et al. [25], we show in Proposition 1 that the relative duality gap diminishes when the total number of resources in the system $F$ increases. The concept of relative duality gap is extended to the relative optimality gap, where $d^\star$ can also be the solution given by JAFMA.

**Proposition 1.** Consider $p^\star$ as the optimal solution of problem P-JAFM in (4), and $d^\star$ as the optimal solution of problem D-JAFM in (21), and that the number of UL users, $I$, and

Table II
SIMULATION PARAMETERS

| Parameter | Value |
|---|---|
| Cell radius | 100 m |
| Number of UL UEs $[I = J]$ | [4 19 25] |
| Monte Carlo iterations | 400 |
| Carrier frequency | 2.5 GHz |
| System bandwidth | 5 MHz |
| Number of freq. channels $[F]$ | [25] |
| LOS path-loss model | $34.96 + 22.7 \log_{10}(d)$ |
| NLOS path-loss model | $33.36 + 38.35 \log_{10}(d)$ |
| LOS probability | $\min(18/d, 1)(1-\exp(-d/36)) + \exp(-d/36)$ |
| Shadowing st. dev. LOS | 3 dB |
| Shadowing st. dev. NLOS | 4 dB |
| Thermal noise power $[\sigma^2]$ | $-116.4$ dBm/channel |
| Average user speed | 3 km/h |
| User antenna height | 1.5 m |
| BS antenna height | 10 m |
| SI cancelling level $[\beta]$ | $[-70\ -100]$ dB |
| UE max power $[P^u_{\max}]$ | 24 dBm |
| BS max power $[P^d_{\max}]$ | 24 dBm |
| Minimum SINR $[\gamma^u_{\text{th}} = \gamma^d_{\text{th}}]$ | [0 5] dB |

DL users, $J$, do not increase with the frequency channels. The duality gap $(p^\star - d^\star)/p^\star$ diminishes as the number of frequency channels increases.

*Proof.* See Appendix D. ■

With the characterization of the duality gap and the performance loss between the proposed greedy algorithm and optimal solution of the dual, we recall here that the proposed algorithm JAFMA is an *approximated primal* solution to problem (4).

## VII. NUMERICAL RESULTS

In this section we consider a single cell system operating in the urban micro environment assuming 2.5 GHz carrier frequency and a system bandwidth of 5 MHz [26]–[28]. The maximum number of frequency channels is $F = 25$ that corresponds to the number of available frequency channel blocks in the frequency domain of a 5 MHz Long Term Evolution (LTE) system [27]. The total number of served UE varies between $I + J = 8...50$, where we assume that after UE pairing the number of UE transmitting in UL ($I$) is equal to the number of UE receiving in DL ($J$). The parameters of this system are set according to Table II.

To evaluate the performance of JAFMA in this environment, we use the RUdimentary Network Emulator (RUNE) as a basic platform for system simulations [29] and extended it to FD cellular networks. The RUNE FD simulation tool allows to generate the environment of Table II and perform Monte Carlo simulations using either an exhaustive search algorithm to solve problem (4) or JAFMA.

In order to understand and quantify the gap between the initially proposed JAFM in (4) (P-JAFM), the dual problem converted to Axial 3-DAP in (21) (D-JAFM) and the greedy solution named JAFMA, we analyse the relative duality gap between the three algorithms in Section VII-A. In Sections VII-B and VII-C we compare the performance of

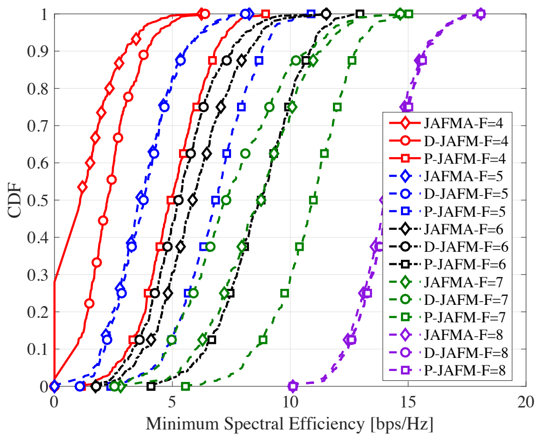



Figure 3. CDF of the minimum spectral efficiency among all users. We notice that as we increase the number of frequency channels in the system, the gap between JAFM and P-JAFM diminishes, where in the 50th percentile this relative gap is approximately 1 %.

JAFMA for different users' loads and SI cancelling levels, respectively, with three other algorithms:

1) Random assignment with EPA, named R-EPA;
2) Assignment according to JAFMA but with EPA, named AF-EPA;
3) Random assignment with power allocation according to JAFMA, named R-FMA.

