Analys och jämförelse av ISDA SIMM och VaR för initiala säkerhetskrav i swap-portföljer

LUDVIG HAMILTON
Analys och jämförelse av ISDA SIMM och VaR för initiala säkerhetskrav i swap-portföljer

LUDVIG HAMILTON
Abstract

In this study the ISDA SIMM and the initial margin requirements for non-centrally cleared over the counter derivatives is investigated and compared with the traditional risk measure value-at-risk.

The empirical results suggest that for swap portfolios the ISDA SIMM achieves its set out purpose of being less volatile and more transparent than the 10-day value-at-risk on a 99% confidence level. However, the SIMM framework will require market participants to be continuously updated and provided calibration parameters which reflect current market conditions from ISDA.
Analys och jämförelse av ISDA SIMM och VaR för initiala säkerhetskrav i swap-portföljer

Sammanfattning

I denna studie jämförs ISDA SIMM initiala säkerhetskrav för ej centralt clearade OTC-derivat med en traditionell riskmätningsmetod vid namn value-at-risk.

Resultatet antyder att ISDA SIMM för enklare swap-portföljer uppfyller kraven satta i form av att vara mer stabil och transparent än en 10 dagars value-at-risk på en 99%ig konfidentsnivå. SIMM-modellen kräver dock att marknadsaktörerna kontinuerligt blir givna kalibreringsparametrarna som återspeglar marknadsförhållanden av ISDA.
Acknowledgements

I would like to thank my supervisor at the Royal Institute of Technology (KTH), Dr Anja Janssen.
## Notations and Abbreviations

Table 1: Abbreviations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EMIR</td>
<td>European Market Infrastructure Regulation</td>
</tr>
<tr>
<td>BCBS</td>
<td>Basel Committee on Banking Supervision</td>
</tr>
<tr>
<td>IOSCO</td>
<td>The International Organization of Securities Comissions</td>
</tr>
<tr>
<td>ISDA</td>
<td>International Swaps and Derivatives Association</td>
</tr>
<tr>
<td>SIMM</td>
<td>Standard Initial Margin Model</td>
</tr>
<tr>
<td>IRS</td>
<td>Interest Rate Swap</td>
</tr>
<tr>
<td>MTM</td>
<td>Mark-to-Market</td>
</tr>
<tr>
<td>IM</td>
<td>Initial Margin</td>
</tr>
<tr>
<td>VM</td>
<td>Variation Margin</td>
</tr>
</tbody>
</table>
## Contents

1 Introduction ................................................................. 8
   1.1 Background ............................................................... 8
   1.2 Research Question ...................................................... 8
   1.3 Goal and Purpose ....................................................... 9
   1.4 Scope and Limitations .................................................. 9

2 Finance and Counterparty Risk Theory .................................. 10
   2.1 Zero Coupon Bond ........................................................ 10
   2.2 The Yield Curve .......................................................... 10
   2.3 Over-the-counter derivatives .......................................... 10
      2.3.1 The Plain Vanilla Interest Rate Swap .......................... 10
   2.4 Counterparty Risk Theory .............................................. 11
      2.4.1 Counterparty versus Lending Risk .............................. 11
      2.4.2 Settlement and Pre-Settlement Risk ............................ 12
      2.4.3 Mitigating Counterparty Risk .................................. 12
      2.4.4 Collateral .......................................................... 12

3 The ISDA SIMM ............................................................... 14
   3.1 Background and Objectives ............................................ 14
   3.2 Criteria and Constraints ............................................... 14
   3.3 Product and Risk Classes .............................................. 15
   3.4 Structure of Margin Calculation ..................................... 15
      3.4.1 Risk Factors ....................................................... 16
      3.4.2 Definition of Sensitivity ....................................... 16
      3.4.3 Delta Margin Calculation ...................................... 16
      3.4.4 Vega Margin and Curvature Margin Calculation ............. 18

4 Mathematical Background .................................................. 20
   4.1 Interest Rate Theory ................................................... 20
      4.1.1 LIBOR Rates ...................................................... 21
      4.1.2 The Overnight Indexed Swap Rate .............................. 21
      4.1.3 Discounting ....................................................... 21
   4.2 Interest Rate Swaps ..................................................... 22
      4.2.1 Swap Rate ........................................................ 22
   4.3 Risk Measure ............................................................ 23
      4.3.1 Properties ........................................................ 23
      4.3.2 The Value at Risk ............................................... 24
      4.3.3 Calculating the Value at Risk ................................. 25

5 Methodology ................................................................. 26
   5.1 Data ................................................................ 26
   5.2 Assumptions and Simplifications ...................................... 26
   5.3 Portfolio Construction .................................................. 26
   5.4 Historical Value at Risk ............................................... 26
5.5 Standard Initial Margin Computation

6 Results
6.1 Backtesting
6.2 Return distribution

7 Discussion
7.1 Evaluation of chosen methods
7.1.1 The Value-at-Risk and Portfolio Linearity
7.1.2 Negative rates
7.1.3 Credit Value Adjustments and the Single Curve Framework
7.2 Criticism of the SIMM
7.3 Conclusion
7.3.1 Further studies

8 Appendix
8.1 Currency Volatilities
8.2 Risk weights per vertex (regular currencies)
8.3 Risk weights per vertex (high-volatility currencies)
8.4 Risk weight per vertex (low-volatility currencies)
8.5 Correlations for Different Maturities
8.6 Interest Rate Risk - Delta Concentration Thresholds
8.7 Interest Rate Risk - Vega Concentration Thresholds
1 Introduction

1.1 Background

On March 15th 2013 the European Union enacted the European market infrastructure regulation ("EMIR") [2]. The objective of this regulation is to reduce risk as well as helping to prevent systemic distress on the financial system. EMIR is enacted in phases with the major entities being subject first, and smaller organizations follow within the next few years.

Furthermore, Article 11 of the EMIR requires in scope entities entering into financial contracts not monitored by a third party to put in place procedures and arrangements to measure, monitor and mitigate associated operational and counterparty risk. Specifically, such entities must:

1. Obtain timely electronic confirmations of the terms of the contracts.
2. Put in place a formal risk management process for such contracts.
3. Have an agreed dispute resolution process with counterparties.
4. Value all contracts daily on a market-value basis.
5. Exchange liquidity when entering into the contract (known as the initial margin) and provide liquidity over the lifetime of the contract (known as the variation margin) depending on the development of its value.

It is point 5. which is of importance for this project. Counterparties might use different mathematical models and calibrations when calculating the liquidity required to enter a financial contract. The international swap and derivative association ("ISDA") has produced a framework named Standard Initial Margin Model ("SIMM") in order to harmonize the calculations of the initial liquidity for such contracts. The ISDA SIMM is based on six risk classes: Interest rate, credit (qualifying), credit (non-qualifying), equity, commodity and foreign exchange. Noteworthy is that banks are not required to follow the exact ISDA SIMM framework, but they are required to post the initial margin based on a 10 day 99 percentile confidence (presented later in the project). However, the purpose of the ISDA SIMM is to make sure that the posting of margin is symmetrical, i.e., that both counterparties provide liquidity which is consistent with the value of their contract. ISDA frameworks have previously been adapted by market participants without legal enforcement with the best example being the ISDA Master Agreement [6].

1.2 Research Question

In this study, I will implement a version of the ISDA SIMM and compute the initial margin. Furthermore I will compare it to other ways of calculating the required collateral between counterparties, namely the one with a historical
value-at-risk model at a 99 percentile confidence. Specifically the research question will be:

*How does the ISDA SIMM variance-covariance approximation compare to a historical value-at-risk when calculating the initial margin, and how will the posting of initial margin affect market participants under the new regulations?*

### 1.3 Goal and Purpose

The purpose of this study is to try and quantify the impact and compare the collateral required by EMIR and the ISDA SIMM with a simple historical value-at-risk model for parties trading relevant financial contracts. By extension this should hint at the impact the regulation framework would have for risk management and control.

