Segmenting Observed Time Series Using Comovement and Complexity Measures

LEE NORGREN
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Supervisor at KTH: Boualem Djehiche
Examiner at KTH: Boualem Djehiche
Abstract

Society depends on unbiased, efficient and replicable measurement tools to tell us more truthfully what is happening when our senses would otherwise fool us. A new approach is made to consistently detect the start and end of historic recessions as defined by the US Federal Reserve. To do this, three measures, correlation (Spearman and Pearson), Baur comovement and Kolmogorov complexity, are used to quantify market behaviour to detect recessions. To compare the effectiveness of each measure the normalized correct Area Under Curve (AUC) fraction $\tilde{AUC}$ is introduced. It is found that for all three measures, the performance is mostly dependent on the type of data and that financial market data does not perform as good as fundamental economical data to detect recessions. Furthermore, comovement is found to be the most efficient individual measure and also most efficient of all measures when compared against several measures merged together.

Keywords: Time Series, Correlation, Baur Comovement, Kolmogorov Complexity
Segmentering av Observerade Tidsserier med hjälp av Comovement- och Komplexitetsmått

Abstract

Nyckelord: Tidserie, Korrelation, Baur Comovement, Kolmogorovkomplexitet
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1 Introduction

1.1 A statement of the problem

Between 1989 and 2001, ownership of financial stocks in American "middle-aged" families grew from 40% to over 60%[1] and has since hovered around this point. This means that on average a family is exposed to the stock market both by private investment and by their pensions that are managed by mutual funds. Increasingly, what the financial markets do today is no longer only a question merely for the financial institutions of the world, but also for the citizen of a modern society. And yet, today we still have great uncertainty of "when the next recession" is coming. This thesis is made as an attempt to move a little bit closer towards classifying the state of a market in an unbiased, efficient and replicable manner, such that anyone interested hopefully can make a little bit more informed decision on their financial future.

1.1.1 Goals and Thesis for Quantification of Market behavior

It has been thought that financial markets can be seen as a good indicator of the condition of a national economy[46]. Since 1950 the central bank of USA, the Federal Reserve (the Fed), has defined nine recession periods[2] during which the economy contracted, as can be seen in Fig 1, and listed in Tab 1 below. Note that among economists there is no absolute definition of what constitutes a recession, but generally speaking it is detectable by the stagnation or decline in economic measures such as Gross Domestic Product (GDP), company order-book and salaries.

<table>
<thead>
<tr>
<th>Recession start</th>
<th>Recession end</th>
<th>Duration (months)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jul – 1953</td>
<td>May – 1954</td>
<td>10</td>
</tr>
<tr>
<td>Aug – 1957</td>
<td>Apr – 1958</td>
<td>8</td>
</tr>
<tr>
<td>Apr – 1960</td>
<td>Feb – 1961</td>
<td>10</td>
</tr>
<tr>
<td>Mar – 2001</td>
<td>Nov – 2001</td>
<td>8</td>
</tr>
<tr>
<td>Dec – 2007</td>
<td>Jun – 2009</td>
<td>18</td>
</tr>
</tbody>
</table>

Tab 1. List of historic recessions since 1950s

As a response during a recession, the Fed lowers the Federal Funds rate (the interest rate which effectively controls the public lending interest rate in the US economy) to stimulate more consumption and help counteract deflation of the currency.

5
The subject specification and goals of this thesis are to build upon the theory discussed in the literature review discussed in the next chapter, to build upon the findings of Longin et al., Brenner et al., Baur, Kolmogorov and Zenil et al. and,

1. try to consistently detect the start/end of historic recessions as defined by the US Federal Reserve\(^2\) using correlation, comovement and complexity measures on available historic market data

2. evaluate the effectiveness of each measure and to compare and if possible, merge the measures to further increase their effectiveness.

The reasons for selecting these measures is that they complement each-others weaknesses and strengths as analytical tools when used with a time series, which can be summarized as,

- **correlation** is well understood in financial markets but needs the assumption that observed data is generated by the same probability distribution

- **Baur comovement** is less understood in financial markets but needs no assumption of probability distribution and can thus measure different data sets jointly

- **Kolmogorov complexity** has been moderately studied in financial markets, needs no assumption of probability distribution and is able to quantify any type of information.

Furthermore, by testing if these measures individually are effective measures, they may also be able to increase the efficiency further when merged. This is analogous to the idea presented in "The Wisdom of Crowds" by Surowiecki\(^3\) which states that the average estimate made by a group (of people, statistical methods, measures etc.) is often more effective than the best individual estimate made within this population.
1.2 Definition of terms

Three measures; Pearson/Spearman correlation, Baur comovement and Kolmogorov complexity are examined and tested. All three have an established utility as tools, but so far only correlation is broadly applied to the financial markets. These tools will be used to more clearly divide a financial market into different phases, also referred to henceforth as "segmenting." Below follows a brief summary of notations and introduction to measures used.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>Stock return, defined as $\frac{Stockprice_{t=1}}{Stockprice_{t=0}} - 1$</td>
<td>[1]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>linear correlation</td>
<td>-1,1</td>
</tr>
<tr>
<td>$\Phi_t$</td>
<td>Baur comovement</td>
<td>[1]</td>
</tr>
<tr>
<td>s</td>
<td>String of data</td>
<td>[bits]</td>
</tr>
<tr>
<td>$K(s)$</td>
<td>Kolmogorov complexity</td>
<td>[bits]</td>
</tr>
</tbody>
</table>

Tab 2. Main notations used

1.2.1 Brief introduction to Correlation

Correlation is a statistical tool used to measure the linear relationship between two variables. It varies between -1 and +1, where +1 indicates a complete positive linear relationship between the studied variables, if one increases, then proportionally so does the other. Conversely, -1 indicates a complete negative (opposite) linear relationship, if one variable increases, then proportionally the other one decreases. A correlation of 0 means that there is no linear relationship between the two studied variables\cite{4}, and anything below $\pm 0.5$ can be considered as having a weak correlation. In this thesis we will use two versions, the most common Pearson correlation and the less frequently used Spearman’s rank correlation. It should be pointed out early that correlation does not imply causation, only a similarity in behaviour, which may be incidental. This is a common error of misunderstanding, where correlation is often erroneously assumed to imply a causal relationship\cite{5}.

