Reinforced Concrete Subjected To Restraint Forces

A comparison with non-linear numerical analyses

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Abstract

In Sweden, it is Eurocode 2 which forms the basis for performing a design of concrete structures, in which methods can be found treating the subject of restrained concrete members and cracking in the serviceability limit state. In the code, both detailed hand calculations procedures as well as simplified methods are described.

Several proposal of how to treat base restrained structures can be found in other codes and reports. Some state that the procedure given in Eurocode 2 is on the unsafe side as the method relies on stabilized cracking, while some say that the method is over conservative as the restraining actions will prevent the cracks from opening.

As these methods are analysed closer and further tested, it is obtained that they all yield different results under the same assumptions. Most of them are within a similar span, and the deviation arises as the various methods takes different aspect into consideration. One method yields a result which is considerably higher than all other, denoted the Chalmers method. As this method is taught at the technical institute of Gothenburg (Chalmers), the large deviation have caused some confusion among Swedish engineers.

As the methods are compared to numerical analyses, it is found that the detailed calculation procedure stated in Eurocode 2 yields fairly good prediction of crack widths for lower levels of strain, while for high levels of strain it is over conservative. The Chalmers method seems to underestimate the number of cracks which occur, and thus give rise to the deviating results. It is further found that in relation to more detailed hand calculations, the simplified procedure stated in Eurocode 2 may not always be on the safe side. The procedure is only valid within a certain range which may be exceeded depending on the magnitude of the load and choice of various design parameters.

The effect creep have on base restrained structures subjected to long term loads such as shrinkage is further discussed and analysed numerically. Various hand calculation methods suggest that creep have a positive influence on base restrained structures in the sense that the crack width become smaller. The numerical results indicates that this is indeed the case, however, uncertainties of these analyses are considered to be large in relation to the short term analyses.

Keywords: Restraint forces, concrete, cracking, restraint degree, crack width, creep, thermal actions, shrinkage
Sammanfattning

I Sverige är det Eurokod 2 som används som basis för dimensionering av betongkonstruktioner, i vilken metoder som beskriver sprickkontroll i bruksgränstadiet för betong utsatt för tvångskrafter återfinns. Både detaljerade handberäkningsmetoder och förenklade metoder beskrivs.

I olika koder och rapporter återfinns ett flertal förslag till hur detta problem ska hanteras. Vissa påstår att metoderna som anges i Eurokod 2 är på osäkra sidan då dessa förlitar sig på stabiliserad sprickbildning, medan andra menar att Eurokod 2 är för konservativ då inspännningen kommer förhindra att sprickorna öppnar sig.

Då metoderna analyseras noggrannare och testas framgår det att alla genererar olika resultat under samma antaganden. De flesta ligger inom samma spann och skillnaderna uppkommer då de olika metoderna beaktar olika aspekter. En metod genererar dock ett resultat som är högre än alla andra, som i denna rapport benämns som Chalmersmetoden. Då denna metod lärs ut på Göteborgs tekniska universitet (Chalmers) så har de utstickande resultatet skapat en viss förvirring bland konstruktörer i Sverige.

Då metoderna jämförs med numeriska analyser framgår det att Eurokod 2 förutsår en rimlig sprickvidd för låga töjningsgrader, medan den verkar vara överkonservativ för höga töjningsgrader. Chalmersmetoden verkar underestimera antalet sprickor som uppkommer i konstruktionen, vilket resulterar i de utstickande resultaten. Fortsättningsvis fastslås det att i relation till en mer detaljerad handberäkning så är den förenklade metoden i Eurokod 2 inte alltid på säkra sidan. Metoden är endast giltig inom ett visst spann, vilket kan överskridas beroende på den egentliga töjningens storlek och valet av dimensioneringsparametrar.

Krypningens effekt på fastinspända betongkonstruktioner då de utsätts för långtidslastar så som krympning har också diskuterats och analyserats numeriskt. Olika handberäkningsmetoder antyder att krypningen har en positiv effekt på så sätt att sprickvidden minskar. Även de numeriska resultaten indikerar att så är fallet, dock anses osäkerheten i dessa analyser vara stor i förhållande till analyser av korttidslastar.

Nyckelord
Tvångskrafter, betong, sprickor, inspänningsgrad, sprickvidder, krypning, termisk last, krympning
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With this thesis, we end our Master studies at the degree program Civil and Architectural Engineering at the Royal Institute of Technology and look brightly upon our future as structural engineers.

Stockholm, June 2017

Niels Brattström

Oliver Hagman
List of symbols

Latin letters

\( A \) \hspace{1cm} \text{Effective area according to ACI}

\( A_b \) \hspace{1cm} \text{Area of one reinforcement bar}

\( A_c \) \hspace{1cm} \text{Section area of concrete}

\( A_{c,ef} \) \hspace{1cm} \text{Effective concrete area in tension}

\( A_{ct} \) \hspace{1cm} \text{Sectional area of concrete is tension}

\( A_F \) \hspace{1cm} \text{Area of the contact surface}

\( A_I \) \hspace{1cm} \text{Equivalent concrete area in stage I}

\( A_{I,ef} \) \hspace{1cm} \text{Long term equivalent concrete area in stage I}

\( A_s \) \hspace{1cm} \text{Reinforcement area}

\( A_{s,\text{min}} \) \hspace{1cm} \text{Minimum reinforcement area}

\( B \) \hspace{1cm} \text{Relative load bearing capacity of concrete and reinforcement}

\( b \) \hspace{1cm} \text{Width of concrete member}

\( C_1 \) \hspace{1cm} \text{Coefficient for creep}

\( C_2 \) \hspace{1cm} \text{Empirical coefficient}

\( C_T \) \hspace{1cm} \text{Coefficient of thermal expansion}

\( c \) \hspace{1cm} \text{Concrete cover}

\( d \) \hspace{1cm} \text{Bar diameter}

\( d_c \) \hspace{1cm} \text{Distance from edge to centre of bar}

\( E_c \) \hspace{1cm} \text{Elastic modulus of concrete}

\( E_{c,ef} \) \hspace{1cm} \text{Effective elastic modulus with respect to creep}

\( E_{cm} \) \hspace{1cm} \text{Mean value of concrete elastic modulus}

\( E_F \) \hspace{1cm} \text{Elastic modulus of contact surface}

\( E_s \) \hspace{1cm} \text{Elastic modulus of steel}

\( F_{cs} \) \hspace{1cm} \text{Reinforcement compressive force}

\( f_b \) \hspace{1cm} \text{Average bond strength between the concrete and the steel}
\( f_{cm} \)  
Mean compressive strength of concrete

\( f_{ct} \)  
Tensile strength for the concrete

\( f_{ctm} \)  
Mean tensile strength for the concrete

\( f_{ct,eff} \)  
Mean tensile strength of concrete at time \( t \)

\( f'_{c} \)  
Tensile strength of concrete

\( f_{y} \)  
Yield stress

\( f_{s} \)  
Steel stress

\( G_{f} \)  
Fracture energy

\( H \)  
Height of concrete segment

\( h \)  
Vertical coordinate along concrete segment

\( K_{1} \)  
Factor taking creep in to account

\( K_{R} \)  
Degree of restraint at a specific height

\( k \)  
Coefficient for non-uniform self-equilibrating stresses

\( k_{1} \)  
Coefficient taking bond properties in to account

\( k_{2} \)  
Coefficient taking strain distribution in to account

\( k_{3} \)  
Coefficient provided in the national annex

\( k_{4} \)  
Coefficient provided in the national annex

\( k_{c} \)  
Coefficient for stress distribution

\( k_{L} \)  
Ratio of crack spacing in unreinforced concrete to height of member

\( k_{t} \)  
Coefficient taking the load duration in to consideration

\( L \)  
Length of concrete segment

\( L' \)  
Average crack spacing

\( l_{e} \)  
Element length

\( l_{t} \)  
Transmission length

\( N \)  
Restraint force

\( N_{cr} \)  
Cracking force

\( N_{H} \)  
Total number of bars

\( n_{cr} \)  
Number of cracks
\( n_{cr,\text{mod}} \)
Modified number of cracks

\( R \)
Degree of restraint

\( R_a \)
Restrained in the concrete member after crack

\( R_{ax} \)
Restraint factor

\( R_b \)
Restrained in the concrete member before crack

\( R_{\text{red}} \)
Reduced degree of restraint

\( S \)
Area of exposed surface

\( S_{r,\text{max}} \)
Maximum crack spacing

\( S_{r,\text{mean}} \)
Mean crack width

\( S_{r,\text{mod}} \)
Modified crack spacing

\( s \)
Slip

\( s_c \)
Coefficient depending on cement class

\( s_{\text{min}} \)
Minimum crack spacing

\( T_0 \)
Initial temperature

\( T_E \)
Temperature difference

\( t \)
Time

\( t_0 \)
Time at which a load is introduced

\( w \)
Crack width

\( w_{TS} \)
Total crack width due to temperature and shrinkage

\( w_T \)
Crack width due to temperature

\( w_S \)
Crack width due to shrinkage

\( w_i \)
Crack opening displacement

\( w_k \)
Characteristic crack width

\( w_{k1} \)
Instant crack width

\( w_{k2} \)
Crack width due to contraction of concrete relative to reinforcement

\( w_{\text{lim}} \)
Limit crack width

\( w_m \)
Mean crack width

\( w_{\text{max}} \)
Maximum crack width
Greek letters

\(\alpha_T\) Thermal coefficient of expansion
\(\alpha_{CT}\) Coefficient of thermal expansion for concrete
\(\alpha_e\) Ratio between elastic modulus of steel and concrete
\(\alpha_{e,ef}\) Long term ratio between elastic modulus of steel and concrete
\(\beta_{cc}\) Function defining the time dependency of concrete material parameters
\(\Delta r\) Length of local cone failure
\(\Delta T\) Temperature difference
\(\varepsilon\) Strain
\(\varepsilon_{c0}\) Peak strain at uniaxial compression
\(\varepsilon_{c1}\) Strain at peak stress according to concrete quality tables
\(\varepsilon_{ca}\) Autogenous shrinkage strain
\(\varepsilon_{c,creep}\) Strain of concrete due to creep
\(\varepsilon_{cd}\) Drying out shrinkage strain
\(\varepsilon_{c,el}\) Elastic strain of concrete
\(\varepsilon_{cm}\) Mean strain in reinforced concrete
\(\varepsilon_{ctu}\) Ultimate tensile strain of concrete
\(\varepsilon_{cs}\) Concrete shrinkage strain
\(\varepsilon_{cu1}\) Nominal ultimate strain according to concrete quality tables
\(\varepsilon_{el}\) Elastic strain
\(\varepsilon_{free}\) Strain for members subjected to no restraining actions
\(\varepsilon_{sm}\) Mean strain in reinforcement
\(\varepsilon_{ult}\) Elastic tensile strain capacity of concrete
\(\varepsilon_{obtained}\) Obtained strain
\(\varepsilon_{pl}\) Plastic strain
\(\varepsilon_T\) Thermal strain
\(\varepsilon_{tot}\) Total strain
\( \rho \)  
*The steel reinforcing ratio*

\( \rho (\text{min}) \)  
*Minimum reinforcement ratio*

\( \rho_{p,ef} \)  
*Ratio between steel area and concrete area in tension*

\( \sigma_c \)  
*Concrete stress*

\( \sigma_s \)  
*Steel stress*

\( \sigma_{s,all} \)  
*Allowable steel stress in reinforcement*

\( \tau_b \)  
*Bond stress*

\( \phi \)  
*Bar diameter*

\( \varphi \)  
*Creep factor*
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Chapter 1 – Background

1. Background

Restraint forces are often difficult to estimate and are known to act differently compared to forces caused by external loading as they are dependent of the structural response (Engström, 2011). In the case of end-restraint, Figure 1-1 (left), extensive research have been performed and is reasonably well understood, compared to the case of edge-restraint, Figure 1-1 (right), which have not been as systematically investigated (EC2-3, 2006).

![Figure 1-1 Two types of restraining situations, end restraint (left) and edge restraint (right) (EC2-3, 2006)](image)

How to conduct a crack control design of an edge restrained structure is vastly debated. Several design approaches exists in the form of design codes and reports. In Sweden, Eurocode 2 is used for performing a design of concrete structures, in which methods treating restrained structures subjected to early age imposed strains can be found.

ELU Konsult AB (Zangeneh, et al., 2013) performed an investigation in association with the Swedish Road Administration in order investigate appropriate finite element recommendations. It was concluded that the method stated in (EC2-3, 2006) is an appropriate design procedure as a linear finite element analyses overestimates the stresses as cracking is not considered. This report was questioned by (Christensen & Ledin, 2015) who stated that the method is on the unsafe side if the structure is very long in relation to its height.

The Chalmers method, proposed by (Engström, 2011), also states that the (EC2-3, 2006) procedure is on the unsafe side as it relies on stabilized cracking. Hence, an alternative design procedure is suggested based on single cracks. This method have been taught at the Technical University of Gothenburg (Chalmers) and result in significantly higher reinforcement ratios compared to (EC2-3, 2006). This have caused some confusion among Swedish engineers.

Further, a revised design approach for the (EC2-3, 2006) procedure is proposed by (Bamforth, et al., 2010). The revised method states that the crack cannot reach its full potential as the restraining member will prevent the crack from opening. Further, the restraining member will act somewhat like reinforcement, distributing the cracks as many small ones. Hence, (Bamforth, et al., 2010) suggests that (EC2-3, 2006) is over conservative for high restraint degrees.

All these statement contradict each other, revealing the disagreement of how base restrained concrete structures should be treated in the serviceability limit state.
1.1. **Aim and scope**

In this report, various methods and theories are presented and compared with non-linear finite element analysis. Focus is put on the method denoted as the Chalmers method, (EC2-3, 2006) and the compendium written by (Zangeneh, et al., 2013). The main aims are as stated below:

- Distinguish the differences in the hand calculation procedures
- Investigate the suitability of the design procedures stated in (EC2-3, 2006) and the compendium written by (Zangeneh, et al., 2013).
- Determine the source of the protruding results obtained when using the procedure denoted as the Chalmers method.
- Determine the effect creep have on base-restrained concrete members and appropriate ways of including it in a hand calculation procedure.

1.2. **Method**

- A literature study is conducted with the purpose to receive relevant background knowledge and to get familiar with the problem of base-restrained concrete members. The literature which is to be studied concerns the material concrete and its general properties, the workings of restraining actions, as well as concept of concrete cracking.

- Design approaches suggested by various codes and reports are analysed. Both detailed hand calculation methods and simplified design procedures are investigated and further tested and compared. Design codes and reports considered are (EC2-3, 2006) and (ACI, 1995), as well as (Engström, 2011), (Zangeneh, et al., 2013), (Bamforth, et al., 2010) and (Kheder, et al., 1994).

- Non-linear finite element analyses are conducted and compared with the results obtained through corresponding hand calculation procedures. Selected cases are tested in combination with different plasticity models.
1.3. Limitations

In this report, analyses and investigations are limited to cracking of base-restrained concrete members and the result may thus differ for other cases of similar nature. The only load considered is a uniform volume changes caused by short term thermal actions or long term shrinkage of concrete.

Non-linear finite element analyses are limited to 2D plane stress. By doing this, some internal restraint factors are neglected and reinforcement bars cannot be arranged properly over the width. Further, the bond-slip relationship between the reinforcement and concrete is neglected throughout the analyses.

1.4. Outline of the report

Chapter 2  A theoretical background is provided which includes general aspects about the material concrete and restraining actions. Further, a description of how cracking caused by restraining actions differs from regular concrete cracking is given.

Chapter 3  A description of design approaches proposed in various design codes and reports is described. Both detailed hand calculations procedures and simplified design procedures are presented.

Chapter 4  A description of finite element modelling, with focus on non-linear modelling, is provided. A brief description of the concept and different material models is given together with common issues which may arise. Further, the general modelling approach used in this report is described.

Chapter 5  Case studies, analyses and results are provided. A discussion is included in each case study, followed by remarks which are considered to be of importance.

Chapter 6  A method criticism together with a final discussion and conclusions is given. Also recommendations based on the obtained results and suggestions for future research is discussed.
2. Theoretical background

In this chapter, a theoretical background concerning general and relevant aspects of concrete and restraining actions is provided. Descriptions are made in accordance with established theories which have been tested and generalized.

2.1. The material concrete

Concrete in its pure form is a composition of cement, aggregate and water. The cement is produced from materials which contain lime, silica, aluminium and iron which is heated to about 1500 °C in rotating kilns which result in small pellets known as Portland cement clinker (Ansell, et al., 2012). These are further grinded in mills and mixed with a limited amount of gypsum in order to enhance the casting process. Further additives may also be added in order to obtain specific properties.

About 60 – 70 % of the concrete mixture consists of aggregate (Ansell, et al., 2012). The aggregate is normally obtained from rock material and consists in a mixture of particles with varying grain size. Preferably, the aggregate should have a round or cubic shape and have good strength, wear strength, durability and purity from humus which typically is found in top soils (Ansell, et al., 2012).

The water used in concrete should be free from organic impurities which can delay the hydration process. Further, it should be free from chlorides in order to reduce the risk of reinforcement corrosion. There are standards describing acceptable quality of water used in concrete, but in general regular tap water or even water which is not suitable for drinking can be used (Ansell, et al., 2012).

2.1.1. Strength properties

In practice, concrete structures are build such that they are loaded in compression in order to utilize the good compressive properties. Hence, it is the compressive strength which usually is the most important material property (Ansell, et al., 2012).

Concrete is a highly non-linear material in a uniaxial compression, see Figure 2-1. Up until 30 % of the ultimate strain, the material behaves more or less linear-elastic (Bangash, 2001), after which the tangent modulus is gradually decreased up until 70 – 90 % of the ultimate strain. As the peak value of the stress-strain curve is reached, the material starts to soften, resulting in an increase in strain even though the stress is reduced. The softening proceeds until the ultimate strain is reached.
In terms of crack design, it is the tensile behaviour of the concrete which have the greatest importance as this is considerably lower than the compressive strength. Many different methods of how to determine the tensile strength have been suggested in various reports for which it have been shown that depending on what method is used, there is a large spreading in the results (Svenskbyggtjänst, 2017).

One method is to perform an indirect splitting test, in which a cube or cylinder is loaded by two line loads, see Figure 2-2. Through the majority of the section, uniform tensile stresses arise which further can be used in order to determine the tensile strength (Ansell, et al., 2012).

The biaxial behaviour of concrete is typically of more ductile nature compared to the uniaxial behaviour (Malm [B], 2016). As concrete is subjected to biaxial loading, the strength properties vary with respect to the loading situation, see illustration in Figure 2-3. For instance, as concrete is loaded in pure biaxial compression the strength become larger compared to its uniaxial strength. According to (Malm [A], 2016), the biaxial compressive strength is typically 16 % higher than the uniaxial compressive strength.
2.1.2. Shrinkage

Shrinkage is a stress-independent deformation which occurs during the hardening process of the concrete and may be divided into two major types; drying shrinkage ($\varepsilon_{cd}$) and autogenous shrinkage ($\varepsilon_{ca}$) (Engström, 2011).

The drying shrinkage occurs due to the evaporation of water from concrete to surrounding air, and is thus influenced by factors such as the thickness and surface area of the element, temperature, wind speed and relative humidity (Ansell, et al., 2012).

The autogenous shrinkage takes place as the hydration process of the concrete starts to slow down. At this point, the concrete have hardened to some degree, but there is still some moisture and cement left which have not yet reacted. As these react, there will be no exchange of water between the concrete and the surrounding air, hence the concrete shrinks due to the chemical reaction itself (Engström, 2011).

