Classification of Financial Instruments

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Abstract

In this thesis a general framework and accompanying guidelines for how to classify financial instruments within the fair value hierarchy (included within IFRS 13) is presented. IFRS 13 introduces a broad and loosely defined regulation of how to classify a financial instrument which leaves room for misinterpretation and uncertainties. In this thesis the pricing of financial instruments and behaviour of the market data used as inputs in the models has been investigated. This is to give better insight into what is classified as significant market data, how it is used and how it is approximated. Instruments that have been investigated are autocalls, swaps, European options and Asian options. The result is presented as general recommendations for how to classify the specified instruments with clearer boarders introduced between the levels in the hierarchy. Methods and deductions introduced in the thesis could also further be implemented in classification of closely related financial instruments but has been limited in this thesis due to time restrictions.
Klassifikation av Finansiella Instrument

Sammanfattning

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Chapter 1

Background and Motivation

1.1 Background

IFRS (International Financial Reporting Standards) is a set of standards set out by the International Accounting Standards Board (IASB) to create a set of consistent reporting standards to be implemented across the financial sector[1]. Within the IFRS framework, fair value is introduced which is a market based-based measurement and to be reported within the fair value hierarchy [2]. The fair value hierarchy declares in which domain the reported fair value has derived input variables used in the valuation. The fair value and accompanying hierarchy is thus set in place for the purpose of giving governments and potential investors a good basis for conclusions about financial entities and their disclosed assets.

1.2 Purpose

The purpose of this paper is to identify a coherent framework which allows for fair value classification of assets and financial instruments. The main problems will be the assessment of instrument level classification and how the handling of lacking data will affect valuation models. Assessment of approximated input parameters such as yield and volatility will be examined and how these approximated parameters will be accurate for longer maturities than observable by the market. The effect of lacking data is also investigated. The main goal of this thesis is thus to clarify where lines between classification levels can be drawn based on the available market data.
1.3 Research Question

"How does lacking market data affect the fair value valuation models and how can classification according to the fair value hierarchy of these valuation models be done?"

- How does inputs in different valuation models affect the overall classification level of the financial instrument?
- How can activity in a market be assessed in accordance to a classification level?
- How far ahead can approximations yield accurate estimates, given varying levels of market data?
Chapter 2

Introduction

2.1 IFRS Policies and Implications

The International Financial Reporting Standards (IFRS) as mentioned in the background, are standards put in place to aid in providing information about the entity and set out to make financial reports readily available in a general standard across the sector [2]. In this report the main regulations which are to be applied and discussed is in regards to fair value and the fair value hierarchy, which are mainly prominent in the IFRS 7, 9 and 13 regulations concerning the pricing of assets and financial instruments [3]. These regulations are set in place to subside investors, since in disclosures of asset and instrument value by financial entities there is often a certain degree of parameter and behaviour assumptions made. This has therefore led IASB to the introduction of IFRS and fair value [3].

Fair value is the value based on data directly from the market. The fair value is thus the approach which makes best use of such available market corroborated inputs in the valuation of financial instruments and assets [2]. The classification level itself is based on an assessment of the approach used and on which assumptions are made regarding the inputs used in the valuation [2]. The classification standard which also forms the fair value hierarchy is made up of three levels which depends on the observability of the input parameters used in the instrument valuation [2]. The hierarchy ranges from level 1 of directly observable prices in an active market, to level 3 where the inputs used are based on assumptions not observable in the market [2]. This also inherently results in a hierarchy which depicts the accuracy of the valuation, where fair value classified as level 1 has a higher accuracy than a level 3 classification.
2.1.1 Fair Value Hierarchy

As mentioned in section 2.1 above, the fair value hierarchy is made up of three levels of inputs. The fair value is to be calculated using the highest available level of input data according to this hierarchy as depicted in Figure 2.1. Therefore the priority is to

"maximizes the use of relevant observable inputs and minimizes the use of unobservable inputs." - IFRS 13 [2]

It also makes a point to declare that

"The fair value hierarchy prioritizes the inputs to valuation techniques, not the valuation techniques used to measure fair value" - IFRS 13 [2]
Level 1 Inputs

"Level 1 inputs are quoted prices (unadjusted) in active markets for identical assets or liabilities that the entity can access at the measurement date." - IFRS 13 [2]

Level 1 inputs are the most reliable inputs in the fair value hierarchy. The challenge in discernment of a level 1 and level 2 input is often in regards to the assessment of activity in the market.