The most frequently used definition of correlation $\rho_{X,Y}$ of random variables $X, Y$ is defined as

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} \quad (1)$$

where $cov(X,Y)$ is the covariance of r.v. $X, Y$ and $\sigma_i$ is the standard deviation of random variable $i$. In a financial setting the standard deviation is often used as a measure of the volatility of an instrument.
1.2.2 Brief introduction to Baur Comovement

Comovement has been defined as a pattern of positive correlation but was in 2003 given a more specific and mathematical definition by Baur: the common movement of returns that is shared by all returns at time \( t \). Within scientific literature there is a very broad definition of what comovement means. Most frequently it is used analogously to correlation, which may be due to its absence in dictionaries. Unlike correlation, which measures a degree of relationship, comovement measures the smallest shared change in value, which is itself often measured as a fraction or percent. Mathematically it is defined below as,

\[
\Phi_t = \max(R_{1t}, R_{2t}, ..., R_{nt}) - \min(R_{1t}, R_{2t}, ..., R_{nt})
\]

where the indicator function \( 1^- (1^+) \) is equal to one if all returns are negative (positive) and zero otherwise.

1.2.3 Brief introduction to Kolmogorov Complexity

Kolmogorov complexity (also referred to as Algorithmic complexity) was discovered independently by Solomonoff, Kolmogorov and Chaitin within a span of 5 years in the late 1960’s. At its essence it can be described by the intuitive approach that the "shortest effective code" (set of instructions) that can reproduce the string (or realisation), is the measure of this string’s complexity. For instance the string ’01010101010101010101’ can be reproduced with the instruction ’01 ten times’, which indicates that string has a low complexity, unlike the string ’00101110110101110101’ which needs a longer "code" to be reproduced, and is thus more complex than the previous string. As a consequence, Kolmogorov complexity is measured in bits, because it is a measure of the shortest length of instructions needed for lossless reproduction.

Formally, the set of instructions that are needed to recreate string \( s \) is referred to as program \( p \) which is then run on machine \( U \) to recreate string \( s \). Furthermore, the length of program \( p \) can be denoted as \( l(p) \) and is measured in bits, i.e. the amount of information contained within the program, and the shortest program is defined below by Kolmogorov complexity \( K(s) \).

\[
K(s) := \min_p \{ l(p) : U(p) = s \}
\]
2 Review of literature

The idea to segment a financial market into different phases has been covered in literature before, such as by Sornette where he explains and elaborates his Dragon-King (DK) theory [34]. The theory postulates that financial crashes belong to a category of phenomena that have unique origins (like dragons) and have large and dominant consequences for the world (like kings). Sornette goes on to state that current extreme value theory (EVT), such as the power law distributions are unable to capture the behaviour found in the tails of a distribution, i.e. the extremely few but significant events. However, Janczura et al. showed that the events that Sornette refers to as Dragon Kings, can in fact be fitted with a Weibull distribution if its parameters are adjusted sufficiently [36]. Furthermore, the Weibull distribution belongs to the family of generalized extreme value (GEV) distributions which are scale-invariant. This means that the phenomena that Sornette claimed to be beyond the current power law distributions [34], are still able to be captured using extreme value theory irregardless how an extreme event is defined relative to the rest of observed data due to the effect of normalization on independent and identically distributed (iid) random variables as is proven by the by Fisher–Tippett–Gnedenko theorem.

2.1 Empirical testing for Quantification of the Dragon King Theory

As a preliminary test to attempt to quantify the Dragon king theory using the arguments of Didier [34] a simple empirical search for possible DK events found in Dow Jones Industrial Average (DJIA) from October 1928 to December 2018 can easily be made, as they should be detectable by their low frequency relative to their extreme value, as can be summarized in Fig 2 [9], below.

![Fig 2. Hypothesis that Dragon-Kings are detectable by not following known distributions](image)

While Janczura et al. chose to create an empirical distribution function $F_n$ to analyze the data [36], when using Didier’s approach and hypothesis, it should be possible to measure the size of all consecutive days where DJIA had an Adjusted Close that was negative and then use a histogram to get an empirical distribution of these. Some of these draw-downs (i.e. negative returns days)
are size-wise large enough to be outliers, and can also be called "Dragon Kings" using Sornette’s definition[34] due to their significant consequences. All negative-days were gathered in a histogram with 40 bins (with similar results for 20 - 100 bins).

As a brief explanatory example, a bullish stock trading at 4 before rising to 6 and closing at 5 the next day before rising again, would have the price vector [4 5 6 6 7]. The example is illustrated in Fig 3. below. Any uninterrupted sequence of consecutive negative returns will be recorded as one event.

Fig 3. A drop from 6 to 5 in the Graph would correspond to one event with a size of 16.6% loss in the Histogram

By using the same method described above for DJIA, but this time using a logarithmic y-axis in the histogram to compensate for the high number of low percentage loss observations, the data is represented Fig 4a, below. It is shown together with the realization of DJIA from October 1928 to December 2018.

Fig 4a. With 40 bins and 5437 loss events, there are 3 extreme events with losses > 20%
By looking at the histogram in Fig 4a, it is possible to see that using the log of the frequency there are extreme outliers present, which may indicate that Sornette may be right to some degree that there may be "life beyond power laws" although these are statistically very few (6 events among 5437 observations) as is shown in Fig 4b. (a zoomed-in version of Fig 4a.)

![Figure 4b](image)

Fig 4b. The extreme values highlighted in red for easier viewing

Because Dragon Kings are outliers by definition this could be interpreted as the observation of them. However, here also the problem arises from the definition of DK when attempting to measure them in historical data. This is because if selecting all historic market crashes since 1928 as Dragon Kings, what would then be the definition of all the other lesser but noticeable crashes that are not significant enough to be Dragon Kings but are still market significant declines that are present in a observed time series, i.e. there will constantly be very few extreme observations disproportionately outnumbered while still within a confidence interval of a GEV-distribution such as Weibull. In contrast, EVT manages to quantify and measure these events by fitting a GEV-distribution to the observed events as shown by Janczura et al., such as those shown in Fig 4b.