The extent to which the shrinkage takes place is highly dependent on the relation between water and cement, the vct – ratio. Low vct-ratios results in that the majority of the water will react with the cement and consequentially the concrete will have a very low permeability. In such a case, only the surface of the concrete member will be subjected to drying shrinkage while the main part of the total shrinkage will is caused by autogenous shrinkage (Engström, 2011). For high vct-ratios, the effect will be the opposite.
2.1.3. Creep

Creep is time-dependent deformation (Ansell, et al., 2012). As a member is loaded, an immediate elastic deformation will occur in accordance with Hook’s law, see Figure 2-4. As the load remains on the member, further deformation will occur even though the stress level remains the same. As the load is removed, the elastic deformation will dissolve, while the permanent deformation (the creep) remains.

Figure 2-4 Response of concrete member subjected to long term loading and unloading (Engström, 2011)

Creep affects the concrete in the sense that the concrete becomes softer, and thus deform more easily, resulting in that second order effects may be introduced (Engström, 2011). In the case of restrained structures, a softer concrete may have a positive effect as the restraint stresses are reduced. Creep is influence by various parameters, of which some are listed below:

- Environment
- Stress level
- Time of loading
- Concrete composition
- Section size

If a concrete member is subjected to a constant stress which is lower than about half of the compressive strength, creep can be assumed to be proportional to the elastic strain, see Equation 2-1 (Engström, 2011). The creep factor \( \varphi \) can thus be determined through experimental procedures. If the stress is larger than about half of the compressive strength, non-linear creep will occur (Engström, 2011).

\[
\varepsilon_{c,\text{creep}} = \varphi(t, t_0) \cdot \varepsilon_{c,\text{el}}
\]  

(2-1)
2.1.4. Time dependency

During the hardening process of the concrete the material experience constant changes of its properties. According to (EC2-1, 2005), the time dependency may be estimated based on the 28-day material properties, provided that the curing occurs in 20 °C and follows the regulations stated in EN 12390. For such a case, the variation of material properties are based on the same shape function, Equation 2-2.

\[
\beta_{cc}(t) = e^{s_c \left[1 - \left(\frac{28}{t}\right)^{1/2}\right]}
\tag{2-2}
\]

Where

- \( s_c \) is \{0.2, 0.25, 0.38\} depending on cement strength class
- \( t \) is the age of concrete in days

Further, the time dependency of the compressive strength, tensile strength and elastic modulus may be estimated through Equation 2-3, 2-4 and 2-5 respectively.

\[
f_{cm}(t) = \beta_{cc}(t) \cdot f_{cm}
\tag{2-3}
\]

\[
f_{ctm}(t) = (\beta_{cc}(t))^{\alpha} \cdot f_{ctm}
\tag{2-4}
\]

\[
E_{cm}(t) = \left(\frac{f_{ctm}(t)}{f_{cm}}\right)^{0.3} \cdot E_{cm}
\tag{2-5}
\]
2.2. Restraint actions

All concrete elements are restrained to some degree as there always is some restraint provided by the support or the element itself (Engström, 2011). The two main parts of restraint are external- and internal restraint. The external restraint could be caused by e.g. surface contact, while internal restraint can be seen as stresses which arise within the element itself due to its components, e.g. bond between concrete and reinforcement (ACI, 1995).

The restraint degree $R$ describes the extent to which a structure is prevented from moving in relation to its supports (Zangeneh, et al., 2013), see Equation 2-6. It is thus obtained as a value of $0 \leq R \leq 1$, i.e. no restraint, partial restraint or full restraint (Engström, 2011).

$$R = 1 - \frac{\varepsilon_{\text{obtained}}}{\varepsilon_{\text{free}}}$$  \hspace{1cm} (2-6)

2.2.1. External restraint

External restraint refers to restraining action caused by attachment or the supports of the concrete member. The restraint degree depends on the stiffness of the member itself, as well as the stiffness of the restraining member and the type of the boundary conditions (Engström, 2011). Figure 2-5 illustrates the variation of restraint degree over the height for a concrete member which is restrained along its bottom edge. It is shown that there is a major decrease over the height of the member.

![Figure 2-5 Variation of restraint degree in a concrete member restrained along its base (Engström, 2011)]

The variation is highly dependent on the ratio between the length and the height of the element, the $L/H$-ratio (Engström, 2011). The larger the ratio is, the less variation there is over the height of the mid-section. According to (ACI, 1995), one may even obtain compressive stresses at the top of member which have a very low $L/H$ – ratio.
As long as only one edge or two perpendicular edges are restrained, the concrete member still has some freedom to allow volume changes. The situation becomes considerably worse as two opposite sides are restrained (Engström, 2011) and thus more measures have to be taken.

2.2.2. Internal restraint

Internal restraint arises as different parts within the structural members are subjected to different strains, resulting in one member restraining another (Engström, 2011). This effect will give rise to internal stresses (eigenstresses) which may cause cracking in the concrete. A typical example of such a situation is as concrete shrinks, in which it is restrained by the reinforcement bond. Based on the internal restraint, the total strain of a reinforced concrete member can be derived according to Figure 2-6 (Engström, 2011).

![Figure 2-6](image)

*Figure 2-6 Reinforced prismatic member subjected to shrinkage, reproduction from (Engström, 2011)*

a) Both concrete and reinforcement is subjected to zero loading

b) The concrete shrinks freely assuming no interaction with reinforcement, resulting in a stress-independent strain $\varepsilon_{cs}$

c) The reinforcement is compressed to the same length by introducing a force $F_{cs}$ and the reinforcement and concrete are coupled together

d) The compressive force is removed, resulting in that the reinforcement wants to elongate, and thus subject the concrete to a tensile force of the same magnitude as $F_{cs}$
If full bond is assumed, the concrete stress can be calculated using Navier’s equation and a transformed concrete section, Equation 2-7 (Engström, 2011):

\[
\sigma_c = \frac{F_{cs}}{A_{I,ef}} \quad \text{where} \quad A_{I,ef} = A_{c,ef} + (\alpha_e - 1)A_s
\]  

The steel stress is found by adding the concrete stress \( \sigma_c \) with step c), Equation 2-8

\[
\sigma_s = \frac{-F_{cs}}{A_s} + \alpha_e \cdot \sigma_c
\]  

If the strain of the concrete and reinforcement is assumed to be equal, the strain of the reinforced concrete member can be expressed in accordance with Equation 2-9

\[
\varepsilon_{cm} = \frac{1}{E_s} \left( \frac{-F_{cs}}{A_s} + \alpha_e \cdot \sigma_c \right)
\]  

The restraint force due to imposed strain may also be expressed in the means of eigenstresses in accordance with Equation 2-10 (Engström, 2011).

\[
\sigma_s \cdot A_s = \sigma_c \cdot A_{c,ef}
\]  

### 2.2.3. Two types of strain

There are two types of strain, stress-dependent and stress-independent strain. The first mentioned is the result of an external load causing a deformation. An initial elastic strain is obtained in accordance with Hook’s law, followed by the time dependent strain due to creep. As creep may be assumed to be proportional to the elastic strain, the total strain can be determined through Equation 2-11.

\[
\varepsilon = \varepsilon_{el} (1 + \varphi)
\]  

The stress-independent strain is caused by factors that do not introduce stresses, such as volume changes due to thermal action (Equation 2-12) or shrinkage (Equation 2-13) (Engström, 2011). As long as the concrete member is free to move, no stresses will arise within the element. If there is some degree of restraint, stresses will occur and thus introduce stress dependent strains. If these stresses become sufficiently large in the sense that the tensile strength of the concrete is exceeded, cracking will occur (Engström, 2011).

\[
\varepsilon_T = \Delta T \cdot \alpha_T
\]
\[ \varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t) \]  

(2.13)

2.3. Concrete cracking

Cracks are not something that one strives to avoid as these appear during normal use of concrete, but rather something that must be taken into consideration in the design procedure in order to estimate the correct response (Engström, 2011). In this sense, cracks can be divided into two groups, normal cracks and damage cracks. According to (Engström, 2011), these two types can be distinguished in the sense that a damage crack is a crack which is "unpredictable or is greater than predicted at the actual loading level".

Cracks are associated with negative influence in the sense that they allow environmental actions on the reinforcement and concrete and thus affect the durability of the structure. This may result in that the structure do not fulfil the requirements on safety during its service life (Engström, 2011). The wider the crack is, the more exposed the reinforcement will be.

Regular reinforcement cannot be used in order to prevent cracks, but only to distribute them such that few large cracks are avoided (Ansell, et al., 2012). This is due the fact that only about 4% of the reinforcement strength is utilized as the first crack appears, under the general assumption that the strain of the reinforcement and concrete is equal during un-cracked conditions (Engström, 2011). In order to have an significant effect of the reinforcement in concrete structures, one must thus either allow cracks such that the steel strain can increase, i.e. the concept of reinforced concrete, or the steel must be strained in advanced, i.e. the concept of pre-stressed concrete (Engström, 2011).

2.3.1. Tensile properties during cracking

The tensile stress-strain relationship consists of a more or less linear elastic part up until a level just before the ultimate tensile strength, see first part in Figure 2-7. As this point is exceeded, the stiffness is reduced due to micro cracks (Malm [A], 2016).

The micro cracking proceeds up until the tensile strength of the concrete is reached, after which the cracking procedure becomes unstable and concentrated to a limited area known as the fracture process zone (Malm [A], 2016). The stress-strain curve starts to descend and the concrete softens as further micro crack arise.
As the micro cracks are concentrated to a limited area, these will eventually form a macro crack, visible to the naked eye, see second part in Figure 2-7. The elongation of the concrete member now consists in two parts, an elastic strain in the un-cracked concrete and the crack opening itself (Malm [A], 2016).

The crack opening is related to the fracture energy $G_f$, which is a material property that defines the amount of energy required in order to “obtain a stress free crack” (Malm [A], 2016), or the energy required to “propagate a tensile crack of unit area” (Model Code, 2010).

The fracture energy is equivalent to the area underneath the crack opening curve. According to (Model Code, 2010), in the absence of experimental data, the fracture energy may be estimated according to Equation 2-14.

$$G_f = 73 \cdot f_{cm}^{0.18}$$

(2-14)

Where $f_{cm}$ is inserted in MPa

### 2.3.2. Cracking of reinforced concrete

As thin members are loaded in tension, see Figure 2-8 (left), a certain length $l_t$ is required in order to transfer the stresses in the reinforcement to the surrounding concrete (Engström, 2011). Within this length, a bond-stress $\tau_b$ arises as a result of a certain slip $s$ as the strain of the reinforcement and concrete is not equal along this length.

This results in a local cone failure of the length $\Delta r$, in which no bond stress can occur, see Figure 2-8 (right). The length over which the applied load is transferred is a function of the load itself in the sense that an increased load results in an increased transmission length.
As illustrated in Figure 2-8 (left), the largest stress in the concrete occurs along the mid-section, hence, this is where the cracking will occur (Engström, 2011). As the first crack has developed, the element is divided into two segments connected by the reinforcement bar. For each segment, a new transmission length and local cone failure arises. As the load is slightly increased, more cracks will appear and the procedure repeats itself, see Figure 2-9.
As no cracks can occur within the transmission length, the cracking procedure will reach a final stage, known as *stabilized cracking*, in which no more cracks will occur even though the load is increased (Engström, 2011). At this stage, the crack spacing can be no shorter than $l_t + \Delta r$ and no larger than $2(l_t + \Delta r)$. However, the crack width will increase further as the load is increased due to an increase of strain in the reinforcement.

**Analysis of thick members**

For thick members, the transmission length may be smaller than the height of the member. In such a case, a certain distance is required in order to distribute the stresses uniformly over the cross-section (Engström, 2011). Along this discontinuity region, there is an uneven stress distribution resulting in that the maximum stresses will act on an area which is smaller than the section area itself, see Figure 2-10.

![Figure 2-10 Illustration of concrete stresses near a crack of a thick concrete member (Engström, 2011)](image)

For such members, the first crack will be a crack through the whole cross-section, through which the total load is carried by the reinforcement only. This major crack will be followed by smaller cracks on each side as the load is increased, as illustrated in Figure 2-11 (Engström, 2011).

![Figure 2-11 Cracking procedure of thick concrete members subjected to tension (Engström, 2011)](image)
In the means of determine the cracking load for thick members, an effective area $A_{c,\text{eff}}$ should be used in order to account for the discontinuity region. In (EC2-2, 2005), this is determined based on the effective height illustrated in Figure 2-12.

![Figure 2-12 Calculation of effective height of thick beams (a) slabs (b) and walls (c) (Engström, 2011)](image)

For restrained concrete members, such as walls that are subjected to volume changes, one effective area for each reinforced face should be used in accordance with case (c). If the thickness $t \leq 2 \cdot 2,5(c + \phi/2)$, the whole section will be in tension and can thus be analysed as a thin member (Engström, 2011).

If the bar spacing is large enough, a discontinuity region must be considered in the perpendicular direction as well. In (EC2-2, 2005), this is taken into account when calculating the crack spacing $S_{r,\text{max}}$. This is further described in chapter 3.1.4.

### 2.3.3. Influence of restraint conditions

Depending on the type of restraint, the stress distribution will act differently. Compare the two unreinforced concrete members displayed in Figure 2-13 and Figure 2-14.

As the upper member is subjected to a stress-independent strain, Figure 2-13, tensile stresses will arise. At one point, the stresses exceed the tensile strength of the concrete, and thus create a through crack. At this point, the stresses are immediately reduced to zero and the member can continue its volume decrease without introducing new stresses or cracks (Engström, 2011).
In Figure 2-14, the member is continually restrained along its base. Even though a crack appears, the member is not free to move and further volume decrease will thus introduce new stresses and cracks.

![Figure 2-13 Member restrained at its end and subjected to a stress-independent strain (Engström, 2011)]

The cracking is also dependent on the stiffness of the restraining boundary. In the numerical analysis performed by (Johannsson & Lantz, 2009), the influence of the base stiffness was studied for an edge beam. The results indicated that the required amount stress-independent strain for the first crack to occur decreases as the stiffness of the base is increased. The crack widths were obtained slightly larger and not as evenly distributed as the stiffness was decreased.

### 2.3.4. Influence of loading type

Displayed in Figure 2-15 is two types of loading situations, *load-controlled* loading and *deformation-controlled* loading. The boundary conditions are the same for both cases, but for member (a) the load is continuously increased while for member (b) the deformation is continuously increased.

![Figure 2-15 Beam subjected to load-controlled loading (a) deformation-controlled loading (b) (Engström, 2011)]

As the first crack occurs, the overall stiffness is suddenly reduced. For the load-controlled member, the tensile load $N$ is now taken by the reinforcement through the crack, resulting in an instant increase in stress (and thus strain) in the reinforcement, see Figure 2-16 (a).
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For the displacement-controlled member however, it is the elongations $a$ which is controlled, which means that at the moment of cracking the elongation is still the same. As equilibrium is required, the stress is instantly reduced in order to compensate for the reduced stiffness, see Figure 2-16 (b) (Engström, 2011). If the tensile force or elongation is further increased, the procedure continues until the stabilized cracking stage describe in chapter 2.3.2 is obtained.

![Diagram of tensile stress and elongation](image1)

**Figure 2-16** Relationship between the tensile stress and elongation in the two beams that are displayed in Figure 2-15 (Engström, 2011)

### 2.3.5. Cracking procedure of restrained concrete walls

As a base-restrained wall is subjected to a stress-independent strain such as a volume decrees due to thermal actions, the first crack will initiate in the base of the restrained edge (ACI, 1995). As described in chapter 2.2.1, the degree of restrain decreases over the height of the element, and thus does the stresses. Hence, the crack will propagate upwards to the point in which the stresses no longer exceeds the tensile strength of the concrete (ACI, 1995).

However, as the concrete cracks the tensile stresses of the cracked region will be transferred to the un-cracked region, increasing the tensile stress above the cracks, see Figure 2-17. According to (ACI, 1995), for higher L/H-ratios ($L/H \geq 2.5$), if there is enough tensile stress to create a crack, the crack may very well propagate through the whole section due to this effect.

![Diagram of stress distribution](image2)

**Figure 2-17** Change in stress distribution due to cracking (Johannsson & Lantz, 2009)
According to (ACI, 1995), the first crack will appear approximately in the middle of the element as the restraint degree is at its largest there. If the volume decrease proceeds, further cracks will appear in between the previous crack and element edge, or in between two previous cracks, as illustrated in Figure 2-18. In an experimental study performed by (Kheder, et al., 1994), a similar cracking procedure was obtained, however with inclined crack close to the free edges.

As Figure 2-18 indicates, the height to which the crack propagates will decrease for every new crack. According to (Kheder, et al., 1994), reinforcement is thus not required over the whole concrete member in order to control cracking as the member will contain crack free zones. For this purpose, (Kheder, et al., 1994) suggests a designing zone for restrained concrete members as illustrated in Figure 2-19.

For a restrained member subjected to stress-independent strains, the elongation will be displacement controlled as described in chapter 2.3.4, in the sense that at the moment of cracking the elongation is the same. As the first crack appears, the stresses will be instantly reduced in order to compensate for the reduced stiffness (Engström, 2011). As the structural stiffness is reduced, the structure will adapt itself to the new conditions. The stiffness might have been reduced to such a magnitude that no further cracks will appear resulting in that the stabilized cracking stage described in chapter 2.3.1 is not reached. This could also be the case if the volume decrease does not proceed after the first few cracks (Engström, 2011).
2.4. Thermal actions according to Eurocode

According to (EC2-1, 2005) thermal effects should be taken in to consideration when verifying serviceability limit state. Further, they should be checked for the ultimate limit state only if they are believed to be significant with respect to e.g. fatigue conditions or verification of second order effects. When the thermal actions are taken into account, they should be considered as variable actions.

In (EC1-1-5, 2005) it is stated that a structure should be designed in such a way that thermal movements do not give arise to overloading, which may be taken in to account by using expansion joints or consider relevant restraint forces. It is further stated that the linear thermal coefficient of expansion should be used in design, which is denoted as $\alpha_{cT} = 10 \cdot 10^{-6} \, 1/\circ C$ for regular concrete in Appendix C of the same code.

For a uniform temperature change, the effects of restraint forces caused by elongation or contraction should be taken in to consideration for the superstructure of a bridge. Measures should be taken with respect to different temperature changes for different structural parts which are connected. According to (EC1-1-5, 2005) the recommended value of a temperature difference between structural parts is $\Delta T = 15 \, ^\circ C$.

A non-linear temperature variation through the concrete section should also be taken in to consideration. This temperature distribution may arise as one side of an element is warmer than the other side. Two methods of calculating the non-linear temperature variation are provided in (EC1-1-5, 2005).
3. Design procedures

In this chapter, various design procedures for crack control is presented. It includes a description of procedures for determining the restraint degree and the minimum reinforcement required in order to control cracking, as well as detailed procedures for determining the maximum crack widths and simplified procedures for a quick and alternative design.

3.1. Eurocode 2

In 1975, the commission of the European Community began the Eurocode program within which initiative were taken to establish technical rules for design of constructional works (EC2-1, 2005). The program consists in several codes used in Swedish design, where Eurocode 2 considers the design of concrete structures.

3.1.1. EC2-3 - Restraint degree

The degree of external restraint may according to (EC2-3, 2006) be estimated through knowledge of the stiffness relation between the restraining member and the member attached to it. Alternately, a restraint factor may be chosen from the tabular values in Table 3-1.

