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Monte Carlo criticality calculations accelerated by a growing neutron population

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Abstract

We propose a fission source convergence acceleration method for Monte Carlo criticality simulation. As the efficiency of Monte Carlo criticality simulations is sensitive to the selected neutron population size, the method attempts to achieve the acceleration via on-the-fly control of the neutron population size. The neutron population size is gradually increased over successive criticality cycles so that the fission source bias amounts to a specific fraction of the total error in the cumulative fission source. An optimal setting then gives a reasonably small neutron population size, allowing for an efficient source iteration; at the same time the neutron population size is chosen large enough to ensure a sufficiently small source bias, such that does not limit accuracy of the simulation.

Key words: Monte Carlo criticality, fission source, convergence, bias, convergence acceleration

1. Introduction

In order to obtain reliable results, the fission source in Monte Carlo criticality calculations must be sampled from its steady-state distribution. As the steady-state fission source is generally not known, a number of inactive cycles are simulated first, during which the fission source is expected to converge from its initial (usually guessed) state to the steady state. Since no results are collected during the inactive cycles, the computing time of the inactive cycles can be considered as lost. Simulation results are combined only over a number of active cycles during which the fission source is assumed to be converged. The efficiency of Monte Carlo criticality calculations thus may be improved by methods that try to reduce the computing cost of the inactive cycles (see the works by Yamamoto and Miyoshi (2004); Kadotani et al. (1991); Kitada and Takeda (2001); Dufek and Gudowski (2009); Brissenden and Garlick (1986)).

Tuttelberg and Dufek (2015) demonstrated that the efficiency of Monte Carlo criticality calculations is sensitive to the selected neutron population size—the number of neutron histories simulated at each cycle (generation), and they suggested an optimisation procedure for choosing the optimal population size. This optimisation was done with respect to the computational time allocated beforehand the simulation, and the population size was set at a fixed value common to all cycles—as is a standard practice in Monte Carlo criticality calculations.

In this paper, we demonstrate that the computing efficiency may be improved furthermore when the neutron population size, m , gradually increases over the successive cycles. The strategy of increasing the neutron population size during the criticality simulation is not new; e.g., Gast and Candelore (1974) tried a linear growth of the neutron population size at the rate of 10 neutron histories added at each cycle. The strategy has a clear benefit: it allows to cut the computing cost on cycles where the iterated fission source contains large errors inherited from the initial cycle source. The bias in fission source of the order $O(1/m)$ (Brissenden and Garlick, 1986) does not need to be kept very small when the fission source contains large errors since these are the limiting factor for achieving a good accuracy; therefore, the neutron population size (and so the computing cost per cycle) can be set relatively small at the beginning of the simulation. Nevertheless, as the errors in the fission source decay over the successive cycles, the bias in the fission source may become the dominant error unless it is reduced by increasing the neutron population size. In this paper we propose an on-the-fly control methodology for the neutron population size according to the actual convergence of the Monte Carlo fission source.

The paper is organised as follows. Section 2 describes the suggested method. Section 3 gives results of the numerical test calculations, and Section 4 summarises our conclusions.

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2. Method

We would like to note that no distinction is made between inactive and active cycles in the following text as the suggested method is supposed to be applied to all inactive as well as active cycles. The neutron population size is controlled during the whole simulation. This does not eliminate the need for inactive cycles in simulations that use the method.

The fission source contains errors of statistical and systematic origins. The statistical error of the order $O(1/\sqrt{m})$ is introduced directly into the fission source by sampling the source at a limited number, m , of fission sites. The statistical error propagates into the results; however, it is reduced to $O(1/\sqrt{n \times m})$ by combining the results over n cycles.

