Analysis and control of boundary layer transition on a NACA 0008 wing profile

by

Arijit Sinha Roy

August 2018
Technical Report
Royal Institute of Technology
Department of Mechanics
SE-100 44 Stockholm, Sweden
Analysis and control of boundary layer transition on a NACA 0008 wing profile

Arijit Sinha Roy
Fluid Physics Laboratory, KTH Mechanics, Royal Institute of Technology
SE-100 44 Stockholm, Sweden

Abstract
The main aim of this thesis was to understand the mechanism behind the classical transition scenario inside the boundary layer over an airfoil and eventually attempting to control this transition utilizing passive devices for transition delay. The initial objective of analyzing the transition phenomenon based on TS wave disturbance growth was conducted at 90 Hz using LDV and CTA measurement techniques at two different angles of attack. This was combined with the studies performed on two other frequencies of 100 and 110 Hz, in order to witness its impact on the neutral stability curve behaviour.

The challenges faced in the next phase of the thesis while trying to control the transition location, was to understand and encompass the effect of adverse pressure gradient before setting up the passive control devices, which in this case was miniature vortex generators. Consequently, several attempts were made to optimize the parameters of the miniature vortex generators depending upon the streak strength and stability. Finally, for 90 Hz a configuration of miniature vortex generators have been found to successfully stabilize the TS wave disturbances below a certain forcing amplitude, which also led to transition delay.

Key words: boundary layer stability, laminar - turbulent transition, laminar flow control, Tollmien-Schlichting waves, streaky boundary layers, miniature vortex generators, Falkner-Skan boundary layer.
## Contents

Abstract ii

Nomenclature iv

Chapter 1. Introduction 1

Chapter 2. Literature Review 2
   2.1. Transition scenarios 2
   2.2. Laminar flow control based on streak interaction 3
   2.3. Scope of this study 4

Chapter 3. Theory governing the flow 6
   3.1. Linear disturbance Equations: 6
   3.2. Falkner Skan Boundary Layer: 8
   3.3. Integral boundary layer equation 10

Chapter 4. Experimental Setup 15
   4.1. Wind tunnel 15
   4.2. Experimental setup 15
   4.3. Measurement techniques 17

Chapter 5. Results and discussions 19
   5.1. TS wave disturbance analysis without MVGs 19
   5.2. TS wave disturbance analysis in the presence of MVGs 29

Chapter 6. Conclusion 42

Chapter 7. Acknowledgements 44

Bibliography 45
**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>AoA</td>
<td>Angle of attack</td>
</tr>
<tr>
<td>c</td>
<td>Chord</td>
</tr>
<tr>
<td>x</td>
<td>Chordwise direction</td>
</tr>
<tr>
<td>y</td>
<td>Wall normal direction</td>
</tr>
<tr>
<td>z</td>
<td>Spanwise direction</td>
</tr>
<tr>
<td>u</td>
<td>Perturbation in the $x$ direction</td>
</tr>
<tr>
<td>v</td>
<td>Perturbation in the $y$ direction</td>
</tr>
<tr>
<td>w</td>
<td>Perturbation in the $z$ direction</td>
</tr>
<tr>
<td>$U$</td>
<td>Mean Flow Velocity</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Free stream velocity</td>
</tr>
<tr>
<td>$MVG$</td>
<td>Miniature vortex generator</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Normalized wall normal coordinate with $\delta$</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Normalized spanwise coordinate with $\lambda$</td>
</tr>
<tr>
<td>$u_e$</td>
<td>Boundary layer edge velocity</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Boundary layer parameter</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Spanwise wavelength of vortex generators</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Viscous fluid flow is associated with boundary layer flow close to the surface which can either be a laminar or a turbulent boundary layer. Consequently, the state of the boundary layer plays a dominant role in producing drag. For instance, in the case of external flows like flow over a wing of a commercial aircraft or gas turbine blades, it is desirable to have a design which would ensure laminar boundary layer for operational angles of attack, as it would ensure lower losses.

A laminar boundary layer can transform into a turbulent boundary layer, which involves significant changes to the base flow inside the boundary layer. The region where this transformation occurs is called the transition region. Some of the parameters that typically affect this transition include - free-stream turbulence, pressure gradient, Reynolds number, Mach number, acoustic disturbances, surface roughness, surface temperature and surface curvature. The transition from laminar to turbulent is usually designated by receptivity, disturbance growth which in turn is characterized by initial amplification of linear instability waves known as T-S (Tollmien-Schlichting) waves which grow exponentially, eventually lead to non-linear disturbances that start amplifying until turbulent spots start appearing and transition is completed. Typically, the receptivity is quite difficult to determine due its dependence on free stream vortices, perturbations and surface roughness of the component. Hydrodynamic stability theory therefore can be considered predominantly with laminar flows in response to disturbances with small to moderate amplitudes. Therefore, it is of utmost importance to control and maintain the disturbance amplitude at a level where the linear stability theory is applicable. Although, it is possible for bypass transition to occur which would lead to flow transition without the generation of T-S waves, it has been neglected by keeping the ambient turbulence to a minimum. But, the investigation done in this thesis involves the analysis of the linear (exponential) growth of disturbances inside the laminar boundary layer over a NACA 0008 wing profile and an attempt has been made to stabilize the linear instabilities by the modification of base flow using miniature vortex generators. This has been done as passive control of linear instabilities are easier than trying to stabilize the non-linear disturbances, due to larger uncertainties associated with the non-linear disturbances.
Chapter 2

Literature Review

2.1. Transition scenarios

2.1.1. Classical transition scenario

The process and physics of transition of boundary layer from laminar into turbulent has been at the epicentre of research for many years, in the aerospace and fluid dynamics discipline. The initial perspective into the physical mechanism that governs this transition process in terms of vorticity was given by Lighthill (1963), a detailed explanation of the phenomenon was later attempted by Betchov & Criminale (1967). Consequently, to better understand the two-dimensional instability of Blasius boundary layers and similar flows, insightful references can be drawn from Baines & Mitsudera (1994). This research explored the relation between the mechanism of the instability, as a consequence of the interaction of two idealized mode systems, involving a neutral inviscid mode and one of the decaying viscous modes resulting from uniform shear and the no-slip boundary condition. This dynamical system had a foundation in the concepts responsible for inviscid shear flow instabilities presented in Baines & Mitsudera (1992), which was extended to viscous flows. The instability had eventually been related to the constructive interference of an inviscid partial mode (away from the wall) and the most weakly damped viscous partial mode (near the wall) as had been demonstrated for inviscid flows in Craik (1985). Consequently, the resulting eigen-mode would only grow if the mutual forcing can overcome the viscous damping. Furthermore Prandtl (1921) also proposed similar ideas based on growth of instability due to positive work (exceeding viscous dissipation) of Reynold’s stress (from viscous modes) against wall normal shear. The classical transition scenario of boundary layers involving TS wave amplification leading to onset of secondary instabilities in low-noise ambient conditions was developed later. This was based on approximate solutions of the Orr-Sommerfeld equations used by Tollmien (1929) to develop the linear viscous instability theory and the validation of its predictions in experiments conducted using vibrating ribbons by G.B. Schubauer (1947).
2.1.2. By-pass transition and streaky structures

An alternative scenario for the transition to turbulence can be attributed to by-pass transition and streaky structures. The presence of free stream turbulence in conjugation to a Blasius boundary layer or similar profiles, can induce disturbances which lead to streamwise low and high speed fluid structures as has been investigated in Jacobs & Durbin (2001), Matsubara & Alfredsson (2001). These streaky structures grow and when they attain a certain amplitude, secondary instabilities start appearing and initiate the breakdown to turbulence. This type of bypass transition can be characterized by the concept of transient growth. This linear mechanism involves initial algebraic growth followed by exponential decay resulting from superposition of non-orthogonal OS and Squire modes as has been studied in Schmid (2001). The lift up effect resulting from the streaky structures contributes to the algebraic growth. Therefore, in this thesis it was imperative to keep a low free-stream turbulence level in order to prevent the by-pass transition scenario from dominating the transition.