<table>
<thead>
<tr>
<th>L/H-ratio</th>
<th>Restraint factor at base</th>
<th>Restraint factor at top</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0,5</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0,5</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0,5</td>
<td>0,05</td>
</tr>
<tr>
<td>4</td>
<td>0,5</td>
<td>0,3</td>
</tr>
<tr>
<td>&gt;8</td>
<td>0,5</td>
<td>0,5</td>
</tr>
</tbody>
</table>

3.1.2. EC2-2 – Minimum reinforcement

According to (EC2-2, 2005), a minimum amount of reinforcement is required in order to control cracking. This may be estimated through equilibrium between the stresses in the concrete just before cracking and reinforcement at yielding, or lower stress if required, see Equation 3-1.

\[ A_{s,min} \cdot \sigma_s = k_c \cdot k \cdot f_{ct,eff} \cdot A_{ct} \]

Equation 3-1
3.1.3. EC2-2 – Control of cracking without direct calculations

According to (EC2-3, 2006), if minimum reinforcement is provided, cracks widths are not likely to be excessive for cracks caused dominantly by restraint if the bar size given in Figure 3-1 is not exceeded, where the steel stress is the value obtained directly after cracking, i.e. $\sigma_s$ in Equation 3-1.

![Figure 3-1 Maximum bar size and steel stress to limit cracks (EC2-3, 2006)](image)

3.1.4. EC2-2 – Crack width calculation

The main expression for calculating the crack width according to (EC2-2, 2005) is based on the crack spacing and the differential mean strain between concrete and reinforcement, see Equation 3-2.

$$w_k = S_{r,\text{max}}(\varepsilon_{sm} - \varepsilon_{cm}) \quad (3-2)$$

The differential mean strain may be calculated according to Equation 3-3

$$\left(\varepsilon_{sm} - \varepsilon_{cm}\right) = \frac{1}{E_s} \left(\sigma_s - \frac{k_t \cdot f_{\alpha,ef}}{\rho_{p,ef}} \left(1 + \alpha_e \cdot \rho_{p,ef}\right)\right) \geq 0.6 \frac{\sigma_s}{E_s} \quad (3-3)$$

Depending on the centre distance between the reinforcement bars, (EC2-2, 2005) states that the maximum crack spacing can be calculated according to Equation 3-4 or Equation 3-5.

$$S_{r,\text{max}} = k_3 c + \frac{k_1 k_2 k_4 \phi}{\rho_{p,ef}} \quad \text{for} \quad c/c \leq 5 \left(c + \frac{\phi}{2}\right) \quad (3-4)$$

$$S_{r,\text{max}} = 1.3(h - x) \quad \text{for} \quad c/c > 5 \left(c + \frac{\phi}{2}\right) \quad (3-5)$$
For walls subjected to early thermal contraction, which do not fulfil the requirements of minimum reinforcement and for which the bottom edge is restrained by previously base cast, the maximum crack spacing may according to (EC2-2, 2005) be assumed to be 1.3 times the height of the wall.

3.1.5. EC2-3 – Crack width calculation according to Annex M

According to (EC2-3, 2006) a reasonable estimation of the differential strain for base-restrained structures such as the one illustrated in Figure 3-2 can be obtained by applying Equation 3-6 as the structure is subjected to early age stress-independent strains. The restrain factor $R_{ax}$ may be obtained from Table 3-1. In order to obtain the crack widths, the result from Equation 3-6 should be inserted in to Equation 3-2.

\[
(\varepsilon_{sm} - \varepsilon_{cm}) = R_{ax} \cdot \varepsilon_{free}
\]

\textit{Figure 3-2 Member restrained along its base (EC2-3, 2006)}
3.2. The Chalmers method

The method which in this report is denoted as the Chalmers method is proposed by (Engström, 2011). According to (Engström, 2011), in the case of restrained structures, the stabilized cracking stage might not be reached, hence the standard methods of crack control calculation stated in EC2 cannot be applied. Instead, the design must be performed with respect to single crack response, in which single cracks refers to those cracks that appear before the stabilized cracking stage.

3.2.1. Chalmers - Restraint degree

In order to estimate the degree of restrain along the height of a wall, Figure 3-3 may be used (Engström, 2011), which displays the variation of the restraint degree over the height at the midsection for different $L/H$ ratios.

![Figure 3-3: Restraint degree as a function of height for different $L/H$-ratios (Engström, 2011)]
3.2.2. Chalmers – Minimum reinforcement

According to (Engström, 2011), the minimum amount of required reinforcement for controlling cracks can be determined through Equation 3-7. The expression is based on the condition that there should be enough reinforcement in order to initiate a new crack within the effective concrete area and that the steel stress should be limited with respect to the allowable crack width.

The limiting steel stress may be obtained by using Equation 3-9 or a design diagram such as the one displayed in Figure 3-4. Provided in (Engström, 2011) are several design diagrams for specific concrete strength classes.

The expression does not include the flexibility of the structural member at cracking, and is thus “somewhat on the safe side” (Engström, 2011). According to (Engström, 2011), the iterative procedure further described in chapter 3.2.3 should be used in order to analyse the actual response.

\[
A_s \geq \frac{f_{ctm} \cdot A_{c,ef}}{\sigma_{s,all} - f_{ctm}(\alpha - 1)} \tag{3-7}
\]

![Figure 3-4 Relation between mean crack width and steel stress for various bar diameters and C30/37 concrete (Engström, 2011)]
3.2.3. Chalmers – Single cracks

The crack width is in this method determined based on the slip which is created along the transmission length described in chapter 2.3.1. The slip occurs due to differential strain between the reinforcement and concrete along this length. In order to estimate the crack width, the relation between the slip $s$ and the bond stress $\tau_b$ must be known (Engström, 2011). According to (Jaccoud, 1997), this relationship can be described by the means of Equation 3-8 for the serviceability limit state, which is also is used in the Chalmers method.

$$\tau_b(s) = 0.22f_{cm} \cdot s^{0.21} \quad (3-8)$$

Further, the mean crack width is derived as a function of the steel stress in the cracked section according to Equation 3-9 (Engström, 2011).

$$w_m(\sigma_s) = 0.42 \cdot \left( \frac{\phi \cdot \sigma_s^2}{0.22f_{cm} \cdot E_s \left( 1 + \frac{E_s}{E_c} \cdot \frac{A_s}{A_{c,ref}} \right)} \right)^{0.826} + \frac{\sigma_s}{E_s} \cdot 4\phi \quad (3-9)$$

The last term of the expression compensates for the elongation which take place along the length of the cone failure $\Delta r$ where no bond exists. The equation is only valid for single cracks during short term loading. If long-term loading is to be considered, the crack width will according to (Engström, 2011) increase with time, which can be taken in to account by multiplying the first part of the expression with a magnification factor of 1,24.

As a restrained wall is analysed, the calculation should be performed for a small strip of the element, see Figure 3-5 as the degree of restraint varies over the height of the member, see chapter 2.2.1.

![Figure 3-5 Design strip of a restrained wall (Engström, 2011)]

As the structure is analysed for single cracks and deformation-controlled loading, (Engström, 2011) proposes an iterative designing procedure based on the deformation condition provided in Equation 3-10.
\[
\frac{\sigma_s \cdot A_s}{E_{cm} \cdot A_I} \cdot L + n_{cr} \cdot w_m(\sigma_s) + R \cdot \varepsilon_{cr} \cdot L = 0
\]  
\[(3-10)\]

\(A\) \(B\) \(C\)

Where

(A) Is the deformation in the concrete due to the restraint force, expressed in the means of eigenstresses (internal restraint), see Equation 2-10

(B) The sum of calculated crack widths

(C) The restrained deformation due to stress-independent thermal actions (external restraint)

For long-term calculations, such as shrinkage, the effective elastic modulus and effective area should be used in (A), the crack width for long-term loading in (B) and the shrinkage strain in (C) (Engström, 2011).

The iterative procedure starts by assuming an initial number of cracks, \(n_{cr} = 1\). Equation 3-10 can thus be solved for the steel stress \(\sigma_s\) which is used to calculate the restraint force \(N\) through Equation 3-11.

\[N = \sigma_s \cdot A_s \]  
\[(3-11)\]

The restraint force is compared to the cracking force \(N_{cr}\), which should be calculated based on the equivalent area for stage I concrete, see Equation 3-12 for short term loading and Equation 3-13 long term loading.

\[N_{cr} = f_{ct} (A_c + (\alpha_e - 1)A_s) \]  
\[(3-12)\]

\[N_{cr} = 0.6f_{ct} (A_c + (\alpha_{e,ef} - 1)A_s) \text{ where } \alpha_{e,ef} = \frac{E_s}{E_c}(1 + \varphi) \]  
\[(3-13)\]

If \(N \geq N_{cr}\) a new crack will appear, and thus the number of cracks is increased to \(n_{cr} = 2\). This procedure is repeated until a sufficient number of cracks is obtained in order to have no further cracks, i.e. \(N < N_{cr}\). The finally obtained steel stress is inserted in to Equation 3-9 in order to calculate the mean crack width \(w_m\).

With the mean crack width as a basis, the characteristic crack width \(w_k\) is determined and compared to a limit value \(w_{lim}\) in order to control that the crack satisfy the condition \(w_k \leq w_{lim}\). The relation between the mean crack width and the characteristic crack width is in (Engström, 2011) denoted as \(w_k = 1.3 \cdot w_m\) for restraint loading and \(w_k = 1.7 \cdot w_m\) for external loading.
Chapter 3 – Design Procedures

Provided in Appendix A is a Matlab code which may be use in order to perform the iterative procedure stated in this chapter.

3.3. The American Concrete Institute

The American Concrete Institute (ACI) provides guides and standard practises which are intended as guidance in the design of concrete structures. (ACI, 1995) considers the concept of crack control in restrained concrete members, subjected to volume changes. Note that equations given in this chapter are based on the United States customary units.

3.3.1. ACI - Restraint degree

The degree of external restraint can according to (ACI, 1995) be approximated by Equation 3-14 and Equation 3-15 along the midsection of a concrete member, by assuming full restraint along the base.

\[
R = \begin{cases} 
\left( \frac{L}{H} - 2 \right)^{h/H} & \text{if } \frac{L}{H} \geq 2.5 \\
\left( \frac{L}{H} - 1 \right)^{h/H} & \text{if } \frac{L}{H} < 2.5 
\end{cases} 
\]

As a result, the degree of restraint can be plotted as a function of the height \( h \) along the member, for different \( L/H \)-ratios, which is shown in Figure 3-6.

![Figure 3-6 Restraint degree as a function of height for various L/H-ratios (ACI, 1995). The degree of restraint is here denoted as \( K_s \).](image)

\( \text{Figure 3-6 Restraint degree as a function of height for various L/H-ratios (ACI, 1995). The degree of restraint is here denoted as } K_s. \)
As mentioned in chapter 2.2.1, the degree of restraint is depends of the geometry and the stiffness of the members. To take this in to account, (ACI, 1995) allows a reduction of the restraint degree according to the relationship stated in Equation 3-16.

\[
R_{\text{red}} = \frac{1}{1 + \frac{2k_c}{E_c A_f E_f}} \cdot R \quad (3-16)
\]

### 3.3.2. ACI - Minimum reinforcement

According to (ACI, 1995), if the expression stated in Equation 3-17 is fulfilled, only minimum reinforcement according to Equation 3-18 and Equation 3-19 is required. It is further stated that the bar size and bar spacing should be no less than Ø20 s300.

\[
K_{RT} C_T E = \frac{f_t}{f_c} < \frac{f_t}{E_c} \quad (3-17)
\]

\[
\rho(\text{min}) = \frac{f_t}{f_s} \quad (3-18)
\]

\[
A_{s, \text{min}} = \frac{f_t}{f_s} \cdot A \quad (3-19)
\]

### 3.3.3. ACI - Crack width calculations

According to (ACI, 1995), the crack width can be determined through the stress in the reinforcement bar after the first crack has occurred in accordance with Equation 3-20. The equation is in turn dependent on the crack width itself, Equation 3-21, hence the calculation procedure requires either a limit steel stress or a limit crack width. It is denoted in (ACI, 1995) that it is common to assume a crack width limit of 0,009 in (0,23 mm) for reinforcement in tension.

The effective height for thick members according to (ACI, 1995) is illustrated in Figure 3-7. Compared to the effective height that has been previously discussed, this is 20 % smaller as (ACI, 1995) use 2(\(c + 0,5\phi\)), see Equation 3-22, while EC2 and Chalmers use 2,5(\(c + 0,5\phi\)), see Figure 2-12.

\[
w = 0.1 \cdot f_s \cdot \sqrt[3]{d_c A} \cdot 10^{-3} \quad (3-20)
\]

\[
f_s = \frac{w \cdot 10^3}{0.076 \sqrt[3]{d_c A}} \quad (3-21)
\]
\[ A = 2 \cdot d_c \cdot \text{spacing} \tag{3-22} \]

The average crack spacing is determined through the obtained or limiting crack width according to Equation 3-23. The expression was developed by measuring the mean value between the crack width and the number of cracks as the stress in the reinforcement varied in-between 207 – 276 MPa (ACI, 1995).

\[
L' = \frac{w}{18 \cdot \left( R_C T_E - \frac{f'_c}{E_c} \right)} \tag{3-23}
\]

(ACI, 1995) proposes a design per zone/height in order to optimize the distribution of reinforcement over the element height, and thus reduce the costs. This is illustrated in Figure 3-8, where the concept becomes clear in the sense that a larger reinforcement content is required in the bottom.

\[ \text{Figure 3-7 Effective area according to (ACI, 1995)} \]

\[ \text{Figure 3-8 Distribution of reinforcement (ACI, 1995)} \]
In order to determine the required amount of reinforcement in each face of the wall, Equation 3.24 is used to calculate the area of one bar surrounding by its effective area, which is distributed over the designing height.

\[
A_b = 0.4 \cdot \frac{f_t' bh}{f_e N_h} (1 - \frac{L'}{2h})
\]

(3.24)

### 3.4. Methods proposed in reports

#### 3.4.1. Kheder – Experimental study

An extensive experimental study has been performed by (Kheder, et al., 1994) in order to evaluate a proper design procedures for base-restrained concrete walls. Equation 3.25 was developed in order to estimate the minimum crack spacing based on the bonding between the concrete and the reinforcement as well as the relationship between crack spacing and member height.

\[
s_{\text{min}} = \frac{k \cdot d \cdot H}{\rho \cdot H + k \cdot d}
\]

(3.25)

Where

- \(k = \frac{f_{ct}}{4f_b}\)
- \(d\) is the bar diameter

(Kheder, et al., 1994) states that the crack spacing should be seen as an interval of \(s_{\text{min}} \leq s \leq 2 \cdot s_{\text{min}}\) as the real crack spacing will be somewhat in-between. It is also stated that the crack spacing range between 1-2 times the wall height for unreinforced walls.

Based on the crack spacing, the crack width may further be calculated according to Equation 3.26. The expression is based on the restrain degree that is obtained before \((R_b)\) and after \((R_a)\) the first crack have occurred in order to capture the most accurate response. Provided in (Kheder, 1997) are design diagrams which can be used in order to estimate \(R_b\) and \(R_a\).

\[
w_{\text{max}} = 2 \cdot s_{\text{min}} \left[ C_1 (R_b - C_2 R_a) \varepsilon_{\text{free}} - \frac{\varepsilon_{\text{ult}}}{2} \right]
\]

(3.26)

Where

- \(C_1\) is a factor of 0.6 taking creep in to account
- \(C_2\) is an empirical value of 0.8
3.4.2. ICE – Revised method

A revised method of the (EC2-3, 2006) procedure is proposed by the Intuition of Civil engineers (ICE) through a research report provided by (Bamforth, et al., 2010). With respect to restrained concrete members, they are surprised that (EC2-3, 2006) does not consider any load transferred through the reinforcement after cracking, despite the fact that this is the only thing considered when estimating the minimum reinforcement. They also point out that (EC2-3, 2006) suggests a linear relationship between the degree of restraint and the crack width, such that increased restraint degree results in an increase in crack width.

According to (Bamforth, et al., 2010), the magnitude of a single crack cannot reach its full potential as the restraining member prevents the crack from opening. Instead, a number of smaller cracks will occur, resulting in the restraining member acting somewhat like reinforcement, see Figure 3-9. Finally, they suggest that the restraining member will absorb some of the force, reducing the stress in the reinforcement.

As a result, (Bamforth, et al., 2010) suggests a two-stage cracking procedure. The total crack width is the sum of an instant crack as the load is transferred from the concrete to the reinforcement, Equation 3-27, and a further opening of the crack as the concrete continues contraction relative to the reinforcement, Equation 3-28.

\[
\varepsilon_{sm} - \varepsilon_{cm} = \frac{0,5\varepsilon_{ctu}(1 - R)B}{1 - SR\left\{1 - 0,5\left(B + \frac{1}{1 - R}\right)\right\}} \quad (3-27)
\]

Where

- \(B = \frac{k \cdot k_c}{a_c \cdot \rho_{ef}} + 1\)
- \(1 \leq k_L \leq 2\)

\[
w_{k2} = S_{r,max}(1 - 0,5R)K_1 \left(\varepsilon_{free} - \frac{\varepsilon_{ctu}}{R \cdot K_1}\right) \quad (3-28)
\]

Where

- \(K_1\) is a factor of 0.65 taking creep in to account
It is further stated in (Bamforth, et al., 2010) that the restraint degree given as tabular values in (EC2-3, 2006) includes a reduction with respect to creep. In order to account for short term restraining actions, a worst case restraint factor of \( R = \frac{0.5}{K_1} = 0.77 \) is thus suggested.

### 3.4.3. ELU – Control of cracking in the transverse direction

The report is part of a larger research project conducted by the Swedish road administration Trafikverket, for which the aim is to investigate appropriate finite element recommendations. The investigation is performed by (Zangeneh, et al., 2013) and the aims to establish an efficient method for determining the crack width and reinforcement content in the transversal direction of restrained concrete members as these are subjected to temperature- and shrinkage loading. Only the effect of short term temperature loading was actually studied in the report.

Based on numerical analyses (Zangeneh, et al., 2013) concluded that the procedure recommended in (EC2-3, 2006) is suitable for crack control in the transverse direction of members restrained along one side. It was shown than as the \( L/H - ratio \) is increased within a reasonable range, the maximum crack width obtained in the numerical analyses become larger, but do not exceed the crack width estimated through (EC2-3, 2006), see Figure 3-10. It was also shown that the (EC2-3, 2006) procedure in 90 % of the cases yield a result on the safe side compared to the experimental investigation performed by (Kheder, 1997), see Figure 3-11.

![Figure 3-10](image-url)  
*Figure 3-10* Comparison between the maximum crack widths obtained in numerical analyses and the procedure stated in EC2-3 for various L/H-ratios (Zangeneh, et al., 2013)
As a result, a step by step solution is suggested by (Zangeneh, et al., 2013) as stated below:

- Determine the load case. A uniform short term temperature load $\Delta T = 15 \, ^\circ C$ should be chosen in-between structural parts in accordance with (EC1-1-5, 2005) and an equivalent temperature due to shrinkage may be estimated through Equation 3-29.

\[
\Delta T = \frac{\varepsilon_{cs}}{\alpha_T (1 + \varphi)} \quad (3-29)
\]

- Determine the minimum reinforcement content according to Equation 3-30 where the tensile strength $f_{ctm}$ should be greater than 2.9 MPa for bridges.

\[
\rho_{s.min} = \frac{k \cdot k_c \cdot f_{ctm}}{f_y} \geq 0.2\% \quad (3-30)
\]

- Determine the restraint degree $R$ in accordance with the tabular values given in (EC2-3, 2006).

