Measurement and valuation of country risk: how to get a right value?

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Supervisor at PwC, France: Karen Adler
Supervisor at KTH: Ali Mohammadi
Examiner: Boualem Djehiche

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Royal Institute of Technology
School of Engineering Sciences
KTH SCI
SE-100 44 Stockholm, Sweden
URL: www.kth.se/sci
Abstract:

The purpose of this master thesis is to focus on country risk and its quantification as a premium. Country risk is an important parameter for investors willing to invest abroad and especially in emerging countries. Indeed, there is additional risk to invest in such countries for numerous reasons. It is thus imperative to be able to quantify it. The actual state of the art about this topic is still at its beginning.

In this master thesis, I have developed two axis of reflection to get a country risk premium. The first one derives from the Capital Asset Pricing Model and related corporate finance theory. The second axis is based on a more mathematical approach.

In the end, I have managed to have a quantified results with those two approaches. They are converging for both methods.

I have applied my results with case studies on two countries: Sweden and Mexico.
Abstrakt:

Syftet med detta examensarbete är att fokusera på landrisken och dess kvantifiering som en premie. Landrisken är en viktig parameter för investerare som är villiga att investera utomlands och i synnerhet i tillväxtländerna. I själva verket finns det ytterligare risk att investera i dessa länder på grund av flera skäl. Det är därför viktigt att kunna kvantifiera det. Forskningen om detta ämne är för närvarande fortfarande i ett begynnelsesstadium.


I slutändan har jag lyckats få ett kvantifierat resultat med dessa två synsätt. De konvergerande för båda metoderna.

Jag har kommit fram till mina resultat genom fallstudier på två länder: Sverige och Mexiko"
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Introduction

We live in a more and more globalized world. International trade is now a common thing and big companies are generally implanted over the five continents. It means, from the perspective of an investor, that there are worldwide investment possibilities. It is thus of major importance to be able to evaluate the risk of investing in a country compared to the same investment in an other country. This risk is the country risk. The oil crisis of the 1970’s as well as the worldwide economic turmoil make economists, researchers and financial practitioners aware of the necessity of a global risk factor for the management of firms and organizations. This is also valuable for the economic development of countries (Kosmidou, Doumpos, Zopounidis 2008). For this reason, the researches about country risk have been a central topic in economy and finance since three decades. These researches were mainly focused on the economic and financial difficulties of countries, their causes and their consequences on country policies and on the business and investment environment. The effervescency around country risk researches is symbolized by the numerous studies and assessments around country risk provided by risk rating agencies (Erb et al 1996).

Moreover, the problematic around country risk appeared also when investors were more and more interested to invest in emerging regions like Asia or South America. Lots of companies consider an additional risk when investing in emerging countries. Thus, they assume that international development is more risky than a domestic investment project. However, the financial theory states diversification reduces risk undertaken. With different assets in a portfolio, one can reduce the risk at constant expected return. The question which arises from that observation is how can one compare gains associated with diversification and additional risks associated with a development project in an emerging country.

This question is central for companies, investment banks as well as financial services institutions. The question is really interesting because, in the facts, every emerging country has its own specificities and represents different risk exposures for investors. Some emerging countries have developed industries or other have a national stock market. One of the major difficulties encountered by academicians about country risk is to integer the different aspects (economic, political, financial, structural,...) of this risk in the same model. As it is often the case in finance, empirical data are not necessarily easy to gather especially for emerging countries. There is a gap between theory and practice. A gap difficult to bridge.

The reason is that country risk has many sides. Indeed, the economic and financial development of a country has many different aspects and the consequences of them are also very various (Kosmidou, Doumpos, Zopounidis 2008). A general definition of country risk is to consider country risk as the
probability that a country will fail to generate enough foreign exchange in order to meet the cash flow expectation of its foreign creditors (Cosset et al, 1992). This definition is exclusively based on economic consideration. Several researchers have completed this definition because they judged a broader view on the issue was needed. Mondt and Despotin (1986), for example, add the necessity to investigate the political environment of the country. For them, the economic aspect only explains the capacity of a country to service its debt. The political aspect is complementary because it takes into account the willingness of a country to service its debt. Calverley (1990) employed this context to define country risk as the potential economic and financial losses due to the difficulties resulting from the macroeconomic and/or political environment of the country.

All things equal except the country of investment, it is obviously safer to invest in Sweden (a country with a stable economic and political situation) than in Mexico (an emerging country) or in Nigeria (an African country with economic growth but a latent political instability). But how to materialize this situation with a premium or a discount rate reflecting the situation in the country?

After this brief introduction, I will more deeply present the topic of interest of this master thesis. The following parts of my thesis will be about explaining methods of calculus of the country risk. I will describe different methods, each time applied to case studies on two countries: Sweden and Mexico. The choice of these two countries is thoughtful. First, I have chosen Sweden as a reference to test the different outcomes of my measures of country risk. Indeed, the Swedish economy is one of the most advanced in the world and the expected country risk premium should be 0%. Sweden is used as a verification to assess the accuracy of the computation methods. The choice of Mexico results from several empirical observations. Mexico is an emerging country, so there is a great interest to compute a country risk premium. However, it is not the only argument which motivated my choice of Mexico. The economic outlooks of Mexico are apparently stable in the future. There are no social nor political turmoil like in Brazil or Argentina. The region of Central America is stable in overall which is not the case of the Middle East (where there are also emerging countries) for example. These non recurring events have a major influence (limited in time though) on the country risk. A last argument which led me to pick up Mexico among emerging economies is the critical size of the economy of the country. Indeed Mexico is big enough to make access easy to useful empirical data. With a smaller country like Thailand it would have been much more difficult. I have bounded my scope of empirical analysis on two countries because I wanted to be able to give results for all my models. This is certainly a subjective choice but I prefer to stay focus on a reduced scope of application. This could always be completed later anyway. Finally, there will be a confrontation between the different outputs obtained and a conclusion.
I/ Presentation of the topic

A) Stakes of an accurate analysis of country risk

In my introduction, the researchers, that I have evoked, tackle the issue of country risk from the general point of view that the concept of country risk is related to the obligation of a country towards its foreign debt (Kosmidou, Doumpos, Zopounidis 2008). However, other research papers have been published on the subject with a different angle. They introduce the analysis from the investment perspective. This perspective is interesting because it is the point of view of the financial analysts within the company, where I am currently working in. More precisely, the investment perspective means the study of the impact of a country economic and political environment on the investment choices made by global companies. Herring (1983) simply analyses the macro and micro risks faced by an international investors and their influence on investment decisions. According Kosmidou, Doumpos and Zopounidis (2008) the macro risks are the following:

- wars, sectarian conflicts, revolution, riots
- country wide price controls
- tax system
- ...

while micro risks concerns industry, firm or project specific parameters (like import/export licenses on some goods, discriminatory taxes, ...). All along my thesis, the country risk will be analyzed from the investment perspective. Indeed, as I am also doing it under the supervision of a manager at PwC, the results of my study should be usable in the every life of analysts at the office.

So far, one can remind that country risk has been a central subject of research since the 80’s and that it remains an open subject because so far, the majority of studies were more qualitative than quantitative. The goal of my thesis is to be able to give a value/percentage of the country risk for different countries.

After this review of some of the existing literature about country risk, it clearly appears that there is a lot of parameters which have an impact on the country risk. The path to obtain a percentage that includes country risk in the cost of capital seems full of hurdles. Then, the entire difficulty is to manage to compute a country risk premium that can be added to the cost of capital or a cost of capital which directly includes country risk. The stake of an accurate analysis of country risk is thus to be able to have an accurate percentage representing the risk of investing in a country.

This leads naturally to the introduction of my research question.
B) Research question

The central point of reflection about country risk is to correctly price it. To do so, it is important to compare additional risks attached to an investment abroad with additional gains. It is the risk/return couple which is fundamental in corporate finance. As a teaser of the research question, I will detail which risks and which gains an investment in an emerging country can bring.

An investment abroad presents the following main risks:

- exchange rate risk: internationalization increases variability of cash flow generated in risky currencies (Kevin 2009).
- political risk: political environment has a huge impact on foreign investment in a country. Government stability, absence of internal conflict and ethnic tensions, basic democratic rights and ensuring law and order are highly significant factors on the amount of foreign investment (Busse and Hefeker, 2005)
- cultural risk: the cultural differences between local management and central executives of the company can cause troubles on which it can be hard to keep a close watch on (Lee et Kwok 1988).
- “prophetic” risk: if an investor requires a high cost of capital for an investment in an emerging country, he will implicitly choose the riskiest ones. As a consequence, this will incite him to require an even higher cost of capital for the next project in the country. And thus, he will choose an even riskier project. In the end, this investor will have the impression to deal with a very risky country whereas he is just choosing riskier and riskier projects (Bancel and Perrotin, 2000).

On the other hand, there are several advantages to invest in emerging countries. It allows to generate cash-flows on different time-frame compared to the country of origin. This diversifies the national risk and makes additional gains. For most academicians, international diversification improves the risk/return couple. The main reason is that covariance between financial markets is low. Correlation factors (Campbell, 1991) between national markets are around 0,40. This conclusion has aged. El Hedi Arouri (2005) states that the correlation between markets has risen in the last years. This reduces the impact on the risk/return couple.

However, Ang and Chen (2002) insist on the fact that during crisis, gains of investment in emerging countries are less significant. Moreover stock markets are more correlated during bear periods than bull periods.
The question of a fair premium or discount rate to reflect the evolution of the risk/return couple while investing in emerging country is fundamental. This is why my research question is the following:

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To answer this question, I will make some choices. There exists indeed numerous ways to tackle this problematic. In the department, where I am working, the current model including the country risk premium is the model of Damodaran (more details below).

Damodaran’s model appeared as a gateway to address the topic of country risk. I have thus initiated my reasoning by gathering information about it. My idea was to understand the theory of this widely used model in the financial industry and see its limits. From this assessment, I have the intention to develop an extension of the existing model to have more accurate outcomes and/or to fill a lack of theory at some points. As Damodaran’s theory is based on the CAPM theory, the first part of my researches is based on the corporate finance theory of the Capital Asset Pricing Model (extension of Damodaran’s theory or adaptation of CAPM to the international context).

To have a counterweight to challenge my results from the first part, I wanted to have an other computation method independent from the CAPM. After a review of literature, I have been seduced by the possibility to use a modified version of Merton’s contingent claim analysis (Ades, 2003) to price country risk. It was really interesting because, the reflection was totally independent from the CAPM. It was entirely adapted to my attempt to have a counterweight to what was accepted among the firm.

I am aware that the choices evoked above are subjective and arbitrary but they appeared naturally all along my process of reflection. For me the most important thing was to stay consistent during my paper and not necessarily seeking completeness at all costs. That is why I have decided to stay focus on some computation theories and methods and did not make an inventory list without logical ties.
II/ The corporate finance approach

The corporate finance approach is a generic term used to designate all the methods to get a country risk premium deriving from the Capital Asset Pricing Model (CAPM).

A) The Capital Asset Pricing Model (Sharpe 1964 and Lintner 1965)

The most widely used model to compute an equity cost of capital is the Capital Asset Pricing Model (CAPM). It is necessary to present it and its parameters before basing a reflection on country risk premium on it.

The basis of this model is that an investor who invests in a project p in a country c can expect the following return:

$$E(R_p) = R_f + \beta_p (E(R_m) - R_f)$$

where $E(R_p)$ is the expected return of portfolio p held by the investor

$R_f$ is the risk-free rate of the country of investment in local currency

$\beta_p$ is the measure of the non-diversifiable risk of investment p

$E(R_m)$ is the expected return of the market portfolio.

The beta is a central parameter in the Capital Asset Pricing Model. According to the CAPM, an investor should have a return solely for the systematic risk because specific risk is diversifiable. Thus, depending on the level of systematic risk, a project is more or less risky. The beta measures the systematic (non-diversifiable) risk and is equal to:

$$\beta_p = \frac{Cov(R_p, R_m)}{Var(R_m)}$$

A small beta means that the systematic risk is low or that correlation with the market portfolio is weak. A highly risky project with an expected return weakly correlated to the market portfolio will have a small beta (below 1). One can compute the beta against a local or an international index. This depends on the segmentation of the market one is looking at. For highly segmented market, one uses local indexes (like CAC 40 in France) whereas international index (like MSCI 500) is recommended for globalized market. The final value of the beta depends on the choice of the reference index.
The second important parameter in the CAPM is the market risk premium. The market risk premium corresponds to the following difference:

\[ E(R_m) - R_f \]

The premium depends strongly on the time frame and computation method (JP Morgan 2008). For long term investment (around 10 years), there is a consensus around 6% for the US market risk premium. The PwC office in Paris has a consensus for the French market risk premium for long term investment between 6% and 7%. For emerging countries, information is still generally rare and unreliable.

