Simulation and study of passive shunt damping of a composite plate by embedded PZT transducers

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Abstract

This report investigates the possibility of simulating a fiber composite plate with embedded piezoelectric transducers connected to shunt circuits in order to control vibration. The simulation model is validated in steps of discipline, both analytic and experimentally. First the piezoelectric material used is studied where comparisons are done between analytic and numerical results to ensure electromechanical coupling. Next the interaction with structures are studied by bonding piezoelectric transducers to an aluminum cantilever and study the structural response, where numerical results are compared to experimental data. Piezoelectric shunt damping is studied by connecting a tuned shunt circuit to a piezoelectric transducer, the results from the simulation model are then compared to experimental data where an attenuation of 12 dB is experimentally verified for the first mode. Lastly, all aspects above are implemented in a simulation model to estimate structural behavior of an arbitrary fiber composite plate with embedded piezoelectric transducers. The software used for numerical finite element simulation is Comsol Multiphysics 5.2.

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1. **INTRODUCTION AND BACKGROUND**

In this work, the technique of using embedded piezoelectric transducers to passively damp composite structures have been studied, which demonstrate a way to further increase the functionality of composite structures. This approach would allow designers to control vibrations and still keep the structural appearance intact. In this introduction, all aspects regarding this work will be introduced in separate subsections: the concept of smart structures, composite materials, the use of piezoelectric transducers and shunt damping.

The phenomenon of mechanical vibration can be desirable, for example when using a tuning fork or for the strings on a guitar vibration is crucial to the design. But when designing most structures vibration is a phenomenon designers want to minimize because of the unpredictability and the effect it can have on the material, in worst case lead to material failure. Another desire often present in design processes is stiffness- and strength to mass ratios; one material which have the possibility to be thin and light, and still very strong, is fiber composites. This advantage allows designers to minimize mass and still withstand static loads of high degree. But thin and light structures are prone to oscillations when subjected to dynamic loads, which could limit the designer and decrease the advantage of fiber composites high specific strength and stiffness. Increased interest in lighter material is another reason for this study, for example, due to increased carbon dioxide in the atmosphere, the vehicle industry have to develop new ways to decrease fuel consumption whether it’s combustion or electrically powered. One effective way to do this is to decrease the mass of the vehicle which would result in lowering the required forces needed to accelerate the mass, which could be done using materials with high specific strength and stiffness. However with a decreased structural mass the structure is more prone to noise and vibration, and with engines moving and contact between tires and the track, as examples for dynamic sources, causing major dynamic loads the structure need sufficient damping to also minimize noise and vibration. Many approaches have been tried to dampen structures prone to vibration, where the most common method is simply to increase the mass of the structure or adding viscoelastic material, others have acknowledged smart structures where damping apparatus have been bonded to the structure.

A simulation model that correspond well with experimental results is crucial in this analysis for further use and implementation on different structures. The software used to simulate the effect described is Comsol Multiphysics 5.2 [24]. Comsol Multiphysics is a FE-modeling software using predefined physics and couple those to simulate a system.

1.1. **Smart structures**

When implementing sensors and/or actuators in a material for structural purposes, it is known as a smart structure. During a workshop organized by the US army research office to define smart structures and materials, I. Ahmad [1] proposed the definition “A system or material which has built-in or intrinsic sensor/s, actuator/s and control mechanism/s whereby it is capable of sensing a stimulus, responding to it in a predetermined manner and extent, in a short/appropriate time and reverting to its original state as soon as the stimulus is removed.” In this work, this is in the form of a CFRP (Carbon Fiber Reinforced Plastic) plate and aluminum beams with embedded or bonded piezoceramic transducers. When the structure is subjected to vibrations by an external source, the piezoceramic transducers transform the strain and via piezoelectric effect minimize these effects. One thing all smart structures have in common when analyzed is that they are multidisciplinary, e.g., static-, electrical- and dynamical loads together with circuitry will be studied in this work which will be presented throughout this report.

1.2. **Piezoelectric transducers**

The smart materials used in this work will express a piezoelectric effect and is thus called piezoelectric transducers. The name *piezoelectricity* can conveniently be translated into pressure electricity. Piezoelectric materials produce an electric charge when subjected to a mechanical stress and in reverse the material develop mechanical strain proportional to an applied electric field. This material phenomenon was first discovered by the brothers Pierre and Jacques Curie in 1880. The discovery was the result of experiments where they measured surface charge appearing on polarized crystals subjected to mechanical stress, however they did not predict the reverse piezoelectric effect, strain as a result of applied electric field. This was first predicted and proposed through mathematical deduction from
fundamental thermodynamic principals by Gabriel Lippmann in 1881 [17]. Shortly after this proposal, the Curie brothers confirmed the existence of the reversed piezoelectric effect through experiments.

The piezoelectric effect was for a long time considered a laboratory curiosity. Until World War I, where piezoelectric transducers was first used in practical applications, namely for sonar to detect submarines. The device contained a piezoelectric transducer which could emit a high frequency pulse and a hydrophone to detect the returning echo, and by measuring the time for the signal to return one can calculate the distance to the target. With ample success, a great interest was developed for piezoelectric transducers which resulted in new application areas and new piezoelectric materials were developed.

Piezoelectricity is found naturally in many mono crystalline materials, like quarts and Rochelle salt. It was from these materials the piezoelectric effect was first discovered. However, these materials are generally not suitable for engineering applications. This changed when poly crystalline materials such as PZT (lead zirconate titanate) was developed. PZT have greater piezoelectric properties like the coupling between voltage and mechanical strain, are more durable and are relatively easy to manufacture. As a result of this development, the piezoelectric transducers used today are plentiful and can be manufactured into several shapes and forms, some examples of applications used today are sound detectors (microphones), airbag sensors, fetal heart monitors, ultrasonic imaging, disc drives and the ignition in some cigarette lighters. In this work the piezoelectric transducers are in the form of PZT discs bonded to- or embedded in structures to excite and dampen vibration at resonance frequencies.

1.3. Shunt damping

The piezoelectric shunt damping technique is characterized by connecting an electrical impedance to a piezoelectric transducer that is bonded to a structure and utilizes its electrical to mechanical coupling to dissipate mechanical energy. Passive piezoelectric shunt damping have been a subject of research for the last couple of decades, where most of the initial credit is given to Hagood and von Flotow [9] for deriving the effective mechanical impedance of resistor and inductor in series connected parallel to a piezoelectric transducer. One benefit with using shunted PZT to damp mechanical systems is that it need no sensors or advanced control systems, like in active vibration control. This allows for simple installation and maintenance as well as minimal added mass to the main structure, which is the main reason for studying this technique.

1.4. Environmental aspect

Actually none of the materials used in this study is in any way good for the local- or global environment in itself. Composite material, using thermoset plastics, still struggle with ‘end-of-life’ issues, i.e. recycling and reuse. The process of cross linking matrices are inherently what gives the material its excellent properties, but mixing plastics makes them highly un-recyclable. Some companies have excelled in this new market where big industries in the vehicle and aerospace sectors use increased amount of thermoset composites, e.g ELG Carbon Fibre, UK, and Reprocover, Belgium, who take on thousand of tonnes of fiber composites and recycle the material to a cost-sensitive market. The process used is milling down thermoset composites into small granules which is then mixed with neat resin to produce a new composite material. It is a sign of advancement that the large industries that use alot of material now have the option to recycle their material, but most of the commercial waste still ends up in a furnace producing electricity or on a landfill. In 2014 the global demand for CFRP was reported by Dr. Elmar Witten (AVK) and Thomas Kraus, Michael KÅïhnel (CCeV) in the Composites Market Report 2015 [3] to have reached 85000 tonnes with an estimated demand for 2021 at 175000 tonnes. With such a high demand from the market, more focus must be placed on end-of-life aspects.

Lead zirconate titanate (PZT) is the most popular material when manufacturing piezoelectric transducers for engineering purposes and it is also the material used and studied in this work. This material contain a high weight percent of lead which lead to both environmental and health issues. According to the RoHS (Restriction of Hazardous Substances) directive, EU, specifies a maximum weight% of lead in a component to 0.1. The usual PZT transducers today contain more than 60 weight% lead which could makes it subject to restriction, and change how the material is used all together.[20] As a result of this fact development of lead free piezocermics has become a hot topic for the last decade and some of the potential materials that have been researched is bismuth-, Alkaline- and KNN-based
ceramics.[8] To this date few of the materials can reach the same electromechanical coupling factor ($k_{PZT} \approx 0.63$, $k_{KNN} \approx 0.36$) and charge constant ($d_{33}^{PZT} \approx 400 \text{ pC/N}$, $d_{33}^{KNN} \approx 80 \text{ pC/N}$), but development continues and if regulations come to effect the industry have to addapt.[14] Recently researchers have developed materials which express higher charge constants than PZT, Liu et. al. [12] document charge constants of 620 pC/N and Xue et. al. [28] report 530 pC/N, which give hope for future research on the subject.