- Determine the maximum crack width according to Equation 3-31 and compare to a limit value.

\[
w_k = S_{r.max} \cdot R(\alpha_T \cdot \Delta T) \quad (3-31)
\]

- The total crack width due to long term shrinkage and short term temperature may further be estimated in the means of Equation 3-32.

\[
w_{TS} = w_T + w_s \quad (3-32)
\]
4. Numerical analyses

In this report, analyses are conducted through non-linear finite element (NLFE) analyses. This chapter contain a short description of NLFE-modelling in general and specific aspect concerning modelling of concrete cracking. The chapter also contain a description of the general setup of the numerical model used in this project.

The numerical software BrigAde/Plus 6.1 is initially used. In the later phase of the thesis, attempts are made to simulate the effect of creep. At this point, the numerical software is switched to COMSOL Multiphysics 5.2a.

4.1. Introduction to non-linear modelling

In this chapter, a brief description of non-linear finite element analyses is presented as it is assumed that the reader has basic knowledge in the concept of finite elements. The chapter also contain a brief description of the non-linear material models used in the analyses.

4.1.1. Accuracy of model

As in any finite element analysis, the choice of elements and mesh size will affect the accuracy of the results, hence a convergence study should always be conducted. Some general rules of thumb are presented by (Malm [A], 2016) which may be considered in order to increase the accuracy:

- Increased number of elements result in higher accuracy
- Higher order of elements result in higher accuracy
- Avoid disorientated elements
- Strive to have elements which H/L-ratio is close to 1

In the case of non-linear analysis of concrete cracking, (Malm [A], 2016) states that the required element size is very much smaller than for a corresponding linear analysis and that it is generally better to use a larger number of low order elements compared to a fewer number of high order elements.
4.1.2. Convergence issues

A non-linear finite element analysis is an iterative procedure as the deformation is not proportional to the load (Malm [A], 2016). For this reason, the load must be divided into small steps (increments) such that the structure is loaded one small step at a time. For each loading step, the corresponding response is iterated based on the non-linear material definitions.

Illustrated in Figure 4-1 is an example of a non-linear load-deflection curve (left) and a corresponding iteration during one load increment (right). As the structure is loaded with a small load increment, the response is determined through the tangent stiffness $K_0$ from the previous load increment. The displacement correction $c_a$ is extrapolated in order to update the configurations to $u_a$ (Malm [A], 2016). Further, the internal force $I_a$ is determined and the residual force is calculated as $R_a = F - I_a$.

![Figure 4-1 Non-linear load-deflection curve (left) and a corresponding iteration during one load increment (right). Reproduction from (Malm [A], 2016)](image)

If the residual force equals zero, the structure is in equilibrium, i.e. external forces equals the internal forces. However, this can never be the case in a non-linear analysis as there always will be some residual force left (Malm [A], 2016). Hence, the residual force is compared to a tolerance value for which the structure is considered being in equilibrium if the residual force is smaller.

If the residual force is larger than the tolerance value, a new attempt is made. In such a case, the stiffness $K_a$ is calculated based on $u_a$, which together with the residual force $R_a$ determines a new displacement correction $c_b$ and residual force $R_b$ (Malm [A], 2016). As the residual force is accepted, the displacement correction is compared to another tolerance value. Both convergence criteria must be accepted before the analysis proceeds to the next load increment (Malm [A], 2016).
This procedure may have to be repeated several times before the structure is considered being in equilibrium for a particular load increment. If the load increment is to large or rapid stiffness changes occur, e.g. an instant crack, the model may not find convergence at all. This is known as a convergence problem and is a common issue within non-linear finite element modelling. In order to avoid this, several methods are suggested by (Malm [B], 2016).

- Choosing suitable elements and mesh size.
- Defining mesh and boundary conditions in such a way that numerical singularities are minimized.
- Changing the level of tolerance for equilibrium. In this case, (Malm [B], 2016) states that the tolerance should not be increased as this may give rise to secondary errors. Instead, it should be decreased while the number of iterations performed before equilibrium is checked is increased.
- Decrease the load increments in order to not approach a change in stiffness to fast.
- Introduce stabilizing damping in the structure such that the energy tolerance is not exceeded as energy is released due to cracking.

4.1.3. Plasticity models

The cracking behaviour and structural response is dependent on what material model is used in the numerical analysis. In this report, three material models are presented, the smeared crack model, the damage plasticity model and the isotropic damage model.

Smeared cracking and damage plasticity are plasticity models provided in the numerical software BrigAde/Plus which is initially used. As the COMSOL Multiphysics analyses are conducted, the cracking behaviour is based on isotropic damage. The post-cracking behaviour of the plasticity models is defined by the crack opening curve and fracture energy described in chapter 2.3.1.

**Smeared cracking**

In the smeared crack material model, the crack appears in the element integration points, and the effect is distributed over the whole element, see Figure 4-2. In this sense, the strain in the elements consists in two parts, an elastic part for the un-cracked concrete and a non-linear part for the cracked concrete, represented by a reduction in the elastic stiffness (Malm [A], 2016) and (Sorenson, 2011). The failure envelop is defined by the biaxial relationship in Figure 2-3.
If the element size used in the model is larger than the crack spacing, the strain in one element may represent two cracks. In order to obtain a crack spacing, it is thus essential to choose a small enough mesh size such that un-cracked element in between two cracks is obtained (Malm [B], 2016). The material model is suitable for monotonic straining with minor unloading as it assumes that there is no permanent strains associated with the cracking (Sorenson, 2011), see Figure 4-3.

**Damage plasticity**

The damage plasticity is a modification of the Mohr-Coulomb and Drucker-Prager yield criteria in the sense that the yield criteria is dependent of a cohesion factor (J, et al., u.d.). As the yield limit of the material is exceeded, the cohesion factor holds back the element and allows a permanent plastic strain. The material is thus modified with a damage such that the characteristic strength is reduced when unloaded, see Figure 4-4. As the model is loaded again, the elastic region will thus be shorter due to the damage.
This behaviour is defined by a damage parameter which can be describes as a reduction of concrete area as the yield limit is exceeded, see Figure 4-5 (Malm [B], 2016). As long as the yield limit is not exceeded the damage parameter will be zero and the concrete will keep its original behaviour.

Isotropic damage models

The isotropic damage model consider damage as a degradation of the stiffness of the material (Oliver, et al., 1990) and is thus very similar to the damage plasticity model used in BrigAde/Plus. The material model used in the COMSOL Multiphysics analyse is based on the Rankine yield criteria (Gasch, 2016) which compared to failure envelopes used in smeared cracking and damage plasticity is fairly simple.
4.1.4. Creep models

In COMSOL Multiphysics, the creep is simulated with a material model which is based on Microprestress-Solidification theory (MPS). An illustration of the concept is displayed in Figure 4-6.

The model have an initial non-aging elastic part which represent the instantons elastic deformation $\varepsilon_a$, (Havlasek, 2014). This is further connected to a chain of solidifying kelvin elements representing the short-term creep $\varepsilon_v$. The kelvin elements consist of an elastic part, represented by a spring, and a linear-viscose dash-pot part representing the delayed deformation which arise due to creep. In the COMSOL material model, a number of 1–10 kelvin elements may be chosen in the chain.

The kelvin elements represents the short term creep such that the viscose dash-dot elements delays the strain and further gradually transfers it to the spring element until the it is fully stretched and maximal strain is reached (Havlasek, 2014). The same concept is applied for the long term creep $\varepsilon_f$, but in this case the dash-dot element is assigned an aging parameter.

The total strain $\varepsilon$ in the creep model is further obtained as the sum of the above stated strains, as well as the strain due to shrinkage $\varepsilon_{sh}$, temperature $\varepsilon_T$, and cracking $\varepsilon_{Cr}$ as illustrated in Figure 4-6.

In COMSOL Multiphysics, the behaviour of the creep model is defined by four parameters denoted $q_1$, $q_2$, $q_3$ and $q_4$ [Pa$^{-1}$]. Information about how to estimate these can be found in (Bazant, 2015).
4.2. General setup of the numerical model

The general geometry of the numerical model is illustrated in Figure 4-7. The concrete member is modelled with 2D plane stress elements while the reinforcement is modelled with truss elements. In cases where two parallel reinforcement bars are to be defined in the direction perpendicular to the plane, only one bar is modelled with a section area which equals the total area of two. The reinforcement is coupled with the concrete as an embedded region in BrigAde/Plus and through a general extrusion in COMSOL Multiphysics.

There are three boundary conditions applied to the model. The base is fully fixed in the horizontal direction, simulating the longitudinal restraint. The top is fixed in the vertical direction, allowing zero movements. By applying the restraining conditions in such a way, numerical singularities are reduced. In order to reduce the size of the model, half of the concrete member is modelled and a symmetry condition is applied on the left edge. Specific material- and input parameters can be found in Appendix B and a convergence study for the model can be found in Appendix C.

4.2.1. Interpretation of results

As the results are evaluated, it is the crack widths, crack spacing and steel stress in the horizontal bars which is of interest. How these parameters are extracted from the results is stated below:

- The crack spacing is evaluated based the prior cracks. Minor cracks and inclined cracks at the end of the member is thus not included in the results. The cracked concrete member from the analysis is inserted in Autodesk AutoCAD Architecture 2016. Based on the known geometry, the member is scaled and the crack spacing is measured.

- The maximum crack width is estimated based on the maximum principal strain obtained in the analysis. The crack is assumed to be smeared out over a length equal to the element
length \( l_e \), consisting in one elastic part and one plastic part. Hence, the crack width is calculated according to Equation 4-1.

- The steel stress in the horizontal bars is simply extracted from the numerical model.

\[
w_k = \left( \varepsilon_{\text{tot}} - \frac{f_{\text{ctm}}}{E_{\text{cm}}} \right) \cdot l_e
\]

\[ (4-1) \]
5. Case studies

In this chapter, a series of case studies are conducted. Numerical results are compared with the various hand calculations methods presented in the report. Every case study includes a discussion and remarks which are considered to be of importance.

5.1. Initial hand calculations

An initial comparison between the hand calculations procedures discussed in chapter 3 is performed. The purpose is to identify the difference in results as the various methods are adopted. The methods discussed in chapter 3 all work in different ways and yield different results under the same assumptions.

5.1.1. Description of problem

The member which is investigated in this chapter is casted in C30/37 concrete, have a sectional area of 500x3000 mm², see Figure 5-1, and a concrete cover of 75 mm. The member is assumed to be fully restrained along its base and specific data depending on the type of analysis is stated in the corresponding subchapter.

![Figure 5-1 Geometry of member](image)

5.1.2. Minimum reinforcement

(EC2-2, 2005), the Chalmers method and (ACI, 1995) all suggest the same concept; the minimum amount of required reinforcement may be determined through equilibrium between the stresses in the concrete just before cracking and yielding of the reinforcement, or lower steel stress if required. How the stress distribution in the concrete is considered differs in-between the three methods, resulting in a different amount of minimum reinforcement for all three cases, see Figure 5-2.
5.1.3. Non-direct calculation procedures

Both EC2-3 and the Chalmers method provide simplified procedures to limit the crack widths with respect to certain reinforcement diameters and steel stress.

Consider the member being reinforced with $\phi_{12}$ bars, and the crack width is to be limited to 0.15 mm. With the non-direct procedure recommended in (EC2-3, 2006), an allowable steel stress of 200 MPa is suggested, Figure 5-3 (left).

The diagram provided by (Engström, 2011), Figure 5-3 (right), displays the mean crack width for a specific concrete quality, in this case C30/37. The characteristic crack width must be reduced according the relationship stated in the Chalmers method in order to use the diagram, such that $w_m = \frac{0.15}{1.3} = 0.115$ mm. For such a case, the steel stress should be limited to about 140 MPa in the Chalmers method, which is less than what (EC2-3, 2006) suggests.
In order to investigate how conservative the non-direct procedure is in relation to more detailed hand calculations according to (EC2-3, 2006), a comparison between these is further conducted.

The allowable steel stress of 200 MPa is used in order to calculate the minimum reinforcement, see Equation 3-1. The crack spacing and corresponding crack width is further calculated as a function of the temperature drop, see Equation 3-2 and Equation 3-6. For the comparison, two cases are tested, one with normal assumptions and one with conservative assumptions for various coefficients of (EC2-2, 2005), see Table 5-1.

The results, which are displayed in Figure 5-4, suggests that the non-direct calculation procedure in this case provide a conservative result for temperatures which are below 35 °C for the normal assumptions and 22 °C for the conservative assumptions. As the detailed procedure stated in (EC2-3, 2006) assumes a linear relationship between the crack width and the thermal strain, the limit crack width is eventually exceeded. It can be seen that as conservative assumptions are made, the tangent to the linear relationship increase, and the limit is faster reached.

**Table 5-1 Input values for Eurocode 2 coefficients**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Normal assumptions</th>
<th>Description</th>
<th>Conservative assumptions</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>0,8</td>
<td>For high bond bars</td>
<td>1,6</td>
<td>For smooth bars</td>
</tr>
<tr>
<td>$k_2$</td>
<td>1,0</td>
<td>For pure tension</td>
<td>1,0</td>
<td>For pure tension</td>
</tr>
<tr>
<td>$k_3c$</td>
<td>7 $\phi$</td>
<td>National annex</td>
<td>3,4 \cdot c</td>
<td>EC2-2</td>
</tr>
<tr>
<td>$k_4$</td>
<td>0,425</td>
<td>National annex/EC2-2</td>
<td>0,425</td>
<td>National annex/EC2-2</td>
</tr>
<tr>
<td>$k$</td>
<td>0,65</td>
<td>For webs with h &gt; 800</td>
<td>1,0</td>
<td>For webs with h &lt; 800 mm</td>
</tr>
<tr>
<td>$k_c$</td>
<td>1,0</td>
<td>For pure tension</td>
<td>1,0</td>
<td>For pure tension</td>
</tr>
</tbody>
</table>

**Figure 5-4 Crack width as a function of temperature drop, comparison between EC2-3 detailed- and simplified procedure**
5.1.4. Detailed hand calculations

Displayed Figure 5-5 is the resulting crack width as the more detailed hand calculation procedures are used. The figures show the obtained crack width as a function of $L/H$ ratios for (EC2-3, 2006), the Chalmers method, (ACI, 1995) and the method proposed by (Kheder, et al., 1994). The corresponding steel stress is displayed in Figure 5-6.

The temperature drop is chosen as $\Delta T = -15 \, ^\circ C$ and the effective reinforcement ratio is set to $\rho_{ef} = 0.6 \%$, which corresponds to $\phi_{12} s/90$ in each face of the wall. This is also the minimum amount of reinforcement that is required according to the Chalmers method.

![Figure 5-5 Crack widths as a function of L/H-ratio](image1)

![Figure 5-6 Steel stress as a function of L/H-ratio](image2)
Calculations according to (EC2-3, 2006) are not influenced by the length of the element, compared to the other methods. Neither does it consider the number of cracks, but instead have a constant crack spacing based on stabilized cracking, and thus a constant crack width.

For the Chalmers method, which is based on single cracks, it can be seen that the second crack occurs at $L/H = 3.2$. At the point of cracking, the yield limit is reached as the minimum reinforcement according to the Chalmers method is used.

The methods provided in (ACI, 1995) takes the variation of the restraint degree over the height of the member into account. Displayed in Figure 5-5 is the maximum crack width, and thus the position over the base differs for the different $L/H$ ratios. No cracks are obtained, or the crack spacing exceeded the physically possible length, until $L/H = 3$. As cracks are obtained, the curve has approximately the same curvature as in between new cracks in the Chalmers method, however with considerably lower crack widths and steel stress.

The lowest crack width is obtained when performing calculations according to the method proposed by (Kheder, et al., 1994) up until $L/H = 6$, after which (EC2-3, 2006) yields a slightly lower crack width. The method is based on design diagrams from which the degree of restraint before and after cracking occur is approximated. This may give rise to some uncertainty in the results. Further, the bond-slip relationship in Equation 3-25 is assumed to be the average value of the ones stated in the report.

**Analysis with modified crack spacing**

As mentioned, (EC2-3, 2006) suggests constant crack spacing, and thus a constant crack width, compared to the Chalmers method in which the crack spacing will change with every new crack that occur. With this as a basis, the two methods are analysed under the assumption that they have the same crack spacing, Figure 5-7 (dashed curves). This is obtained by modifying either the EC2-3 crack spacing such that it equals the Chalmers method (Equation 5-1), or by modifying the number of cracks in the Chalmers method such that the crack spacing is approximately the same as in EC2-3 (Equation 5-2).

\[
S_{r,mod} = \frac{L}{n_{cr} + 1} \quad (5-1)
\]

\[
n_{cr,mod} = \text{floor}\left(\frac{L}{S_{r,\text{max}}} - 1\right) \quad (5-2)
\]
As the crack spacing obtained in (EC2-2, 2005) is adopted in the Chalmers method, the resulting crack widths are quite close in magnitude. The crack width of the Chalmers method is converging towards a final value, slightly higher than EC2-3.

As the crack spacing obtained in the Chalmers method is adopted into the (EC2-3, 2006) calculation procedure, the resulting crack width quite quickly exceed the values obtained in the Chalmers method.

By producing modified crack spacing's in this way, no consideration is taken to the actual cracking procedure of the concrete member. It does however point out the importance and the influence the assumed crack spacing has on the resulting crack width of these methods.

**Comparison with revised approach proposed by ICE**

The refined method proposed by ICE is based on the assumption that highly restraining member will absorb forces and distribute the cracks as many small cracks. For the fully restrained member described above, the resulting crack width is thus of very small magnitude. For this reason, a separate case is illustrated in Figure 5-8 for which the crack width is plotted as a function of the restraint degree.

It is not clearly stated how the debonding length \( S \) in Equation 3-27 should be determined for the refined method. Based on results that are given in (Bamforth, et al., 2010), the debonding length is estimated to 250 mm in the case of a similar concrete member with Ø20 s250 bars.
Further, their results are based on in-situ material properties which are lower than the 28-day martial properties for $C30/37$. Hence, two cases are displayed in Figure 5-8, one for the concrete properties used in (Bamforth, et al., 2010) and one for the full 28-day concrete properties, under the assumption of a debonding length of 250 mm. For the second mentioned case, the effect of creep is neglected as the short term response of thermal strains is analysed, and thus $K_1 = 1$ in Equation 3-28.

It can be seen that the revised method suggests that the maximum crack width is obtained as the restraint degree $R \approx 0.3$. The method implies that this is the degree of restraint at which there is an optimum in between the amount of force absorbed by the restraining member and the distribution of cracks in relation to crack widths.

However, no cracking will occur if the condition stated in Equation 5-3 is fulfilled (Bamforth, et al., 2010). Hence, for $R < 0.60$ and $R < 0.79$, no cracks will occur for the case of 28-day and reduced concrete properties respectively. The maximum crack width would thus not be at the peak, but rather 0.19 mm and 0.09 mm for the two cases.

$$R \cdot \varepsilon_{\text{free}} \leq \varepsilon_{\text{ctu}} \quad (5-3)$$

Unlike the other methods discussed, the revised method according to ICE suggests a decrease in crack width as the degree of restraint is increased for $R > 0.3$. Further, the method implies that for a high restraint degrees, (EC2-3, 2006) is very much on the safe side.
5.1.5. Crack spacing

The crack spacing for restrained concrete members may be obtained through the calculations procedures provided by (EC2-2, 2005), (ACI, 1995) and the experimental study performed by (Kheder, et al., 1994). For the concrete member described above, the corresponding crack spacing for the different methods is illustrated in Figure 5-9.