### 2.2 Current State of the Art and Quantification

By using correlation and "comovement" (different from Baur comovement) measures Longin et al. and Brenner et al. have established that it is possible to consistently measure a change in markets during financial crashes (i.e. bear markets). It is important to note that the authors use different mathematical definitions for computing comovement but generally find similar results of increased comovement during bear markets. However, there are also papers criticizing correlation as a measure because, for instance even when markets that have a high degree of integration they can still exhibit a low correlation due to varying degrees of sensitivities to volatility and returns, specific to each market. And, although there is sensitivity found in correlation dependent on how it is measured, it does not contradict the findings of Longin et al., but it does however highlight the importance of data selection (such as window length) as mentioned for instance by Hult et al. Since Longin et al. use monthly data, this would imply 20 trading days, which can result in lot of noise compared to for example a longer moving window of 100 days. Some testing with correlation using NDX (Nasdaq100) and SPX (S&P 500) to quantify any
market behavior building on the paper by Longin et. al. can be found in Fig 5 below. Though not conclusive by any standards, an eyeball-inspection of Fig 5. does show an interesting change in correlation spanning a 3 year interval during financial crash of 2000 and again in 2018, which shows a presence of the findings by Longin et. al.[37] and Brenner et. al.[40].

More examples of correlation as a measure of market behavior can be found for NDX and SPX in the Appendix using a 20 and 200 days moving correlation for the the years 1985 – 2019.

2.3 Markets as Endogenous- / Exogenous processes

Here the words endogenous and exogenous refer to the internal and external origins respectively for behaviour in financial markets. The words are frequently used in many different settings within different scientific fields. Within stock markets it has been found that at any moment among institutional investors there are a few but critical participants who hold extremely pessimistic views in contrast to the rest of their peers[10]. The actions of these participants, whose decisions involve a relatively large share of the markets, will amplify any already existing pattern of beliefs held by investors. Other studies have also suggested that with the prescience of certain "signatures," financial can be measured as endogenous[38], and thus acting more or less independently of fundamental financial data, such as quarterly earnings. Currently there exists no clear consensus on financial markets as endogenous, exogenous processes or if it is alternating in-between. This is mainly due to the sharp divide within both academia and the financial sector on the question of inefficient[11] versus efficient[12] markets. Black stated that the presence of noise, which he mentions as an important factor for the liquidity of a market, also makes it "somewhat inefficient"[39]. This divide is further evidenced by the 2013 Nobel prize which in the same year was shared between Fama (efficient) and Shiller (inefficient).
2.3.1 Information and participants in the market

There have been significant studies on the effect that new information has on the financial markets. When quantifying this as the volatility and comovement (different from correlation and Baur comovement) of index returns, Brenner et. al. found that new information related to the FED (target rates), Consumer Price Index, unemployment rates and non-farm payroll had a significant statistical affect on the markets\[40\]. However, they also found that the type of reaction to the new information was very dependent on the asset type, such as bonds versus stocks seeing increased and lowered daily volatility respectively in anticipation of new information\[40\].

However, the extreme events that are of interest in this report are more equivalent to the onset of a Bear market that have significant persistent consequences, rather than the smaller shocks in the daily change of return volatility as examined by Brenner et. al, which are only persistent in the short run\[40\]. For these events of longer duration, Longin et al. have examined the correlation of financial markets where they used monthly index returns from 5 major countries with observations from 1959 to 1996. They found that for extreme events such as Bear markets, the correlation increases, but that conversely this behavior is not present in Bull markets\[37\]. These findings of increased correlation echo an earlier paper by Wu et al. where they found that after the infamous October 1987 crash "black monday," the stock markets showed a substantial increase in correlation\[41\].

These findings of Brenner et. al. and Wu et. al. could imply that Bear markets indeed are more endogenous in a "self-organizing"\[34\] sense during their crashes, but that the claim that some financial crashes are exogenous processes may be more difficult to confirm since there is such a strong similarity in behavior of stock returns on a global scale during Bear markets\[37\]\[41\]. If markets would be exogenous during some crashes, then the high correlation found by Longin et. al. would imply that new macroeconomic information such as higher unemployment rates in USA have an immediate affect on the price of a stock in Japan from a fundamental (e.g. earnings etc.) analysis aspect in the valuation of that Japanese stock, even when that company has no business exposure to the USA. This has however been dis-proven to some extent as there are indications that a crisis spreads through the portfolio of international investors rather than through changes in local fundamentals\[43\], which is more in-line with findings by others\[10\]. At a basic level, there is still a lot of uncertainty regarding how market mechanisms operate, but there is a growing interest for capturing and quantifying market behaviour\[11\]\[43\]\[10\], rather than established fundamental financial/economic metrics.
By using correlation or comovement as a measure for similarity when comparing financial markets over time, then maybe the argument could be made that although every market has unique fundamentals intrinsic to its geographic location etc., an increase in similarity in behaviour (e.g. increase in correlation during Bear markets) could imply that the markets then also become to some degree endogenous. Conversely, when markets have low correlation, this could imply that they at that time they are more open to “local” fundamentals or noise[39] and thus exogenous. Both correlation and comovement is easy to quantify and to compare for different markets for different periods, as has been done by Longin et al. where they used monthly equity index returns[37]. However, while true for any measure, correlation is sensitive to how the data is selected (e.g. moving or interval) and how many days are included for each correlation measurement has a large impact on its representation as can be seen below in Fig 6. For simplification within this report, a moving window[42] will be used because it allows for a continuous daily measurement without the discrete “jumps” present when measuring with an interval.