Recall that other algorithms previously proposed for rate maximization, e.g. [12], are not considered here because our scenario does not suit the solution proposed by them. Although the sum rate maximization and fairness trade-off is not analysed, we aim to solve an essential problem, which is how to achieve fairness and how fair can we be using full-duplex communications. For a careful analysis of the rate-fairness trade-off in general wireless networks, see for example [30].

### A. Analysis of Optimality Gap

To evaluate the optimality gap between the proposed JAFMA, P-JAFM and D-JAFM, we evaluate the objective function of problem (4), i.e., the minimum spectral efficiency of UL and DL users.

Recall from Section III-B and Section VI-B that the P-JAFM and D-JAFM are NP-hard, thus we solve them by exhaustive search for scenarios with a small number of users and frequency channels. Specifically, for P-JAFM and D-JAFM, we consider a small system with reduced number of users, 4 UL and DL users, and frequency channels, where we increase its number from 4 to 8. Moreover, we consider a SI cancelling level of $-100$ dB, i.e., with $\beta = -100$ dB and $\gamma^u_{\text{th}} = \gamma^d_{\text{th}} = 0$ dB.

Figure 3 shows the CDF of minimum spectral efficiency among all UL and DL users, which is the objective function of problem (4), and an important performance indicator of cellular networks. At full load, when all users need to use FD transmission, the relative gap between JAFMA and P-JAFM is approximately 69 % at the 50th percentile. Similarly, the gap between D-JAFM and P-JAFM is approximately 57 %.

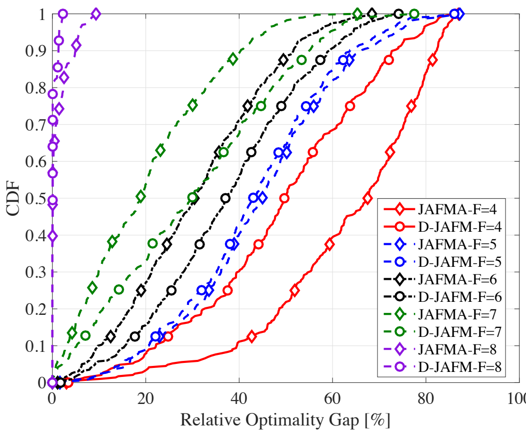



Figure 4. CDF of the relative optimality gap of JAFMA and D-JFMA with P-JFMA. We clearly see that the optimality gap diminishes when the number of frequency channels is increased, where in 57 % of the cases the gap is approximately zero for JAFMA.

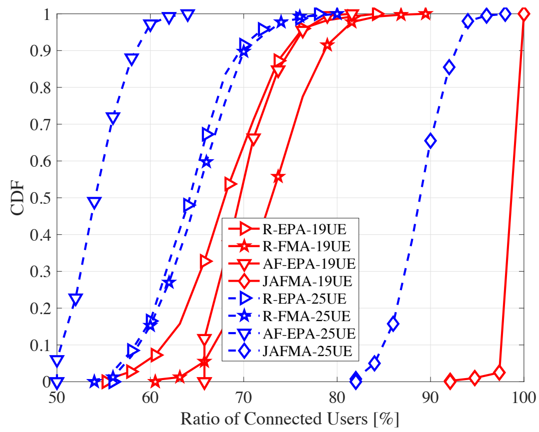



Figure 5. CDF of the ratio of connected users in the system for different users' load. Notice that JAFMA guarantees connection to at least 92 % in a system with 19 UL and DL users and 82 % in a system with 25 UL and DL users. Thus, JAFMA is able to maintain a high ratio of connected users although the system is completely loaded.

Moreover, notice that in approximately 30 % of the Monte Carlo iterations at least one user was disconnected. However, the number of resources increases, both JAFMA and D-JAFM get closer to P-JAFM: approximately 1 % for JAFMA and 0.6 % for D-JAFM at the 50th percentile.

Figure 4 shows the CDF of the relative optimality gap for the same configuration as that used in Figure 3, and here we highlight how close JAFMA is to the optimal solution achieved by P-JAFM. At high load the relative optimality gap is approximately 67 % for JAFMA and 50 % for D-JAFM at the 50th percentile. When the number of channels is increased to 8, in 59 % of the cases the gap is approximately zero for JAFMA and at most 14 %. These figures clearly show that the optimality gap diminishes with an increasing number of resources as proved in Appendix D.

### *B. Fairness Performance Analysis for Different Users' Loads*

In this subsection we study the performance of JAFMA and compare it with R-EPA, AF-EPA and R-FMA in terms of the achieved ratio of connected users and the achieved fairness measured by the well known Jain's fairness index [7]. To characterize the fairness achieved by this system, we employ a modified versions of Jain's index that is weighed by the number of connected users [7]. In simple terms, the modified Jain's fairness index punishes the algorithms that disconnect users rather than considering the connected users only, which is given by

$$J_{mod} = \left(1 - \frac{N_d}{I+J}\right) \frac{\left(\sum_k^{I+J} C_k\right)^2}{(I+J)\sum_k^{I+J} C_k^2}, \tag{22}$$

where $C_k$ is the spectral efficiency of user $k$ (either UL or DL) and $N_d$ is the number of disconnected users. Note that $N_d$ represents how many users, within the ones that are scheduled to transmit, cannot communicate with the BS fulfilling a minimum SINR threshold $\gamma_{\text{th}}$.