Unlike the statistical error, the systematic errors are present in Monte Carlo calculations due to peculiarities of the specific computing procedures involved. In case of criticality Monte Carlo calculations, a systematic bias of the order $O(1/m)$ is present in the fission source (hence, in the results). Brissenden and Garlick (1986) explained and demonstrated that the source bias is caused by the normalisation of the fission source size to the required value m at each cycle. This bias does not decay over the cycles; hence, it may become the dominant error in results of large simulations where the statistical and other errors have decayed. To prevent the source bias from limiting the accuracy of results, it is often recommended to use a very large m in Monte Carlo criticality calculations.

Another source of errors is caused by a cycle-to-cycle propagation of errors originating from the guessed fission source sampled at the very first cycle; the decay of this error over the cycles is governed by the dominance ratio of the system (Ueki et al., 2003).

Tuttelberg and Dufek (2015) showed that the efficiency of the criticality calculation is affected by the selected neutron population size m . There are few factors that decide whether the calculation can benefit from setting a small or large value of m . While a large m ensures a small bias in the fission source, it restricts the number of cycles that can be simulated within a certain computing time; a small number of inactive cycles may be then insufficient for decaying the error originating from the initial fission source. On the other hand, choosing a small m increases the systematic bias in the fission source, which corrupts the results as well. Choosing the suitable value of m thus represents an optimisation problem: m should be small enough to ensure an acceptably small computing time per cycle, yet m should be large enough to limit the bias in the fission source.

Tuttelberg and Dufek (2015) derived a simplified formula for optimising the value of m , taking into account the computing time allocated for the whole simulation, the estimated dominance ratio of the system and the estimated error in the initial fission source sampled at the beginning of the simulation. The final formula was derived

with an assumption that m remains fixed at all cycles, as it is common in standard Monte Carlo criticality calculations.

In this paper, we abandon the practice of keeping the neutron population size fixed over all cycles; instead, we suggest to gradually increase the neutron population size over the successive cycles. This approach has a direct impact on the efficiency of the criticality calculation. A small population size allows iterating the fission source rapidly during the initial cycles (in terms of the wall-clock time), which helps to decrease the error coming from the initial fission source. The fairly large bias associated with a small population size has no significance during these cycles since the total error in the fission source is dominated by the error coming from the source sampled at the initial cycle. As the error originating from the initial fission source decays over the successive cycles, we suggest to gradually increase the neutron population size in order to balance the source bias with other decaying errors. When the neutron population grows over the successive cycles, all sources of errors decay, which makes it possible to achieve any required accuracy.

In other words, we suggest that at any cycle the neutron population size is set as small as possible—yet without compromising the accuracy of the results by the fission source bias. This can be achieved when the source bias is, in a way, balanced to other errors in the fission source, so that neither the bias nor other types of errors dominate in the final results. Here, we must consider that the final results are tallied over a number of cycles; hence, the impact of the statistical error (that is introduced in the fission source at each cycle) on the final results is reduced by averaging over the cycles. It would, therefore, be unreasonable to balance the source bias to the statistical error in the fission source of a single cycle—unlike the statistical error, the source bias is not reduced by averaging over a number of cycles. Consequently it appears reasonable to assume that the bias should be balanced to other errors in the cumulative fission source, i.e., in the fission source combined over all cycles (and not in the fission source of a single cycle).

Therefore, in order to decide the maximal acceptable source bias (hence the minimal size of the neutron population) we suggest to estimate the total error in the cumulative fission source, and decide a certain ratio of the source bias to the total error. The neutron population size can then be controlled in such a way that this ratio is maintained at the intended level during the whole simulation.

