Alternative Methods of Estimating Investor’s Risk Appetite

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Degree Projects in Financial Mathematics (30 ECTS credits)
Master’s Programme in Applied and Computational Mathematics (120 credits)
KTH Royal Institute of Technology year 2019
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Abstract

In this thesis three risk appetite indexes are derived and measured from the beginning of 2006 to the end of the first quarter in 2019. One of the risk appetite indexes relies on annualized returns and volatilities from risky and safe assets while the others relies on subjective and risk neutral probability distributions. The distributions are obtained from historical data on equity indexes and from a wide spectrum of option prices with one month until the options expires. All data is provided by Refinitiv through Ohman Fonder.

The indexes studied throughout the thesis is provided by authors from financial institutions such as Bank of England, Bank of International Settlements and Credit Suisse First Boston.

I conclude in this thesis that the Credit Suisse First Boston index and the Bank of International Settlements index generated the most intuitive result regarding expected response after major financial events. A principal component analysis demonstrated that the Credit Suisse First Boston index held most of the information in terms of explanation of variance. At last, the indexes was used as a trend-following strategy for asset allocation for switching between a safe versus a risky portfolio. A trend in the risk appetite was studied for 2 to 12 months back in time and resulted in that all of the risk appetite indexes studied throughout the thesis can be a helpful tool to asset allocation.
Alternativa metoder för att mäta investerares riskaptit

Sammanfattning

I denna rapport studeras tre riskaptit index från början av år 2006 till slutet av första kvartalet år 2019. Ett av riskaptit indexen är beroende av årlig avkastning och volatilitet hos flera olika riskabla och säkra tillgångar medan de andra två är beroende på subjektiva och risk neutrala sannolikhets-fördelningar. Fördelningarna erhålls från historisk data från olika aktieindex och från ett brett spektrum av options priser med en månad till optionerna förfaller. All data kommer från Refinitiv genom Öhman Fonder.


I denna rapport kommer jag fram till att indexen från Credit Suisse First Boston och Bank of International Settlements genererar det mest intuitiva resultatet beträffande förväntningar efter större finansiella händelser. En principal komponent analys visade på att Credit Suisse First Bostons index innehåller mest information i form av förklaring av variansen. Tills sist så användes riskaptit indexen som en trendföljande strategi för tillgångsallokering mellan en säker och en riskfylld portfölj. Trenden i riskaptiten studerades från 2 till 12 månader bak i tiden och resultatet visar på att alla undersökta riskaptit index i denna rapport kan fungera som ett verktyg för tillgångsallokering.
Acknowledgments

I would like to give a big thanks both to my supervisor at KTH, Boualem Djehiche, and to my supervisor at Öhman Fonder, Jamal Abida Norling. Jamal has had many useful inputs to the thesis in times when I got stuck and needed help. I would also like to thank the company Öhman fonder and all the colleagues for providing me with the data and knowledge needed to accomplish the thesis. At last, but not to forget, I would like to thank my friends and family for being supportive and helped me a lot during the time at KTH.
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1 Introduction

1.1 Background

The market sentiment, investors attitude, is often seen as a key factor driving various trends in the financial market. Increasing asset prices indicates a positive attitude towards the market, which in financial terms is known as a bull-market, while falling asset prices indicates a negative attitude and is known as a bear-market. A good illustration of a shifting market sentiment is the "dotcom-bubble" in the late 1990s until the early 2000s. The market shifted from a euphoric behaviour to excessive pessimism in a short period of time. Another example of shifting market sentiment was the financial crisis with its outburst in 2008. Experienced market practitioners know that the market tend to overshoot its expectations, both up and down, and in later days it has been of interest to measure the market sentiment. The task might sound easier than what it is and several articles has been published about the subject with various views describing the market sentiment. They are summarized in the articles "A Brief Survey of Risk-Appetite Indexes" and "Measuring Investors Risk Appetite” written in 2005 and 2007.[1][2]. Concepts such as risk aversion, risk tolerance and risk appetite is mentioned in these reports with purpose to describe the market sentiment, though the importance of keeping them distinct can not be neglected.

The survey of risk appetite indexes is a brief summary and was written in 2005 which means that the underlying data is only for years before 2005. It is of great interest to see how the risk appetite indexes has developed after this year and how they responded to, specifically, the big crisis in 2008.

The general idea with the risk appetite indexes is to capture when investors experience both excessive fear and when they experience a euphoric behaviour and then take advantage of it. A period of low risk appetite is often followed by a period with with high risk appetite, which gives opportunities to use the indexes as trend-following strategies.

1.2 Purpose

In this thesis I will with the help of the fund company Öhman Fonder investigate the concept risk appetite by looking at alternative methods from the survey "A Brief Survey of Risk-Appetite Indexes” with my own refinements. The purpose of this thesis is to measure investors risk appetite by alternative methods and analyze them. This includes the implementation of several risk appetite measures and analyzing them with the help of mathematical tools.

1.3 Research Question

In summary the following research questions will be treated in the thesis,

• Which of the risk appetite indexes gives best response to euphoric periods and excessive pessimism according to what we expect?
• Which risk appetite index holds most of the information?
• Can the risk appetite indexes studied throughout the thesis work together as a better alternative?
• By implementing the risk appetite indexes as a trend-following strategy, which of them yields the best result in terms of return, sharpe ratio, sortino ratio and max-drawdown?
1.4 Structure

The thesis is structured as follows. The underlying theory used through the thesis is presented in section 2. Section 3 presents the methodology for each method used to measure the risk appetite with my own refinements and with the associated data. At last in section 3 I describe how I will validate my measurements. Further on the result is presented in section 5 followed up by the discussion in section 6. At last in section 7 I summarize my conclusions from the thesis. An appendix is included in the end of the report.

1.5 Delimitations

I have chosen to delimit this thesis by studying 3 of the risk appetite indexes from the survey. This is due to time limitation. Another limitation lies within each of the index studied according to each of their underlying data.

The methods provided by Gai & Vause and Tarashev et al. relies on monthly prices on a wide spectrum of options. The number of data points increases fast by shortening the time-period studied and thus these methods is limited to only provide monthly values.

The method provided by Credit Suisse First Boston relies on many assets and this is the only limitation in this method. David Holst and Anton Norberg compares how this index is affected by only include 6 assets and the result is closely related to the index with 51 assets.[3] In this thesis the index relies on 57 underlying assets and the risk appetite values are daily values.

1.6 Findings

In this thesis three risk appetite indexes is derived and measured from the beginning of year 2006 until the first quarter in 2019. The risk appetite indexes studied responded differently to major financial events during the time period studied and all together the result was plausible.

A principal component analysis was made on the three risk appetite indexes and the result showed that the index provided by Credit Suisse First Boston held most of the information. A high correlation was found between the Tarashev’s and Credit Suisse First Boston’s index, although the methods is independent and relies on distinct datasets.

Looking at the trend in the risk appetite indexes has proved to be a good tool for asset allocation, in terms of when to switch between risky and safe assets. Implementing a trend-following strategy on all indexes confirmed that Gai & Vause’s risk appetite index, with a 4-month trend, was the best tool for asset allocation in terms of a portfolios mean annual return for the time period studied.
2 Theory

The first subsection introduces definitions and the notation through the thesis. The second subsection presents general theory that the thesis relies on.

2.1 Definitions and Notation

Risk - The possibility of losing something of value  
Risk appetite - Investors willingness to bear risk  
Risk aversion - Investors uncertainty to bear risk  
Risk premium - The return the investor is compensated with for taking risk  
Return - The profit or loss of an underlying asset  
Volatility - The fluctuation of an assets price

2.2 General theory

2.2.1 Principal Components Analysis

Principal Component Analysis (PCA) is a statistical method to transform possibly correlated variables into a set of values that are linearly uncorrelated. These linearly uncorrelated variables are called the principal components. It is an unsupervised method since it only includes the explaining features, \(X_1, X_2, ..., X_p\) with \(n\) observations each, and does not include any response variable, \(Y\). Notice that each of the features must be centered with mean zero. The main purpose of the PCA is to find a small number of factors that explains most of the variation in the original dataset.