As stated before, none of the materials used in this work is in any way good for the local- or global environment in itself. But combined and utilized correctly one could achieve attractive solutions to both local- and global environmental issues. The big and obvious global benefit of using fiber composite material is the specific strength and stiffness which allow designers to minimize mass and as a result lower fuel consumption for cars, airplanes and other vehicles. Implementation of piezoelectric actuators and sensors could also be used to decrease drag and increase aerodynamics, which result in further decreased fuel consumption. Smart materials can improve local environment through damping of vibration and noise, for example a floor that is actively or passively damped by piezoelectric transducers could result in a Karaoke bar to be situated on top of a yoga studio, or a wall that actively or passively damp noise is set up along a railroad track to improve the environment for residents in the area. For example where such research is conducted one can look into the ‘Smart Helicopter Rotor’-project conducted by NASA in collaboration with Boeing, DARPA and the U.S. Army, which is a project that uses piezoelectric material to alter the shape of the rotor blade during operations, and resulting in lower fuel consumption and decreased noise for the passengers and the surrounding environment.

1.5. Order of dissertation

1. The second section of this report will present the study of piezoelectric material through theoretical expressions and implementations, method of usage, model implementation in FEM, experimental setup and data and end with a separate results section.

2. The third section of the report will handle piezoelectric shunt damping, where a theoretical analogy to classical tuned mass damping is presented, a subsection where the method is presented showing the impact of design parameters, model implementation in FEM, experimental setup and data acquired and end with a separate results subsection.

3. The fourth section of this report will explain the use of fiber composites in this work and how it is implemented, both from a theoretical viewpoint and how it is implemented in FEM. The results from analytic and numerical calculations will be compared in a separate results subsection.

4. The fifth section of the report will bundle all of the previous studies and explain how a smart structure can be modeled. This section will also describe the method and FEM implementations, and end with a separate results subsection.

5. The sixth section of the report is dedicated to a discussion on the work done.

6. Lastly a section that will conclude the work will be found together with the authors thoughts on future work.
2. PZT Actuator/Sensor

In this section a study of piezoelectric transducers will be conducted. Both its behavior in free space, but also when it is bonded to a structure, this is important ground work when implementing this in more complex structures. The piezoelectric transducers chosen in this work are in the form of discs distributed by APC International, Ltd.[2] model 850 with material data found in Appendix A. The structure chosen to study piezo-structure interaction in this section is an aluminum cantilever beam. Main reasons being that a majority of reference material use this structure, material is easily available and typical material data is widely documented.

2.1. Theory

This report will focus on using PZT-material in the form of mechanical actuators and sensors to excite and increase damping in structures. It will be shown in the following sections that by passively using the inherent capacitance of the PZT, one could control or minimize vibration.

As indicated in the introduction, piezoelectric material have both direct- and reverse effects. Direct piezoelectric effect allows the component to work as a sensor or energy harvester, i.e. it generates an electrical charge when a mechanical force is applied. The reverse piezoelectric effect is the opposite, generation of strain when an electrical field is applied, acting as an actuator. These properties behave differently depending on the polarization of the piezoelectric material, during manufacturing the material is polarized under high temperatures to ensure the piezoelectric effect. The polarization direction for piezoelectric transducers in this work is in the 3-direction, i.e. thickness direction, see Figure 1 for material directions.

To estimate and model piezoelectric behavior one needs to define the constitutive equations. The piezoelectric constitutive relations can be expressed in strain-charge form as follows in piezoelectric theory [6]

\[ S = s^E T + d^E E \]  
\[ D = d T + \varepsilon^T E \]

where the second equation describe the direct piezoelectric effect and the first describe the converse piezoelectric effect. Subscript \((\ )^E\) indicates that the value is measured at constant electrical field and \((\ )^T\) that is measured at constant mechanical stress. Subscript \((\ )^T\) indicate transposed.

<table>
<thead>
<tr>
<th>( S )</th>
<th>( T )</th>
<th>( s )</th>
<th>( d )</th>
<th>( D )</th>
<th>( E )</th>
<th>( \varepsilon )</th>
<th>( c )</th>
<th>( e )</th>
</tr>
</thead>
<tbody>
<tr>
<td>mechanical strain</td>
<td>mechanical stress</td>
<td>elastic compliance</td>
<td>piezoelectric coefficients</td>
<td>electric displacement</td>
<td>electric field</td>
<td>dielectric permittivity</td>
<td>stiffness coefficient</td>
<td>coupling coefficients</td>
</tr>
<tr>
<td>( [\text{N/m}^2] )</td>
<td>( [\text{Pa}^{-1}] )</td>
<td>( [\text{C/N}] )</td>
<td>( [\text{C/m}^2] )</td>
<td>( [\text{V/m}] )</td>
<td>( [\text{F/m}] )</td>
<td>( [\text{N/m}^2] )</td>
<td>( [\text{C/m}^2] )</td>
<td></td>
</tr>
</tbody>
</table>

The constitutive relations can also be expressed in stress-charge form as follows

\[ T = \varepsilon^T S - \varepsilon^T E \]  
\[ D = \varepsilon S + \varepsilon^T E \]

The choice of constitutive relations to use is based on what material data is available or the application used. Due to the relations \( \varepsilon^T = s^{E^{-1}} \), \( \varepsilon = ds^{E^{-1}} \) and \( \varepsilon S = -ds^{E^{-1}} d^T \), either can be applied. Most material data from manufacturers is given in strain-charge form. The piezoelectric materials used in this work are orthotropic, i.e. identical properties in 1 and 2 directions, see Figure 1 for coordinate system used.

![Piezoelectric material directions](image)

This results in symmetrical form to the compliance- and dielectric permittivity matrices. These are structured, in that order, as follows

\[ s^E = \begin{bmatrix} s_{11}^E & s_{12}^E & s_{13}^E & 0 & 0 & 0 \\ s_{21}^E & s_{22}^E & s_{23}^E & 0 & 0 & 0 \\ s_{31}^E & s_{32}^E & s_{33}^E & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{55}^E & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{55}^E & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{66}^E \end{bmatrix} Pa^{-1} \]
where each component in the compliance matrix is the inverse of the materials modulus,

\[
d = \begin{bmatrix}
  0 & 0 & 0 & 0 & d_{15} & 0 \\
  0 & 0 & 0 & d_{15} & 0 & 0 \\
  d_{31} & d_{31} & d_{33} & 0 & 0 & 0
\end{bmatrix} \text{ C/N} \tag{6}
\]

where each component are given by the manufacturer,

\[
\varepsilon^T / \varepsilon_0 = \begin{bmatrix}
  \varepsilon_{11}^T / \varepsilon_0 & 0 & 0 \\
  0 & \varepsilon_{11}^T / \varepsilon_0 & 0 \\
  0 & 0 & \varepsilon_{33}^T / \varepsilon_0
\end{bmatrix} \tag{7}
\]

where \( \varepsilon_0 = 8.854 \text{ pF/m} \) is the permittivity of free space. The most popular piezoceramics used in engineering purposes are PZT-5A and PZT-5H, typical values for those can be found in Table 1 [7].

<table>
<thead>
<tr>
<th>PZT-5A</th>
<th>PZT-5H</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_{11} ) (pPa(^{-1}))</td>
<td>16.4</td>
</tr>
<tr>
<td>( s_{33} ) (pPa(^{-1}))</td>
<td>5.74</td>
</tr>
<tr>
<td>( s_{31} ) (pPa(^{-1}))</td>
<td>7.22</td>
</tr>
<tr>
<td>( s_{15} ) (pPa(^{-1}))</td>
<td>18.8</td>
</tr>
<tr>
<td>( s_{35} ) (pPa(^{-1}))</td>
<td>47.5</td>
</tr>
<tr>
<td>( s_{66} ) (pPa(^{-1}))</td>
<td>44.3</td>
</tr>
<tr>
<td>( d_{31} ) (pC/N)</td>
<td>171</td>
</tr>
<tr>
<td>( d_{33} ) (pC/N)</td>
<td>374</td>
</tr>
<tr>
<td>( d_{15} ) (pC/N)</td>
<td>584</td>
</tr>
<tr>
<td>( \varepsilon_{11}^T / \varepsilon_0 )</td>
<td>1730</td>
</tr>
<tr>
<td>( \varepsilon_{33}^T / \varepsilon_0 )</td>
<td>1700</td>
</tr>
<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>7750</td>
</tr>
</tbody>
</table>

Table 1: Typical three-dimensional piezoelectric properties for PZT-5A and PZT-5H.

Typical values of dielectric loss in PZT are about 1 – 3% [23]. The piezoceramics used for experimental testing and simulations in this work is APC 850 which specifications can be found in Appendix A which are similar to PZT-5A. These similarities is analyzed in the next subsection to determine which material model to use.

2.2. Method

The method can, somewhat simplified, be summeri-est into three steps:

1. Develop a simulation model in FE-software Comsol multiphysics to estimate structural response. The model will be covered in a separate subsection.
2. Conduct experiments and extract results and data. This will also be covered in separate subsections.
3. Calibrate model using the data acquired analytically or from experiments.