![Fig 6. 20 Days (left) and 100 Days (right) moving Correlation](image)

### 2.3.2 Algorithmic Information Theory

Although different from "classical" statistics, because there is no need to make any assumptions on underlying distributions when working with the data, there is still the possibility of information asymmetry as discussed by Battiston et. al. when analyzing the systemic nature of the 2007-2008 financial crisis[28]. This asymmetry may for instance be caused by one bank that is in business with-, but unaware of the troubled assets, of another bank. This lack of information may thus also be a possibility within a financial market and thus not captured by a complexity measure (nor any other measure). However, Algorithmic Information Theory (AIT) is still more robust than most other statistical measures (such as correlation), and has already seen some applications within finance, e.g. as a measure of randomness. The main foundation of AIT was independently
laid by Solomonoff, Kolmogorov and Chaitin. It is centered around Kolmogorov Complexity $K(s)$ as the measure of information string $s$, with key branches extending into Solomonoff Probability $P(s)$, a measure of the probability (referred to by Solomonoff as the confidence of our estimate\cite{32}) of observing string $s$, and Levin Search (an approach to search for the most efficient algorithm to yield a solution). The relationship between complexity $K(s)$ of string $s$ its algorithmic probability $P(s)$ is given by,

$$P(s) = 2^{-K(s)} \quad (4)$$

where the base 2 is a consequence of information having a binary representation of a 0 or a 1. When applying the complexity measure to time series analysis, Chen et. al. built upon the idea introduced by Chaitin that if a time series is random, then the complexity should grow in proportion to the sample size\cite{29}. This can be graphically visualized as a 45 degree line between the y-axis of complexity and the x-axis of the sample size. As a consequence, the experimenter should thus not lose nor gain any information by adjusting the sample size. Chen et. al. found that when testing different observed time series against generated (iid normal) random walks, there was a lower complexity (and thus randomness) for the observed data than for the generated random walks. Similarly, Bonanno et. al. found that complexity of observed time series varies depending on the volatility of a trading day, i.e. if it was a normal or extreme trading day\cite{30}. These findings indicate that at times there is a presence of regularities within an observed time series and that there will different levels of how random this time series is. However, there have been similar findings where the authors instead see the periods of high complexity (randomness) as efficient flow of information and a confirmation of the Efficient Market Hypothesis\cite{31}. 


3 Methods used

3.1 Conditioning w.r.t. observations

Analysis of data to make a prediction or draw a conclusion can be simplified to being a case of conditional probability, e.g. the confidence (likelihood) that a hypothesized event will occur conditional upon the observed data. Without any data, the prediction of an event would only be a prescribed probability, also known as prior probability. By using Bayes Rule the posterior likelihood (or confidence), given data, could be computed. To detect the start of a recession, an assigned hypothesis of a recession, here denoted $P_{\text{prior}}(\text{recession})$, can be computed given observed data, denoted $P_{\text{posterior}}(\text{observed data})$, when using Bayes Rule to give the likelihood of recession given observed data, $P_{\text{posterior}}(\text{recession|observed data})$, by the equation below,

$$P_{\text{posterior}}(\text{recession|observed data}) = \frac{P_{\text{posterior}}(\text{observed data|recession}) \cdot P_{\text{prior}}(\text{recession})}{P_{\text{posterior}}(\text{observed data})}. \quad (5)$$

If the experimenter is able to assign the prior and posterior probabilities as given above, it would be a relatively simple matter to compute the likelihood of a recession. However, to calculate $P_{\text{posterior}}(\text{observed data|recession})$, a method is used to first measure the historical data using correlation, comovement and/or complexity, and secondly calculate the effectiveness of the measure given the historic dates that are defined as recession periods. As an illustrative example, the measure $M$ can be interpreted as the process of measuring the data (using correlation/comovement/complexity) and evaluating its effectiveness (using Area Under Curve), which yields the conditional generalization below,

$$M(\text{recession|observed data}) = \frac{M_{\text{correct}}(\text{data})}{M(\text{data})}. \quad (6)$$

3.2 Pearson- and Spearman Correlation

Correlation is a statistical tool used to measure the linear relationship between two variables. It varies between -1 and +1, where +1 indicates a complete positive linear relationship between the studied variables, if one increases, then proportionally so does the other. Conversely, -1 indicates a complete negative (opposite) linear relationship, if one variable increases, then proportionally the other one decreases. A correlation of 0 means that there is no linear relationship between the two studied variables, and anything below ±0.5 can be considered as having a weak correlation. In this thesis we will use two versions, the most common Pearson correlation and the less frequently used Spearman’s rank correlation. It should be pointed out early that correlation does not imply causation, only a similarity in behaviour, which may be incidental. This is a common error of
misunderstanding, where correlation is often erroneously assumed to imply a causal relationship.\[5\]

Within this project correlation is computed, either as Pearson correlation,

$$\rho_{A,B} = \frac{1}{N-1} \sum_{i=1}^{N} \left( \frac{A_i - \mu_A}{\sigma_A} \right) \left( \frac{B_i - \mu_B}{\sigma_B} \right)$$

(7)

for data sets $A_i, B_i$ each with mean value $\mu$ and standard deviation $\sigma$. As a comparison, Spearman’s rank correlation is used, where Pearson measures the linear relationship between two variables, Spearman measures if the relation is monotonic, and is thus defined as,

$$r_{A,B} = 1 - \frac{6 \sum_{i=1}^{N} d_i^2}{N(N^2 - 1)}$$

(8)

where $d_i = rg(A_i) - rg(B_i)$ is the difference in rank $rg$ (i.e. growth order) between observation $A_i, B_i$. Due to how Pearson and Spearman are defined they will measure the same observations differently as is exhibited by Fig 7 [13] below,

![Fig 7. Spearman (monotonic) vs. Pearson (linear) correlation](image)

While both Spearman (monotonic) and Pearson (linear) will assign a relatively low value (e.g. approx. 0.3 to 0.4) to seemingly uncorrelated data, Spearman is more sensitive to monotonically increasing functions, such as those shown above, which may or may not be desirable if the linear relationship is of less value.

3.2.1 Correlation and distribution

Since financial time series are not stationery, meaning that its descriptive statistics such as mean and variance are not independent of time, there is no guarantee that two different observed time series share the same random distribution. However with correlation, it is assumed that the random variables share the same sample space $\Omega$. As a consequence, when using correlation, this statement has to be assumed to be true, although that does not have to be the case. Another issue with standard statistical tools, as stated by Kolmogorov is to "distinguish between randomness proper (as absence of any regularity) and stochastic randomness (which is the subject of probability
theory).” This, he felt raised a question of the applicability of probability theory to real world problems.