The relative error in the cumulative fission source, ε , can be estimated using the fundamental-mode eigenvector of the fission matrix. Tuttelberg and Dufek (2014) suggest to estimate the relative error ε in the cumulative fission source $\mathbf{s}^{(n)}$ in the n^{th} criticality cycle by $\hat{\varepsilon}$ as

$$\hat{\varepsilon} = \left\| \tilde{\mathbf{s}}^{(n)} - \tilde{\mathbf{h}}^{(n)} \right\|_1, \quad (1)$$

where $\mathbf{h}^{(n)}$ is the fundamental-mode eigenvector of the fission matrix $\mathbf{H}^{(n)}$ that is tallied over all cycles of the Monte

Carlo calculation (Carter and McCormick, 1969), and $\sim$ denotes a normalisation operator defined for any vector $\mathbf{x}$ as

$$\tilde{\mathbf{x}} = \frac{\mathbf{x}}{\|\mathbf{x}\|_1}.$$

Note that $\hat{\varepsilon}$ defined by Eq. (1) may take any value from the interval $[0, 2]$, which is not convenient. Here, we therefore propose to scale the maximal value of $\hat{\varepsilon}$ to unity, which changes Eq. (1) into

$$\hat{\varepsilon} = \left\| \tilde{\mathbf{s}}^{(n)} - \tilde{\mathbf{h}}^{(n)} \right\|_1 / 2. \quad (2)$$

The fission matrix $\mathbf{H}$ can be computed during a standard Monte Carlo criticality simulation using a spatial mesh superimposed over the whole system. The $(i, j)^{\text{th}}$ element of $\mathbf{H}$ represents the probability that a fission neutron born in space zone j causes a subsequent birth of a fission neutron in space zone i . The fission matrix is always combined over all simulated cycles, so statistical errors in the fission matrix decay inversely to the square root of the total number of all neutron histories simulated during all cycles. The fundamental-mode eigenvalue of $\mathbf{H}$ equals the multiplication factor k_{eff} , and the corresponding eigenvector $\mathbf{h}$ equals the discretised fundamental-mode fission source.

The calculation of the fundamental-mode eigenvector of the fission matrix comes at an additional computing cost that could worsen the computing efficiency if it is performed at every cycle. Therefore, we suggest to update the neutron population size in selected cycles only, and to carry out a certain number, c , of cycles with a fixed neutron population size. We shall refer to this as to the multi-stage approach, with the term “stage” denoting the set of c cycles with the same neutron population size. The number c should be sufficiently large to keep the eigenvector computing cost small in the overall simulation cost. The neutron population size must be guessed for the first stage, before the first estimate of the error in the cumulative fission source becomes available.

As the source bias is of the order $O(1/m)$, for our needs we choose to approximate the relative value of the source bias b simply as

$$b = 1/m; \quad (3)$$

then, the maximal value of b is unity that is reached for $m = 1$. It is understood that Eq. (3) is a rough approximation that may not be optimal for small values of m . The source bias can also be affected by system properties, which is also neglected in Eq. (3).

We suggest to keep the fission source bias b to a fraction of the estimated relative total error of the cumulative fission source $\hat{\varepsilon}$; i.e. we postulate that

$$b = \frac{\hat{\varepsilon}}{r} \quad (4)$$

where $1/r$ is the specified fraction, $r > 1$. Eqs. (3) and (4) give the neutron population size as

$$m = \frac{r}{\hat{\varepsilon}}. \quad (5)$$

The optimal value of r is studied in Sec. 3.

The condition that the source bias does not exceed a specific fraction of the error in the cumulative fission source should be satisfied during all cycles of each stage. Nevertheless, when the neutron population size is updated at the beginning of a stage according to Eq. (4) then the same equation may not be satisfied by the end of the stage. As the error in the cumulative fission source naturally decays over the cycles, $\hat{\varepsilon}/r$ may naturally get smaller than b by the end of the stage. Therefore, m must be set so that Eq. (4) is satisfied primarily at the end of the stage; i.e., we postulate that

$$m_i = \frac{r}{\hat{\varepsilon}_i}, \quad (6)$$

where m_i is the neutron population size set for the i^{th} stage, and $\hat{\varepsilon}_i$ is the relative error in the cumulative fission source at the end of the i^{th} stage.