To introduce the way of dealing with country risk within the frame of corporate finance theory, we will first consider the risk of a project in an emerging country and then broaden to the overall country risk.

B) Different ways to take into account the risk of a project in an emerging country

Basically, there are two different ways to take into account the risk to invest in a project in an emerging country. The first choice is to adjust cash flows with their certain equivalents. The second is to discount risky cash flows with risky discount rates.

i) Cash flow adjustments

Supporters of cash flow adjustments justify their position with the following arguments: some risk cannot be priced into the discount rate. There exists models able to quantify political risks (Clark 2003) but they are very theoretical. In practice, the point is to adjust cash flows in terms of certain equivalent. Cash flows are corrected with probabilities (defined formerly) which anticipate major risks (political or exchange risk for example).

Generally financial analysts and bankers uses measures computed and sold by specialized institutes (Economist Intelligence Unit among others) to evaluate political risk.

For exchange risk, an investor should use cash flows and discount rates in the same currency. If cash flows are in local currency, it is then necessary to convert them in the investor currency. The risk (mainly difference in inflation) has to be taken into account while converting from a currency to another.

Let’s take an example to highlight this approach. One considers a Swedish company specialized in house furniture retail. This company has the intention to buy an other company in the same sector in Mexico to expand its business in Central America. There is a probability \( \lambda = 5\% \) that the local
A company is nationalized by the authority. This means the loss of the cash flows for the Swedish company. In the coming years, the Mexican peso should appreciate itself of 1% against the SEK. The cost of capital is 10%. Cash flows generated by the Mexican company for the Swedish shareholders are 80 millions MXN over 5 years. 1 MXN is almost 0,5 SEK.

In SEK, one get the following computation:

\[
V = \sum_{t=1}^{5} \left(1 - 5\% \right) \frac{80}{2} \left(1 + 1\% \right) \frac{1}{(1 + 10\%)^t} = 145 \text{ million}
\]

At first sight, this way of reasoning seems convincing but a simple consideration is enough to show that it has limits.

One considers two countries A and B. A is a really safe country while B is much more riskier. We are facing two investment opportunities:

- investment in country A: expected cash flow is SEK 900 known with certainty
- investment in country B: expected cash flow is SEK 1 000 with 90% certainty and SEK 0 with 10% certainty. In the end, the expected cash flow is SEK 90

In the two investments, cash flows are adjusted for the country risk (10% of default in country B). An investor should make a neutral choice between the two investments as the expected cash flows are the same. However, in reality, all reasonable (risk-adverse) investors will choose the investment in the safe country A.

The conclusion is that cash flow adjustments are not enough to value the risk of a project in an emerging country and by consequence to value overall country risk.

ii) Adjusted Net Present Value (ANPV)

The ANPV is about discounting cash flows resulting from the economic activity of the company with a different discount rate than for cash flows resulting from financing choice or hosting country incentives. So, unlike the NPV, the ANPV needs several discount rates (equity cost of capital, debt cost of capital, risk free rate). This way of valuing an investment project in an emerging country is really interesting because it allows to take into account the whole risk. But the problem remains how to include country risk in the equity cost of capital.

The computation is as follow:
\[ \text{ANPV} = \sum_{t=1}^{n} \frac{FCFF_t}{(1 + k_e)^t} + \sum_{t=1}^{n} \frac{D_t \cdot k_d \cdot CIT}{(1 + k_d)^t} \]

\(k_e\) is the equity cost of capital and \(k_d\) is the debt cost of capital.

\(FCFF_t\) are the Free Cash Flows to the Firm at time \(t\), \(D_t\) the amount of debt at time \(t\), CIT the corporate income tax rate. Thus \(D_t \cdot k_d \cdot CIT\) are the income savings linked to the cost of debt.

Let’s looks at the same example as in the part above (the Swedish company investing in Mexico). If one makes the additional assumption that the target has been valued at 100 million SEK. Half of the financing is in equity \((k_e=12\%)\) and the other half is debt \((k_d=8\%)\) with maturity 10 years. Moreover the CIT in Mexico is 30\% (PwC Tax Summaries).

For this project the ANPV is:

\[
V = -100 + \sum_{t=1}^{5} \frac{(1 - 5\%) \cdot 80}{2 \cdot (1 + 1\%)^t} + \sum_{t=1}^{10} \frac{50 \cdot 8\% \cdot 30\%}{(1 + 8\%(1 - 30\%))^t}
\]

\[
V = 54 \text{ million}
\]

The acquisition of the target generate 54 million SEK for ther buyers. The real value of the target is equal to 154 million SEK. Fortunately, the two results converge around the same value (around 150 million SEK) but there is a limit. The limit with the two computation methods developed above is that it works well when you consider a specific project. However, as they take into account the specificities of the project, one cannot generalize to a country. Moreover, those methods do not result in a discount rate which includes the additional risk of an emerging economy. The benefit of this introduction has allow us to eliminate cash flows adjustments for country risk analysis. Indeed, in the facts, the reasoning of practitioners is based upon a discount rate.

Anyway, before directly looking at practitioners’ reflections, which can present a lack of theoretical background, we decided along with my manager at PwC to focus on what is called the International Capital Asset Pricing Model (ICAPM). Indeed, as I will develop it deeper after, this recent extension of the CAPM has been the topic of lots of research papers. Moreover, Korkmaz, Cevik and Gürkan (2010) have made empirical and statistical studies on the ICAPM. Our hope is to be able to infer a country risk premium from their conclusions.
C) The International Capital Asset Pricing Model (ICAPM)

i) The theoretical background of the ICAPM

The ICAPM derives from the CAPM (Sharpe 1964, Lintner 1965) and is based on the general idea that the structure of the theory of international finance reflects the theory of domestic finance (Adler and Dumas, 1983). Thus, the basic version of the ICAPM inserts in the CAPM the following parameters (Korkmaz, Cevik and Gürkan, 2010):

- the expected return of the country replaces the expected return of the portfolio
- the expected world index return replaces the expected return of the market portfolio
- the global risk free rate replaces the local risk free rate

To sum up the ICAPM generally takes into account the world market portfolio instead of the domestic market portfolio. The CAPM commonly appears in finance papers as an explanation of the differences in risk premium across assets (Korkmaz, Cevik and Gürkan, 2010). The domestic CAPM suggests that the expected return on any portfolio is proportional to the systematic risk of the asset (Beta parameter) (De Santis and Gerard, 1997).

Since the beginning of the 70’s, lots of studies have been published about the ICAPM (Agmon 1972, Solnik 1974, Lessard 1974, Harvey 1991, Bekaert and Harvey 1995 among others). All those papers differ in methods concerning the ICAPM but they are all intended to explain the conditionality of betas and risk factors (Korkmaz, Cevik and Gürkan, 2010). Some researchers opt for multi-factor models by taking into account the currency risk and/or the inflation risk (Adler and Dumas 1983, Dumas and Solnik 1995, Phylaktis and Ravazzolo 2004, Wu 2008). But most articles on the ICAPM stay on a single-factor models: market risk (Lessard 1974, Bekaert and Harvey 1995, De Santis and Gerard 1997, Ramchand and Susmel 1998). Korajczyk and Viallet (1989) show that multi-factor models tend to over perform single-factor models.

There is an ambivalence concerning multi-factor models. Indeed, Solnik (1974) writes that complex model should be preferred to study international finance. This appears as a promotion of multi-factor models. However, the same Solnik (1977) lays importance on the fact that assumptions should remain simple (inter allia simple enough to be able to be tested with empirical data). He suggests that it must not be forgotten that complex models are compelling for researchers but lack of empirical traceability. Thus, Korkmaz, Cevik and Gürkan (2010) chose to test ICAPM under the single-factor model (no currency nor inflation risk).
Concerning our research question, the ICAPM is even more interesting because the earlier literature was centered on developed markets (Agmon 1972, Solnik 1974, Adler and Dumas 1983, Korajczyk and Viallet 1989, Engel and Rodrigues 1989, Ramchand and Susmel 1998) but the second wave of studies address emerging countries (De Santis and Imrohoroglu 1997, Jan, Chou and Hung 2000, Gerard, Thanyalakpark and Batten 2003, Phylaktis and Ravazzolo 2004, Chi, Li and Young 2006, Chen and Huang 2007, Jacobsen and Liu 2008, Tai 2007). The growing number of studies on emerging countries can be explained by the fact that investors have larger and larger flows of investment in those countries since the 90’s.

An other point on the ICAPM is central. This is to know whether the beta is time-varying or not. In the CAPM, there is a linear relationship between the expected return and systematic risk (Sharpe 1964 and Lintner 1965). However there is an increasing number of proofs showing that the relationship is time varying (Blume 1971, Levy 1971, Fabozzi and Francis 1978, Chen 1981, Ferson and Harvey 1991/93, Ferson and Korajczyk 1995).

ii) Empirical model of the ICAPM (Korkmaz, Cevik and Gürkan 2010)

I will review one model of Korkmaz, Cevik and Gürkan (2010) on the ICAPM. Indeed, the idea is to use their findings to derive a country risk premium. As we are not sure to get to a conclusion, I will first concentrate on the most simple of their test.

First of all, Korkmaz, Cevik and Gürkan (2010) define the ICAPM where currency risk and inflation risk is not considered as follows:

\[ R_{it} - R_{ft} = \alpha_i + \beta_i (R_{mt} - R_{ft}) + \varepsilon_{it} \]

where \( R_{i} \) is the return of the index of country \( i \), \( R_{mt} \) is the return of the world index and \( R_{ft} \) is the risk free rate. \( \varepsilon_{it} \) follows a probability law \( N(0,\sigma^2) \).

The beta depends on three parameters in the ICAPM (Korkmaz, Cevik and Gürkan 2010):

- correlation between the country \( i \) and the world index
- volatility of the country index return
- volatility of world index return

The model is the linear ICAPM. As in the CAPM, \( \alpha \) and \( \beta \) are not time varying, so we get the same linear relationship between expected return and systematic risk:

\[ E(R_{it} - R_{ft}) = \beta_i E(R_{mt} - R_{ft}) \]
a. Empirical results

Korkmaz, Cevik and Gürkan (2010) made observations on a sample of 23 emerging countries between January 1995 and April 2009 (172 monthly observations). They managed to get an $\alpha$ and a $\beta$ for each country. More precisely their statistical study leads to the following conclusions:

- Stock market returns in emerging countries tend to fatter tail distribution than a normal distribution (Kurtosis test)
- According to the Jarque-Bera normality test statistics, all of the return series of emerging markets exhibit significant deviation from normality
- The expected values of $\alpha$ parameter are not statistically significant
- The expected values of $\beta$ parameter are statistically significant at 1% level for all countries

iii) Connection with the country risk premium

Let’s now try to infer a country risk premium from the observation made by Korkmaz, Cevik and Gürkan (2010). As said in the introduction, I have focused my thesis on two countries for the case studies (Sweden and Mexico). Luckily, Mexico is among the 23 emerging countries of the study. We, then, have information about its international beta. The confidence on this information is moreover important.

Nevertheless, we are interested in getting a country risk premium. How can we infer a country risk premium from what we know about Mexico in the framework of the ICAPM?

One can first rewrite the equation in the case of Mexico:

$$R_{Mex} - R_{ft} = \alpha_{Mex} + \beta_{Mex}(R_{mt} - R_{ft}) + \varepsilon_{Mex}$$

In this equation, everything is know except $\varepsilon$ (Korkmaz, Cevik and Gürkan 2010)

One can also write the same equation for Sweden:

$$R_{Swe} - R_{ft} = \alpha_{Swe} + \beta_{Swe}(R_{mt} - R_{ft}) + \varepsilon_{Swe}$$

In this equation nothing is known because Sweden is not in the 23 countries of the study. Anyway, if we make the assumption that country risk premium for Sweden is 0% because the Swedish economy is developed (demonstrated later), one could figure out the country risk premium for Mexico from a comparison between the two countries. In the ICAPM formula, the specificity of the country stock market is included within the $\beta$ (volatility of the country index return). This means, if we are able to
have an approximate value for the beta for Sweden, we have a basis to infer the country risk premium of Mexico.

