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Energy efficient D2D communications in dynamic TDD systems

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Abstract

Device-to-Device (D2D) communication is a promising technology for improving the performance of proximity-based services. This paper demonstrates how the integration of D2D communication in cellular systems operating under dynamic Time Division Duplex (TDD) can improve energy efficiency. We perform joint optimization of mode selection, uplink/downlink transmission period, and power allocation to minimize the transmission energy consumption while satisfying a traffic requirement. Solutions are developed for two scenarios: with and without interference among D2D communications. Both formulations are expressed as mixed-integer nonlinear programming problems, which are NP-hard in general. We exploit problem structure to develop efficient solutions for both scenarios. For the interference-free case, we develop algorithms that find the optimal solution in polynomial time. When considering interference, we propose a customized solver based on branch-and-bound that reduces the search complexity by taking advantage of the problem-specific proprieties. We complement this solver by a more practical heuristic algorithm. Simulation results demonstrate that D2D communications in dynamic TDD systems can yield significant energy savings and improved spectral efficiency compared to traditional cellular communication. Furthermore, we give analytical characterizations of the receiver locations relative to a given transmitter where D2D communication is optimal. These regions can be surprisingly large and not necessarily circular.

Index Terms

D2D communication, energy-efficient network, dynamic TDD, mode selection, power control.

I. INTRODUCTION

During the last decade, wireless communications have experienced an explosive growth, both in the number of mobile subscribers and in the data traffic demands. This phenomenon is expected...
to continue [1], mainly driven by an increasingly rich web content, video- and audio-streaming, and file sharing. Meeting this increasing demand causes the energy consumption of wireless systems to escalate. The growing energy bills of operators, limited battery lifetime of mobile devices, and environmental concerns are progressively steering the research community towards the development of energy-efficient wireless communications [2]. Efficiency and scalability are therefore becoming the key criteria for the development of the next generation (5G) systems, where D2D communication is recognized as one of its promising technology components [3,4].

A natural question in the context of D2D communication is under which condition two users should communicate through a direct link rather than via the base station (BS). This problem is known as the mode selection problem. In this paper, we formulate and solve the (transmission) energy-optimal mode selection problem for D2D communications in cellular systems operating under dynamic TDD. Dynamic TDD systems, in fact, are expected to be prominent in future networks, mainly because of their ability to handle traffic level fluctuations between uplink (UL) and downlink (DL), and to allow channel reciprocity to be exploited to reduce signaling and RF front-end complexity, compared to Frequency Division Duplex (FDD) systems.

Several variations of the mode selection problem have already been studied in the literature. Examples include mode selection for maximizing user rate while satisfying SINR constraints on active cellular links [5], or for maximizing the power-efficiency of the network [6,7]. Authors of [8] address the problem with two dual performance objectives: to maximize the Quality-of-Service (QoS) for a given transmit power, and to minimize the power for a given QoS. The latter formulation has later been extended to the multiple link case [9]. To realize the full potential of D2D communications, the mode selection should be done jointly with the radio resource allocation. Several works have investigated this joint problem [6,10,11], mainly developing mixed-integer programming models and solving these off-line to gain insight into the potential gains of D2D communications. In addition, heuristics [9,12,13] and game-theoretic approaches [14] have also been explored to develop more practical, albeit suboptimal, algorithms.

Performance improvement that can be obtained by integrating D2D communications in cellular systems with flexible TDD have only recently gained attention in literature. Frameworks for D2D enhanced TDD networks are proposed in [15–17]. However, they do not account for the mode selection and focus mainly on the adaptive UL/DL slot allocation to D2D pairs, so as to balance the traffic load, coordinate the interference, improve coverage probability and sum-rate. Recently,
the authors of [18] have extended the resource allocation problem introduced in [15] to include mode selection. Yet, the mode selection decision is based on the instantaneous SINR, and not in joint consideration with the power and transmission time allocation.

In this work, instead, we aim to reduce the transmission energy consumption of cellular system by jointly optimizing mode selection, power control, as well as the time allocation for UL/DL and direct transmissions. Significant energy reduction can be achieved by exploiting the benefits brought by both the better channel gain of the direct D2D link and the adaptive transmission time allocation of dynamic TDD technology. This paper extends our previous study [19] to a more general multi-link scenario, where we consider two spectrum allocation strategies: an interference-free case where all the D2D links use orthogonal frequency channels, and a channel-reuse case where all D2D connections share the same frequency resource to improve the spectral efficiency. To the best of our knowledge, this paper is the first to address the joint mode selection and resource allocation problem for energy-optimal operation of D2D-enabled cellular systems with dynamic TDD. We make the following contributions:

• We formulate several variations of the joint mode selection, transmit power, and transmission time allocation problem, targeting both total system energy and mobile device energy, as mixed-integer nonlinear programs (MINLPs).
• Although MINLPs are NP-hard in general, we demonstrate how the mathematical structure in our problem can be exploited to solve several classes of the general resource allocation in polynomial (in some cases linear) time.
• When multiple D2D links share the same frequency resource, finding the optimal solution remains challenging. However,
  – we design a customized branch and bound (B&B) solver, with novel variable selection and branching rules, and efficient procedures for infeasibility detection and performance bound computations. This solver allows us to find provably optimal solutions much more efficiently than using generic B&B solvers or naive exhaustive search;
  – we propose a heuristic algorithm for computing near-optimal solutions while respecting practical constraints in terms of execution times and signalling overhead.

The rest of the paper is structured as follows. The system model and the problem formulation are introduced in § II. § III elaborates the optimal solution for the interference-free case, while the
B&B algorithm and heuristics for minimizing total device energy when all D2D communications share a single frequency channel are given § IV. Numerical results are presented in § V and concluding remarks are given in § VI.

II. SYSTEM MODEL AND PROBLEM STATEMENT

A. System model and assumptions

We consider a single cell network where infrastructure-assisted D2D communication is enabled. Inter-cell interference (that occurs in multi-cell scenarios) is thus neglected, and our focus is on managing transmit powers, transmit durations, and interference within an isolated cell. Communication between in-cell users is done in one of two possible modes:

1) **Cellular mode**, where the transmitter first sends the data to the BS, which then forwards the message to the intended receiver (Fig. 1(a) left);

2) **D2D mode**, where a dedicated direct link is set up between the transmitter and receiver and allows the users to communicate without involving the BS (Fig. 1(a) right).

In the cell, we consider a set \( \mathcal{L} \) of user pairs that wish to communicate. Each user pair constitutes a logical link that we label by an integer taking value from \( \{1, 2, \ldots, L\} \). The BS is indexed as 0 and we refer to the users in pair-\( l \) as transmitter-\( l \) (Tx-\( l \)) and receiver-\( l \) (Rx-\( l \)). The mode selection policy divides the set of all user pairs \( \mathcal{L} \) into two subsets: \( \mathcal{D} \), the user pairs that should communicate in D2D mode, and \( \mathcal{C} = \mathcal{L} \setminus \mathcal{D} \), the user pairs that should communicate in cellular mode. The system bandwidth is divided into a fixed number \( F = L \) of orthogonal channels of size \( W \) Hz, and time is divided into frames of fixed length of \( T \) seconds; see Fig. 1(b). The BS manages the spectrum by assigning to each logical link, a time-frequency physical resource, consisting of one time frame and one frequency channel. The transmit power of each pair, together with the transmission duration and the communication mode, are updated at the beginning of each frame on the basis of the estimated large-scale fading. We assume that the large-scale fading remains constant within the frame duration \( T \) and changes slowly among consecutive frames. The advantage of assigning the communication mode based on the large-scale fading, rather than on the fast fading measurement, is to reduce the number of unnecessary communication mode switches due to a temporary changes of the channel condition. In fact,

\[^1\text{This assumption is valid with low users mobility such that the coherence time is larger than } T.\]
if the channel gains do not change fast, it is likely that the user pairs would prefer the same communication mode for longer time. This is beneficial for the link configuration procedure and signalling overhead. On the other hand, to exploit or compensate for the small-scale fading and frequency diversity of the channels, we assume that a proper frequency channel assignment is applied after the communication mode selection, to further improve the user performance [20]. Solving this additional resource allocation problem is out of the scope of this work.