2.2. Laminar flow control based on streak interaction

The concept that has been investigated here is Laminar flow control (LFC), which has been an area of significant research interest in the recent decades. But the LFC method utilized here is based on the concept of delaying the laminar to turbulent transition and not on relaminarization of the flow, as the energy costs are higher for the latter case. It is very difficult to observe a two-dimensional boundary layer unless in extremely well controlled situations. Small amounts of noise, such as free-stream turbulence or wall imperfections, are able to induce non-negligible spanwise variations of the boundary layer profiles. This sensitivity is due to the “lift-up” effect, streamwise vortices of small amplitude, living in a high shear region such as the boundary layer, are able to mix very efficiently low momentum and high momentum fluid. This eventually leads to large elongated spanwise modulations of the streamwise velocity field called streamwise streaks. For the LFC method chosen here, the streamwise vortices are forced with the miniature vortex generators based on the classical vortex generators. These kind of classical vortex generators have been experimented on flat plate boundary layers primarily as has been described in Shahinfar et al. (2012). In the case of streaky boundary layers on flat plates, the onset of inflectional instabilities have been reported for streak amplitude at around 26% of free-stream velocity, initiated by unstable sinous (anti-symmetric) modes of the streaks as documented in Hoepffner et al. (2005). It was concluded the breakdown to turbulence could have resulted from the strong shear layers associated with streaks and the significance of the interaction between high and low speed streaks. The interaction between 2D finite TS waves and streaks was studied by Komoda (1967) to observe any destabilizing resonance between the waves. Subsequently, a study including the three dimensional boundary layer with linear three dimensional waves was required and such studies were
2.3. Scope of this study

The initial aim is to understand and document the influence of streamwise varying pressure gradient associated with flow over airfoils (NACA 0008), using linear stability theory for two dimensional TS waves. Additionally, the frequency dependence of the instability growth would be explored as well.

The stabilization effect evidenced due to the presence of roughness elements to generate streamwise streaks in Fransson et al. (2005) and the subsequent success at delaying transition in Fransson et al. (2006) was shown to be more robust using miniature vortex generators. Additionally, the work of Shahinfar et al. (2013), involving triangular MVGs also provided the necessary inspiration and confidence to attempt at delaying transition on an airfoil. Thus having realized the implications and potential of transitional delay the eventual aim was to control the transitional delay passively using MVGs, while taking into account the difference with the Blasius studies previously conducted, due to the presence of varying pressure gradients. There are a lot of techniques that could be utilized for this purpose including high favourable pressure gradients, wall suction, fluid heating or cooling. But this thesis explores the prospect of utilizing miniature vortex generators for creating streaks and suppressing the exponential growth of linear disturbances, as this technique would involved placement of actuators upstream of the unstable region and require no external energy source as it would conducted including the work of Kachanov & Tararykin (1987). They observed the development of three dimensional waves with M-shaped structures and phase speed as TS waves for Blasius case. However, even though transitional delay was never confirmed, they essentially witnessed the absence of amplification of the streaky TS waves in contrast to their two dimensional behaviour. Eventually, Cossu & Brandt (2002) conducted spatial and temporal numerical simulations to visualize the stabilization effects that streaks of sufficient amplitude can have on linearly growing viscous instabilities. The concept of perturbation kinetic energy as shown in equation[2.2.1] was utilized to understand the stabilizing effect especially in the temporal domain.

\[
\frac{\partial E}{\partial t} = T_y + T_z - D \tag{2.2.1}
\]

In the above equation the \( T_y \) and \( T_z \) are the perturbation energy production terms comprising of the work of the Reynold's stress against the wall normal shear \( \frac{\partial U}{\partial y} \) and \( \frac{\partial U}{\partial z} \) respectively. \( D \) is the viscous dissipation term in terms of square of the perturbation vorticities. The stabilization was finally attributed to the presence of the growth of the stabilizing contribution from \( (T_z - D) \) over the destabilizing term \( T_y \), due to the introduction of spanwise velocity gradients in the form of streaks. Consequently, it was the presence of this spanwise component in the three-dimensional case with streaky structures which led to the stabilization of the TS wave disturbances.
use the lift up effect to extract energy for the streaks. The introduction of the MVG into the base-flow transformed the 2D disturbance (from TS waves) into a 3D disturbance due to the formation of low speed and high speed streaks. Hence, the method for calculating the TS wave amplitude and the streak amplitude had been altered as shown in equation [2.3.2] and equation [2.3.1] respectively,

\[ A_{ST}(x) = \frac{1}{U_\infty} \int_{-1/2}^{1/2} \int_0^\eta |U(x, \eta, \zeta) - U(x, \eta) Z| d\eta d\zeta, \]  

(2.3.1)

\[ A_{TS}(x) = \int_{-1/2}^{1/2} \int_0^\eta \frac{u_{rms}^*(x, \eta, \zeta)}{U_\infty} d\eta d\zeta. \]  

(2.3.2)

Note, \( \zeta = z/\lambda \), where \( \lambda \) is the spanwise wavelength of the miniature vortex generators, and \( \eta = y/\delta \).
3.1. Linear disturbance Equations:

In order to obtain the linear disturbance equations, the 2D Navier Stokes equation for steady incompressible flow equation has been used in conjunction with linear perturbation theory of small disturbances and parallel flow assumption which replaces $u = U + u', v = v', w = w'$ and $p = P + p'$. The perturbations have assumed to be small enough to neglect the non-linear terms resulting from the substitution.

\[
\begin{align*}
\frac{\partial u'}{\partial t} + U \frac{\partial u'}{\partial x} + v \frac{\partial U}{\partial y} + (1/\rho) \frac{\partial p'}{\partial x} &= \nu \Delta u' \quad (3.1.1) \\
\frac{\partial v'}{\partial t} + U \frac{\partial v'}{\partial x} + (1/\rho) \frac{\partial p'}{\partial y} &= \nu \Delta v' \quad (3.1.2) \\
\frac{\partial w'}{\partial t} + U \frac{\partial w'}{\partial x} + (1/\rho) \frac{\partial p'}{\partial z} &= \nu \Delta w' \quad (3.1.3) \\
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} &= 0 \quad (3.1.4)
\end{align*}
\]

The aforementioned equations are used to derive:

\[
\Delta P = -2U' \partial u' \partial x \quad (3.1.5)
\]

and equation[3.1.5] is replaced in the equation for the normal velocity equation to derive:

\[
[(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x})\nabla^2 - U'' \frac{\partial}{\partial x} - 1/Re\nabla^4]v = 0 \quad (3.1.6)
\]

In order to capture the 3D nature of the perturbation a second equation for the normal vorticity($\varphi$) has been derived:

\[
\varphi = \frac{\partial u'}{\partial z} - \frac{\partial w'}{\partial x} \quad (3.1.7)
\]

so that the second equation can be derived to:

\[
[(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x}) - 1/Re\nabla^2]\varphi = -U' \frac{\partial v}{\partial z} \quad (3.1.8)
\]
3.1. Linear disturbance Equations:

The pair of equations [3.1.6] and [3.1.8] satisfy the boundary conditions \( v = v' = \eta = 0 \) at the wall and in the far field. Additionally, the introduction of wave-like equations like:

\[
v(x, y, z, t) = \hat{v}(y)e^{i(\alpha x + \beta y - \omega t)}
\]

\[
\varphi(x, y, z, t) = \hat{\varphi}(y)e^{i(\alpha x + \beta y - \omega t)}
\]

Replacing these into equations [3.1.6] and [3.1.8], the Orr-Sommerfield and Squire Equation can be derived as shown in equation [3.1.11] and equation [3.1.12] respectively,

\[
\left( -i\omega + i\alpha U \right) (D^2 - k^2) \hat{v} - \frac{1}{i\alpha Re} (D^2 - k^2)^2 \hat{v} = 0,
\]

\[
\left( -i\omega + i\alpha U \right) - \frac{1}{Re} (D^2 - k^2)^2 \hat{\varphi} = -i\beta U' \hat{v}.
\]

In the above mentioned equations \( D = \frac{d}{dy} \) and \( k = \sqrt{\alpha^2 + \beta^2} \) and \( U' = \frac{\partial U}{\partial y} \).