The first principal component can be derived from the normalized linear combination of the explaining features

\[
Z_1 = \phi_{11}X_1 + \phi_{21}X_2 + ... + \phi_{p1}X_p
\]  

that has the largest variance. The elements \(\phi_{11}, \phi_{21}, ..., \phi_{p1}\) is called the loadings of the first principal component and by normalized it means that the sum of squares of these elements shall be equal to one. The principal component loading vector consists of all these loading elements and is defined as,

\[
\phi_1 = (\phi_{11}, \phi_{21}, ..., \phi_{p1})^T
\]  

The loading vectors can thus be achieved by solving the maximization problem,

\[
\text{maximize}_{\phi_{11}, \phi_{21}, ..., \phi_{p1}} \left\{ \frac{1}{n} \sum_{i=1}^{n} \left( \sum_{j=1}^{p} \phi_{j1}x_{ij} \right)^2 \right\} \text{ subject to } \sum_{j=1}^{p} \phi_{j1}^2 = 1
\]  

and with the help of linear algebra it can be shown that the loading vectors is equal to the eigenvectors to the matrix \(X^TX\) and the eigenvalues correspond to the variance of each component vector. The interpretation of these together is that the original data set vary the most for the highest eigenvalue obtained in the direction of the the corresponding eigenvector. [4]
2.3 Options

2.3.1 European call option

A European call option on \( S_T \) with strike price \( K \) is a contract that gives the owner the right, but not the obligation, to buy the underlying asset at time \( T \) for the price \( K \). The payoff function for a European call option is given as follows,

\[
f(S_T) = \max(S_T - K, 0)
\]  

(4)

2.3.2 European put option

A European put option on \( S_T \) with strike price \( K \) is a contract that gives the owner the right, but not the obligation, to sell the underlying asset at time \( T \) for the price \( K \). The payoff function for a European put option is given as follows,

\[
f(S_T) = \max(K - S_T, 0)
\]  

(5)

2.3.3 Put-call parity

The put-call parity is a relation between the price of a European put option and a European call option with same strike price \( K \) and time to maturity \( T \) and the relationship is defined as follows,

\[
P(t, S(t)) = Ke^{-r(T-t)} + C(t, S(t)) - S(t)
\]  

(6)

The relation implies that we can replicate a European put option by buying \( K \) zero coupon bonds with maturity \( T \), and a call option with same strike price \( K \) and time to maturity as the put option, and selling the underlying.

The relation also implies determining the interest rate, \( r \), by knowing the other variables and it is given below,

\[
r = -\frac{\ln\left(\frac{P(t, S(t)) + S(t) - C(t, S(t))}{K}\right)}{(T-t)}
\]  

(7)

2.3.4 Black-Scholes formula

Black-Scholes formula provides a risk neutral valuation of European call options and the definiton is given below with dividends included,

The price of a European call option with time to maturity \( T-t \), continuous dividend yield \( \delta \), and strike price \( K \) is given by the formula \( \Pi(t) = C(t, S(t)) \), where

\[
C(t, s) = se^{-\delta(T-t)} N[d_1(t, s)] - e^{-r(T-t)} KN[d_2(t, s)]
\]  

(8)

where \( S(t) \) is the spot price of the underlying at time \( t \), \( r \) is the risk free interest rate, \( N \) denotes the cumulative distribution function for the \( N[0,1] \) distribution and \( d_1, d_2 \) is given by,

\[
d_1(t, s) = \frac{1}{\sigma\sqrt{T-t}} \left\{ \ln\left(\frac{s}{K}\right) + \left( r - \delta + \frac{1}{2}\sigma^2\right)(T-t) \right\}
\]  

(9)

\[
d_2(t, s) = d_1(t, s) - \sigma\sqrt{T-t}
\]  

(10)

where \( \sigma \) is the implied volatility.
2.3.5 Newton Raphson's Method and Implied Volatility

Newton - Raphson's method is a numerical approach to find the zero roots for a given function, \( f(x) \)[5]. The recursive formula used in the method is defined as,

\[
x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}
\]  (11)

To find the implied volatility from Black-Scholes framework we introduce the function \( f(\sigma) \) (where \( \sigma \) is the unknown parameter) as,

\[
f(\sigma) = sN[d_1(\sigma)] - e^{-r(T-t)}KN[d_2(\sigma)] - C
\]  (12)

and the derivative with respect to the volatility,

\[
f'(\sigma) = s\phi(d_1(\sigma))\sqrt{T-t} = \mathcal{V}(\sigma)
\]  (13)

Thus we obtain the recursive formula to find the implied volatility as,

\[
\sigma_{n+1} = \sigma_n + \frac{f(\sigma_n)}{\mathcal{V}(\sigma_n)}
\]  (14)

Notice that it is understood that the spot price of the underlying, the risk free interest rate, strike price and time to maturity is well known parameters.

The method needs a start guess of the implied volatility, \( \sigma_0 \), and a tolerance for when to stop iterate. In this thesis I used the values,

\[
\sigma_0 = 0.30 = 30\%
\]

\[
e_{tol} = 10^{-4}
\]

The choice of these parameters are arbitrary. The start guess of the volatility is a natural guess.
2.4 Probabilities

2.4.1 Implied forward probabilities

The implied forward probabilities can be obtained from considering a wide spectrum of call options that expires at the same time. If we let the call price, $C$, only be dependent on the strike price, $C(k)$, the forward probability density can be derived from the general pricing formula as,

$$q(k) = \frac{1}{\text{e}^{r(T-t)}} \frac{d^2C}{dK^2}$$

This was at first introduced by Breeden and Litzenberger in 1978. [6] A disadvantage with this method is that the volatility is assumed to be constant which implies the underlying asset to be log-normally distributed in Black-Scholes framework. To handle this issue we let the volatility be dependent on the strike price $K$. The adoption of the strike price dependence in the volatility then leads to an observed volatility smile which is described by John C. Hull [7]. The volatility smile can then be fitted to a second order polynomial,

$$\sigma(k) = a_0 + a_1k + a_2k^2$$

Note that the choice to fit the volatilities with a second order polynomial is nothing but a crude guess and an arbitrary choice, though it still satisfies the pattern of the implied volatilities.1

The dependence of $K$ in the volatility then implies that the forward probability density is given by,

$$q(k) = \frac{1}{\text{e}^{r(T-t)}} \left( \frac{\partial^2 C}{\partial K^2}(k, \sigma(k)) + 2 \frac{\partial^2 C}{\partial K \partial \sigma}(k, \sigma(k)) \frac{d\sigma}{dK}(k) \right.
+ \frac{\partial C}{\partial \sigma}(k, \sigma(k)) \frac{d^2 \sigma}{dK^2}(k) + \left. \frac{\partial^2 C}{\partial \sigma^2}(k, \sigma(k)) \left( \frac{d\sigma}{dK}(k) \right)^2 \right)$$

The derivatives in equation (17) is given by,

$$\frac{\partial^2 C}{\partial K^2}(k, \sigma(k)) = \frac{e^{-r(T-t)}}{K\sigma \sqrt{T-t}} \phi(-d_2)$$

$$\frac{\partial^2 C}{\partial K \partial \sigma}(k, \sigma(k)) = \frac{e^{-r(T-t)}G_0}{K \sigma} \phi(d_1)$$

$$\frac{\partial^2 C}{\partial \sigma^2}(k, \sigma(k)) = \frac{e^{-r(T-t)}G_0 \sqrt{T-t}}{\sigma} d_1 d_2 \phi(d_1)$$

$$\frac{\partial C}{\partial \sigma}(k, \sigma(k)) = e^{-r(T-t)}G_0 \phi(d_1) \sqrt{T-t}$$

$$\frac{d^2 \sigma}{dK^2}(k) = a_2k + a_1$$

$$\frac{d\sigma}{dK}(k) = a_2$$

1See the appendix for examples of the implied forward probabilities and the volatility smile.
where $G_0$ equals the forward price of the underlying, which in this case is given by the relation,

$$G_0 = S_0 e^{(r - \delta)(T - t)}$$

where $\delta$ is the annual dividend yield for the underlying and $\phi(x)$ is the probability density function for a standard normal distribution[8].