To satisfy Eq. (6), the error $\hat{\varepsilon}_i$ in the cumulative fission source at the end of the i^{th} stage must be predicted even before the stage is simulated. The value of $\hat{\varepsilon}_i$ can be predicted based on the error estimated at the beginning of the stage by Eq. (2) and based on the rate at which the error in the cumulative fission source decays. While a complex model for the decay rate could be made, considering the dominance ratio and possibly other system properties, for our purpose we suggest to use simply the rate at which the statistical error decays in the cumulative fission source. Assuming the statistical error in the cumulative fission source decays at the rate $1/\sqrt{h}$ where h is the total number of neutron histories simulated during the whole simulation, we can approximate the decay of the error in the cumulative fission source at the i^{th} stage as

$$\frac{\varepsilon_i}{\varepsilon_{i-1}} = \frac{\sqrt{h_{i-1}}}{\sqrt{h_i}}, \quad (7)$$

where ε_{i-1} and ε_i is the error in the cumulative fission source at the beginning and the end of the i^{th} stage, respectively; and h_{i-1} and h_i is the total number of neutron histories simulated during the whole simulation until the beginning and the end of the i^{th} stage, respectively.

Note that

$$h_i = c \sum_{j=1}^i m_j,$$

where j sums over the stages, and

$$h_i = h_{i-1} + c \times m_i; \quad (8)$$

therefore, Eq. (7) can be rewritten as

$$\varepsilon_i = \varepsilon_{i-1} \frac{\sqrt{h_{i-1}}}{\sqrt{h_{i-1} + c \times m_i}}, \quad (9)$$

When ε_i and ε_{i-1} are substituted by their respective estimates $\hat{\varepsilon}_i$ and $\hat{\varepsilon}_{i-1}$ then Eqs. (6) and (9) give

$$m_i = \frac{r}{\hat{\varepsilon}_{i-1}} \sqrt{1 + \frac{c \times m_i}{h_{i-1}}}, \quad (10)$$

that is a quadratic equation for m_i :

$$m_i^2 - \frac{r^2}{\hat{\varepsilon}_{i-1}^2} \frac{c}{h_{i-1}} m_i - \frac{r^2}{\hat{\varepsilon}_{i-1}^2} = 0, \quad (11)$$

which has a physical solution

$$m_i = \frac{b_i + \sqrt{b_i^2 + 4 \frac{r^2}{\hat{\varepsilon}_{i-1}^2}}}{2}, \quad (12)$$

where

$$b_i = \frac{r^2}{\hat{\varepsilon}_{i-1}^2} \frac{c}{h_{i-1}}.$$

Let us summarise the method. First, the user selects the neutron population size m_1 for the first stage, as well as the r and c constants. We discuss the selection of r and c in Sec. 3. We recommend to set m_1 to a minimal value acceptable for the user. At the beginning of the i^{th} stage (for $i > 1$) the estimated relative error, $\hat{\varepsilon}_{i-1}$, in the cumulative fission source is obtained by Eq. (2), and the new neutron population size m_i is determined by Eq. (12). It may happen, depending on the actual estimate of the relative error in the cumulative fission source, that Eq. (12) suggests to lower the neutron population size from its previous stage value. In that case, we accept the new neutron population size unless it drops below m_1 —the smallest population size acceptable for the user. In case Eq. (12) gives the neutron population size below m_1 , we suggest that the neutron population size m_i is assigned the m_1 value.

The new neutron population size can be enforced by Monte Carlo criticality codes via their standard normalisation procedures. At the beginning of each criticality cycle, Monte Carlo codes commonly normalise the combined statistical weight of all fission neutrons to the required neutron population size m . This procedure sets the statistical weight w of each fission neutron to

$$w = \frac{m}{x},$$

where x is the actual number of fission neutrons. Note that the actual number of fission neutrons varies from cycle to cycle in all Monte Carlo criticality calculations as it is a stochastic variable; however, it is the combined statistical weight of fission neutrons that matters in the simulation, and this variable can be set to any number by the normalisation above.