The boundary conditions that have been satisfied are \( \hat{v} = \hat{D} \hat{v} = \hat{\varphi} = 0 \). For the temporal stability of TS waves \( \alpha \) and \( \beta \) are purely real and they can interpreted as wave number of the TS wave in the corresponding direction. For spatial stability the \( \omega \) is purely real, which represents the frequency of the TS wave. Hence the stability of the TS wave is determined by \( \alpha \) and \( \beta \). By the aid of the Squire’s transformation

\[
(U - c)(D^2 - k^2) \hat{v} - U'' \hat{v} - \frac{1}{i\alpha Re} (D^2 - k^2)^2 \hat{v} = 0
\]

For the 2D case, \( \beta = 0 \) hence the equation [3.1.11] can be expressed as

\[
(U - c)(D^2 - \alpha_{2D}^2) \hat{v} - U'' \hat{v} - \frac{1}{i\alpha_{2D} Re_{2D}} (D^2 - \alpha_{2D}^2)^2 \hat{v} = 0
\]

Comparing equations [3.1.13] and [3.1.14], for identical solutions, it can be derived that \( \alpha_{2D} = k = \sqrt{\alpha^2 + \beta^2} \) and \( \alpha_{2D} Re_{2D} = \alpha Re \). Hence \( Re_{2D} \) is lower than \( Re_{3D} \), which means that same modal instability would occur at a lower Reynolds number for the 2D case. The Orr-Sommerfeld and Squire equation have been posed as an eigenvalue problem, which for the temporal stability case would provide a complex valued \( c = c_r + ic_i \) where \( c = \omega/\alpha \). Subsequently for the spatial stability the eigenvalue of the equations would provide a complex valued \( \alpha = \alpha_r + i\alpha_i \), where \( \alpha_r \) dictates the wave number and the imaginary part dictates the growth of the TS disturbance as shown below.

\[
Real(v) = Real(\hat{v}e^{i(\alpha x - \omega t)})
\]

\[
Real(v) = \hat{v}e^{-\alpha_r x}
\]

Hence a negative imaginary part of \( \alpha \) would correspond to an unstable exponentially growing TS wave.
3.2. Falkner Skan Boundary Layer:

The two-dimensional boundary layer equations for steady incompressible flow can be reduced to

\[
\begin{align*}
&u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U \frac{\partial U}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \\
&\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\end{align*}
\]  

(3.2.1)

(3.2.2)

using the boundary layer approximation concept which assumes that variations across the boundary layer (\(y\) direction) are much faster than variations along the boundary layer (\(x\) direction) and with boundary conditions at \(y = 0, u = 0, v = 0\); and as \(y \to \infty, u = U_\infty\). In accordance to the equation[3.2.2] the stream function had been introduced into equation[3.2.1] to arrive at:

\[
\frac{\partial \psi}{\partial y} \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \frac{\partial^2 \psi}{\partial y^2} = U \frac{\partial U}{\partial x} + \nu \frac{\partial^3 \psi}{\partial y^3}
\]  

(3.2.3)

After this non dimensional coordinate system can be introduced \(\epsilon = x/c\), and \(\eta = y/\delta(\epsilon)\) where \(\delta(\epsilon)\) can interpreted as boundary layer scale. A trial solution for the stream function like

\[
\psi(\epsilon, \eta) = \frac{U(\epsilon)c}{\sqrt{Re}} \delta_1(\epsilon) f(\epsilon, \eta),
\]  

(3.2.4)

\[
\frac{u}{U_N(\epsilon)} = f'(\epsilon, \eta),
\]  

(3.2.5)

\[
\delta = \frac{\nu}{U_\infty x}.
\]  

(3.2.6)

along with the similarity variables had been introduced into equation[3.2.3] to arrive at

\[
f''' + \alpha_1 ff'' + \alpha_2 - \alpha_3 f'^2 = \delta^3 U_N U_\infty \left( f' \frac{\partial f'}{\partial \epsilon} - f'' \frac{\partial f}{\partial \epsilon} \right)
\]  

(3.2.7)

where \(U_N(\zeta)\) is the mean velocity in the outer flow.

\[
\alpha_1 = \frac{\delta}{U_\infty} \frac{d}{d\epsilon} (U\delta_1)
\]  

(3.2.8)

\[
\alpha_2 = \frac{\delta^2}{U_\infty} \frac{U}{U_N} \frac{dU}{d\epsilon}
\]  

(3.2.9)

\[
\alpha_3 = \frac{\delta^2}{U_\infty} \frac{dU}{d\epsilon}
\]  

(3.2.10)
If $\alpha_1, \alpha_2$ and $\alpha_3$ are constants $f(\eta, \epsilon)$ becomes independent of $\epsilon$ while satisfying the boundary conditions at $\eta = 0$, $f= 0$, $f’=0$, and as $\eta \to \infty$ and equation [3.2.7] reduces to

$$f''' + \alpha_1 f f'' + \alpha_2 - \alpha_3 f'^2$$  \hspace{1cm} (3.2.11)

Introducing the Faulkner Skan boundary condition $U_N(\epsilon) = U(\epsilon)=U_\infty \epsilon^m$ and $\alpha_1=1$, $\alpha_2=\alpha_3=\beta$, equation [3.2.11] reduces to

$$f''' + f f'' + \beta(1 - f'^2) = 0$$  \hspace{1cm} (3.2.12)

Equation[3.2.12] proposed by V.M Falkner and S.W. Skan was examined by D.R. Hartree. Consequently,

$$1 = \frac{\delta}{U_\infty} \frac{d}{d\epsilon}(U\delta)$$  \hspace{1cm} (3.2.13)

$$\beta = \frac{\partial^2 U}{U_\infty U_N} \frac{dU}{d\epsilon}$$  \hspace{1cm} (3.2.14)

where $m = \beta/(2 - \beta)$.

Boundary layer plots have been generated utilizing the Faulkner Skan Method for the NACA 0008 airfoil using data generated from Xfoil have been shown in figure[3.2.2].

![Figure 3.2.1: Velocity profile for $\alpha = 0$](image)

![Figure 3.2.2: Velocity profile for $u=5$ m/s using the Falkner Skan boundary layer theory](image)
The Falkner-Skan method has been developed into an in-house code and this method is necessary for calculations in the post processing of the experiments conducted in this thesis. As the readings taken need to be corrected, which requires a prediction of the wall normal position using the fitted profile. Consequently, Falkner-Skan method was investigated as a potential method for this purpose. However, the Falkner Skan solver was used to predict the boundary development, it failed to do so accurately at high angles of attack and at far-downstream locations due to the highly inflectional profile caused by high adverse pressure gradients especially below \( \text{m} = -0.0905 \). Consequently, other methods of predicting the boundary layer development were investigated to get more accurate results and hence Pohlhausen method was explored for possible solutions.

### 3.3. Integral boundary layer equation

The integral boundary layer equation has been derived from the 2D Navier Stokes equation[3.2.1] by integrating it with respect to \( y \) upto any height of \( h^* \) and introducing the displacement thickness(\( \delta_1 \)) and momentum thickness(\( \theta_1 \))

\[
\delta_1 = \int_0^{h^*} (1 - \frac{u}{U})dy \tag{3.3.1}
\]

\[
\delta_2 = \int_0^{h^*} \frac{u}{U}(1 - \frac{u}{U})dy \tag{3.3.2}
\]

\[
\int_0^{h^*} (u) \frac{\partial u}{\partial x} + (v) \frac{\partial u}{\partial y} = \int_0^{h^*} \nu \frac{\partial^2 u}{\partial y^2} = \frac{\mu}{\rho} \frac{\partial u}{\partial y} \bigg|_{0}^{h^*} = \frac{\tau_w}{\rho} \tag{3.3.3}
\]

The second on the left hand side of the equation [3.3.3] has been replaced as shown below,

\[
\int_0^{h^*} (v) \frac{\partial u}{\partial y} dy = [uv] \bigg|_0^{h^*} - \int_0^{h^*} (u) \frac{\partial u}{\partial x} dy = -U \int_0^{h^*} \frac{\partial u}{\partial x} dy + \int_0^{h^*} (u) \frac{\partial u}{\partial x} dy \tag{3.3.4}
\]

Using limiting condition that as \( h^* \rightarrow \infty, \frac{\partial u}{\partial y} \rightarrow 0 \). Utilizing this with the displacement and momentum thickness mentioned in equations [3.3.1] and [3.3.2] in equation [3.3.3], the integral momentum equation has been derived, where \( \tau_w(x) \) is the wall shear stress.

\[
\frac{d}{dx}(U^2 \delta_2) + \delta_1 U \frac{dU}{dx} = \frac{\tau_w}{\rho} \tag{3.3.5}
\]
### 3.3. Integral boundary layer equation