### 1.4 Scope and Limitations

To limit the size of this study, I have posed some limitations on its scope, primarily related to the vast amount of portfolios one can form using various financial contracts. I will limit the study to only consider six different portfolios. The first three portfolios will consist of a single common derivative, namely plain vanilla interest rate swaps (also called ordinary interest rate swap and presented in Section 2) with maturity 2, 5 and 10 years respectively. Furthermore, the last three portfolios will consider different positions in all the previously mentioned swaps, presented more thoroughly in Section 5. Lastly, there are many ways of calculating financial risk and required collateral within the given framework provided by the EMIR regulation. This project will only (apart from the ISDA SIMM) consider the historical value-at-risk model at the 99th percentile.
2 Finance and Counterparty Risk Theory

In this section, the theory and some financial concepts fundamental to the project will be introduced followed by counterparty risk theory. Section 2.4 Counterparty Risk Theory is based upon the thesis [9] sections 2.1.1 - 2.1.2, originally from [6].

2.1 Zero Coupon Bond

A bond is a debt security which repays borrowed money with interest known as coupons at a fixed interval and with the principal repayment at the expiration date called maturity. The zero coupon bond is the simplest type of bond. The zero coupon bond consist only of a single payment at maturity where buyer pays $Z_0$ at time $t = 0$ and receives 1 at maturity. The most common zero coupon bond is the U.S. T-bill [5].

Zero coupon bonds are often used to analyze the yield curve. That is, the relationship between the interest rates and different times of maturity for fixed income securities [5].

2.2 The Yield Curve

The yield curve is the relationship between interest rates and time to maturity of debt securities and is generally a flattening curve, but depending on the macro-economic environment it could take a different shape (such as flat or decreasing). From this relationship it is possible to derive the yields for an equivalent bond which does not pay coupons, i.e., the zero coupon bond [5].

2.3 Over-the-counter derivatives

Financial derivatives are financial contracts which derive their value from the performance of an underlying asset of some sort. Common derivatives are forwards, futures and swaps [5]. In this report interest rate swaps are of particular interest and will be discussed further in Section 2.3.1. Financial derivatives can broadly be categorized into two sets, that is exchange-traded derivatives and over-the-counter derivatives. The difference between those two type of contracts is that exchange traded derivatives are traded on a derivatives exchange such as the Chicago Board Options Exchange, whilst OTC derivatives are negotiated privately between two counterparties without going through an exchange or intermediary [6].

2.3.1 The Plain Vanilla Interest Rate Swap

Swaps are, unlike most standardized derivatives, such as options and futures, customized OTC derivatives negotiated between private parties. There are
many different types of swap contracts such as the interest rate swap or currency swap.

The plain vanilla interest rate swap is an agreement between two parties to exchange interest rate payments (cash flows) on predetermined settlement dates in the same currency. Party A agrees to pay Party B a predetermined fixed rate of interest on a notional principal and concurrently, Party B agrees to pay Party A a floating interest rate on the same notional principal under the same period of time. The dates on which the cash flow occurs are called the settlement dates and the periods in between are called settlement periods [5].

The floating interest payment Party A makes to Party B is generally based on the market index LIBOR [5]. The theoretical pricing of swaps will be discussed under Mathematical Background, in Section 4.1.

2.4 Counterparty Risk Theory

Counterparty credit risk (commonly referred to as counterparty risk) is defined as the risk arising from the possibility that the counterparty may default on amounts owed in a transaction, and hence not being able to fulfill their contractual obligation due to insolvency. Broadly speaking, counterparty risk arises from several classes of financial products: OTC derivatives (e.g. interest rate swaps), exchange traded products and securities financial transactions where the OTC derivatives will be under consideration of this study.

2.4.1 Counterparty versus Lending Risk

Generally counterparty risk can be thought of as being similar to lending risk. One party owes an amount to his or her counterpart and may because of circumstances and insolvency be unable to pay the counterparty. This concept can be applied to everyday transactions such as loans, bonds, mortgages and so on. Lending risk is characterized by two things, the first is that the notional amount of risk during the lending period is known to a certain degree. Market variables such as interest rate can only create a limited amount of uncertainty over the borrowed amount. The second characterization is that only one of the parties takes lending risk as opposed to both being exposed to that specific risk. As with counterparty risk, the loss is inflicted because the obligor is unable to meet contractual obligations, i.e., pay up. There are however two aspects which contrast counterparty risk. The first is that the value of the contract in the future is significantly uncertain and the second one is that since the value of the contract can be negative counterparty risk is bilateral, i.e., it has two sides where both parties carry exposure [6].
2.4.2 Settlement and Pre-Settlement Risk

Counterparty risk is mainly associated with pre-settlement risk, and we will henceforth use the terms indifferently. Pre-settlement risk is the risk of default prior to expiration (settlement) of the contract. But one must also consider settlement risk, the risk that arises at settlement time due to timing differences between each party fulfills its contractual obligations. [6]

Unlike counterparty risk the settlement risk is characterized by a very large exposure, potentially the whole notional amount of the transaction. In contrast, the probability of default before settlement is much more likely than during the settlement period, but settlement risk can be more complex and span out over only a small amount of time (often just days or hours). An example could be a (physical) delivery of a commodity during a specified time period. Generally, the balance between settlement and pre-settlement risk depends on the contract. Spot contracts, for instance, carry more settlement risk whilst long-dated swaps carry mainly pre-settlement risk [6].

2.4.3 Mitigating Counterparty Risk

There are a number of ways a market participant can reduce counterparty risk. Some are rather simple contractual mitigants whilst other methods are more complex and costly to implement. It’s apparent that no mitigant is perfect and there will always exist residual counterparty risk. It’s noteworthy that risk mitigants do not remove counterparty risk but rather convert it into various forms of financial risk in the same way that insurance doesn’t remove the risk, but rather portions it out. Some natural ways to reduce counterparty risk is through usage of netting¹, collateral, other contractual clauses or requirements, hedging or run the derivatives through central counterparties. However, the mitigation of counterparty risk is two faced in the sense that it does reduce the risk (or rather converts it) to improve financial stability, but it may also lead to a reduction in constraints such as liquidity requirements and grow out of control putting the financial stability in place. Specifically, this project will consider collateral [6].

2.4.4 Collateral

Collateral, also known as margin, is one of the tools mentioned above to reduce counterparty risk. Specifically, collateral is defined as an asset posted by one counterparty to the other which supports a risk in a legally enforceable way and may be liquidated immediately in the case of a default. There are two different types of collateral relating to OTC derivatives, the initial margin and variation margin.

Variation Margin

¹Aggregating two or more obligations to achieve a reduced net obligation
The variation margin (sometimes called "mark-to-market margin") is a variable market payment made by counterparties to each other based on adverse price movements of the agreement to bring the equity in a transaction up to the minimum level agreed upon. The variation margin is based on the idea that collateral should reflect the mark-to-market value of the underlying transaction which can be either negative or positive from each party’s point of view. However, in a situation where one party is declared insolvent the variation margin might not be sufficient due to e.g. close-out or bankruptcy costs, additional collateral is sometimes used in the form of initial margin [6].

**Initial Margin**

The initial margin is the required amount (or minimum amount) of collateral to enter into an agreement with a counterparty. Historically the OTC derivatives market has almost entirely relied on the variation margin and usage of initial margin has been rare. However, new regulations such as the EMIR covering collateral requirements in bilateral markets will make the initial margin much more common. Specifically, the initial margin will be the margin requirement to open a position in the appropriate market [6].
3 The ISDA SIMM

This section will describe the purpose of the ISDA SIMM as well as its components. If nothing else is specified, the theory in this section is based upon the research fellowship [15] project, originally from [12, 13].

3.1 Background and Objectives

In 2011, the G20 called upon the Basel Committee on Banking Supervision and The International Organization of Securities Commissions to develop consistent global standards for margin calculations for non-centrally cleared derivatives. In order to comply with the newly established guidelines the ISDA announced in 2013 the start of an industry-wide initiative to develop a standard initial margin model which could be used by relevant participants to determine the calculations of the initial margin.

The common initial margin framework would have several benefits such as more efficient planning and management of liquidity need for margin calls, transparent conflict resolutions and consistent government and regulatory oversight [11]. In the absence of a common framework each participant would have to build IM models for each of its counterparties which would put stress on the participants’ day-to-day operations.

3.2 Criteria and Constraints

The ISDA identified nine core criteria to which an initial margin model should adhere to satisfy the BCBS-IOSCO rules. These include:

1. **Non-procyclicality** - margins are not subject to continuous change due to changes in market volatility.

2. **Ease of replication** - easy to replicate calculations performed by a counterparty.