Examples included within the level 1 classification is:

- Stocks and bonds traded on an active exchange.
- Financial instruments (e.g. Options) traded on an active market.

Level 2 Inputs

"Level 2 inputs are inputs other than quoted prices included within Level 1 that are observable for the asset or liability, either directly or indirectly." - IFRS 13 [2]

If a financial instrument is classified as level 2, the valuation of the instrument is done with the help of level 2 input variables. These are inputs which needs a valuation model to provide a fair value. Such input variables are for example yield curves, implied volatility surfaces or observable prices of the equity when the market is inactive[4].

Examples included within the level 2 classification is:

- Pricing of OTC options in which approximated data such as volatility, discount curves and other market corroborated inputs are used in the valuation model.
- Pricing of swaps where forward curves derived from market data have been used.
- Prices derived from similar products which are actively traded.
Level 3 Inputs

"Unobservable inputs for the asset or liability." - IFRS 13 [2]

Level 3 inputs are inputs based on assumptions and used when the availability of observable inputs are no longer feasible. These can be for example very forward looking volatility or yield curve predictions not directly observable for the maturity of the financial instrument while also affecting the fair Value in a significant way.[3]

Examples included within the level 3 classification is:

- Pricing of options with long expiry which cant be appropriately approximated by observable data
- Complex correlation products.
- Pricing of equity which is not publicly traded.

Fair Value Valuation Techniques

Three suggested approaches by IFRS when measuring fair Value is the market approach, the income approach and the cost approach [2].

- The market approach provides the fair Value based on market variables such as pricing of equivalent or identical products.
- The income approach provides the fair value from valuation techniques to put a present value of predicted future incomes and costs.
- The cost approach provides the fair value based on how much the cost of current replacement is of the asset to be valued.
Chapter 3

Literature and Theory

Regarding Valuation models, multiple different means of valuation methods have been composed to price current financial instruments on the market. In order to ultimately derive a fair value and a classification within the fair value hierarchy investigation of these valuation models must be performed. The focus of this thesis will be on financial instruments for which the fundamental theoretical framework is presented below.

3.1 Valuation Models

3.1.1 Risk neutral measure

The Risk neutral measure is a key component in pricing of financial products and stems from wanting to exclude individual investors risk preferences. The assumptions is thus based on a market which is free from arbitrage opportunities and complete with only one price for the product [5].

This yields a risk neutral measure in which the expected value of the product today is equal to the discounted product value in the future [5]. In modelling this behavior the martingale measure provided below is used to derive the risk neutral measure. In Options, Futures, and Other Derivatives [6] two properties are introduced that simplify the pricing of derivatives:

- "The expected return on an investment is the risk-free rate" [6].
- "The discount rate used for the expected payoff on an instrument is the risk-free rate" [6].
The properties of the risk-neutral measure is one of the most critical components used in valuation theory. It declares that when the assumption of risk-neutrality is made, one can derive the correct price in all worlds (not just the risk-neutral world) [6]. The use of the measure in valuation is thus done by calculating the probabilities of different results in the risk-neutral world, leading to the calculation of the expected payoff from the investment. Lastly this payoff is discounted from the risk-free zero rate, to then calculate the present value of the investment. This can be viewed in the mathematical terms as:

\[ X_0 = P(0, t)E_Q[X_t], \]

where \( X_0 \) is the present value of observation \( X \), \( P(0, t) \) is the discount factor for the time period from 0 to \( t \) and \( E_Q[X_t] \) is the risk neutral expectation of the future observation of \( X \) at time \( t \). This stems from the martingale measure which has the property that the expected future value is the same as the current value [6]. Where the general martingale measure can be seen as

\[ E[\theta_t] = \theta_0, \]

which is interpreted as the expected future value of \( \theta \) is equal to the present value of \( \theta \).