Both (EC2-2, 2005) and (Kheder, et al., 1994) suggests constant crack spacing for various $L/H$ ratios, while (ACI, 1995) suggests a decrease in crack spacing as the $L/H$ ratio is increased, reaching towards a limit value.

Assuming that the degree of restraint is larger over the height for high $L/H$ ratios, as described in chapter 2.1.4, the decrease in crack spacing according to (ACI, 1995) is consistent with the revised method proposed by (Bamforth, et al., 2010) in the sense that a larger degree of restraint results in a finer distribution of cracks, and thus smaller crack spacing. (ACI, 1995) also takes the distribution of restraint degree over the height into account to find the maximum crack spacing, for which the results imply that the crack spacing is larger in the top of the element, hence the cracks are inclined.
5.1.6. Remarks

- All proposed methods yield different results under the same assumptions.

- There is an agreement in-between codes in how to determine the minimum required reinforcement. How the stress distribution in the concrete is considered differs in-between the methods, resulting in different reinforcement content.

- The detailed procedure stated in EC2-3 assumes a linear relationship between the crack width and stress-independent strain. A detailed hand calculation may thus exceed a corresponding analysis with the non-direct procedure as this is only valid within a certain range of strain.

- Depending on the choice of design coefficients and concrete quality, the tangent to the linear relationship changes. Hence, for high concrete qualities and conservative assumptions of parameters, the range within which the non-direct procedure is valid decrease.

- EC2-3 is the only method which does not consider the member length. Instead stabilized cracking is assumed, hence a constant crack width is obtained for various $L/H$ – ratios.

- The Chalmers method result in considerably larger crack widths compared to all other methods.

- The Chalmers method yield results similar to EC2-3 as the crack spacing is chosen to be the same.

- The revised method proposed by ICE is the only method which suggest a decrease in crack widths for an increase in restraint degree. This is because the restraining member is assumed to absorb forces and act somewhat like reinforcement such that a finer distribution of cracks is obtained.

- The crack spacing obtained in ACI is shows agreement with the concept described in the revised approach suggested by ICE in the sense that a higher restraint degree result in a finer crack distribution. All other methods suggest a constant crack spacing for various $L/H$ – ratios.
5.2. Numerical comparison with initial hand calculations

In this chapter, a comparison with the results obtained with the hand calculations in chapter 5.1 have been made with results obtained in numerical analyses.

5.2.1. Problem description

The problem consider the concrete member described in chapter 5.1. Only the results as \( L/H = 8 \) are compared as the (EC2-3, 2006) procedure have been questioned for high \( L/H - \text{ratios} \).

The length is 24 m, the height is 3 m and the thickness is 0,5 m, see Figure 5-10. The concrete quality is \( C30/37 \) and the steel quality is set to \( B500B \) with an elastic modulus of 205 GPa. The reinforcement is chosen as \( \phi12 \) s90 in both the horizontal and vertical direction in each face of the wall, which correspond to the minimum reinforcement according to the Chalmers method. The member is fully restrained and subjected to a temperature drop of \( \Delta T = -15^\circ C \).

![Figure 5-10 Geometry of member](image)

5.2.2. Method

The analysis is performed in BrigAde/Plus 6.1. The results obtained in chapter 5.1 is compared with the corresponding results from the non-linear numerical analysis. Both the smeared crack material model and the damage plasticity material model are compared. The results which are to be analysed are:

- Maximum crack width
- Crack spacing
- Steel stress in horizontal reinforcement bars

Further, the steel stress is checked with the non-direct calculation procedures that are stated in (EC2-3, 2006), see chapter 3.1.3.
5.2.3. Setup of numerical model

The numerical model is modelled as described in chapter 4.2 and the mesh size is chosen in accordance with the convergence study in Appendix C. In order to avoid convergence issues, some damping is applied to the model. This is chosen as the default damping provided in BrigAde/Plus.

5.2.4. Results

The resulting crack pattern is displayed in Figure 5-11 and Figure 5-12 for the smeared crack and damage plasticity material model respectively. A comparison with the results obtained in the hand calculations is displayed in Figure 5-13 for the crack widths and Figure 5-14 for the crack spacing. It should be noted that the crack spacing for the Chalmers method is estimated based on Equation 5-1.

![Crack pattern obtained with the smeared crack material model](image1)

![Crack pattern obtained with the damage plasticity material model](image2)

![Comparison between crack widths obtained from hand calculation and NLFE-analysis](image3)
5.2.5. Discussion

The number of cracks, crack spacing and the location of cracks is very similar when comparing the two material models smeared crack and damage plasticity. However, in the smeared crack method a larger number of through cracks is obtained. The location at which the maximum crack width is obtained differs. In the smeared crack method, the largest crack width is obtained in the crack which is closest to the symmetry line, i.e. the middle of the concrete element. As for the damage plasticity model, the largest crack width is obtained in the inclined cracks at the sides.

If directly comparing the maximum crack widths, the damage plasticity model give rise to a crack width which is about 2.3 times larger than what is obtained in the smeared crack model. However, in this report it is the primary through cracks that are of interest. If neglecting the inclined cracks in the damage plasticity model, the largest crack width is obtained in the crack closest to the symmetry line, which is consistent which the smeared crack approach. The resulting crack widths are then very close in magnitude as seen in Figure 5-13.

As the results from the numerical analyses are compared with the hand calculations, the FE-analysis result in the smallest crack widths, slightly lower than what is predicted with (EC2-3, 2006). This is consistent with the results obtained in (Zangeneh, et al., 2013).

The Chalmers method give rise to cracks widths which are about 9.6 times larger than what is obtained when performing the numerical analysis. Obtained is that the deformation conditions requires two cracks in order to be fulfilled. The number of cracks obtained in the numerical analysis, taking the symmetry in to account, is approximately 18, i.e. 9 times more than what the Chalmers method predict. This suggest that the Chalmers method underestimates the number of

![Comparison between crack spacing obtained from hand calculation and NLFE-analysis](image_url)
cracks that occur. If further entering 18 crack into the Chalmers method, the characteristic crack width is reduced by 78 %, to 0.213 mm.

As for the crack spacing, (EC2-2, 2005) predict the smallest value. If studying the maximum crack spacing, all other methods as well as the FE-analysis predict a crack spacing of about twice the magnitude. The average crack spacing is closer to (EC2-2, 2005), but still larger. That a larger crack spacing is obtained with all other methods is consistent with the theory the stabilizing cracking stage on which EC2-2 is based on is not reached.

Displayed in Figure 5-15 is the steel stress in a reinforcement bar as a function of the temperature decrease at the point where the maximum crack occurs. Initially, there is a very small compressive stress as the temperature is increased. As the crack appears at around 9 °C, the stress is instantly increased. The damage plasticity model suggest that the stress peaks directly after the crack occurs, after which a small decrease is obtained, while the smeared crack model implies an initial decrease in stress after the crack occurs, after which it increases again.

![Figure 5-15 Steel stress as a function of temperature](image)

The steel stress obtained directly after the point of cracking is about 345 MPa and 412 MPa for the smeared crack and damage plasticity model respectively. Table 5-2 contains the crack widths that are obtained if using these stresses in the non-direct calculation procedure stated in (EC2-3, 2006), see chapter 3.1.3. It should be noted that the crack width is extrapolated as the magnitude of the steel stress is not within the limits of the method, and thus they are approximate.

| Table 5-2 Crack width of non-direct calculations procedures based on steel stress obtained in analysis |
| --- | --- |
| **EC2-3 Non-direct calculations** |  |
| Smeared Crack | 0.3 – 0.4 mm |
| Damage plasticity | 0.3 – 0.4 mm |
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The non-direct calculation procedures should be a quick and alternative way of performing a design on the safe side. The results show that the crack width in this case indeed is larger than what is obtained in the non-linear analysis as well as all hand calculation procedures, part from the Chalmers method. This implies that even though a conservative design approach is used, Chalmers method still suggest a larger crack width.

5.2.6. Remarks

- The two material models give rise to very similar results. As the damage plasticity material model contain more uncertainties in the parameters, the smeared crack approach will be used for future analyses.

- The analysis show good agreement with EC2-3 as it comes to crack widths for the short term temperature drop of $\Delta T = -15$ °C. This is consistent with the results obtained in (Zangeneh, et al., 2013).

- The Chalmers method suggests considerably higher values compared to the numerical results, analytical results and the non-direct calculation procedure stated in EC2-3.

- EC2-3 result in the lowest crack spacing which is consistent with the theory that stabilized cracking is not reached.

- The numerical results imply that the Chalmers method underestimates the number of cracks that will occur and thus over estimates the maximum crack width.

- The non-direct calculation procedures provided in (EC2-3, 2006) suggests a larger crack width than what is obtained for the same steel stress in most other methods, as well as in the numerical analysis. This implies that the method in this case is on the safe side.
5.3. **Deeper analysis of non-direct calculations procedures**

In this chapter, a deeper analysis of the non-direct calculations procedures stated in (EC2-3, 2006) and (Engström, 2011) is conducted. The purpose is to verify that the non-direct calculations procedure which is stated in (EC2-3, 2006) is indeed a method on the safe side and that it can be used as an alternative and quick way of performing a design with respect to crack control.

5.3.1. **Problem description**

In this case study, a member with an $L/H$ ratio = 8 and a restrain degree of $R = 1$ is used. The length is 8 m, the height is 1 m and the thickness is 0,4 m, see Figure 5-16. The concrete quality is set to $C40/50$ and the steel quality is set to $B500B$ with an elastic modulus of 205 GPA. The reinforcement is chosen as $\phi16$ in both the horizontal and vertical direction in each face of the wall, with a concrete cover of $c = 75 \text{ mm}$ on each side.

![Figure 5-16 Geometry of member](image)

5.3.2. **Method**

The numerical analysis is performed in BrigAde/Plus 6.1. The non-direct calculation procedure stated in (EC2-3, 2006) is used as a basis for all other procedures. A crack width of 0,2 mm is chosen as a limit, and the investigation is further performed as stated below:

1. Determine the allowable steel stress according to Figure 3-1 in order to obtain a crack width of $w_k \leq 0,2 \text{ mm}$ for $\phi16 - bars$

2. Based on the allowed steel stress $\sigma_s$, calculate:

   2.1. Characteristic crack width according to the Chalmers non-direct calculation procedure, see chapter 3.2.2

   2.2. Minimum required reinforcement according to the Chalmers method, see chapter 3.2.2

   2.3. Minimum reinforcement according to (EC2-2, 2005), see chapter 3.1.2
2.3.1. With normal assumptions, see Table 5-1

2.3.2. With conservative assumptions, see Table 5-1

2.4. Characteristic crack width according to (EC2-3, 2006), see chapter 3.1.5

2.4.1. With normal assumptions, see Table 5-1

2.4.2. With conservative assumptions, see Table 5-1

3. Perform a non-linear finite element analysis as the member is reinforced according to the results obtained from the:

3.1. Chalmers method

3.2. (EC2-2, 2005)- Normal assumptions

3.3. (EC2-2, 2005)- Conservative assumptions

4. Compare the results

Two analyses are conducted in order to see the difference in results:

- **Analysis 1**
  Short term response for a temperature drop of \( \Delta T = -15^\circ C \).

- **Analysis 2**
  Long term response for a shrinkage strain of \( \varepsilon_{cs} = 2,576 \cdot 10^{-4} \) combined with a short term temperature drop of \( \Delta T = -15^\circ C \)

In the second mentioned case, the effects of creep should be included for the shrinkage strain. A correct representation of the creep behaviour is however of great difficulty to define in BrigAde/Plus, especially as a plastic material model is used. Hence, creep will be taken into account by reducing the shrinkage strain by a factor of \( 1 + \varphi \) as proposed by (Zangeneh, et al., 2013), see Equation 3-29. The final creep factor is estimated as \( \varphi = 2.1 \).

This method will not capture the real effect of creep as both the shrinkage strain and creep factor are time dependent parameters. In fact, the analysis is only performed for a higher strain as the shrinkage is included. It will however imply whether the calculations procedure stated in (EC2-3, 2006) in relation to a corresponding numerical analysis yield a result on the safe side. For comparable reasons, the method proposed by (Zangeneh, et al., 2013) is used also in the hand calculation procedure in order to account for the creep.
5.3.3. Setup of the numerical model

The numerical model is modelled as described in chapter 4.2. Mesh size and element type is chosen in accordance with the convergence study in Appendix C. The smeared crack material model is used in accordance with the results obtained in chapter 5.1.5. In this case, no damping is required in order to avoid convergence issues.

5.3.4. Results

As $\phi 16 - bars$ are used and the crack width is to be limited to 0,2 mm, the non-direct calculation procedure stated in (EC2-3, 2006) suggest a maximum steel stress of 218 MPa, see Figure 5-17.

![Figure 5-17 Non-direct calculation procedure (EC2-3, 2006)](image1)

As the steel stress is adopted into the non-direct calculation procedure stated in the Chalmers method, the obtained mean crack width equals $w_m = 0,25 mm$, see Figure 5-18. The method states that the characteristic crack width is obtained by multiplying the mean crack width with a factor of 1,3. Hence, the characteristic crack width is $w_k = 1,3 \cdot 0,25 = 0,33 mm$.

![Figure 5-18 Crack width as a function of steel stress for C40/50 concrete (Engström, 2011)](image2)
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**Analysis 1 - Short term response**

Provided in Table 5-3 is the reinforcement content which is determined through the various methods in the short term analysis. The table also contain the crack width obtained in the hand calculations, as well as the crack width and steel stress obtained in the numerical analyses.

| Table 5-3 Short term response obtained from EC2, Chalmers and the numerical analysis |
|---------------------------------|---------------------------------|---------------------------------|
| Normal assumptions | Conservative assumptions | Chalmers method |
| \( A_{s,min} \) | 2\( \phi \)16 s100 | 2\( \phi \)16 s65 | 2\( \phi \)16 s95 |
| \( w_k^* \) | 0,07 mm | 0,13 mm | 0,33 mm |
| \( w_k \text{ NLFE} \) | 0,05 mm | 0,04 mm | 0,05 mm |
| \( \sigma_s \text{ NLFE} \) | 203 MPa | 171 MPa | 198 MPa |

*Crack width determined through hand calculations*

The result show that conservative assumptions for (EC2-2, 2005) and (EC2-3, 2006) results in a higher reinforcement ratio as well as a larger crack width when performing a hand calculation. The Chalmers method suggest a crack width which is 2,5 times larger than what is obtained with the most conservative parameters in EC2-3, under the same assumption that the steel stress is to be limited to 218 MPa. It is also larger than the limit value of 0,2 mm which was assumed in the non-direct calculation procedure stated in (EC2-3, 2006).

**Analysis 2 - Long term response**

Provided in Table 5-4 is the reinforcement content which is determined through the various methods in the long term analysis. The table also contain the crack width obtained through the hand calculations, as well as the crack width and steel stress obtained in the numerical analyses.

| Table 5-4 Long term response obtained from EC2, Chalmers and the numerical analysis |
|---------------------------------|---------------------------------|---------------------------------|
| Normal assumption | Conservative assumption | Chalmers method |
| \( A_{s,min} \) | 2\( \phi \)16 s100 | 2\( \phi \)16 s65 | 2\( \phi \)16 s95 |
| \( w_k^* \) | 0,11 mm | 0,20 mm | 0,33 mm |
| \( w_k \text{ NLFE} \) | 0,06 mm | 0,04 mm | 0,05 mm |
| \( \sigma_s \text{ NLFE} \) | 216 MPa | 158 MPa | 199 MPa |

*Crack width determined through hand calculations*

The Chalmers method in this case suggest a crack width which is 1,65 times larger than what is obtained with the most conservative parameters in (EC2-3, 2006), under the same assumption that the steel stress is to be limited to 218 MPa. It is also larger than the limit value of 0,2 mm which was assumed in the non-direct calculation procedure stated in (EC2-3, 2006).
5.3.5. Discussion

Displayed in Figure 5-19 and Figure 5-20 is a graphical illustration of the crack width obtained in the short term analysis and the long term analysis respectively. The non-direct calculation procedure stated in (EC2-3, 2006) should be compatible with any load case. This means that it should still be on the safe side even though more loads than the temperature drop are considered, e.g. shrinkage and external loading.

Figure 5-19 Crack widths obtained in the short term analysis

Figure 5-20 Crack widths obtained in the long term analysis

For the short term analysis, there is a good margin between the most conservative hand calculation according to (EC2-3, 2006) and the 0,2 mm limit used in the non-direct calculation procedure. As the shrinkage strain is included in the long term analysis the limit crack width is reached, leaving no room for external load cases. As for the normal assumption case, there is still a fairly large difference in the crack width as the shrinkage is applied.

Crack widths obtained in the numerical analyses are lower than corresponding hand EC2-3 calculations, which is consistent with the results obtained in chapter 5.2. It can however be seen that the deviation between numerical results and hand calculations become larger as the
shrinkage strain is included in the long term analyses. The procedure stated in (EC2-3, 2006) assumes a linear relationship between the crack width and stress-independent strain. The numerical results however indicate that an increase in strain rather introduce more cracks of similar magnitude than increasing the single crack width, see Figure 5-21. Hence, the deviation in results become larger for higher levels of strains.

For the conservative assumption, the factor which have the largest impact on the crack width is the factor $k_1$, see Table 5-1, as the crack spacing is determined. The most conservative value for this factor requires that smooth bars are assumed, i.e. the bond between the reinforcement and concrete is significantly reduced. As the numerical analysis assumes full interaction between these two components, the results are not comparable with the hand calculations in this case.

The steel stress is used in order to determine the minimum required reinforcement for equilibrium between the concrete just before cracking and the steel stress directly after a crack have appeared. The steel stress from the numerical analyses is fairly close in magnitude to the limiting stress assumed in the non-direct calculation procedure stated in (EC2-3, 2006), especially for the normal assumptions, see Figure 5-22. Hence, the numerical results imply that the calculation procedure for minimum reinforcement according to (EC2-2, 2005) is reasonable as a somewhat expected steel stress is obtained for a given reinforcement ratio, depending on the choice of parameters $k$ and $k_c$ in the hand calculations.

![Figure 5-21 Crack pattern for short term response (left) and long term response (right)](image)

![Figure 5-22 Steel stress obtained in the horizontal reinforcement bars](image)
5.3.6. Remarks

- The non-direct calculation procedure yields a result on the safe side in relation to more detailed hand calculations and numerical results in the case of normal assumptions of EC2-3 coefficients.

- The limiting crack width used to determine the minimum reinforcement is reached and leave no room for further load cases as conservative assumptions are made for the detailed EC2-3 hand calculation. As smooth bars are assume in the hand calculations, a comparison with the numerical results is not applicable.

- The Chalmers method suggest a crack width larger than all other all other methods and the numerical analyses.

- An increase in stress-independent strain rather introduce more cracks of similar magnitude than increasing the maximum crack width. EC2-3 assumes a linear relationship between crack width and strain, resulting in that the deviation between numerical results and hand calculations become larger for higher levels of strain.

- The numerically obtain steel stress correspond fairly well with the steel stress that is assumed in order to determine the required minimum reinforcement, depending on the choice of design parameters.
5.4. Implementation of creep

In this chapter, a deeper analysis of the creep effect is performed. However, as pointed out in chapter 5.3, the influence of creep is of great difficulty to apply in BrigAde/Plus when it comes to varying stresses and plastic material models. Hence, an attempt of capturing this effect is made in another finite element software, COMSOL Multiphysics 5.2a.