These steps are used throughout this work with small variations. To begin, the PZT disc and its electromechanical coupling will be studied. Later a study on placement will be carried out, ending with calibration method of the model when experimental data is acquired from structural interaction.

A simple model using the software Comsol Multiphysics [24] is developed to examine the direct- and converse piezoelectric effect under static load and applied voltage. The direct effect, voltage as result of applied force, can be estimated analytically as

\[
V = \frac{g_{33}F}{2\pi r} \tag{8}
\]

where \( g_{33} \) is the piezoelectric constant in m\(^2\)/C which is a constant given by manufacturers to calculate said voltage\(^1\), \( F \) is applied force in Newton and \( r \) is the radius of the piezoelectric disc. The converse effect, strain as result of applied voltage, can be estimated analytically as

\[
\Delta d = \frac{2d_{31}V}{h} \tag{9}
\]

where \( d_{31} \) is the piezoelectric charge constant in C/N, \( V \) is voltage applied and \( h \) is the thickness of the piezoelectric disc. These values are calculated using APC 850 material data then compared to the model developed. The results between using data given by manufacturers and the PZT-A5 material given by Comsol can be seen in Figure 2 and 3 where 10 N is applied to study the direct effect, and 10 V is applied to demonstrate the converse effect.

\(^1\) \( g_{33} \) is derived from the relation \( E = -g_{33} T \), where \( E \) is electric field and \( T \) is mechanical stress. This constant is not an input in Comsol multiphysics because it uses the constitutive relation explained in subsection 2.1.
The differences for the two materials, given in values are found in Table 2. One should also note that in Figure 2 the polling direction is visible due to the change in electric potential, i.e. 3-direction.

<table>
<thead>
<tr>
<th>Material</th>
<th>( V(F) )</th>
<th>( \Delta d(V) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT-A5</td>
<td>-0.2452 V</td>
<td>0.0869 ( \mu m )</td>
</tr>
<tr>
<td>APC 850</td>
<td>-0.2446 V</td>
<td>0.0898 ( \mu m )</td>
</tr>
<tr>
<td>Analytic</td>
<td>-0.2447 V</td>
<td>0.0889 ( \mu m )</td>
</tr>
</tbody>
</table>

Table 2: Values attained from studying two material models in direct- and converse piezoelectric effect.

As can be seen in Table 2, the difference is small, but the values acquired using APC 850 material data for the model are closer to what were analytically acquired, therefor this material data is used throughout this work. Also the capacitance of the piezo was measured at 16.353 nF.

Next the behavior of a piezoelectric transducer bonded to a structure will be studied. To make sure a placement of the piezo is decided so that it maximize the frequency response on the structure a study of placement is carried out. The optimal placement of the piezoelectric actuator is considered to be where it maximizes the frequency response function, acceleration over voltage, of the structure for a specific point of measuring acceleration. This study for cantilever beams is carried out in a 2D-environment. The PZT actuator is continuously exciting the first, second and third resonance frequency while the position of the piezoelectric actuator is moved along the length of the beam, see Figure 4.
From Figure 4 one can see that to excite all three resonance frequencies adequately, placement is important. This will for example cause problem when trying to damp a certain mode using shunt damping, in this work the installation of the piezoelectric actuator for experimental purposes only took the first mode in regard, which caused problems damping the second mode which will be evident in subsection 3.2.

The experimental results, that will be evident in the second to last subsection of this section, was used to calibrate the model. The boundary conditions and damping coefficients was modified to achieve an adequate coherence. The fixed end-boundary condition resulted in resonance frequencies positioned at frequencies higher than what was acquired from the experiment. To match the eigen-frequency values that was received in the experiment the boundary condition was changed from a fixed end to being clamped between two elastic blocks that could vary in stiffness to match the experimental results. This transaction is illustrated in Figure 5.

A second boundary condition to consider is the placement of the piezo and the anti-resonance that is a result of this placement. To achieve a coherence in results the placement of the piezo was moved from its original position closer to the clamped end by 4 mm in the model.

For the model to behave as the experimental setup, correct damping coefficients need to be applied. Due to the dynamic nature, and frequency dependent damping, Rayleigh Damping is used which is expressed as

$$[C] = \alpha[M] + \beta[K]$$  \hspace{1cm} (10)
(10) can be simplified as

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2}$$  \hspace{1cm} (11)$$

or on matrix form for the two first natural frequencies of the system

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \frac{1}{\omega_1^2} & \omega_1 \\ \omega_2 & \frac{1}{\omega_2^2} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$  \hspace{1cm} (12)$$

where $\zeta$ is the damping ratio given in fractions of critical damping and $\omega = 2\pi f$, where $f$ is frequency.$^2$ If no experimental data is available, damping ratio for aluminium can be approximated as equal to 0.02% [21].

### 2.3. Model

**Converse piezoelectric effect** The setup to study the converse piezoelectric effect when it is bonded to a structure is an aluminum cantilever beam with a PZT-transducer bonded to the surface to excite the structure and an accelerometer to record the acceleration response, which is later used to calculate the frequency response, acceleration over voltage. The PZT’s are polarized in the 3-direction, i.e. the electric potential is developed in the thickness direction. The schematic can be found in Figure 6. Material data for the model can be found in Table 3.

![Figure 6: Setup for study of piezo actuator.](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Alu 6063</th>
<th>APC 850</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free length</td>
<td>270 mm</td>
<td>–</td>
</tr>
<tr>
<td>Width</td>
<td>30 mm</td>
<td>–</td>
</tr>
<tr>
<td>Thickness</td>
<td>3 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Diameter</td>
<td>–</td>
<td>25.4 mm</td>
</tr>
<tr>
<td>Density</td>
<td>2700</td>
<td>7600</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>69 GPa</td>
<td>63 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.33</td>
<td>–</td>
</tr>
</tbody>
</table>

Table 3: Model parameters for simulations. see Appendix A for full specifications of APC 850.

The interface used in the simulation are ‘Piezoelectric devices’ which combines and couples solid mechanics and electrostatics which allows to simulate piezoelectric effects using the same constitutive relations described in subsection 2.1. Studies selected are ‘Eigenfrequency’, which solves for the natural frequencies and the corresponding mode shapes, and ‘Frequency domain’, which solves the frequency response over a range of frequencies with selected resolution. Materials selected for the model are Aluminum 6063-T83, which is a common form of aluminum that is used in everything from buildings to electrical components. The material data for the PZT disc are added in a ‘blank material’, where input data can be found in the material sheet, see Appendix A, using theory from subsection 2.1. Fixed boundary conditions are added to blocks clamping one end of the beam to attain the clamped condition seen in Figure 6. The top boundary of the piezo is connected to a ground node and a terminal boundary, with a given voltage, is applied to the bottom boundary.

**Direct piezoelectric effect** The setup when studying the direct piezoelectric effect can be found in Figure 7. It uses the same aluminum beam as for the previous model, but now with an additional piezoelectric transducer acting as sensor with the same material, APC 850.

![Figure 7: Setup for study of piezo sensor.](image)

A voltage is applied to the terminal boundary of the actuating piezo and a ‘floating potential’-boundary to the piezo acting as sensor, this means that the electric potential will be uniform along this boundary but can take any value and be measured in voltage. Ground nodes are connected to the opposite side of the terminal- and floating potential boundaries. A study in frequency domain is done to estimate the frequency response, voltage received

$^2$Two specific damping ratios are maximum to implement in Comsol Multiphysics.
from the sensor over voltage applied to the actuator.

2.4. Experiment

To validate and calibrate the Comsol Multiphysics FE-model, extensive experiments was carried out using aluminum structure with bonded PZT transducers, which was later compared to the results given by the simulation. An aluminum beam was attached to a steel structure, sandwiched in one end by two aluminum plates and clamped by a sash clamp. Figure 8 show a photo of the structure with two piezo’s and accelerometer attached. The adhesive used to bond the PZT-disc to the aluminum beam is HBM X60 which is a 2-component fast curing adhesive consisting of liquid- and powder components. The accelerometer is secured by applying Petro wax on the boundary.

![Figure 8: Structure used in experiments. Two PZT discs and an accelerometer bonded to an aluminum cantilever.](image)

The experiments were carried out in the lab of Creo Dynamics in Linköping, Sweden. The aim with the experiment was to use the results and compare with the Comsol Multiphysics model and later calibrate it. First, a frequency response is calculated using a PZT actuator and accelerometer to ensure that the structural response correspond to an appropriate level with the model. To acquire the FRF of a structure excited by a piezoelectric actuator the setup found in Figure 9 was used where the output signal to the actuating PZT is generated by LMS hardware using LMS software installed on a Windows 10 laptop. The signal is white noise with a chosen bandwidth, which should allow sufficient frequency-resolution, that is passed through an amplifier and is simultaneously acquired by the LMS hardware as a reference input signal. The dotted sections in Figure 9 are added to study the direct piezoelectric effect where PZT₂ acts as a sensor.