### 3.2 Geometric Brownian Motion

Geometric Brownian motion is a widely used notion within quantitative finance in which the logarithm of the random varying phenomenon follows an ordinary Brownian motion. Geometric Brownian motion is often used in the modeling of stock prices assuming the stock return can be expressed as a log-normal distribution [6]. The general model can be written as

\[ dS_t = \mu S_t dt + \sigma S_t dW_t. \]

This process can be described as follows. At each time step the process will move in the direction of \( \mu \) with a random component of \( \sigma \) up or down which is normal distributed with mean 0 and variance 1. This last part is thus simply a Brownian motion which exhibits a stochastic behaviour as described in equation 3.2. Further the expression can be rewritten as

\[ \frac{dS_t}{S_t} = \mu dt + \sigma dW_t \]
Where \( \frac{dS_t}{S_t} \) can be seen as the derivative of \( \ln(S_t) \). Applying Ito’s lemma which is used to differentiate a time dependent stochastic function in time [6], leads to

\[
d(\ln(S_t)) = (\ln(S_t))' \mu S_t dt + (\ln(S_t))' \sigma S_t dW_t + \frac{1}{2} (\ln(S_t))'' \sigma^2 S_t^2 dt \tag{3.5}
\]

which can be written as

\[
d(\ln(S_t)) = \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t \tag{3.6}
\]

and leads to

\[
\ln\left( \frac{S_t}{S_0} \right) = \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \sigma dW_t. \tag{3.7}
\]

Exponentiating both sides and multiplying by \( S_0 \) then leads to

\[
S_t = S_0 e^{\left( \left( \mu - \sigma^2/2 \right) t + \sigma W_t \right)}, \tag{3.8}
\]

where \( W_t \) is a wiener process or Brownian motion as previously mentioned, \( S_0 \) the current price of the asset, \( \mu \) is the mean drift of the asset and \( \sigma \) the standard deviation or volatility of the asset. This can then be used in simulations to investigate the probable asset paths of \( S_t \) by averaging the path over multiple Monte Carlo simulations [6].

### 3.3 Black-Scholes formula

The Black-Scholes model is widely used formula in finance for pricing the fair value of European options on various underlyings [6]. The formula which assumes that percentage changes in a short period of time is normal distributed is used as a good estimate in calculation of option prices but are subjected to constraints which limits the model from being empirically verifiable [6]. The defining formula for a an underlying paying a continuous dividend is defined as [6]

\[
C = S_0 e^{-qT} N(d_1) - Ke^{-rT} N(d_2) \tag{3.9}
\]

\[
P = Ke^{-qT} N(-d_2) - S_0 e^{-rT} N(-d_1) \tag{3.10}
\]

\[
d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma \sqrt{T}} \tag{3.11}
\]

\[
d_2 = \frac{\ln(S_0/K) + (r - q \sigma^2/2)T}{\sigma \sqrt{T}} = d_1 - \sigma \sqrt{T} \tag{3.12}
\]

Where
• $C$ is the obtained call price
• $P$ is the obtained put price
• $S_0$ is the current underlying price
• $r$ is the risk-free zero rate continuously compounded
• $K$ is the listed strike price
• $T$ is the remaining time to maturity in years
• $N()$ is the distribution function of the normal distribution
• $q$ is the dividend yield of the underlying
• $\sigma$ is the volatility of underlying

which together also constitutes the put-call parity on the form

$$C + Ke^{-rT} = P + S_0e^{-qT}.$$  \hspace{1cm} (3.13)

The method is as mentioned based on several assumptions not empirically verifiable but necessary to obtain a pricing of the option. Such assumptions are:

1. There are no transaction cost in obtaining the option
2. The risk-free zero-rate is constant and known
3. Dividends for the whole maturity of the option are known
4. The log returns of the underlying are normal distributed.
5. Volatility is constant and is not subject to change in time
6. There exists no arbitrage opportunities on the market

Nevertheless the model provides a good approximation of the European option price. The model is however not used in the pricing of an American option as the American option can be exercised at any time prior to the maturity. To price the American option the application of the finite difference method as presented in section 3.4 is used. The method is based on the same dynamics as mentioned in this section but the price is calculated continuously throughout the life of the option to better reflect the property of exercise prior to maturity.
3.3.1 Black-76 model

The Black-76 model is an extension of the previously mentioned Black-Scholes by utilizing the discounted future price in place of the spot price \[6\]. This can be derived from the future price of an asset paying dividend \( q \) and has zero rate \( r \) defined as

\[
F_0 = S_0 e^{(r-q)T}, \quad (3.14)
\]

then rewritten as

\[
S_0 = F_0 e^{(q-r)T}, \quad (3.15)
\]

and inserted in the original Black-Scholes model as presented in equation 3.9 and 3.10. This leads to Black’s model as presented below.

\[
C = e^{-rT} [F_0 N(d_1) - KN(d_2)] \quad (3.16)
\]

\[
P = e^{-rT} [KN(-d_2) - F_0 N(-d_1)] \quad (3.17)
\]

\[
d_1 = \frac{\ln(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \quad (3.18)
\]

\[
d_2 = \frac{\ln(S_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \quad (3.19)
\]