We analyse how the fairness of the system changes with different users' load considering $\gamma_{\text{th}}^u = \gamma_{\text{th}}^d = 5\,\text{dB}$ and a SI cancelling level of $-70\,\text{dB}$, i.e., with $\beta = -70\,\text{dB}$. Recall that the SINR threshold $\gamma_{\text{th}}$ also impacts the number of disconnected users. Due to this sensitivity, we consider a high minimum SINR threshold, which allows us to evaluate if full-duplex communications can guarantee such threshold.

Figure 5 shows the CDF of the ratio of connected users, where 100 % means that all users are connected and use a frequency channel. Notice that for 19 UL and DL users the percentage of connected users with JAFMA is at least 92 % (lowest ratio of connected users in the right axis), while for AF-EPA is at least 66 %, for R-FMA is at least 61 % for R-EPA is at least 55 %. Notice that R-FMA crosses AF-EPA, meaning that the power allocation for pairs randomly assigned can improve connection of the users, instead of assigning them in a proper manner and allocating maximum power. For 25 UL and DL users, the performance of all algorithms is degraded, but JAFMA still shows superior performance guaranteeing connection to at least 82 % of all users. In contrast, AF-EPA has at least 50 %, R-FMA has at least 54 % and R-EPA at least 56 % connected users. Notice that R-EPA has a higher connection ratio than AF-EPA, meaning that considering only the assignment of JAFMA should not be taken into consideration alone, but also with the power allocation. Moreover, R-FMA crosses R-EPA, but they are still close, showing that in a high load the gain of optimizing the power allocation for a pair randomly allocated is negligible. This result shows that JAFMA maintains a high ratio of connected users although the system is fully loaded and that the fairness assignments and power allocation of JAFMA should not be split and merged with other techniques.

Figure 6 shows the CDF of modified Jain's fairness index among all UL and DL users. Notice that for 19 UL and DL users the relative difference is already clear between JAFMA, AF-EPA and R-FMA, approximately 23 % at the 50th percentile. The performance between AF-EPA and R-FMA is similar for at least 50 % of the cases (see above 50th percentile), thus regardless of how good the assignment or power allocation are, if they are taken into consideration alone they will have the same effect in the fairness. As expected,

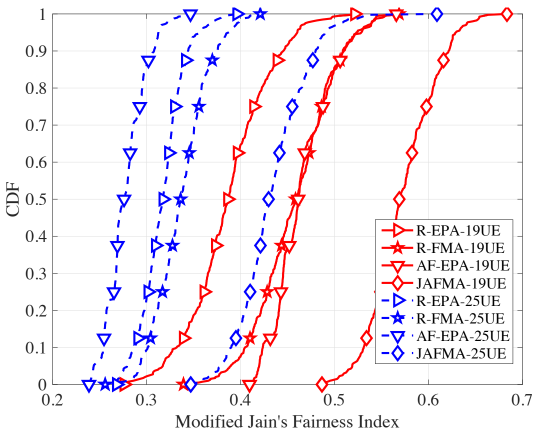



Figure 6. CDF of the modified Jain's fairness index among all UL and DL users for different users' load. We notice that as we increase the number of users JAFMA increases its relative difference to AF-EPA and R-FMA, with 35 % at the 50th percentile.

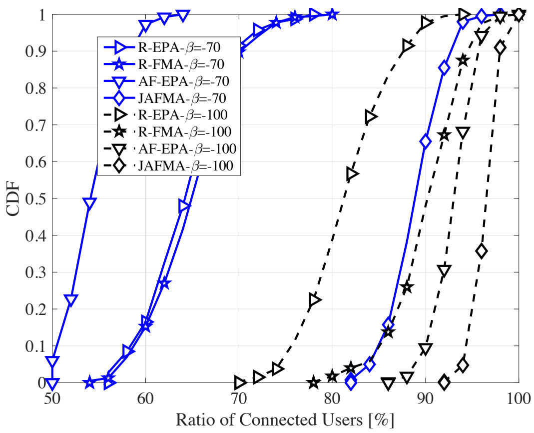



Figure 7. CDF of the ratio of connected users in the system for different values of $\beta$. Notice that JAFMA guarantees connection to at least 82 % in a system with high SI level, i.e., JAFMA still guarantees a high connection ratio to users in system with severe SI.