For the analysis, a plasticity model and a creep model developed by Tobias Gash, PhD student at the division of Concrete Structures at the Royal Institute of Technology Stockholm, is used. With these material models, it is possible to take the effect of creep in combination with plasticity and varying stresses into account. The creep model is verified with respect to a reference experiment, further described in chapter 5.4.1. A verification of the plasticity model can be found in Appendix D.

In COMSOL Multiphysics, it is possible to simulate moisture transport, which in turn can be used in order to simulate the shrinkage of concrete. However, as 2D plane stress analysis in conducted, moisture transport perpendicular to the plane face is not possible, resulting in that the moisture will only dissipate along the edges.

As a wall is rather thin in relation to its length and height, the majority of the moisture transport would occur through the wall faces. For this reason, the shrinkage strain is applied as a time dependent equivalent temperature. By doing this, internal restraint factors such as differential shrinkage through the section are neglected. These would however been neglected anyway as the analysis is conducted in 2D plane stress.

5.4.1. Verification of creep model

In this chapter, a description of the reference experiment is provided together with results obtained in the creep model verification. For a full description of the experimental setup, see (Nejadi, 2005). Note that material data presented in this chapter will be used in the modelling of the base-restrained concrete member as well.

Description of reference experiment

The creep model is compared to an experiment performed by (Nejadi, 2005) as a part of a PhD thesis at the University of New South Wales. The experiment was performed such that slabs fixed in the ends, see Figure 5-23, were casted and further subjected to shrinkage as the formwork is removed after three days. Various material properties, such as creep, elastic modulus, tensile strength, compressive strength etc. was measured throughout the experiment, which lasted in a total of 122 days.
The geometry of the test slabs is displayed in Figure 5-24. The width of the slab is locally reduced by 75 mm on each side at the middle (section A-A in Figure 5-24) in order to initiate the first crack at this point. The experiment is performed for various reinforcement content, but the comparison with the creep model is done for the case with 3φ12 s185 i.e. slab RS1-a and RS1-b in (Nejadi, 2005). Both slabs are casted with the same batch of concrete (Batch I) and are identical part from small variations in thickness which arise during casting.

The resulting material properties are displayed in Table 5-5 and Table 5-6. The measured shrinkage-, creep-, elastic- and total strain is displayed in Figure 5-25. The resulting test data suggest that the concrete is of a type close to strength class C16/20. It should be noted that the creep factor and its development is based on loading of the concrete at the age of 3 days.
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Table 5-5 Measured material properties (Nejadi, 2005)

<table>
<thead>
<tr>
<th>Material Property</th>
<th>3</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>28</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compressive strength (MPa)</td>
<td>8,17</td>
<td>13,7</td>
<td>20,7</td>
<td>22,9</td>
<td>24,3</td>
</tr>
<tr>
<td>Indirect tensile strength (Brazil) (MPa)</td>
<td>-</td>
<td>1,55</td>
<td>-</td>
<td>-</td>
<td>1,97</td>
</tr>
<tr>
<td>Modulus of elasticity (GPa)</td>
<td>13,24</td>
<td>17,13</td>
<td>21,08</td>
<td>22,15</td>
<td>22,81</td>
</tr>
</tbody>
</table>

Table 5-6 Creep factor and shrinkage strain measured throughout the experiment (Nejadi, 2005)

<table>
<thead>
<tr>
<th>Age (days)</th>
<th>7</th>
<th>14</th>
<th>21</th>
<th>28</th>
<th>36</th>
<th>43</th>
<th>53</th>
<th>77</th>
<th>100</th>
<th>122</th>
</tr>
</thead>
<tbody>
<tr>
<td>ϕ</td>
<td>0,38</td>
<td>0,60</td>
<td>0,68</td>
<td>0,69</td>
<td>0,73</td>
<td>0,84</td>
<td>0,86</td>
<td>0,93</td>
<td>0,97</td>
<td>0,98</td>
</tr>
<tr>
<td>εcs · 10⁻⁶</td>
<td>66</td>
<td>115</td>
<td>154</td>
<td>208</td>
<td>244</td>
<td>313</td>
<td>327</td>
<td>342</td>
<td>421</td>
<td>457</td>
</tr>
</tbody>
</table>

Figure 5-25 Strains measured throughout the experiment (Nejadi, 2005)

Determining creep parameters

In order to isolate the creep behaviour and determine input parameters $q_1$, $q_2$, $q_3$ and $q_4$ mentioned in chapter 4.1.4, a test specimen is initially modelled in COMSOL Multiphysics, see upper right corner of Figure 5-26, from which the creep strain is extracted and compared to the experimental results. The input parameters of the creep model are initially calculated based on the methods given in (Bazant, et al., 2015), and further slightly manipulated in order to match the experimental results.
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As in the experiment, the specimen is loaded with 5 MPa at the age of 3 – 122 days. A comparison with the final result is displayed in Figure 5-26 and show a good overall similarity, with a slightly over estimated elastic strain at the time of loading. Note that the figure displays the summation of the elastic- and creep strain, with the elastic strain $\varepsilon_{el} = 464 \cdot 10^{-6}$ as a reference point.

![Comparison between creep strain obtained in experiment and FE-analysis](image)

**Figure 5-26** Comparison between creep strain obtained in experiment and FE-analysis

**Modelling of the experimental slab**

The input parameters which were determined for the creep model are further used in a second verification, in which the experimental slab is modelled. Here, the creep model is combined with the plasticity model in a coupled analysis in order to verify the cracking. A comparison is also made for a model in which no creep is taken into consideration.

The extra reinforcement close to the supports are assumed to have the same diameter as the main reinforcement, and the location is approximated based on Figure 5-24. The shrinkage strain is applied as an equivalent temperature, following the shrinkage development reported in the experiment.

Due to the geometry of the slab, a uniform element size is not possible to obtain in the concrete. Hence, the concrete element size varies, while for the straight reinforcement bars it remains constant. The 1000x1000x600 supports are not modelled, but a fixed boundary conditions are applied to the slab ends. In the report provided by (Nejadi, 2005) it is stated that there is a displacement in point B and C in Figure 5-24 as the supports shrink. This displacement is applied...
in the fixed ends as a time dependent prescribed displacement, estimated such that the displacement follows the curvature of the shrinkage strain up until the final displacement.

The varying material properties are taken into account throughout the analysis, for which values in between measurements are interpolated and values for which few measurements are made are assumed to be constant after the last measurement, e.g., the elastic modulus and tensile strength. The fracture energy is calculated based on the varying compressive strength according to Equation 2-14. Some damping is applied in order to avoid numerical issues and no consideration is taken to variation in temperature or relative humidity.

**Comparison between experiment and FE-analysis**

According to (EC2-3, 2006), in the case of end-restrained member the crack width may be estimated according to Equation 5-4. For such a case, assuming that creep is implemented in the equation, the resulting crack width would be \( w_k = 0.72 \text{ mm} \). Based on the interpretation of EC2-3, the factor \( k \) is in this case set to \( k = 1 \). In previous analyses, it has been chosen as \( k = 0.65 \) for webs > 800 mm, which in this case would result in \( w_k = 0.47 \text{ mm} \).

\[
 w_k = S_{r,\text{max}} \cdot 0.5 \alpha_e \cdot k_c \cdot k \cdot f_{ct,\text{eff}} \left( 1 + \frac{1}{\alpha_e \cdot \rho} \right) \cdot \frac{1}{E_s} \tag{5-4}
\]

Displayed in Figure 5-27 is the resulting crack pattern and crack widths from the experiment. It can be seen that even though the slabs are identical to a large extent, the crack pattern differs, while the crack widths are within a similar span. It can also be seen that none of the cracks exceed the crack width estimated through the hand calculation procedure stated in (EC2-3, 2006).

![Figure 5-27 Crack pattern and crack width of slab RS1-a (left) and RS1-b (right) (Nejadi, 2005)](image)

Displayed in Figure 5-27 and Figure 5-28 is the corresponding results obtained in two COMSOL Multiphysics analysis as creep is excluded and included respectively. The two analyses have different mesh size for the reinforcement, and it can clearly be seen that it has an effect. If choosing to fine of a mesh for the bars result in unclear crack patterns smeared out over the model, similar to what is described in Appendix C, resulting in a pattern which is hard to interpret. It can be seen
that this effect initiates in Figure 5-29. Provided in Table 5-7 is the average value of the crack widths obtained the experiment and the numerical analyses.

**Figure 5-28** Crack pattern and crack widths obtained in the FE-analysis excluding creep (left) and including creep (right) for reinforcement mesh 0.13 m

**Figure 5-29** Crack pattern and crack widths obtained in the FE-analysis excluding creep (left) and including creep (right) for reinforcement mesh 0.06 m

**Table 5-7** Average crack width (mm), cracks are numbered from the left in corresponding figures

<table>
<thead>
<tr>
<th>Crack</th>
<th>RS1-a</th>
<th>RS1-b</th>
<th>Figure 5-28</th>
<th>Figure 5-29</th>
<th>Figure 5-28</th>
<th>Figure 5-29</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.21</td>
<td>0.10</td>
<td>0.17</td>
<td>0.30</td>
<td>0.15</td>
<td>0.14</td>
</tr>
<tr>
<td>2</td>
<td>0.37</td>
<td>0.34</td>
<td>0.43</td>
<td>0.24</td>
<td>0.19</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
<td>0.21</td>
<td>0.21</td>
<td>0.17</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0.15</td>
<td>0.12</td>
<td>0.23</td>
<td>0.25</td>
<td>0.18</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>0.13</td>
<td>0.23</td>
<td>0.30</td>
<td>0.16</td>
<td>0.07</td>
</tr>
<tr>
<td>6</td>
<td>-</td>
<td>-</td>
<td>0.48</td>
<td>0.74</td>
<td>-</td>
<td>0.21</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
<td>0.21</td>
<td>0.19</td>
<td>-</td>
<td>0.07</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>-</td>
<td>0.49</td>
<td>0.21</td>
<td>-</td>
<td>0.13</td>
</tr>
<tr>
<td>9</td>
<td>-</td>
<td>-</td>
<td>0.39</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Sum</td>
<td>0.86</td>
<td>0.90</td>
<td>2.84</td>
<td>2.40</td>
<td>0.90</td>
<td>0.92</td>
</tr>
<tr>
<td>$n_{cr}$</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>7</td>
</tr>
</tbody>
</table>
Discussion

As the identical experimental slabs yield different results, further experiments would most likely result in even more variations of crack patterns. Further, depending on the choice of reinforcement element size, different crack patterns are obtained in the numerical analysis. With respect to the verification, focus is thus put on the sum of all cracks along the slab length.

It can be seen in Figure 5-30 that as creep is excluded in the numerical analysis, the results become very large compared the experiment. As creep is included, the sum of the crack widths along the slab decrease and the numerical- and experimental results are very close in magnitude. This implies that the model reflects the correct response of creep and yield reasonable crack widths.

5.4.2. Main analysis – Problem description

In this case study, a member with an $L/H$ – ratio = 8 and a degree of restraint of $R = 1$ is used. The length is 8 m, the height is 1 m and the thickness is 0.1 m, see Figure 5-31. The member contain $\phi 8$- bars in one layer in both the vertical and horizontal direction, compared to previous analyses where the member contain one reinforcement layer in each face of the wall.
5.4.3. Method
The reinforcement content is determined by assuming a limiting crack width with the non-direct calculation procedure according to (EC2-3, 2006) and further calculating the minimum reinforcement based on the steel stress. In this case, the limiting crack width is chosen as 0,1 mm and the bar diameter is chosen as φ8. The reason for this is to obtain a sufficiently small spacing between reinforcement bars in order to avoid the numerical issues described in Appendix C. The analysis is performed under normal assumptions of hand calculation parameters only.

Stated in (Zangeneh, et al., 2013) is that a restraint factor $R_{ax}$ should be chosen in accordance with (EC2-3, 2006) and that the shrinkage may be reduced by a factor $(1 + \varphi)$ in order to account for creep. However, (Bamforth, et al., 2010) states that the restraint factor $R_{ax}$ given in (EC2-3, 2006) already includes a reduction with respect to creep. In this sense, the creep effect is taken into account twice in the hand calculation procedure stated in (Zangeneh, et al., 2013).

One of these reductions should be excluded. In order to determine which one is the most suitable, two ways of considering creep in the (EC2-3, 2006) hand calculations are investigated:

a) A restraint factor $R_{ax}$ is chosen in accordance with (EC2-3, 2006) and a full shrinkage strain is applied. (Bamforth, et al., 2010) states that the restraint factors given in (EC2-3, 2006) follows the relationship $R_{ax} = K_1 \cdot R$ where $K_1 = 0,65$. As the member is assumed to be fully restrained, the resulting restraint factor with respect to creep is $R_{ax} = 0,65$ in this case.

b) A restraint factor of $R = 1$ is used and the creep is taken into account by reducing the shrinkage strain by a factor of $(1 + \varphi)$.

Two numerical analyses are conducted and compared to hand calculations:

- **Analysis 1**

The same time dependent material properties, shrinkage strain and creep strain as reported in the experiment is applied to the concrete member. In this sense, the verified parameters are tested in the case of interest, for which conclusions can be drawn about the creep effect on base-restrained members. The wall thickness is chosen to be the same as the slab thickness in the experiment. As the majority of moisture dissipates through the wall faces, it is reasonable to assume that the same shrinkage strain can be applied.
Analysis 2

The material time dependency is calculated in accordance with (EC2-1, 2005). This includes the time-dependent creep, shrinkage strain, elastic modulus and strength properties. The concrete is assumed to have the same 28-day material properties as in the experiment, which is used as a basis. The aim is to see the difference in results as a procedure according to Eurocode is chosen.

Overview of execution procedure

1. Determine the allowable steel stress according to Figure 3-1 in order to obtain a crack width of \( w_k \leq 0,1 \text{ mm} \) for \( \phi 8 - \text{bars} \)

2. Based on the allowable steel stress \( \sigma_s \), calculate minimum reinforcement according to (EC2-2, 2005), see chapter 3.1.2

3. Apply the reinforcement at the base restrained member and determine the characteristic crack width according to:
   3.1. (EC2-3, 2006), see chapter 3.1.5
      3.1.1. As creep is excluded
      3.1.2. As creep is included
         3.1.2.1. By using a restraint factor of \( R_{ax} = 0,65 \)
         3.1.2.2. By applying a shrinkage strain of \( \frac{\epsilon_{cs}}{(1+\phi)} \)

3.2. The Chalmers method, see chapter 3.2.3
   3.2.1. As creep is excluded
   3.2.2. As creep is included

4. Conduct a numerical analysis of the concrete member as
   4.1. Creep is excluded
   4.2. Creep is included

5. Compare the results
5.4.4. Setup of the numerical model

The numerical model is modelled as described in chapter 4.2 and the mesh size is chosen in accordance with the convergence study in Appendix C. Results from the verification in chapter 5.4.1 indicate that a triangular elements yields the most suitable results, hence this will be used in this model. It should be noted that the verification of the plasticity model in COMSOL is made for rectangular elements and that it is assumed that the mesh size chosen in the convergence analysis is sufficiently small also for triangular elements.

5.4.5. Results

As $\phi_8$ – bars are used and the crack width is to be limited to 0,1 mm, the non-direct calculation procedure stated in (EC2-3, 2006) suggest a maximum steel stress of 178 MPa, see Figure 5-32. Further, the minimum reinforcement content is determined as $\phi_8 s 70$ in accordance with (EC2-2, 2005).

![Figure 5-32 Non-direct calculation procedure (EC2-3, 2006)](image)

**Analysis 1 - Experiment based analysis**

Displayed in Figure 5-33 and Figure 5-34 is the resulting crack pattern as creep is excluded and included respectively in the experiment based analysis. The material properties, creep- and shrinkage strain is chosen in accordance with the results obtained in the experiment in chapter 5.4.1. The final shrinkage strain is $\varepsilon_{sh} = 457 \cdot 10^{-6}$, which is equivalent to a temperature drop of $\Delta T = -45,7 \degree C$. 

---

*Figure 5-32 Non-direct calculation procedure (EC2-3, 2006)*
The resulting crack width, steel stress, number of cracks and crack spacing can be found in Table 5-8 and Table 5-9 as creep is excluded and included respectively. Note that all hand calculations are based on the 28-day material properties and the final creep factor obtained in the experiment, i.e. no time dependency is taken in to account in the hand calculations. The number of cracks in EC2-3 and the crack spacing in the Chalmers method is approximated based on Equation 5-1 and Equation 5-2 respectively.

**Table 5-8 Crack widths obtained in hand calculations and analysis as creep is excluded**

<table>
<thead>
<tr>
<th></th>
<th>EC2-3</th>
<th>Chalmers method</th>
<th>FE-analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_k$ (mm)</td>
<td>0,191</td>
<td>0,354</td>
<td>0,140</td>
</tr>
<tr>
<td>$S_{r,mean}$ (mm)</td>
<td>-</td>
<td>-</td>
<td>200</td>
</tr>
<tr>
<td>$S_{r,max}$ (mm)</td>
<td>417</td>
<td>667</td>
<td>462</td>
</tr>
<tr>
<td>$n_{cr}$</td>
<td>21</td>
<td>11</td>
<td>~34</td>
</tr>
<tr>
<td>$\sigma_s$ (MPa)</td>
<td>178</td>
<td>266</td>
<td>208</td>
</tr>
</tbody>
</table>

**Table 5-9 Crack widths obtained in hand calculations and analysis as creep is included**

<table>
<thead>
<tr>
<th></th>
<th>EC2-3 $R = 0.65$</th>
<th>EC2-3 $\frac{\varepsilon_{cr}}{1 + \varphi}$</th>
<th>Chalmers method</th>
<th>FE-analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_k$ (mm)</td>
<td>0,124</td>
<td>0,096</td>
<td>0,206</td>
<td>0,0350</td>
</tr>
<tr>
<td>$S_{r,mean}$ (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>172</td>
</tr>
<tr>
<td>$S_{r,max}$ (mm)</td>
<td>417</td>
<td>417</td>
<td>421</td>
<td>255</td>
</tr>
<tr>
<td>$n_{cr}$</td>
<td>21</td>
<td>21</td>
<td>18</td>
<td>~34</td>
</tr>
<tr>
<td>$\sigma_s$ (MPa)</td>
<td>178</td>
<td>178</td>
<td>172</td>
<td>76</td>
</tr>
</tbody>
</table>
Analysis 2 – Eurocode based analysis

In this analysis, the creep factor as well as the shrinkage strain is based on the hand calculations suggested by (EC2-1, 2005). In order to reach a value close to the final shrinkage strain the analysis is extended to 500 days, see Figure 5-35. Loading at the concrete age of 3 days is assumed as in the previous analysis.

As the time dependent creep factor is updated, the creep curve in the numerical analysis is refitted in order to match the hand calculations, see Figure 5-36. Note that the figure displays the summation of the elastic- and creep strain, with the elastic strain $\varepsilon_{el} = 220 \cdot 10^{-6}$ as a reference point. The same test specimen as in previous analyses is used, and the obtained fitting correspond very well. As the creep factor in Eurocode is calculated based on the 28-day elastic modulus, the elastic strain is now smaller compared to Analysis 1.

Figure 5-35 Shrinkage strain obtained in hand calculations

Figure 5-36 Comparison between creep strain obtained in hand calculations and FE-analysis
Chapter 5 – Case Studies

The time dependency of the material is determined through the procedure stated in (EC2-1, 2005), see chapter 2.1.4. Illustrated in Figure 5-37 is the time dependent elastic modulus and tensile strength used in the analysis.