### 2.4.2 Historical probabilities for monthly returns

The historical probabilities for monthly returns are derived from the monthly log-return. The monthly log-returns are calculated for each month interval under the time-period investigated. The monthly log-return are calculated as,

$$R_m(t_i) = \ln \left( \frac{S(t_i)}{S(t_{i-20})} \right) \quad i = 20, 21, ..., N \quad (18)$$

where $N$ corresponds to the index of the last day in the time period investigated.\(^2\)

The next step is to fit the log-returns to a specified distribution. In this thesis I choose to fit the log-returns to a Student’s $t$ distribution. Typically is that one choose to fit the log-returns to a normal distribution, but it tends to give a poor fit.\(^3\) The Student’s $t$ distribution has the density function

$$\frac{\Gamma((\nu + 1)/2)}{\sqrt{\nu \pi} \Gamma(\nu/2)} \left( 1 + \frac{(x - \mu)^2}{\nu \sigma^2} \right)^{-(\nu+1)/2} \quad \text{for all } x,$$

where $\Gamma$ is the gamma function, the parameter $\nu$ is the degrees of freedom, $\mu$ is the location parameter and $\sigma$ is the scale parameter. The estimated parameters, $(\hat{\mu}, \hat{\sigma}, \hat{\nu})$, is obtained from maximizing the likelihood function throughout the thesis.

---

\(^2\)The choice of the the interval twenty days is due to match the time period between the option price and the time to maturity of the option.

\(^3\)See the appendix for the argument to use a Student’s $t$ distribution instead of a normal distribution. The log-returns for each equity index that I used is both fitted to a Student’s $t$ distribution and a normal distribution to indicate the difference between these two.
2.5 Portfolio evaluation measures

The portfolio evaluation measures is estimated annually for the the considered portfolio and then presented as the mean of the included years.

2.5.1 Sharpe ratio

The sharpe ratio is a common value in finance to evaluate a portfolio. It both takes the return and the volatility of the portfolio into account. The ratio was introduced by William Sharpe and it is defined as,

\[ SR = \frac{R_p - r_f}{\sigma_p} \]  \hspace{1cm} (19)

where \( R_p \) is the portfolio return, \( r_f \) is the risk free interest rate and \( \sigma_p \) is the volatility of the portfolio. It is understood that the investor wants a high sharpe ratio for the given portfolio.

2.5.2 Sortino ratio

The sortino ratio is a natural extension of the sharpe ratio. Since the sharpe ratio takes both positive and negative returns into account in the volatility, one is interested to only look at the downside risk. Thus, the sortino ratio is defined as,

\[ SOR = \frac{R_p - r_f}{\sigma_{pd}} \]  \hspace{1cm} (20)

where \( \sigma_{pd} \) is the volatility of the negative returns. A high value of the Sortino ratio is wanted from the investor.

2.5.3 Max-drawdown

The max-drawdown value is defined as the maximum loss an investor can be exposed to by investing in the given portfolio. The max-drawdown value is defined as,

\[ MDD = \frac{P_{min} - P_{max}}{P_{max}} \]  \hspace{1cm} (21)

where \( P_{max} \) and \( P_{min} \) is the maximum and minimum portfolio value in the time period studied. The time period is one year in this thesis for for the max-drawdown value. A high value of the max-drawdown is wanted from the investor since this ensures a more stable portfolio.[9]
3 Methodology & Data

3.1 Methodology

In this section I first describe each method separately, then with their underlying data and at last how I will validate these methods. The methods names used throughout the thesis is given below in Table 1 with the financial institution it was written for.

<table>
<thead>
<tr>
<th>Method</th>
<th>Financial Institution</th>
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<tr>
<td>Gai &amp; Vause</td>
<td>Bank of England</td>
</tr>
<tr>
<td>Tarashev</td>
<td>Bank of International Settlements</td>
</tr>
<tr>
<td>CSFB</td>
<td>Credit Suisse First Boston</td>
</tr>
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</table>

Table 1: Shortnames for institutions

3.1.1 Gai & Vause

This method was proposed by Prassana Gai and Nikolas Vause in 2005[10] and it starts from standard asset pricing theory from the literature written by John H. Cochrane [11]. The methodology follows Gai & Vause with some modifications to finally arrive at an expression for the risk appetite.

Standard asset pricing theory states that the current price of an asset, \( p_t \), is equal to the expected value of the discounted possible future payoffs, \( m_{t+1} \cdot x_{t+1} \), with the condition that all investors are fully informed and rational in an efficient market. More formally we can write the current price with the following equation,

\[
p_t = E[m_{t+1} \cdot x_{t+1}]
\]  

(22)

where the current price is given as \( p_t \), the stochastic discount factor is given as \( m_{t+1} \) and the possible future payoffs is given as \( x_{t+1} \). Note that \( x_{t+1} \) is the future value of the investment and not the return of the investment. If the asset is paying future dividends it will be included in the future payoff value, for example \( x_{t+1} = p_{t+1} + d_{t+1} \).

Let us take a closer look at the introduced stochastic discount factor \( m_{t+1} \), also called marginal rate of substitution. It measures how much an investor is willing to substitute consumption at time \( t + 1 \) for consumption at time \( t \) and thus it is dependent on the investors utility function, \( u(x) \). We write the stochastic discount factor as,

\[
m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}
\]  

(23)

where \( \beta \) is the subjective discount factor and \( c_t \) is the investors consumption at time \( t \). Following Bliss and Panigirtzoglou [12] there is a common relation between the marginal rate of substitution, investors subjective density function, \( p(S_{t+1}) \) and the risk-neutral density function, \( q(S_{t+1}) \). Assuming a complete and frictionless market and a single asset the relation holds as follows,

\[
\frac{u'(S_{t+1})}{u'(S_t)} = k \frac{p(S_{t+1})}{q(S_{t+1})}
\]  

(24)

where \( k \) is a constant. The following expression for the stochastic discount factor is then obtained,

\[
m_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)} = k \beta \frac{p(S_{t+1})}{q(S_{t+1})} = K \frac{p(S_{t+1})}{q(S_{t+1})}
\]  

(25)
where $K$ is a new constant. I will not put further investigation in investors utility function, since it can vary a lot between different type of investors and might be dependent on the considered market. Supposing the asset is risk-free we define the expected value of the stochastic discount factor as,

$$E[m_{t+1}] = \frac{1}{R^f_{t+1}}$$  \hspace{1cm} (26)

where $R^f_{t+1}$ is the gross risk-free interest rate.

Following up equation (22) with the formula for covariance (see equation (40) in the appendix) and the above expression for the expected value of the stochastic discount factor we can rewrite the current price of the asset as,

$$p_t = E[x_{t+1}] \frac{R^f_{t+1}}{R^f_{t+1}} + \text{Cov}(m_{t+1}, x_{t+1})$$  \hspace{1cm} (27)

The first term in the right-handside of the equation is the assets price as if we would live in a risk-neutral world. The second and last term in the right-handside of the equation is the compensation the investor get for taking risk. Dividing the above equation with the current price and then inserting the expression for the expected value of the stochastic discount factor from equation (26) we arrive at a common expression,

$$E[R_{t+1}] - R^f_{t+1} = -R^f_{t+1} \text{Cov}(m_{t+1}, R_{t+1})$$  \hspace{1cm} (28)

where $R_{t+1}$ is the return of the investment and the left-handside is known as the "risk premium" of the investment.