Following Korkmaz, Cevik and Gürkan (2010) method and sources for the empirical data, one can easily get a value for the beta of Sweden. I made a first quick estimation using the same sources (MSCI-Barra’s official website for Swedish OMX 30 index return and Kenneth W. French’s official website for the monthly US T-Bill) as Korkmaz, Cevik and Gürkan (2010) but without all the statistical analyses. Firstly, because all the tests they made were good on emerging countries, so there is no reason that it does not work with developed countries. Secondly, as I am not sure of the result yet, I want to go straightforward to a result.

We then have the following results for the beta of Mexico (Korkmaz, Cevik and Gürkan 2010) and Sweden (own calculus).

<table>
<thead>
<tr>
<th>Country</th>
<th>Beta</th>
<th>Kustosis test</th>
<th>J-B normality test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>1,417</td>
<td>6,428</td>
<td>132,407</td>
</tr>
<tr>
<td>Sweden</td>
<td>0,763</td>
<td>ns</td>
<td>ns</td>
</tr>
</tbody>
</table>

The first thing one can notice is that the beta of Mexico is larger than the beta of Sweden. This makes sense because one expects more volatility in Mexico than in Sweden. The country risk is included within the beta of Mexico. The difficulty is to get a value for the country risk. Indeed, this is not the only parameter taken into account in the beta. The beta, as mentioned above, is influenced by the correlation between the local index and the world index.

It seems extremely difficult to quantify a premium for country risk with this approach. I am positive that the information is taken into account but it is mixed with other. It cannot be extracted easily.

The strong advantage of this approach is that it is based on a solid theoretical background. It is unfortunately not suitable to get a value of country risk.

Let’s now move to practitioner’s reflections. They surely lack of theoretical background but country risk is a parameter which is used a lot by professional. So if the justification is not theoretical, it comes from the best practices of the finance industry.
D) Godfrey and Espinosa model

According Godfrey and Espinosa (1996), the expected return for an American investor (they set up their reflection by considering the USA as the reference market but it can be any developed countries) in an emerging country can be expressed as follow:

\[ \text{Cost of equity} = (R_{fUS} + \text{Credit Spread}) + \beta_{US} \times \text{US equity premium} \]

The cost of equity is equal to the American risk free rate plus a premium representing the sovereign risk. This premium is in USD and not in local currency, the exchange risk is considered within the cash flows. The beta is actually an “adjusted” beta and is expressed as follow:

\[ \beta_{US} = \frac{\sigma_i}{\sigma_m} \]

where \( \sigma_i \) is the standard deviation of return for a proxy of the local stock market (e.g. the local stock market index) and \( \sigma_m \) is the standard deviation of return for the US stock market (e.g. S&P 500 index).

Godfrey and Espinosa suggest to multiply the beta obtain above by 0,6. The justification is that there exists a correlation between the sovereign risk and the market risk. They have demonstrated that 40% of market risk can be explained by sovereign risk.

Nevertheless, one can express some limits to Godfrey and Espinosa methodology. First, to take into consideration the total risk \( \frac{\sigma_i}{\sigma_m} \) is hard to justify. Indeed, it does not take the portfolio diversification into account at all. The second point concerns, the measure of the local risk \( \sigma_i \). There is obviously an issue to get data for emerging countries. They generally have a narrow stock market (if any). Thirdly, the question of the correlation between the sovereign risk and the market risk cannot be evacuate simply with a 0,6 factor. The correlation factor has to be precisely computed for each country.

E) Damodaran model

Aswath Damodaran is an Indian professor of finance at the Stern School of Business at New York University. His specialization in finance concerns valuation, corporate finance and investment management. He has written lots of books and articles on these subjects. Among these publications, some are about country risk premium. Every year, Aswath Damodaran publishes an update table with country risk premium computed for the great majority of countries around the world. Along the
years, Damodaran has become a reference among practitioners in banks for the computation of country risk premium. Let’s have a closer sight on his theory.

Damodaran focuses on the idea of a country risk premium. The idea is to add to the equity cost of capital (computed with the CAPM) and the debt cost of capital a country risk premium. This premium will reflect the additional risk to invest in an emerging or less advanced country.

A guideline in this idea is to propose a model easy to use and to understand by practitioners. Indeed, practitioners in banks or financial institutions will compute the equity cost of capital and the debt cost of capital as they ever do and after apply a country risk premium (proposed by Damodaran) to each of this cost for the adjustment.

Before starting to develop Damodaran’s country risk premium theory, I want to make some precisions:

- as equity market is riskier than debt market the premium add to the equity cost of capital is higher than the premium added to the debt cost of capital. Both premia are however correlated.
- I will explain Damodaran’s theory with the computation of the country risk premium for debt cost of capital.
- I will not do the same demonstration for the country risk premium added to the equity cost of capital. For this premium, I will solely explain the multiplying factor that allows to switch from the debt premium to the equity premium
- In all this part, I will denominate the country risk premium added to the debt cost of capital by the expression debt risk premium
- In all this part, I will denominate the country risk premium added to the equity cost of capital by the expression equity risk premium

i) Debt risk premium

The easiest way to compute a debt risk premium is to find available market data. For this reason, Damodaran’s debt risk premium is computed with market based proxies of country risk.

The first and the more commonly used proxy of the country risk is the bond default spread. The idea is to say that the difference between the rate of a long term bond issued by the country and denominated in international currency (USD or EUR) and the risk-free rate of a same maturity zero coupon bond corresponds to the debt risk premium. Generally, we use a maturity of 10 years. The risk free rate is either the 10 year US Treasury bond or the 10 year German bond. The main limit of
The second proxy of the country risk is the credit default swap spread. With this proxy, the idea is exactly the same as with bond default spread. I will thus not explain again the logic behind. The interesting aspect with credit default swaps (CDS) compared to sovereign bond is that the market has developed rapidly in the last years. In particular, data about CDS market tend to be more updated and precise than for bond market. In the end, in the case of lack of data about bonds for some emerging or African countries, it is likely to find to find data about CDS.

However, sometimes, it happens that one cannot find either information about bonds nor CDS. Generally, it concerns interesting countries with regard to the computation of debt risk premium. In that case, Damodaran suggests an alternative way of proceeding. His idea is to establish a table that summarize debt risk premium depending on the Moody’s/S&P notation of the country. The table is constructed from countries with available data on bonds/CDS. Once the table constructed, one is able to find a debt risk premium for a country only by looking at his Moody’s/S&P notation. Damodaran updates his table on an annual basis. Below is the latest table proposed by Damodaran on his website.
Once the debt risk premium is computed, one still need the value of the equity risk premium. At first sight, the equity risk premium is different from the debt risk premium. This observation is obvious since debt and stocks are not traded on the same market. It still remains some tricky questions on the subject. Will the equity risk premium be larger than the debt risk premium? And how much larger or smaller?

Intuitively, one would argue that the equity risk premium should be larger than the debt risk premium. Indeed, the volatility of the equity market is higher than the debt market. More volatility means more risk embedded on the securities traded and thus a higher risk premium.

To quantify by how much the equity risk premium should be compared to the debt risk premium, Damodaran suggests a multiplying factor. This multiplying factor is the relative volatility of the equity market on the debt market. This results in the following formula:

\[
\text{Equity Risk Premium} = \text{Debt Risk Premium} \times \text{Multiplying Factor}
\]
So, once the debt risk premium is calculated for a country, the equity risk premium derives simply from it. The only obstacle between us and a final value for the equity risk premium is the availability of data to compute both volatilities.

With data, one faces the same problem that with the debt risk premium. Indeed, debt market volatility requires that long term government bonds not only exist but are traded on a regular basis. This is the problem of a liquid market mentioned above. Damodaran proposes a solution to avoid this problem (which is common for emerging or less advanced countries). The solution is to consider that the volatility of corporate debt is a good proxy for the volatility of government debt. To be effective, this solution requires the existence of large companies in the country. In a sense, this does not really solve the problem. Moreover it is not generally accepted that corporate debt is a good proxy of government debt.

The problem is also the same concerning equity market.

With that ascertainment, Damodaran decides to compute equity market volatility and debt market volatility for countries with available data. Then he takes the average value of the volatilities. He gets an average ratio, he then applies as a constant factor to the debt risk premium without regards to the country. He arrives at the average ratio of 1,5.

\[
\frac{\sigma_{\text{equity market}}}{\sigma_{\text{debt market}}} = 1,5
\]

This results in the end in the following table for equity risk premium.
With the two summary tables, one is able to get by a direct reading the debt risk premium and the equity risk premium for a country. One gets then an equity cost of capital, debt cost of capital or weighted average cost of capital (WACC) adjusted for the country risk.

Now that I have explained Damodaran’s theory on country risk, I will use it as a starting point for the development of my own tool.

I would like to mention some remarks about Damodaran’s model.

- Gains associated with portfolio diversification are once again not taken into account.

Investing in an emerging country with economic cycles correlated to the economic cycles of the country of origin has not the same consequences on expected return as if they were not.

- Damodaran’s model does not contain theoretical assumptions and is not linked to anterior researches in finance. It is striking but there is almost zero references in Damodaran’s bibliography.

From the assessment of the state of the research in corporate finance concerning country risk, I will make two proposals about the topic. One suggestion is an extension of Damodaran’s work. I am
aware that Damodaran’s model has strong theoretical weakness but it is widely use in practice. As I am working in an financial services company, PwC, analysts refers to the best practice of the industry. In our case, this is Damodaran’s model for the country risk premium. This is a practical reflection and the goal is to implement this extension of Damodaran’s model in Excel tools within my department. The second suggestion is more theoretical and is based on my observations of the two model described above (Godfrey and Espinosa / Damodaran)s.

F) An extension of Damodaran’s theory.

Damodaran’s theory is among the most used method to compute country risk premium in the financial world (Naumoski, 2011). His name is synonym of reliability among bankers and financial analysts. However, I have notice several limits with Damodaran’s model.

The first thing is that this model is based on Moody’s and Standard and Poors (S&P) ratings. In fact, lots of countries in Africa, Middle East, South East Asia and even South America do not have a rating (134 countries for S&P: https://www.standardandpoors.com/en_US/web/guest/entity-browse). There are several reasons for that but this is not the topic of this paper. Anyway, without a Moody’s or S&P rating, it is not possible to get an estimation of country risk premium. This is a pity knowing that the concerned countries are countries where information about country risk is extremely relevant.

The second thing is that by using rating agencies, Damodaran reduces the country risk to the risk of default on sovereign debt (Damodaran, 2008) (this is on this parameter that rating agencies focus on). He may forget to take into account other parameters that influence country risk. Country risk is indeed a set of lots of parameters among other sovereign financial risk, financial market risk, political risk, business environment risk (Bouchet, Clark and Groslambert 2003). I am not saying that Damodaran estimation of country risk is false but one may broaden the scope of parameters that are taken into account for the estimation of the premium.

A third point, that strokes me, is that Damodaran’s classification according ratings is discrete. There is only 25 different country risk premium. I think that this is a little bit reductive. A continuous approach is more suitable according me. Indeed, each country has its own specific characteristics (Bancel and Perrotin, 2000) and thus I think that it will be better to have specific country risk premium for each.

All these ascertainments, led me to develop a tool in order to have a more accurate value for the country risk premium.
I have constructed my tool as follow:

- I have found on the Internet a group of economists (Economist Intelligence Unit) that have developed their own score to evaluate country risk. They take much more parameters than ratings agencies (more below). The score is between 0 and 100
- I made a regression of the EIU score of the countries against the corresponding debt risk premium (computed with Damodaran’s method).
- From that regression, I then got a new debt risk premium influenced by the EIU score.

My idea behind this extension is double:

- If a country does not have a Moody’s or S&P rating, we are now able to find a country risk premium with its EIU score.
- For countries with a Moody’s or S&P rating, the regression allows to find a debt risk premium influenced by the EIU score of the country. By doing that, one take into account all the risk included in the EIU score (more details below) and then has a broader view on the country risk. By doing the average between Damodaran’s debt risk premium and the newly computed risk premium, one becomes closer to a debt risk premium really specific to the country of study.

Naturally, the reasoning is absolutely the same for equity risk premium. And as said before, to get the equity risk premium from the debt risk premium, we just need to multiply by 1,5.

i) The Economist Intelligence Unit Score for country risk (EIU score)

The Economist Intelligence Unit (EIU) is a business unit from the Economist Group. The EIU provides advisory services and analysis on large business and economic subjects. The EIU publishes inter alia reports on Country Risk. Along the years, the EIU has developed a score to evaluate country risk. The scale is between 0 and 100. 0 means 0 risk and 100 is the most risky country.