1) Communication in cellular mode: We consider the dynamic TDD scheme, where the UL and DL transmissions for a user pair occur on the same frequency channel but alternate in time. The portioning of resources for UL and DL can be reconfigured in each time frame, but it is assumed to be the same for all communications within the same cell. This intra-cell UL/DL synchronization is usually applied in practice because it reduces the complexity of the inter-cell interference management in the more general multi-cell network [21][22]. Furthermore, to prevent intra-cell interference between concurrent transmissions in cellular mode, the BS follows the channel allocation policy of legacy LTE systems and assigns a separate channel to each cellular user pair (see Fig. [1(b)]. We denote by $t_{ul}$ and $t_{dl}$ the portion of the time frame allocated to the UL and DL transmissions, respectively. Let $p_{0l}$ and $p_{0l}$ be the transmit power levels used by Tx-$l$ in UL, and used by the BS in DL to Rx-$l$, which are subject to upper bounds $P_{l}^{\text{max}}$ and $P_{0}^{\text{max}}$, respectively. The instantaneous rates $r_{l0}$ and $r_{0l}$ achieved in UL and DL,
respectively, are assumed to follow Shannon’s capacity formula:

\[ r_{l0} = W \log \left( 1 + \frac{p_{l0}G_{l0}}{\sigma^2} \right), \quad r_{0l} = W \log \left( 1 + \frac{p_{0l}G_{0l}}{\sigma^2} \right). \]  

(1)

Here, \( G_{l0} \) is the channel gain between Tx-\( l \) and the BS, \( G_{0l} \) is the channel gain between the BS and Rx-\( l \), and \( \sigma^2 \) is the noise power. The maximum instantaneous rates (corresponding to maximum power transmissions) in (1) are denoted by \( r_{l0}^\text{max} \) and \( r_{0l}^\text{max} \), respectively.

2) Communication in D2D mode: In D2D mode, each pair can use the full frame duration for its single-hop transmission (Fig. 1). Let \( t_l \leq T \) denote the active time of pair-\( l \) in D2D mode.

A large body of work considers underlay in-band D2D communication, where D2D transmitters opportunistically access the radio resources occupied by cellular users. A disadvantage of underlay D2D communication in TDD systems is that the receiver of the D2D link perceives a rapid change of the interference power in one time frame when the cellular pair switches between UL and DL transmission. It is difficult to compensate for this effect without resorting to complex interference management algorithms that require detailed cross-gain knowledge and have high signalling load. In this paper, we therefore focus on overlay in-band D2D communication, where D2D communications and cellular communications are allocated different frequency channels so that they do not cause interference to each other (Fig. 1(b)). Nevertheless, resource reuse among D2D communications has been investigated in our studies. In particular, we consider two channel allocation strategies for D2D pairs:

- **Full Orthogonality (FO):** All D2D communications are assigned orthogonal frequency channels. Hence, no receiver is interfered by other transmissions within the cell.

- **D2D Resource Sharing (RS):** All D2D communications are assigned to the same frequency channel, hence interfere with each other.

Let \( r_{ll} \) denote the instantaneous rate between users of pair-\( l \) when transmitting in D2D mode:

\[ r_{ll} = W \log \left( 1 + \frac{p_{ll}G_{ll}}{\sigma^2 + I_l} \right), \]  

(2)

where \( G_{ll} \) is the direct channel gain between Tx-\( l \) and its intended receiver Rx-\( l \), \( p_{ll} \) is the transmit power level (upper bounded by \( p_{ll}^\text{max} \)), and \( I_l = \sum_{j \neq l} p_{jj}G_{jl} \) is the interference power level experienced at Rx-\( l \), due to other concurrent D2D transmissions (\( I_l = 0 \) in the FO case). For a given interference level, we use \( r_{ll}^\text{max} \) to denote the maximum instantaneous achievable rate corresponding to \( p_{ll}^\text{max} \).
3) Rate constraint, power feasibility and energy cost: To guarantee a certain QoS, each pair-$l$ has a traffic requirement of $b_l$ nats per time frame, irrespective of the communication mode. This QoS requirement can be translated into a session rate requirement, which depends on the communication mode and on the time allocated to transmissions. Specifically, if pair-$l$ is in cellular mode, the transmission times for UL and DL along with the corresponding instantaneous transmission rates must satisfy

$$r_{l0} t_{ul} \geq b_l \quad \text{and} \quad r_{0l} t_{dl} \geq b_l. \quad (3)$$

Similarly, if pair-$l$ is in D2D mode, $t_l$ and $r_{ll}$ must satisfy

$$r_{ll} t_l \geq b_l. \quad (4)$$

The limitation on the transmission power levels, together with the session rate requirements above, entail the need to verify under which conditions the communication of a pair can be supported by the network. To this end, we introduce the concept of power-feasibility:

**Definition 1** (Power feasibility). We say that user pair-$l$ is power-feasible

(a) in D2D mode if $r_{ll}^{\max} T \geq b_l$;

(b) in cellular mode if there exists a time allocation $(t_{ul}, t_{dl})$ such that

$$t_{ul} \geq \frac{b_l}{r_{l0}^{\max}}, \quad t_{dl} \geq \frac{b_l}{r_{0l}^{\max}}, \quad t_{ul} + t_{dl} \leq T. \quad (5)$$

**Assumption 1.** There exists a time allocation $(t_{ul}, t_{dl})$ which can support the communication of all users in cellular mode.

**Remark 1.** The power-feasibility conditions (5) imply that $t_{ul}$ must satisfy

$$\frac{b_l}{r_{l0}^{\max}} \leq t_{ul} \leq T - \frac{b_l}{r_{0l}^{\max}}. \quad (6)$$

Thus, under Assumption 1, it must hold that $\max_l \{\frac{b_l}{r_{l0}^{\max}}\} \leq \min_l \{T - \frac{b_l}{r_{0l}^{\max}}\}$.

By rewriting the power-rate relationships (1) and (2), we find the minimum transmission energy required to satisfy the traffic rate requirement $b_l$ on link-$l$ for a given time allocation:
\[ E_{D2D}(t, I_l) = p_{ll} t = \left( \exp \left( \frac{b_I}{W_{ll}} \right) - 1 \right) \frac{\sigma^2 + I_l}{G_{ll}^2} t, \]  \hspace{1cm} \text{D2D mode.} \]

These functions are convex and monotonically decreasing in the transmission time (see [23] and reference therein). This observation leads to the following result:

**Lemma 1.** When minimizing the transmission energy, any optimal solution must allocate the full frame duration for communication. For the D2D mode, this implies that

\[ \min_{t, I_l} E_{D2D}^{D2D}(t, I_l) = \min_{I_l} E_{D2D}^{D2D}(T, I_l), \]

while for cellular communication it must hold that \( t_{ul} + t_{dl} = T \).

The transmission energy cost for communicating in cellular mode includes both the energy cost of the transmitting mobile device and that of the BS. However, the BS often has access to cheap and abundant energy in comparison with the user equipment, in which case it is relevant to only focus on the device energy. To this end, we consider the following two definitions of energy consumption for a generic user pair-\( \text{-}l \) in cellular mode:

- **The System Energy Consumption (SEC):** is the energy consumed by both Tx-\( l \) in UL and the BS in DL. By Lemma [1], the total energy cost is obtained by

\[ E_{ll}^{CELL}(t_{ul}) = E_{l0}(t_{ul}) + E_{0l}(T - t_{ul}), \] 

which is a convex function of \( t_{ul} \).

- **The User Energy Consumption (UCE):** is the energy consumed by Tx-\( l \) in UL transmission, disregarding the energy spent by the BS, that is

\[ E_{ll}^{CELL}(t_{ul}) = E_{l0}(t_{ul}), \] 

which is a convex and monotonically decreasing function of \( t_{ul} \).

This work focuses on minimizing the transmission energy, neglecting the circuit and the idle power consumption of the transmitters. We believe that this is a reasonable simplification for
finding practical low-complexity solutions to the considered problems. The idling power, in fact, is usually much smaller than the circuit power, and the transmission power is dominating unless we consider low rate or very short range (under 10 m) connections [24]. Since D2D communications are foreseen to complement existing technologies for direct communication, such as Bluetooth, supporting longer range links (up to 1 km) and higher data rates (up to 1 Gbps) [25], we deem that the computational benefits of considering the transmission energy as the main contribution to the energy consumption is well motivated.