Further on we do a simple trick to lead us to the more general expression,

$$E[R_{t+1}] - R^f_{t+1} = -\frac{\text{Cov}(m_{t+1}, R_{t+1})}{\text{Var}(m_{t+1})} \cdot \frac{\text{Var}(m_{t+1})}{E[m_{t+1}]} \equiv \beta \lambda$$  \hspace{1cm} (29)

where $\lambda$ can be rewritten with equation (26) as,

$$\lambda = \frac{\text{Var}(m_{t+1})}{E[m_{t+1}]} = \text{Var}(m_{t+1})R^f_{t+1}$$  \hspace{1cm} (30)

The introduced parameter $\lambda$ is known as the "price of risk". The higher the risk the investor is willing to take, the more the investor wants to be compensated in form of excess return. Furthermore the risk appetite is defined as the inverse of the price of risk parameter - "The willingness of investors to bear risk". It then follows by combining equation (25) with equation (30) that the following expression is a measure of investors risk appetite,

$$\frac{1}{\text{Var}(p(S_{t+1}/q(S_{t+1})))} \cdot \frac{1}{R_{t+1}}$$  \hspace{1cm} (31)

From the expression above one can expect the risk appetite to be low when the subjective probabilities of the investor varies a lot from the risk-neutral probabilities. The constant $K$ disappears in the expression for the risk appetite since the unit of the risk appetite is arbitrary and will not change the interpretation of the estimation. The risk appetite values can thus be normalized to values between 0 and 1 where 0 indicates minimum risk appetite and 1 indicates maximum risk appetite.\(^4\)

\(^4\)The way of interpreting the risk appetite in levels rather than its real values is also discussed in the article "A Brief Survey of Risk-Appetite Indexes".
The risk-neutral probabilities are obtained from a cross-section of put and call options and the underlying math is described in the theory section as the "Implied forward probabilities" while the subjective probabilities are based on historical monthly returns. The risk free interest rate is obtained using the put-call parity for options maturing at the same date with the same strike. Since there is many strikes for one maturity, the risk free interest rate is determined as the mean of all rates from the put call parity. All options considered in this thesis is of the exercise type "European" with a given exercise date.

In this study it is considered to measure the risk appetite values from various assets and then combine them by a simple market-cap weighting. The risk appetite values for each equity index will be weighted with their market-cap. If monthly values is missing for any of the indexes, then only the existing values will be considered.

---

5Gai & Vause estimated the subjective probabilities following Hayes et al approach with a threshold GARCH model based on historical returns.

6There are many ways to derive the risk free interest rate for example it can be derived from treasury bills etc. The argument of using the mean of the risk free interest rates from the put-call parity on options maturing at the same date is also drawn by Hult et al.

7American options are much more complex in pricing theory, but under some conditions the price of an American call option coincides with a European call option price. This is discussed by Tomas Björk in his book "Arbitrage Theory in Continuous Time."[13]

8The study by Gai & Vause only covered data on the equity index S&P500
3.1.2 Tarashev

The method provided by Nikola Tarashev, Kostas Tsatsaronis and Dimitrios Karampatos[14] is closely related to Gai & Vause’s method\(^9\). Tarashev et al. studies the ratio of the risk-neutral distribution and the subjective distribution, but in comparison to Gai & Vause they assume that the subjective distribution is the one obtained from the cross section of options and that the risk-neutral distribution is the one derived from a GARCH model. In this thesis I follow their methodology with two exceptions. The first exception is that I look on the historical distribution for monthly returns instead of a GARCH model. The second is that I measure the inverse of their interpretation of investors risk aversion, which in my interpretation is a measure of investors risk appetite. The measure of the risk appetite is then given as the ratio between the two distributions,

\[
\frac{Q_1}{Q_1 + Q_2} \div \frac{P_1}{P_1 + P_2} \tag{32}
\]

where \(P\) and \(Q\) is given from Figure 1 below,

![Figure 1: Distribution plots](image-url)

Since no extrapolation outside the range of strikes, \(K\), is desirable the subjective distribution constraints the historical distribution to a decline at most to the minimum strike for the maturity date and to an increase of the maximum strike. The measure shall both capture periods when investors expresses pessimism and the opposite i.e when investors indicate too much self confidence.

---

\(^9\)Gai & Vause’s method is based on the method introduced by Tarashev et al. but they look at the whole distribution instead of the tails.
3.1.3 CSFB

The Credit Suisse First Boston Risk-Appetite Index was introduced by Jonathan Wilmut, Paul Mielczarski and James Sweeney [15]. They consider to model the risk appetite by studying a large set of financial data from various asset markets and comparing their performance against risk taken. They separate these assets in two groups, one group containing risky assets and the other one containing safe assets. Both of the groups are then divided in to two subgroups which are "Developed Equities" and "Emerging Equities" for the risky asset group and "Developed Fixed Income" and "Emerging Market Bonds" for the safe asset group. They then argue that the risk appetite is high during periods with high returns for the risky asset group and low returns for the safe asset group. The opposite holds for a low risk appetite i.e when the safe asset group outperforms the risky asset group. Then for each asset, the six-month excess return over cash and the corresponding 12-month volatility is calculated and annualized and then presented in a graph where the excess return is the dependent variable (i.e y-axis) and the volatility is the independent variable (i.e x-axis). The risk appetite is then interpreted as the slope coefficient from a simple linear regression between the two variables.

The value of the risk appetite is flexible in a way that it is possible to separate the continents from the global risk appetite index and look at them individually. It is however important that the underlying assets for the continent both belong to the risky and the safe asset group since the theory of the index relies of the slope between these two groups. Another flexible aspect of the index is how the returns and the volatilities are constructed. Dependent on how these two variables are constructed the index tend to shift more rapidly with a short time period investigated, and more slowly with a wider time period. In this thesis the return for each asset is constructed the following way,

\[ R_{\text{annual}}^*(t_i) = \frac{N_{\text{year}}}{N} \cdot \sum_{j=i-N}^{i} R_{\text{exc}}(t_j) \]  

(33)

where \( N_{\text{year}} \) is the total number of business days in a year, \( N \) is the length of the time period (in days) investigated and the excess return is calculated as the daily log-return adjusted with the risk-free interest rate for the assets currency, \( r_{\text{curr}}(t_i) \),

\[ R_{\text{t,exc}} = \ln \left( \frac{S(t_i)}{S(t_{i-1})} \right) - r_{\text{curr}}(t_i) \]

(34)

where \( S(t_i) \) is the spot price of the asset at time \( t_i \). The risk-free interest rate is achieved from overnight deposit rates for each currency.

The volatility for each asset is calculated by first introducing the variable \( Z_i \) which is equal to the first term in the excess return i.e the log-return of the spot prices,

\[ Z_i = \ln \left( \frac{S(t_i)}{S(t_{i-1})} \right) \]

(35)

The standard deviation of \( Z_i \) is expressed as follows

\[ S_{Z_i} = \sqrt{\frac{1}{N-1} \sum_{j=i-N}^{i} (Z_j - \bar{Z})^2} \]

(36)

and the estimate of the annualized volatility is then obtained as,

\[ \sigma^*_{\text{annual}}(t_i) = \frac{S_{Z_i}}{\sqrt{N_{\text{year}}}} = \sqrt{N_{\text{year}}} \cdot S_{Z_i} \]

(37)
Furthermore when the annualized values of the return and the volatility is calculated for each asset, the risk appetite value at time $t_i$ is obtained as the slope coefficient $\hat{\beta}_1$ from the linear equation,

$$\bar{R}_{\text{annual}}(t_i) = \hat{\beta}_0 + \hat{\beta}_1 \bar{\sigma}_{\text{annual}}(t_i) + \bar{\epsilon}$$

by minimizing the error vector, $\bar{\epsilon}$.