R-EPA has poor performance, not only because of the random assignment of frequency channels but also because of using EPA with maximum power, which increases the SI in the UL. As the load of the system increases to 25 UL and DL users, the relative difference between JAFMA and AF-EPA increases to 35 % at the 50th percentile. JAFMA achieves the highest modified Jain's fairness index for different users' load, meaning that not only the fairness of the system is maintained but also the ratio of connected users is improved. Again, we notice that considering only one feature of JAFMA, either fairness assignment or power allocation, does not perform much better than a simple random allocation with maximum power, thus JAFMA should be used as it is.

### C. *Fairness Performance Analysis for SI Cancelling Levels*

It is important to understand the impact of SI on the performance of JAFMA To this end, we now assume a high user load with 25 UL and DL users and vary $\beta$ as $-70\,$dB and $-100\,$dB, i.e., from a system with severe SI level to a modest SI level.

Figure 7 shows the CDF of the ratio of connected users for

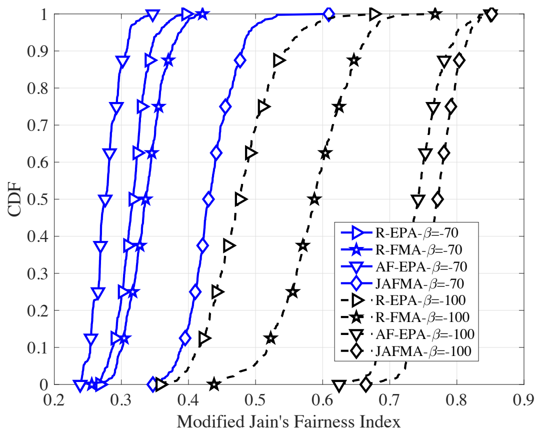



Figure 8. CDF of the modified Jain's index among all UL and DL users for different SI cancelling levels. We notice that as $\beta$ increases, the relative difference between JAFMA and AF-EPA also increases.

different SI cancelling levels, i.e., for different values of $\beta$. Notice that when $\beta$ is $-100\,$dB, JAFMA guarantees connection to at least 92 % of all users, while AF-EPA has at least 86 %, R-FMA has at least 78 % and R-EPA guarantees at least 70 %. The difference between AF-EPA and R-FMA is more clear now, which shows that with modest SI cancelling there is an improvement in optimizing only the assignment instead of only the power allocation. When $\beta$ increases to $-70\,$dB, all algorithms guarantee connection to fewer users than before. However, JAFMA clearly outperforms the other algorithms, guaranteeing connection to at least 82 % of all users, while AF-EPA decreases to 50 %, R-FMA decreases to 55 % and R-EPA to 56 %. Again, we see that considering only the assignment or power allocation of JAFMA, the performance is worse or the same as using a random assignment, as the performance of AF-EPA and R-FMA show.

Figure 8 shows the modified Jain's fairness index and we notice that for $\beta$ =$-100\,$dB the relative difference between JAFMA and AF-EPA is approximately 4 % while between JAFMA and R-EPA is 62 % at the 50th percentile. Notice that as expected from Figure 7, there is a relative gain of approximately 26 % of optimizing the assignment instead of the power allocation, as it is shown by the difference between AF-EPA and R-FMA at the 50th percentile. However, when we increase $\beta$ to $-70\,$dB, the relative difference between JAFMA and AF-EPA at the 50th percentile is approximately 56 % and higher than the relative difference between JAFMA and R-EPA, which is approximately 35 % and the relative difference between JAFMA and R-FMA (approximately 28 %). Notice that as $\beta$ increases, the relative difference between JAFMA with AF-EPA and R-FMA increase, where AF-EPA becomes worse than R-EPA and R-FMA becomes close to R-EPA. Therefore, JAFMA should be taken into account as a joint algorithm for different SI levels, where neither the assignment nor the power allocation should be split to create new techniques.

## VIII. Conclusion

In this paper we considered the joint problem of user pairing and frequency channel selection in FD cellular networks. Specifically, our objective was to maximize the minimum spectral efficiency of the user with the lowest achieved spectral efficiency. This problem was posed as a mixed integer nonlinear optimization, called JAFM, which was shown to be NP-hard. We resorted to Lagrangian duality and developed a closed-form solution for the assignment and found the optimal power allocation. Since the closed-form solution could not be solved explicitly, we used duality theory, combined with a greedy solution algorithm, JAFMA, to approximate the primal problem. We derived the duality gap between JAFMA and the primal problem, and we showed that JAFMA has a guaranteed performance, which allowed us to denominate as a greedy approximation to the primal problem. The numerical results showed that JAFMA improved the spectral efficiency of the users with low spectral efficiency and achieved the highest ratio of connected users in a wide range of scenarios.