B. Problem statement

We consider the problem of minimizing the transmission energy consumption of a D2D-enabled cellular network with dynamic TDD system, by jointly optimizing: i) the communication mode of each user pair, ii) the UL/DL time configuration, and iii) the powers allocated to all transmitters. Since we are interested in two possible energy cost functions (SEC and UEC) and two channel allocation strategies for D2D communications (FO and RS), we obtain four variations of the energy minimization problem, which all can be formulated as MINLPs. Table I summarizes the main results for these four cases, with the section number for easy reference.

<table>
<thead>
<tr>
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<th>UEC</th>
<th>SEC</th>
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</thead>
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<tr>
<td><strong>FO</strong></td>
<td>Optimal solution in linear time (§ III)</td>
<td>Optimal solution in polynomial time (§ III)</td>
</tr>
<tr>
<td><strong>RS</strong></td>
<td>Optimal solution with B&amp;B (§ IV-B)</td>
<td>Optimal solution with B&amp;B (§ IV-B)</td>
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<td></td>
<td>Suboptimal solution with heuristic (§ IV-C)</td>
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</tbody>
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III. MINIMIZING THE ENERGY CONSUMPTION WITH FO

In this section, we show how the jointly optimal mode selection and resource (time and power) allocation with FO can be found in polynomial time, even though the overall problem is not convex. For ease of exposition, we first derive the optimal solution for a single user pair, and then extend the results to multiple user pairs.
A. Single user pair

We first characterize the minimum energy cost for a pair to fulfil the rate requirement when in cellular and when in D2D mode. We show that the minimal SEC can be found by solving a simple convex optimization problem, while the minimal UEC admits an explicit expression.

1) Minimum energy cost for communication in cellular mode: In cellular mode, the UL/DL time allocation is chosen to minimize one of the following two objectives:

- Minimizing SEC: The minimum amount of energy of pair-$l$ in cellular mode can be determined by solving the following single-variable convex optimization problem:

\[
\begin{align*}
\text{minimize} & \quad E_{l0}(t_{ul}) + E_{0l}(T - t_{ul}) \\
\text{subject to} & \quad \frac{b_l}{r_{l0}^{\max}} \leq t_{ul} \leq T - \frac{b_l}{r_{0l}^{\max}},
\end{align*}
\]

where constraint (10b) ensures power feasibility in the sense of Definition 1. Problem (10) can be solved efficiently using a wide variety of methods, e.g. bisection search [26]. We denote with $t_{ul}^\star$ the optimal solution to (10), and with $E_{l0}^{\text{CELL}}(t_{ul}^\star)$ the minimum energy cost.

- Minimizing UEC: Here, the only difference from the problem formulation in (10) is that the objective function reduces to $E_{l0}(t_{ul})$. By monotonicity of the objective function, the optimal solution is attained by $t_{ul}^\star = T - \frac{b_l}{r_{0l}^{\max}}$. We indicate with $E_{l0}^{\text{CELL}}(t_{ul}^\star)$ the corresponding optimal energy cost.

2) Minimum energy cost for communication in D2D mode: In D2D mode, no traffic is forwarded through the BS and only the user equipment consumes energy for the connection. The minimum energy cost follows from Lemma 1 with $I_l = 0$:

\[
E_{ll}^{\text{D2D}} = \left( \exp \left( \frac{b_l}{WT} \right) - 1 \right) \frac{\sigma^2}{G_{ll}} T.
\]

Equation (11) is only valid when the D2D mode is power feasible. Since we have ensured power feasibility only for communications in cellular mode, we need to verify that $r_{ll}^{\max} T \geq b_l$ before applying (11). It turns out to be more convenient to work with the extended value function

\[
\bar{E}_{ll}^{\text{D2D}} = \begin{cases} 
\left( \exp \left( \frac{b_l}{WT} \right) - 1 \right) \frac{\sigma^2}{G_{ll}} T & \text{if } r_{ll}^{\max} T \geq b_l \\
+\infty & \text{otherwise}.
\end{cases}
\]
3) **Optimal mode selection policy and resource allocation:** The optimal mode selection policy consists in estimating the energy cost for cellular mode (either for the UEC or for the SEC case), and then comparing it with the energy cost for D2D mode. The optimal communication mode is simply the one that requires the least amount of energy. Once the optimal communication mode and transmission time has been selected, the corresponding optimal powers are easily derived as

\[
p_{l0} = \left( \exp\left( \frac{b_{l}}{W t_{ul}^*} \right) - 1 \right) \frac{\sigma^2}{G_{l0}}, \quad \text{UL, cellular mode;}
\]

\[
p_{0l} = \left( \exp\left( \frac{b_{l}}{W(T - t_{ul}^*)} \right) - 1 \right) \frac{\sigma^2}{G_{0l}}, \quad \text{DL, cellular mode;}
\]

\[
p_{ll} = \left( \exp\left( \frac{b_{l}}{W T} \right) - 1 \right) \frac{\sigma^2}{G_{ll}}, \quad \text{D2D mode.}
\]

In § V we explore the geometric interpretation of the mode selection policy for a single link, interpreted in terms of physical distance between the two communicating devices.

**B. Multiple user pairs**

The main challenge with multiple user pairs in FO operation is that all cellular connections must use a common UL/DL time allocation, which accounts for the energy consumption of all users. To formulate the joint mode selection and resource allocation problem for multiple pairs, we introduce the *mode selection vector* \( \mathbf{m} \in \{0, 1\}^L \), whose entries satisfy

\[
m_{l} = \begin{cases} 
0 & \text{if pair-}l \text{ is in cellular mode,} \\
1 & \text{if pair-}l \text{ is in D2D mode,}
\end{cases}
\]

and consider the following MINLP problem

\[
\begin{align*}
\text{minimize} & \quad \sum_{l=1}^{L} E_{D2D}^{D2D} m_{l} + E_{ll}^{CELL}(t_{ul})(1 - m_{l}) \\
\text{subject to} & \quad \frac{b_{l}}{r_{l0}^{\max}} - T m_{l} \leq t_{ul} \leq T - \frac{b_{l}}{r_{0l}^{\max}} + T m_{l}, \quad \forall l, \\
& \quad t_{ul} \in [0, T], \quad m_{l} \in \{0, 1\}, \quad \forall l.
\end{align*}
\]

The objective function in (15) is the total energy consumption of all the \( L \) user pairs, with \( E_{ll}^{CELL}(t_{ul}) \) given by (8) or (9) under FO-SEC or FO-UEC, respectively. Here, we assume that D2D pairs do not interfere with each other, and their minimal energy consumption is the same constant value (12) as in the single user pair case. Constraints (15b) ensure that pair-\( l \) can only be assigned to cellular mode if it is power feasible in the sense of Definition 1.
Since (15a) is separable in $m_l$, we can express the objective in terms of the transmission energy for each device when operating in its optimal mode. In other words, we rewrite (15) as

$$\min_{t_{ul} \in [0,T]} F(t_{ul}),$$

where $F(t_{ul}) = \sum_{i=1}^{L} E_i(t_{ul})$, and $E_i(t_{ul})$ denotes the minimum energy-cost for the single pair-$l$ when the UL time is fixed to $t_{ul}$, that is

$$E_i(t_{ul}) \triangleq \begin{cases} \min\{E_{iD2D}^{D2D}, E_{iCELL}(t_{ul})\} & \text{if } t_{ul} \in [\frac{b_l}{r_l} \max, T - \frac{b_l}{r_l} \max] \\ E_{iD2D}^{D2D} & \text{otherwise.} \end{cases}$$

(17)

Equation (17) reveals the piecewise nature of $E_i(t_{ul})$. For $t_{ul} < \frac{b_l}{r_l} \max$ and $t_{ul} > T - \frac{b_l}{r_l} \max$, we have $E_i(t_{ul}) = E_{iD2D}^{D2D}$, which is a finite constant if user pair-$l$ is power feasible in D2D mode, and $+\infty$ otherwise. In the interval $[\frac{b_l}{r_l} \max, T - \frac{b_l}{r_l} \max]$, $E_i(t_{ul})$ is either equal to the constant $E_{iD2D}^{D2D}$, or given by $E_{iCELL}(t_{ul})$, depending on whether and at which points the graphs of $E_{iCELL}(t_{ul})$ and $E_{iD2D}^{D2D}$ intersect. Note that the two graphs can intersect only once if $E_{iCELL}(t_{ul})$ is monotonically decreasing (i.e. when minimizing UEC) or twice if it is convex (i.e. when minimizing SEC).