![Figure 9: The complete setup to calculate FRF, acceleration over voltage. The dotted section is added to study the direct piezoelectric effect.](image)

2.5. Results

The clamped condition was as mentioned assumed as a sandwich between two layers of varying stiffness. The stiffness of the blocks, resulting in matching eigenfrequencies for the structure, was 0.3 GPa, which can be compared to solid, low-density polyethylene. The placement of the piezoelectric actuator was moved 4 mm from 85 mm to 81 mm, this was to ensure that the anti-resonance was located more correctly and as a result, attenuate the second resonance peak matching experimental results.

The converse piezoelectric effect was compared between FE-model and experimental data. With damping factors extracted from experimental results as $\xi_1 = 0.42\%$ and $\xi_2 = 0.16\%$ the frequency response as acceleration over volt for the model and experiment is compared in Figure 10.

The direct piezoelectric effect was handled similarly. When the converse piezoelectric effect modeled was verified, the direct effect could be com-
pared with a frequency response showing voltage from the passive PZT (sensor) over voltage sent to the actuating PZT (actuator). The result can be found in Figure 11 where the FE-model is plotted together with experimental results. No separate calibration was needed to increase correlation when studying the direct piezoelectric effect.

The results found in Figure 10 and 11, together with analytic comparison done on neat PZT under subsection 2.2, show that modeling direct- and converse piezoelectric effect correlate well with experimental results.

**Figure 10:** Frequency response function, acceleration over voltage, demonstrating the converse piezoelectric effect from Comsol Multiphysics and experimental.

**Figure 11:** Frequency response function, voltage in over voltage out, comparing the direct piezoelectric effect between FE-simulations and experiment.
3. SHUNT DAMPING

As stated in subsection 1.3, the piezoelectric shunt damping technique is characterized by connecting an electrical impedance to a piezoelectric transducer that is bonded to- or embedded in a structure and utilizes its mechanical to electrical coupling to dissipate mechanical energy. In Figure 12 a summary of different shunt techniques are illustrated. The circuits studied in this work are single-mode shunts which dampens a narrow bandwidth around a resonance frequency when tuned correctly. Several approaches to describe the tuning of a shunt circuit have been presented [25][27][11] since Hagood and von Flotow first derived the phenomenon [9], with small variations.

3.1. Theory

To introduce and explain the shunt damping technique an analogy written by Matthew V. Kozlowski [11] will be presented. The study shows that a piezoelectric transducer and its damping performance, using the series shunt circuit presented by Hagood and von Flotow, can be explained through an analogy with the skyhook damper absorber presented by M. Z. Ren [18], see Figure 13. This section will also shine some light on the difference in performance between the two systems.3

3The analogy use some non-conventional nomenclature so for further explanation on the specific analogy the written article is highly recommended.

Figure 12: Passive piezoelectric shunt damping techniques (S.O. Reza. Moheimani and Andrew J Fleming [13]).

Figure 13: The two systems referred to in the analogy. Left: Vibration absorber. Right: Resonant shunt circuit, where R is a resistor, L an inductor and C_p a capacitor.

To start, the equation for a piezstructure as sensor is, in this study, presented by Hagood and von
where the non-dimensional gain $\zeta$ is the electromechanical coupling matrix which is later defined by Hagood in equation (21), $r$ is a mechanical coordinate, $C_p$ is capacitance of the piezoelectric transducer. In equation (13) the charge can be divided into mechanical and electrical charge as $q = q_{\text{mech}} + q_{\text{elec}}$. The transfer function between the mechanical contribution to the charge, $q_{\text{mech}}$, and voltage can now be expressed as

\[ \frac{q_{\text{mech}}}{V} = C_p K^2 \frac{\omega_n^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \]  

where $\omega_n$ is the natural frequency, $\zeta$ is the damping and $K$ is the non-dimensional electromechanical coupling factor, which Kozlowski defines as

\[ K^2 = \frac{\theta^2}{C_p k} = \frac{\theta^2 L}{M} \frac{\omega_n^2}{\omega_n^2} = \beta \gamma_c^2 \]  

This factor is shown to be the constant that defines the performance that can be expected from a piezo, i.e. it is the ratio of mechanical energy in the structure that can be turned into electrical energy and stored in the capacitor, this will later be shown to be the relation between natural frequencies of the structure. By use of Kirchhoff’s voltage law, including the resistor and inductor, the transfer function for mechanical charge between output of voltage and input of mechanical charge is found as

\[ \frac{V}{q_{\text{mech}}} = -\frac{1}{C_p \xi_c} \frac{s^2 + 2\zeta_\omega_s s + \omega_s^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \]  

where $\xi_c = \frac{R}{s \omega_c}$ is the damping and $\omega_c = \sqrt{\frac{1}{LC}}$ is the electrical resonance frequency. To show the similarities to the skyhook damper absorber the same theory is adopted to this system: the transfer function for the main mass between position and force can be found as

\[ \frac{x}{F} = \frac{1}{k} \frac{\omega_n^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \]  

And the transfer function for the absorber system between output of force and input of position is

\[ \frac{F}{x} = -k_s \frac{s^2 + 2\zeta_\omega s}{s^2 + 2\zeta_\omega s + \omega_s^2} \]  

where the non-dimensional gain $k_s/k = m_s/M \times \omega_s^2/\omega_n^2$ will be shown to be important when showing that the mass ratio $\beta = m_s/m$ determines the upper limit of performance that the absorber can achieve. Now similarities between the two damping systems can be identified by analyzing equation (14)/(16) and (17)/(18). Further the similarities between the mass ratio, $\beta$, and the coupling coefficient $K^2$ that determine the upper level of performance for the two systems can be seen by examining the loop-gain. The loop-gain for the vibration absorber system can be expressed from equation (17) and (18) as

\[ l_{\text{sky}} = -\beta \frac{\omega_n^2}{\omega_n^2} \left( \frac{\omega_n^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \right) \left( \frac{s^2 + 2\zeta_\omega s + \omega_s^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \right) \]  

and the same for the piezo-shunt system using equation (14) and (16) as

\[ l_{\text{shunt}} = -K^2 \left( \frac{\omega_n^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \right) \left( \frac{s^2 + 2\zeta_\omega s + \omega_s^2}{s^2 + 2\zeta_\omega_n s + \omega_n^2} \right) \]  

From these equations the performance factors can be extracted, $\beta \omega_n^2$ and $K^2$. It should now be noted that the upper limit of the absorber is determined by the applied mass, or size, of the absorber, i.e. more mass lends more attenuation. This is a tradeoff that needs to be considered on a case to case basis. The upper limit of the piezo-shunt system is determined by the inherent capacitance of the piezo, i.e. lower capacitance allows more energy to be dissipated. A lower capacitance piezo can be achieved by increasing its thickness or lowering the diameter, which both effect the strain capabilities in a negative way. If again looking at the expression for $K^2$ in equation (15), there is only one straightforward way to increase the coupling: maximize the mechanical coupling matrix, $\theta$, which is defined by Hagood et al. [15] (p.333) as

\[ \theta = \int_{\mathcal{V}} N_t^T R_{\mathcal{V}}^T \mathcal{C} R_{\mathcal{V}} N_t \]  

where $N_t = L_t \Psi_t$ where $L_t$ is the differential operator acting on the mode shape $\Psi_t$. Here Kozlowski points out that the operator is the second derivative, which express the curvature of the piezo. The curvature is closely related to the strain, which gives that a way to increase the performance factor of piezoelectric shunt system is to position the piezo where strain in the main structure is maximized.

When position of the shunted piezo is determined, tuning of circuit-parameters is needed. One established approaches is used in this work, single mode series shunt damping.
Hagood and von Flotow [9] derived an expression for tuning the series shunt circuit as follows

\[ R_{opt}^S = \frac{\sqrt{2}K_{31}}{C_p^2 \omega_n^D (1 + K_{31}^2)} \] (22)

and

\[ L_{opt}^S = \frac{1}{C_p^S (\omega_n^E)^2} \] (23)

where \( K_{31} \) is the electromechanical coupling factor which is found as

\[ K_{31} = \sqrt{\frac{(\omega_n^D)^2 - (\omega_n^E)^2}{(\omega_n^E)^2}} \] (24)

where \( \omega_n^D \) is the natural frequency of the structure corresponding to the resonance frequency that are to be damped when the circuit is open, \( \omega_n^E \) is the natural frequency of the structure corresponding to the same resonance frequency when the circuit is shortened, and \( C_p^S \) is the capacitance of the PZT measured at constant strain which is calculated

\[ C_p^S = C_p^T (1 - k_{31}^2) \] (25)

where \( C_p^T \) is the capacitance of the PZT measured at constant mechanical stress and \( k_{31} \) is a electromechanical coupling factor provided by the manufacturer.

In the next subsection the series shunt method will be demonstrated in terms of tuning design parameters and implementation.

3.2. Method

In this subsection a single mode series R-L shunt circuit will be considered connected to a PZT disc to dissipate energy and damp a structure, see Figure 14. The significance of correct tuning of the shunt design parameters will be presented through plots showing the influence on the frequency response. The method are applied to both simulation and experiments.