Down below in Figure 2 is a scatter-plot where the global risk appetite reached its minimum value. The linear coefficient in this case indicates a low risk appetite.

![Scatterplot for date 2008-10-28](image)

Figure 2: Scatterplot for date 2008-10-28
3.2 Data

The data I used in this thesis was provided by Öhman Fonder from the data supplier Refinitiv, also known as old Thomson Reuters. Gai & Vause’s method and Tarashev’s method for calculating the risk appetite indexes relies on the same distributions and therefore they also rely on the same underlying data.

3.2.1 Gai & Vause and Tarashev

The included equity indexes I used for these methods was,

<table>
<thead>
<tr>
<th>Country</th>
<th>Equity Index</th>
<th>Short code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>OMX Stockholm 30</td>
<td>OMXS30</td>
</tr>
<tr>
<td>China</td>
<td>Hang Seng Index</td>
<td>HSI</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>Financial Times Stock Exchange 100 Index</td>
<td>FTSE100</td>
</tr>
<tr>
<td>United States</td>
<td>Standard &amp; Poor’s 500</td>
<td>S&amp;P500</td>
</tr>
</tbody>
</table>

Table 2: Included equity indexes

Historical prices was retrieved for each index from date 1980-01-01 or from the date when the index started. The historical prices was used to determine the historical monthly return distribution.

The number of put and call options with the same strike price used for these equity indexes is shown in Table 3.

<table>
<thead>
<tr>
<th>Year</th>
<th>OMXS30</th>
<th>HSI</th>
<th>FTSE100</th>
<th>S&amp;P500</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>-</td>
<td>-</td>
<td>458</td>
<td>589</td>
</tr>
<tr>
<td>2007</td>
<td>104</td>
<td>395</td>
<td>660</td>
<td>1014</td>
</tr>
<tr>
<td>2008</td>
<td>279</td>
<td>940</td>
<td>553</td>
<td>1516</td>
</tr>
<tr>
<td>2009</td>
<td>272</td>
<td>842</td>
<td>605</td>
<td>1663</td>
</tr>
<tr>
<td>2010</td>
<td>264</td>
<td>675</td>
<td>670</td>
<td>1835</td>
</tr>
<tr>
<td>2011</td>
<td>227</td>
<td>916</td>
<td>586</td>
<td>1875</td>
</tr>
<tr>
<td>2012</td>
<td>222</td>
<td>937</td>
<td>521</td>
<td>1954</td>
</tr>
<tr>
<td>2013</td>
<td>206</td>
<td>664</td>
<td>526</td>
<td>2039</td>
</tr>
<tr>
<td>2014</td>
<td>376</td>
<td>590</td>
<td>472</td>
<td>2290</td>
</tr>
<tr>
<td>2015</td>
<td>766</td>
<td>874</td>
<td>730</td>
<td>2663</td>
</tr>
<tr>
<td>2016</td>
<td>705</td>
<td>982</td>
<td>983</td>
<td>2688</td>
</tr>
<tr>
<td>2017</td>
<td>886</td>
<td>831</td>
<td>936</td>
<td>2684</td>
</tr>
<tr>
<td>2018</td>
<td>567</td>
<td>1087</td>
<td>885</td>
<td>3150</td>
</tr>
<tr>
<td>2019</td>
<td>76</td>
<td>243</td>
<td>258</td>
<td>1195</td>
</tr>
</tbody>
</table>

Table 3: Number of options for the included years for each index

In Table 3 we can see that the number of options increases with time. This was a limitation regarding data accessibility. The option data could only be retrieved from 2006 or later. The number of options for 2019 is small because it only covers the first quarter.
### 3.2.2 CSFB

The included constituents throughout this report regarding the CFSB risk appetite is shown in Table 4 where the country’s name represents the underlying asset. The commodities included is shown in Table 16.

<table>
<thead>
<tr>
<th>Risky Assets</th>
<th>Safe Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Developed Equitys</td>
<td>Emerging Equitys</td>
</tr>
<tr>
<td>Australia</td>
<td>Argentine</td>
</tr>
<tr>
<td>Austria</td>
<td>Brazil</td>
</tr>
<tr>
<td>Belgium</td>
<td>Chile</td>
</tr>
<tr>
<td>Canada</td>
<td>Czech Republic</td>
</tr>
<tr>
<td>Denmark</td>
<td>Hong Kong</td>
</tr>
<tr>
<td>France</td>
<td>Hungary</td>
</tr>
<tr>
<td>Germany</td>
<td>India</td>
</tr>
<tr>
<td>Greece</td>
<td>Indonesia</td>
</tr>
<tr>
<td>Ireland</td>
<td>Israel</td>
</tr>
<tr>
<td>Italy</td>
<td>Korea</td>
</tr>
<tr>
<td>Japan</td>
<td>Mexico</td>
</tr>
<tr>
<td>Netherlands</td>
<td>Philippines</td>
</tr>
<tr>
<td>New Zealand</td>
<td>Poland</td>
</tr>
<tr>
<td>Norway</td>
<td>Russia</td>
</tr>
<tr>
<td>Portugal</td>
<td>Taiwan</td>
</tr>
<tr>
<td>Spain</td>
<td>Thailand</td>
</tr>
<tr>
<td>Sweden</td>
<td>Turkey</td>
</tr>
<tr>
<td>Switzerland</td>
<td></td>
</tr>
<tr>
<td>United Kingdom</td>
<td></td>
</tr>
<tr>
<td>USA</td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Constituents for CSFB Risk Appetite Index

<table>
<thead>
<tr>
<th>Commodities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crude oil</td>
</tr>
<tr>
<td>Gold</td>
</tr>
<tr>
<td>Commodity Research Bureau</td>
</tr>
</tbody>
</table>

Table 5: Commodities for CSFB Risk Appetite Index

---

10Due to restriction of data the section “Emerging Fixed Income” was neglected compared to Wilmut et al. studied.
3.3 Validation

3.3.1 Response

The risk appetite indexes will be validated in two steps. The first step is to check how the indexes respond to major financial events and with their expected signal. One could expect in general that investors risk appetite is low after dramatic events, such as the financial crisis in 2008. It is harder to expect when the risk appetite should have been high but one could for example expect a high risk appetite right before the dotcom-bubble in the late 1990’s. The use and adoption of the internet made investors eager to invest, at any valuation, in a dotcom company and thus they where willing to bear a very high risk. The events considered in this thesis will only take respect to periods indicating a low risk appetite, and I let the periods of high risk appetite be shown by themself in the result. The expected signals is given below in Table 6.

<table>
<thead>
<tr>
<th>Date</th>
<th>Description</th>
<th>Expected signal of risk appetite</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007 - February</td>
<td>Chinese stock bubble</td>
<td>↓</td>
</tr>
<tr>
<td>2008 - September</td>
<td>Financial crisis of 2008</td>
<td>↓</td>
</tr>
<tr>
<td>2009 - November</td>
<td>Dubai debt standstill</td>
<td>↓</td>
</tr>
<tr>
<td>2010 - April</td>
<td>Euro-crisis</td>
<td>↓</td>
</tr>
<tr>
<td>2015 - June</td>
<td>Chinese stock market crash</td>
<td>↓</td>
</tr>
<tr>
<td>2018 - September</td>
<td>US bear market</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Major financial events

In this thesis the signal is given by looking at constant percentiles of the risk appetite values since I study the signal back in time. When future values is obtained (for each of the indexes) the time-series needs to be rescaled. A low risk appetite signal is achieved when the value is under the 10-percentile and a high risk appetite is achieved when the value is over the 90-percentile.