To better describe the piecewise nature of $E_i(t_{ul})$, we introduce $\Delta_l = [\tau_l \min, \tau_l \max]$ as the interval of $t_{ul}$ during which $E_i(t_{ul}) = E_{iCELL}(t_{ul})$. If, for a pair-$l$, such an interval does not exist (i.e., $\Delta_l = \emptyset$), it is always more efficient for the pair to operate in D2D mode. See Fig. 2.

Figures 3 show the minimum energy-cost of three user pairs and the corresponding total energy $F(t_{ul})$ under both FO-UEC and FO-SEC. Note that $F(t_{ul})$ is non-convex on $[0,T]$. However, the following lemma establishes a key property of $F(t_{ul})$, useful later to solve Problem (16).

**Lemma 2.** (a) In the FO-SEC scenario, $F(t_{ul})$ is a piecewise convex function.

(b) In the FO-UEC scenario, $F(t_{ul})$ is a piecewise decreasing function.

**Proof:** For each pair-$l$, if $\Delta_l = \emptyset$, by its definition, $E_{iCELL}(t_{ul})$ is a constant value on $[0,T]$. Otherwise, $E_{iCELL}(t_{ul})$ is a constant value or $+\infty$ in the two intervals $[0, \tau_l \min)$ and $(\tau_l \max, T]$. During the interval $[\tau_l \min, \tau_l \max]$, in the FO-SEC case $E_{iCELL}(t_{ul})$ is a convex function of $t_{ul}$ (given by (8)); in the FO-UEC case, $E_{iCELL}(t_{ul})$ is a monotonically decreasing function in $t_{ul}$ (given by (9)). The function $F(t_{ul})$ is obtained as the sum of $E_{iCELL}(t_{ul})$ of all $L$ pairs. Hence, the whole interval $[0,T]$ is divided into $J \leq 2L + 1$ adjacent intervals. In the FO-UEC case, $F(t_{ul})$ is the sum of constants and convex functions in each interval, which makes $F(t_{ul})$ piecewise
convex. In the FO-UEC case, on the other hand, $F(t_{ul})$ is the sum of constants and monotonically decreasing functions in each interval, which makes $F(t_{ul})$ piecewise decreasing.

Based on Lemma 2, the optimal solution to (16) can be computed efficiently.

**Proposition 1.** (a) In the FO-UEC case, let $\bigcup_{j=1}^{J} \Gamma_j$ be a partition of $[0, T]$ induced by the points $\{\tau_{1}^{\min}, \tau_{1}^{\max}, \ldots, \tau_{L}^{\min}, \tau_{L}^{\max}\}$. Then, the optimal UL time allocation $t_{ul}^*$ can be found by solving at most $2L - 1$ single-variable convex optimization problems of the form

$$\min_{t_{ul} \in \Gamma_j} \sum_{l \in L} E_l(t_{ul})$$

(b) In the FO-UEC case, $t_{ul}^* \in \{\tau_{1}^{\max}, \tau_{2}^{\max}, \ldots, \tau_{L}^{\max}\}$. Moreover, if $\max_{l} \{\tau_{l}^{\min}\} \leq \min_{l} \{\tau_{l}^{\max}\}$, then $t_{ul}^* = \min_{l} \{\tau_{l}^{\max}\} = \min_{l} \{T - \frac{b_{l}}{r_{ul}^{\max}}\}$.

**Proof:** Since for each user pair-$l$, $E_{ul}^{\text{CELL}}(t_{ul})$ reaches its maximum value during the two intervals $[0, \tau_{l}^{\min}]$ and $(\tau_{l}^{\max}, T]$, $F(t_{ul})$ achieves its maximum value in the two intervals $\Gamma_1 = [0, \min_{l} \{\tau_{l}^{\min}\}]$ and $\Gamma_J = [\max_{l} \{\tau_{l}^{\max}\}, T]$. Hence, $t_{ul}^*$ is not in $\Gamma_1$ and $\Gamma_J$, but must be found in one of the remaining (at most) $2L - 1$ intervals. By Lemma 2 in the FO-SEC case, $F(t_{ul})$ is
Fig. 3. FO-UEC case (a) and (b) and FO-SEC case (c) and (d) with three links and frame duration $T = 1$ time unit.

piecewise convex. Hence, its global minimum can be found among its $2L - 1$ local minima in each interval. In the FO-UEC case we know that, from Lemma 2, $F(t_{ul})$ is piecewise decreasing. Thus, its global minimum can be found in the set $\bigcup_{l \in L} \{\tau^{\min}_l, \tau^{\max}_l\}$. However, for each $\tau^{\min}_l$, there is at least one component $E_l(t_{ul})$ in the sum defining $F(t_{ul})$, that decreases for $t_{ul} \geq \tau^{\min}_l$. Therefore, the global minimum can only be found in the set $\{\tau^{\max}_l, l \in L\}$. Furthermore, if $\max_l \{\tau^{\min}_l\} \leq \min_l \{\tau^{\max}_l\}$, then $t_{ul}^* = \min_l \{\tau^{\max}_l\}$.

Given the optimal solution $t_{ul}^*$ to problem (16), the optimal mode selection vector $m^*$ of
Problem (15) is then given by setting, $\forall l \in L$:

$$m_l^* = \begin{cases} 
0 & \text{if } E_{ll}^{CELL}(t_{ul}^*) \leq \bar{E}_{ll}^{D2D} \\
1 & \text{otherwise,}
\end{cases}$$

and the corresponding optimal transmission powers are derived as in (13).

**Remark 2.** The proposed solutions in the FO case are centralized at the BS. To collect the information about the link gains, we assume an approach similar to one proposed for handoff procedure [27]. Specifically, channel gains are estimated by averaging the received pilot signals’ strength over multiple frames, in order to diminish the undesirable effect of the fast fading component. To do so, the BS has to assign a dedicated beacon resource to the D2D users [28].

**IV. MINIMIZING THE ENERGY CONSUMPTION WITH D2D RS**

To increase the cell capacity and the spectral efficiency of the system, we consider the RS strategy, where all communications in D2D mode are assigned the same channel. Due to the cross interference among D2D pairs, the optimal solutions for this scenario are more complicated to compute. Nevertheless, we develop a combinatorial optimization algorithm that is guaranteed to find the optimal solution, and often does so very quickly. This off-line algorithm is complemented by a heuristic, suitable for real-time implementation under practical signalling constraints.

**A. Optimal resource allocation via mixed-integer nonlinear programming**

The effect of interference on the energy consumption in D2D mode appears explicitly in (7). Due to the fixed time allocation $T$ (see Lemma 1), minimizing the energy consumption of D2D communications is equivalent to minimizing the transmission powers. To meet the minimum rate requirement in (4), the transmission power of any pair-$l$ in D2D mode must be such that

$$p_{ll} \geq \left[ \exp \left( \frac{b_l}{WT} \right) - 1 \right] \frac{\sigma^2 + I_{ll}}{G_{ll}}.$$

Introducing $\gamma_{ll}^{tgt} = \left[ \exp \left( \frac{b_l}{WT} \right) - 1 \right]$ as the target signal-to-interference-plus-noise ratio (SINR) required to satisfy the session rate requirement of pair-$l$, $\eta_l = \frac{\gamma_{ll}^{tgt} \sigma^2}{G_{ll}}$, and $h_{lj} = \gamma_{lj}^{tgt} G_{lj}$, we can rewrite this inequality as

$$p_{ll} \geq \eta_l + \sum_{j \neq l} p_{jj} h_{lj}. \quad (18)$$
The joint mode selection, power and time allocation problem can now be formulated as the following MINLP problem:

\[
\begin{align*}
\text{minimize} & \quad \sum_{l=1}^{L} (T_{pl} m_l + E_{CELL}^{pl}(t_{ul}) (1 - m_l)) \\
\text{subject to} & \quad \frac{b_l}{T_{l0}^{max}} - T m_l \leq t_{ul} \leq T - \frac{b_l}{T_{l0}^{max}} + T m_l, \quad \forall l, \\
& \quad \left[\eta_l + \sum_{j \neq l} p_{jj} h_{lj}\right] - C (1 - m_l) \leq p_{ul}, \quad \forall l, \\
& \quad p_{ul} \leq p_l^{max} m_l, \quad \forall l, \\
& \quad m \in \{0, 1\}^L, \quad t_{ul} \in [0, T], \quad p_{ul} \geq 0.
\end{align*}
\]

Here, \(m\) is the mode selection vector defined in (14). Constraint (19b) ensures power feasibility for pairs in cellular mode, while (19c) guarantees that the rate requirement is satisfied for each pair in D2D mode. The constant \(C\) in (19c) is a large number (\(C = \max_l \{\eta_l + \sum_j p_{jj}^{max}\}\), for example) ensuring that the constraint is only enforced for users in D2D mode. The expression for \(E_{CELL}^{pl}(t_{ul})\) in the objective function is either (8) or (9), depending on whether we are interested in the RS-SEC or RS-UEC problem, respectively.