#### 3.3.1. Pohlhausen Method:

In order to solve the dimensionless integral boundary layer equation \[3.3.5\] a quartic polynomial can be assumed for the velocity profile.

\[
\frac{u}{U} = f(\eta_{99}) = a + b\eta_{99} + c\eta_{99}^2 + d\eta_{99}^3 + e\eta_{99}^4
\]  \hspace{1cm} \text{(3.3.6)}

where \(\eta_{99} = y/\delta_{99}\), and \(\delta_{99}\) is the boundary layer thickness. To obtain the coefficients the boundary conditions that need to be used have been mentioned as follows:

\[
y = 0, u(0) = 0, \Rightarrow a = 0;
\]

\[
y = \delta_{99}, u(\delta_{99}) = U; \Rightarrow b + c + d + e = 1
\]

\[
y = 0; 0 = U \frac{dU}{dx} + \nu \frac{\partial^2 u}{\partial y^2} \bigg|_{y=0}; \Rightarrow c = -\frac{1}{2} \frac{\delta_{99}^2}{\nu} \frac{dU}{dx}
\]

\[
y = \delta_{99}, \frac{\partial U}{\partial y} \bigg|_{y=\delta_{99}} = 0; \Rightarrow b + 2c + 3d + 4e = 0
\]

\[
y = \delta_{99}, \frac{\partial^2 U}{\partial y^2} \bigg|_{y=\delta_{99}} = 0; \Rightarrow b + 2c + 3d + 4e = 0
\]

Furthermore, a dimensionless pressure gradient parameter (\(\Lambda\)) called Pohlhausen parameter has been introduced such that:

\[
\Lambda(x) = \frac{\delta_{99}^2}{\nu} \frac{dU}{dx} = -\frac{\delta_{99}^2}{U \mu} \frac{dp}{dx}.
\]  \hspace{1cm} \text{(3.3.7)}

Having utilized the aforementioned boundary conditions in the equation[3.3.6], the coefficients can be derived to be: \(a = 0, b = 2 + \frac{\Lambda}{6}, c = -\frac{\Lambda}{2}, d = -2 + \frac{\Lambda}{2}, e = 1 - \frac{\Lambda}{6}\) Hence the equation reduces to:

\[
\frac{u}{U} = f(\eta_{99}) = 2\eta_{99} - 2\eta_{99}^3 + \frac{\Lambda}{6} \eta_{99}^2 (1 - \eta_{99}^3) + \eta_{99}^4
\]  \hspace{1cm} \text{(3.3.8)}

Consequently the Pohlhausen Method has been used to generate the velocity profiles inside the boundary layer at different normalized chord positions on the airfoil.

#### 3.3.2. Comparison of Falkner-Skan and Pohlhausen profiles

A comparison of the mean velocity variation with non-dimensionalised wall normal coordinate have been conducted between the results from the Falkner-Skan, Pohlhausen method and experimental data. This has been done since flow over an is typically characterized by favourable and adverse pressure gradient flows, which cannot be captured using the Blasius profile. Hence, a comparison has been done between two methods- one from similarity method (Falkner-Skan)
Figure 3.3.1: Velocity profiles for $u = 5$ m/s at different $x/c$ positions (Pohlhausen method) for different angles of attack.

and the other from integral boundary layer equations (Pohlhausen). The figure [3.3.2] can be used to conclude that the Falkner-Skan method provides a more accurate fitting with the experimental data, hence this method has been used for obtaining the corrected wall positions for all the laminar profiles in the subsequent experiments.

Figure 3.3.2: Comparison of velocity profiles for $\alpha = 0^\circ$ at $U_\infty = 5$ m/s.
3.3.3. Neutral Stability Curve

The neutral stability curve can be utilized to demonstrate the areas in parameter space where perturbations may or may not exponentially grow. The figure [3.3.3a] shows the amplitude growth curves for various frequencies, generated using an in-house code developed by Prof. Ardeshir Hanifi, on a NACA 0008 airfoil at 0° angle of attack. From this plot four frequencies were chosen and their branch I and branch II locations were recorded. Branch I location in a neutral stability curve corresponds to initial point where for a certain frequency intersects with the $\alpha_i = 0$ contour, hence a disturbance would behave as a LCO (limit cycle oscillation) response and branch II location is point beyond which the amplitude starts decreasing again as it is the second intersection of the same frequency with the $\alpha_i = 0$ contour. The neutral stability curve in figure [3.3.3b] shows the branch I and branch II locations for each of the chosen frequencies. Consequently, between these locations the amplitude of the disturbance grows exponentially as this region comprises of contours corresponding to $\alpha_i < 0$, hence this is the unstable region, the region in the parameter space beyond the area of the neutral stability curve represents the stable region, where any disturbance would decay. These plots were utilized to get an indication of the branch I and branch II locations before the amplitude measurements were conducted at 0°. The branch I and branch II locations were then used to decide on the streamwise positions for the measurement coordinates that have been supplied to the traverse mechanism.

Figure 3.3.3: (a) Growth curves generated for different frequencies at $U_\infty = 5$ m/s, (b) Neutral Stability curve for the growth curves generated for four chosen frequencies in Figure [3.3.3a].
Table 3.1: $U_\infty = 5$ m/s Neutral Stability curve data points

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Branch 1 (x/c)</th>
<th>Branch 2 (x/c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.2114</td>
<td>0.5315</td>
</tr>
<tr>
<td>90</td>
<td>0.2057</td>
<td>0.4946</td>
</tr>
<tr>
<td>100</td>
<td>0.1932</td>
<td>0.4063</td>
</tr>
<tr>
<td>110</td>
<td>0.1930</td>
<td>0.3305</td>
</tr>
<tr>
<td>120</td>
<td>0.1917</td>
<td>0.2803</td>
</tr>
</tbody>
</table>
Chapter 4

Experimental Setup

4.1. Wind tunnel
The wind tunnel that has been used for this experiment is the BL wind tunnel located at KTH Mechanics, Stockholm. The test section shown in figure [4.2.2] has a cross sectional area of $0.5 \times 0.75 \text{ m}^2$ and a length of 4.2 m. The maximum achievable flow velocity is 48 m/s. The turbulence intensity of all three components is less than 0.04% of the free-stream velocity.

4.2. Experimental setup
Figure [4.2.2] shows the test section of the wind tunnel along with the wing profile and the LDV probe. The LDV probe in turn has been attached to a traverse mechanism capable of measuring in all three planar directions. Figure[4.2.1] provides a cross sectional view of the wing profile which has a chord length of 0.8 m and span of about 0.75 m. It also shows the locations of the TS wave generator slot and the 2 hot wires in the streamwise direction. The T-S wave has been generated using a signal generator which feeds the signal into an amplifier which in turn is connected to a loudspeaker that has plexi glass on top in order to serve as a periodic blowing and suction mechanism for generating the disturbances. These wave-like disturbances are introduced into boundary layer flow through slots at about 10% chord position.

![Sectional view of the wing with slots for T-S Wave and hot wires.](image)

Figure 4.2.1: Sectional view of the wing with slots for T-S Wave and hot wires.
4.2. Experimental setup

4.2.1. Measurement planes

The initial part of the thesis involved conducting analysis of TS wave development within the Falkner-Skan boundary layer on the wing profile. The two angles of attack for which this experiment had been conducted was $0^\circ$ and $2^\circ$. For this purpose the measurements were conducted as shown in figure [4.2.3a], which included 12 and 15 streamwise locations for $0^\circ$ and $2^\circ$ angles of attack respectively. At each of these streamwise locations 40 wall normal positions were chosen and they were distributed in a logarithmic fashion in order to better capture the fluctuations near the wall.

4.2.2. Experimental setup for the miniature vortex generators:

The second part of the thesis involved conducting experiments on the stabilization effects of miniature vortex generators (MVGs) attached near the leading edge on the TS wave disturbances. For this case, seven streamwise positions were chosen and each position corresponded to a measurement plane normal to
4.3. Measurement techniques

The mean-flow direction as shown in figure [4.2.3b]. Each of these planes in turn consisted of 11 spanwise location so as to accommodate at least one spanwise wavelength between a pair of MVGs. Subsequently, at each of the spanwise locations 25 wall normal points were distributed again in a logarithmic fashion in order to capture the streaky base-flow structures accurately.