As the EIU sells its EIU Country Risk Score, it does not communicate precisely about their methodology. However, it gives indication about the parameters taken into account in their calculation as well as all the variables included in the model.

The EIU uses 6 parameters into account which globally converge with Bouchet, Clark and Groslambert (2003):
- Sovereign risk measures the risk of a build-up in arrears of principal and/or interest on foreign- and/or local-currency debt that is the direct obligation of the sovereign or guaranteed by the sovereign.

- Currency risk measures the risk of devaluation against the reference currency (usually the US dollar, occasionally the Euro) of 25% or more in nominal terms over the next 12-month period.

- Banking sector risk gauges the risk of a systemic crisis whereby bank(s) holding 10% or more of total bank assets become insolvent and unable to discharge their obligations to depositors and/or creditors.

- Political risk evaluates a range of political factors relating to political stability and effectiveness that could affect a country’s ability and/or commitment to service its debt obligations and/or cause turbulence in the foreign exchange market. This rating informs the first three.

- Economic structure risk is derived from a series of macroeconomic variables of a structural rather than a cyclical nature. Consequently, the rating for economic structure risk will tend to be relatively stable, evolving in line with structural changes in the economy. This rating informs the first three.

- Overall country risk is derived by taking a simple average of the scores for sovereign risk, currency risk, and banking sector risk.
And the variables included in the model are the following:

<table>
<thead>
<tr>
<th>Politics/institutions</th>
<th>Macroeconomic</th>
</tr>
</thead>
<tbody>
<tr>
<td>External conflict</td>
<td>Real OECD GDP growth</td>
</tr>
<tr>
<td>Governability/social unrest</td>
<td>Credit as % of GDP growth</td>
</tr>
<tr>
<td>Electoral cycle</td>
<td>Real GDP growth, 48 months</td>
</tr>
<tr>
<td>Orderly transfers</td>
<td>Real GDP growth, 12 months</td>
</tr>
<tr>
<td>Event risk</td>
<td>Inflation, 48 months</td>
</tr>
<tr>
<td>Sovereignty risk</td>
<td>Inflation, direction</td>
</tr>
<tr>
<td>Institutional effectiveness</td>
<td>Trade-weighted real exchange rate</td>
</tr>
<tr>
<td>Corruption</td>
<td>Exchange-rate misalignment</td>
</tr>
<tr>
<td>Corruption in the banking sector</td>
<td>Exchange-rate volatility</td>
</tr>
<tr>
<td>Commitment to pay</td>
<td>Export receipts growth, 12 months</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Economic policy</th>
<th>Financial and liquidity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quality of policymaking/policy mix</td>
<td>Transfer and convertibility risk</td>
</tr>
<tr>
<td>Monetary stability</td>
<td>IMF programme</td>
</tr>
<tr>
<td>Use of indirect instruments</td>
<td>International financial support</td>
</tr>
<tr>
<td>Real interest rates</td>
<td>Access to financing</td>
</tr>
<tr>
<td>Fiscal balance/GDP</td>
<td>Gross external financing requirement</td>
</tr>
<tr>
<td>Fiscal policy flexibility</td>
<td>Debt-service ratio</td>
</tr>
<tr>
<td>Transparency of public finances</td>
<td>Interest rate/exports</td>
</tr>
<tr>
<td>Domestic debt</td>
<td>External short-term debt/foreign exchange reserves</td>
</tr>
<tr>
<td>Unfunded pension and healthcare liabilities</td>
<td>Change in foreign exchange reserves</td>
</tr>
<tr>
<td>Exchange-rate regime</td>
<td>Net external debt/exports</td>
</tr>
<tr>
<td>Black-market/dual exchange rate</td>
<td>FX/gross financing requirement</td>
</tr>
</tbody>
</table>

As we can see, the model takes much more parameter into account to get a score than Damodaran who only focuses on sovereign debt default.

An other interesting point with the EIU Country Risk Score is that the data are easily available. Indeed, the scores are directly available on Bloomberg terminals. From a practical point of view, it is really interesting.
ii) Regression of EIU Country Risk Score

The first step of the regression is to collect the EIU Country Risk Score for the maximum of countries. On Bloomberg, I have found the EIU score for 121 countries. As EIU Country Risk Score is a dynamic index, I have decided to take January 1st, 2016 as reference date. This will also make our results comparable with the country risk computed in part III (because the DCF model uses this date as reference as well).

Once the list of countries established, the second step is to extract countries with a Moody’s or S&P Long-Term rating (they generally have both). The list is now reduced to 101 countries. We now have a list of countries with a Damodaran debt (resp. equity) risk premium and a EIU score. The regression is possible.

I will do the regression of the EIU score against equity risk premium but the result is absolutely the same with debt risk premium.

A linear regression gives a $R^2$, coefficient of correlation of 0.75. For empirical data, this can be considered as a good correlation.
iii) Statistical analysis of the regression

Now to have more confidence about the accuracy of the regression made, I will do some statistics. I will first analyze the variance. It will give us the information about what is explained and what is the residual information not taken into account by the regression.

We recall that the regression results from the least squares method. This means that for all elements \( i \) of the sample we have:

\[
y_i = ax_i + b + e_i
\]

in our case \( y_i \) is the EIU Country Risk Score and \( x_i \) is Damodaran country risk premium. \( e_i \) is the residual error not explained by the regression.

Numerically \( a=314,67 \) and \( b=29,762 \).

To get a more accurate information on what is explained by the regression and what is residual (in other word the variance). One can write the following equality:

\[
y_i - \bar{y} = a(x_i - \bar{x}) + e_i
\]

where \( \bar{y} \) is the average of the observations of EIU Country Risk Score. \( \bar{x} \) is the same thing for Damodaran country risk premium.

According Verdel (2015), by using squares and sums we get the equation of analysis of the variance.

\[
\sum_{i=1}^{n} (y_i - \bar{y})^2 = a^2 \sum_{i=1}^{n} (x_i - \bar{x})^2 - \sum_{i=1}^{n} e_i^2
\]

This not exactly the mathematical definition of the variance (one should divide by \( n \)) but it is an interesting equation since one can write it as follow:

\[
\text{sum of least squares} = \text{sum of explained square} - \text{sum of residual square}
\]

also written

\[
\text{SLS} = \text{SES} - \text{SRS}
\]

The SRS/SLS ration is very interesting because it gives the % of the information of the sample lost with the regression. We have thus a concrete information about accuracy and relevancy.

After computation we have: \( \text{SLS}=17\,543, \text{SES}=13\,235 \) and \( \text{SRS}=4\,308 \).
SRS/SLS = 24.6%. This means that almost on quarter of information is lost with the regression.

Moreover, Verdel (2015) describes also the computation of R2. R2 = SES/SLS. We can check if the value of Excel is the good one. With Verdel formula, one get R2 = 75.4% which is fortunately the same as in the Excel sheet.

So far, we made an analytical description of the empirical data gathered. It will be more interesting to focus on statistical properties of the regression (least squares) curve. We have observed points but one could generate statistical sample from that observations.

a and b are coefficient determined experimentally. But let's consider a random individual i in the sample. x_i is known with certainty while y_i is the outcome of the random variable Y_i = αx_i + β + ε_i.

Where α and β are certain parameters to be determined and ε is random variable with the following properties (Verdel):

- E(ε_i) = 0
- E(ε_i^2) = σ_i^2
- E(ε_iε_j) = 0 if i different from j.

I will not get into the detail of the calculation but Verdel (2015) states that a and b (the coefficient of the least squares curve) are outcomes of two random variables A and B.

A and B are unbiased (E(A) = α and E(B) = β) and converging (σ_A^2 and σ_B^2 tend to 0) estimators of α and β.

Now, that we know the properties of α and β which remains unknown. It appears interesting to me to determine the interval of confidence of a point of the regression curve y(x) = αx + β. The determination of this interval of confidence will tells us if the least squares curve is a good approximation of the regression.

The outcome of the interval of confidence is a frame for the unknown regression curve. If the least squares curve is also in that interval we can consider it as a good approximation.

The equation of the regression curve is:

\[ y(x) = ax + \beta \]

And the equation of the least squares curve is:

\[ y^*(x) = ax + b \]
Considered as an outcome of the random variable:

\[ Y^*(x) = Ax + B \]

To determine the interval of confidence, Verdel suggests to compute the variance of \( Y^*(x) \) and to write it as follow:

\[ Y^*(x) = A(x - \bar{x}) + \bar{Y} \]

\[ \sigma^2(Y^*(x)) = (x - \bar{x})^2\sigma^2(A) + \sigma^2(\bar{Y}) \]

because \( A \) and \( \bar{Y} \) are independent (Verdel). Moreover, in the demonstration of Verdel \( \sigma^2(A) = \frac{\sigma^2}{\sum_{i=1}(x_i - \bar{x})^2} \) and \( \sigma^2(B) = \bar{x}\sigma^2(A) + \frac{\sigma^2}{n} \) where \( \sigma^2 \) is the variance corresponding to the dispersion around the regression curve. The SRS is an estimator of \( \sigma^2 \).

Knowing that, it comes \( \sigma^2(Y^*(x)) = (x - \bar{x})^2\frac{\sigma^2}{\sum_{i=1}(x_i - \bar{x})^2} + \frac{\sigma^2}{n} \).

Verdel (2015) demonstrates that \( \frac{Y^*(x) - y(x)}{\sigma(Y^*(x))} \) follows a standard normal distribution and as a consequence the quotient

\[ T = \frac{Y^*(x) - y(x)}{\sigma \sqrt{\frac{(x - \bar{x})^2}{\sum_{i=1}(x_i - \bar{x})^2} + \frac{1}{n}}} \]

follows a Student-Fisher distribution with \((n-2)\) degrees of freedom. This property allows us to find an interval of confidence for \( y(x) \). At a given risk \( \Omega \), we have:

\[ y(x) \in ax + b \pm t_{\Omega/2} \sigma \sqrt{\frac{(x - \bar{x})^2}{\sum_{i=1}(x_i - \bar{x})^2} + \frac{1}{n}} \]

where \( t_{\Omega/2} \) is the quantile of order \( \Omega/2 \). When \( x \) varies, \( y(x) \) is included within the two branches of an hyperbolas.
For $\Omega=0.05$ (interval of confidence at 95%), we get the following plot:

The least square curve (in red) is totally within the border of the two branches of the hyperbole. Moreover, we can say that the regression is good because the space between the two branches is the narrowest around 5% (Damodaran’s country risk premium) which is the value for which there is the most of data. In conclusion, the statistical analysis of the sample has convinced us that the linear regression was adapted to this sample.

Anyway, this extension is still based on a poor theoretical model (Damodaran’s model), that is why I have chosen to think about an alternative model.

G) Limits of the extension

As the proposed extension is based upon Damodaran’s theory, the limits of the extension are actually the limits of Damodaran’s method. Indeed, from the starting point of his reflection, Damodaran wanted to keep his country risk premium (debt and equity) easy to use by practitioners. For this reason, Damodaran’s country risk premium is broadly accepted among practitioners but it is far less the case in the academic world. Academicians have raised a certain number of theoretical limits with Damodaran’s theory.
The first criticism is the lack of formal definition of what country risk premium is. Indeed, for Damodaran the country risk premium (either the debt risk premium or the equity risk premium) reflects the surplus that investors are requesting to invest in emerging or less advanced countries. Next to that informal definition, Damodaran does not give clearer definition. Unlike the market risk premium computed with the Capital Asset Pricing Model, there is no clear mathematical definition of the debt (resp. equity) risk premium (Kruschwitz, Löffler and Mandl 2010).

The second criticism concerns one pillar of Damodaran’s theory: the debt (resp. equity) risk premium is not diversifiable. On the subject, Damodaran says that diversification is possible only if countries are “uncorrelated”. First of all, this is not really a precise definition but academicians suppose that this means that the coefficient of correlation is 0. However, it has already been demonstrated that we are able to create portfolio of positively correlated assets that are less risky than the less risky asset. Actually the only condition on the correlation is that the coefficient should be different from 1. It is clear that the effects of diversification increases while correlation decreases. It is yet not necessary to have zero correlation (Kruschwitz, Löffler and Mandl 2010).

The third point raised by academicians is even more theoretical than the two points above. This is an observation directly linked to the framework of Damodaran’s work: the Capital Asset Pricing Method (CAPM).