The tuning parameters in equation (22) and (23) work as initial guesses, they give a good first approximation which results in an iterative evaluation around these values. The FRF in Figure 15 are acquired from a 2D-model include material damping acquired experimentally from previous section 2.5.

**Resistance** Left column of Figure 15 show the frequency response of the first to third resonance when different resistor-values are connected to the individual circuits, showing that an optimal value can be obtained for the resonance. When the resistance is too large, the damping is not sufficient, and when too low the resonance peak is split in two new resonance peaks. To optimize the damping a specific resistor is to be implemented in the circuit.

**Inductance** It is shown in the right column of Figure 15 that the inductance, \( L \), can be tuned for a specific resonance frequency where the damping is to be applied for a narrow bandwidth.
(a) Frequency response showing the influence of changing the Resistance, R around its optimum, for calibrated $L_1^{\ast}$ to the first resonance.

(b) Frequency response showing the influence of changing the inductance, in the series circuit around the first resonance.

(c) Frequency response showing the influence of changing the Resistance, R around its optimum, for calibrated $L_2^{\ast}$ to the second resonance.

(d) Frequency response showing the influence of changing the inductance, in the series circuit around the second resonance.

(e) Frequency response showing the influence of changing the Resistance, R around its optimum, for calibrated $L_3^{\ast}$ to the third resonance.

(f) Frequency response showing the influence of changing the inductance, in the series circuit around the third resonance.

Figure 15: Demonstration of the influence on damping when changing the resistance and inductance in a single mode series circuit. Material damping factor $\zeta_1 = 0.42\%$ and $\zeta_2 = 0.16\%$. 
The difficulties when damping the second resonance is explained by the placement of the PZT’s, remembering the results from studying the direct effect in the previous section, showing very low sensing-capabilities for the second resonance, which is also the reason for low damping-capabilities for the same resonance.

3.3. Model

The setup for the series shunt circuit model can be found in Figure 16 which is the same physical setup that was used in previous section to estimate the direct piezoelectric effect. Two models are made. One in a 2D environment to illustrate the behavior of changing design parameters, this model is used to produce the curves in Figure 15. Another is done in a 3D environment to estimate the structural response and compare to experimental results which will be shown in the next subsection.

![Figure 16: Experimental setup of shunt damping of aluminum cantilever beam.](image)

To model a shunt circuit in Comsol Multiphysics one need to, in addition to the physics used in the actuator/sensor-model, add the ‘Electrical circuit’ module. This module allows implementation of electric circuits that can interact with the solid model. In this model that is in the form of a shunt circuit including a resistor and a inductor. To implement an electric circuit that can dissipate the mechanical energy from the structure, one needs to add a terminal boundary to the shunted piezo, then select the terminal to act as a circuit connection. Second one adds the components in the circuit and connect them via a nodes-system. The circuit contains a ground (node 0), inductor (node 0 – 1), resistor (node 1 – 2) and a ’connection’ node (node 2) that is selected to connect to the terminal boundary of the structure.

![Figure 17: Riordan gyrator circuit [19]](image)

3.4. Experiment

In Figure 15 the corresponding values of inductance is listed for the three resonances demonstrated when a series circuit is applied. It can be concluded that the shunt circuit require unrealistic inductance for this application. Common inductors have values that range from 1 μH to 1 H which is much lower than the desired value. For applicational use, this higher value of inductance can be achieved by replacing the passive inductor with active components acting as an inductor with the required inductance. For this work, the Riordan gyrator was implemented [19] when conducting experiments. The gyrator is a circuit containing a capacitor, resistors and operational amplifiers to act as impedance, this circuit is represented in Figure 17. The Riordan gyrator can be shown to have the impedance of

\[ Z = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} \]  

(26)

where either \( Z_2 \) or \( Z_4 \) is a capacitor, \( C \), and the others are resistors, \( R \). Now the impedance can be expressed as

\[ Z = j\omega CR^2 \]  

(27)

so that the circuit behaves as an inductance

\[ L = CR^2 \]  

(28)

To be able to tune the impedance, one of the resistors is chosen as variable, \( Z_3 = R_v \).

<table>
<thead>
<tr>
<th>( Z )</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Z_1 )</td>
<td>10 kΩ</td>
</tr>
<tr>
<td>( Z_2 )</td>
<td>100.6 nF</td>
</tr>
<tr>
<td>( Z_3 )</td>
<td>1 – 10 kΩ</td>
</tr>
<tr>
<td>( Z_4 )</td>
<td>10 kΩ</td>
</tr>
<tr>
<td>( Z_5 )</td>
<td>10 kΩ</td>
</tr>
</tbody>
</table>

Table 4: Impedance values for the Riordan gyrator circuit.

With the design values given in Table 4, the inductance, \( L \), can be varied to match the inductance given by \( L \omega \) using equation (23) for frequencies 400 \( < f < 1300 \) Hz which is illustrated in Figure 18. By increasing the variable resistance further in the Riordan gyrator circuit, damping of resonances located at lower frequencies is possible.
One thing should be mentioned about the Riordan gyrator in relation to the shunt configuration: the implemented Riordan gyrator will result in parasite resistance in the circuit. Because of low resistance values needed to damp low frequency resonances, the influence of this parasite resistance affect experimental results. The experimental setup to effectively damp and capture the structural response is illustrated in Figure 19.

The capture equipment is the same that was used in the previous section. The output signal to the actuating PZT is generated by LMS hardware using LMS software installed on a Windows 10 laptop. The signal is white noise with a chosen bandwidth, which should allow sufficient frequency-resolution, that is passed through an amplifier and is simultaneously acquired by the LMS hardware as a reference input signal. Other input signals are acquired from an accelerometer to measure acceleration and the voltage over the resistor in the series shunt circuit. The FRF (acceleration/volt) is then calculated through LMS software using averaging of multiple samples.

### 3.5. Results

Experimental results when tuning the design parameters in the shunt circuit can be found in Figure 20 to 22 for the first three resonance frequencies. Material data can be found in Table 5.

![Figure 18: Impedance as function of frequency for a range of resistance $R_v$ for the gyrator, matched with the passive inductance.](image)

Table 5: Model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Beam</th>
<th>APC 850</th>
</tr>
</thead>
<tbody>
<tr>
<td>Free length [mm]</td>
<td>270</td>
<td>25.4</td>
</tr>
<tr>
<td>Thickness [mm]</td>
<td>3</td>
<td>0.5</td>
</tr>
<tr>
<td>Density [kg/m$^3$]</td>
<td>2700</td>
<td>7600</td>
</tr>
<tr>
<td>Young’s modulus [GPa]</td>
<td>69</td>
<td>63</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.33</td>
<td>–</td>
</tr>
<tr>
<td>Loss factor $\eta$</td>
<td>–</td>
<td>$1/Q_m$</td>
</tr>
<tr>
<td>Damping factor $\zeta_1$</td>
<td>0.42%</td>
<td>–</td>
</tr>
<tr>
<td>Damping factor $\zeta_2$</td>
<td>0.16%</td>
<td>–</td>
</tr>
<tr>
<td>Dielectric loss</td>
<td>–</td>
<td>1%</td>
</tr>
<tr>
<td>$k_{31}$</td>
<td>0.36</td>
<td></td>
</tr>
</tbody>
</table>

$Q_m = 80$, found in Table 5, is the mechanical quality factor of the PZT. These plots show the same behavior as simulations in regard to changing parameters when the shunt circuit is of series configuration, the damping is increased for lower values of resistance, and the damping is applied for a frequency band depending on the inductance.

![Figure 19: The complete setup using shunt circuit.](image)

Table 6: Parameters that correspond to the attenuation highlighted in Figures 20 to 22.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>#1</th>
<th>#2</th>
<th>#3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{\text{experiment}}$</td>
<td>20 kΩ</td>
<td>275 Ω</td>
<td>1 kΩ</td>
</tr>
<tr>
<td>$L_{\text{experiment}}$</td>
<td>2.1 kH</td>
<td>60 H</td>
<td>7.6 H</td>
</tr>
<tr>
<td>Attenuation</td>
<td>12.18 dB</td>
<td>14.88 dB</td>
<td>11.65 dB</td>
</tr>
<tr>
<td>$R_{\text{model}}$</td>
<td>20 kΩ</td>
<td>300 Ω</td>
<td>1 kΩ</td>
</tr>
<tr>
<td>$L_{\text{model}}$</td>
<td>2.2 kH</td>
<td>60 H</td>
<td>7.5 H</td>
</tr>
<tr>
<td>Attenuation</td>
<td>12.44 dB</td>
<td>22.15 dB</td>
<td>14.37 dB</td>
</tr>
</tbody>
</table>
Figure 20: Frequency response showing increased damping of the first resonance.

Figure 21: Frequency response showing increased damping of the second resonance.

Figure 22: Frequency response showing increased damping of the third resonance.
4. Fiber composites

Carbon fiber composite is the structural material that will embed the piezoelectric transducers in the smart structure that are to be simulated. In this section classical lamina theory will be presented, a model will be presented and results will be compared between the two.