3.2.2 MATLAB code implementation

To compute the correlation on day $i$ (denoted $\text{movCORRLp}(i)$ below) of observed time series data the standard corr function can be used for Pearson in MATLAB, with a certain window length as shown below,

**Pearson**

$$\text{movCORRLp}(i) = \text{corr}(R1(i:i-1+\text{windowL}),R2(i:i-1+\text{windowL}))$$

where variables $R1$ and $R2$ correspond to stock returns for each respective time series. For Spearman rank correlation on day $i$ (denoted $\text{movCORRLs}(i)$) the implementation is very similar, as shown below,

**Spearman**

$$\text{movCORRLs}(i) = \text{corr}(R1(i:i-1+\text{windowL}),R2(i:i-1+\text{windowL}),'Type','Spearman')$$
3.3 Baur Comovement

Comovement has been defined as a pattern of positive correlation but was in 2003 given a more specific and mathematical definition by Baur as "the common movement of returns that is shared by all returns at time $t$". Within scientific literature there is a very broad definition of what comovement means. Most frequently it is used analogously to correlation, which may be due to its absence in dictionaries. Unlike correlation, which measures a degree of relationship, comovement measures the smallest shared change in value, which is itself often measured as a return of a stock in fraction or percent. Comovement as defined by Baur has the property of only measuring the smallest shared movement between two or more time series ($1, 2, ..., n$) when all returns $R_t$ move in the same direction, as can be seen in the equation below.

$$\Phi_t = \max(R_{1t}, R_{2t}, ..., R_{nt}) - \min(R_{1t}, R_{2t}, ..., R_{nt})$$  \hspace{1cm} (9)

Here, the indicator function $1^{-}(1^{+})$ is equal to one if all returns are negative (positive) and zero otherwise. As a consequence comovement is defined as 0 during periods of different directions (later referred to as "zero days" in ch. 4) of returns and will thus measure behaviour less frequently than correlation. However, unlike correlation, which will increase in value four-fold when measuring a standardized shock that is increased two-fold, comovement is linear for increases/decreases. It is this linearity and lack of assumption on sample space that is necessary for correlation, that can be argued to make comovement a better tool for measuring behaviour in financial markets. Comovement is also found to be a more consistent tool due to it not having the heteroscedastic (dispersion) bias of data as has been found to be present in correlation.

To further illustrate the concept of comovement, a collection of example return vectors and their resulting comovement value are given below. These also illustrate the weakness of comovement, in a scenario where there is no common smallest joint movement and it therefore is measured as zero.

<table>
<thead>
<tr>
<th>daily return vector</th>
<th>Comovement $\Phi_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 1</td>
<td>1</td>
</tr>
<tr>
<td>2, 1</td>
<td>1</td>
</tr>
<tr>
<td>1, -1</td>
<td>0</td>
</tr>
<tr>
<td>3, 2, 4</td>
<td>2</td>
</tr>
<tr>
<td>-3, -2, -4</td>
<td>-2</td>
</tr>
<tr>
<td>3, -2, -4</td>
<td>0</td>
</tr>
</tbody>
</table>

Tab 3. Examples of different returns and their comovement
3.3.1 The special feature of comovement

Because comovement does not need to make any statistical assumptions regarding a time series, any data of interest that can be quantified can be grouped together and measured in comovement. Within the scope of this project price changes are measured as returns $R$ (or fractions), which also acts as a tool to normalize any data. Consequently, as an example the returns of a stock market can also be measured against changes in interest rate or credit availability. This makes comovement a more versatile tool when comparing market behavior because these are metrics that also have a fundamental basis in economics because interest rates and credit availability are a measure of willingness to lend and thus a market sentiment on the future.[15]

3.3.2 MATLAB code implementation

To compute the comovement in MATLAB of observed time series data there is no standard function that can be used, and thus a simple code to define it was written to define comovement on day $i$ as,

```matlab
for i=1:length(RET)
    COMVMT(i) = max(RET(i,:))*all(RET(i,:) < 0)+min(RET(i,:))*all(RET(i,:) > 0)
end
```

where $RET$ is the vector of returns or changes for markets and instruments being measured.
3.4 Kolmogorov Complexity

Another measure that does not need any assumption of distribution is Kolmogorov complexity (also referred to as Algorithmic complexity). It was discovered independently by Solomonoff, Kolmogorov and Chaitin within a span of 5 years in the late 1960’s. At its essence it can be described by the intuitive approach that the ”shortest effective code” (set of instructions) that can reproduce the string (or realisation) is the measure of this string’s complexity\[8\]. For instance the string ‘01010101010101010101’ can be reproduced with the instruction ‘01 ten times’, which indicates that string has a low complexity, unlike the string ‘00101110110101110101’ which needs a longer ”code” to be reproduced, and is thus more complex than the previous string. As a consequence, Kolmogorov complexity is measured in bits $b$, because it is a measure of the shortest length of instructions needed for lossless reproduction. In its application within Algorithmic Probability, Solomonoff\[32\] states that there are four main properties to note;

1. **Completeness**: this allows a complexity measure to find regularities in small amounts of data

2. **Incomputability**: Completeness means that the algorithm must be incomputable.

3. Incomputability does not limit practical induction due to ability to be approximated.

4. **Subjectivity**: additional a priori (i.e. before observation of data) information, even if subjective, can still give a critical advantage in prediction

Formally, the set of instructions that are needed to recreate string $s$ is referred to as program $p$ which is then run on (Turing) machine $U$ (e.g. a theoretical computer) to output string $s$. This can be viewed as running a program on a computer to output a string of data, and can be written as $U(p) = s$. Furthermore, the length of program $p$ can be denoted as $l(p)$ and is measured in bits, i.e. the amount of information contained within the program. Thus the length of the shortest set of instructions to completely recreate a string is the most efficient allocation of information for that string and is referred to as the Kolmogorov complexity $K(s)$, and is defined below\[8\].

$$K(s) := \min_p \{l(p) : U(p) = s\}$$  

This is a very powerful statement, because if computed $K$ will be able to measure if any string $s$ is optimally structured, i.e. ”random”, or if it contains regularities (”patterns”) that are redundant and thus predictable. Unfortunately, the Kolmogorov complexity of a string $K(s)$ can only be approximated because of the infinite amount of work required to find the shortest program $p$ to completely recreate string $s$. Fortunately there has been progress on approximating $K(s)$ as $c(n)$.
using the asymptotic property shown by Ziv et. al.\cite{16} to be:

\[
\lim_{n \to \infty} c(n) = \frac{n}{\log_2(n)}
\]