3. Numerical test calculations

3.1. Numerical model

The numerical test model represents a fuel pin cell with parameters summarised in Table 1. Reflective boundary conditions are applied to the radial faces; non-reflective void boundary conditions are applied to the axial faces. The model is based on a common PWR fuel pin cell, with the exception of the pin length that is increased to 10 m in order to achieve a dominance ratio of a large core.

Table 1: Specifications of the pin cell model.

Fuel	UO ₂
Cladding material	Zr
Moderator	light water
Radius of fuel pellets	0.41 cm
Outer radius of cladding	0.475 cm
Rod pitch	1.26 cm
Length of the fuel rod	1000 cm
²³⁵ U enrichment	3.1 wt%
Fuel density	10 g/cm ³
Moderator (water) density	0.7 g/cm ³

The dominance ratio of this model is close to unity (we evaluate it at 0.9982 using the fission matrix obtained from the test calculations) which ensures a fission source convergence comparable to that in large reactor core criticality simulations.

Due to the neutron leakage at the axial faces the system actually represents a slab reactor with a close-to-cosine axial distribution of its fundamental-mode fission source. This axial distribution can be biased by a small neutron population size, as it is demonstrated in Sec. 3.3, which allows to test the effectiveness of the suggested method. Nevertheless, the fundamental mode is cosine only in one-group diffusion approximation; in reality it slightly differs to it, therefore, a reference solution is needed for evaluating the real errors in numerical test solutions.

All numerical calculations are performed by an in-house non-analogue continuous-energy 3D Monte Carlo criticality code using the JEFF3.1 point-wise neutron cross-section library. For the purpose of computing the fission matrix a uniform axial mesh with 100 zones is superimposed over the system.

3.2. Reference solution

The fission source in all calculations is discretised into a vector form using a uniform mesh with 100 axial zones. The reference fundamental-mode fission source, $\mathbf{s}_{\text{ref}}$, is combined over active cycles of the reference Monte Carlo calculation specified in Table 2. The initial fission source is sampled from a flat distribution, and the neutron population size is fixed over all cycles. As the test model is axially symmetrical, we further improve the accuracy of $\mathbf{s}_{\text{ref}}$ by averaging its symmetrical elements.

Table 2: Specifications of the reference calculation.

Number of inactive cycles	10 ³
Number of active cycles	10 ⁶
Neutron population size	10 ⁶
Total number of active histories	10 ¹²

The reference solution $\mathbf{s}_{\text{ref}}$ serves in evaluating the fission source convergence and computing efficiency of all test calculations. The fission source convergence is evaluated based on the real relative error in the cumulative fission

source at various stages in various test calculations. The real relative error ε in the cumulative fission source distribution $\mathbf{s}$ in a certain stage of a certain test calculation is evaluated as

$$\varepsilon = \|\tilde{\mathbf{s}} - \tilde{\mathbf{s}}_{\text{ref}}\|_1 / 2. \quad (13)$$

The computing efficiency corresponding to the same stage of the same simulation is then evaluated by the figure-of-merit FOM,

$$\text{FOM} = \frac{1}{\varepsilon^2 \times t}, \quad (14)$$

where the CPU time t covers the neutron transport simulation as well as the running time of the eigenvector solver.

3.3. Results

The suggested acceleration method is tested on calculations specified in Table 3. A test calculation is done for each possible combination of the c and r values in Table 3; totally, eight combinations of parameters are tested. Each test calculation is repeated $100\times$ with a unique random number generator seed; the presented results are averaged. Hence, 800 test calculations with the variable neutron population size are processed in total. Note that results of an individual calculation may significantly differ to the presented averaged results. The initial fission source is sampled asymmetrically in a single point 1 cm from an axial boundary; this is done in order to impede the fission source convergence. Division of cycles into inactive and active is irrelevant here as these tests study the fission source convergence only.