3.3.2 Trend-following strategy

The next step is to validate each of the risk appetite indexes separately, by letting them work as a trend-following strategy. Holst and Norberg used the CSFB risk appetite index as a contradictory signal to the well known strategy SMA10. The SMA10 is a trend-following strategy that makes decisions based on a 10-month Simple Moving Average.[16] However, it is considered in this thesis to let the risk appetite index be the only signal to the strategy. Wilmut *et al.* discusses that their risk appetite index can be very helpful for when to switch between safe and risky assets and this will be considered in the strategy for all risk appetite indexes.

The strategy itself takes the trend of the risk appetite index into account for $m$ months back in time. The trend is defined as the slope coefficient, $\hat{k}$, from the linear equation,

$$ y(t_i) = \hat{k} t_i + c_1 + \epsilon_i \quad i = 2, 3, ..., m $$

by minimizing the error, $\epsilon_i$. The risk appetite value is given by $y(t_i)$, $t_i$ is time and $c_1$ is the intercept. The strategy starts by holding the normal portfolio. Further on, if the trend goes from negative to positive, we want to hold the risky portfolio and the opposite holds i.e when the trend goes from positive to negative we want to hold the safe portfolio. The months back for the trend will be tested for months in the interval 2 to 12 and the different type of portfolios is defined below in Table 7 with its corresponding weights.

---

11This is due to data limitation. The longest time interval for the option data coincides with the shaky period before 2008 - in the continuous case this would indicate that low risk appetite values would be seen as normal, thus the constant case is considered.
<table>
<thead>
<tr>
<th>Portfolio type</th>
<th>Risky Assets</th>
<th>Safe Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky</td>
<td>90 %</td>
<td>10 %</td>
</tr>
<tr>
<td>Normal</td>
<td>60 %</td>
<td>40 %</td>
</tr>
<tr>
<td>Safe</td>
<td>10 %</td>
<td>90 %</td>
</tr>
</tbody>
</table>

Table 7: Portfolios

The risky and the safe assets is the underlying assets for the CSFB risk appetite index given in section 3.2.2.\textsuperscript{12} The portfolios will be constructed from the start date 2006-01-01 to the end date 2019-03-27.\textsuperscript{13}

**Algorithm 1** Trend-following strategy for risk appetite indexes

1: \textbf{procedure} PortfolioPerformance
2: 
3: \hspace{1em} newTrend $\leftarrow$ Normal
4: \hspace{1em} oldTrend $\leftarrow$ Normal
5: \hspace{1em} for date in dates do
6: \hspace{2em} newTrend $\leftarrow$ riskappetite_trend(date)
7: \hspace{1em} if newTrend = Positive and newTrend $\neq$ oldTrend then
8: \hspace{2em} portfolio $\leftarrow$ risky
9: \hspace{1em} else if newTrend = Negative and newTrend $\neq$ oldTrend then
10: \hspace{2em} portfolio $\leftarrow$ safe
11: \hspace{1em} calcPerformance(portfolio)
12: \hspace{1em} oldTrend $\leftarrow$ newTrend
13: \hspace{1em} return portfolio

\textsuperscript{12}The same assets was used for all tests for the trend-following strategy.
\textsuperscript{13}No transaction costs was included for rebalancing the portfolio.
4 Results

4.1 Response

The response from the risk appetite indexes is shown below.

<table>
<thead>
<tr>
<th>Date</th>
<th>Expected signal</th>
<th>Gai &amp; Vause</th>
<th>Tarashev</th>
<th>CSFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>2007 - February</td>
<td>↘</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2008 - September</td>
<td>↘</td>
<td>↘</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2009 - November</td>
<td>↘</td>
<td>↘</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2010 - April</td>
<td>↘</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2015 - June</td>
<td>↘</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2018 - September</td>
<td>↘</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 8: Response from the risk appetite indexes with respect to their percentiles

The risk appetite indexes plotted together,

![Global risk appetite indexes](image)

Figure 3: The risk appetite indexes plotted together

4.2 Correlation and PCA

The correlation matrix,

<table>
<thead>
<tr>
<th></th>
<th>Gai &amp; Vause</th>
<th>Tarashev</th>
<th>CSFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gai &amp; Vause</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tarashev</td>
<td>0.25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>CSFB</td>
<td>0.13</td>
<td>0.55</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 9: Correlation matrix
Results from Principal Component Analysis

<table>
<thead>
<tr>
<th>Method</th>
<th>Eigenvalue</th>
<th>Value</th>
<th>Proportion of variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>CSFB</td>
<td>1</td>
<td>1.6745</td>
<td>55.46%</td>
</tr>
<tr>
<td>Tarashev</td>
<td>2</td>
<td>0.9094</td>
<td>30.11%</td>
</tr>
<tr>
<td>Gai &amp; Vause</td>
<td>3</td>
<td>0.4356</td>
<td>14.43%</td>
</tr>
</tbody>
</table>

Table 10: Results PCA

4.3 Trend-following strategy

Down below is a summary of the portfolios obtained from the trend-following for each risk appetite index.

Static portfolios

<table>
<thead>
<tr>
<th>Type of portfolio</th>
<th>$R_{\text{annual}}$</th>
<th>$\sigma_{\text{annual}}$</th>
<th>$SR$</th>
<th>$SOR$</th>
<th>$MDD$</th>
<th>$P_{\text{end}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risky</td>
<td>6.19%</td>
<td>11.48%</td>
<td>0.88</td>
<td>1.22</td>
<td>-19.40%</td>
<td>190.84</td>
</tr>
<tr>
<td>Normal</td>
<td>4.12%</td>
<td>7.82%</td>
<td>0.88</td>
<td>1.22</td>
<td>-14.06%</td>
<td>164.75</td>
</tr>
<tr>
<td>Safe</td>
<td>1.29%</td>
<td>2.98%</td>
<td>0.53</td>
<td>0.74</td>
<td>-5.80%</td>
<td>121.27</td>
</tr>
</tbody>
</table>

Table 11: Result from trend-following strategy

Portfolios based on Gai & Vause’s risk appetite index

<table>
<thead>
<tr>
<th>$t$ (months)</th>
<th>$R_{\text{annual}}$</th>
<th>$\sigma_{\text{annual}}$</th>
<th>$SR$</th>
<th>$SOR$</th>
<th>$MDD$</th>
<th>$P_{\text{end}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7.83%</td>
<td>8.28%</td>
<td>0.92</td>
<td>1.29</td>
<td>14.08%</td>
<td>269.72</td>
</tr>
<tr>
<td>7</td>
<td>7.50%</td>
<td>7.45%</td>
<td>1.13</td>
<td>1.58</td>
<td>12.87%</td>
<td>264.42</td>
</tr>
<tr>
<td>8</td>
<td>7.36%</td>
<td>7.26%</td>
<td>1.27</td>
<td>1.83</td>
<td>11.78%</td>
<td>251.53</td>
</tr>
<tr>
<td>9</td>
<td>6.89%</td>
<td>7.01%</td>
<td>1.21</td>
<td>1.71</td>
<td>11.57%</td>
<td>239.57</td>
</tr>
<tr>
<td>11</td>
<td>5.87%</td>
<td>7.38%</td>
<td>1.02</td>
<td>1.47</td>
<td>10.95%</td>
<td>213.57</td>
</tr>
<tr>
<td>12</td>
<td>5.72%</td>
<td>7.35%</td>
<td>1.00</td>
<td>1.47</td>
<td>10.94%</td>
<td>209.09</td>
</tr>
<tr>
<td>3</td>
<td>5.61%</td>
<td>8.37%</td>
<td>0.71</td>
<td>1.00</td>
<td>13.02%</td>
<td>208.94</td>
</tr>
<tr>
<td>6</td>
<td>5.56%</td>
<td>8.4%</td>
<td>0.8</td>
<td>1.11</td>
<td>13.08%</td>
<td>196.51</td>
</tr>
<tr>
<td>10</td>
<td>5.45%</td>
<td>7.21%</td>
<td>0.92</td>
<td>1.3</td>
<td>10.75%</td>
<td>202.89</td>
</tr>
<tr>
<td>5</td>
<td>3.78%</td>
<td>8.12%</td>
<td>0.59</td>
<td>0.83</td>
<td>12.74%</td>
<td>158.67</td>
</tr>
<tr>
<td>2</td>
<td>-0.93%</td>
<td>8.98%</td>
<td>0.25</td>
<td>0.39</td>
<td>14.48%</td>
<td>82.95</td>
</tr>
</tbody>
</table>