4.1. Theory

Carbon fiber is known to be used in high performance applications such as the automotive- and aerospace industry. Due to the material high specific strength and stiffness and ability to be molded into complex shapes it has reached its high popularity in areas where low mass is desirable or even crucial. Nowadays fiber composite material is increasingly used in consumer products. Until recently structural fiber composite materials have been the luxury of cost insensitive markets such as Formula 1- or expensive sports cars, expensive yachts or sailing boats and aerospace applications. Today, carbon reinforced plastic is built into cars that are in series production for a broad consumer market, for example the BMW i3 which has most of its interior structure made from CFRP [26], bicycles that common consumers can afford and airplanes designed for commercial flights e.g. Boeing 787 Dreamliner which is 50% composite by weight [5]. All applications mentioned above, and especially in the vehicle market, components prone to vibrations are considered a problem. Carbon fiber composite materials are often considered because of its superior strength to weight ratio, which often results in the desire to manufacture and use thin, prone to vibration, parts. To enable the use of thin components and still achieve adequate damping, external components could be the solution.

To verify the FE-model, results will be compared to classical lamina theory. To estimate the behavior of a specific composite material one need to consider its anisotropic or orthotropic nature. For orthotropic materials, which is considered in this work, one needs the value of the elastic constants that builds the local compliance matrix $S_l$, given for each layer as shown in equation (29).

$$S = \begin{bmatrix} \frac{1}{E_{11}} & -\frac{1}{E_{12}} & \frac{1}{E_{13}} & 0 & 0 & 0 \\ -\frac{1}{E_{12}} & \frac{1}{E_{22}} & -\frac{1}{E_{23}} & 0 & 0 & 0 \\ -\frac{1}{E_{13}} & -\frac{1}{E_{23}} & \frac{1}{E_{33}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{13}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{23}} \end{bmatrix}$$

(29)

where $E_{ij}$ is the Young’s modulus in the fiber direction and $E_{23}$ is the transverse equivalent, $\nu$ is the Poisson’s ratio and $G$ is the shear modulus. The inverse of the compliance matrix is the local lamina stiffness matrix, $Q_l$. To take each layers fiber direction, $\theta$, into consideration a transformation matrix on the form

$$T = \begin{bmatrix} c^2 & s^2 & 0 & 0 & 0 & 2cs \\ s^2 & c^2 & 0 & 0 & 0 & -2cs \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & c & s & 0 & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ -cs & cs & 0 & 0 & 0 & c^2 - s^2 \end{bmatrix}$$

(30)

is calculated, where $c = \cos(\theta)$ and $s = \sin(\theta)$ where $\theta$ is the fiber direction. For each layer, the global stiffness matrix $Q$ can now be calculated

$$Q = TQ_lT^t$$

(31)

which is used to calculate structural response exposed to external forces of a composite lamina. To be able to take the whole laminate into account, the ‘$ABD$’ matrix needs to be calculated

$$[A, B, D] = \sum_{i=1}^{n} Q_l \left[ (z_i - z_{i-1}), \frac{1}{2} (z_i^2 - z_{i-1}^2), \frac{1}{3} (z_i^3 - z_{i-1}^3) \right]$$

(32)

which describe the relation between applied forces and moments to the strains and curvatures of the whole laminate, in matrix form this is expressed as

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} t_0 \\ k \end{bmatrix}$$

(33)

where $A$ is called the extensional stiffness matrix, $B$ the extension-bending coupling matrix and $D$ the bending stiffness matrix.

4.2. Method

To validate the simulation-model, the classical laminate theory found in Zenkert [29], is used to calculate results that are later compared to the results acquired from the model. Two comparisons are made,
maximum displacement of a composite plate and the natural frequencies. The general plate equations for anisotropic plates is expressed

\[
D_{11} \frac{\partial^4 w}{\partial x^4} + 4D_{16} \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2(D_{12} + 2D_{66}) \frac{\partial^4 w}{\partial y^4} + 4D_{26} \frac{\partial^4 w}{\partial x \partial y^3} + D_{22} \frac{\partial^4 w}{\partial y^4} = q^* \tag{34}
\]

where \(q^*\) are loads applied to the plate. Derived from (34), the displacement of an orthotropic plate subjected to a uniformly distributed load \(q\), while simply supported is calculated

\[
w = \frac{16qb^4}{\pi^6} \sum_{n=1,3,5} \sum_{m=1,3,5} \sin \frac{mn \pi x}{a} \sin \frac{mn \pi y}{b} \left[ D_{11}\left(\frac{mn}{\pi}\right)^4 + 2(D_{12} + 2D_{66})\left(\frac{mn}{\pi}\right)^2 + D_{22}\left(\frac{mn}{\pi}\right)^4 \right] \tag{35}
\]

where \(a\) and \(b\) is the length and width of the plate, \(q\) is the uniform pressure in Pa and \(m, n\) are summation indexes. Max displacement is measured at the midpoint, i.e. \(x = a/2, y = b/2\). The natural angular frequency is for an orthotropic- and simply supported plate, derived from (34), is calculated

\[
\omega_{mn}^2 = \frac{D_{11}\left(\frac{mn}{\pi}\right)^4}{\rho^*} + \frac{2(D_{12} + 2D_{66})\left(\frac{mn}{\pi}\right)^2 + D_{22}\left(\frac{mn}{\pi}\right)^4}{\rho^*} \tag{36}
\]

where \(a\) and \(b\) is the length and width of the plate and \(\rho^*\) is the mass per unit area of the plate, kg/m². Note that \(m\) and \(n\) in equation (4.2) refer to different modes of vibration and are not summation indexes.

It should be noted that classic laminate theory is based on a set of assumptions, namely

- It is only valid for thin laminates, i.e. span is much larger than thickness.
- It is only valid for small displacements in transverse direction (z-direction).
- Each lamina is considered to be a homogeneous layer such that its effective properties are known.
- The laminate consists of perfectly bonded layers. There is no slip between the adjacent layers. In other words, it is equivalent to saying that the displacement components are continuous through the thickness.

• The laminate deforms according to the Kirchhoff - Love assumptions for bending and stretching of thin plates (as assumed in classical plate theory).

4.3. Model

The FE-model is built up as an square 100x100 mm, 8-layer CFRP-composite with layup \([0/90]_{2s}\) using solid elements, see Figure 23. All layers are assumed to be in perfect contact. Solid elements are used instead of shells because strains through the thickness is of interest when later embedding PZT-discs.

Material data for the model can be found in Table 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>CFRP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ply thickness [mm]</td>
<td>0.25</td>
</tr>
<tr>
<td>(\rho) [kg/m³]</td>
<td>1600</td>
</tr>
<tr>
<td>(E_1) [GPa]</td>
<td>181</td>
</tr>
<tr>
<td>(E_2) [GPa]</td>
<td>10.3</td>
</tr>
<tr>
<td>(G_{12}) [GPa]</td>
<td>7.17</td>
</tr>
<tr>
<td>(\nu_{12})</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Table 7: Material data for unidirectional CFRP used in model.

Material implementation in Comsol Multiphysics for an orthotropic material studied in 3D is in the set of nine parameters

\[
E = [E_1, E_2, E_3] \\
\nu = [\nu_{12}, \nu_{23}, \nu_{13}] \tag{37} \\
G = [G_{12}, G_{23}, G_{13}]
\]
where the remaining components can be estimated as

\[
\begin{align*}
E_3 &= E_2 \\
\nu_{23} &= \frac{\nu_{12}}{1 - \nu_{12}^2} \\
\nu_{13} &= \nu_{12} \\
G_{23} &= \frac{E_2}{2(1 + \nu_{23})} \\
G_{13} &= G_{12}
\end{align*}
\]  
(38)

The fiber direction of the layers are applied by a rotated coordinate system applied to the 90°-layers. The model is applied with simply supported boundary conditions. This is done by applying a zero prescribed displacement in thickness- and perpendicular length-directions (x or y, z) of the lower edge of the plate edges. A boundary load of 100 kPa is applied on the top boundary of the plate. A stationary study results in the displacement of the plate as a result of the distributed load and an eigenfrequency study results in the natural frequencies and the structural mode shapes.

4.4. Results

Here will be presented the results acquired using the analytic expressions explained under subsection 4.2 and the numerical FE-model explained under subsection 4.3. Figure 24 show the displacement of the composite plate when subjected to a static distributed load of 100 kPa. Figure 25 show the mode shape of the fundamental frequency at 1070 Hz. Table 8 a comparison between analytic results and what was acquired from the FE-model is listed.

<table>
<thead>
<tr>
<th>w_{max}</th>
<th>\omega</th>
</tr>
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<tbody>
<tr>
<td>Analytic</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>FE-model</td>
<td>1.1 mm</td>
</tr>
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</table>

Table 8: Comparison of analytic and FE-model, showing results of max deflection and natural frequencies of a composite plate with layup [0/90]^2s.

Figure 24: Plot showing displacement as result of distributed load (100 kPa), the plate is simply supported and max-displacement is located at x = y = 50 mm.