Problem (19) belongs to the class of mixed boolean-convex problems, where for each fixed \(m \in \{0, 1\}^L\) the objective function is convex in the continuous variables. In general, MINLPs are NP-hard problems [29], combining the combinatorial difficulty of optimizing over discrete variable sets, with the challenges of handling nonlinear functions. Their solution times grow exponentially with the problem dimension. In particular, if \(L\) in (19) is very small (i.e., smaller than 15), the optimization problem can be solved exactly by exhaustive enumeration of the \(2^L\) possible mode selection vectors. However, realistic cellular networks might consist of a large number of user pairs. For this reason, we develop a customized solver based on B&B (see, e.g., [30]). Although B&B is a general framework for finding solutions to MINLP problems, its efficiency depends strongly on how well it is tailored to the specific problem structure. Thus, we give special attention to developing novel variable selection and branching rules, along with efficient procedures for infeasibility detection and performance bound computations. The proposed solver allows us to find provably optimal solutions much more efficiently than using generic B&B solvers or naive exhaustive search. Due to space restrictions, in the sequel we will
focus on algorithms that solve the RS-UEC minimization problem, considering that the mobile devices are the most energy-sensitive component of the network. However, the reasoning behind the proposed approaches and design choices are still valid for the RS-SEC case, although other alternative choices, tailored for the specific SEC case, can be investigated to further improve the solution efficiency of the approaches. It is worth mentioning that by assigning only one channel to D2D pairs, there will be $L - |\mathcal{C}| + 1$ unused channels. This choice might not be optimal from the energy efficiency perspective, but it leads to another promised gain of D2D communications, namely, improved cell capacity. Indeed, by assigning multiple D2D pairs to the same channel, it is possible to accommodate a larger number of communication requests compared to the FO case. Moreover, the unused resources in a cell can be used to reduce the intercell interference in the multicell scenario. For example, a Fractional Frequency Reuse scheme could be employed to assign the unused resources of a cell to the cell-edge users in the neighbouring cells [31].

**B. A branch-and-bound approach for finding the optimal solution**

Before describing the proposed B&B algorithm, it is convenient to introduce a definition and two useful propositions on the feasibility of the mode selection vector $\mathbf{m}$. Let $\mathbf{H}$ be a non-negative matrix with entries $H_{lj} = h_{lj}$ if $l \neq j$, and zero otherwise, and let us introduce the vectors $\mathbf{p} = (p_l, \forall l \in \mathcal{L})^\top$, $\mathbf{p}^{\max} = (p^{\max}_l, \forall l \in \mathcal{L})^\top$ and $\mathbf{\eta} = (\eta_l, \forall l \in \mathcal{L})^\top$. For each mode selection vector $\mathbf{m}$, we can define the corresponding set of pairs assigned to D2D mode and to cellular mode as $\mathcal{D}_m$ and $\mathcal{C}_m$, respectively. Let $\mathbf{A}_m$ denote the $L \times |\mathcal{D}_m|$ incidence matrix, which is formed by removing the $l$-th column from the $L \times L$ identity matrix if $m_l = 0$. We define $\mathbf{H}_m = \mathbf{A}_m^\top \mathbf{H} \mathbf{A}_m$, $\mathbf{p}_m = \mathbf{A}_m^\top \mathbf{p}$, $\mathbf{I}_m = \mathbf{A}_m^\top \mathbf{I}_m$, $\mathbf{p}^{\max}_m = \mathbf{A}_m^\top \mathbf{p}^{\max}$, and $\mathbf{\eta}_m = \mathbf{A}_m^\top \mathbf{\eta}$, and we rewrite Constraints (19c) and (19d) in matrix form as

$$
(I_m - \mathbf{H}_m)\mathbf{p}_m \geq \mathbf{\eta}_m \quad \text{and} \quad \mathbf{p}_m \leq \mathbf{p}^{\max}_m,
$$

where the inequalities are component-wise. The matrix $\mathbf{H}_m$ has strictly positive off-diagonal elements, and we can assume that it is irreducible because we do not consider totally isolated groups of pairs that do not interact with each other. Let $\rho(\mathbf{H}_m)$ denote the largest real eigenvalue of $\mathbf{H}_m$. From the Perron-Frobenius theorem [32], we have the following proposition:
Proposition 2 ([33], Ch. 2). For a given mode selection vector \( \mathbf{m} \), the necessary and sufficient condition for the existence of a positive \( p_m \) to solve inequality \( (\mathbf{I}_m - \mathbf{H}_m)p_m \geq \eta_m \) is that

\[
\rho(\mathbf{H}_m) < 1.
\]

Moreover, \( p_m^* = (\mathbf{I}_m - \mathbf{H}_m)^{-1}\eta_m \) is its component-wise minimum solution.

Proposition 2 provides an easy condition to verify if a mode selection vector \( \mathbf{m} \) is feasible.

Definition 2 (Feasible mode selection vector). A mode selection vector \( \mathbf{m} \) is feasible if both condition (21) and \( (\mathbf{I}_m - \mathbf{H}_m)^{-1}\eta_m \leq p_m^{\max} \) are verified.

Proposition 3 ([34]). If \( \mathbf{m} \) is not feasible, then every other mode selection vector \( \tilde{\mathbf{m}} \) with additional users assigned to D2D mode, i.e. such that \( \{l \in \mathcal{L} : m_l = 1\} \subseteq \{l \in \mathcal{L} : \tilde{m}_l = 1\} \), is not feasible.

We now use the results above to design a B&B algorithm that solves the MINLP problem in (19). When using a B&B approach, all possible mode selection vectors \( \mathbf{m} \) are explored through a binary tree. Each node of the tree (except the root) represents a subproblem where one of the mode selection variables \( m_l \) is set to either 0 or 1. Thus, to each node corresponds a partial mode selection vector with some components already defined (fixed) and forming the set \( \mathcal{F} \), while others are still undetermined and represent the set \( \mathcal{U} \). Therefore, each branch of the three corresponds to a subset of the possible mode selection vectors. The main idea of B&B is to only explore branches that have the potential to produce better solutions than the best solution found so far, and disregard (prune) the others. This is done by computing upper and lower bounds on the optimal value at each node. If the lower bound of a node is larger than the current upper bound, then there is no need to explore its branches. To achieve a good performance of B&B, it is essential to select the branching rule and tree exploration strategies carefully, and to have efficient methods for computing good (tight) upper and lower bounds.

The flowchart in Fig. 4 summarizes the proposed B&B algorithm. It is based on the following four choices: 1) the computation of the initial upper bound (UB), where we assume all pairs in cellular mode; 2) the branching rule that selects the variable to fix at each node as the one that increases the likelihood of finding infeasible mode selection vectors, to exploit Proposition 3; 3) the tree exploration strategy that assigns the selected branching variable to 1 first (i.e., D2D
mode comes first as a choice); and, finally, 4) the computation of the upper and lower bounds (Node-UB and Node-LB) as the sum of the minimum energy cost of the pairs in $\mathcal{F}$ plus an upper and lower bound of the energy cost of the pairs in $\mathcal{U}$, respectively. We refer to the Appendix for a detailed description and motivation of each such choice. Once the optimal mode selection vector and transmission time have been found, the power levels are obtained as in (13) for cellular users, and as in Proposition 2 for D2D users.