Table 3: Specifications of test calculations with variable neutron population size.

Minimal neutron population size m_1	50
Total number of neutron histories	10^8
Number of cycles per stage, c	50; 100
Value of the r constant	10; 20; 50; 200
Number of repetitions of each test	100

The above test simulations are compared to standard Monte Carlo criticality simulations with a fixed neutron population size; specifications of these tests are summarised in Table 4. The initial fission source is sampled asymmetrically in a single point 1 cm from an axial boundary, same as in the other calculations. Tests are carried out for three various values of the neutron population size. Each of the three configurations is repeated $100\times$ with a unique random number generator seed, and the results are averaged within the tests of the same configuration.

Table 4: Specifications of test calculations with fixed neutron population size.

Neutron population size	50; 5×10^3 ; 5×10^5
Total number of active histories	10^8
Number of repetitions of each test	100

Figure 1: Comparison of convergence of the cumulative fission source among fixed and variable neutron population size calculations.

Figure 1 depicts the convergence of the cumulative fission source in the three test simulations with a fixed neutron population size; a test calculation with variable neutron population size ($c = 100$, $r = 50$) is included as well. The simulation with the fixed population size of 50 neutrons converges rapidly until the total number of simulated neutron histories h reaches about 3×10^6 ; a large bias associated with the small population size then prevents ε from getting below about 0.1. The simulation with the fixed population size of 5×10^3 neutrons takes more time to simulate each cycle, so the simulation converges slower; however, due to a much smaller bias it can eventually significantly outperform the former simulation. Due to the large cost of each cycle, the simulation with the fixed population size of 5×10^5 neutrons can iterate the fission source only $200\times$ within the total allocated computing cost, which is not sufficient to significantly reduce the error introduced into the fission source at the first cycle. The test simulation with the variable neutron population size ($c = 100$ and $r = 50$) gradually grew the population size from its minimal value of 50 to about 4000 neutrons by the end of the simulation; the simulation converges rapidly at the beginning while it maintains the same convergence rate till the end.

Figure 2 depicts the figure-of-merit of the variable neutron population size test calculation with $c = 100$ and $r = 50$, and compares it to the fixed neutron population size calculations. The efficiency of the variable neutron population size calculation outperforms all the fixed neutron population size calculations; the eigenvector computing cost is negligible in the studied case when $c = 100$.

Figure 3 and 4 show the convergence of the cumulative fission source in all variable neutron population size test simulations with 50 and 100 cycles per stage, respectively. As for the source convergence, the test calculations with

Figure 2: Comparison of efficiency of calculations with fixed and variable neutron population size (100 cycles per stage).

Figure 3: Convergence of the cumulative fission source in the variable neutron population size calculations with 50 cycles per stage.

100 cycles per stage perform almost as well as the calculations with 50 cycles per stage. As the source convergence does not appear to differ significantly between these two tested c values, the larger c value can be preferred. The figures also show that the r values of 10 and 200 are not optimal for the test simulations. The r value of 10 is too small; the large source bias (of approximately 1/10-th of ε) negatively affects the results at the latest stages of the simulation. On the contrary, the r value of 200 seems to be a bit large; the small bias (of approximately 1/200-th of ε) requires a large neutron population size that slows down the convergence during a part of the simulation, which is visible on Figure 4.

Figure 5 compares the computing efficiency among the variable neutron population size calculations with 100 cy-

Figure 4: Convergence of the cumulative fission source in the variable neutron population size calculations with 100 cycles per stage.

Figure 5: The figure-of-merit of variable neutron population size calculations with 100 cycles per stage.

cles per stage. While the r values of 20 and 50 allow the cumulative fission source to converge similarly well with respect to the total number of simulated neutron histories (as shown on Figures 3 and 4), the simulation with the r value of 20 uses a smaller neutron population size, and thus it performs more stages in total than the simulation with the r value of 50. As an eigenvector is computed at each stage, the eigenvector computing cost in the simulation with the r value of 20 results in a low computing efficiency. The test simulation with the r value around 50 performs best.