Table 12: Result from trend-following strategy
Portfolios based on Tarashev’s risk appetite index

<table>
<thead>
<tr>
<th>$t$ (months)</th>
<th>$R_{\text{annual}}$</th>
<th>$\sigma_{\text{annual}}$</th>
<th>$SR$</th>
<th>$SOR$</th>
<th>$MDD$</th>
<th>$P_{\text{end}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6.50%</td>
<td>7.67%</td>
<td>0.74</td>
<td>1.04</td>
<td>12.75%</td>
<td>225.84</td>
</tr>
<tr>
<td>12</td>
<td>5.68%</td>
<td>7.46%</td>
<td>0.79</td>
<td>1.12</td>
<td>11.74%</td>
<td>207.46</td>
</tr>
<tr>
<td>5</td>
<td>4.88%</td>
<td>7.66%</td>
<td>0.53</td>
<td>0.75</td>
<td>12.2%</td>
<td>181.97</td>
</tr>
<tr>
<td>11</td>
<td>4.71%</td>
<td>8.34%</td>
<td>0.81</td>
<td>1.15</td>
<td>13.27%</td>
<td>173.94</td>
</tr>
<tr>
<td>10</td>
<td>4.09%</td>
<td>8.07%</td>
<td>0.75</td>
<td>1.08</td>
<td>13.23%</td>
<td>166.19</td>
</tr>
<tr>
<td>9</td>
<td>3.90%</td>
<td>8.4%</td>
<td>0.73</td>
<td>1.04</td>
<td>14.15%</td>
<td>151.63</td>
</tr>
<tr>
<td>3</td>
<td>3.27%</td>
<td>7.75%</td>
<td>0.48</td>
<td>0.68</td>
<td>12.39%</td>
<td>152.86</td>
</tr>
<tr>
<td>7</td>
<td>3.08%</td>
<td>7.84%</td>
<td>0.63</td>
<td>0.88</td>
<td>13.19%</td>
<td>136.31</td>
</tr>
<tr>
<td>8</td>
<td>2.88%</td>
<td>8.09%</td>
<td>0.60</td>
<td>0.85</td>
<td>13.55%</td>
<td>132.49</td>
</tr>
<tr>
<td>6</td>
<td>2.84%</td>
<td>8.19%</td>
<td>0.47</td>
<td>0.68</td>
<td>14.05%</td>
<td>126.97</td>
</tr>
<tr>
<td>2</td>
<td>1.06%</td>
<td>8.45%</td>
<td>0.36</td>
<td>0.54</td>
<td>14.93%</td>
<td>109.04</td>
</tr>
</tbody>
</table>

Table 13: Result from trend-following strategy

Portfolios based on CSFB’s risk appetite index

<table>
<thead>
<tr>
<th>$t$ (months)</th>
<th>$R_{\text{annual}}$</th>
<th>$\sigma_{\text{annual}}$</th>
<th>$SR$</th>
<th>$SOR$</th>
<th>$MDD$</th>
<th>$P_{\text{end}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>6.37%</td>
<td>8.18%</td>
<td>0.80</td>
<td>1.15</td>
<td>13.55%</td>
<td>222.5</td>
</tr>
<tr>
<td>4</td>
<td>6.34%</td>
<td>7.89%</td>
<td>0.77</td>
<td>1.12</td>
<td>13.13%</td>
<td>215.53</td>
</tr>
<tr>
<td>10</td>
<td>5.65%</td>
<td>6.79%</td>
<td>0.86</td>
<td>1.18</td>
<td>10.52%</td>
<td>207.07</td>
</tr>
<tr>
<td>9</td>
<td>5.43%</td>
<td>6.82%</td>
<td>0.79</td>
<td>1.1</td>
<td>10.54%</td>
<td>201.22</td>
</tr>
<tr>
<td>12</td>
<td>5.11%</td>
<td>7.02%</td>
<td>0.81</td>
<td>1.16</td>
<td>11.39%</td>
<td>188.9</td>
</tr>
<tr>
<td>8</td>
<td>4.83%</td>
<td>6.76%</td>
<td>0.66</td>
<td>0.92</td>
<td>10.67%</td>
<td>185.7</td>
</tr>
<tr>
<td>3</td>
<td>4.82%</td>
<td>7.81%</td>
<td>0.83</td>
<td>1.29</td>
<td>14.38%</td>
<td>169.01</td>
</tr>
<tr>
<td>11</td>
<td>4.67%</td>
<td>6.99%</td>
<td>0.72</td>
<td>1.02</td>
<td>11.64%</td>
<td>180.6</td>
</tr>
<tr>
<td>6</td>
<td>3.85%</td>
<td>7.58%</td>
<td>0.68</td>
<td>0.98</td>
<td>12.66%</td>
<td>151.18</td>
</tr>
<tr>
<td>5</td>
<td>3.44%</td>
<td>7.77%</td>
<td>0.53</td>
<td>0.77</td>
<td>13.55%</td>
<td>142.73</td>
</tr>
<tr>
<td>7</td>
<td>3.33%</td>
<td>8.03%</td>
<td>0.59</td>
<td>0.88</td>
<td>13.78%</td>
<td>141.01</td>
</tr>
</tbody>
</table>

Table 14: Result from trend-following strategy
5 Discussion

First of all, the concept “investors risk appetite” is broad and hard to measure. There is no general method to measure the concept even though it has been studied by authors from several financial institutions. The term “appetite” itself is a commonly used word to describe the level of hunger for food. Similar to risk appetite, there is no general method to put a value on the concept without including subjective preferences. All the different methods in the article “A Brief Survey of Risk-Appetite Indexes” has the common factor with the aim to measure the market sentiment, but they all have different methods to describe the same phenomenon.

5.1 Response

The values generated from each of the risk appetite index is far from easy to interpret without normalizing the data. Without normalization, each of the index took values in ranges given in Table 15 below,

<table>
<thead>
<tr>
<th>Risk Appetite index</th>
<th>Min Value</th>
<th>Max Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gai &amp; Vause</td>
<td>$2.08 \times 10^{-3}$</td>
<td>$1.81 \times 10^{9}$</td>
</tr>
<tr>
<td>Tarashev</td>
<td>0.14</td>
<td>0.86</td>
</tr>
<tr>
<td>CSFB</td>
<td>-2.74</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Table 15: Maximum and minimum values

Because of this it was considered to normalize all of the values for each risk appetite index and to interpret it in levels in a range from 0 to 1, rather than its actual value to do a reasonable comparison. However, all of the indexes responded with its maximum value before the crisis in 2008 and with its minimum value in a short time afterwards. From figure 3 it is obvious that Tarashev’s and CSFB’s index gave a more intuitive picture of the risk appetite in comparison to Gai & Vause’s index. Tarashev’s and CSFB’s index was highly correlated, but they clearly differ regarding the recovery after 2008. CSFB’s index shifted more rapidly from pessimistic to euphoric behaviour when Tarashev’s index recovered more slowly due to heavier left tails in the distributions obtained from the options.

The data limitation regarding Gai & Vause’s and Tarashev’s risk appetite clearly restricted the result. At first, the option prices was retrieved a month before the option itself expired, which implied that a lot of information was lost in the time period between these two dates. Investors preferences can change significantly in the short term and this would have been captured by using daily option prices for the time period between these dates, with the same methodology applied. Unfortunately, this was a limitation and it would exceed the maximum number of data points allowed to be retrieved from the suppliers perspective. Secondly, the available options from the data supplier was only retrievable from 2006 meaning that important periods of shifting market sentiment could not be studied and analyzed. At last, these two indexes only relies on 4 risky assets, where the equity index S&P500 has a clear overweight compared to the other equity indexes. Moreover, CSFB’s index covered a wider spectrum of assets and thus giving a more general measure of investors risk appetite.