The B&B algorithm is guaranteed to find the optimal solution, and does so much faster than the exhaustive search, as shown in § V-B2. However, for large networks it can still have impractical running times. In addition, the optimization formulation assumes that all cross-gains between users are known, which in turn would impose significant communication overhead. We therefore turn our attention to heuristics that can be run in real-time and do not assume centralized knowledge of all the channel gains.

C. A heuristic approach to achieve a practical sub-optimal solution

In this subsection, we present a heuristic algorithm that achieves a near-optimal solution to (19) in a more practical and scalable way than the B&B approach. Again, we focus on the UEC case.
The key idea of this algorithm is to first determine an initial mode selection vector, together with the corresponding power and time allocation, and then improve this solution by means of a distributed power control algorithm based only on local measurements. The heuristic algorithm, described in Algorithm 1, consists of the following three main steps:

**Algorithm 1: Heuristic approach for RS-UEC minimization**

**Input:** \((\gamma_t^{tgt}, G_l, G_l^0, G_0) \forall l \in \mathcal{L}, \theta_l\)

**Output:** \(m^*, p^*\)

1. \((m^{FO}, t_{ul}(m^{FO}), p(m^{FO})) \leftarrow \) solution to FO-UEC problem;
2. each \(l \in \mathcal{D}_{m^{FO}}\) acquires \(E_{CELL}^{CELL}(t_{ul}(m^{FO}))\) from the BS;
3. \(p^{(0)} \leftarrow p(m^{FO}), \ m^{(0)} \leftarrow m^{FO}, \ k = 0;\)
4. each \(l \in \mathcal{D}_{m^{FO}}\) computes \(\gamma_l^{(0)};\)
5. convergence \(\leftarrow\) False;
6. while \(\neg\) convergence do
7. \(m^{(k+1)} \leftarrow m^{(k)};\)
8. for each \(l \in \mathcal{D}_{m^{(k)}}\) do
9. \(P_{il}^{(k+1)} \leftarrow \frac{\gamma_l^{tgt}}{\gamma_l^{(k)}} P_{il}^{(k)};\)
10. if \(P_{il}^{(k+1)} > \min\left\{ T E_{CELL}^{CELL}(t_{ul}(m^{FO})), P_l^{max}\right\}\) then
11. \(m_l^{(k+1)} \leftarrow 0, \ \mathcal{D}_{m^{(k+1)}} \leftarrow \mathcal{D}_{m^{(k)}} \setminus \{l\};\)
12. each \(l \in \mathcal{D}_{m^{(k+1)}}\) computes \(\gamma_l^{(k+1)};\)
13. if \(\gamma_l^{(k+1)} \geq \gamma_l^{tgt}, \forall l \in \mathcal{D}_{m^{(k+1)}}\) then
14. convergence \(\leftarrow\) True;
15. \(p^* \leftarrow p^{(k+1)}, \ m^* \leftarrow m^{(k+1)};\)

1) **Initial phase:** We adopt the optimal solution to the FO-UEC problem in § III as the initial solution, denoted by \((m^{FO}, t_{ul}(m^{FO}), p(m^{FO}))\). The FO-UEC problem is solved by the BS. For each pair-\(l \in \mathcal{D}_{m^{FO}}\) (i.e., assigned to D2D mode), the BS also computes the energy it would consume if in cellular mode, that is \(E_{CELL}^{CELL}(t_{ul}(m^{FO}))\) from (9), and broadcasts \(m^{FO}\) and \(E_{CELL}^{CELL}(t_{ul}(m^{FO}))\) to each Tx-\(l \in \mathcal{D}_{m^{FO}}\). The initial mode selection vector \(m^{FO}\) is obtained under the assumption of no interference among the D2D pairs. However, under the RS scenario, all the D2D pairs share the same channel, thus \(m^{FO}\) may be energy inefficient or even infeasible, due to the interference. Therefore, a distributed power control algorithm is then executed by the D2D pairs to find a feasible and more energy-efficient solution:

2) **Iterative distributed power control for D2D pairs:** Using the iterative power control method
originally proposed by Foschini and Miljanic in [35], each Tx-$l$ in D2D mode can achieve its target SINR $\gamma_{l}^{tgt}$ by updating its transmit power as follows

$$p_{ll}^{(k+1)} = \frac{\gamma_{l}^{tgt}}{\gamma_{l}^{(k)}} p_{ll}^{(k)},$$

where $p^{(0)} = p(m^{FO})$ and $\gamma_{l}^{(k)}$ is the perceived SINR for pair-$l \in D_{m^{FO}}$ in iteration-$k$, defined as $\gamma_{l}^{(k)} = \frac{p_{ll}^{(k)} G_{ll}}{\sigma^2 + \sum_{j \in D_{m^{FO}}, j \neq l} p_{jj}^{(k)} G_{jl}}$. To achieve a feasible mode selection vector and to further reduce the energy cost, some links in D2D mode need to switch to cellular mode. Specifically, pair-$l$ in D2D mode will switch to cellular mode if its transmit power level exceeds its maximum limit or if it is more energy efficient for it to communicate in cellular mode, that is,

$$p_{ll}^{(k)} > \min \left\{ \frac{\theta}{T} E_{ll}^{CELL} \left( t_{ul}(m^{FO}) \right), p_{l}^{max} \right\},$$

where we have introduced a design parameter $\theta \geq 1$ (see discussion in subsection below). During the power update (22), if Tx-$l$ finds that condition (23) is fulfilled, it asks the BS to switch it to cellular mode and to assign it an orthogonal frequency channel. Otherwise, it keeps updating its power according to (22). The BS keeps track of the pairs changing communication mode, and updates the mode selection vector. This power control algorithm converges to the minimum power levels that the user pairs remaining in D2D mode need to fulfil the rate requirement.

3) Final phase: Once the algorithm converges, the BS recomputes the optimal power and time allocation for the user pairs in cellular mode, broadcasting this information.

Note that all computations are based on the assumption that the final duration for the data transmission is $T$. For this reason, the power control iterations should not be performed within the frame dedicated to the data transmission. We assume that the power control runs either on a dedicated time interval before the data frame, or, alternatively, on a separate control channel.

**On the selection of parameter $\theta$:** The selection of parameter $\theta$ accounts for the following key aspects of the possible practical implementation of the proposed heuristic algorithm:

- Trade-off between signalling overhead and energy gain: Communication mode switches incur additional signalling overhead between mobile devices and the BS to coordinate the re-allocation of radio resources. Hence, by setting $\theta > 1$, mode switches will occur only if they result in a significant energy gain.
• Trade-off between channel reuse and energy consumption: Since moving a user pair from D2D mode to cellular mode requires another orthogonal channel, a large value of $\theta$ can enforce more pairs to communicate in D2D mode and thus increase the channel reuse, even if this comes at the cost of a higher energy consumption due to the interference.

• Accounting for the mis-estimation of the energy cost: D2D pairs base their selection to switch communication mode on an under-estimate of the energy consumption in cellular mode (Eq. (23)). Since the optimal UL transmission time computed when Algorithm 1 has converged will be greater or equal to the initial $t_{ul}(m^{FO})$, the actual energy consumption in cellular mode can be larger than expected. Using $\theta > 1$ reserves a margin for mis-estimation so that only connections that truly gain by being in cellular switch to cellular.

**Remark 3.** The initial phase of the RS heuristic needs the same state information at the BS as the FO algorithms. This information can be acquired using the strategy described in Remark 2. The remaining steps of the RS heuristic are fully distributed, and driven by local measurements (i.e., the SINR at the receivers) without any need for additional channel gain estimation.

V. SIMULATIONS AND DISCUSSION

This section presents simulation results that validate our theoretical findings and evaluate our proposed algorithms. We consider a single cell with a BS, equipped with an omnidirectional antenna, positioned in the center. We assume a path loss channel model $G_{ij} = G_0 D_{ij}^{-\alpha}$, where $D_{ij}$ is the physical distance between Tx-$i$ and Rx-$j$ and $G_0$ is the path gain at a reference distance of 1 m. By neglecting small scale and frequency selective fading, in the following simulations we assume that frequency channels are assigned to the users in a round-robin fashion. The main simulation parameters are listed in Table II.