4. Discussion and conclusions

In this paper we demonstrate that letting the neutron population size grow over cycles can improve the fission source convergence and efficiency of Monte Carlo criticality calculations. We suggest that the neutron population size is gradually increased so that the associated source bias is kept in a suitable proportion to the total error in the cumulative fission source. The neutron population size is then optimised during the whole simulation, so the method does not require any estimate of the allocated computing time before the simulation.

The suggested method adds a certain computing cost to the simulation, primarily due to tallying the fission matrix and computing its fundamental-mode eigenvector. Nevertheless, the fission matrix is typically very sparse; its sparsity can be very effectively used to lower the eigensolver cost down to a minimum (Saad, 1992). Moreover, the fission matrix and its eigenvector change only little over the stages. Therefore when e.g. the iterative Arnoldi solver (Saad, 1992) is employed, the previous-stage eigenvector solution can be re-used as a starting point and iterated only few-times. The cost is lowered furthermore by introducing stages with a certain number of cycles during which the neutron population size is not updated.

In terms of the source convergence and computing efficiency, the growing neutron population size calculations outperformed the standard Monte Carlo calculations in our tests despite the certain eigensolver cost. The computing efficiency is affected by the number of cycles per stage. Optimal results were achieved with 100 cycles per stage in our tests; however, this number may be different in simulations of other systems. A very large fission matrix may be necessary for estimating the error in the cumulative fission source in large systems, which may increase the eigenvector solver cost. The computing cost of the eigenvector solver should be reflected in the chosen number of cycles per stage; it may therefore be suitable to increase this number in criticality simulations of large systems.

The optimal value of the proportionality constant $1/r$ (that represents the desired ratio of the source bias to the total error in the cumulative fission source) also depends on the system; that is because various systems exhibit the bias in their simulations to various degrees. For instance a homogeneous system with no neutron leakage has no source bias associated with a small neutron population size; then, the neutron population size does not need to be large in order to produce unbiased results. In this case, r can therefore be set small. On the contrary, systems with a cosine-like fission source distribution (like the system in our test simulations) are likely to benefit from the r value of 50 or so. The study of optimal r values for various systems may be a subject of the future research.

The choice of the neutron population size for the first stage, m_1 , should reflect the error in the distribution from which the initial fission source is sampled. When the distribution approximates the correct fundamental-mode well

then it is reasonable to set a large value for m_1 , otherwise the useful information contained in the distribution from which the source is sampled would be lost in the large statistical noise of the sampled source. For this purpose Eq. (5) can be helpful when choosing the m_1 value. For instance, when the distribution is guessed then we may estimate the relative error in the distribution to be close to unity; the neutron population size can be then set to the ratio of the suitable r value (e.g. 50) and the estimated relative error, which gives the population size of 50 neutrons. If we could sample the initial fission source from a more accurate distribution having an error of e.g. 1% (such a distribution would be based on a previous Monte Carlo or deterministic calculation) then it would be more suitable to set the initial population size at 5000 neutrons.

Here we wish to make a general note about source convergence acceleration techniques for the Monte Carlo criticality calculations. Many of the current acceleration techniques, such as the Wielandt's method adapted for Monte Carlo criticality calculation (Yamamoto and Miyoshi, 2004) or the fission matrix acceleration method (Kadotani et al., 1991), have been designed for improving the cycle-to-cycle convergence of the fission source. Unfortunately, these and similar techniques are often compared to standard Monte Carlo criticality calculations with a very large neutron population size. This may be viewed as an unfair comparison since the standard source iteration is then purposely slowed down by the choice of the large neutron population size. We wish to see new acceleration techniques being compared to standard Monte Carlo criticality calculations running at optimal settings.

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