3.4 Finite Difference Method

The finite method is a numerical method for solving differential equations, the method can be utilized in pricing of both American and European options. For which the difference between the options is the possibility to exercise the American option prior to maturity. This means that while European options can be priced using a closed form solution (as presented in the Section 3.3) this is not possible for American options. Pricing American options thus utilize the aforementioned finite difference method (FDM). The formula solves the Black-Scholes partial difference equation

\[
\frac{\partial f}{\partial t} + (r - q)S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = rf. \quad (3.20)
\]

To solve this formula the central difference formula of second order is introduced[7]. The first derivative in \( t \) is derived as

\[
\frac{\partial f}{\partial t} = \frac{f_{i,j+1} - f_{i,j-1}}{2h_t} + O(h_t^2) \quad (3.21)
\]
and the first order derivative in $S$ is presented as
\[
\frac{\partial f}{\partial S} = \frac{f_{i+1,j} - f_{i-1,j}}{2h_S} + O\left(h_S^2\right).
\] (3.22)

The second order derivative in $S$ is
\[
\frac{\partial^2 f}{\partial S^2} = \frac{f_{i+1,j} + f_{i-1,j} - 2f_{i,j}}{h_S^2} + O\left(h_S^2\right).
\] (3.23)

These equations are inserted into the original Black-Scholes differential equation as
\[
\frac{f_{i,j} - f_{i,j-1}}{h_t} + (r-q)ih_S \left(\frac{f_{i+1,j} + f_{i-1,j}}{2h_S} + \frac{1}{2} \sigma^2 i^2 h_S^2 \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{h_S^2}\right) = rf_{i,j},
\] (3.24)

where the boundary conditions are given by
\[
f(0, t) = 0
\] (3.25)
and
\[
f(S_{max}, t) = S_{max} - Ke^{(q-r)(T-t)}.
\] (3.26)

To calculate a price from these equations the implicit finite difference method or the Crank–Nicolson method can be used [6].

### 3.5 Correlation

The measure of correlation which defines relating properties in objects is a necessary measure in financial instruments composed of multiple assets. Correlation most notably expressed using the Pearson correlation coefficient is expressed as
\[
\rho_{X,Y} = \frac{\text{cov}(X,Y)}{\sigma_X \sigma_Y}
\] (3.27)

between two entities $X$ and $Y$. Estimation of such a property is thus simple when looking at historical correlation using historical data. The sample correlation coefficient $r$ can be calculated as [8]
\[
r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\left(\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2\right)^{1/2}}
\] (3.28)

where $\bar{x}$ and $\bar{y}$ is the different sample means.
Nevertheless, historical data is not sufficient in a correct estimation of future values. Thus, several methods have been brought forth to deal with this and make a more accurate estimation. Such methods often utilize the implied volatility of a basket option (i.e., an option on multiple assets) and that the variance of a portfolio containing multiple assets can be expressed as [9]

\[
\sigma_{\text{portfolio}}^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} \rho_{i,j} w_i w_j \sigma_i \sigma_j
\]  

(3.29)

where \( w \) is the individual weights of contained assets, \( \sigma \) the individual volatility of the assets, \( \rho \) the correlation factor between assets and \( \sigma_{\text{portfolio}} \) the volatility of the portfolio. Using the implied volatility for the included assets and the overall basket option containing the portfolio can thus yield the extraction of the implied correlation factor between the assets.
Chapter 4

Methodology

The core methodology for this thesis was divided up in steps to easier obtain clear goals and how to compare different scenarios. The main structure and methodology was along the structure presented below.

1. Literature review of IFRS, the fair value hierarchy and related components.

2. Literature review of financial instruments of which to assess and investigate.

3. Literature review of how input parameters are extrapolated from market data.

4. Modelling of financial instruments and comparison of test-scenarios varying the level of input parameters.

5. General reflection of assumptions made and their corresponding pros and cons for the tested scenarios.

4.1 Fair Value Hierarchy Interpretation

As presented in the introduction chapter, the structure of the fair value hierarchy is made up of three levels corresponding to the level of input used in the valuation approach. From level 1, where prices are deemed to be most accurate of fair value to level 3, where the fair value is based on inputs not directly observable and thus might result in a less accurate fair value. As financial entities strive to report with the highest level in the hierarchy a simplified
and general decision tree for classification of financial instruments was developed from the information provided by the International Accounting Standards Board included in IFRS 13. The acquired interpretation of the IFRS 13 fair value hierarchy is presented below.