3. **Transparency** - calculations provide contribution of different components to enable dispute resolutions.

4. **Quick to calculate** - low analytical overhead to enable fast primary and re-run calculations.

5. **Extensible** - the methodology is conducive to additions of new risk factors or products required by the industry or regulators.

6. **Predictability** - the initial margin demands need to be predictable and consistent.

7. **Costs** - reasonably low cost and operational burden on the market participants and regulators.
8. Governance - recognize an appropriate role between regulators and the industry.

The calculations of the IM could involve shocks and recalibrations depending on the underlying assets or liabilities. ISDA deemed the best way to approximate a contracts’ response to shock was to compute a sensitivity (delta) of a contract for each risk factor, and multiplying each sensitivity by the respective risk factor shock size.

3.3 Product and Risk Classes

The SIMM requires every trade to be assigned a specific product class. For each of these four product classes the IM is calculated separately. The four different product classes are (i) commodity, (ii) credit, (iii) equity and (iv) interest rate and foreign exchange.

After the appropriate product class is identified the portfolio is broken down into one or more of following six risk classes: (i) Credit (qualifying), (ii) credit (non-qualifying), (iii) commodity, (iv) equity, (v) interest rate and (vi) foreign exchange risk. The difference between credit (qualifying) and credit (non-qualifying) is that the former consist of investment grade and high yield credit issued by sovereigns and corporations, whereas the latter consist of mortgage backed securities.

3.4 Structure of Margin Calculation

The SIMM utilizes a sequence of nested variance-covariance relations presented thoroughly in [12] to calculate capital and margin requirements. In the SIMM model, the IM is calculated for each risk class $X$ using the delta, vega and curvature margin presented in equation (1).

$$IM_X = \text{DeltaMargin}_X + \text{VegaMargin}_X + \text{CurvatureMargin}_X.$$  (1)

The IM for each product class is then obtained by equation (2).

$$\text{SIMM}_{\text{Product}} = \sqrt{\sum_r IM_r^2 + \sum_{r \neq s} \phi_{r,s} IM_r IM_s}$$  (2)

Where $r$ and $s$ are summed over each of the risk classes and $\phi_{r,s}$ is the correlation between the risk factors which has been pre-computed by ISDA, and for the specific products analyzed in this report can be found in Appendix Section 8.4 Table 5. The overall initial margin to be posted is then obtained by summing over all four product classes presented in equation (3).

$$\text{SIMM} = \text{SIMM}_{\text{Credit}} + \text{SIMM}_{\text{Commodity}} + \text{SIMM}_{\text{Equity}} + \text{SIMM}_{\text{RatesFx}}.$$  (3)
This approach forms what ISDA refers to as a margin cover assertion, which achieves a cover standard for all portfolios given it does so for single risk factor, while avoiding the need to cross check the margin scheme against all possible portfolios.

3.4.1 Risk Factors

One of the conditions that the margin cover assertion standard is required to satisfy is that is should span all the randomness or risk of all portfolios under consideration. For interest rate derivatives a list of the most commonly traded tenors such as 5 year or 10 year maturities or principal component analysis could be used as proxies for the risk factors. The ISDA SIMM uses first three principal components which they found to have an explanatory power of around 99.5% and which represents a parallel shift in the relevant interest rate curve, such as the swap or yield curve.

3.4.2 Definition of Sensitivity

The main input of the SIMM model are the sensitivities \( s \) to different risk factors. If we let the value of the instrument be \( V(x) \), where \( x \) is the given value of a risk factor, then for interest rate and credit the sensitivity is defined as

\[
s = V(x + 1\text{bp}) - V(x). \tag{4}
\]

For equity, commodity and foreign exchange:

\[
s = V(x + 1\%x) - V(x).
\]

Where the shift of 1bp (one basis point and is equivalent to 0.01%) or 1% represent a shock to the value of the underlying.

3.4.3 Delta Margin Calculation

There are two ways to calculate the delta margin. One is for the interest rate risk class and the second one is for the non-interest rate risk classes. For the interest risk class the procedure is as follows:

1. Calculate the net sensitivity \( s_{m,i} \) for each of the risk factors \( (m, i) \) where \( m \) is the rate tenor and \( i \) is the index name of the sub yield curve.

2. Calculate the weighted net sensitivity.

\[
WS_{m,i} = RW_m \cdot s_{m,i} \cdot CR_b, \tag{5}
\]

where \( RW_m \) is the risk weight pre-defined by ISDA from Appendix Section 8.1 - 8.3 depending on the volatility of the currency and \( CR_b \) is the concentration risk factor.
where $T_b$ is the predefined concentration threshold for each currency $b$ presented in Appendix section 8.6 Interest Rate Risk - Delta Concentration Thresholds. The maximum function and the concentration threshold ensure that the concentration factor is capped at one, which represents no real benefits to portfolios which are finely concentrated while simultaneously ensuring that the risk weight increases if a portfolio is highly concentrated with respect to the risk factors.

3. Aggregate the weighted sensitivities within each currency by equation (8).

$$K = \sqrt{\sum_{i,m} WS_{m,i}^2 + \sum_{i,m} \sum_{(j,n) \neq (i,m)} \phi_{i,j} \rho_{m,n} WS_{m,i} WS_{n,j}}$$

where $\phi_{i,j}$ is the sub-curve correlation parameters and $\rho_{m,n}$ are the tenor correlation parameters which can be found in Appendix section 8.5 Correlations for Different Maturities, and are set by ISDA. The purpose of the correlation parameters is to ensure that diversification is rewarded within risk factor classes of a particular currency.

4. Calculate the Delta Margin by aggregating the currency level sensitivity within the interest rate risk class across currencies according to equation (9).

$$\text{DeltaMargin} = \sqrt{\sum_b K_b^2 + \sum_{b \neq c} \gamma_{bc} g_{bc} S_b S_c}$$

where

$$S_b = \max\left(\min\left(\sum_{i,m} WS_{m,i} K_b, K_b\right), -K_b\right)$$

$$g_{bc} = \frac{\min(CR_b, CR_c)}{\max(CR_b, CR_c)}$$

and $\gamma = 27\%$ for regular volatility currencies.

For non-interest rate risk classes the calculations are similar, but carry different calculations of sensitivities with respect to different risk factors. The details can be found in [13].
3.4.4 Vega Margin and Curvature Margin Calculation

Instruments that are options or include an option are subject to additional margin requirements because one has to account for vega and curvature risk. Vega risk is the instrument’s sensitivity to changes in the volatility whereas curvature risk is the profit or loss due to a specified shock in the underlying asset not explained by the local delta of the position. The vega margin can be calculated in the following way.

1. The vega risk for each instrument $i$ with respect to different risk factors $k$ is estimated by

$$VR_{i,k} = HVC_c \sum_j \sigma_{k,j} \frac{\partial V_i}{\partial \sigma},$$

where $\sigma_{k,j}$ is the implied at-the-money volatility of the risk factor $k$ at each volatility tenor $j$ for interest rate and credit instruments and $\partial V_i/\partial \sigma$ is the sensitivity of the price for instrument $i$ with respect to the volatility, i.e., the vega. The term $HVC_c$ is the historical volatility rate for asset class $c$, which for interest-rate instruments is set to equal 1. For other product classes the volatility can be calculated as:

$$\sigma_{k,j} = RW_k \sqrt{\frac{365}{14} \Phi^{-1}(0.99)}$$

where $\Phi^{-1}(0.99)$ is the inverse cumulative distribution function of the standard normal distribution.

2. Find the net vega risk exposure across instruments to each risk factor. For interest rate vega risk the net vega can be calculated by

$$VR_k = VR \left( \sum_i VR_{i,k} \right) VCR_b,$$

where $VCR_b = \max(1, \sqrt{|\sum_{i,k} VR_{i,k}/VT_b|})$ and $VT_b$ is the vega concentration threshold.