<table>
<thead>
<tr>
<th><strong>Parameter</strong></th>
<th><strong>Value</strong></th>
<th><strong>Time frame duration (T)</strong></th>
<th><strong>1 s</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier frequency</td>
<td>1 GHz</td>
<td>Max Tx power for transmitter-$l$ ($p^{max}_l$)</td>
<td>0.25 W</td>
</tr>
<tr>
<td>Cell radius</td>
<td>500 m</td>
<td>Max Tx power for the BS ($p^{max}_0$)</td>
<td>40 W</td>
</tr>
<tr>
<td>Frequency channel bandwidth (W')</td>
<td>5 MHz</td>
<td>Path gain at reference distance of 1 m ($G_0$)</td>
<td>$5.7 \times 10^{-4}$</td>
</tr>
<tr>
<td>Noise power spectrum density ($\sigma^2$)</td>
<td>-174 dBm/Hz</td>
<td>Path loss exponent ($\alpha$)</td>
<td>4</td>
</tr>
</tbody>
</table>
A. Single link analysis: geometrical interpretation

We develop a geometrical interpretation of the optimal mode selection policy for the single link case. To ensure that Assumption 1 is satisfied, we set $b_l$ to its maximum value that guarantees that the constraint set of Problem (10) is never empty, i.e., $b_l = \frac{r_{0l}^{\text{max}} + r_{0l}^{\text{max}}}{r_{0l}^{\text{max}} + r_{0l}^{\text{max}}} T$, where the maximum rates $r_{0l}^{\text{max}}$ and $r_{0l}^{\text{max}}$ are functions of gains $G_{l0} = G_0 D_{l0}^{-\alpha}$ and $G_{0l} = G_0 R_{\text{cell}}^{-\alpha}$, respectively, with $R_{\text{cell}}$ being the cell radius. We first consider the UEC case. D2D communication is energy-optimal when $E_{ll}^{\text{D2D}}(T) \leq E_{ll}^{\text{CELL}}(t_{ul}^*)$. Under the path loss model, the mode selection policy is equivalent to the following condition:

$$D_{ll} \leq \left( \frac{e^{b_l/WT} - 1}{e^{b_l/Wt_{ul}^*} - 1} \cdot \frac{T}{t_{ul}^*} \right)^{-1/\alpha} D_{l0} = \kappa(D_{0l}) D_{l0},$$

(24)

where $\kappa(\cdot)$ is a function of $D_{0l}$, since $D_{0l}$ affects $r_{0l}^{\text{max}}$ and thereby $t_{ul}^* = T - b_l/r_{0l}^{\text{max}}$. Thus, even though in the UEC case the energy cost for the DL transmission is neglected, $D_{0l}$ still plays a role in the optimal mode selection. To characterize the region where D2D mode is preferable, we fix the position of Tx-$l$ (and therefore $D_{l0}$) and then move Rx-$l$ along a circle centred at the BS, thus keeping $D_{0l}$ (and $\kappa(D_{0l})$) constant. According to (24), D2D mode is energy-optimal when Rx-$l$ is located in the arc defined by the intersection of the circle of radius $D_{0l}$ centered at the BS, and the disc of radius $\kappa(D_{0l}) D_{l0}$ centered at Tx-$l$. The D2D-optimal area can be constructed by tracing out these arcs for different distances $D_{0l}$.

Fig. 5 illustrates the D2D-optimal area in red and the D2D power-feasible are in light blue, for two different locations of Tx-$l$. Although (24) does not formally define a disc around Tx-$l$, the D2D-optimal area is close to circular. This is due to the power imbalance between the user equipment and the BS. For the UEC case, the BS transmits at its maximum power, with $p_{0l}^{\text{max}} \gg p_{0l}$, which makes $b_l/r_{0l}^{\text{max}}$ very small, $t_{ul}^* \approx T$ and $\kappa \approx 1$ practically independently of $D_{0l}$. For the SEC case, similar calculations and arguments can be made. As shown in Fig. 6, the D2D-optimal area is no longer circular and it covers a large portion of the cell. The expression for $\kappa(D_{0l})$ in the SEC case, and more results on the D2D-optimal area for different values of $b_l$ can be found in [19].
Fig. 5. UEC case. Fixing the position of Tx-$l$, the red area represents the positions of Rx-$l$ for which D2D mode is energy-optimal, while the light blue disk represents the area within which Tx-$l$ can satisfy the power feasibility in D2D mode.

(a) Tx-$l$ is 250 m away from the BS.  
(b) Tx-$l$ is 450 m away from the BS.

Fig. 6. SEC case. Fixing the position of Tx-$l$, the red area represents the positions of Rx-$l$ for which D2D mode is energy-optimal, while the light blue disk represents the area within which Tx-$l$ can satisfy the power feasibility in D2D mode.

(a) Tx-$l$ is 250 m away from the BS.  
(b) Tx-$l$ is 450 m away from the BS.

B. Multiple link analysis: energy savings and algorithm performance

In the multi-link case, the transmitters and receivers are uniformly placed within the cell. An example network with 10 user pairs is given in Fig. 7. We assume large-scale path loss model, and adopt the same parameter setup as in Section V-A (cf. Table II). For a given number of user pairs, we investigate 1000 random networks and present the averaged results.

1) Energy gain by enabling D2D communications in a fully orthogonal system: To quantify the energy savings that can be obtained by using D2D communication, we compare the energy cost of the optimal FO-UEC solution ($E_{ModeSelection}$) with the one where all pairs are forced to communicate in cellular mode ($E_{Cellular}$). In particular, we define the percentage energy saving
of a single user as $\frac{E_{\text{Cellular}} - E_{\text{ModeSelection}}}{E_{\text{Cellular}}} \times 100\%$. For each random configuration used in our Monte Carlo study, we sort the links in order of increasing energy saving, and then average over the 1000 values coming from the different simulations. Fig. 8 shows the results for networks with 10 and 30 user pairs. We observe that when D2D communication is enabled, the average per user energy saving is $\sim 40\%$. In particular, one third of the user pairs have an energy saving
larger than 60%, and half of the transmitters achieve an energy saving larger than 20%. We highlight that these gains are not only a consequence of the proximity of the users but also stem from the more advantageous full time frame allocation to the single-hop D2D connections.

2) Performance evaluation of the B&B algorithm for RS-UEC: The difficulty in solving Problem (19) lies mainly in the possibly large search space of integer feasible points. Table III shows the average number of mode selection vectors explored by different strategies, before finding the optimal solution. We consider the B&B algorithm described in Section IV-B both with the proposed branching rule and with a random selection of the branching variable. We compare the two algorithms to the naive exhaustive search approach, where infeasible solutions are eliminated using Proposition 3. The results clearly show that the customized design of the branching rules have a strong effect in reducing the search space. Furthermore, we evaluate the runtime performance of the aforementioned two B&B approaches and that of the proposed heuristic. Table IV shows the averaged results over 1000 independent simulations and for different network sizes. The B&B algorithm with random branching rule requires an unacceptable runtime as the number of links in the network increases, while our customized B&B finds the optimal solution much faster. On the other hand, the heuristic algorithm, although suboptimal and iterative, shows much more efficient and scalable runtime, which makes it more suitable for practical implementation.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>10 user pairs</th>
<th>15 user pairs</th>
<th>20 user pairs</th>
<th>30 user pairs</th>
<th>40 user pairs</th>
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<tbody>
<tr>
<td>Exhaustive enumeration with Proposition 3</td>
<td>472.975</td>
<td>7.46 × 10^3</td>
<td>NA</td>
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<td>NA</td>
</tr>
<tr>
<td>B&amp;B - Random branching rule</td>
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<td>251.21</td>
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<tr>
<td>B&amp;B - Proposed branching rule</td>
<td>25.57</td>
<td>54.72</td>
<td>120.15</td>
<td>579</td>
<td>3.08 × 10^3</td>
</tr>
</tbody>
</table>

“NA (not available) denotes the case when the optimal solution was not found within 8 hours on a standard PC.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>10 user pairs</th>
<th>20 user pairs</th>
<th>30 user pairs</th>
<th>40 user pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>B&amp;B - Random branching rule</td>
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<td>0.2442</td>
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<td>0.0043</td>
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</tbody>
</table>
3) Performance evaluation of the heuristic mode selection algorithm for RS-UEC: In this section, we evaluate the performance of the proposed heuristic algorithm. The histograms in Fig. 9 show the additional energy cost of the heuristic relative to the optimal solution computed using the B&B solver. For networks with 10 user pairs, the heuristic is within 10% of the optimal solution for almost all network configurations; see Fig. 9(a). For networks with 30 user pairs, the performance of the heuristic is slightly worse. This degradation is due to the larger degree of freedom in packing D2D links on the same frequency channel. However, the solutions are still within the 10% suboptimality for most configurations.