The CAPM is the specific case of the risk-return model when one considers only one factor. This factor is the market risk premium. Damodaran adds an equity risk premium to this model. So, de facto, we are not in a one factor model anymore but in a two factor model. The problem is that Damodaran still considers being in the framework of the CAPM. This problem may be abstract for practitioners but it did not go unnoticed from academicians (Kruschwitz, Löffler and Mandl 2010).

This part is, in the end, not intended to show that Damodaran’s model is false on a theoretical basis. This was only to underline that even if the theory may have some weakness, Damodaran is able to provide practically a country risk premium that is coherent with the overall situation of the financial industry.

H) An alternative model

By taking into account the limits of Godfrey and Espinosa and Damodaran’s model, I have developed a model to determine the expected return of an investor in an emerging country.

My model is based on three different risks associated with an investment in an emerging country.

- exchange risk
- sovereign (or political) risk
- economic risk

For most academicians and among them Godfrey and Espinosa (1996), exchange rate must be integrated into the cash flows. If you have cash flows in USD or in Euro, they will be discounted with a rate in the same currency. This is also my choice. There only remain the sovereign risk and the economic risk. The first thing to notice is that they are strongly correlated.

I propose the following model to measure the equity cost of capital in an emerging country:

\[ E(R_p) = (R_f + \text{Sovereign Spread}) + \beta_p(E(R_m) - R_f) \]

where \( R_f \) is the risk free rate of the country of reference (a suitable developed country). Expressed in USD or Euro. The Sovereign Spread is a measure of the political risk assumed by the investor. \( \beta_p \) is a measure of the non-diversifiable economic risk in the emerging country of investment. \( E(R_m) - R_f \) is the market risk premium.

Here comes the explanations of the model. Fundamentally, for investment projects in globalized sector (Oil & Gas industry for example), the economic risk does not depend on the country. The exploitation of a refinery or an oil plant in Norway is as risky as the same industrial installation in Venezuela. To avoid arbitrage opportunity, one should not advantage or disadvantage Venezuela because it is an emerging country. However, the political risk is very different between the two countries. This risk in Venezuela could trigger an unexpected expropriation of the shareholders or the shutting down of the infrastructures.

For this reason, I am in line with Shapiro (1990) assumption. It states that the discount rate of an investment in an emerging country should be the same as in the country of reference. Yet, emerging countries are very diversified and some have very high political risk. That is why, one is still obliged to include a political risk premium. For me, the best possible proxy of this risk is the sovereign spread and not the local stock market volatility.

Evaluating the sovereign risk is not a problem. For a great number of countries, one has access to the sovereign spreads (with Bloomberg terminal for example). They are generally expressed in hard currency, to understand in USD, and do not include the exchange rate risk (which is included in the cash flows).

The local economic risk is much more sensitive to get. There exists a local economic risk when a project is not in a globalized sector. Indeed, we have to evaluate the local volatility of the stock market \( \sigma_p \), the correlation \( \rho_{pm} \) between the local market, the reference market and the volatility of
the reference stock market $\sigma_m$ in order to get a local beta ($\beta = \rho_{im} \frac{\sigma_i}{\sigma_m}$). As we have already mentioned several times in this report stock markets in emerging markets are small and few companies represent the quasi-totality of the market. Thus, I think it is very hazardous to use this type of measure for the economic risk.

Obviously, the principal limit of this model is when one is dealing with a non-globalized sector (retail sector for example). I think that we are at the boundaries of the model even if we should not necessarily change the methodology. But, even if we would be able to compute a local premium, the unreliable data will make the result questionable.

An other question arises if the sovereign spread is not available. I suggest to use country debt’s rating (S&P or Moody’s) and the average corporate spread linked to the rating. It is better to use this method than the spread on the latest issued sovereign debt. Indeed, rating have an historical approach, it avoids taking into account the specificities of the latest debt issuance (liquidity, country reputation at the time of issuance,…). If the country has no rating, one can still proceed with analogy with similar countries.
I) Application to the country of our case studies.

Going back to the case studies, I will apply with data the regression resulting from the extension of Damodaran’s theory.

i) Sweden

We can sum up the results on the following Excel sheet:

<table>
<thead>
<tr>
<th>Long-Term Moody’s</th>
<th>Long-Term S&amp;P</th>
<th>EIU Score</th>
<th>Debt risk premium Moody’s</th>
<th>Debt risk premium S&amp;P</th>
<th>Debt risk premium EIU Score</th>
<th>Average Debt Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>Aaa</td>
<td>16</td>
<td>0,0%</td>
<td>-0,9%</td>
<td>-0,5%</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long-Term Moody’s</th>
<th>Long-Term S&amp;P</th>
<th>EIU Score</th>
<th>Equity risk premium Moody’s</th>
<th>Equity risk premium S&amp;P</th>
<th>Equity risk premium EIU Score</th>
<th>Average Equity Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>Aaa</td>
<td>16</td>
<td>0,0%</td>
<td>-1,4%</td>
<td>-0,7%</td>
<td></td>
</tr>
</tbody>
</table>

Polynomial of degree 2:

\[ y = -2424.6x^2 + 605.09x + 25.216 \]

Debt risk premium verifies the following system:

\[ x \geq 0 \]

\[ 16 = -2424.6x^2 + 605.09x + 25.216 \]

Solution:

\[ \Delta = 455,514 \]

\[ x_1 = -1.4\% \]

\[ x_2 = 26.4\% \]

On that sheet, one realizes that none of the two roots give an acceptable result. \( x_2 \) is obviously way too high for Sweden. \( x_1 \) is negative so it does not respect the condition of positivity. What can we deduct from that? That our EIU score debt (resp. equity) risk premium is not working?
No, the explication comes from the part below of the trend line.

On that screenshot, one notices that some points are below the trend line at the origin. The trend line crosses the vertical axis at around 25. Sweden EIU Country Risk Score is 16. Sweden are among the points that are below the trend line at the origin (Damodaran debt risk premium is 0,0%). The negative value of one root simply results from the fact that the trend line cannot totally represent the correlation between Damodaran’s debt (resp. equity) risk premium and the EIU Country Risk Score. A regression is by definition an approximation at some point.

I have decided to replace negative roots by the value 0,0% by default. This value is in line with the observations made so far.
After retreatment, we have the following Excel sheet for Sweden.

<table>
<thead>
<tr>
<th></th>
<th>Long-Term Moody's</th>
<th>Long-Term S&amp;P</th>
<th>EIU Score</th>
<th>Debt risk premium Moody's</th>
<th>Debt risk premium S&amp;P</th>
<th>Debt risk premium EIU Score</th>
<th>Average Debt Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sweden</strong></td>
<td>Aaa</td>
<td>16</td>
<td></td>
<td>0,0%</td>
<td>0,0%</td>
<td>0,0%</td>
<td>0,0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Long-Term Moody's</th>
<th>Long-Term S&amp;P</th>
<th>EIU Score</th>
<th>Equity risk premium Moody's</th>
<th>Equity risk premium S&amp;P</th>
<th>Equity risk premium EIU Score</th>
<th>Average Equity Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sweden</strong></td>
<td>Aaa</td>
<td>16</td>
<td></td>
<td>0,0%</td>
<td>0,0%</td>
<td>0,0%</td>
<td>0,0%</td>
</tr>
</tbody>
</table>

Polynomial of degree 2:
\[ y = -2424,6x^2 + 605,09x + 25,216 \]

Debt risk premium verifies the following system:
\[ x \geq 0 \]
\[ 16 = -2424,6x^2 + 605,09x + 25,216 \]

Solution:
\[ \Delta = 455 514 \]
\[ x_1 = -1,4\% \quad 0,0\% \quad \text{(after retreatment)} \]
\[ x_2 = 26,4\% \]

The average debt (resp. equity) risk premium for Sweden is thus 0,0%.
ii) Mexico

The same Excel sheet but adapted to the case of Mexico:

<table>
<thead>
<tr>
<th>Long-Term Moody’s</th>
<th>Long-Term S&amp;P</th>
<th>EIU Score</th>
<th>Debt risk premium Moody’s</th>
<th>Debt risk premium S&amp;P</th>
<th>Debt risk premium EIU Score</th>
<th>Average Debt Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>A3</td>
<td>37</td>
<td>1,2%</td>
<td>1,4%</td>
<td></td>
<td>1,3%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long-Term Moody’s</th>
<th>Long-Term S&amp;P</th>
<th>EIU Score</th>
<th>Equity risk premium Moody’s</th>
<th>Equity risk premium S&amp;P</th>
<th>Equity risk premium EIU Score</th>
<th>Average Equity Risk Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mexico</td>
<td>A3</td>
<td>37</td>
<td>1,8%</td>
<td>2,1%</td>
<td></td>
<td>2,0%</td>
</tr>
</tbody>
</table>

Polynomial of degree 2:

\[ y = -2424,6x^2 + 605,09x + 25,216 \]

Debt risk premium verifies the following system:

\[ x \geq 0 \]

37 = -2424,6x^2 + 605,09x + 25,216

Solution:

\[ \Delta = 251,848 \]

\[ x_1 = 2,1\% \]

\[ x_2 = 22,8\% \]

In the case of Mexico, x1 appears to be a suitable result for the EIU score equity risk premium. The tool gives us an average equity (resp. debt) risk premium of 2,0% (resp. 1,3%) for Mexico.
III/ The modified Merton’s contingent claim analysis approach

Before going into the mathematics, I would like to give an overview of this approach. The general idea from Merton with his Contingent Claim Analysis theory (CCA) is that the price of a corporate bond is the same as a portfolio consisting of a long position in the same maturity risk-free zero coupon bond and a short position in a put option.

The parameters of the put option in that case are the following:

- the underlying “asset” is the company
- the strike price is the value of the total outstanding net liabilities of the company
- the maturity corresponds to the weighted average duration of the debt
- the volatility derives from the volatility of an option on a stock with the same maturity as the weighted average duration of the debt

Naturally, the mathematical background will be developed much more in a dedicated part after but let’s continue with the overview. Now we are able to price the corporate debt of the company. When we deal with debt or bonds, once we have the price, we are able to talk in terms of yields too. This means that if we subtract the yield-to-maturity of the risk-free zero coupon bond to the yield-to-maturity of the corporate debt computed according Merton’s CCA, we get the risk premium of the corporate debt.

This is an overall explanation of Merton’s CCA. The point is that Merton is dealing with corporate debt and then computes a risk premium. My goal is to compute a country risk premium related to the sovereign debt of a country. What I want to do is to apply Merton’s Contingent Claim Analysis to a country (instead of a company) and gets a country risk premium (instead of a corporate debt risk premium). This will require a certain number of adjustments.

A) Fundamentals: Black Scholes option valuation model

To clearly understand Merton’s Contingent Claim Analysis (CCA), it is important to first understand the fundamental theory which is behind. As Merton’s Contingent Claim Analysis derives from Black Scholes option valuation model, we will first have a review of this theory. In a second time, we will bridge the gap between Black Scholes and Merton theories.

I will develop the demonstration with a call option but similarly we can have a result for a put option using the put-call parity formula.
In the early 70s, Fischer Black and Myron Scholes set up a theoretical valuation formula for options. This research work is even more interesting because it allows to derive the price of numerous securities or portfolio of securities. Indeed a lot of them can be seen as a combination of options among other thing.

i) The hypotheses

The first step of the demonstration is the clarification of the hypothesis. Black Scholes option valuation model is based on “perfect” market assumptions. More precisely, the hypotheses are the following:

- there are no transaction costs linked to any operations undertaken on the market
- the information about market securities is the same for all investors
- it is allowed to buy, sell or borrow any fraction of the price of any security
- there is no bid-ask spread
- short selling is authorized without any additional costs
- trading of assets is continuous in time
- the short term interest rates are known with certainty and constant

These hypotheses are general hypotheses about the market and are the hypotheses that make the market “perfect”.

Besides these hypotheses, there are some additional conditions specific to the theory:

- stocks do not pay any dividends or other cash distribution
- the distribution of the stock price is lognormal
- the option is “European”. It can only be exercised at maturity time

ii) Delta hedging

With regards to all the hypotheses highlighted above, the price of the call option will only depends on the underlying stock and on time.

Thus, we can write the price of the call option as follow:

\[ C(S_t, t) \quad \text{with } S_t \text{ price of the underlying stock at time } t \text{ and } t \text{ the time} \]

Now, let’s build a hedged position consisting of a long position in the stock and a short position in the option.
To simplify computations, we consider that our long position in the stock is 1 unit. With a Taylor expansion around \( t=0 \) of the call option price, we get:

\[
C(S_t, t) = C(S_0, 0) + \frac{\partial C}{\partial S}(S_t - S_0) \quad (1)
\]

which is equivalent to \( \Delta S_t = \Delta C \times \frac{1}{\frac{\partial C}{\partial S}} \quad (2) \)

This is our delta hedge position. The change in value of the stock price will be offset by a change in value of the short position in \( \frac{\partial C}{\partial S} \Delta S_t \) options.