3. Aggregate the vega risk exposure within each bucket.

$$K_b = \sqrt{\sum_k VR_k^2 + \sum_k \sum_{l,k} \rho_{k,l} f_{k,l} VR_k VR_l},$$

where the inner correlation adjustment factor $f_{k,l} = 1$ for interest rate risk class and for other risk classes are defined by:

$$f_{k,l} = \frac{\min(VCR_k, VCR_l)}{\min(VCR_k, VCR_l)}.$$
4. Aggregate the vega risk exposure across buckets. Within each risk class according to equation (13).

\[
\text{VegaMargin} = \sqrt{\sum_b K_b^2 + \sum_b \sum_{c \neq b} \gamma_{b,c}g_{b,c}S_bS_c + K_{\text{residual}}}
\]

where \(S_b = \max\left(\min\left(\sum_{k=1}^K VR_k, K_b\right), -K_b\right)\). The outer correlation adjustment factors \(g_{b,c}\) equals one for all risk classes other than interest rate risk where it is defined as:

\[
g_{b,c} = \frac{\min(VCR_b, VCR_c)}{\min(VCR_b, VCR_c)}.
\]

Similarly we can calculate the curvature margin, for details see [13].
4 Mathematical Background

This section will describe the mathematical background and theory that is used in the quantitative part of the study. Section 4.1 Interest Rate Theory and Section 4.2 Interest Rate Swaps is based upon the thesis [3] sections 3.1 and 3.2, originally from [14].

Let $(\Omega, \mathcal{G}, \mathcal{G}_t, Q)$ be a probability space where $\Omega$ is the outcome space and contains the possible outcome $\omega \in \Omega$. $\mathcal{G}$ is the $\sigma$-algebra and $\mathcal{G}_t$ is the filtration up to time $t$ where for $t \leq u$ it holds that $\mathcal{G}_t \subseteq \mathcal{G}_u \subseteq \mathcal{G}$. The interpretation of the filtration $\mathcal{G}_t$ is that it contains all available market information up to time $t$. Furthermore, let $\mathcal{F}$ denote the $\sigma$-algebra such that the filtration $\mathcal{F}_t$ contains the same information as $\mathcal{G}_t$ except the default events. This means that $\mathcal{F}_t$ is a sub-filtration of $\mathcal{G}_t$, i.e., $\mathcal{F}_t \subseteq \mathcal{G}_t$ where $\mathcal{F}_t$ only contains the available risk-free market information. The probability measure $Q$ is the risk-neutral arbitrage free probability measure such that under the risk-free bank account, the process

$$dB_t = B_t r_t dt, \quad B_0 = 1,$$

all tradable assets divided by $B_t$, i.e., discounted are martingales [14].

4.1 Interest Rate Theory

Define a continuously compounded forward rate $F(t; S, T)$ at time $t$ for a period $[S, T]$ as

$$e^{F(t; S, T)(T - S)} = \frac{P(t, S)}{P(t, T)}.$$

Now, introduce an instantaneous forward rate such that

$$f(t, T) = \lim_{\Delta \to 0} F(t; T, T + \Delta).$$

The continuously compounded short rate is defined as $r(t) = f(t, t)$ so that a risk-free bank account evolves with the process $B_t = \exp(\int_0^t r(s)ds)$. For simplification the discount rate at time $t$ for a cash flow received at time $T$ will be denoted by

$$D(t, T) = \frac{B_t}{B_T} = \exp\left(-\int_t^T r(s)ds\right).$$

The price of a risk-free zero coupon bond at time $t$ received at time $T$ is hence given by

$$P(t, T) = \exp\left(-\int_t^T f(t, s)ds\right),$$

where $f(t, s)$ is the instantaneous forward rate, or equivalent using the short rate
\[ P(t, T) = \mathbb{E}^Q \left[ \exp \left( - \int_t^T r(s) ds \right) \bigg| \mathcal{F}_t \right]. \]

### 4.1.1 LIBOR Rates

The London Interbank Offered Rate ("LIBOR") is a reference rate for which leading banks in London borrow from other banks [14]. The LIBOR rate is calculated daily for five different currencies and seven maturities as the average of the estimated bank rates. Since banks and financial institutions are not risk free credit risk is reflected in the rates. The LIBOR spot rate is defined as

\[ L(t, T) = \frac{1}{T-t} \left( \frac{1}{P(t, T)} - 1 \right), \quad (8) \]

and the forward LIBOR rate is defined as

\[ L(t; T, S) = \frac{1}{T-S} \left( \frac{P(t, T)}{P(T, S)} - 1 \right). \quad (9) \]

A 360 day-count convention is used for the LIBOR rates as opposed to the approximately 250 trading day regular. For more convenient notation, the forward LIBOR rate at \( t \) between time \( T_{i-1} \) and \( T_i \) will be denoted

\[ L_i(t) = L(t; T_{i-1}, T_i). \]

### 4.1.2 The Overnight Indexed Swap Rate

Overnight Indexed Swap ("OIS") are interest rate swaps where two counterparties exchange a fixed rate for a floating rate based overnight index rate. This index rate is a weighted average of the unsecured overnight lending rates in the interbank market. The overnight index rate for the U.S. dollar is the effective federal funds rate and in the Euro zone the Euro Overnight Index Average ("EONIA") [6].

### 4.1.3 Discounting

Traditionally market participants have often used the LIBOR and LIBOR-swap rates as proxies for risk free rates when valuing financial derivatives with future cash flow. However, during the credit crisis in 2007 and 2008 the LIBOR rates were quoted with a large spread compared to treasury bills and OIS rates. The 3-month U.S. dollar LIBOR Treasury spread, also known as the TED spread, which usually had been in the 50 basis point range peaked at 450 basis points during the crisis. Similarly, the EURIBOR-OIS spread which historically had been in the within the 10 basis point range reached a peak of 350 basis points. However, according to Hull and White in [7] there's no “perfect” risk free rate (further amplified under stressed market conditions), but the OIS rate is the best proxy currently available.
4.2 Interest Rate Swaps

The most common interest rate swap is the fixed-for-floating same currency interest rate swap. One party A pays a floating rate \( L \), typically a 3- or 6-month LIBOR rate, to party B while simultaneously pays a predetermined fixed swap rate \( S \) to party A. The direction of the swap is often referred to as either a payer or a receiver swap where the party who receives the fixed swap rate has a receiver swap and the counterparty has a payer swap [14].

Let payments of the fixed leg occur at the dates \( T_1, ..., T_n \) and payments of the floating leg occur at the dates \( T'_1, ..., T'_N \) and the initiation of the swap is at time \( T_0 \). Furthermore, let the notional amount of the swap be \( K \) and let \( \Delta T_i = T_i - T_{i-1} \) and \( \Delta T'_i = T'_i - T'_{i-1} \). Then at time \( T'_i \) party A will pay

\[
KL(T'_{i-1}, T_i)\Delta T'_i
\]

and at time \( T_i \) receive

\[
KS\Delta T_i.
\]

If \( T_i = T'_i \) and \( \Delta T_i = \Delta T'_i \) the net cash flow at time \( T_i \) is

\[
K\Delta T_i (L(T_{i-1}, T_i) - S).
\]

4.2.1 Swap Rate

A swap can be regarded as two bonds [14]. With this view party A has bought a fixed rate bond with a return \( S \) issued by \( B \) and receives coupons based on the fixed rate. Simultaneously, party B has bought a floating rate bond issued by \( A \) and receive coupons according to the prevailing spot rate. Usually the swap rate \( S \) is determined such that the initial value of the swap is zero. With the swap rate starting at \( T_0 \) and payments of the fixed leg occurring at \( T_1 < ... < T_n \) and payments of the floating leg at \( T'_1 < ... < T'_N \), using the previously introduced notation, \( \Delta T_i = T_i - T_{i-1} \) and \( \Delta T'_i = T'_i - T'_{i-1} \) we must for a fair swap find a rate \( S \) such that at time \( t < T_0 \)

\[
\Pi(t;\text{swap}) = P_{fix}(t) - P_{float}(t) = 0. \tag{10}
\]

Utilizing the fact that the payback of the notional of the swap should be the same and the risk-neutral valuation framework

\[
KS \sum_{i=1}^{n} P(t, T_i)\Delta T_i - KB \sum_{i=1}^{N} E^Q \left[ \frac{1}{B_{T'_i}} L(T'_{i-1}; T'_{i-1}, T'_i)\Delta T'_i \mid \mathcal{F}_i \right] = 0 \tag{11}
\]

where \( P(t, T_i) \) is the discount rate for the fixed leg and using that the LIBOR forward rate \( L(T'_{i-1}; T'_{i-1}, T'_i) \) is a martingale under the \( Q^{T'_i} \)-measure
\[ KS \sum_{i=1}^{n} P(t, T_i) \Delta T_i - K \sum_{i=1}^{N} P(t, T'_i) L(t; T'_{i-1}, T'_i) \Delta T'_i = 0 \]

using the definition of the LIBOR forward rate from equation (9)

\[ KS \sum_{i=1}^{n} P(t, T_i) \Delta T_i - K \sum_{i=1}^{N} P(t, T'_i) \left( \frac{P(t, T'_{i-1})}{P(t, T'_i)} - 1 \right) = 0. \]

Simplifying yields

\[ KS \sum_{i=1}^{n} P(t, T_i) \Delta T_i - KP(t, T_0) + KP(t, T'_n) = 0, \]

and solving this for \( S \) yields the fair swpa rate

\[ S = \frac{P(t, T_0) - P(t, T'_n)}{\sum_{i=1}^{n} P(t, T_i) \Delta T_i}. \]

### 4.3 Risk Measure

To determine the amount of liquidity an investor (or regulator) require in order to make a financial position acceptable a risk measure is widely used. The purpose of a risk measure is to measure the liquidity required as a function of the positions risk.