![Figure 5-37 Time dependency of elastic modulus (left) and tensile strength (right) according to Eurocode](image)

Displayed in Figure 5-38 and Figure 5-39 is the resulting crack pattern as creep is excluded and included respectively. The final shrinkage strain is \( \varepsilon_{cs} = 207 \cdot 10^{-6} \), which is equivalent to a temperature drop of \( \Delta T = -20.7 \, ^\circ C \). It can be seen that as creep is included, no cracks are obtained.

![Figure 5-38 Resulting crack pattern as creep is excluded in Analysis 2](image)

![Figure 5-39 Resulting crack pattern as creep is included in Analysis 2](image)

The resulting crack widths, steel stress, crack spacing and number of cracks can be found in Table 5-10 and Table 5-11 as creep is excluded and included respectively. The crack spacing and number of cracks for the Chalmers methods and EC2-3 are approximated in the same ways as in previous analysis.
Table 5-10 Crack widths obtained in hand calculations and analysis as creep is excluded

<table>
<thead>
<tr>
<th></th>
<th>EC2-3</th>
<th>Chalmers method</th>
<th>FE-analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_k$ (mm)</td>
<td>0,086</td>
<td>0,33</td>
<td>0,12</td>
</tr>
<tr>
<td>$S_{r,mean}$ (mm)</td>
<td></td>
<td>-</td>
<td>834</td>
</tr>
<tr>
<td>$S_{r,max}$ (mm)</td>
<td>417</td>
<td>1600</td>
<td>890</td>
</tr>
<tr>
<td>$n_{cr}$</td>
<td>21</td>
<td>4</td>
<td>~10</td>
</tr>
<tr>
<td>$\sigma_s$ (MPa)</td>
<td>178</td>
<td>255</td>
<td>216</td>
</tr>
</tbody>
</table>

Table 5-11 Crack widths obtained in hand calculations and analysis as creep is included

<table>
<thead>
<tr>
<th></th>
<th>EC2-3</th>
<th>Chalmers method</th>
<th>FE-analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R = 0,65$</td>
<td>$0,056$</td>
<td>$0,022$</td>
<td>-</td>
</tr>
<tr>
<td>$\frac{\varepsilon_{cs}}{1 + \varphi}$</td>
<td>$0,203$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$w_k$ (mm)</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$S_{r,mean}$ (mm)</td>
<td></td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$S_{r,max}$ (mm)</td>
<td>417</td>
<td>417</td>
<td>4000</td>
</tr>
<tr>
<td>$n_{cr}$</td>
<td>21</td>
<td>21</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_s$ (MPa)</td>
<td>178</td>
<td>178</td>
<td>-</td>
</tr>
</tbody>
</table>

5.4.7. Discussion

Initially, it is worth mentioning that the shrinkage strain as well as the creep factor reported in the experiment differ significantly to what is obtain when performing corresponding hand calculations according to Eurocode. This can clearly be seen in Figure 5-40 and Figure 5-41 for the shrinkage strain and creep factor respectively.

In terms of equivalent temperature, the experimental slab is subjected to a temperature drop of $\Delta T = -45,7 \, ^\circ C$ after 122 days. A corresponding Eurocode calculation result in an equivalent temperature drop of $\Delta T = -17,3 \, ^\circ C$ at the same age, which is 2,7 times smaller. The final shrinkage obtained through Eurocode results is equivalent to a temperature drop of $\Delta T = -20,7 \, ^\circ C$ at 500 days which is not even half of the magnitude obtained during the 122 days in the experiment.

The creep factor at the age of 122 days is $\varphi = 0,98$ and $\varphi = 2,28$ for the experiment and hand calculations respectively, i.e. they differ with a factor of 2,6. The final creep factor at 500 days is $\varphi = 3,01$ in the hand calculations.
Figure 5-40 Shrinkage strain obtained in the experiment and through hand calculations according to Eurocode

Figure 5-41 Creep factor obtained in the experiment and through hand calculations according to Eurocode

Displayed in Figure 5-42 and Figure 5-43 is a comparison between the obtained maximum crack widths from the hand calculations and numerical analyses. In Analysis 2, it can be seen that as creep is excluded, the FE-analysis predict a crack width which is 1.4 times larger than the (EC2-3, 2006) hand calculation. It is the first time during this thesis that such a result is obtained, and contradict conclusions drawn earlier in this report. However, this analysis differ from previous analyses in several ways:

- Material properties are time dependent, compared to previous analyses where they were held constant.

- It have been shown that in COMSOL, the crack pattern and crack widths are sensitive to the choice of reinforcement element size.

- The plasticity model in COMSOL was verified with respect to rectangular elements and the mesh size was assumed to be sufficiently small for triangular elements in the creep analysis. A convergence study for triangular elements was thus not performed.
Further, the verification of the creep model is performed with respect to an end-restrained concrete member. The result become more similar as creep is included in the analysis, implying that the response due to creep is reasonable, but in order to verify it with respect to base-restrained members a comparison with a corresponding experiment is required.

For these reasons, focus will be put on the effect creep have on base-restrained structures, rather than drawing conclusions about maximum crack widths and whether (EC2-3, 2006) yields a conservative results in relation to the numerical results.

Figure 5-42 Crack widths obtained in the experiment based analysis. As creep is included in Eurocode calculations, it is taken in to account by $R_{ax} = 0.65^*$ and $\varepsilon_{cs}/(1 + \varphi)^{**}$.

Figure 5-43 Crack widths obtained in the Eurocode based analysis. As creep is included in Eurocode calculations, it is taken in to account by $R_{ax} = 0.65^*$ and $\varepsilon_{cs}/(1 + \varphi)^{**}$.
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It can be seen in the results that the numerical analyses suggests a reduction of the crack width as creep is included. This is consistent with the various hand calculation procedures treated in this report. This is also consistent with chapter 2.1.3 in which it is stated that creep have a positive influence on the restrained structures as softening due to creep reduces the restraining stresses.

(Bamforth, et al., 2010) states that the restraint factors given in (EC2-3, 2006) includes creep and follow the relationship $R_{ax} = 0,65 \cdot R$. If comparing this with a reduction of $(1 + \varphi)$, it can be seen that for $\varphi > 0,54$, the method proposed by (Bamforth, et al., 2010) yields a more conservative results.

The Chalmers methods suggests that the tensile strength is reduced by 40 % and the elastic modulus is reduced by a factor of $(1 + \varphi)$ as long term loading is considered. At the same time, the single crack is suggested to increase due to the effect of creep as the crack width for sustained loading is increased by a factor of 1,24. The effect of these three modifications due to long term loading result in a reduction of the final crack width. The method overshoots (EC2-3, 2006) with a good margin, which is consistent with the results obtained throughout this report.

In the experiment based analysis, it can be seen that as creep is excluded in the (EC2-3, 2006) hand calculations, the resulting crack width exceeds the chosen limiting crack width $0,1 \, mm$ on which the required minimum reinforcement in based on. Further, it can be seen that as creep is included in the hand calculations, the resulting crack width is very close to the limiting crack width and give very little rooms for other load cases such as short term temperature and external loading.

Concluded in chapter 5.1.3 is that the non-direct calculation procedure is only valid up until a certain strain. Analysis 1 is based on the experimental results, for which the shrinkage strain is considerably larger than corresponding Eurocode hand calculations. Hence, the strain for which the non-direct calculation procedure is valid for is almost reached for the shrinkage only. The non-direct calculation procedure would in this case not be on the safe side in relation to more detailed hand calculations.
5.4.8. Remarks

- Compared to the reference experiment, Eurocode seems to underestimate the shrinkage strain and overestimate the time-dependent creep.

- Due to the uncertainty in the numerical analysis, no conclusions can be drawn whether EC2-3 yields a conservative results in relation to a FE- analysis. Results can only be seen as indications of the creep effect.

- All calculation methods, as well as numerical analyses, implies that creep have a positive influence on the crack widths in the sense that it become smaller.

- The Chalmers method yield results which are significantly larger than the other calculation methods.

- The non-direct calculation procedure stated in (EC2-3, 2006) is in this case only on the safe side in relation to more detailed hand calculations if performing a design within the limits of Eurocode.

- Choosing a restraint factor of $R_{ax} = 0,65 \cdot R$ is on the safe side in relation to reducing the strain by a factor of $(1 + \varphi)$ if the condition $\frac{1}{1+\varphi} \leq 0,65$ is fulfilled.
6. Final remarks

It is clear that the problem treated in this report have more complexity than what one initially may think. Opinions of how to treat cracking of base restrained concrete members differ, and several hand calculation methods are suggested. In this report, attempts have been made to get one step closer of understanding the concept through non-linear finite element analyses.

6.1. Method criticism

Throughout the report, assumptions and simplifications have been made in order to conduct a numerical analysis with reasonable CPU-time and interpretable results. In this chapter, a brief discussion of suitable modifications to the method is presented.

6.1.1. Mesh dependency

As the COMSOL model is compared to the experiment, it is shown that the summation of all cracks was about the same as obtained in the experiment, but the single crack width and crack pattern is dependent on the choice of reinforcement element size. The cause is believed to be that a finer reinforcement mesh result in a finer distribution of the stresses between concrete and reinforcement, which in turn result in a finer crack distribution, see illustration in Figure 6-1.

![Illustration of reinforcement and concrete coupling](image)

In this sense, the crack pattern can more or less be chosen by the one conducting the analysis by regulating the mesh size in COMSOL such that it fits the purpose. Hence, it is hard to draw any conclusions at all about the maximum crack widths and crack spacing without a suitable reference experiment.

To overcome this problem in future research, it may be favourable to look upon cracks as the sum over a certain length, either the member length or an assumed crack spacing based on e.g. EC2-2, or to compare the model with a suitable experiment.
6.1.2. Coupling of reinforcement

In the two numerical software that were used, the reinforcement is connected to the concrete through some type of embedment. In BrigAde/Plus, an embedded region was used, while in COMSOL Multiphysics a general extrusion was used.

The embedded region in BrigAde/Plus works in such a way that the program searches for a geometrical relationship between the nodes of the reinforcement and the concrete. If the reinforcement node lies within an element of the concrete, the degrees of freedom are eliminated and constrained to the corresponding degrees of freedom of the concrete element (Sorenson, 2011).

The general extrusion in COMSOL works in another way. According to Tobias Gash the displacement field of the concrete is in this case mapped to the reinforcement. In this sense, it is not as important to have the same mesh size for the reinforcement and concrete.

These two methods are simple ways of modelling the reinforcement bars in 2D plane stress concrete analyses. It does however not consider one important aspect of reinforced concrete, the bond-slip relationship. For future research it is suggested to find another way of connecting the reinforcement and concrete such that this aspect is taken into account.

One method may be to connect the reinforcement elements to the concrete elements through springs as illustrated in Figure 6-2. Concrete node $n$ should then be connected to reinforcement node $n + 1$ along the horizontal plane. The spring could further be assigned a non-linear stiffness which correspond to the bond-slip relationship of interest.

![Figure 6-2 Illustration of coupling through non-linear springs](image-url)
6.2. Discussion

6.2.1. Detailed hand calculation procedures

All numerical analyses as well as hand calculations have indicated that the method denoted as the Chalmers method result in very high crack widths. Even though an analysis through the non-direct calculation procedure stated in (EC2-3, 2006) is conducted under reasonable assumptions, the Chalmers method overshoots the results with good margin.

The method itself is rather detailed, taking many reasonable factors into account. It is well backed up by theory and is based on an equilibrium condition which theoretically should be valid for the cases under investigations. Some of the factors which are believed to give rise to the protruding results are stated below:

- The Chalmers method states a relationship between the calculated mean crack width \( w_m \) and the characteristic crack width \( w_k \). In the case of internal loading, such as thermal actions and shrinkage, the relationship is \( w_k = 1.3 \cdot w_m \), i.e. the calculated crack width is increased by 30%. For external loading, the corresponding relationship is \( w_k = 1.7 \cdot w_m \).

- In chapter 5.1 it is concluded that as the crack spacing is chosen the same as for EC2-3, and hence the number of cracks in the Chalmers method is increased, the resulting crack width is very close in magnitude to what is obtained when performing calculations according to EC2-3, see Figure 6-3. In chapter 5.2 it is further shown that a larger number of cracks is predicted by the FE-analyses compared to the Chalmers method.

In the revised method proposed by ICE it is stated that the single crack cannot reach its full potential as the restraining member will act somewhat like reinforcement, distributing the cracks. This is not taken into account in the Chalmers method, resulting in that the number of cracks may be underestimated such that fewer but larger cracks are obtained.
In relation to the numerical analyses performed in BrigAde/Plus, the detailed hand calculation procedure stated in (EC2-3, 2006) yield results on the safe side. This is consistent with the numerical results reported by (Zangeneh, et al., 2013). In the same report, it was also shown that the EC2-3 procedure yield a conservative result in 90% of the cases when comparing it to the experiment provided by (Kheder, et al., 1994).

The (EC2-3, 2006) procedure is based on stabilized cracking and suggests an increase in crack width which is proportional to the increase in strain. Numerical results obtained in chapter 5.3 implies that an increase in strain rather result in more cracks of similar magnitude than increasing the single crack width. As a result the deviation between numerical results and results predicted through EC2-3 become larger for higher levels of strain.

This implies that the hand calculation procedure stated in (EC2-3, 2006) result in reasonable crack widths for lower levels of strain such as the short term temperature drop of $\Delta T = -15$ °C, while for higher levels of strain the crack width is overestimated, at least up until stabilized cracking is reached. A higher level of strain may be introduced through external loading or shrinkage.

### 6.2.2. Control of cracking without direct calculations

Denoted in (EC2-1, 2005) is that the crack width is unlikely to be exceeded as the non-direct calculation procedure is applied. Hence, the method should be applicable as an alternative way of performing a design on the safe side for any combination of loads.

In relation to the numerical results, this statement may very well be correct. In all numerical analyses performed in BrigAde/Plus, the crack width is considerably lower than assumed through
the non-direct calculation procedure. The question is whether EC2 refers to the actual crack width or the crack width which would be obtained if performing a more detailed hand calculation.

As the procedure is compared to detailed hand calculations based on the experimental material properties given in chapter 5.4.1, the method did not yield a result on the safe side. When applying the shrinkage strain only, very little room was left for other load cases such as short term temperature and external loading. It was further shown in chapter 5.3 that as conservative assumptions are made for the hand calculation procedure, the limiting crack width on which the minimum reinforcement is based on is reached.

Concluded in chapter 5.1 is that in relation to more detailed hand calculations, the non-direct calculation procedure is only valid up until a certain limit strain, see Figure 6-4. It is also concluded that if conservative assumptions are made for EC2-2 and EC2-3 coefficients or a high concrete quality is used, the limit strain decrease. Hence, the designer should be aware of the assumptions made and the magnitude of the serviceability load when making a choice in-between the methods.

![Figure 6-4 Crack width as a function of temperature drop, from chapter 5.1](image)

**6.2.3. The effect of creep**

Analyses regarding creep are in this report seen as indications of the effect it has on base-restrained concrete members. This is mainly due to the following reasons:

- As creep is included in the numerical analysis, the results become more similar to the results reported in the reference experiment. Hence, the effect creep have on the response is reasonable. However, the reference experiment treats the concept of end restrained members. To verify the results of the base-restrained members a comparison with a corresponding experiment is required.
• The plasticity model in COMSOL was verified with respect to rectangular elements and the mesh size was assumed to be sufficiently small for triangular elements in the creep analysis. A convergence study for triangular elements was thus not performed.

• The crack pattern and crack width is dependent on the mesh size of the reinforcement in COMSOL. The summation of crack widths is very close to what is obtained in the reference experiment, but conclusions regarding the maximum crack width and crack spacing is hard to draw.

As the creep model is adopted to the base-restrained member, the numerical results indicate that creep have favourable effect as the maximum crack width decrease. A reduction of the crack widths is also consistent with the various hand calculation method presented in this report as well as chapter 2.1.3 in which it is stated that creep will reduce the restraint stresses as the material softens.

Suggested by (Zangeneh, et al., 2013) is that the effect of creep in hand calculation may be considered by reducing the shrinkage strain by a factor of \((1 + \varphi)\). It is also stated that the restraint factor should be chosen in accordance with (EC2-3, 2006). However, (Bamforth, et al., 2010) states that the restraint degrees given in (EC2-3, 2006) includes creep, hence the effect is accounted for twice.

For this reason, one of the reductions should be excluded. It can be seen in Figure 6-5 that a reduction factor of 0.65 as proposed by (Bamforth, et al., 2010) is on the safe side in relation to a reduction of \((1 + \varphi)\) if the creep factor is larger than 0.54.

![Figure 6-5](image_url)

*Figure 6-5 Comparison between methods of considering creep in hand calculations*
6.3. Conclusions

Conclusions drawn in this report are made under the assumption that the results obtained in the numerical software BrigAde/Plus are more reliable than the results obtained in COMSOL Multiphysics due to the reasons discussed in previous chapters.

- Numerical results as well as various hand calculation methods suggest that the method denoted as the Chalmers method results in crack widths which are very much on the safe side. The main reason for this is believed to be that the Chalmers method underestimates the number of cracks that will occur in base-restrained structures and thus overestimates the crack widths.

- Numerical results indicate that the hand calculation procedure stated in EC2-3 give reasonable crack widths for lower levels of strain, while overestimating the crack widths for high levels of strain. The reason for this is that the EC2-3 procedure assumes a linear relationship between the crack width and stress-independent strain while the numerical results imply that an increase in strain rather introduce more cracks of similar magnitude as stabilized cracking on which EC2-3 is based on is not reached.

- In relation to more detailed hand calculations, the non-direct calculation procedure stated in EC2-3 is not always on the safe side. Usage of the method should be based on judgement regarding the magnitude of serviceability load, concrete quality and the choice of various design coefficients. In this report it have been shown that the method only yields a result on the safe side up until a limit value of the loading, which may be exceeded when performing more detailed hand calculations.

- Numerical results as well as various hand calculation methods suggest that creep have a favourable effect on crack width of base-restrained concrete members in the sense that the maximum crack width decrease.

- Using the restraint factors provided in EC2-3, which includes creep, is on the safe side in relation reducing the shrinkage strain by a factor of \((1 + \varphi)\) if \(\varphi > 0,54\). It is in the authors opinion that a creep factor larger than 0,54 is not uncommon in practical design situations.
6.4. **Recommendations**

Based on the results obtained in this report, it is in the authors opinion that the detailed hand calculations procedure recommended in (EC2-3, 2006) is a suitable method for estimating the crack width of base restrained concrete members. This is consistent with the results obtained in (Zangeneh, et al., 2013) and hence the design procedure stated the compendium may be used for crack control is the transverse direction. It is however recommended that the way of considering creep is reconsidered in the compendium.

Stated in (Zangeneh, et al., 2013) is that the degree of restraint should be chosen in accordance with (EC2-3, 2006) such that a maximum restraint factor of $R_{ax} = 0.5$ is used. It is further stated that a reduction with respect to creep may be estimated by reducing the shrinkage strain by a factor of $(1 + \varphi)$.

Staten in (Bamforth, et al., 2010) is that the restraint factors given in (EC2-3, 2006) already includes the effect of creep, hence the creep is taken into account twice in the procedure proposed by (Zangeneh, et al., 2013).