In a “perfect” world, the delta hedge position is adjusted continuously and the risk is thus 0.

I will justify more precisely the computation later, but so far it is enough to demonstrate Black and Scholes option valuation formula.

iii) Value of the delta hedge position

To obtain the price of the call option \( C \), we will first value our delta hedge portfolio. Within a short time interval \( \Delta t \), the change in the value of the position is as follow:

\[
\Delta S_t - \Delta C \times \frac{1}{\frac{\partial C}{\partial S}} \quad (3)
\]

Assuming the continuous update, we can use stochastic calculus (McKean 1969) to expand the \( \Delta C \) as shown below:

\[
\Delta C = \frac{\partial C}{\partial S} \Delta S + \frac{1}{2} \frac{\partial^2 C}{\partial (S)^2} \sigma^2 S^2 \Delta t + \frac{\partial C}{\partial t} \Delta t \quad (4)
\]

\( \sigma^2 \) is the variance of the underlying stock.

We, then, replace the expression of equation (4) in the equation of change in equity value (3). We find then that the change in equity value verifies:

\[
-\left(\frac{1}{2} \frac{\partial^2 C}{\partial (S)^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} \right) \frac{\Delta t}{\frac{\partial C}{\partial S}} \quad (5)
\]
Knowing the perfect market hypotheses, the return on the equity in the hedge position is not stochastic. As a consequence the return must be equal to \( r_f \Delta t \), where \( r_f \) is the instantaneous expected risk free rate of return. Indeed the hedge position is updated continuously.

So equation (5) must be equal to equation (2) times \( r_f \Delta t \), which results in

\[
- \left( \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 + \frac{\partial C}{\partial t} \right) \frac{\Delta t}{\partial S} = (S - C + \frac{1}{2} \frac{\partial C}{\partial S}) r_f \Delta t \quad (6)
\]

After simplification, we have the following differential equation:

\[
\frac{\partial C}{\partial t} = r_f C - r_f S \frac{\partial C}{\partial S} - \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \quad (7)
\]

To be able to resolve this equation (7) and find a solution, we need boundary condition. If we call the maturity time \( t^* \) and the strike price \( K \), we have the boundary condition:

\[
C(S, t^*) = \max(S - K, 0) \quad (8)
\]

There is only one solution to the couple (7) and (8) and this is the option valuation formula.

iv) The option valuation formula

To solve the couple (7) and (8) and get the option valuation formula, Black and Scholes suggest to write \( C \) as follow:

\[
C(S, t) = e^{r_f(t-t^*)} y \left( \frac{2}{\sigma^2} \left( r_f - \frac{\sigma^2}{2} \right) \left( \ln \frac{S}{K} - \left( r_f - \frac{\sigma^2}{2} \right) (t - t^*) \right) - \frac{2}{\sigma^2} \left( r_f - \frac{\sigma^2}{2} \right) (t - t^*) \right) \quad (9)
\]

In expression (9), \( y \) represents an unknown function of the couple of variables \( (S,t) \). At first sight, this expression seems to come out of nowhere, but this is a clever move from Black and Scholes because \( y \) verifies the heat-transfer equation of physics.

Heat-transfer equation:

\[
\frac{\partial y}{\partial t} = \frac{\partial^2 y}{\partial S^2} \quad (10)
\]

with the boundaries condition:

\[
y(S, 0) = 0 \text{ if } S < 0
\]
By solving (10) and replacing the result in equation (9), we finally find the option valuation formula that we all know. I have intentionally not developed the detail of the computation because it is not in the topic of my thesis. I wanted yet to finish the demonstration.

The resulting option valuation formula:

\[ C(S, t) = SN(d_1) - Ke^{r(t-t^*)}N(d_2) \quad (11) \]

\( N \) is the cumulative normal density function and:

\[ d_1 = \frac{\ln \frac{S}{K} + (r + \frac{\sigma^2}{2})(t^* - t)}{\sigma \sqrt{t^* - t}} \]

\[ d_2 = d_1 - \sigma \sqrt{t^* - t} \]

Equation (11) along with the definition of \( d_1 \) and \( d_2 \) is widely used to price option. We can derive the price of a put option from equation (10) thanks to the put-call parity.

v) More precision about the delta and justification of the delta hedge

Now that Black and Scholes option valuation formula is demonstrated, one can include this formula in the reflection. Racicot and Théoret (2006) suggest indeed to start the demonstration with Black and Scholes option valuation. They add that, by definition, the delta of a call option is the partial derivative of the price of the call option against the price of the underlying.

In our case this results in the following formula:

\[ \Delta = \frac{\partial C}{\partial S} = N(d_1) + S\frac{\partial N(d_1)}{\partial S} - Ke^{r(t-t^*)}\frac{\partial N(d_2)}{\partial S} \quad (12) \]

\[ = N(d_1) + SN'(d_1)\frac{\partial d_1}{\partial S} - Ke^{r(t-t^*)}N'(d_2)\frac{\partial d_2}{\partial S} \quad (13) \]

where \( \frac{\partial d_1}{\partial S} = \frac{1}{\sigma \sqrt{(t^* - t)}} \) and \( \frac{\partial d_2}{\partial S} = \frac{1}{\sigma \sqrt{(t^* - t)}} \), one can infer the following equation:

\[ \Delta = N(d_1) + (SN'(d_1) - Ke^{r(t-t^*)}N'(d_2))\frac{1}{3\sigma \sqrt{t^* - t}} \quad (14) \]

To simplify (14), Racicot and Théoret suggest to demonstrate that \( SN'(d_1) = Ke^{r(t-t^*)}N'(d_2) \) (15)
We know that \( N(d) \) is a bounded integral (the infinite limit is 1). An analysis theorem states that the derivative of a bounded integral is the primitive function evaluated at the boundaries.

Thus \( N'(d_1) = \frac{e^{-\frac{(d_1)^2}{2}}}{\sqrt{2\pi}} \) and \( N'(d_2) = \frac{e^{-\frac{(d_2)^2}{2}}}{\sqrt{2\pi}} \)

Equation (15) can be written as follow: \( S \frac{N'(d_1)}{N'(d_2)} = Ke^{rf(t-t^*)} \) (16) by replacing the terms of the fraction by their value, it comes:

\[
Se^{\frac{1}{2}(d_2^2-d_1^2)} = Ke^{rf(t-t^*)} \quad (17)
\]

This is this equation (16) that Racicot and Théoret demonstrate. I will not give here the details of the calculation because the expansion of the squares is very long. But equation (17) is verified and this results in the following value of the delta.

\[
\Delta = N(d_1)
\]

The delta is of major importance for derivative securities because it is commonly used for hedge operations. To hedge a portfolio of one stock, one need to short sell \( 1/\Delta \) call option to build the hedge. \( 1/\Delta \) is the ratio of the hedging.

To establish this relation, one suppose that we have one stock \( S \) and that one have short \( sC \) call options. One wants to find the “s” that gives the delta hedging, i.e that verifies:

\[
dS - sdC = 0 \quad (18)
\]

The approximation of the first degree of \( dC \) is equal, thanks to Taylor expansion to:

\[
dC = \Delta dS \quad (19)
\]

Finally, one get \( s=1/\Delta \).

So, to have a delta-hedged portfolio, one needs to have a stock and have short \( 1/\Delta \) call options. Alternatively, one needs to short one call option and have \( \Delta \) stocks for the same result.

For the portfolio to remain hedged, it is necessary to update continuously the composition. Indeed, the delta of a call option keeps changing. The delta changes when the stock price changes. A rebalancing is then needed. To make optimized rebalancing, one should look at an other parameter of the call option: the gamma. This is not the topic of this paper.
This demonstration of the delta hedge is useful because it justifies the assumption made to compute Black and Scholes valuation formula. From now on, one can move to Merton’s contingent claim analysis.

B) Theoretical background of the model: Merton’s contingent claim analysis

The idea of this part is to link Merton’s contingent claim analysis with Black Scholes option valuation model. To do so, I will develop Merton’s reflection until the convergence with Black Scholes.

i) The hypotheses

Merton has based his work on the same assumptions that Black and Scholes but has also used two additional hypotheses:

- application of the Modigliani-Miller theorem. The value of company does not depend on its leverage.
- the dynamic for the value of a company, V, follows a diffusion-type stochastic process with a stochastic differential equation:

\[ dV = (rV - C)dt + \sigma Vdz \] (20)

where \( r \) is the instantaneous expected rate of return on the company per unit of time, \( C \) is the total payout of the firm per unit of time, \( \sigma \) is the standard deviation of the return and \( dz \) a standard Gauss-Wiener process.

ii) Convergence with Black Scholes model

To start the demonstration, we construct a portfolio of three different securities:

- the firm
- a risk-free debt
- a specific security, Y, whose value depends only on the value of the firm and time.

According to the last hypothesis made above, we can write the following stochastic equation for Y:

\[ dY = (r_y Y - C_y)dt + \sigma_y Ydz_y \] (21)

The parameters are the same as for the company V but applied to the security.

Next to this formula, we can also write \( Y = F(V, t) \). And Itô’s Lemma tells us:
\[ dY = \left( \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 F}{(\partial V)^2} + (rV - C) \frac{\partial F}{\partial V} + \frac{\partial F}{\partial t} \right) dt + \sigma V \frac{\partial F}{\partial V} dz \]  

(22)

By identification between equation (10) and (11):

\[ r_y Y - C_y = \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 F}{(\partial V)^2} + (rV - C) \frac{\partial F}{\partial V} + \frac{\partial F}{\partial t} \]  

(23)

\[ \sigma_y Y = \sigma V \frac{\partial F}{\partial V} \]  

(24)

\[ dz_y = dz \]  

(25)

Now, that we know the properties of the securities forming the portfolio, we define a position such that the aggregate investment in the portfolio is zero.

Let \( W_1 \) be the investment in the firm, \( W_2 \) the amount put in the specific security and \( W_3 \) the short position in the risk free debt such that \( W_3 = -(W_1 + W_2) \). If we name \( dx \) the instantaneous expected return of the portfolio then:

\[ dx = W_1 (r + \sigma dz) + W_2 (r_y + \sigma_y dz_y) + W_3 \gamma dt \]  

(26)

and combined with equation (20), (21) and (25),

\[ dx = (W_1 (r - \gamma) + W_2 (r_y - \gamma)) dt + (W_1 \sigma + W_2 \sigma_y) dz \]  

(27)

where \( \gamma \) is the instantaneous risk free rate.

We will make an other assumption. We will choose a portfolio strategy such that the stochastic coefficient of \( dz \) is always 0. The return of the portfolio is by matter of fact non stochastic. Thus, to respect an arbitrage free market and since the portfolio requires zero net investment, the associated return has no other choice than being equal to 0. In terms of equation, this can be written as follow:

\[ (W_1 \sigma + W_2 \sigma_y) = 0 \text{ (no risk)} \]  

(28)

\[ W_1 (r - \gamma) + W_2 (r_y - \gamma) = 0 \text{ (no arbitrage)} \]  

(29)

A solution \( (W_1^*, W_2^*) \) different from zero exists if and only if

\[ \frac{(r-\gamma)}{\sigma} = \frac{(r_y-\gamma)}{\sigma_y} \]  

(30)

Using equation (23) and (24) for substitution of \( r \) and \( r_y \), we obtain a parabolic partial differential equation for \( F \). More generally, the following equation must be satisfied by any security whose value is a function solely of the value of the company and time.
\[ 0 = \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 F}{(\partial V)^2} + (rV - C) \frac{\partial F}{\partial V} - \gamma F + \frac{\partial F}{\partial \tau} + C_y \] (31)

As this equation is a second degree one, we need two boundaries conditions and an initial condition to define a unique security.

We will now create those boundaries and initial conditions in the case of a corporate bond, which does not pay coupon. We just define a maturity time \( T \) and an amount \( B \) to be paid to the bondholders at maturity time. \( B \) is the outstanding liabilities of the company.

\( Y = F(V,t) \) is the value of the debt at time \( t \). To have a proper initial condition, one introduces the variable \( \tau = T-t \).

- **Boundaries conditions**

\( F(0,\tau) = 0 \) if the value of the firm is zero, the value of the debt is necessarily 0.