![Figure 4.1: General decision chart for classification according to the fair value hierarchy](image)

The decision tree presented is as mentioned very general which is a product of definitions in the regulation being loosely defined and because valuation of different financial instruments vary depending on which input parameters are used. More distinct lines will have to be drawn for each instrument class.

### 4.2 Examined pricing methods and Financial Products

In this thesis the investigated financial instruments and their respective primary valuation techniques are presented below. The instruments considered are bonds, floating rate notes, interest rate swaps, total return swap, currency swaps, future/forward, plain vanilla European options, the autocallable structured product and Asian options.

#### 4.2.1 Bond

Bonds are issued products which can consist of one or several payments from the issuer to the buyer. The fair value price of a bond is easiest depicted as the sum of all cash flows discounted\cite{10}. The price can therefore be calculated as

\[
BPV = \sum_{k=1}^{n} c_k d_k, \quad (4.1)
\]
where $B_{PV}$ is the present value of the bond, $c_k$ is the cash flow at time step $k$ (which usually equals the face value of the bond at the last payout i.e. $k = n$), and $d_k$ is the corresponding discount factor for the specific time. The discount factor can be calculated from market observed zero rates $r_k$ as

$$
d_k = e^{-r_k t_k},
$$

(4.2)

where $t_k$ is the time to maturity for the specified observation.

### 4.2.2 Floating Rate Note

A floating rate note (FRN) or also referred to as a bond paying a varying coupon is an instrument which is beneficial because it has limited interest risk. The coupon is often on a varying floating rate such as the LIBOR rate plus a quoted margin. The general bond price is calculate as above in equation 4.1 where the coupon varies each payout depending on the future rate. Future rates or also known as forward rates can be derived from market data by interpolating between given rates of different maturities. The forward rate $F(T - t, T)$ between the maturities $T - t$ and $T$ can be calculated from the continuously compounded rates $r(0, T - t)$ and $r(0, T)$ as [6]

$$
F(T - t, T) = \frac{r(0, T)T - r(0, T - t)(T - t)}{t}.
$$

(4.3)

For a floating rate note or bond with a floating rate, the payout coupons are often structured in a way such that the spot interest on the paid face value of the bond over each accrual period is paid each payout date, and for the last payout face value is paid. Thus the present value of the note can thus be calculated using equation 4.1 where the paid coupons are

$$
c_k = F_i \tau_i P \quad if \quad k < n
$$

(4.4)

and

$$
c_k = P \quad if \quad k = n.
$$

(4.5)

Where $F_i$ is the forward rate of the $i$:th period, $\tau_i$ is the $i$:th accrual period and $P$ is the face value or principal amount paid for the note.

### 4.2.3 Interest rate Swap

Interest rate swaps are normally trading a floating interest rate and fixed interest (fixed-for-floating) rate or also common, exchange of a float-for-floating interest
swap. As mentioned in section 4.2.2, the floating interest rate is varying over time and as such the swap will be varying in its present value as the rate changes [6]. The present value $V_{PV}$ of a fixed-for-floating swap for the party paying the floating rate can be depicted as [6]

$$V_{PV} = B_{fixed} - B_{float} \quad (4.6)$$

and in the case of two floating rates a float-for-float is depicted as [6]

$$V_{PV} = B_{float1} - B_{float2} \quad (4.7)$$

The bond prices are calculated as per equation 4.1 using the discount factor retrieved via the fixed or the floating rates.

### 4.2.4 Currency Swap

A currency swap between parties is a common method to hedge risk or to exchange favorable situations between parties in different markets wanting to borrow money from each respective markets and the native company having an edge in the local market. The pricing method most commonly used for this agreement is based on the local zero rates and at the initial agreement the swap is usually valued at zero unless in an off market swap where it can differ. The present value of a vanilla currency swap can be seen as the difference between bonds in the exchanging markets and one bond transformed to the corresponding currency with the use of the present exchange rate. Below is a present value of fixed-for-fixed swap where foreign payments are received and local is paid [6].

$$V_{PV} = B_F - Y_0 B_L \quad (4.8)$$

and

$$Y_0 = \frac{C_{urr_f}}{C_{urr_L}} \quad (4.9)$$

Where $B_F$ is the bond value of the current cash flows in the foreign currency, $B_L$ is the bond value of the local currency and $Y_0$ is the current spot exchange rate between the currencies, the present value of the formula is thus presented in the currency of the foreign market.