![Diagram](image)

Figure 4.2.4: (a) MVG pairs placed on the wing profile about 24% chord, (b) Close up of an MVG pair showing the parameters of the MVGs.

The miniature vortex generators had been attached 50 mm upstream of the maximum thickness position (30% chord), which corresponds to about 24% chord position. This setup was chosen in order to have a better representation of a real-life transitional case, as the source of the TS wave disturbance is located upstream of the vortex generators. The length of the miniature vortex generators (MVG) depicted as ‘L’ and the distance between the mid-points of each MVG depicted as ‘d’ in figure[4.2.4b] has been kept constant for all the setups as 3.25 mm and 3.02 mm respectively.

4.3. Measurement techniques

The two measurement techniques that have been used are constant temperature anemometry (CTA) and laser doppler velocimetry (LDV). The locations of the hot wires have been shown in the sectional view of the wing profile as shown in fig[4.2.1]. The CTA has been used to determine the phase speed of the TS wave which is calculated from the phase difference of the signal between the consequent hot-wire ports. Additionally, the CTA was also used to measure intermittency of the non-linearities in the transitional zone, which have later been used to determine the intermittency factor (γ). For different angles of attack, the phase signals from the two CTA hot wires have been utilized to compute the phase velocity of the TS wave disturbance using the formula shown
4.3. Measurement techniques

in equation [4.3.2].

\[ \alpha_r = \frac{-d\phi + 2n\pi}{dx}, \]  

(4.3.1)

\[ \frac{c_r}{U_\infty} = \frac{2\pi f}{\alpha_r U_\infty} \]  

(4.3.2)

Here, \( \alpha_r \) represents the wavenumber of the TS wave disturbance, and consequently is the real part of the complex streamwise wavelength. \( d\phi \) is the phase difference calculated from the two signals acquired at 30% and 40% chord.

The Laser Doppler Velocimetry (LDV) is based on the principle of Mie scattering which involves the interaction of a monochromatic, coherent and collimated beam of light, with particles similar in dimension to the wavelength of the light, to result in scattering of the light. The LDV system used in this experiment is a single probe LDV, which generates two beams of equal intensity, which are focused and crossed at an angle at the focal length of the lens to form the measurement volume (MV). The interference of the two beams lead to the formation of fringe patterns within the MV, with alternating dark and bright fringes. Particles passing through the MV result in the actual scattering and the frequency is generated from the intermittent back scattered light that the photo-detector receives when the particle passes through the bright fringe. Furthermore, the fringe spacing depends on the wavelength of the incident light. This method thus requires seeding with particles smaller or equal to the fringe spacing and in turn the wavelength. But if the two incident beams have the same frequency then a stationary fringe pattern is formed inside the MV, which can useful for measuring speed of the flow. But the presence of Bragg cell in the setup used for the experiment, resulted in a frequency shift in one of the beams. This results in a moving fringe pattern, which in turn would measure the relative velocity, which means the flow direction can be accounted for. Since the experiment involved only measurement of velocity in the streamwise direction, a single probe LDV system was sufficient, for more velocity components the number of probes would have to be increased. In this experiments the LDV used a one dimensional laser-optics unit, including a 10mW He-Ne laser of wavelength of 532 nm. The measurement volume can be approximated to an ellipsoid with axes lengths of 0.14 mm and 2.4 mm. Advantages of using the LDV includes non-interference with the flow properties, independence of fluid temperature and pressure. But the drawbacks of using the LDV measurement system include its inability to provide statistics like phase speed, due to the random sampling frequency of the LDV.
5.1. TS wave disturbance analysis without MVGs

The TS wave frequency utilized for this experiment was set to 90 Hz and the initial amplitude fed from the signal generator for the initial part of experiment was 90 mV which was amplified to approximately 465.6 mV by the amplifier and fed into the loudspeaker to create the forcing amplitude of the TS wave disturbance.

5.1.1. Results for AoA = 0°

At AoA = 0°, the phase speed to free-stream velocity ratio \( \frac{c_r}{U_\infty} \) was computed to be 0.46 using equation [4.3.2]. For this case the plots of the mean velocity and the disturbance amplitude profiles have been provided in figure [5.1.1]. The \( u_{rms}^* \) that has been utilized here and in all the subsequent calculations is the free stream corrected disturbance amplitudes as shown in equation [5.1.1]. Here, \( u_{rms_{fs}} \) is the mean free stream disturbance amplitude which was calculated from the measurements taken at far-field wall normal positions along the chordwise locations in order to remove the free-stream energy on the TS wave disturbance amplitudes.

\[
 u_{rms}^* = \sqrt{u_{rms}^2 - u_{rms_{fs}}^2}
\]  

(5.1.1)

The 'mean' velocity profiles shown in figure [5.1.1b] almost look self similar when they have non-dimensionalised with displacement thickness(\( \delta_1 \)) which is defined in equation [3.3.1] and boundary layer edge velocity. However, on closer inspection of the mean velocity profiles would reveal that in the inner part (close to the wall) of the profile, one may discern differences due to the presence of inflectional profiles resulting from the growth of the external adverse pressure gradient.

The amplitude plot for the 0° case is shown Figure [5.1.2], the curve has been fitted using a third order polynomial to find the branch I and branch II locations for TS-wave frequency of 90 Hz. Similarly, the fig[5.1.4a] shows the variation of \( \alpha_i \), which is the primary contributor to exponential growth of disturbance amplitude from the Orr-Sommerfeld equations as the amplitude
5.1. TS wave disturbance analysis without MVGs

Figure 5.1.1: Plots of the data measured from the experiment at 0° angle of attack: Fig(5.1.1b) shows mean flow velocity normalized with free-stream velocity, Fig(5.1.1a) shows variation of $u_{rms}$ normalized with the maximum at the corresponding chord position.

growth is proportional to $e^{\alpha_i}$. Consequently, from the same plot it can be noted that $\alpha_i$ is negative between the branch I and branch II locations.
5.1. TS wave disturbance analysis without MVGs

The figure[5.1.3] shows the development of the disturbance energy which has been calculated as shown below.

\[ E_u = \int_0^{\eta^*} \frac{u_{rms}^2}{U_\infty^2} d\eta \]  
(5.1.2)

The \( \eta^* \) that has been used for this purpose was extended upto 9, but it can be changed to only capture the variation of the disturbance energy within the boundary layer if required.
5.1. TS wave disturbance analysis without MVGs

5.1.2. Results for AoA = 2°

At AoA = 2°, the phase speed to free stream velocity ratio \( \frac{c_r}{U_\infty} \) had been computed to be 0.49 using equation[4.3.2]. The figure[5.1.5a] shows the development of the disturbance amplitude profile along the chordwise locations. However, unlike the 0°, the disturbance amplitude profiles do not resemble the expected eigen-mode of the Orr-sommerfeld equation beyond the 48.5% chord position. This can be correlated to the mean velocity profile shown in fig[5.1.5b]. In this plot it is more evident that the mean velocity profiles are not self-similar for the 2° case. From the same figure, it can be witnessed that at 48.5% chord position the mean velocity profile started getting distorted, which can be concluded to be an effect of the boundary layer transition. Consequently, the disturbance amplitude profile also started getting distorted at the same position, which may be due to the growth the inflectional instability due to the high adverse pressure gradients at higher angles of attack.

The amplitude plot for the 2° case is shown Fig5.1.7, the curve has been fitted using a third order polynomial to find the branch I and branch II locations for TS-wave frequency of 90 Hz. Similarly, the fig[5.1.6a] shows the variation of the \( \alpha_i \), which is the primary contributor to exponential growth of disturbance amplitude from the Orr-Sommerfeld equations as the amplitude growth is proportional to \( e^{\alpha_i} \). Consequently, from the same plot it can be noted that \( \alpha_i \) is negative between the branch I locations. However, for the 2° case, the branch I location as shown in fig[5.1.7] is an approximation that had to be made as conducting measurements further upstream of that location were not possible, as a result of which the values of \( \alpha_i \) starts from a negative value. This can
be explained from the fact that at a higher angle of attack due to the higher and earlier onset of adverse pressure gradient, the neutral stability curve moves further upstream, which results in a more upstream branch I and branch II location. Additionally, the higher streamwise adverse pressure gradient also caused a much higher amplification of the disturbance amplitude, which is
evident from the fig[5.1.7].