Figure 25: Plot showing the first natural mode shape at 1070 Hz, which correspond to the analytic solution using m = n = 1.
5. Smart Structures

Now that a method to simulate both piezoelectric and fiber composite behavior individually has been developed, it is time to build a FE-model that can help when designing and estimate structural behavior of smart structures. The model will feature a clamped CFRP plate with embedded PZT discs to excite and damp the structure using shunted circuits. Before the design methodology is presented it should be noted that the structural integrity is changed when embedding PZT transducers, therefore this section will begin by illustrating some of those aspects.

5.1. Structural influence of embedding PZT

The composite plate used to illustrate the influence of embedding PZT is a clamped CFRP plate, dimensions 100x100x2 mm excited at its fundamental frequency expressing the first mode shape. The study will be presented by slice plots along y = 50 mm (across the anti-node) to better visualize the structural response through the thickness, the embedded PZT’s are also placed along this line. To study the influence of embedding PZT discs in the fiber composite a plate with, and without, embedded disc are studied and compared when subjected to a sinusoidal load. The PZT is embedded at \([x, y, z] = [20, 50, 1.25]\). The results can be found in visualized plot in Figure 41, in Appendix B, where maximum and minimum values are pointed out, strains are given in principal strains. Numerical values are gathered in Table 9 and 10 for the composite plate and smart plate respectively.

<table>
<thead>
<tr>
<th>Composite plate [0/90]_2s</th>
<th>Value [x,z] mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>2080.8 Hz</td>
</tr>
<tr>
<td>(w_{\text{max}})</td>
<td>0.26 µm [50,1]</td>
</tr>
<tr>
<td>(\sigma_{\text{max}})</td>
<td>82.9 kPa [100,0]</td>
</tr>
<tr>
<td>(\varepsilon_{\text{min}})</td>
<td>20.4 Pa [50,1]</td>
</tr>
<tr>
<td>(\varepsilon_{\text{max}})</td>
<td>4.6e(^{-7}) [50,2]</td>
</tr>
<tr>
<td>(\varepsilon_{\text{min}})</td>
<td>-4.6e(^{-7}) [50,0]</td>
</tr>
</tbody>
</table>

Table 9: Structural response of neat CFRP plate when subjected to a sinusoidal load at fundamental frequency with amplitude 1 N.

<table>
<thead>
<tr>
<th>Smart plate [0/90]_2s</th>
<th>Value [x,z] mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f_1)</td>
<td>2025.1 Hz</td>
</tr>
<tr>
<td>(w_{\text{max}})</td>
<td>0.31 µm [47,1]</td>
</tr>
<tr>
<td>(\sigma_{\text{max}})</td>
<td>136.5 kPa [0,0]</td>
</tr>
<tr>
<td>(\varepsilon_{\text{min}})</td>
<td>2.2 Pa [28,1]</td>
</tr>
<tr>
<td>(\varepsilon_{\text{max}})</td>
<td>5.1e(^{-7}) [50,2]</td>
</tr>
<tr>
<td>(\varepsilon_{\text{min}})</td>
<td>-5.1e(^{-7}) [50,0]</td>
</tr>
</tbody>
</table>

Table 10: Structural response of CFRP plate with an embedded PZT subjected to a sinusoidal load at fundamental frequency with amplitude 1 N.

Figure 26: Fundamental frequency compared between neat CFRP and with embedded PZT for different plate dimensions.

It can be concluded from the study that embedding PZT discs in the composite affect the structural behavior. The fundamental frequency is lowered due to the higher density of the PZT disc together with lower stiffness in the radial direction than in the fiber direction of a lamina, but higher in the transverse fiber direction. One can also see that the stresses are altered in the smart panel, now measuring minimum stress on the lower boundary of the piezo PZT connected to a transverse ply, again because of different stiffness and density of the PZT. Using this tool the model can now be optimized to find suitable placement dependent on an application. Specifically the placement of the shunted PZT is crucial because of reasons discussed in subsection 3.1, which will also be studied in the next subsection. It should be noted that with increased plate-dimensions the general structural influence is smaller, an example of this is illustrated in Figure 26 where the fundamental fre-
frequency is plotted as function of plate dimension, compared to a smart plate with an embedded PZT positioned at the center of the plate showing that influence of embedding PZT is smaller for larger plates using the same layup and thickness, which is expected. Stress concentrations and delamination are also present concerns when embedding transducers in composites but is not covered in this work. Different studies show different results regarding the structural influence of embedding PZT in composites. Christophe A. Paget and Klas Levin [16] made tensile and compression test to a composite plate, with layup $[0_4/90_4/0_4/90_4/0_2/PZT]_s$, showing small differences in either compression- and tensile strength, with final failure not coinciding with location of embedded PZT. M. Torres [22] show that cracks and delamination propagate from location of the cavity made from embedding sensors studying tension and 4-point bending.

5.2. Composite layup and structural response

The layup of the composite plate that are to be excited by embedded PZT is shown to be important. Different layups located on the boundary of the PZT results in different resonance amplitudes acquireable. To best illustrate this the PZT is modeled as a rectangular plate instead of a round disc positioned between a node and anti-node for the first mode, see Figure 27 and 28. The plate will express different strains in local 1- and 2-directions instead of equal strain radially. The embedded PZT-plate is then rotated in the plane around its axis. For degrees, $0^\circ$ to $90^\circ$, the resonance peak amplitude is measured as acceleration over applied voltage, exciting the first mode.

The measured values are illustrated by a 1D plot in Figure 29 for different layups. The study show similarity to buckling, i.e. it is easier to buckle a plate with lower stiffness, which is in the case of laminated composites in the direction where the fibers are transverse.

The placement of the PZT is $[x, y, z] = [20, 50, 1.25]$, which has boundaries connected to the top four layers. This study shows that fibers aligned transverse to the larger PZT-strain is preferable and a "wrong" choice between the three examples of layup can result in losses of up to 6 dB.
5.3. Method

This subsection will present a method for designing a smart CFRP plate, within the scope of this report, containing embedded PZT transducers that are to excite and damp the fourth mode, note that the same methodology can be applied to any mode. Dimensions of the plate are 100x100x2 mm and the PZT disc have diameter of 25.4 mm and thickness is 0.5 mm, layup sequence is \([0/90]_2\). The method will go through the process of how to determine placement of a PZT actuator to achieve the best results when exciting a chosen resonance frequency both in the plane and in the thickness direction, and how to determine placement of a PZT to dissipate energy once an actuating PZT is already placed in the composite. To begin this design method a neat fiber composite plate is studied using modal analysis. The fourth mode shape of the plate can be found in Figure 30 showing four anti-nodes symmetrically around nodal-lines.

Figure 30: Structure mode shape of a symmetric composite plate for the fourth resonance frequency.

Placement is crucial to excite a mode. If the excitation is positioned at a nodal line, minimal energy will be transferred to the plate. Figure 31 show this by plotting resonance amplitude versus position of the actuating PZT in x-direction along the plate for y-coordinate 70 mm (over two anti-nodes) which show correlation between resonance amplitude and position of anti-nodes, i.e. optimal position for an actuating PZT is on an anti-node.

Figure 31: Resonance amplitude as function of placement of a PZT along the x-axis over two anti-nodes. \([y,z] = [70, 1.25] \text{ mm}\).

Placing the actuating PZT change the mode shape of the plate which explain why the curve in Figure 31 is not all symmetrical around x = 50 mm for one single frequency. When the position in the plane is determined, the placement in thickness direction is studied.

Figure 32 show how strain is developed through the thickness, \(\varepsilon\), normalized with maximum strain, of the neat composite plate at an anti-node, which show linear strain variation. These results can be compared with theory explained in Zenkert [29] regarding bending of composites p.3.23.

Figure 32: Strain throughout the thickness of the composite plate at anti-node for the fourth mode.

Figure 33 show resonance amplitude as function of placement in thickness direction, where it is shown as absolute value of acceleration of the plate at a anti-node over voltage applied to the PZT versus
position in z-direction. The circles indicate possible positions for the PZT disc, i.e. between layers.

Figure 33: Resonance amplitude as function of placement in the thickness direction for the fourth mode. Circles indicate possible placements of PZT.

Comparing Figure 32 and 33 one can notice that the placement of the actuating piezos ability to excite the structure is in direct correlation with the strain in the neat fiber composite structure. It should here be noted that the layup is symmetric, for certain unsymmetrical laminates, the strain is not symmetric with respect to the neutral plane.

As mentioned, there is a change in mode shape when embedding a PZT. Figure 34 show the mode shape of the plate when a PZT is embedded in the CFRP plate located at the anti-node in the top corner \([x, y, z] = [70, 70, 1.25]\) mm.

Next a study when the plate is actuated by the PZT disc is carried out to determine placement of the second, shunted PZT. By applying voltage to the actuator at the fourth structural resonance frequency we can measure strain throughout the structure. These results are important when deciding the position of a sensor- or shunted PZT because of reasons discussed in subsection 3.1, i.e. the shunt performance factor is directly related to mechanical strain of main structure.