### 4.2.5 Future/Forward

A future or forward contract is the agreement between parties to at a later stage buy or sell commodity at a specific price [6]. The future price is calculated as
\[ F_0 = S_0 e^{rt} \]

if the contract includes assets paying dividends this would yield a price depicted as

\[ F_0 = (S_0 - I) e^{rt}. \]

(4.11)

Where \( I \) is the sum of the discounted future paid dividends during the life of the contract, if there are continuous dividends the formula is expressed as

\[ F_0 = S_0 e^{(r-q)t}. \]

(4.12)

Where \( q \) is the continuously compounding dividend yield, \( S_0 \) is the current price of the commodity and \( F_0 \) is the current futures price. For a long forward contract the price can be calculated as

\[ f = (F_0 - K) e^{-rt} \]

(4.13)

and for a short forward contract calculated as

\[ f = (K - F_0) e^{-rt}. \]

(4.14)

Where \( K \) would be the the strike price (i.e. the agreed upon price to buy for a long and sell for a short).

### 4.2.6 Total Return Swap

A total return swap is a swap between two parties in which one party pays the total return of an asset (e.g. stock or index, including both generated income and any capital gains) and the other pays a set fixed or floating rate in return. The swap can be of interest for financial entities which are interested in the return of the asset while not directly wanting to own the asset. The present value of such a financial instrument can thus be seen as the difference between a fixed or floating bond and the expected return of the asset for the period [10].

In the event of only one payment at the end of maturity for the total return leg, the formula simplifies as the dividend payments doesn’t have to be considered in the formula since they are included within the present value of the asset (i.e. the present value of \( S_0 \) is priced including the expected dividend payments) and also included within the total return payment. Thus the formula for the total return leg simplifies as

\[ \text{RP}_{PV} = P \left( \frac{S_0 e^{rT}}{S_0} - 1 \right) e^{-rT} \]

(4.15)
where, $P$ is the face value or principal of the financial instrument, $S_0$ is the current value of the asset, $r$ is the continuously compounding risk-free rate for the period $[0, T]$ and $T$ is the time to maturity, which can also be rewritten as

$$RP_{PV} = P \left(1 - e^{-rT}\right). \quad (4.16)$$

For an asset with multiple payments on the return leg of the asset which institutes re-balancing days (i.e. a day for which a new reference value for the asset is set between payments), the present value to be paid of the total return on the asset can be described by the future price of the asset divided by the re-balanced reference value (i.e. the future price at the previous re-balancing day). Thus for $n$ re-balancing days paying the return of the asset times the notional can be depicted as

$$RP_{PV} = P \sum_{i=1}^{n} \left(\frac{F_{0,i-1}e^{f(t_{i-1}, t_i)(t_i - t_{i-1})}}{F_{0,i-1}} - 1\right) e^{-r_i t_i}, \quad (4.17)$$

where $f(t_{i-1}, t_i)$ the forward rate for period $[t_{i-1}, t_i]$ and $F_{0,i-1}$ is the forward price for the period $[0, t_{i-1}]$ as presented in equations 4.10, 4.11 and 4.12.

The present value of the leg paying a set rate (floating or fixed) is described using valuation according to equation 4.1. Thus the present value of the swap for the party paying the total return leg is

$$V_{PV} = B_{PV} - RP_{PV}. \quad (4.18)$$

### 4.2.7 Option pricing

The standard European option is an agreement between parties for the buyer of the option to further buy (call option) or sell (put option) an underlying to a specific price (strike price) at a future date. For plain vanilla European put- and call options the Black-Scholes formula as described in section 3.3 is frequently used to determine the value of the option and is the one of the valuation models which will be investigated in this thesis. The investigated parameters and how they affect the price will be the volatility, risk-free rate and the dividend. Pricing of an American option will follow the same characteristics and the evaluation regarding appropriate classification levels will thus be extended to cover both types of options.
4.2.8 Autocallable Products

In recent years the market for structured products has increased and there is possibly an infinite amount of ways to structure a product. An intriguing product for investors predicting a market in a standstill is the autocallable product. The autocallable product provides a fixed coupon payoff and is partly secured against negative price movements to a predetermined level. The potential payoff is retrieved as a coupon percentage or fixed sum if the underlying or multiple of underlying equities are above a certain payoff-barrier at specified observation times. The coupon can at times be increasing if it is not at once redeemed by the barrier i.e., it meets the barrier at one of the more later observation times. If the product has not meet the barrier at any of the observation times, then at the end of it's maturity the initial investment is returned to the investor if the worst performing equity is above another threshold called the "risk barrier". If the worst performing equity is below this barrier this would result in the payoff as the return of the worst performing underlying times the initial investment.