It is clear from Table 8 that the indexes work better together, rather than on its own, according to obtain the correct risk appetite signals that was expected from the section validation. The CSFB’s index misses half of the expected signals with respect to the condition that the value should be below the 10-percentile, although clear drops is seen for the missed signals.

\[14\text{For example the period around year 2000.}\]
5.2 Trend-following strategies

The adoption of shifting trends as a strategy for the risk appetite clearly proves to outperform all of the static portfolios, with the right trend implemented. The 4-month trend gave best result in terms of mean annual return for Gai & Vause’s index and Tarashev’s index, compared to CSFB’s index that showed best result for the 2-month trend. Clearly, from Figure 16 in the Appendix, the best portfolios avoided the big crisis in 2008 by rebalance to the safe portfolio at some time during that year, dependent on their underlying risk appetite trend. Comparing the portfolios, with highest mean annual return, the portfolio based on Gai & Vause’s index showed best result for the Sharpe ratio and the Sortino ratio. On the other hand it also showed the biggest maximum-drawdown.

The usage of the risk appetite in a trend-following strategy clearly improves the portfolio compared to the static portfolios and can be used as a helpful tool for asset allocation.

5.3 Other methods

The methods to measure the risk appetite are many and they all can not be discussed. Thus, the most interesting methods (that was not included in this thesis) in my point of view will be discussed below.

5.3.1 Bollerslev, Gibson and Zhou

Tim Bollerslev, Michael Gibson and Hao Zhou[17] start by assuming the continuous-time stochastic volatility model. Further on they introduce that the link between the stochastic volatility risk premium and the representative agent’s risk aversion coefficient, with the condition that the agent has a power utility function, is directly proportional. The stochastic volatility risk premium is then modeled with an autoregressive, AR(1), model where the time-variation in the risk premium is driven by the fitted error between realized and implied volatility. The realized volatilities is based on high frequency data on the S&P500 equity index with intraday prices with 5 minute intervals.

The methodology proposed by Bollerslev et al. is clearly interesting though assuming a power utility function for the representative investor can be discussed. Other than that, the high frequency data was unfortunately not retrievable for this thesis which is why the method was not further investigated.

5.3.2 Kumar and Persaud

Mannmohan Kumar and Avinash Persaud[18] constructs their global risk appetite index by looking at past excess returns in several assets and their riskiness proxied by the variance of past returns. The change in the risk appetite is then captured by ranking each asset by their past return and risk and measure the correlation between these rankings.

A discussion whether the variance of past returns is a good risk measure for this purpose is up to discussion, since both positive and negative returns are included. However, the method relies on simple theory and further investigation is of interest.
5.4 Further research

I encourage readers of this thesis to put further research in how the other risk appetite indexes from the survey has developed since year 2004. The methods mentioned above is clearly of interest, assuming that intraday pricing for a long time period is retrievable.

As mentioned in the methodology for CSFB’s risk appetite, the index is flexible in a way that the time period back in time can be adjusted to retrieve a “fast” risk appetite. The shorter the time period the more the annualized returns and volatility deviates. The fast risk appetite is thus an alternative to the other indexes when assessing the risk appetite measures in the short term. The index can also be divided into separate continents with the condition that data for the underlying assets is accessible. Further investigation for the CSFB’s risk appetite and its other branches is thus of interest.\textsuperscript{15}

6 Conclusion

This thesis has derived three ways to measure the concept risk appetite. The three risk appetite indexes can be used seperately to see how investor’s appetite for risk has evolved over time. The indexes provided by Credit Suisse First Boston and Tarashev appeared to generate a more intuitive result to the reader in a manner that they responded to major financial events, compared to the index provided by Gai & Vause. A principal component analysis demonstrated that the index provided by Credit Suisse First Boston explained most of the variance out of the three indexes. Implementing a trend-following strategy with the risk appetite indexes as input shows that the indexes can work as a useful tool for asset allocation.

\textsuperscript{15}See the appendix, Figure 17b where the fast risk appetite is illustrated.
References


A Appendix

The weights used for Gai & Vause’s method and Tarashev et al. was retrieved for date 2019-04-04 and was kept constant for the whole period for these two methods.

<table>
<thead>
<tr>
<th>Country</th>
<th>Equity Index</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>OMXS30</td>
<td>1.83%</td>
</tr>
<tr>
<td>China</td>
<td>HangSeng</td>
<td>8.07%</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>FTSE100</td>
<td>8.45%</td>
</tr>
<tr>
<td>United States</td>
<td>S&amp;P500</td>
<td>81.65%</td>
</tr>
</tbody>
</table>

Table 16: Weights for the equity indexes

Below is the argument for fitting a Student’s $t$ distribution instead of a normal distribution.

![Graphs](omx_stockholm_30.png) ![Graphs](hang_seng_index.png) ![Graphs](financial_times_stock_exchange_100.png) ![Graphs](standard_and_poor's_500.png)

(a) OMX Stockholm 30  
(b) Hang Seng Index  
(c) Financial Times Stock Exchange 100  
(d) Standard & Poor’s 500

Figure 4: Fitting log-returns to a Student’s $t$ distribution compared to a normal distribution
Down below is examples of the implied volatilities and the implied forward distribution for each index. The figures is only for the first option maturity for each index.

**OMX Stockholm 30**

Index: OMX  
SpotPrice: 1254.49  
CurrentDate: 2007-07-02  
MaturityDate: 2007-07-27

Figure 5: Volatility smile and implied forward distribution

**Hang Seng Index**

Index: HangSeng  
SpotPrice: 22252.99  
CurrentDate: 2007-07-05  
MaturityDate: 2007-07-30

Figure 6: Volatility smile and implied forward distribution
Financial Times Stock Exchange 100

Index: FTSE  SpotPrice: 6098.71  CurrentDate: 2006-04-24  MaturityDate: 2006-05-19

Figure 7: Volatility smile and implied forward distribution

Standard & Poor’s 500

Index: S&P  SpotPrice: 1308.11  CurrentDate: 2006-04-24  MaturityDate: 2006-05-18

Figure 8: Volatility smile and implied forward distribution

The following figures is a graphical view of all distributions used throughout the report.
A.0.1 Covariance

The covariance between two random variables \( X \) and \( Y \) is given as,

\[
\text{Cov}(X, Y) = E[XY] - E[X]E[Y]
\] (40)

and it is a measure of the joint variability of the two variables. Note that equation (40) can be rewritten as

\[
E[XY] = \text{Cov}(X, Y) + E[X]E[Y]
\] (41)

A.0.2 Normalization of data

For a given dataset,

\[
x = (x_1, \ldots, x_n) \quad x_i \in \mathbb{R} \quad i = 1, \ldots, n
\] (42)

one can normalize the given data by following equation,

\[
z_i = \frac{x_i - \min(x)}{\max(x) - \min(x)} \quad z_i \in [0, 1]
\] (43)
OMX Stockholm 30

(a) Implied forward probabilities

(b) Historical distribution
Hang Seng Index

(a) Implied forward probabilities

(b) Historical distribution
Financial Times Stock Exchange 100

(a) Implied forward probabilities

(b) Historical distribution
Standard & Poor’s 500

(a) Implied forward probabilities

(b) Historical distribution
Figure 13: Risk appetite with percentiles

Figure 14: Risk appetite with percentiles

Figure 15: Risk appetite with percentiles
Figure 16: Best portfolios in terms of mean annual return
Other branches of the CSFB’s risk appetite

(a) 6-month interval for different continents

(b) 1-month interval for different continents

Figure 17: Other branches of CSFB