If again study Figure 34, with knowledge of relation between curvature and strain, the optimal placement should be on the anti-node in the lower corner. This is, similarly to previous step, shown by letting the sensing PZT change position along the x-axis for \(y = 30\) mm (over two anti-nodes) and measuring the voltage, see Figure 35 which show that optimal placement is in direct correlation to maximal strain, as expected. Optimal position is considered where the electric potential is large, i.e maximum mechanical energy is converted to electrical energy.

Through the thickness strain, when position of sensing/shunted PZT the strain variation is linear identically to what is shown in Figure 32.

5.4. Model

Using the method presented in previous subsection, the placement of the embedded PZT’s in the composite plate is determined, Table 11 list the coordinates of said PZT’s. The FE-model, using Comsol
Multiphysics, can be seen in Figure 36 containing an eight layer CFRP plate and two embedded PZT transducers.

<table>
<thead>
<tr>
<th>Transducer</th>
<th>Placement [x,y,z] mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>PZT\text{actuator}</td>
<td>[70,70,1.25]</td>
</tr>
<tr>
<td>PZT\text{shunt}</td>
<td>[30,30,1.25]</td>
</tr>
</tbody>
</table>

Table 11: Position coordinates for the piezo's determined by the method in previous subsection.

![Figure 36](image1)

Figure 36: The model featuring two embedded PZT discs to simulate shunt damping.

The layup for the composite plate is found in Figure 37 where the layers at the same z-coordinate as the PZT are modeled with cutouts to make room for the discs. The material data is found in Table 12.

![Figure 37](image2)

Figure 37: Stacking sequence and placement of the PZT actuator.

Material damping ratio of the composite plate is approximated as 0.5% based on the work by H. Hanselka, U. Hoffmann [10] and B A Ben et. al [4].

5.5. Results

Here will be presented the results from exciting and damping of a fiber composite plate using embedded PZT transducers (smart plate). Figure 38 show the mode shape of the fourth resonance mode that are excited and shunt damped. The positioning of the PZT-discs follow the method presented in subsection 5.3 and are found in Table 11.

![Figure 38](image3)

Figure 38: Mode shape for the fourth resonance frequency of the smart plate.

Figure 39 show an attenuation of 21.94 dB using tuned shunt parameters \( R = 400 \, \Omega \) and \( L = 86 \, \text{mH} \).
(a) FRF showing the first four resonances of the smart structure. The fourth mode is damped using single mode series shunt technique.

(b) Fourth resonance of the smart plate damped using PZT shunt highlighting the attenuation.

Figure 39: Frequency response for first four modes and the fourth mode to highlight the attenuation. Layup [0/90]_2s.
To illustrate that the methodology is applicable to any mode number, Figure 40 shows the damping predicted by FE-model when a series shunt circuit that is tuned for the first and third mode using the layup displayed in Figure 37 and material data found in Table 12 but now the placement for the actuating and shunted PZT are \([x,y,z] = [20,50,1.25]\) and \([50,50,1.25]\) respectively. Here should be noted the importance of placement when damping structures through shunt technique. For the third mode the shunted PZT is placed on a nodal line, which vastly decrease its damping capabilities.

(a) Left: First resonance damped. Right: First mode shape.

(b) Left: Third resonance damped. Right: Third mode shape.

Figure 40: Frequency response for first and third mode. Layup \([0/90]_2s\).
6. Discussion

The purpose of this report is to develop a finite element simulation model that can estimate structural response of fiber composite plates with embedded piezoelectric transducers and also simulate piezoelectric shunt damping.

Initially this work should have been carried out studying parallel shunt circuits as these circuits are, theoretically, more stable and are more easily tuned. Unfortunately, because of limiting knowledge of circuitry, one could not be successfully manufactured for experiments. However a series shunt circuit that responded as expected was manufactured which changed the focus of the report toward this configuration. This left the author with a decent amount of work prepared that unfortunately could not be compared with experimental results. A parallel shunt circuit is still on the "to-do"-list for the future of this project. Material data for APC 850 Navy II, given by the distributor (see Appendix A), is incomplete in terms of implementing the material in Comsol Multiphysics. Especially the elastic modulus which builds the elastic compliance matrix is lacking. Therefore, from the data that could explicitly be calculated, $s_{11}^E$ and $s_{22}^E$, a factor is calculated between APC 850 Navy II and PZT-A5, a material that can be found in Comsol Multiphysics, which is then multiplied with the remaining terms to form a full compliance matrix. This shows sufficient due to the good results acquired in Section 2 where analytic and experimental results are compared to numerical.

The experiments was a time consuming process due to the delicate tuning laws represented by RLC-circuits. Small shifts in natural frequencies give different results which leads to a very iterative and lengthy process. In theory shunt damping is a robust method, but in practice it benefits from some kind of control filters due to this instability. A possible solution to this, including control systems, is proposed under next section regarding future enhancements.

Due to expensive computations when working with large dimensional ratios, all simulations regarding composite plates was carried out on small plates (100x100 mm) which lets the embedding of PZT influence the structural response to a greater degree than if more realistic dimensions are considered. There are aspects in designing a smart plate that is not covered in this work, namely the implementation of physical wires and connection interfaces. These components are necessary and will affect the structure further. The final model was planned to be compared and tuned using experimental results but because of time constraints was not manufactured, this would have helped with calibrating the model for further use and given the report more significance, this is left as another point on the "to-do"-list.

7. Conclusion and future enhancements

A simulation model of a composite plate with embedded PZT transducers for shunt damping was developed. Successful comparisons with analytic and experimental results throughout the process allow good platform for estimating structural response. This model can in future research be used to determine placement of piezo's for several applications in any layup sequence of layered plates. Plenty of research papers cover optimization of placement of bonded piezo's in the plane of a structure. This report propose a method to determine placement in three directions by including through-the-thickness simulations showing that the best placement for a shunted PZT is where absolute strain is maximum in the structure.

These methods if implemented enhance the local environment due to lower noise and vibrations, implementation of lighter material allows, for example, lower fuel consumption in the vehicle industry.

Comsol Multiphysics show to be an adequate tool to use when simulating multidisciplinary systems such as piezoelectricity. The software defaults to conventional constitutive relations which makes it easy for implementation and coupling between disciplines are parallel between physics.

The possible future enhancements are plenty to this work, where the most pressing one is to acquire experimental results to compare the smart structures model. Due to the inconsistency in material properties inherent in manufacturing of fiber composites and embedding wires it is likely that the model can only roughly estimate structural response. A future enhancement to be studied is applying control systems for tuning the shunt circuits design variables to stabilize the system when small variations are present. This is almost necessary to study if to implement this technique for application use. Another branch of shunt damping is energy harvesting, where the energy dissipated through shunt circuit is stored are used to drive small electrically driven devices, which is a logical continuation of shunt damping where energy is essentially wasted.
REFERENCES


# Appendices

## A. APC materials properties

### Physical and Piezoelectric Properties of APC Materials

<table>
<thead>
<tr>
<th>APC Material:</th>
<th>840</th>
<th>841</th>
<th>850</th>
<th>855</th>
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<tr>
<td>Navy Equivalent</td>
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<td>–</td>
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<td>Navy VI</td>
<td>Navy III</td>
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<td>1375</td>
<td>1900</td>
<td>3300</td>
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<tr>
<td>Dielectric Dissipation Factor (Dielectric Loss(%))*</td>
<td>tan δ</td>
<td>0.60</td>
<td>0.40</td>
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<td>≤ 2.50</td>
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<td>Curie Point (°C)**</td>
<td>Tc</td>
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<td>320</td>
<td>360</td>
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<td>290</td>
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<td>400</td>
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<td>-d33</td>
<td>125</td>
<td>109</td>
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<td>Young's Modulus (10^{10} N/m²)</td>
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<td>1500</td>
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The values listed above pertain to test specimens. They are for reference purposes only and cannot be applied unconditionally to other shapes and dimensions. In practice, piezoelectric materials show varying values depending on their thickness, actual shape, surface finish, shaping process and post-processing.

Note: measurements made 24 hours after polarization.

Maximum voltage: 5.7 VAC / mil for 850, 851, 855, Type VI VOC ~2X.

9-11 VAC /mil for 840, 841, 842, 844, 880, 881 VOC ~2X.

*At 1 kHz, low field.

**Maximum operating temperature = Curie point/2.

Standard Tolerances

- Capacitance: ±20%
- Frequency: ±5% (to ±0.5% on request)

Updated: Nov. 2013 QF-MP Rev. 2
B. STRUCTURAL INFLUENCE OF EMBEDDING PZT - FIGURES

(a) Comparison of displacement throughout a plate. Left: Neat CFRP, Right: Smart plate.

(b) Comparison of stresses throughout a plate. Left: Neat CFRP, Right: Smart plate.

(c) Comparison of strain throughout a plate. Left: Neat CFRP, Right: Smart plate.

Figure 41: Structural response of a CFRP plate (y = 50 mm) with (left column) and without (right column) embedded PZT when subjected to a sinusoidal exciting distributed boundary load of $F_z = -1$ N at fundamental frequency. Layup [0/90]$_{2s}$. 