The payoff is best explained in three steps for an autocall with multiple underlying as

- At any observation date the lowest returning asset is above the payout barrier, this would yield the autocall to expire and pay the initial investment plus the coupon premium.

- If at maturity the the autocall has not pre-expired and the lowest asset is below the payout barrier but above the risk barrier this would yield a return of the initial investment.

- If at maturity the lowest returning asset is below the pay barrier this would yield this return times the initial investment to be payed out.

The payoff structure can be visualized as in the Figure 4.2 below, where a payoff is evaluated in each observatory time if the value of the worst underlying equity is above the payoff barrier. In the figure two coupon structures are presented one which grows every observatory date vs one which is set to a constant rate.
Pricing

Pricing of such products is most straightforwardly done by simulating the underlying a number of times and calculating the mean payoff from the investment. This is an income approach which takes advantage of the known payoff structure of the autocallable and uses stochastic calculations as presented in the valuation theory section to retrieve a discounted expected value of the product. This is simply done by simulating the assets with a Geometric Brownian motion and the correlation between them can be modelled by assigning a correlation to the randomly simulated Wiener process $W_t$ between the assets. Thus the present value can be calculated as

$$AC_{PV_t} = E_Q[d_k H(S_1,t, ..., S_N,t)].$$  \hspace{1cm} (4.19)

If there is $n$ underlyings and $H$ is the defined payoff structure, calculating this numerically can be seen as averaging the payoff over multiple simulations i.e. calculating the mean payoff over $J$ Monte Carlo simulations

$$AC_{PV_t} \approx \frac{1}{J} \sum_{i=1}^{J} [d_k H(S_{1,t}, ..., S_{N,t})].$$  \hspace{1cm} (4.20)

In this case for the autocall product, the eventual payoff is not set at one specific time. This results in a "tree" of possible payoffs where the expected
value of the possibilities is calculated to form a present value for the product
this is depicted in the formula as $H$. $H$ is modelled as explained in Figure 4.2
and mathematically $H$ can be presented as the payoff function which at the
discrete points in time $t < n$ has the potential payoff $c_t$ (i.e. the predetermined
coupon payout) if the worst asset return is above the payout barrier $PB$. This
can be depicted as

$$c_t I \left\{ \min \left( \frac{S_{1,t}, \ldots, S_{N,t}}{S_{1,0}, \ldots, S_{N,0}} \right) > PB \right\}$$ \quad (4.21)

where $I$ is the indicator function and defined as 1 if the condition within
the bracket is fulfilled and else zero. For the last observation date $t = n$ the
payout is depicted as

$$\max \left[ c_t I \left\{ \min \left( \frac{S_{1,t}, \ldots, S_{N,t}}{S_{1,0}, \ldots, S_{N,0}} \right) > PB \right\},
I \left\{ RB < \min \left( \frac{S_{1,t}, \ldots, S_{N,t}}{S_{1,0}, \ldots, S_{N,0}} \right) < PB \right\} \right], \quad (4.22)$$

If the payoff for the product at any earlier time $t < n$ is larger than zero this
calls the product and no further payouts will be paid out.

In order to assess impact of input parameters a simplified model of the ex-
pected payoff was used. The model utilized stochastic variables to simulate
the returns of the investment and in such a way could measure the potential
impact of inaccurate input parameters. The model which was constructed in
R Studio [11], simulated the four underlying stock returns payoff structure as
depicted in Figure 4.2 with a growing coupon for each observation time if the
payoff barrier wasn’t reached the past year. The main objective of this model
was to measure the impact of implied correlation, implied volatility, discount
factor and dividend of the underlying equity on the value of the product.

In pricing of a product with a payoff structure such as an autocall which can
yield a payoff at several points in time, it can be useful to model the stochastic
behaviour of the inputs. For this, models such as the Heston model for volatil-
ity [12] and the Hull-White model [10] for interest rates can be used. These
models works by fitting market data to the stochastic models which thus leads
to a model which takes into account both the observed data and stochastic be-
hoavour of the market. Such models does however introduce another dimen-
sion of complexity to the model. In evaluation of the parameter significance
for this thesis these models are not used since the point of the evaluation is to
see how much the price varies when altering the input and not necessarily the accuracy of the input in the specific model. The models are presented briefly below.