\( F(V, \tau) \leq V \) bondholders cannot claim more value than the firm value

- **Initial condition**

\( F(V, 0) = \min[V,B] \) this condition means that if the firm goes bankrupt at maturity date, bondholders claim on the firm assets else they get the promise payout \( B \).

Equation (31) becomes:

\[ 0 = \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 F}{(\partial V)^2} + rV \frac{\partial F}{\partial V} - \gamma F - \frac{\partial F}{\partial \tau} \] (32)

The reasons are:

- \( C_y = 0 \) because it is a zero coupon bond
- \( C = 0 \) because the firm does not pay dividends
- \( \frac{\partial F}{\partial t} = - \frac{\partial F}{\partial \tau} \) because \( \tau = T-t \)

With all the conditions mentioned above, it becomes possible to solve equation (32) directly with Fourier transforms or separation of variables. But there is a clever way of proceeding. It is at this moment that we converge with Black Scholes model.

If \( f(V, \tau) \) is the corresponding equity value of the company \( V \), one get the following relation with debt value:
\[ f(V, \tau) = V - F(V, \tau) \quad (33) \]

And by replacing (33) in the equation (32), one obtains

\[
0 = \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 f}{(\partial V)^2} + rV \frac{\partial f}{\partial V} - r_f f - \frac{\partial f}{\partial \tau} \quad (34) \text{ or } 
\]

\[
0 = \frac{1}{2} \sigma^2 V^2 \frac{\partial^2 f}{(\partial V)^2} + rV \frac{\partial f}{\partial V} - r_f f + \frac{\partial f}{\partial \tau} \quad (35) \text{ because } \frac{\partial f}{\partial t} = -\frac{\partial f}{\partial \tau} \]

Moreover the initial condition becomes \( f(V, 0) = \max[V-B,0] \).

With equation (35) and the new initial condition on \( f \), we recognize the same equation as in Black Scholes demonstration (7) and the same initial condition (8). The two reflections converge from that moment. Thus, equity of a company can be viewed and consequently priced as a call option.

The call option has the following parameters:

- the underlying “asset” is the company V
- the strike price is the value of the total outstanding net liabilities of the company B
- the maturity corresponds to the weighted average duration of the debt \( T \)
- the volatility derives from the volatility of an option on a stock with the same maturity as the weighted average duration of the debt

We are just one step away to be able to value corporate debt with an option valuation formula.

Equation (33) tells us that the value of the corporate debt is the difference between the company value \( V \) (the “underlying”) and the equity value (the “call option”). This configuration lets us think to the put-call parity.

According the put-call parity, we have the following equality:

\[
C_t + e^{-r_f(T-t)}B_0 = P_t + S_t \quad (36)
\]

where \( C_t \) is the call option price at time \( t \) with maturity \( T \)

\( r_f \) is the risk free rate

\( B_0 \) is the face value of a risk free zero coupon bond with maturity \( T \)

\( P_t \) is the put option price at time \( t \) with maturity \( T \)

\( S_t \) is the underlying price a time \( t \)
By identifying the value of equity \( f(V,t) \) to \( C_t \) and the company \( V \) to \( S_t \) and reorganizing the put-call equality, it comes that the corporate debt value can be seen as the difference between the zero coupon bond and the put option.

As unveiled at the beginning of this part, we finally have the demonstration that the price of a corporate bond is the same as a portfolio consisting of a long position in the same maturity risk-free zero coupon bond and a short position in a put option. The put option has the same parameters than the call option mentioned above.

C) Adaptation to the estimation of country risk

So far, it is has been demonstrated that a corporate debt can be priced with a portfolio consisting of a risk-free zero coupon bond and a put option.

Just as a reminder, the parameters of the put option in that case are:

- the underlying “asset” is the company
- the strike price is the value of the total outstanding net liabilities of the company
- the maturity corresponds to the weighted average duration of the debt
- the volatility derives from the volatility of an option on a stock with the same maturity as the weighted average duration of the debt

There is a final step to get a country risk premium with this approach. Indeed, we need to transpose the corporate debt into a sovereign debt and find an equivalent for the enterprise value in the case of a country.

The simplest way to proceed is to breakdown the parameters of the put option one by one and adapt them to the study of a country.

i) The underlying “asset”: the company becomes the country

The price of an option depends on the underlying asset on which the derivative is built. Thus, we have to find a good proxy for the “enterprise value” of a country. The country is the underlying in our case.

The idea is to value the country’s possibility to generate surplus discounted at the cost it requires (over a certain number of years). This surplus can be seen as Free Cash-Flow. We can then apply the Discounted Cash-Flow valuation method and get the “enterprise value” of the country.
The Discounted Cash-Flow method is a widely used method to value assets and companies. The method appeared in the aftermath of the Great Depression. Irvine Fisher (1930) and John Burr William (1938) were the first researchers to formally defined this method in modern economic terms.

From this period, there have been continuous researches and extensions on this method. Gordon (1962) extended William model by introducing a dividend growth component. More recently, in the 90’s and early 2000’s, several papers have been published with a special focus on modelling free cash flow (Coppeland, Koller and Murrin, 2000 – Rappaport 1998).

In concrete terms, the Discounted Cash-Flow method (DCF method) is clearly explained in the book Corporate Finance 3rd Edition of Berk and DeMarzo (2014). The method is explained for the valuation of a random company. I will give details in the coming paragraphs what concerns the adaptation to the “valuation” of a country.

Berk and DeMarzo defines the DCF method as follow: to estimate the value of a company, we calculate the present value of the free cash flow. By free cash flow, they mean the cash available to pay the investors (both debt and equity). The second point is to determine which discount rate to use. In the case of the valuation of a company, it is the Weight Average Cost of Capital (WACC). The adaptation to the “valuation” of a country will also come after. The last point, we need to know about the DCF method is how to compute the terminal year value.

For Berk and DeMarzo, it should be computed as follow:

\[ \text{Terminal year value} = \frac{FCF_{\text{terminal year}}}{\text{wacc} - g} \]

where \( g \) is the long term growth prevision

a. the primary surplus

The question is now to precisely define how to compute this surplus. This surplus is simply the subtraction of country’s income less country’s outcome. The income is quite easy to define. As we are dealing with a country, the income comes from the taxation of goods and services on the national territory. The question is much more difficult when it comes to the country’s outcome. Indeed, spendings of a country are multiple. It ranges from military to education over administration.

If not impossible, it will surely take too much time to get to a value. Fortunately, we can bypass the direct computation of the subtraction. On the website of the great majority of Central Bank, we can find projections about the primary surplus (deficit). See definition below.
Definition of the primary surplus:

Countries’ primary surplus (deficit) refers to the component of the fiscal surplus (deficit) that is comprised of current government spending less current income from taxes, and excludes interest paid on government debt. If a country has larger levels of income relative to current spending, it is said to have a primary surplus; if a country has larger levels of current spending relative to income, it is said to have a primary deficit.

Source: http://www.merkfunds.com/currency-asset-class/glossary/primary-surplus.html

This definition of primary surplus seems totally suitable to the country’s surplus we are looking for for our valuation. From now on, when we refer to surplus it is the primary surplus. Once we have the surplus, we need to find the appropriate rate to discount it.

b. the discount rate

Finding the most suitable discount rate is probably the trickiest point among all the hypotheses (from a theoretical point of view). As we will use Discounted Cash-Flow valuation method, we make easily an analogy with the discount rate used when valuing companies. This discount rate is the Weighted Average Cost of Capital (WACC). However, unlike private or public companies, a country can raise funds only through debt and bond issues. A country cannot issue equity. The WACC seems in such circumstances not the appropriate discount rate because it takes into account the debt cost of capital as well as the equity cost of capital.

Thus, the discount rate can only be a cost of debt. If we look precisely to the components of the sovereign debt, we realize that the largest amount is domestic debt (in local currency). Moreover domestic debt is much more liquid than foreign debt. The domestic interest rate is the rate we will thus use for our valuation of the “enterprise value” of the company.

Knowing the primary surplus and the domestic interest rate, we can compute the “enterprise value” of the company. I will give further details in the case studies of different countries in part III/.

ii) The strike price: the total outstanding net liabilities of the company becomes the total outstanding net liabilities of the country

Determining the total outstanding liabilities of a country is not as straightforward as one may expect. The information cannot be found one a couple of line in a balance sheet like in the case of traded companies. There are lots of different types of public debt. It is important to consider all those debts. It is also important to do the distinction between internal and foreign liabilities. Both kinds of
liabilities should be taken into account. It is of major importance to have an exhaustive vision on the subject. Indeed, it is this parameter which will tell us if the country is “bankrupt” or not.

Practically, the information on the total amount of internal and foreign liabilities are given on the website of all the Central Banks. The information is generally given as a percent of the GDP.

iii) The maturity

The maturity of the put option corresponds to the weighted average duration of the liabilities mentioned above. There are not many more things to say about this parameter. The main hurdle is the difficulty to find data on it. I will develop this point later in the case studies.

iv) The volatility

The last parameter to explain is the volatility of the option. There are two possibilities to tackle the issue of the volatility. The first one, and the most natural too, is to take the volatility of the stock market of the country as a proxy of the volatility of the underlying. But this solution presents a major flaw. Indeed, the volatility of the market depends on the leverage of the traded companies. If we want to have an accurate value of the volatility for the country, we need to de-lever the market volatility and then re-lever it with the leverage level of the sovereign. This task is far from being easy.

The second possibility is to use the foreign exchange market as a proxy. Of course, this possibility works only if the country pursues a floating exchange. The volatility of the exchange rate is a good proxy for the volatility of the underlying asset (in our case the country) because a strong volatility of the exchange rate in a country against a reference currency (in our case the reference currency is the US dollar) is the consequence of large variation in the economic health of the country.

The decision to choose one possibility rather than another depends on the country we are analyzing but generally the second possibility is preferred. The volatility of the exchange market is directly observable and do not need a de-leverage and re-leverage process. All parameters are now defined, before moving to the case studies, I would like to introduce an alternative model.

D) An alternative way to mobilize financial mathematics for country risk

One considers a given country and a random financial security, whose price at time $t$ is $Z_t$.

One suppose that the historical probability $P$ of this security has the following instantaneous parameters: a return $m_t^Z$ and a volatility vector $\sigma_t^Z$. Moreover, one defines $\hat{W}_t = (\hat{W}_t^1, ..., \hat{W}_t^n)^T$, the column vector of the $n$ independent standard brownian processes, representing the $n$ sources of hazards which affect the price.
Under the objective probability $P$, the process $Z_t$ of the asset price follows the following dynamic (El Karoui 2004):

$$\frac{dZ_t}{Z_t} = m_t dt + \langle \sigma_t^Z, d\tilde{W}_t \rangle$$

Without arbitrage opportunity, there exists a risk premium $\lambda_t$, so that for any security $Z_t$,

$$m_t^Z = r_t + \langle \sigma_t^Z, \lambda_t \rangle.$$ 

One then defines a risk neutral probability $Q$, as the probability under which the process $W_t$ defined by $dW_t = d\tilde{W}_t + \lambda_t dt$ is a standard $Q$-Brownian motion.

In this universe, called risk neutral universe, all securities have for instantaneous return the spot risk free rate $r_t$.

$$\frac{dZ_t}{Z_t} = r_t dt + \langle \sigma_t^Z, d\tilde{W}_t + \lambda_t dt \rangle = r_t dt + \langle \sigma_t^Z, dW_t \rangle$$

This approach is interesting on a mathematical basis but the major limit is that it is almost impossible to put it in practice. There are too many parameters. Moreover the risk premium $\lambda_t$ certainly includes country risk but not only. This is the overall risk of the security. One could assume that one could have a good proxy for the country risk by considering the market portfolio instead of a specific security. This is theoretical and in practice it is not the same thing. Besides, this strategy will work only with mature economy (economy where one can build a market portfolio) while the interest of country risk is for emerging countries. For all these reasons, I chose to concentrate the case study on the modified contingent claim analysis.
IV/ The modified Merton’s contingent claim analysis approach: case studies

The goal of this master thesis is to accurately compute a country risk premium. The mathematical approach has been theoretically developed. We will now confront it with the data available. I will make two case studies. One with a developed country (Sweden) and one with an emerging country (Mexico).

The case study on Sweden will serve to validate the mathematical approach. Data are not too difficult to find and we expect the country risk premium that should be found (around 0%).

The case study on Mexico has much more interest because it will give us an estimation of the country risk premium of this country. We will then be able to confront it with the country risk premium found with the corporate finance approach. And then draw conclusions.