The results also indicated that the optimization of the assignment and power allocation should be solved jointly; otherwise, a random allocation with EPA achieves a similar performance. In future works, we intend to study distributed schemes to jointly assign UL and DL users to frequency channels, to inspect in details the rate-fairness trade-off, and to analyse the impact of multi-cell interference in the fairness of FD systems.

## Appendix A
## Proof of Result 1

To check if the problem (12) is convex, we need to preliminary check that all the inequality constraints are convex, thus we start with constraint (12b). Suppose that user $i$ in the UL is interfering with user $j$ in the DL, where user $i$ is transmitting only on frequency channel $f$, i.e., $x^u_{if} = 1$. Thus, constraint (12b) can be rewritten as

$$(2^t - 1)(\sigma^2 + \sum\nolimits_{j=1}^{J} x^d_{jf} P^d_j \beta) - P^u_i G_{ibf} \leq 0. \tag{23}$$

If the Hessian of inequality (23) with respect to variables $t, x_{jf}, P^u_i, P^d_j$ is positive semi-definite, then problem (12) is convex. However, the second leading minor is not positive, which implies that the matrix is not positive semi-definite, i.e., constraint (23) is not convex. Since we proved that a simpler version of the relaxed problem is not convex, the general version cannot be convex either.

## Appendix B
## Proof of Lemma 4

We take the first derivative of $O(P^u_i, P^d_{\text{max}})$ with respect to $P^u_i$, which is given by

$$\frac{\partial O(P^u_i, P^d_{\text{max}})}{\partial P^u_i} = \frac{\delta^u_i G_{ibf}}{\sigma^2 + P^d_{\text{max}}\beta} - \frac{\delta^d_j P^d_{\text{max}} G_{ijf} G_{bjf}}{(\sigma^2 + P^u_i G_{ijf})^2} + \frac{\lambda^u_i G_{ibf}}{\ln(2)(P^u_i G_{ibf} + P^d_{\text{max}}\beta + \sigma^2)} - \frac{\lambda^d_j P^d_{\text{max}} G_{ijf} G_{bjf}}{\ln(2)(P^u_i G_{ijf} + P^d_{\text{max}} G_{bjf} + \sigma^2)(\sigma^2 + P^u_i G_{ijf})} \tag{24}$$

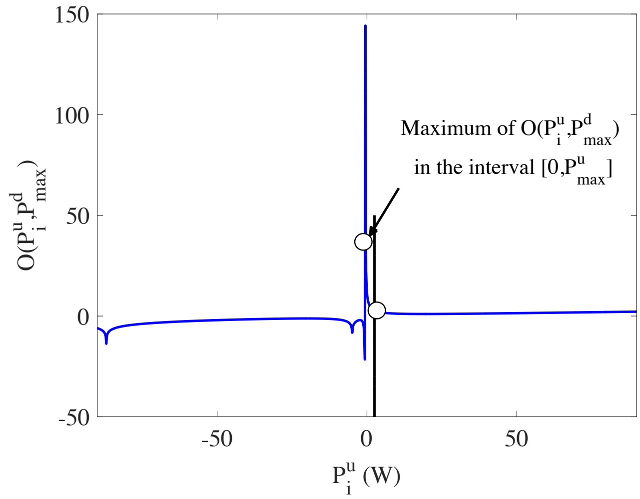



Figure 9. Illustrative plot of $O(P^u_i, P^d_{\text{max}})$ that shows the possible maxima of the function and its transitions in the poles.

We notice from Eq. (24) that the derivative has three distinct poles all of which are located at negative powers, namely $P^u_{i,l}$ tends to $\infty$ for $l = 1, 2, 3$, since the function is continuous for positive powers, tends to infinity as $P^u_i$ tends to infinity, and is given by the sum of a strictly decreasing function and a strictly increasing function, thus it must have a U shape for positive $P^u_i$. These are inflection points for the derivative. Therefore, the function must look like Figure 9, and thus for $P^u_i > 0$ the function is convex and it is maximized in the corner points of the interval $[0, P^u_{\text{max}}]$.

## Appendix C
## Proof of Lemma 2

In Figure 2, we can find the coordinates of point $K$ $(P_{jK}, P_{iK})$ by solving the system of equations

$$\frac{P^u_{iK} G_{ibf}}{\sigma^2 + P^d_{jK}\beta} = \gamma^u_{\text{th}}, \qquad \frac{P^d_{jK} G_{bjf}}{\sigma^2 + P^u_{iK} G_{ijf}} = \gamma^d_{\text{th}}, \tag{25}$$

which gives the following coordinates

$$P^u_{iK} = \frac{\gamma^u_{\text{th}}\sigma^2(G_{bjKf} + \beta\gamma^d_{\text{th}})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma^u_{\text{th}}\gamma^d_{\text{th}}}, \tag{26a}$$

$$P^d_{jK} = \frac{\gamma^d_{\text{th}}\sigma^2(G_{iKjKf}\gamma^u_{\text{th}} + G_{iKbf})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma^u_{\text{th}}\gamma^d_{\text{th}}}. \tag{26b}$$