#### 4.3.1 Properties

Let \( \rho(X) \) be the risk measure as a function of the net worth \( X \) of an asset between times \( t_0 \) and \( t_1 \). The following properties are natural requirements for a good risk measure. \([4]\)

1. **Translation invariance** can be written as

   \[ \rho(X + cR_0) = \rho(X) - c \]

   which states that adding a certain amount \( c \) of cash and using it to buy a risk free asset, reduce the risk by the same amount.

2. **Monotonicity** can be written as

   \[ \rho(X_1) \leq \rho(X_2) \]

   if \( X_2 \leq X_1 \) and states that if one of the two positions have a greater future net worth then that position is less risky.
3. **Positive homogeneity** can be written as

\[ \rho(\lambda X) = \lambda \rho(X) \]

for all \( \lambda \geq 0 \) and states that the risk increases linearly proportional to the positions size, i.e. if we double the position we double the risk.

4. **Subadditivity** can be written as

\[ \rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2) \]

which states that diversification is rewarded.

A risk measure that satisfies properties 1 through 3 is known as a coherent risk measure and it follows that it’s a useful tool for managing risk. A common and widely used risk measure is the *value at risk* presented in the section below. However, the value at risk does not satisfy property 3, to fix this problem one can use the expected shortfall (also known as the conditional value at risk). 

### 4.3.2 The Value at Risk

The Value-at-Risk (“VaR”) is a risk measure for investment loss. It estimates how much a set of investments might lose given market conditions in a pre-defined time frame such as a day or a week.

Given a portfolio of assets and liabilities, a time frame and a probability \( \alpha \) the VaR is informally defined as the worst possible loss given that the outcomes has a probability greater than \( \alpha \). The method assumes mark-to-market accounting and that no trading is done during the given time frame. Mathematically, the VaR for a portfolio \( X \) at time 1 according to level \( \alpha \in (0,1) \) with a risk free interest \( R_f \) is expressed the following way:

\[ \text{VaR}_\alpha(X) = \min(m : P(mR_f + X < 0) \leq \alpha) \]

It is often convenient to express the VaR in terms of the cumulative distribution function of the losses. To do so define the loss \( L = -X/R_f \) and the net gain from the investment \( X = V_1 - V_0 R_f \). Hence it follows that \( L = V_0 - V_1/R_f \) and the VaR in term of the cumulative distribution function of the losses at level \( \alpha \) can be expressed as:

\[ \text{VaR}_\alpha(X) = \min(m : P(L \leq m) \geq 1 - \alpha) = F^{-1}_L(1 - \alpha) \]

Where \( F^{-1}_L(x) \) is the ordinary inverse of \( F_L \), the distribution function of \( L \), or the generalized ordinary inverse if the former doesn’t exist. Furthermore, the value-at-risk satisfies the properties translation invariance, monotonicity and positive homogeneity presented in section 4.4.1 [4].
4.3.3 Calculating the Value at Risk

There are three widely used approaches to compute the VaR. The methods are the empirical method (also known as historical simulation), the variance-covariance method and Monte Carlo method. This report will use the empirical method for which the procedure is such that one assumes that the next time period can be described by the past, i.e., history repeats. The advantage with the historical method is that it’s non-parametric, meaning it does not require any assumptions of the underlying distribution function. However, this implies that the largest possible loss can not be larger than the historical maximum loss. The unknown empirical distribution of the random sample \( \{X_1, \ldots, X_n\} \) is given by

\[
F_{n,X}(x) = \frac{1}{n} \sum_{k=1}^{n} I\{X_k \leq x\}
\]

where \( I \) denotes the indicator function. It can be shown that the value at risk at level \( \alpha \in (0,1) \) can consistently be estimated by

\[
\text{VaR}_\alpha(X) = L_{\lfloor \alpha n \rfloor + 1, n}
\]

where \( L_{1,n} \geq \ldots \geq L_{n,n} \) is the ordered loss sample. The SIMM however, is derived from the variance-covariance method and discussion regarding the two approaches will be discussed in the end of the report. Briefly the variance-covariance method rests on the assumption that short term changes are normally distributed which isn’t always necessarily true and tends to ignore tail risk, which is better represented by distribution’s such as the student’s \( t \) [4].
5 Methodology

In this section, the methodology used to examine the research question of the study will be presented.

5.1 Data

The data consists of historical swap rates from January 1st 2007 to May 3rd 2018. Furthermore, several parameters introduced in Section 3, the ISDA SIMM, is provided in the various Appendix sections.

5.2 Assumptions and Simplifications

In order to manipulate a relatively constrained data set into a tangible result some assumptions regarding the swaps are made. The first assumption is that we work in a mono-curve (also known as single curve) framework as opposed to todays more common multi-curve framework presented and discussed in [3].

The second assumption is that the swap rates can be used as proxy for the discount rates is discussed in further detail in section 4.1.3. The third assumption is that coupons are paid out annually, the notional of the swap is €1,000,000 and there are 250 trading days in a year. Lastly the floating leg of the swap is defined as nil. The reason for this is that when a swap-contract is agreed upon the present value for each counterparty is zero under the no arbitrage conditions. However, the no arbitrage conditions also implies that the initial margin posted when entering into the contract by the fixed receiver will be equal to that of the floating receiver.

5.3 Portfolio Construction

There are many different portfolios that can be constructed from interest rate swaps with different maturities. In this report six different portfolios will be considered. The first three portfolios are simple one-derivative portfolios consisting of a same currency single long swap position with maturity 2, 5 and 10 years respectively. The fourth portfolio will consist of a so called steepener portfolio consisting of being short a 10 year and long a 2 year swap and is a strategy that seeks to exploit escalating yield differences. The fifth portfolio will be short a 10 year and 2 year swap while being long a 5 year. Finally, the sixth portfolio consist of long position in all three swap contracts. The definition of each portfolio is specified in Table 2: The portfolios.

5.4 Historical Value at Risk

In order to appropriately compare the ISDA SIMM with the historical value we need to work in the same time span and at an appropriate confidence level. This report consider a 10-day value at risk based on one year (250 trading days) of
<table>
<thead>
<tr>
<th></th>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
<th>Portfolio 3</th>
<th>Portfolio 4</th>
<th>Portfolio 5</th>
<th>Portfolio 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Year Swap</td>
<td>Long</td>
<td>Long</td>
<td>Long</td>
<td>Short</td>
<td>Long</td>
<td></td>
</tr>
<tr>
<td>5 Year Swap</td>
<td>Long</td>
<td></td>
<td>Long</td>
<td>Long</td>
<td>Long</td>
<td></td>
</tr>
<tr>
<td>10 Year Swap</td>
<td></td>
<td>Long</td>
<td>Short</td>
<td>Short</td>
<td>Long</td>
<td></td>
</tr>
</tbody>
</table>
historical data. The value of the fixed leg of the swap, defining the floating legs value as nil, at time \( t = 0 \), i.e., when entering into the contract, and maturity \( n \) years is as previously defined in equation (10) and (11)

\[
\Pi(0; \text{fixed leg}) = P_{fix}(0) - P_{float}(0) = KS \sum_{i=1}^{n} P(0, T_i) \Delta T_i.
\]