(11)

where \( n = l(s) \). This property allows Kolmogorov complexity to be approximated as outlined by Kaspar et. al. as a normalized measure of complexity by iterative checking how far the first part of a binary string \( s \) can reconstruct the remaining part of itself\cite{17}. Basically, the normalized measure \( h(n) \) is a ratio to the approximated Kolmogorov complexity \( c(n) \) to the information in bits \( b(n) \), as shown below.

\[
h(n) = \frac{c(n)}{b(n)}
\]

(12)

\[
b(n) = \frac{l(s)}{\log_2(l(s))}
\]

(13)

The Kolmogorov complexity is approximated with an iterative function counting the number of passes needed to fully recreate string \( s \). In addition to measuring for regularities in the data ("patterns"), complexity is also a tool for measuring the randomness of a string \( s \), as was discovered by Martin-Löf and is defined by the intuitive notion that if the Kolmogorov complexity is equal to or longer than the length of the actual string itself, the string is said to be random. This definition is given by the following equation, where string \( s \) is random iff,

\[
K(s) \geq l(s).
\]

(14)

A collection of examples are shown below to give a more intuitive understanding of the relation between any arbitrary string \( s \) and its complexity. Under the definition of Martin-Löf, the final example can be said to be an example of a random string because the approximated complexity is higher than its own description.

<table>
<thead>
<tr>
<th>data string ( s )</th>
<th>shorter description</th>
<th>approx. norm. complexity ( h(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>00000000000000000000</td>
<td>'22 times 0'</td>
<td>0.21</td>
</tr>
<tr>
<td>11111111111111111111</td>
<td>'22 times 1'</td>
<td>0.21</td>
</tr>
<tr>
<td>01010101010101010101</td>
<td>'11 times 01'</td>
<td>0.25</td>
</tr>
<tr>
<td>00110101010011101011</td>
<td>-</td>
<td>0.42</td>
</tr>
<tr>
<td>3941380273294204298731</td>
<td>-</td>
<td>0.93</td>
</tr>
<tr>
<td>sX(f7dHfM4f0dSoeQ-3)</td>
<td>-</td>
<td>1.14</td>
</tr>
</tbody>
</table>

Tab 4. Examples of different strings and their complexity
### 3.4.1 Entropy is not the same as Complexity

Shannon Entropy, which is a measure of uncertainty (or chaos), is not analogous with Kolmogorov Complexity, which is a measure of regularities (and randomness)\(^8\)\(^,\)\(^19\) within data. While both are measured in bits, there is some ambiguity in some works where the assumption is made that entropy is a measure of presence or absence of regularities. While traditionally there has been strong arguments to use compression\(^20\), recent studies have increasingly shifted away from this idea. This has led to a popular notion that compression algorithms such as \texttt{gZip} are a good way to measure the complexity of data, however this is not necessarily the case, because a compression algorithm uses exploitable statistical regularities\(^19\), unlike complexity which is a specific value of the structure of the regularities. Instead, a compression algorithm that compresses string \(s\) into a compressed string \(s_c\), will at best serve as a measure of the \textit{upper bound} of the complexity of string \(s\)\(^21\), more specifically; the length of \(s_c\) is the upper bound of the complexity of \(s\).

### 3.4.2 MATLAB code implementation

To approximate the Kolmogorov complexity of a string \(s\) in MATLAB there is no standard function that can be used. Fortunately there is an implementation of the method by Kaspar et. al developed by Faul\(^18\). To remove any redundant structures the data first has to be prepared by removing any predictable sequences, such as dates and time. Then any repetitive symbols that do not convey any actual information has to be removed, such as periods, commas and blank spaces. Finally any repetitive rounding errors have to be removed, and in this project has been set as a standard to round to nearest cent (i.e. nearest \(1/100\)th of a USD). Using "raw" time series data (i.e. prices) not its return to not add an additional layer of artificial complexity, the MATLAB code can be roughly outlined as shown below.

```matlab
TimeSeriesRound = round(TimeSeriesData*100)
stringInt = sprintf(’%d’, TimeSeriesRound)
stringIntBinary = reshape(dec2bin(stringInt, 8))
Ncplxty = kolmogorov(binary)
```

A basic explanation is that first the time series is rounded to nearest cent with \texttt{round} and formatted to integers with \texttt{sprintf}, then converted to binary with \texttt{dec2bin}. The reason for converting data to binary is because with a financial time series any relevant data will be comprised of numbers in the range 0 to 9, while the ASCII system contains 256 different symbols. Thus less than 4% of the actual sample space is used unless converted to binary representation. Finally the binary string is approximated with \texttt{kolmogorov}. An outline to the code of \texttt{kolmogorov} can be found in the Appendix.
3.5 Quantifying the Effectiveness of each Measure

To evaluate the effectiveness of each measure $M$, and to be able to compare these results, it is necessary to use a non-parametric test. This means that the results will be free from any assumptions of similarities or symmetries in distribution. For these reasons, at first a simple Sign test was considered. It tests if the hypothesis is false that two sets of data when compared pairwise against each-other have persistent differences, and uses as only assumption that the underlying distribution be continuous. Unfortunately, during empirical testing of the measures against the observed data, the sign test would for any confidence level reject the hypothesis due to rapid shifts in standard deviations of the observed data in relation to the measure. These properties also caused issues when tried with a generalized Kalman Filter known as a Particle Filter. This filter is more suitable for financial data due to the lack of necessity to assume errors are Normally distributed. The aim was using the Kalman controller gain $K_K$ and posterior error co-variance $P_K$, in an attempt to find the errors of each measure. Unfortunately, the Particle Filter could not converge to a stable solution. This issue between data and measure was solved by using an even more robust method, which was developed by using inspiration from Hajian-Tilaki et. al.\[22\] who use Area Under Curve (AUC) as a non-parametric approach to test data. This approach is more robust and has been adjusted to be better suited to measure the accuracy of detecting the historic recessions as defined by the US Federal Reserve. By calculating the AUC that is found within historic periods of defined recessions $AUC_{correct}$ and and then normalize it by comparing it against the AUC for the whole time series $AUC_{total}$, a fraction if computed that measures the (weighted) correct detection rate of historic recessions. Thus the normalized correct AUC fraction $\tilde{AUC}$ is given by,

$$\tilde{AUC} = \frac{AUC_{correct}(M)}{AUC_{total}(M)}$$

(15)

where a high $\tilde{AUC}$ fraction indicates a high correct detection rate, i.e. a good measure $M$. Within MATLAB $AUC_{correct}$ and $AUC_{total}$ can both be computed using the \texttt{trapz} function and using the Federal Reserve dates to find the correct region of detection for each measure.