Fig. 7 is an example of those few network configurations where the heuristic performs much worse than optimal algorithm (indicated with a red circle in Fig. 9). Under FO, both pair-9 and pair-4 in Fig. 7 would be assigned to D2D mode. When the heuristic initially attempts to assign these links to the same channel, it encounters an infeasible configuration due to the high interference that Rx-4 perceives from Tx-9. Therefore, only one of the two pairs can be assigned to D2D mode. The optimal decision is to let pair-4 in D2D mode, but the heuristic makes a wrong decision during the first iterations, when a temporarily high interference from Rx-9 leads pair-4 to leave the shared channel and switch to cellular mode.

![Histograms](image)

(a) Networks with 10 user pairs.  
(b) Networks with 30 user pairs.

Fig. 9. RS-UEC: Histograms of the additional energy cost of the heuristic algorithm relative to the optimal solution achieved with B&B. Red circles represent cases where the heuristic performs much worse than the B&B (an example of such a case is given in Fig. 7).
4) Energy and spectral efficiency of D2D in dynamic TDD: We conclude this section by a comparison of the different mode selection policies proposed in this paper, both in terms of their energy efficiency and their spectral efficiency. Fig. 10 shows the energy-channel performance of the proposed algorithms for networks with 10 and 30 user pairs, respectively. We note that FO-UEC represents the energy-optimal solution, with significant energy savings compared to the purely cellular solution (labelled FO-cellular mode) and the same spectral efficiency (measured in number of allocated channels to all communications). Resource sharing (RS-UEC-B&B) yields big improvements in spectral efficiency at the cost of a slight increase in energy consumption when D2D transmitters need to compensate for interference. The reason that the energy increase is so small is that the transmission powers assigned to D2D pairs are generally small, partly because D2D pairs typically have high direct gains (since the transmitter and receiver often are in close proximity of each other), and partly because D2D connections can use the full frame duration (see Eq. (7)). These results demonstrate that for some user pairs, communicating in D2D mode and interfering with each other is still more energy efficient than communicating in cellular mode on an exclusive frequency channel. Thus, in dynamic TDD systems, D2D communications have the potential to improve both spectrum and energy efficiency over a traditional cellular solution. Finally, we evaluate the performance of the heuristic method for different values of the threshold \( \theta \). As expected, large values of \( \theta \) decrease the number of channels used at the expense of a slightly increased energy consumption. For this reason, \( \theta \) is an important design parameter for finding a suitable trade-off between energy consumption and channel use.

VI. CONCLUSION AND FUTURE WORK

We investigated the problem of energy efficient mode selection and resource (power and time) allocation for network-assisted D2D communications in dynamic TDD systems. We analyzed the problem under two frequency channel allocation strategies (with and without interference among D2D pairs) and with two objectives (total user energy and total system energy). For each configuration we derived the optimal solution to the corresponding MINLP formulation. In particular, for the interference free case, we demonstrated how the optimal solution can be obtained in polynomial time. On the other hand, when D2D pairs interfere with each other, finding the optimal solution in an efficient manner in terms of execution time and cross-gains knowledge is much harder. Therefore, a customized branch-and-bound solver was complemented by a more
Fig. 10. Performance evaluation for different algorithms. Total energy consumption and number of orthogonal frequency channels needed to accommodate all the communication requests within the cell (i.e., both cellular and D2D users).

practical low-complexity heuristic. Through simulations, we demonstrated that significant energy savings can be achieved by exploiting the benefits brought both by the better channel gain of D2D links, and by the adaptive transmission time of the dynamic TDD technology. Moreover, leveraging on the observation that large channel gain and long transmission duration for D2D links lead to a low transmit power (and hence low interference), we showed the potential of D2D communications to improve also the spectrum efficiency over traditional cellular communications. Finally, we presented analytical characterizations of the D2D-optimal areas for a transmitter-receiver pair, showing that these regions can be surprisingly large and not necessarily circular.

There are several interesting extensions of this work that we hope to return to in the future. For example, the multi-cell scenario where different cells optimize their individual UL/DL time allocation adds a non-trivial inter-cell interference that needs to be managed. Combining the considered time (and power) allocation problem with a channel assignment that accounts for frequency selective fading could yield additional energy savings. How to distribute resources when the number of channels exceeds the number of user pairs is another challenging problem. Finally, it could be useful to explore advanced concepts, such as network coding, and consider the energy models which also account for circuit and idle power consumption of transmitters.
In the design of our B&B algorithm in § IV-B, we have made the following choices:

1) *Initial upper bound.* By assumption, letting all pairs communicate in cellular mode (i.e., \( m_l = 0 \forall l \)) is always feasible. We therefore take the corresponding energy cost as an initial upper bound on the optimal cost, computed by solving

\[
\begin{align*}
\text{minimize} & \quad \sum_{l \in \mathcal{L}} E_{d_{ul}}^\text{CELL}(t_{ul}) \\
\text{subject to} & \quad \max_{l \in \mathcal{L}} \left\{ \frac{b_l}{r_{l0}^\text{max}} \right\} \leq t_{ul} \leq \min_{l \in \mathcal{L}} \left\{ T - \frac{b_l}{r_{l0}^\text{max}} \right\},
\end{align*}
\]

where \( E_{d_{ul}}^\text{CELL}(t_{ul}) \) is given by (9).

2) *Branching rule.* The branching rule selects the next variable to fix at each node of the tree. The goal is to identify the branching variable that changes the problem the most, either to quickly detect branches that can be cut, or to significantly improve the current solution. Our branching strategy is based on first solving the FO-UEC problem described in § III. The corresponding optimal mode selection vector \( \mathbf{m}^\text{FO} \) reveals the set \( \mathcal{D}_{\mathbf{m}^\text{FO}} = \{ l \mid m_l^\text{FO} = 1 \} \) of pairs that prefer to communicate in D2D mode in an interference-free environment. For each link-\( l \in \mathcal{D}_{\mathbf{m}^\text{FO}} \), we define a measure of its strength \( s_l = \sum_{i \in \mathcal{D}_{\mathbf{m}^\text{FO}}, i \neq l} G_{i_l}/G_{ll} \). This measure attempts to account for both the interference that pair-\( l \) produces on the shared frequency resource and its own direct gain. The branching rule first selects variables in \( \mathcal{D}_{\mathbf{m}^\text{FO}} \) in order of decreasing \( s_l \), and then considers the remaining variables in an arbitrary order. This rule exploits the fact that pairs that prefer to be in cellular mode in FO are also likely to prefer cellular mode in RS. By fixing D2D pairs with high interference strength first, we increase the likelihood of finding infeasible solutions quickly.

3) *Tree exploration strategy.* Once a branching variable \( m_l \) has been selected by the branching rule, the tree exploration strategy determines if the child node to investigate next should be in cellular or D2D mode. We allocate users to D2D mode first (i.e., setting \( m_l = 1 \) first).

4) *Upper and lower bounds.* When we consider a node, we first verify if the set \( \mathcal{F} \) forms a feasible mode selection vector in the sense of Proposition 2. If not, then the node and all branches below it can be disregarded by making use of Proposition 3. Otherwise, upper and lower bounds are calculated. To compute an upper bound, we assign all pairs in \( \mathcal{U} \) to
cellular mode and we solve the problem formulation in (25) where $L$ is replaced by the union of $U$ with the subset of $F$ representing users already assigned to cellular mode. To determine a lower bound, we solve the FO-UEC problem over only the unassigned pairs. This is a lower bound because the computation is a relaxation of the MINLP, where the UL times of fixed and unassigned pairs are allowed to differ, and where the unassigned links that end up in D2D mode do not suffer interference. Furthermore, we strengthen the lower bound by increasing the noise power of pairs in $U$ by the interference that the transmitters fixed to D2D communication in $F$ incur on them.

REFERENCES