Since the coupons are paid annually we have \( \Delta T_i = 1 \) for all \( i \). Furthermore, since this report assumes discrete discounting (and compounding) as opposed to continuous and that the swap rates can be used as proxy \( P(0, T_i) \) can be written as

\[
P(0, i) = \frac{1}{(1 + S_{i,0})^i}
\]

where the subscript \( i, 0 \) of \( S \) means that it’s the swap rate for a swap of maturity \( i \in \{1, 2, \ldots, n\} \) at current time zero. Hence, the fixed leg of a swap of maturity in \( n \) years has the present value

\[
\Pi(0, \text{fixed leg}) = P_{fix}(0) = KS_n \sum_{i=1}^{n} \frac{1}{(1 + S_{i,0})^i}.
\]

(13)

To exemplify consider the fixed leg of a swap with maturity in two years with the current annual one and two year market swap rates of 4% and 5% respectively. The present value of the swaps fixed leg would then be

\[
P_{fix}(0) = \€1,000,000 \left( \frac{5\%}{(1 + 4\%)^1} + \frac{5\%}{(1 + 5\%)^2} + \frac{1}{(1 + 5\%)^2} \right)
\]

\[
\approx \€1,000,458.
\]

Now, to determine the 10-day VaR consider the historical shifts in the discount curve and let the swaps value under current conditions be denoted by \( P_{fix}^{(0)}(0) \). Let for the previous 250 trading days \( j \in \{-250, -249, \ldots, -1\} \) the historical swap rate with maturity \( n \) be \( S_{n,j} \). Define the proxy discount shift

\[
\text{shift}_{n,j} = \frac{S_{n,j+1}}{S_{n,j}}
\]

then revalue the swap given the possible historical shifts in the discount curve according to

\[
P_{fix}^{(j)}(0) = KS_{n,0} \sum_{i=1}^{n} \frac{1}{(1 + \text{shift}_{i,j} \cdot S_{i,0})^i}
\]

to obtain a possible one day value change. For the portfolios consisting of several derivatives the values are netted. Lastly, define the 250 absolute returns given
the simulated possible values as \( L_j = P_{j|\text{fix}}^{(j)}(0) - P_{j|\text{fix}}^{(0)}(0) \) and order the sample \( L_{1,j} \geq ... \geq L_{250,j} \), then the empirical estimate of the one day value at risk at level \( \alpha \in \{0.01, 0.05, 0.10\} \) is given by

\[
\text{VaR}_\alpha = L_{[\alpha]+1,j}
\]

as proposed in equation (12). The daily VaR is then scaled with \( \sqrt{10} \) to obtain the 10-day VaR.

### 5.5 Standard Initial Margin Computation

The SIMM computation, as opposed to the value at risk, doesn’t rely on historical simulation in order to determine the initial margin. However, the first step is similar in the sense that the value of the swap is determined using (10) and (11). Vanilla swaps are also fairly linear since they do not have embedded optionality, and hence no vega and curvature risk. The delta sensitivity of the swap of \( n \) years maturity with respect each tenor \( m \) is obtained by "shocking" it with one basis point as defined in equation (4). Under our framework the equation would be

\[
s_{m,n} = \Pi^{(m)}(0; \text{fixed leg} + 1\text{bps}) - \Pi(0; \text{fixed leg})
\]

Then the weighted net sensitivity for each tenor is decided with the help of equation (5) with predefined values in Appendix section 8.2 Risk weights per vertex (regular currencies). The concentration threshold \( CR_b \) can be set equal to one since for all portfolios the net weighted sensitivity does not exceed \$250 million per bps (or similarly €250 million per bps). The squared weighted sensitivities are aggregated across the different tenors according to equation (6). The correlation between different tenors are as previously mentioned presented in Appendix section 8.5 Correlations for Different Maturities to obtain the initial margin.

If we consider the same example as presented in the previous section the present value of the swap would at time zero is approximately \( \approx €1,000,458 \). Now, compute the value of the swap with respect to the one (1Y) and two (2Y) year tenor "shock".

\[
\Pi^{(1Y)}(0; \text{fixed leg}) = €1,000,000 \left( \frac{5\%}{(1 + 4\% + 1\text{bps})} + \frac{5\%}{(1 + 5\%)^2} + \frac{1}{(1 + 5%)^2} \right)
\]

\( \approx €1,000,453 \)

\[
\Pi^{(2Y)}(0; \text{fixed leg}) = €1,000,000 \left( \frac{5\%}{(1 + 4\%)} + \frac{5\%}{(1 + 5\% + 1\text{bps})^2} + \frac{1}{(1 + 5% + 1\text{bps})^2} \right)
\]

\( \approx €1,000,276 \)
yielding the sensitivities \( s_{1,2} \approx -\varepsilon 5 \) and \( s_{2,2} \approx -\varepsilon 181 \). Using the scaled volatilities presented in Appendix section 8.2 this yields the net weighted sensitivities \( W_{S1,2} \approx -\varepsilon 259 \) and \( W_{S2,2} \approx -\varepsilon 9432 \). Aggregating then gives

\[
K = \sqrt{\sum_{i,m} WS_{m,i}^2 + \sum_{i,m} \sum_{(j,n) \neq (i,m)} \phi_{i,j} \rho_{m,n} WS_{m,i} WS_{n,j}}
\]

\[
= \sqrt{(-259)^2 + (-9432)^2} \approx \varepsilon 9435.
\]

Hence

\[
IM = \text{DeltaMargin} = \sqrt{K^2 + \sum_{b \neq c} \gamma_{bc} g_{bc} S_b S_c} = K \approx \varepsilon 9435,
\]

which is a couple of basis points short of one percent of the notional outstanding and thus consistent with what other studies have concluded for a swap of that maturity, discussed further in Section 7.
6 Results

In this section, the results and model obtained from the methodology previous section will be described. The results will be presented with respect to the SIMM criteria presented in Section 3.2.

6.1 Backtesting

The result for the backtesting of the ISDA SIMM and value at risk at the three given levels for the six different portfolios are presented in Figure 1. With respect to criteria (1) non-procyclicality it’s found that the SIMM as opposed to the 10-day value-at-risk is much more resilient to changes in market volatility and thus carries a lower standard deviation. The standard deviations for the portfolios are presented in Table 3: Margin standard deviations of the portfolios. In general, the maximum 10-day value at risk on level $\alpha = 0.01$ during the major volatility period of 2008 correspond to the margin computed through the SIMM reflecting that when producing the model SIMM had distressed market conditions in mind. Moreover, Portfolio 4 and 5 which consist of short positions present behaviour consistent with the netting and correlation approach established in equation (6). That is, the short position could be viewed as a not perfect hedge, and is netted versus the long positions reducing the total exposure. The exposure is then adjusted by the asset correlations and the initial margin is obtained. If the two assets where exactly the same and had perfect correlation the initial margin would be zero and correspond to a long and short of the same underlying which would have a netted exposure of zero.

With respect to criteria (2) ease of replication and (3) transparency it’s clear that for simple portfolios consisting of derivatives with an established valuation framework, such as the discounted cash flow method, will be easily replicable and transparent. However, the valuation and categorization of more exotic derivatives into risk classes will require communication between the counterparties or alternatively provided by ISDA in line with criteria (5) that the model should be extensible for market participants. Criteria (4) quick to calculate is satisfied under similar conditions.