One of these reductions should be excluded. It have been shown that by choosing a restraint factor in accordance with (EC2-3, 2006) typically yields a safer result in relation to a reduction of the shrinkage strain.
6.5. Future research

- Investigation of bond-slip relationship

In all analyses, the bond-slip relationship between reinforcement and concrete, as well as between member and foundations, have been neglected. Introducing these factors in the analyses would most likely reflect a more correct behaviours of the base-restrained member.

- Further analyses of the effect of creep

Attempts to implement creep in the numerical analyses was made rather late in the thesis. Factors such as switching to a new numerical software, introducing time dependency and the overall complexity of the problem have resulted in that no certain conclusions can be drawn. For a future report, it may be a good idea fully focus on the concept of creep. Further, it may be of interest to investigate the effect of creep under varying climate such as temperature and relative humidity.

- Detailed analysis if the revised method proposed by ICE

The main methods under investigation in this report, i.e. EC2-3 and the Chalmers method, suggest that the crack width is reduced whenever the restraint degree is reduced. For this reason the member have been assumed to be fully restrained throughout this report. The revised method proposed by ICE suggest that the maximum crack width occurs as the restraint degree is $R < 1$. Hence, a detailed analysis of this concept may be of interest for future research in order to determine the suitability of the revised method.

- Investigation of inclined cracks on the sides

In this report, the inclined cracks close to the free edge of the member have been neglected although the maximum crack width is found there in several cases. As future research it may be of interest to investigate these cracks closer.

- Analysis of non-linear temperature change

In this report, only a uniform temperature change have been considered. According to EC1-1-5, a non-linear temperature change should also be checked in the serviceability limit state for bridges.
References


Bamforth, P., Denton, S. & Shave, J., 2010. *The development of a revised unified approach for the design of reinforcement to control cracking in concrete resulting from restrained contraction*, s.l.: Institute of Civil Engineers - ICE Research project 0706.


Gasch, T., 2016. *Concrete as a multi-physical material with applications to hydro power facilities*, Stockholm, Sweden: Royal Institute of Technology.


clc
clear all

%% Parameters

% Material concrete
Ec=32e3;             % Concrete elastic modules [MPa]
fctm=2.9;            % Concrete tensile strength [MPa]
fcm=38;              % Concrete compressive strength [MPa]

% Implementation of creep
cp=0;                % Creep factor
MF=1;                % Magnification factor for crack width, 1 for 
% short term, 1.24 for long term loading
ac=1;                % Reduction factor for tensile strength, 1 for 
% short term, 0.6 for long term loading

% Steel
Es=205e3;            % Steel elastic modules [MPa]
fs=178;              % Steel stress [MPa]
phi=12;              % Diameter of reinforcement [mm]
c=75;                % cover for reinforcements [mm]
n_w=2;               % Number of sides with reinforcement bars

% Geometry
H=1000;              % Height of element [mm]
L=8000;              % Length of the element [mm]
t=100;               % Thickness of element [mm]

%thermal strain
R=1.0;               % Degree of restraint at base
alpha_t=10e-6;       % Coefficient of thermal expansion [1/degC]
delta_t=-15;         % Temperature drop [degC]

% Factor for non-uniform self-equilibrating stresses, EC2
k=1;                 %For web with h < 300 and flanges with b < 300
k=0.65;              %For webs with h>800 and flanges with web > 800

% Coefficient taking in to account the stress distribution prior cracking
kc=1;                % For pure tension

%% ===================================================================
Appendix A – Matlab code for the Chalmers method

% Calculate thermal strain

strain_ct=alpha_t*delta_t; % Thermal strain
strain_cs=R*strain_ct; % Differential strain EC2-3

% Calculate effective area

h_eff=min(2.5*(c+phi/2),t/2); % Effective height [mm]
alpha_ef=(Es/Ec)*(1+cp); % Effective modulus ratio
Ac=t*H; % Section area [mm^2]

As=k*kc*fctm*Ac/fs; % EC2-1 Minimum reinforcement [mm^2]
Ac_eff=n_w*h_eff*H; % Effective area [mm^2]
p_seff=(As/Ac_eff); % Effective reinforcement ratio

% Assign reinforcement manually

p_seff=0.6/100; % Effective reinforcement ratio
As=p_seff*Ac_eff; % Reinforcement area [mm^2]

A1=Ac+(alpha_ef-1)*As; % Equivalent area for stage I concrete
%% Appendix A – Matlab code for the Chalmers method

% Solving the steel stress through the iterative procedure

syms sigma_s;

N_i=0;
N_cr_red=0;
n_cr=1;

while N_i >= N_cr_red
    for i = 1:3
        w_sigma(i) = vpasolve(L*((sigma_s*As)/(Ec/(1+cp)*A1)) + n_cr*(MF*0.42*(((phi*sigma_s^2)/(0.22*fcm*Es*(1+p_seff*alpha_ef)))^(0.826))+(sigma_s/Es)*4*phi) + strain_cs*L == 0,sigma_s,'Random',true);
    end

    w_sigma = max(w_sigma);
    N_i = As*w_sigma;
    N_cr_red=ac*fctm*(Ac_eff+(alpha_ef-1)*As);

    if (N_i >= N_cr_red)
        n_cr = 1+n_cr;
        sigma_s = Sym('sigma_s');
        A1 = Ac_eff+(alpha_ef-1)*As;
    else
        sigma_s=w_sigma;
    end
end

% Calculating the final crack width

wm=MF*0.42*(((phi*sigma_s^2)/(0.22*fcm*Es*(1+p_seff*alpha_ef)))^(0.826)) + (sigma_s/Es)*4*phi;
wk_c=wm*1.3;

%% Print result

disp('Diameter for the reinforcement bars');
disp(phi);
disp('Chalmers calculated crack width');
disp('Wk Chalmers');
disp(wk_c);
disp('Number of cracks Chalmers');
disp(n_cr);
Appendix B – General input for numerical models

**Concrete compressive properties**

In BrigAde/Plus, the uniaxial compressive curve must be defined. For non-linear structural analysis, (EC2-1, 2005) proposes a stress-strain relationship as illustrated in Figure B-1 for concrete subjected to short term uniaxial compression, for which Equation B-1 is valid.

![Stress-strain relationship for non-linear structural analysis (EC2-1, 2005)](image)

\[
\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta} \tag{B-1}
\]

Where

- \( \eta = \frac{\varepsilon_c}{\varepsilon_{c1}} \)
- \( k = \frac{1.05 f_{cm} \varepsilon_{c1}}{f_{cm}} \)
- \( 0 < |\varepsilon_{c1}| < |\varepsilon_{cu1}| \)

The majority of the stresses will be tensile stresses, and the compressive stress that arise is expected to be very low. In this sense, a detailed description of the non-linear compressive behaviour will result in a larger CPU-time with no significant effect on the results. As the concrete behaves more or less linear elastic at low compressive stress levels, the uniaxial relationship is simplified as a bilinear curve, see illustration in Figure B-2.
The mean elastic modulus of the 28-day material properties is assumed up until 40 % of the compressive strength, after which a linear relationship is assumed up until the ultimate strain is reached. In BrigAde/Plus, it is the plastic strain which should be defined, which is obtained by applying Equation B-2.

\[ \varepsilon_{pl} = \varepsilon_{tot} - \frac{\sigma_c}{E_{cm}} \]  

**Concrete tensile properties**

The post-cracking behaviour is defined by the crack opening curve described in chapter 2.3.1. Illustrated in Figure B-3 are three crack opening curves based on the fracture energy provide by (Malm [A], 2016), which can be used in numerical analysis. The choice of curve should be based on the required accuracy of the analysis, where the linear curve represents the simplest case, and the exponential represents in the most advanced.

**Figure B-2** Illustration of a uniaxial compression curve and corresponding input curve

**Figure B-3** A Linear, bilinear and exponential crack opening curve (Malm [A], 2016)
Appendix B – General input for numerical models

For the analyses, the tensile behaviour after cracking is defined by the bilinear crack opening curve displayed in Figure B-4. In BrigAde/Plus, the relationship may be defined in the means of strains instead of crack displacement. This is achieved by applying Equation B-3, resulting in the tensile behaviour being mesh dependent. In COMSOL Multiphysics, the fracture energy itself is entered in to the program and the type of tension stiffening curve may be chosen by the user.

\[ \varepsilon = \frac{w_i}{l_e} \]  \hspace{1cm} (B-3)

![Bilinear crack opening curve](image)

**Figure B-4** Bilinear crack opening curve (Model Code, 2010)

**Steel properties**

The steel quality of the reinforcement is chosen as B500B throughout the report, with an elastic modulus of 205 \( GPa \). The stress-strain curve is assumed to be linear elastic up until the yield stress of 500 \( MPa \), after which perfect plasticity is assumed. Steel stresses are not expected to exceed the yield limit.

**Smeared cracking**

In order to obtain the failure envelope describing the biaxial stress state, BrigAde/Plus requires the following failure ratios in the smeared crack material model:

a) Ratio between biaxial and uniaxial compressive strength

b) Ratio between uniaxial tensile strength and uniaxial compressive strength

c) Ratio between principal plastic strain at uniaxial and biaxial compressive strength

d) Ratio between tensile strength and the maximum uniaxial tensile strength
Ratio a), c) and d) are chosen as the standardized default values, see Table B-1. According to (Malm [A], 2016), these ratios are equal for almost all types of concrete. Ratio b) is calculated as described above.

Table B-1 Failure ratios used in the smeared crack model

<table>
<thead>
<tr>
<th>Ratio:</th>
<th>a)</th>
<th>b) (\frac{f_{ct}}{f_{cm}})</th>
<th>c)</th>
<th>d)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1,16</td>
<td>1,28</td>
<td>0,33</td>
<td></td>
</tr>
</tbody>
</table>

**Damage plasticity**

In order to obtain the failure envelope describing the biaxial stress state, BrigAde/Plus requires the following parameters in the damage plasticity material model:

a) Dilation angel

b) Eccentricity

c) Ratio between the equibiaxial and uniaxial compressive stress

d) The ratio of the second stress invariant on the tensile meridian

e) Viscosity parameter

Parameter a) is chosen based on recommendation found in (Malm, 2009). Parameter b) – e) is chosen as the default values in BrigAde/Plus, see Table B-2.

Table B-2 Input data used in the damage plasticity model

<table>
<thead>
<tr>
<th>Parameter:</th>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>39°</td>
<td>0,1</td>
<td>1,16</td>
<td>0,6667</td>
<td>1 \cdot 10^{-10}</td>
</tr>
</tbody>
</table>

**Isotropic damage model**

The Rankine failure envelop in COMSOL Multiphysics does not consider multiaxial behaviour. Hence, only the tensile strength \(f_{ct}\) must be defined in the plasticity model.
Appendix C – Convergence Study

The results obtained in a FE-analysis are dependent of the mesh size. Generally, the smaller the mesh is, the more accurate the results will be. However, as the mesh size is decreased, the CPU-time increases. The same concept applies for different integration methods and element types.

In the case of non-linear analysis, the CPU-time is already very high. For this purpose, a convergence study is performed in order to find the optimum between the level of accuracy in the result and the CPU-time required in order to perform the analysis. It should be noted that the convergence analysis is performed only for rectangular elements in BrigAde/Plus 6.1.

**Geometrical limitations**

For both the smeared crack method and the damage plasticity method the reinforcement has a large importance, as well as the concrete element size in relation to the bar centre distance. If the member is unreinforced or have very little reinforcement, there will be elements which do not contain any reinforcement. In such a case the results do not converge towards a unique solution when the mesh is refined as it leads to narrow crack bands (Sorenson, 2011).

In order to reduce this effect, it is essential to define the mesh such that every element of the concrete is in contact with a reinforcement bar. The discretization is thus suffering from a geometrical limitation. Illustrated in Figure C-1 is the results of a convergence analysis as this effect is not taken into consideration for the smeared crack method. For the finest mesh, the strain is smeared out over large portions of the concrete member, resulting in low unevaluable cracks.

![Figure C-1 Illustration of results as the mesh size is reduced from 100 to 50 to 20 mm in the convergence study](image)

**Analysis**

In the convergence study, a concrete member with an $L/H \ - ratio = 8$ and $C30/37$ concrete is used. The member is $1\ m$ high and $6\ m$ long and modelled with symmetry conditions as described in chapter 4.2. The member contains $2\ \phi 12\ s90$ reinforcement bars in both horizontal and vertical direction. It is performed for 4-nonded 2D plane stress element with full integration scheme for the concrete member, and two-dimensional truss elements for the reinforcement bars. The results are displayed in Figure C-2.
It is found that the geometrical condition of the concrete mesh in relation to the reinforcement spacing is fulfilled only for the mesh of 100 mm. In the case of a 50 mm mesh size, every third element is in no contact with a reinforcement bar. However, these elements are surrounded by elements that fulfilled the condition, and the obtained crack pattern was reasonable and accepted. In the case of 30 mm mesh, a slightly smaller mesh size had to be chosen for the reinforcement (28 mm) in order to obtain a clear crack pattern.

The results show that there is an 96 % accordance between the 30 mm mesh and the 50 mm mesh in the case of the smeared crack method. The results is considered to have converged, and the mesh of 50 mm is chosen without having to large errors in the results and still obtain a reasonable CPU-time for the future analyses.

For the damage plasticity methods, no clear convergence is obtained. However, after a mesh size of 50 mm the resulting crack widths are of the same magnitude. With respect to the convergence of the smeared crack method, the same mesh size is considered to deliver reasonable results for the damage plasticity method as well.
Appendix D – Verification of plasticity model in COMSOL

As a new numerical software is used for the creep analysis, the new model must also be verified. For this purpose, an initial analysis is performed in which no creep is taken into account, but only the uniform temperature drop of \( \Delta T = -15 ^\circ \text{C} \). The analysis is performed on the same member as described in chapter 5.3 containing \( 2\phi 16 \times 65 \) in horizontal and vertical direction. A comparison is made with results from both the smeared crack and the damage plasticity material model in BrigAde/Plus.

The definition of the plasticity model differs for BrigAde/Plus and COMSOL. Two major differences which are believed to influence the results are stated below:

- **The failure envelope**
  A more complex description of the multiaxial behaviour is used in BrigAde/Plus compared to COMSOL, see illustration in Figure D-1. For the COMSOL analysis, the simplest version known as the Rankine yield surface is used, while in BrigAde/Plus the multiaxial behaviour is defined in more detail, with some difference depending on choice of plasticity model.

![Figure D-1 Illustration of yield surface used in BrigAde/Plus and COMSOL, modification of (Sorensen, 2011)](image-url)
• **The tension stiffening curve**

In BrigAde/Plus, the tension stiffening have been defined as a bilinear curve, calculated with respect to the fracture energy and element size, with (Model Code, 2010) as a basis, see chapter 2.3.1. In COMSOL, the fracture energy is directly entered in to the material model and the tension stiffening curve is calculated automatically. To define the type of curve, the user may choose in-between three sub options; *exponential*, *linear* or *user defined*. Attempts were made to define the bilinear curve as a user defined curve, however with no success.

With respect to the difference in material definitions, two analyses are performed for the verification. As the yield surface cannot be changed to match, focus is put on the tension stiffening curve.

**Analysis 1 – Different tension stiffening curves**

In this analysis, an exponential tension softening curve is applied to the COMSOL model while the previously used bilinear curve is used in BrigAde/Plus. The analysis will imply whether the results in the COMSOL analysis are comparable with previous assumptions made. The resulting crack pattern is displayed in Figure D-2 – D-4 for the smeared crack, damage plasticity and COMSOL analysis respectively.

![Figure D-2 BrigAde/Plus - Smeared cracking](image)

![Figure D-3 BrigAde/Plus - Damage plasticity](image)

![Figure D-4 COMSOL Multiphysics](image)
Appendix D – Verification of plasticity model in COMSOL

It is obtained that the crack pattern differs in all analyses. As BrigAde/Plus is used, two primary through cracks appears, while for the COMSOL analysis there are three. As the damage plasticity material model us used in BrigAde/Plus, an inclined crack is obtained close to the free edge. This can also be seen in the analysis performed in COMSOL Multiphysics.

It is also in the inclined crack the maximum crack width is found for these analyses, which is 0,066 mm and 0,092 mm for the damage plasticity analysis and COMSOL analysis respectively. That the maximum crack width is found in the inclined crack is consistent with results obtained in earlier analyses in this report. However, as it is the primary through cracks which are of interest, the crack width of the inclined cracks will be neglected also in this analysis. For the primary through cracks, the resulting crack width is displayed in Table D-1. Obtained is a crack width of very similar magnitude for all analyses.

\[ \text{Table D-1 Crack widths obtained in BrigAde/Plus and COMSOL Multiphysics} \]

<table>
<thead>
<tr>
<th>Smeared crack</th>
<th>Damage plasticity</th>
<th>COMSOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_{\text{max}} ) (mm)</td>
<td>0,037</td>
<td>0,041</td>
</tr>
</tbody>
</table>

**Analysis 2 – Linear tension stiffening curve**

In this analysis, the linear tension stiffening curve is applied in all models. The analysis will imply whether the two programs yield similar results under the same assumption of tension stiffening. The resulting crack pattern is displayed in Figure D-5 – D-7 for the smeared crack, damage plasticity and COMSOL analysis respectively.

*Figure D-5 BrigAde/Plus - Smeared cracking*

*Figure D-6 BrigAde/Plus - Damage plasticity*
Appendix D – Verification of plasticity model in COMSOL

As seen, none of the performed analysis results in any major cracks. What is seen in the figures is small micro cracks initiating in the base of the concrete member. According to (Malm [A], 2016), a linear tension stiffening curve tends to overestimate the stiffness of the concrete material. The results in this analysis confirms this statement as the member seems stiff enough to resist cracks for the applied strain.

**Analysis 2 – Reduced reinforcement content**

In order to be able to check the model as the linear tensions stiffening curve is used it is essential to obtain a major cracks with a comparable crack width. Hence, the same analysis is performed again with a reduced reinforcement area in order to force a crack. For this purpose, the bar diameter is reduced to 12 mm while the centre distance is kept, such that the member contain $\phi_{12 \times 65}$ in both faces. The resulting crack pattern is displayed in Figure D-8 - D-10 for the smeared crack analysis, damage plasticity analysis and COMSOL analysis respectively.
As a linear tension stiffening curve is used in all models, similar crack patterns are obtained. In the smeared crack analysis, one major crack is obtained, and an initiation of a second can be seen. This second crack can be seen more clearly in the damage plasticity analyses, while in the COMSOL analysis, it have fully developed.

The resulting maximum crack widths are found in Table D-2. It is obtained that in this case, the COMSOL analysis predict the largest crack width while the damage plasticity analysis result in the smallest.

**Table D-2 Resulting crack widths from the analyses in BrigAde/Plus and COMSOL Multiphysics**

<table>
<thead>
<tr>
<th></th>
<th>Smeared crack</th>
<th>Damage plasticity</th>
<th>COMSOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{\max}$ (mm)</td>
<td>0.080</td>
<td>0.075</td>
<td>0.085</td>
</tr>
</tbody>
</table>

**Discussion**

Displayed in Figure D-11 is a comparison between the two analyses. The values which are displayed are normalized with respect to the crack widths obtained in the COMSOL analyses. No consistency is obtained for in which model the largest crack is found, but it can be seen that the maximum deviation obtained is 13\% for the damage plasticity material model in analysis 1.

![Figure D-11 Obtained crack widths normalized with respect to the crack obtained in COMSOL](image)

The crack width obtained in COMSOL Multiphysics correspond fairly well to the results obtained with the smeared crack material model in BrigAde/Plus. However, the crack pattern seems to correspond better with the results obtained with the damage plasticity model.