Similarly, this pattern is also evident in the figure [5.1.5b], which shows that

(a) Variation of $\alpha_i$ in the streamwise direction. (b) Variation of maximum of $u_{rms}$ with chordwise location.

Figure 5.1.6: Results calculated from the post processing of data measured from LDV for AoA = $2^\circ$.

Figure 5.1.7: Plot of amplitude(N-factor) variation of the TS-wave disturbance.

Beyond the 48.5% chord position, the mean velocity profile start to transition towards a turbulent profile evidenced by its deviation from the self-similar laminar profile, as the disturbance amplitude starts showing non-linear growth as shown in figure [5.1.5a].
5.1. TS wave disturbance analysis without MVGs

5.1.3. Comparison between 0° and 2° results

The figure [5.1.9] can be used to draw a comparison between the behaviour of the momentum thickness and displacement thickness at different angles of attack. As is evident from the plot both displacement thickness(δ₁) and momentum thickness(δ₂) are higher for 2°, which is due to the higher adverse pressure gradient in the streamwise direction at higher angles of attack. But the shape factor(H₁₂) is lower for 2°. Additionally, the transition location moves upstream for the 2° is around 375mm, which is evident from the sudden decrease in shape factor at that position due to the fact that the displacement thickness is much lower for a turbulent profile than its corresponding laminar equivalent.

The neutral stability curves shown in figure [5.1.10] can be used to visualize, the effect that the increasing angle of attack of the airfoil can have on the branch I and branch II locations. Accordingly, it can be witnessed that the distance between the branch I and branch II locations increase due to their shift further upstream and downstream respectively. This behaviour can be attributed to the higher adverse pressure gradients at 2° than at 0°. Adverse pressure gradients would result in decelerating flow, which results in a lower critical Reynolds’s number. This is due to the fact that, the Falkner skan solutions for m < 0 due to adverse pressure gradient would lead to an inflection point in the velocity profile which could make the profile more unstable. Subsequently, this can lead to an inflectional instability in the inviscid limit in accordance to the Rayleigh’s inflection point criteria. Hence, the earlier onset of inflectional instability can attributed to as the cause for the upstream shift of the branch I location. Furthermore, the branch II location also shifts downstream as expected when the adverse pressure gradient.

To substantiate on this claim, the figure [5.1.5b] can be used to effectively display the impact that the varying adverse pressure gradient had on the velocity.
profile, leading to inflectional behaviour close to the wall. This eventually affects and distorts the eigen-modes of TS-wave disturbance. From the plot for $0^\circ$ shown in figure [5.1.11a] it can concluded that the TS-wave disturbance introduced at $x/c = 0.1$ developed further downstream into the expected eigen-function derived from the Orr-Sommerfeld solution. However, on moving further
downstream, the effect of adverse pressure gradient is evident, due to increased disturbances leading to distorted $u_{rms}$ profiles. Although, in figure [5.1.11b], the disturbance growth rate seems to be more prominent as the two peaks of the disturbance amplitude are a lot more pronounced at around the 30% chordwise position than for the $0^\circ$ case. At around 52% chord, the $u_{rms}$ profile appears a lot more distorted and much higher second peak with a maximum $u_{rms}$ value of about $12\% \times U_\infty$, which may be indicative of the initiation and prevalence of mixing within the boundary layer caused by the transition towards turbulence.

Figure 5.1.11: Plots showing spatial growth and development of disturbance amplitude along the streamwise direction, (a) For $\alpha = 0^\circ$ and (b) For $\alpha = 2^\circ$.

5.1.4. Velocity bias effects in flow fluctuations

The LDV method of flow measurement involves random sampling of velocity from the passing of particles through the measurement volume, hence time intervals between consecutive measurements may not be equidistant. It is dependant on parameters like the particle concentration, particle dimension, flow velocity and wavelength. If the sampling rate is significantly higher than the flow fluctuation frequency, then inaccuracies are bound to show up, as velocities of large magnitudes will be sampled more frequently than lower velocity magnitudes. For flow measurements related to turbulent flows, they usually involve momentum fluxes (Reynolds stresses) and kinetic energy, hence measurements involving velocity bias from LDV may not necessarily be deemed as measurement errors. Instead it would be rather interesting understanding the influence of velocity bias and the extent to which it may affect the accuracy and then decide on the requirement of correction factor.
5.1. TS wave disturbance analysis without MVGs

The biased velocity is calculated by the integral:

\[ \overline{U}_b = \int_{-\infty}^{\infty} p_b UdU \] (5.1.3)

where \( p_b \) is the probability density function which is defined as:

\[ p_b = \frac{k|U|}{\sqrt{2\pi}\sigma} e^{-\frac{(U - \overline{U})^2}{2\sigma^2}} \] (5.1.4)

where \( k \) is a constant which is a function of the turbulence intensity \( Tu (\sigma/\overline{U}) \) can be determined from the following equation:

\[ \frac{1}{k\sigma} = \sqrt{\frac{2}{\pi}} e^{-\frac{\sigma^2}{2\sigma^2}} + \frac{\overline{U}}{\sigma} erf \left( \frac{\overline{U}}{\sqrt{2}\sigma} \right) \] (5.1.5)

Using the aforementioned equation the biased velocity integral can be reduced to:

\[ \frac{\overline{U}_b}{\overline{U}} = 1 + k\sigma \frac{\overline{U}}{\sigma} erf \left( \frac{\overline{U}}{\sqrt{2}\sigma} \right) \] (5.1.6)

Similarly the biased standard deviation is calculated as:

\[ \frac{\sigma^2_b}{\sigma^2} = 1 \int_{-\infty}^{\infty} p_b (\overline{U} - \overline{U}_b)^2 dU \] (5.1.7)

Replacing \( \overline{U}_b \) into equation[5.1.7], the integral can be resolved as,

\[ \frac{\sigma^2_b}{\sigma^2} = 2 - (k\sigma)^2 [erf \left( \frac{\overline{U}}{\sqrt{2}\sigma} \right)] - k\sigma \overline{U} erf \left( \frac{\overline{U}}{\sqrt{2}\sigma} \right) \] (5.1.8)

For this purpose the raw data from the 0o has been analyzed to check the velocity bias extent of the LDV.

Figure 5.1.12: Variation of turbulence intensity at the diaturbance peak location for different streamwise position.
From figure [5.1.12] it can be concluded that the turbulent intensity calculated from the raw data for 0° case are below 0.5. Hence, in accordance with Zhang (2010) for flows with 0.01 < \( T_u \) < 10, the error function tends to unity and \( k \) can be approximated to \( 1/U \). Consequently the equations for velocity and standard deviation bias reduce to,

\[
\frac{U_b}{U} \approx 1 + \frac{\sigma^2}{U^2}
\]

\[
\frac{\sigma^2_b}{\sigma^2} \approx 1 - \frac{\sigma^2}{U^2}
\]

Figure 5.1.13: (a) Comparison between bias corrected standard deviation and the experimentally recorded standard deviation, (b) Comparison between bias corrected mean velocity and the experimentally recorded mean velocity, for 0° at the 50% chord position.

Utilizing the equations [5.1.9] and [5.1.10] the velocity bias ratio and the standard deviation bias ratio have been calculated for the 0° angles of attack, at the 50% chordwise location and shown in figure [5.1.13b] and figure [5.1.13a] respectively. It can be evidenced that for the mean velocity case the bias corrected values are co-incident on the profile generated from the experimental data, which is an indication of a good level of accuracy especially due to the fact that usually the velocity bias is accounted for by the Flowsizer software using the gate time residual weighing function and hence the accuracy of the results have not been compromised.

**5.2. TS wave disturbance analysis in the presence of MVGs**

The case that has been investigated here pertained to control of laminar flow transition over an wing profile. Hence, the streamwise pressure gradient varies with every chord position. The experiment has been conducted at an angle of
5.2. TS wave disturbance analysis in the presence of MVGs

attack of $0^\circ$, consequently, downstream of the maximum thickness position (30% chord), adverse pressure gradient keeps increasing which had the tendency to cause earlier transition. Hence the challenge was to design miniature vortex generators that would work accordingly.