3.6 Merging the measures

To further maximize the effectiveness to detect recessions in observed data, several measures are used together by utilizing the special property of comovement that allows it to measure different types of data jointly. This extends the use of comovement further as a merging function by including different measures $M$ instead of only using observed data, which yields the following equation of
comovement,

\[ \Phi_{M(t)} = \max\{M_1(t), M_2(t), \ldots, M_n(t)\} \downarrow + \min\{M_1(t), M_2(t), \ldots, M_n(t)\} \uparrow. \]  

(16)

where measure \( M_i \) is either the correlation, comovement or complexity of observed data.

### 3.7 Data

Observed data from the financial markets was collected using the services provided by Yahoo Finance\[23\] and Measuring Worth\[24\]. Any economic data was collected using the FRED\[25\]\[26\] database for economic research that belongs to the St. Louis Federal Reserve.
4 Numerical Testing and Results

4.1 Results Correlation

As can be seen in Fig 8 below, there is a strong similarity between financial markets even between indices that cover different sectors such as S&P500 (500 largest companies, including banking) and Nasdaq100 (primarily technology sector, no banking). Still, there is a very measurable change in correlation depending on market sentiment as can be seen in Fig 9 below.

Fig 8. Standard and Poor 500 (SPX) in red and Nasdaq100 (NDX) in blue using semi-log plot

Fig 9 below, which is a detail of the last three recessions, shows empirically the behaviour of increased correlation during bear market, as has already been discussed and illustrated in Chapter 2. It is interesting to note that although 2015 is not defined as a recession-year, the financial markets did briefly decline by approximately 20%, which is detected with the correlation measure.
Fig 9. Correlation SPX and NDX, window length 50 days

The normalized correct AUC fraction \( \hat{AUC} \) was measured for the correlation coefficient comparing S&P500 and Dow Jones (DJIA) from 1960 until 2019 which includes 8 recessions. Because all 30 stocks that make up DJIA are also found among the 505 stocks (as of 2019) that make up S&P500, these two indices will have some automatic correlation. But due to the diversity of the S&P500, this effect will be very small, while still offering two statistical "portfolios" that replicate the average market sentiment well. As can be seen in Tab 5 below, the average \( \hat{AUC} \) fraction of approx. 0.13 is almost independent of measure type or window length.

<table>
<thead>
<tr>
<th>Window Length</th>
<th>Pearson ( AUC )</th>
<th>Spearman ( AUC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1357</td>
<td>0.1366</td>
</tr>
<tr>
<td>20</td>
<td>0.1360</td>
<td>0.1365</td>
</tr>
<tr>
<td>50</td>
<td>0.1361</td>
<td>0.1367</td>
</tr>
<tr>
<td>100</td>
<td>0.1339</td>
<td>0.1345</td>
</tr>
<tr>
<td>200</td>
<td>0.1272</td>
<td>0.1278</td>
</tr>
</tbody>
</table>

Tab 5. Comparison of \( \hat{AUC} \) when measuring S&P500 and Dow Jones
4.2 Results Baur Comovement

In Fig 10 below, the comovement of 5 prominent stocks (General Electric, Coca Cola, IBM, Boeing and CAT) is measured to illustrate with small stock selection and consequently a low $\tilde{AUC}$ fraction of 0.156 and a high level of noise as can be seen in the lower figure. Here the fraction zero days refers to the fraction of days where the comovement cannot measure any joint movement due to opposite direction of instruments. For 5 stocks, this fraction is more than 90% of the time as can be seen in Tab 6, below. When using S&P500, the broader stock selection is noticed with a 33% better $\tilde{AUC}$ fraction at 0.208 and no zero days due to being an index.

The flexibility of the comovement measure is shown by the ability to measure economic data, such as Available Credit (AC) or Initial Claims (ICSA) with financial data, and produce a better $\tilde{AUC}$ fraction, as can be seen in Tab 6 below.

<table>
<thead>
<tr>
<th>Data measured</th>
<th>$\Phi_t \tilde{AUC}$</th>
<th>fraction zero days</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 stocks</td>
<td>0.156</td>
<td>0.937</td>
</tr>
<tr>
<td>S&amp;P500</td>
<td>0.208</td>
<td>0</td>
</tr>
<tr>
<td>S&amp;P500 and 5 stocks</td>
<td>0.179</td>
<td>0.937</td>
</tr>
<tr>
<td>S&amp;P500 and AC</td>
<td>0.256</td>
<td>0.563</td>
</tr>
<tr>
<td>AC and ICSA</td>
<td>0.348</td>
<td>0.510</td>
</tr>
</tbody>
</table>

Tab 6. Comparison of $\tilde{AUC}$ for varying data
When measuring financial and economic data, the noise level is significantly decreased as can be seen when comparing Fig 10 above and Fig 11 below. When using S&P500 and Available Credit, the \( \tilde{AUC} \) fraction becomes 0.256 and the number of zero days still remain relatively low, at about 56%.

Fig 11. Comovement with Standard and Poor 500 in blue and Available Credit in red.
4.3 Results Kolmogorov Complexity

The Kolmogorov complexity measure, which can be seen in Fig 12 below, shows many similar traits to the correlation measure (see Fig 9). This is also empirically evident when comparing the $\tilde{AUC}$ fraction, see Tab 7 below, with that of correlation, as both tend to hover with significant stability around 0.13 in fraction.

![Graph showing normalized complexity measure with shaded areas indicating Fed defined recessions.](image)

Fig 12. Approximated Normalized Kolmogorov complexity of SPX, string length 50 days

<table>
<thead>
<tr>
<th>Window Length</th>
<th>$\tilde{AUC}$ S&amp;P500</th>
<th>$\tilde{AUC}$ DJIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1343</td>
<td>0.1352</td>
</tr>
<tr>
<td>20</td>
<td>0.1348</td>
<td>0.1354</td>
</tr>
<tr>
<td>50</td>
<td>0.1348</td>
<td>0.1354</td>
</tr>
<tr>
<td>100</td>
<td>0.1315</td>
<td>0.1323</td>
</tr>
<tr>
<td>200</td>
<td>0.1236</td>
<td>0.1252</td>
</tr>
</tbody>
</table>

Tab 7. $\tilde{AUC}$ when measuring SP500 and Dow Jones

When increasing window length, similar to correlation, the perceived noise is decreased which is evident when comparing Fig 12 (50 days window length) above and Fig 13 (200 days window length) below. However, interestingly the $\tilde{AUC}$ fraction is slightly worsened with 8% as is shown in Tab 7 above.