However, the point $K$ needs to be constrained to the boxed region defined by the power constraints, thus we get the following inequalities

$$0 < \frac{\gamma^u_{\text{th}}\sigma^2(G_{bjKf} + \beta\gamma^d_{\text{th}})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma^u_{\text{th}}\gamma^d_{\text{th}}} \leq P^u_{\text{max}}, \tag{27a}$$

$$0 < \frac{\gamma^d_{\text{th}}\sigma^2(G_{iKjKf}\gamma^u_{\text{th}} + G_{iKbf})}{G_{bjKf}G_{iKbf} - \beta G_{iKjKf}\gamma^u_{\text{th}}\gamma^d_{\text{th}}} \leq P^d_{\text{max}}. \tag{27b}$$

Inspired by [31] and using inequalities (27), we can derive now an upper bound for the SI cancellation term for any two users $i$ and $j$ that share a frequency channel $f$ and fulfil constraints (4b)-(4e), i.e., any user in the area with parallel lines defined by points above $K$ and within the boxed area. The bound is $\beta \leq \beta_{ijf}^{\max}$, where $\beta_{ijf}^{\max}$ is given by

$$\beta_{ijf}^{\max} = \begin{cases} \beta_u \triangleq \dfrac{G_{bjf}(P_{\max}^u G_{ibf} - \gamma_{\text{th}}^u \sigma^2)}{\gamma_{\text{th}}^u \gamma_{\text{th}}^d (P_{\max}^u G_{ijf} + \sigma^2)}, \\ \beta_d \triangleq \dfrac{P_{\max}^d G_{bjf} G_{ibf} - \sigma^2 \gamma_{\text{th}}^d (G_{ijf}\gamma_{\text{th}}^u + G_{ibf})}{\gamma_{\text{th}}^u \gamma_{\text{th}}^d P_{\max}^d G_{ijf}}. \end{cases} \tag{28}$$

Thus, $\beta \leq \beta_{ijf}^{\max} = \min\{\beta_u, \beta_d\}$. To understand when $\beta_u$ and $\beta_d$ are chosen, we see that $\beta_u > \beta_d$ when $\gamma_{jMf}^d \leq \gamma_{\text{th}}^d$, i.e., $\min\{\beta_u, \beta_d\} = \beta_d$. In the other case, $\beta_u \leq \beta_d$ when $\gamma_{jMf}^d > \gamma_{\text{th}}^d$, i.e., $\min\{\beta_u, \beta_d\} = \beta_u$. Therefore,

$$\beta_{ijf}^{\max} = \begin{cases} \beta_u, \text{if } \gamma_{jMf}^d > \gamma_{\text{th}}^d, \\ \beta_d, \text{if } \gamma_{jMf}^d \leq \gamma_{\text{th}}^d. \end{cases} \tag{29}$$

Therefore, a pair of UL and DL users is admissible on frequency channel $f$ if the evaluated $\beta_{ijf}$ fulfils inequality $\beta \leq \beta_{ijf}^{\max}$.

## Appendix D
## Proof of Proposition 1

Based on the work of Weeraddana et al. [25], the proof of the duality gap relies on three assumptions of an equivalent version of problem (12). Let us define it as

$$\text{minimize} \quad -\sum_{f=1}^{F} \min\{t_f^u, t_f^d\} \tag{30a}$$

$$\text{subject to} \quad \sum_{f=1}^{F} C_{if}^u \geq \sum_{f=1}^{F} t_f^u, \forall i, \tag{30b}$$

$$\sum_{f=1}^{F} C_{jf}^d \geq \sum_{f=1}^{F} t_f^d, \forall j \tag{30c}$$

$$\text{Constraints (4b)-(4j)} \tag{30d}$$

$$0 \leq t_f^u \leq t_{\max}^u + C_{n_f^u f}^u, \forall f \tag{30e}$$

$$0 \leq t_f^d \leq t_{\max}^d + C_{n_f^d f}^d, \forall f \tag{30f}$$

where the variables are $\mathbf{t}^{u,d}$, $\mathbf{X}^{u,d}$ and $\mathbf{p}^{u,d}$. We define $n_f^u = \arg\max_{i\in\mathcal{I}} C_{if}^u$ and $n_f^d = \arg\max_{j\in\mathcal{J}} C_{jf}^d$, with $t_{\max}^u, t_{\max}^d < \infty$ as upper bound of both optimal solution components $t_f^{u\star}$ and $t_f^{d\star}$ of problem (30). For example, we use $t_{\max}^u = \max_{i\in\mathcal{I}, f\in\mathcal{F}} C_{if}^u$ and $t_{\max}^d = \max_{j\in\mathcal{J}, f\in\mathcal{F}} C_{jf}^u$ throughout this article. Similarly to [25], we can show that problem (30) is equivalent to the original MINLP (12) and the optimal value $P^\star$ of (30) is equal to the optimal value $p^\star$ of MINLP (12), i.e., $P^\star = p^\star$.