Lastly, tests were run on different notionals to investigate the predictability of the model and the results were consistent with respect to the predictability criteria (6), i.e., the initial margin as a percentage of the outstanding notional will stay fairly constant with a couple of basis points variation. However, the report relies on approximations as well as utilizing the fact that a swap can be represented as a bond from each counterparty’s point of view. The larger standard deviation of Portfolio 2 and 3 when compared to the SIMM model for Portfolio 1 is most likely a result of the larger duration as well as the recent years negative interest rates slowly inflating the current value of the swap. Whether it would be appropriate to use a discount rate of zero for negative rates is discussed further in Section 7.1.
Table 3: Margin standard deviations of the portfolios

<table>
<thead>
<tr>
<th></th>
<th>Portfolio 1</th>
<th>Portfolio 2</th>
<th>Portfolio 3</th>
<th>Portfolio 4</th>
<th>Portfolio 5</th>
<th>Portfolio 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>SIMM</td>
<td>€263</td>
<td>€1,522</td>
<td>€5,090</td>
<td>€375</td>
<td>€1,604</td>
<td>€5,587</td>
</tr>
<tr>
<td>VaR$_{\alpha=0.01}$</td>
<td>€2,785</td>
<td>€4,682</td>
<td>€6,290</td>
<td>€7,931</td>
<td>€6,890</td>
<td>€11,945</td>
</tr>
<tr>
<td>VaR$_{\alpha=0.05}$</td>
<td>€1,701</td>
<td>€2,716</td>
<td>€3,893</td>
<td>€4,264</td>
<td>€3,693</td>
<td>€7,367</td>
</tr>
<tr>
<td>VaR$_{\alpha=0.10}$</td>
<td>€1,176</td>
<td>€2,111</td>
<td>€2,710</td>
<td>€2,744</td>
<td>€2,575</td>
<td>€5,403</td>
</tr>
</tbody>
</table>
Figure 1: IM for different portfolios based of VaR and SIMM
6.2 Return distribution

Since the SIMM model is derived from a nested variance-covariance framework assuming that returns are normally distributed and then scaling the volatility a large simulation of possible returns for the different portfolios were conducted and then fitted and compared to the normal and student’s $t$-distribution. The histogram and distribution fit is presented in Figure 2 and the parameters for each portfolio is presented in Table 4: Return distribution fit and parameters. Generally, it appears that the student’s $t$-distribution provides a better fit thus better capturing the distribution mass around the mean and the tails and thus better factors in tail risk or three sigma risk - events such as the financial meltdown. However, the SIMM compensate for that through volatility scaling, essentially artificially increasing the variance with the parameters presented in Appendix section 8.2 Risk weights per vertex (regular currencies). The point of the SIMM is that it should be a versatile and adaptive model, and the scaling parameters can continuously be updated thus adapt to capture current market conditions and risk.

Furthermore, inspecting the fitted distributions we can see that the zero-mean assumption of the model, derived from the expectation that markets are efficient and arbitrage free, is fair. Considering that the portfolios had a notional outstanding of €1,000,000, €2,000,000 and €3,000,000, the deviation from a zero mean does not represent a value more than a couple of fractions of a percentage further strengthening the zero mean normal distribution assumption which is the foundation of the model.
Table 4: Return distribution fit and parameters (rounded to nearest integer, unit € except $\nu$).

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$N(\mu, \sigma)$</th>
<th>$t(\mu; \sigma; \nu)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(33, 668)</td>
<td>(22, 305, 2)</td>
</tr>
<tr>
<td>2</td>
<td>(75, 1847)</td>
<td>(78, 1330, 4)</td>
</tr>
<tr>
<td>3</td>
<td>(129, 3690)</td>
<td>(149, 2907, 5)</td>
</tr>
<tr>
<td>4</td>
<td>(-96, 3294)</td>
<td>(-119, 2590, 5)</td>
</tr>
<tr>
<td>5</td>
<td>(-88, 2547)</td>
<td>(-88, 1958, 5)</td>
</tr>
<tr>
<td>6</td>
<td>(237, 5906)</td>
<td>(258, 4513, 5)</td>
</tr>
</tbody>
</table>
Figure 2: Distribution of daily gains (losses) for the portfolios as well as the normal (left) and student’s $t$-distribution (right) fitted.
7 Discussion

This section will discuss the results obtained in the previous section. Furthermore, a discussion will be held considering the chosen methodology and the framework established.

7.1 Evaluation of chosen methods

7.1.1 The Value-at-Risk and Portfolio Linearity

The choice of historical value-at-risk rather than Monte Carlo simulation or variance-covariance method provide more flexibility given the relatively constrained data set which only consisted of historical swap rates for different tenors. The big benefit of the historical simulation method is that it does not assume any underling return distribution, contrasted with the SIMM which assumes the returns are normally distributed. This project considered swap portfolios, which are fairly linear in their behaviour. If non-linear securities such as derivatives with optionality were added, the historical simulation method would handle their behaviour better than the variance-covariance approach [8], something ISDA have combated through the introduction of the vega and curvature sensitivities. However, since the historical simulation approach only use one sample path of the historical outcomes as opposed to the self-determined number of possible paths in the Monte Carlo-method, the adequacy in the correctness of its future prediction may falter [8].

7.1.2 Negative rates

Another point of interest is whether using zero as discount rate would be more appropriate as opposed to the negative proxy-rate used. A negative discount rate implies that the future cash flow has negative risk and thus one pay up front to receive the cash flow. Although it’s theoretically possible, it doesn’t make economic sense. Even more un-intuitive would be if you’re an insurer who, when discounting future liabilities and possible losses essentially rebate the insurance premium to acquire the risk. The use of negative discounts rates is being widely discussed and in [10] one interviewee makes the argument that negative yields are a reality and should be henceforth be treated as such whereas another argues that it should exist a lower bound of zero. Some experts take this a bit further and argue for the scrapping of discounted cash flow valuations, which have been used throughout this report, entirely [1]. But since the same valuation approach of absolute returns in equations (12) and (4) were used for the underlying swaps in the portfolios in the value-at-risk and SIMM backtesting the impact of the comparison was likely reduced.

7.1.3 Credit Value Adjustments and the Single Curve Framework

Another assumption which affect the result is the assumption of the pre-crisis single-curve framework. Today more sophisticated multi-curve frameworks [3]
have been developed when valuing derivatives. Further valuation adjustments also made through so called credit value adjustments (CVA) since counterparties are often not risk free. The purpose of the CVA is essentially to price counterparty risk, and it’s not an easy task. Furthermore, today’s regulatory framework require that market participants use CVA on their positions to hedge losses such as counterparty defaults [6]. The swap rates used in this report were historical market quotes and is what market participants were quoted. Since participants (excluding stable risk free governments such as the U.S. or Swedish government) can not be considered risk free, a CVA on the swaps coupled with a treasury and swap spread analysis and would most likely give a more true valuation and by extension initial margin consistent with today’s regulatory burden. The SIMM models also handles counterparty credit risk under the risk classes (i) credit (qualifying) and (ii) credit (non-qualifying), so even though these risk classes are not considered in this report, inclusion would most likely increase the initial margin requirements presented in Figure 1 for both methodologies.

7.2 Criticism of the SIMM

As discussed in [6] some of the main criticisms to the SIMM model is that by inflating the initial margin requirements via the scaling procedure, market will be drained from liquidity. That the SIMM overestimate the initial margin requirements compared to the value-at-risk is further emphasized by this study’s findings presented in Figure 1. The SIMM initial margin requirement for a single interest rate bearing derivative is around the 99% value-at-risk in stressed market conditions, see years 2008 and 2009. Draining of liquidity could increase in higher bid-ask spreads and less turnover between market participants leading to price dispersion. Another point made in [6] is the static nature of the SIMM model in the sense that it relies on calibration parameters to be provided ISDA. ISDA must continuously provide market participants with a number of parameters reflecting the current market environment. However, since the model is developed with extensibility and flexibility as fundamental criteria presented in section 3.2, market participants should expect them to be provided continuously.

This study only considered relatively simple portfolios consisting of swap(s), a regularly traded and common derivative where the identification of product and risk class is relatively easy. However, if more complex derivatives were introduced the model lacks a clear framework for breaking down instruments into the different product and risk classes presented in section 3.3. The fact subjectivity might be part of the categorization between two counterparties could result in different initial margins even though the same model was used.
7.3 Conclusion

This study analyzed and compared the ISDA SIMM and initial margin requirements for non-centrally cleared OTC with the value-at-risk method.

The result suggest that for swap portfolios the ISDA SIMM achieves its set out purpose of being less volatile and more transparent than the 10-day value-at-risk on a 99% confidence level. The SIMM assumption that returns are Gaussian is investigated and its found that a student’s $t$-distribution generally fit the daily returns better, since it carries more tail risk. However, the SIMM compensate for the lack of tail risk by scaling up the volatility with a set of provided calibration parameters, and thus increasing the initial margin requirements.

However, the SIMM framework will require market participants to be continuously updated and provided with a set of calibration parameters which reflect the current market conditions. But given ISDA’s previous historical success with the implementation of the ISDA Master Agreement, which broadly can be defined as a framework between two counterparties, the SIMM is likely to become the de facto market standard for initial margin computations given its flexibility and simplicity.