<table>
<thead>
<tr>
<th>Case</th>
<th>$U_\infty$ (m/s)</th>
<th>h (mm)</th>
<th>$\lambda$ (mm)</th>
<th>$h/\delta$</th>
<th>$Re_h$</th>
<th>$Re_\delta$</th>
<th>$\beta$</th>
<th>$A_{ST%}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVG1</td>
<td>5</td>
<td>1.2</td>
<td>13.6</td>
<td>0.918</td>
<td>240.77</td>
<td>262.83</td>
<td>0.3142</td>
<td>20.29</td>
</tr>
<tr>
<td>MVG2</td>
<td>5</td>
<td>1.4</td>
<td>13.6</td>
<td>1.0708</td>
<td>519.5</td>
<td>262.83</td>
<td>0.3142</td>
<td>28.5</td>
</tr>
<tr>
<td>MVG3</td>
<td>5</td>
<td>1.5</td>
<td>11</td>
<td>1.1473</td>
<td>556.62</td>
<td>262.83</td>
<td>0.3142</td>
<td>37.22</td>
</tr>
</tbody>
</table>

Table 5.1: Table showing the three different setups of MVGs that have been used.

5.2.1. First setup with MVG1

The initial setup for the miniature vortex generators was 1.2 mm, and $\lambda$=13.6 mm, the angle of attack of the miniature vortex generators with respect to the streamwise axis was set at $9^\circ$. This setup was utilized to conduct a preliminary analysis in order to understand the behaviour of the streaks on an airfoil. In this case however, the stabilization effect on the TS wave amplitude was not evidenced from the results as shown in figure [5.2.1b]. The plot of the streak amplitude shown in figure [5.2.1a] can used to surmise the reason for the absence of stabilization, as the maximum streak amplitude is only about $0.19 U_\infty$. Thus the vortices formed due to the MVGs are not strong enough to counter the destabilization effect of the adverse pressure, as a result of which beyond 400mm position the streak amplitude starts growing, which may be due to development.
5.2. TS wave disturbance analysis in the presence of MVGs

of non-linear and sub-harmonic growth of disturbances within the streaks due
to adverse pressure gradients. Additionally, the Hence further modifications
have been made to improve the performance of the miniature vortex generators.

5.2.2. Second setup with MVG2 case

As has been evidenced in the last trial, the vortex strength needed to be
increased in order to increase the streak amplitude. Consequently, the height of
the mvg was increased to 1.4mm and the second experiment with MVGs was
conducted. The figure [5.2.2a] shows a much higher maximum streak amplitude
of about $0.285U_{\infty}$. In figure [5.2.2b] the TS wave amplitude can be witnessed
to have reduced due to the stronger streaks.

![Figure 5.2.2: Comparison of (a) Streamwise variation of streak amplitude,
(b) Variation of TS wave amplitude with streamwise location, between MVG ( h
= 1.4 mm ) and MVG( h=1.2 mm).](image)

Although beyond the 39.37% chord which corresponds to $Re_x = 1.03 \times 10^5$,
the TS wave amplitude starts increasing again. This behaviour of the plot has
been analyzed along with the velocity contours as shown in figure [5.2.3]. The
contour plots show the variation of streamwise velocity of the flow in the plane
orthogonal to both the planform plane and the streamwise flow, at different
chordwise locations. The increase in the TS wave amplitude can attributed
to the fact that the low speed streak zones depicted by dark blue zones in the
plots. The low speed streaks have not merged hence the high speed streaks are
further apart at the upstream locations and they slowly develop and come closer
further downstream. The merging point of the low speed streaks as shown in
figure [5.2.3e] and figure [5.2.3f] would be around 50%chord. This might be
the reason why the streaks start to destabilize the flow which may account for
the higher TS- wave amplitude. Subsequently, the next step of the experiment
involved not only increasing the height of the MVG array to make stronger
streaks, but also bring them closer which means the $\lambda_z$ had to be reduced.
Figure 5.2.3: Contour plots of streamwise velocity in the \( \eta-\zeta \) plane, (a) At 28.81% chord, (b) At 34.1% chord, (c) At 39.38% chord, (d) At 44.67% chord, (e) At 49.96% chord, (f) At 52.89% chord.
5.2.3. Third setup with MVG3 case

In accordance to the inferences from the last setup of the MVGs, the third setup uses MVG of height \( h = 1.5 \) mm and \( \lambda_z = 11 \) mm. Initially, due to the increase in the height of the MVG, the resulting streak amplitude are much higher. Additionally, the reduction of the spanwise streak wavelength has also shown promise. The plot of the TS wave amplitude as shown in figure [5.2.5], can be used to conduct a preliminary analysis of the performance of the new MVG setup. Although the initial amplitude of the TS wave is quite high due to the presence of the streamwise vortices close to the wake region of the MVGs, the disturbance started reducing beyond this point which may be indicative of the combined stabilization effect of the higher streak amplitudes and reduced spanwise wavelength. Additionally, in figure [5.2.4] there seems to be the development of a plateau region between 275-325 mm, where the streak amplitude approximately constant. This is different from the second MVG case, as increasing the height of the MVG not resulted in higher streak amplitudes, but also a larger wake region which may explain the plateau behaviour. Although, beyond this position the streak amplitude subsides and shows a gradual decrease as the flow moves further downstream due to the combined effect of viscous forces and adverse pressure gradients.

The contour plots of the streamwise velocity presented in figure [5.2.7] shows a stark difference from the previous setup. As can be witnessed the low speed streak zones have already started to merge at the first streamwise location due to the reduction of the spanwise wavelength of the streaks. As a result of which, the high speed streaks appear to be closer and this may be utilized to account for the better performance of the new MVG setup, although due to the increase
5.2. TS wave disturbance analysis in the presence of MVGs

Figure 5.2.5: Variation of TS wave amplitude with streamwise location between MVG1, MVG2 and MVG3 cases.

in height of MVGs have also resulted in slightly higher streak amplitudes as well as shown in figure [5.2.4].

A further comparison between the streak amplitude has been shown in figure [5.2.9] to illustrate the effect of reducing the distance between the pairs of the MVG, which in turn would reduce the wavelength of the streaks. It is evident from the figure [5.2.6b] that not only are the amplitudes of the streaks higher for the the third MVG case, but also the high speed streaks are closer to one another and hence this quickens the merging of the low streak zones. Whereas for the second MVG case with $\lambda = 13.6$ mm, only one high speed streak is visible from the figure [5.2.6a], which means that the high speed streaks are further apart. The $U_{\text{streak}}$ has been calculated as shown below.

$$U_{\text{streak}} = (U_{yz} - U_z)/\max(\{|U_{yz} - U_z|\})$$

(5.2.1)

Here $U_{yz}$ represent the mean velocities measured at each wall normal position of one streamwise plane along all the spanwise coordinates and $U_z$ is the mean of those velocities at that wall normal position for all the spanwise positions. This has been done in order to better visualize the deficit and the excess in the mean velocity profiles due to the low and high speed streaks respectively for different chord positions. Thus the reduction of the spanwise wavelength seems to have had a stabilizing effect on the TS wave disturbance in the baseflow.
Figure 5.2.6: Comparison of the streak amplitudes generated due to change in the spanwise distance between pairs of MVG. (a) For MVG2 case with \( h = 1.4 \text{mm} \) and \( \lambda = 13.6 \text{ mm} \), (b) For MVG3 case with \( h = 1.5 \text{mm} \) and \( \lambda = 11 \text{ mm} \).
Figure 5.2.7: Contour plots of streamwise velocity in the $\eta$-$\zeta$ plane for MVG3 ($h = 1.5$ mm and $\lambda_z = 11$ mm), (a) At 28.81% chord, (b) At 34.1% chord, (c) At 39.38% chord, (d) At 44.67% chord, (e) At 49.96% chord, (f) At 52.89% chord.