From now on, we refer to problem (30) as the *modified MINLP* from (12), which is in a similar form of Proposition 4 from [25] (indices $i$ and $j$ have a different meaning in that work), where

1) $\mathcal{J} = \mathcal{F}$ and $J = F$;
2) $\mathbf{y}_f = (\mathbf{z}_f^u, \mathbf{z}_f^d, \mathbf{p}^u, \mathbf{p}^d, t_f^u, t_f^d) \in \mathbb{R}^y$, with $\mathbf{z}_f^u = (x_{if}^u)_{i\in\mathcal{I}}$, $\mathbf{z}_f^d = (x_{jf}^d)_{j\in\mathcal{J}}$ and $y = 2(I+J+1)$;
3) $f_f(\mathbf{y}_f) = -\sum_{f=1}^{F} \min\{t_f^u, t_f^d\}$ ;
4) $\mathcal{Y}_f = \Big\{ \Big( (x_{if}^u)_{i\in\mathcal{I}}, (x_{jf}^d)_{j\in\mathcal{J}}, (P_i^u)_{i\in\mathcal{I}}, (P_j^d)_{j\in\mathcal{J}}, t_f^u, t_f^d \Big) \ \Big|\ \sum_{i=1}^{I} x_{if}^u \leq 1, \sum_{j=1}^{J} x_{jf}^d \leq 1,\ \forall f, x_{if}^u,\ x_{jf}^d \in \{0,1\}, P_i^u \leq P_{\max}^u, \forall i, P_j^d \leq P_{\max}^d, \forall j,\ t_f^u \in \big[0, t_{\max}^u + C_{n_f^u f}^u\big]\ t_f^d \in \big[0, t_{\max}^d + C_{n_f^d f}^d\big] \Big\}$
5) $\mathbf{h}_f^1(\mathbf{y}_f) = \Big(t_f^u - C_{1f}^u, \dots, t_f^u - C_{n_f^u f}^u, \dots, t_f^u - C_{If}^u\Big) \in \mathbb{R}^{Q_1}$, with $Q_1 = I$,
$\mathbf{h}_f^2(\mathbf{y}_f) = \Big(t_f^d - C_{1f}^d, \dots, t_f^d - C_{n_f^d f}^d, \dots, t_f^d - C_{Jf}^d\Big) \in \mathbb{R}^{Q_2}$, with $Q_2 = J$,
$\mathbf{h}_f^3(\mathbf{y}_f) = \Big(-\gamma_{1f}^u, \dots, -\gamma_{n_f^u f}^u, \dots, -\gamma_{If}^u\Big) \in \mathbb{R}^{Q_3}$, with $Q_3 = I$,
$\mathbf{h}_f^4(\mathbf{y}_f) = \Big(-\gamma_{1f}^d, \dots, -\gamma_{n_f^d f}^d, \dots, -\gamma_{Jf}^d\Big) \in \mathbb{R}^{Q_4}$, with $Q_4 = J$;
6) $\mathbf{b}^1 = \mathbf{b}^2 = 0$ and $\mathbf{b}^3 = \Big(-\gamma_{\text{th}}^u, \dots, -\gamma_{\text{th}}^u\Big) \in \mathbb{R}^{Q_3}, \mathbf{b}^4 = \Big(-\gamma_{\text{th}}^d, \dots, -\gamma_{\text{th}}^d\Big) \in \mathbb{R}^{Q_4}$.

Assumptions 1 and 2 from [25] can be proved without problems, but not Assumption 3. Since we have 4 different inequality constraints, we need to prove that every one holds, because we can imagine a concatenation of the inequality constraints with dimension $Q_1 + Q_2 + Q_3 + Q_4$ that fulfils Assumption 3.