A) Practical computation method of the country risk premium

This part is intended to explain in a clear way how we get to the country risk premium. First, some parameters are defined:

- \( L_{tot} \) is the total outstanding liabilities of the country.
- \( MV_r \) is the market value of the risk free security
- \( MV_c \) is the market value of the debt of the country
- \( r_f \) is the risk-free rate
- \( CRP \) is the country risk premium
- \( T \) is the average duration of the sovereign debt
- \( P \) is the put option. The put option has the following parameters:
  - \( L_{tot} \) is the strike price
  - The “Enterprise Value” of the country is the underlying
  - \( T \) is the maturity
  - \( \sigma \) is the volatility of the underlying

Thanks to the demonstration made in part II/, we can assert that:

\[
MV_c = MV_{rf} - P \quad (37)
\]

Moreover, we have:

\[
MV_{rf} = \frac{L_{tot}}{(1+r_f)^T} \quad (38)
\]
and

\[ MV_c = \frac{L_{\text{tot}}}{(1 + rf + CRP)^T} \]  \hspace{1cm} (39)

From the first equality, it comes:

\[ \frac{L_{\text{tot}}}{(1 + rf + CRP)^T} = \frac{L_{\text{tot}}}{(1 + rf)^T} - P \]  \hspace{1cm} (40)

\( L_{\text{tot}} \), \( rf \) and \( T \) are input data. So, once we have computed the value of \( P \), we can isolate \( CRP \) and get a value.

Before going further, I would like to give a precision about the country risk premium. Actually, this is the country risk premium for investment in debt. This country risk premium can also be called debt risk premium. This detail will be important in a second time, while comparing this premium with the premium computed according the corporate finance approach.

To compute \( P \), we just need to quantify all the parameters of the put option and then apply Black-Scholes option valuation formula. Below is a review of the parameters.

i) “Enterprise Value” of the country

The “Enterprise Value” is a parameter used as the underlying value for the Black-Scholes Valuation of the put option.

   a. construction of the DCF model

As said in II/C(ii), the “Enterprise Value” is computed with the Discounted Cash-Flow valuation method. To do this valuation, we need to find information about the primary surplus of the country and the discount rate.

I will construct the model over 4 years (2016-2019) with a terminal year in 2020. I choose this time period because it is the time period commonly used to value companies.

The reference date used for the valuation is January 1st, 2016. It is an arbitrary choice but it allows to have four entire years to discount (2016-2019). If I would have chosen September 30th, 2015 for example, I would have taken into account only a quarter of 2015. The impact on the value would have been limited. The reason, why I chose January 1st, 2016, is to avoid any useless complication of the model.
I have also retained a “mid-year convention”. This means that 2016 will be discounted with the following coefficient:

\[
\frac{1}{(1+\text{discount rate})^{0.5}} \quad (41)
\]

Instead of the commonly used coefficient (“end of year convention”):

\[
\frac{1}{(1+\text{discount rate})^{1}} \quad (42)
\]

For 2017, it will be:

\[
\frac{1}{(1+\text{discount rate})^{1.5}} \quad (43)
\]

Instead of:

\[
\frac{1}{(1+\text{discount rate})^{2}} \quad (44)
\]

And so on for 2018 and 2019.

The signification of this assumption is that the cash-flow generated by a country is spread over the year. Thus, in average, this cash-flow occurs at the middle of the year. That is why the power at the denominator are 0,5 1,5 2,5 ...

b. Sources of the parameters

Now that the model is constructed, let’s focus on the parameters that are used in (discount rate, GDP, primary surplus and inflation).

I will not detail those parameters (it is already done in the previous part).

The discount rate comes from a Bloomberg extraction.

For the others parameters (GDP, primary surplus and inflation), all the information come from IMF forecasts. I use directly the data for GDP and inflation.

Concerning the primary surplus, there is a little difference between Sweden and Mexico.
For Sweden:

- I make an adjustment for the primary surplus. In the IMF data, I only have *General Government Revenue* and *General Government Total Expenditure*. By subtracting the two value, I consider that I have a good proxy of the primary surplus of the country.

For Mexico:

- I make the same adjustment than for Sweden. But as Mexico is an emerging country, I use a more accurate proxy for the primary surplus. The second adjustment, I make, is that I retreat the *General Government Total Expenditure* from the interests paid on the debt (information provided by the World Bank Data base).

  ii) The strike price $L_{tot}$

The IMF provides also country’s outstanding liabilities each year. This is the source for $L_{tot}$

  iii) The maturity $T$

This is the country’s average debt maturity. This is the most difficult information to find. Indeed, the OECD study of this parameter stopped in 2010. The information exists but is unfortunately not updated since 5 years. I made the decision to retain the 2010 value rounded to the closer integer. I have not found more recent information so far.

  iv) The volatility $\sigma$

The information about the volatility of the exchange rate can be found on the following website:
http://vlab.stern.nyu.edu

B) The first case study: Sweden

For Sweden, we have the following data:

- $L_{tot} = 722,722$ SEK
- $T = 5$ years
- $r_i$ is the rate of the 5 year US Treasury bond
- $\sigma = 11,85\%$
We also have the following DCF model for the “Enterprise Value”:

The “Enterprise Value” of Sweden is thus 11 162 SEKb.

Now, that we have gathered all the necessary information, we can compute the price $P$ of the put option with Black Scholes formula.

To do so, I will use R. This results in the following price $P$ for the put option.

\[
P = 1.44e-27\text{ SEKb.}
\]

To make it simple, the price of the put option is 0 SEK.

Besides that,

\[
MV_{rf} = \frac{722,722}{(1 + 0.0228)^5}
\]
The market value of the risk-free debt is thus 646,681 SEKb.

It results that

\[ MV_c = 646,681 - 1,44e^{-27} \]

We can conclude that

\[ MV_{rf} = MV_c \]

With (38) and (39), we get

\[ \frac{L_{tot}}{(1 + r_f)^T} = \frac{L_{tot}}{(1 + r_f + CRP)^T} \]

Hence, for Sweden, CRP = 0,0%.

One might think that all this reflection is useless if we find a premium which is 0. On the contrary, it is really interesting. Indeed, Sweden is one of the most advanced economy in the world (along with Western Europe, the UK, the USA, ...). It is then logical that the Country Risk Premium of Sweden is 0.

The interest of performing a case study on Sweden is to demonstrate that the mathematical approach works. We found the expected Country Risk Premium. From now on, we can use this tool with confidence for emerging countries.

I will now make the same analysis with a Central American country, Mexico. This is an emerging country with a growing economy but with still a lot of inequality. A Country Risk Premium is thus expected. Let’s have a look at it.

C) Second case study: Mexico

For Mexico, we have the following data:

- \( L_{tot} = 8993,631 \text{ MXN}b \)
- \( T = 8 \text{ years} \)
- \( r_f \) is the rate of the 8 year US Treasury bond
- \( \sigma = 12,72\% \)
We also have the following DCF model for the “Enterprise Value”:

\[
P = \frac{1745.35\text{ MXNb}}{0.79}
\]

Besides that,

\[
MV_{rf} = \frac{8993.631}{(1 + 0.0290)^9}
\]

The market value of the risk-free debt is thus 7 155,040 MXNb.

It results that

\[
MV_c = 8993.631 - 1745.35
\]
We can conclude that

\[ MV_c = 5,409,690 \text{ MXN} \]

With (27) and (28), we get

\[
\frac{L_{tot}}{(1 + r_f)^T} = \frac{L_{tot}}{(1 + r_f + CRP)^T}
\]

Hence, for Mexico, CRP = 3.7%.
V/ Confrontation of the two approaches

Below is a summary table of the premia calculated with the two approaches (modified Merton’s contingent claim analysis and Damodaran’s extension).

<table>
<thead>
<tr>
<th></th>
<th>Modified Merton’s contingent claim analysis</th>
<th>Damodaran’s extension</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Debt risk premium</td>
<td>Equity risk premium</td>
</tr>
<tr>
<td>Mexico</td>
<td>3,7%</td>
<td></td>
</tr>
<tr>
<td>Sweden</td>
<td>0,0%</td>
<td></td>
</tr>
</tbody>
</table>

On that table, one first notices that the results of the two methods globally converge. For both Sweden (a developed country) and Mexico (an emerging country) the premia are in the same magnitude. This is even more striking for Sweden because the two methods result in the same premia: 0,0%. In the case of Mexico, the modified Merton’s contingent claim analysis approach gives a premia that is 2,4% larger than Damodaran’s extension.

Before addressing the possible explanation of the gap between the premia for Mexico, I would like to underline an advantage of Damodaran’s extension. This is striking if one looks to the table above. Damodaran’s extension allows to compute a debt risk premium as well as an equity risk premium while the modified Merton’s contingent claim analysis approach only focuses on the debt risk approach. From that point of view, Damodaran’s extension has more content.

Besides that, what can explain the difference between the modified Merton’s contingent claim analysis and Damodaran’s extension in the case of Mexico? To my mind, the explanation comes from the fact that the modified Merton’s contingent claim analysis approach is more theoretical than the second approach. Indeed, by addressing the problem of country risk premium from a theoretical angle, one tends to browse all the hypotheses from the worst to the better. Thus, one tends to adapt a wise position. However, this is just a supposition. To conclude that the modified Merton’s contingent claim analysis approach is more careful than Damodaran’s extension, we would need to compare both approaches on a large sample of various countries. This could be the topic of an other paper.

Moreover, there is also the following explanation: the logic behind the two approaches are really different. In the modified Merton’s contingent claim analysis, the idea is to look at the debt of the country in the whole. One says such country has such amount of debt with such characteristics and thus it results in such risk profile. The risk profile is then quantified by the debt risk premium. In
Damodaran’s extension (and more generally in the whole Damodaran reflection process), The logic is the opposite. The reflection starts from information about the risk profile of the country in general (rating agencies or EIU Country Risk Score). The risk profile is then impacted on the debt of the country and results in the debt risk premium. To sum up, in the modified Merton’s contingent claim analysis, one goes from the debt to the risk and in Damodaran’s extension, one goes in the other direction (from the risk to the debt). The first approach is totally focused on the debt and the model is only suitable to deal with debt risk issue, while Damodaran’s approach sees the debt as a part of the risk associated to a country.

What is certain is the difference of philosophy between the two approaches. The strength of the modified Merton’s contingent claim analysis approach is its strong theoretical background while the strength of the corporate value is the reliability of its data.

Indeed, even if the theory is accurate in the modified Merton’s contingent claim analysis approach, it still remains an uncertainty on the data. As this approach requires lots of data that are not easily available, one can be tempted to use incomplete data, unreliable sources, not updated data or other data that are similar. Even in this paper I have been confronted to this problem.

In the end, I do not think that both methods are comparable. Their use depends on the context of work. I consider those methods complementary. The modified Merton’s contingent claim analysis approach brings the theoretical justification while Damodaran’s extension gives an indication on the value of the premia used in the financial industry.
VI/ Conclusion

Measurement and valuation of country risk: how to get a right value?

This question leads my entire reflection along this thesis. It appears that there is no unique answer. Actually, the subject of country risk is very vast and can be addressed from different points of view. I have chosen two approaches in my thesis. I am not exhaustive at all but I had to make a choice. I wanted to avoid to lose myself in too many explanations.

So what can we say to the research question? First, both methods, even if they do not give exactly the same results, give the same magnitude for the country risk value (for more confidence about the subject, one could test the approach on a larger sample of countries). The lesson we get from this observation is that an “academician” method and a “practitioner” method can converge in the conclusion. There is no ranking between the two types of methods. Both have their pros and cons.

Secondly, it seems rather difficult so far to get a unique value for the country risk premium. Using different methods is good because it gives a range of possible value. For example, in our case study with Mexico, we finally ends up with a range of possible value for the debt risk premium included between 1.4% and 3.7%. This range can, of course, be reduced. Discussion with local financial analysts, specific information about the deal conducted in the country, benchmark with comparable deals or countries are kind of things that can be done to reduce this range.

In the end, my conviction is that it is impossible to get a “right” value only with calculus. Calculus, like those I have done, give a good insight on the situation in a country. To really get close to the “right” (by “right” I mean which better reflect the overall situation of the country) value, it is necessary to understand the cultural specificities of a country and their impact on the business environment. This cannot be done in a thesis nor theorize in a paper. Those things are to be found on the ground with experience.

That is why this topic is particularly interesting. Indeed, it gathers intellectual reflection and cultural opening. In this thesis, I stay within the intellectual side of the problem. There is on this topic still lots of thing to dig in and the question finally remains open for more accurate analysis.
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