Fig 13. Approximated Normalized Kolmogorov complexity of SPX, string length 200 days

4.4 Results Merging the Measures

When using the best $\tilde{AUC}$ results from each individual measure and then merging them together using comovement as a merging function, the results in Tab 8 show that the effectiveness of the measures together is slightly higher than the least efficient individual measure (complexity with 50 days) but significantly lower than the most effective individual measure (comovement with AC and ICSA).

<table>
<thead>
<tr>
<th>Measures</th>
<th>$AUC$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_M(\Phi_t, \rho)$</td>
<td>0.1734</td>
</tr>
<tr>
<td>$\Phi_M(h(s))$</td>
<td>0.1554</td>
</tr>
<tr>
<td>$\Phi_M(h(s), \rho)$</td>
<td>0.1218</td>
</tr>
</tbody>
</table>

Tab 8. Comparison of $\tilde{AUC}$ when merging different measures
5 Conclusion and Discussion

5.1 Statement of results

The $\tilde{AUC}$ fraction empirically shows that the most efficient measure was comovement $\Phi_t$ and that when segmenting any data against economical recession periods, the best data is fundamental economical data such as Available Credit or Initial Claims, not financial data such as stocks or indices. This is shown in Tab 6 above, where the $\tilde{AUC}$ fraction of comovement when only using economical data is 36% better than any other combination of data, and 67% better when compared against the best financial data. This was probably likely to be inevitable when using an economic definition with financial data. Furthermore, this also shows that there is significant difference between what happens on financial markets and national economies. As an example, the S&P500 peaked nearly 7 months before the Fed defined recession started in 2001, but in 2007 the difference was less than 2 months. Furthermore, in all cases recession started after the financial markets had begun to decline, as is illustrated with a generalization of each measures average detection spike in Fig 14, below.

![Fig 14. Generalization of measures over time (declining market in cyan) and Fed defined recession (grey)](image)

Because the idea of "Wisdom of Crowds" by Surowiecki works best if measures $M$ are centred around what is to be estimated, not before as was found here, the use of comovement as a merging function $\Phi_{M(t)}$ therefore yields a maximum efficiency that is lower ($\tilde{AUC}$ 0.173 in Tab 8) when compared to the best individual measure ($\tilde{AUC}$ 0.348 in Tab 6).

5.2 Statement of problems left unsolved

For this thesis the use of recessions as a definition to measure against was mainly driven by the lack on any recognized alternatives. Today a common equivalent of recession on a financial market is "bear market," which is frequently defined as a decline of 20% or more[^27]. However, this definition is very prone to false positives, such as the declines found during 2015 and 2018. Therefore it would be interesting to further research the possibility of a more robust definition of a bear market, such that financial data can have its own rigorous definition of a recession. This would also facilitate further research into finance in general, to serve as a benchmark for segmenting observed market data.
5.3 Discussion

The performance of the evaluated measures in this thesis should not be taken as a suggestion that they are not good. When testing against All Time High points in financial data, all measures scored at least 30% better than during their evaluation against recession periods. This further enhances the statement made above regarding the need for a better definition of market declines.

It should also be highlighted that the results presented here are very dependent on the method used to quantify the effectiveness of the measures. Under a Sign test all measures failed the hypothesis irregardless of circumstance, and the Particle Filter was systematically unable to converge to any solution. The use of the $\tilde{AUC}$ fraction is mainly due to its robustness and non-parametric nature which removes any need for assumptions of distribution. Furthermore, it often manages to quantify what is intuitively understood from a graphical plot. This does however not signify that it is a perfect way to quantify the effectiveness of each measure $M$. There is still relatively little literature on these implementations and thus the results should be understood to be open to further improvement and development. In conclusion, it is worth noting that in the course of this thesis many papers were found with different and sometimes opposing results or that consensus has changed over the years since publishing, and thus an attempt has been made to balance any findings and methods herein against admitting to no more truths than are able to be explained.
6 References

References


7 Appendix

7.1 Additional graphs

7.1.1 Correlation

Fig 15. Correlation SPX and NDX, window length 20 days

Fig 16. Correlation SPX and NDX, window length 200 days
7.1.2 Comovement

Fig 17. Comovement with Standard and Poor 500 (blue) and 5 stocks

Fig 18. Comovement with Standard and Poor 500 (blue), Credit Availability and 5 stocks
7.1.3 Complexity from 1990 until today

Fig 19. Standard and Poor 500 (SPX) in semi-log plot

Fig 20. Approximated Normalized Kolmogorov complexity of SPX, string length 20 days
Fig 21. Approximated Normalized Kolmogorov complexity of SPX, string length 50 days

Fig 22. Approximated Normalized Kolmogorov complexity of SPX, string length 200 days
7.1.4 Complexity from 1950 until today

Fig 23. Standard and Poor 500 (SPX) in semi-log plot

Fig 24. Approximated Normalized Kolmogorov complexity of SPX, string length 20 days
7.2 Code comments

**Approximation of Kolmogorov complexity**

Approximation of Kolmogorov complexity using the method by Kaspar et. al. as developed by Faul.[18]

```matlab
function complexity = kolmogorov(binary);
    n = length(binary);
    c = 1;
    l = 1;
    i = 0;
    k = 1;
    k_max = 1;
    stop = 0;
    while stop == 0
        if s(i+k) ~= s(l+k)
            if k > k_max
                k_max = k;
            end
            i = i + 1;
        end
        if i == l
            c = c + 1;
            l = l + k_max;
            if l + 1 > n
                stop = 1;
            else
                i = 0;
                k = 1;
                k_max = 1;
            end
        else
            k = 1;
        end
    end
    b = n / log2(n);
    complexity = c / b;
```