Let $\tilde{\mathbf{y}}$ be any vector in conv($\mathcal{Y}_f$) and let us first analyse the constraint related to $\mathbf{h}_f^1(\mathbf{y}_f)$. From the definition of $\tilde{\mathbf{h}}(\tilde{\mathbf{y}})$ [25, Eq. (28)], we can express it as $\tilde{\mathbf{h}}_f^1(\tilde{\mathbf{y}}) = \sum_{k=1}^{y+1} \alpha^k \mathbf{h}_f^1(\mathbf{y}^k)$, for some $\mathbf{y}^k \in \mathcal{Y}_f$ and $\alpha^k$ such that $\tilde{\mathbf{y}} = \sum_{k=1}^{y+1} \alpha^k \mathbf{y}$, $\sum_{k=1}^{y+1} \alpha^k = 1$, $\alpha^k \geq 0$. For problem (12), we have

$$\tilde{\mathbf{h}}_f^1(\tilde{\mathbf{y}}) = \sum_k \alpha^k \Big(t_f^{u,k} - C_{1f}^{u,k}, \dots, t_f^{u,k} - C_{n_f^u f}^{u,k}, \dots, t_f^{u,k} - C_{If}^{u,k}\Big), \tag{31a}$$

$$= \sum_k \Big(\alpha^k\ t_f^{u,k} - \alpha^k\ C_{1f}^{u,k}, \dots, \alpha^k\ t_f^{u,k} - \alpha^k\ C_{n_f^u f}^{u,k}, \dots, \alpha^k\ t_f^{u,k} - \alpha^k\ C_{If}^{u,k}\Big), \tag{31b}$$

$$= \sum_k \Big(\alpha^k\ t_f^{u,k} - 0, \dots, \alpha^k\ t_f^{u,k} - \alpha^k\ C_{n_f^u f}^{u,k}, \dots, \alpha^k\ t_f^{u,k} - 0\Big), \tag{31c}$$

$$\geq \sum_k \Big(0, \dots, -\alpha^k\ C_{n_f^u f}^{u,k}, \dots, 0\Big), \tag{31d}$$

$$\geq \Big(0, \dots, -C_{n_f^u f}^{u}, \dots, 0\Big), \tag{31e}$$

$$= \mathbf{h}_f^1(\mathbf{y}), \tag{31f}$$

where $\mathbf{z}_f^u = (0, \dots, 1, \dots, 0)_{i\in\mathcal{I}}$ and $t_f^u = 0$, with the other vectors of the solution $\mathbf{y}$ defined in accordance with any feasible solution. Note that (31c) follows from the assumption that a feasible solution for $x_{if}^u$ is to assign the user $n_f^u = \arg\max_{i\in\mathcal{I}} C_{if}^{u,k}$, which leaves $x_{if}^u = 0$ for $i \neq n_f^u$ and if we take the minimum value of $t_f$, we will have (31d). Moreover, since $\sum_{k=1}^{y+1} \alpha^k = 1$, $\alpha^k \geq 0$, we have that $-\alpha^k\ C_{n_f^u f}^{u,k} > -C_{n_f^u f}^{u,k}$. Thus, we have proved that

$\tilde{\mathbf{h}}_f^1(\tilde{\mathbf{y}}) \geq \mathbf{h}_f^1(\mathbf{y})$, and a similar proof can be done for $\mathbf{h}_f^2$.

Now, we have that for $\mathbf{h}_f^3$

$$\tilde{\mathbf{h}}_f^3(\tilde{\mathbf{y}}) = \sum\nolimits_k \alpha^k \Big( -\gamma_{1f}^{u,k}, \ldots, -\gamma_{n_f^u f}^{u,k}, \ldots, -\gamma_{If}^{u,k} \Big), \quad (32a)$$

$$= \sum\nolimits_k \Big( -\alpha^k 0, \ldots, \alpha^k - \gamma_{n_f^u f}^{u,k}, \ldots, -\alpha^k 0 \Big), \quad (32b)$$

$$\geq \Big( 0, \ldots, -\gamma_{n_f^u f}^{u}, \ldots, -0 \Big), \quad (32c)$$

$$= \mathbf{h}_f^3(\mathbf{y}), \quad (32d)$$

and in a similar manner as before, we have proved that $\tilde{\mathbf{h}}_f^3(\tilde{\mathbf{y}}) \geq \mathbf{h}_f^3(\mathbf{y})$. The same idea can be applied to $\mathbf{h}_f^4(\mathbf{y})$ and therefore, all inequalities fulfil Assumption 3 of [25].

Therefore, since we proved that Assumptions 1-3 hold for the modified MINLP (30) and together with Proposition 4, we have

$$p^\star - d^\star \leq (2(I+J)+1)\Big( \min\{t_{\max}^u, t_{\max}^d\} + \rho_f \Big), \quad (33)$$

where the inequality follows from $\sup_f f(y_f) = 0$ and $\inf_f f(y_f) = -\min\Big\{ \max_{i,f} C_{if}^u, \max_{j,f} C_{jf}^d \Big\}$. Inequality (33) ensures that the numerator of the relative duality gap is bounded above by a fixed number, due to power constraints the maximum spectral efficiency is bounded above, which is dependent of the total number of frequency channels $F$, and because $I$ and $J$ do not increase with $F$. Moreover, we note that when $F \to \infty$, $p^\star$ increases due to more resources available. Therefore, we conclude that the relative duality gap $(p^\star - d^\star)/p^\star$ diminishes as $F \to \infty$.

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