7.3.1 Further studies

Since this study only consider swap portfolios, which are relatively linear in behaviour, further studies could incorporate non-linear derivatives spanning several product classes. Since the SIMM model is based on a nested variance-covariance approach it could be investigated how its issues with non-linearity are handled by the vega and curvature sensitivities compared to non-linearity in the value-at-risk or other desired risk measures.

Furthermore, since the valuation used in this study relies on approximations and proxies which in turn filter through to the initial margin, the impact of CVA and the multi-curve framework could be investigated. Lastly, it could be considered how a wrong calibration of the SIMM given certain risk factors and market environment could impact the required collateral and market liquidity.
8 Appendix

8.1 Currency Volatilities

Table 5: Currency Grouping

<table>
<thead>
<tr>
<th>Currency Risk Group</th>
<th>Currencies</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Volatility</td>
<td>All other currencies</td>
</tr>
<tr>
<td>Regular volatility, well-traded</td>
<td>USD, EUR, GBP</td>
</tr>
<tr>
<td>Regular volatility, less well-traded</td>
<td>AUD, CAD, CHF, DKK, HKD, KRW, NOK, NZD, NZD, SEK, SGD, TWD</td>
</tr>
<tr>
<td>Low Volatility</td>
<td>JPY</td>
</tr>
</tbody>
</table>

8.2 Risk weights per vertex (regular currencies)

Table 6: Risk weights per vertex (regular currencies)

<table>
<thead>
<tr>
<th>2w</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>1yr</th>
<th>2yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
<th>15yr</th>
<th>20yr</th>
<th>30yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>113</td>
<td>113</td>
<td>98</td>
<td>69</td>
<td>56</td>
<td>52</td>
<td>51</td>
<td>51</td>
<td>51</td>
<td>53</td>
<td>56</td>
<td>64</td>
</tr>
</tbody>
</table>

8.3 Risk weights per vertex (high-volatility currencies)

Table 7: Risk weights per vertex (high-volatility currencies)

<table>
<thead>
<tr>
<th>2w</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>1yr</th>
<th>2yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
<th>15yr</th>
<th>20yr</th>
<th>30yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
<td>93</td>
<td>90</td>
<td>94</td>
<td>97</td>
<td>103</td>
<td>101</td>
<td>103</td>
<td>102</td>
<td>101</td>
<td>102</td>
<td>101</td>
</tr>
</tbody>
</table>

8.4 Risk weight per vertex (low-volatility currencies)

Table 8: Risk weight per vertex (low-volatility currencies)

<table>
<thead>
<tr>
<th>2w</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>1yr</th>
<th>2yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
<th>15yr</th>
<th>20yr</th>
<th>30yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>21</td>
<td>10</td>
<td>11</td>
<td>15</td>
<td>20</td>
<td>22</td>
<td>21</td>
<td>19</td>
<td>20</td>
<td>23</td>
<td>27</td>
</tr>
</tbody>
</table>
8.5 Correlations for Different Maturities

Table 9: Table of correlations

<table>
<thead>
<tr>
<th></th>
<th>2w</th>
<th>1m</th>
<th>3m</th>
<th>6m</th>
<th>1yr</th>
<th>2yr</th>
<th>3yr</th>
<th>5yr</th>
<th>10yr</th>
<th>15yr</th>
<th>20yr</th>
<th>30yr</th>
</tr>
</thead>
<tbody>
<tr>
<td>2w</td>
<td>-</td>
<td>1</td>
<td>0.79</td>
<td>0.67</td>
<td>0.53</td>
<td>0.42</td>
<td>0.37</td>
<td>0.30</td>
<td>0.22</td>
<td>0.18</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>1m</td>
<td>1</td>
<td>-</td>
<td>0.79</td>
<td>0.67</td>
<td>0.53</td>
<td>0.42</td>
<td>0.37</td>
<td>0.30</td>
<td>0.22</td>
<td>0.18</td>
<td>0.16</td>
<td>0.12</td>
</tr>
<tr>
<td>3m</td>
<td>0.79</td>
<td>0.79</td>
<td>-</td>
<td>0.85</td>
<td>0.69</td>
<td>0.57</td>
<td>0.50</td>
<td>0.42</td>
<td>0.32</td>
<td>0.25</td>
<td>0.23</td>
<td>0.20</td>
</tr>
<tr>
<td>6m</td>
<td>0.67</td>
<td>0.67</td>
<td>0.85</td>
<td>-</td>
<td>0.86</td>
<td>0.76</td>
<td>0.69</td>
<td>0.59</td>
<td>0.47</td>
<td>0.40</td>
<td>0.37</td>
<td>0.32</td>
</tr>
<tr>
<td>1yr</td>
<td>0.53</td>
<td>0.53</td>
<td>0.69</td>
<td>0.86</td>
<td>-</td>
<td>0.93</td>
<td>0.87</td>
<td>0.77</td>
<td>0.63</td>
<td>0.57</td>
<td>0.54</td>
<td>0.50</td>
</tr>
<tr>
<td>2yr</td>
<td>0.42</td>
<td>0.42</td>
<td>0.57</td>
<td>0.76</td>
<td>0.93</td>
<td>-</td>
<td>0.98</td>
<td>0.90</td>
<td>0.77</td>
<td>0.70</td>
<td>0.67</td>
<td>0.63</td>
</tr>
<tr>
<td>3yr</td>
<td>0.37</td>
<td>0.37</td>
<td>0.50</td>
<td>0.69</td>
<td>0.87</td>
<td>0.98</td>
<td>-</td>
<td>0.96</td>
<td>0.84</td>
<td>0.78</td>
<td>0.75</td>
<td>0.71</td>
</tr>
<tr>
<td>5yr</td>
<td>0.30</td>
<td>0.30</td>
<td>0.42</td>
<td>0.59</td>
<td>0.77</td>
<td>0.90</td>
<td>0.96</td>
<td>-</td>
<td>0.93</td>
<td>0.89</td>
<td>0.86</td>
<td>0.82</td>
</tr>
<tr>
<td>10yr</td>
<td>0.22</td>
<td>0.22</td>
<td>0.32</td>
<td>0.47</td>
<td>0.63</td>
<td>0.77</td>
<td>0.84</td>
<td>0.93</td>
<td>-</td>
<td>0.98</td>
<td>0.96</td>
<td>0.94</td>
</tr>
<tr>
<td>15yr</td>
<td>0.18</td>
<td>0.18</td>
<td>0.25</td>
<td>0.40</td>
<td>0.57</td>
<td>0.70</td>
<td>0.78</td>
<td>0.89</td>
<td>0.98</td>
<td>-</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>20yr</td>
<td>0.16</td>
<td>0.16</td>
<td>0.23</td>
<td>0.37</td>
<td>0.54</td>
<td>0.67</td>
<td>0.75</td>
<td>0.86</td>
<td>0.96</td>
<td>0.99</td>
<td>-</td>
<td>0.99</td>
</tr>
<tr>
<td>30yr</td>
<td>0.12</td>
<td>0.12</td>
<td>0.20</td>
<td>0.32</td>
<td>0.50</td>
<td>0.63</td>
<td>0.71</td>
<td>0.82</td>
<td>0.94</td>
<td>0.98</td>
<td>0.99</td>
<td>-</td>
</tr>
</tbody>
</table>

8.6 Interest Rate Risk - Delta Concentration Thresholds

Table 10: Delta Concentration Thresholds

<table>
<thead>
<tr>
<th>Currency Risk Group</th>
<th>Concentration Threshold (USD mm/bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Volatility</td>
<td>8.0</td>
</tr>
<tr>
<td>Regular volatility, well-traded</td>
<td>230</td>
</tr>
<tr>
<td>Regular volatility, less well-traded</td>
<td>28</td>
</tr>
<tr>
<td>Low Volatility</td>
<td>82</td>
</tr>
</tbody>
</table>

8.7 Interest Rate Risk - Vega Concentration Thresholds

Table 11: Vega Concentration Thresholds

<table>
<thead>
<tr>
<th>Currency Risk Group</th>
<th>Concentration Threshold (USD mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Volatility</td>
<td>110</td>
</tr>
<tr>
<td>Regular volatility, well-traded</td>
<td>2,700</td>
</tr>
<tr>
<td>Regular volatility, less well-traded</td>
<td>150</td>
</tr>
<tr>
<td>Low Volatility</td>
<td>960</td>
</tr>
</tbody>
</table>
References