5.2.4. **Test to find the critical forcing amplitude for the MVG3 case**

The initial aim was to find a forcing amplitude of the TS wave disturbance for which the flow transitions even with the presence of the MVG. Consequently,
a downstream position was chosen for conducting this experiment, which was 70% chord. The first step involved finding the distance of the maximum $u_{rms}$ position from the wall (i.e. the wing surface) at 70% chord. Once this had been computed the experiment was conducted by fixing the traverse at this position with a spanwise position such that it lay in between a pair of MVGs. The figure [5.2.8] can be utilized to visualize the flow transition from laminar to turbulent as the forcing amplitude from the signal generator was increased. As is evident from the plot, for 90 Hz disturbance frequency, the flow starts transition in-between 80 and 100 mV amplitudes. Additionally, the flow seems to have transitioned into turbulent around 110 mV amplitude. The voltage input from the signal generator was processed through the amplifier. The amplified output is dependant on the input frequency hence it was necessary to individually note the amplified voltages for the 90Hz case. Thus the critical forcing amplitude was between 376.1 mV and 465.6 mV, which correspond to 80 and 100 mV input voltages respectively. Hence the critical amplitude of disturbance for 90 Hz, was chosen to be 90 mV and thus in turn the actual forcing amplitude that was delivered to the TS wave disturbance was 423.4 mV. Two different forcing amplitudes have been chosen in order to conduct
the validation of the transitional delay which are 90 mV and 130 mV in order to visualize the effect of exceeding the critical forcing amplitude. The next step involved finding the distances of the maximum $u_{rms}$ position in the wall normal direction from the airfoil surface for every streamwise location along the same spanwise coordinate and record the values of the $u_{rms}$. This experiment was conducted three successive times in order to check the precision of the measurement and check for the repeatability of the experiment.

5.2.5. Check for repeatability of experiments

![Figure 5.2.9: Plots of $(u_{rms}/U_{\infty})^2$ to check repeatability. (a) 90 mV forcing amplitude without MVG array, (b) 90 mV forcing amplitude with MVG array, (c) 130 mV forcing amplitude without MVG array, (d) 130 mV forcing amplitude with MVG array.](image)

In order to check for the repeatability of the experiment measurements were conducted three successive times. The initial part of this measurement included
5.2. TS wave disturbance analysis in the presence of MVGs

Choosing several streamwise positions and finding the wall normal position of the disturbance peak at each of these positions. Once this was completed, the measurement was conducted which involved measuring the \( u_{rms} \) at these locations. The same measurement was conducted at the same positions with and without the MVG. Additionally, the measurement was repeated for both the chosen forcing amplitudes 90 mV and 130 mV. The error bars that have been shown in the figure have a range of \( \pm 2.5\% \), hence due to the fact the low deviation in the readings, the experiment can be deemed repeatable.

5.2.6. Comparison between MVG and base case with the chosen forcing amplitudes

Figure [5.2.10a] shows the variation of the maximum disturbance energy within the boundary layer with and without the MVG. To find the maximum disturbance energy measurements had been conducted at the wall normal positions for the disturbance peaks in accordance to the repeatability measurements showed above. As shown in Figure [5.2.8] the critical amplitude is in-between 80-100 mV. Hence 130 mV is a lot higher the critical amplitude. Consequently, it can be seen from the plot that the flow transitions into turbulence even under the presence of the MVG.

However, for the 90 mV case a different can be evidenced. As can be witnessed in figure [5.2.10b], when the forcing amplitude is set to 90 mV. Without the presence of the MVG the flow seems to remain laminar upto around 530 mm which is approximately 66% chord, beyond this streamwise position the disturbance energy starts growing rapidly, indicating the initiation of transition towards turbulence. However, for the case with the MVG (h=1.5 mm and \( \lambda = 11 \) mm) attached the initial amplitude of the disturbance energy is quite higher than no MVG case. This can be explained due to the presence of the
streamwise vortices just downstream of the MVG, which lead to the formation of the streaks. These vortices contribute to increasing the fluctuations in the streamwise velocity within the boundary. But on moving further downstream the decay of the vortices lead to reduction of that fluctuation and it can evidenced that even beyond the 530 mm (66% chord), the flow remains laminar due the presence of the streaks and the proximity of the high speed streaks seem to have stabilized the and energize the boundary layer flow, such that even at high adverse pressure gradient zones, as far downstream as 580 mm (72.5%), the flow remains laminar. The physical mechanism explaining the stabilization effect can be understood using the perturbation energy equation\[5.2.2\], where $\omega$ represents the perturbation vorticity vector, the square of which represents the viscous dissipation term.

\[
K_t = \int \left( -uv \frac{\partial U}{\partial y} - uw \frac{\partial U}{\partial z} - \omega . \omega / Re \right) dx dy dz \tag{5.2.2}
\]

Due to the presence of the MVG's, the spanwise component is introduced due to the streaks. Hence, in addition to the wall normal production term $-uv \frac{\partial U}{\partial y}$, a Reynold’s stress term is introduced $-uw \frac{\partial U}{\partial z}$ is introduced into the baseflow. The stabilization may be attributed to the quenching provided by the combination of the viscous dissipation and the spanwise production term which turns out to be of negative sign of the wall-normal production term as explained in Cossu & Brandt (2004).
5.2. TS wave disturbance analysis in the presence of MVGs

(a) Input voltage 130mV and $f = 90$Hz

(b) Input voltage 90mV and $f = 90$Hz

(c) Input voltage 90mV and $f = 90$Hz

Figure 5.2.10: Plot of $(\frac{u_{rms}}{U_\infty})^2$ variation with streamwise position for (a) 130 mV forcing amplitude, (b) 90 mV forcing amplitude, (c) A closer look at the 90 mV case.
In this thesis, results from experiments conducted on the stability analysis of Tollmien-Schlichtling wave induced transition on the NACA 0008 wing profile have been reported. For this purpose, the 2D development of TS wave disturbances in the streamwise direction were monitored with increase in angle of attack. Upon measuring the streamwise velocity and its rms velocity ($u_{rms}$) profiles for different chord positions for various angles of attack, it could be concluded that increasing the angle of attack resulted in a more upstream transition location.

To demonstrate the physical significance behind the shift of the transition location, the disturbance amplitude (N-factor) were calculated and compared for both angles of attack. It was concluded that the increase in the adverse pressure gradient resulted in larger and quicker amplification of disturbances. Consequently, the maximum amplitude of disturbances are higher for the higher angle of attack. Additionally, the area under the neutral stability curve also increased, due to the upstream and downstream shift of the branch I and branch II locations respectively for the higher angle of attack.

The second part of the thesis involved developing a passive control mechanism using miniature vortex generators and conducting experiments to examine the possible stabilization effect on the TS wave disturbances. For this purpose, cross plane measurements were conducted in order to capture the development of the streaks from the streamwise vortices of the MVGs. The initial setup of MVGs with the lowest height was unsuccessful, which can be attributed to the relatively weak streamwise vortices and consequently, quicker breakdown due to the adverse pressure gradient. Furthermore, TS wave amplitudes were seen to grow more in the absence of high speed streaks. Consequently, decreasing the spanwise wavelength of the streaks by reducing the distance between a pair of MVGs seemed to have a much more profound stabilization effect on the TS-wave disturbances. The streak amplitudes and TS wave amplitudes have been calculated in this case with use of integral in the cross plane ($\eta - \zeta$ plane), which seems to have been accurate, but it is dependant on the wall normal position and the correction required to include the no-slip condition at the wall. The 3D development of waves have been investigated here due to effect of the
high and low speed streaks. The stabilization effect was tested for different forcing amplitudes in order to find the operational limit of the MVG. Subsequently, at 90 mV (critical forcing amplitude) the stabilization effect was evident from the results obtained from the experiments, however for higher forcing amplitude of 130 mV the stabilization effect was absent. Further investigations are required in order to better understand the impact that higher forcing amplitudes can have on the baseflow parameters and also taking into account the non-linear growth of sub-harmonic disturbances that have been observed at downstream location especially for higher angles of attack.
Chapter 7

Acknowledgements

My supervisor Prof. Jens H. M. Fransson, has been an inspirational figure for me in the discipline of fluid dynamics, especially his willingness to share knowledge in relation to stabilization of hydrodynamic instabilities using MVGs. I would also like to thank MSc. Santhosh B.M for his constant support and help throughout the duration of the thesis, including everything from literature sources to identifying errors during measurements. Additionally, the latter part of the thesis would not have been possible without the help of Dr. Bengt Fallenius, who spent all those painstaking hours to manufacture the miniature vortex generators and setting them up. Another person whom I would like to thank is Prof. Ardeshir Hanifi, for providing his numerically generated growth curves, and for his vital inputs especially regarding sub-harmonic instability growths at high angle of attacks. Last but not the least, I would also like to thank my thesis partners Shubham Poptani and Siddhant Tripathi, not only for their interesting ideas during brainstorming sessions, but also for providing an amicable and enjoyable environment to work in. Finally, I would like to thank SAAB AB for letting me perform my master thesis work on their newly designed and manufactured NACA 0008 wing profile at KTH, before being shipped to Brazil.