**Heston Model**

In the Heston model the volatility is expressed as a stochastic process [12]. The asset path can be expressed as

\[
dS_t = \mu S_t dt + \sqrt{\nu_t} S_t dW^S_t
\]  
(4.23)

where the the variance or volatility squared noted as \(\nu_t\) is modelled as

\[
d\nu_t = \kappa (\theta - \nu_t) dt + \xi \sqrt{\nu_t} dW^{\nu}_t
\]  
(4.24)

where \(\mu\) is the mean return of the asset, \(\theta\) is the variance as \(t\) grows large, \(\kappa\) is the rate at which \(\nu_t\) goes towards \(\theta\), \(\xi\) is the variance of the variance and \(W^S_t\) and \(W^{\nu}_t\) are wiener processes with a correlation factor of \(\rho\). Thus as \(t\) goes towards infinity the expected value of the volatility \(\nu_t\) goes towards the long run variance \(\theta\).

**Hull-White model**

The Hull-White model also known as the extended Vasicek model is a popular model which enables calculation of a theoretical curve that is identical to yields in the market and can be used in pricing of financial instruments [10]. It can be modelled as

\[
dr = a \left( \frac{b(t)}{a} - r \right) dt + \sigma dz
\]  
(4.25)

where \(a\) is the speed of the mean reversion and \(b(t)/a\) is the time dependent mean reversion parameter[10].

**4.2.9 Asian Option**

An Asian option is a type of option dependent on the average underlying price over a predetermined interval in time. Thus the option value is less dependent on volatility and less affected by price fluctuations in the underlying late in the maturity. An advantage to Asian options is the reduced risk of market
manipulation at maturity since as mentioned the price is averaged over time [13]. For this thesis the present value of the option was calculated in the same manner as the autocallable product using Monte Carlo simulations. An Asian call option is thus calculated as [13]

$$C_{PV} = d_k E_Q [\max(M(S_t) - k, 0)]$$  \hspace{1cm} (4.26)

where $M(S_t)$ is the simulated arithmetic mean value of the underlying $S$ over the time period $[0, t]$ which in simulation is calculated as

$$C_{PV} \approx d_k \frac{1}{N} \sum_{i=1}^{N} [\max(M(S_t) - k, 0)]$$  \hspace{1cm} (4.27)

the price of Asian put option would thus be calculated as

$$P_{PV} \approx d_k \frac{1}{N} \sum_{i=1}^{N} [\max(k - M(S_t), 0)]$$  \hspace{1cm} (4.28)

$M(S_t)$ can be expressed as

$$M(S_t) = \sum_{i=0}^{n} \frac{S_{t,i}}{n + 1}$$  \hspace{1cm} (4.29)

for $i$ time steps over the options maturity.

### 4.3 Volatility

Volatility is an important parameter in pricing methods for different financial instruments. The volatility can be explained as a model for the fluctuations of an observed asset and a larger volatility would therefore be expected to yield larger fluctuations of price while a lower volatility would suggest a more stable price. This is an important measure, and in the pricing of vanilla options a higher volatility implies a higher option price. This is due to the fact that the option has a higher chance to clear the strike price and thus yield a higher payoff.

In finance estimating the future volatility is common practice and a necessity since it is widely suggested that past volatility is not indicative of market participants expectation of future volatility [2]. Therefore volatility models are used to estimate future volatility. While historical volatility can be directly
observed from past movements of the asset, future or implied volatility is the volatility which is suggested by the market from the current price of an option. In the framework of Black-Scholes as presented in chapter 3.3 one assumes a constant volatility in the underlying. This can however by falsified by looking at fluctuations of implied volatility extracted from option prices and charts such as the VIX chart (Volatility Index) which is an index consisting of the markets thirty day implied volatility deduced from option prices of the SP 500 Index (SPX) [14]. An extract of the VIX chart dating back to mid 1997 is presented below from Yahoo Finance 23/4/2019 [15].

![VIX Chart from Yahoo Finance 23/4/2019](image)

Figure 4.3: VIX Chart from Yahoo Finance 23/4/2019

By inspection of the VIX Chart in Figure 4.3 it is noted that the implied volatility is not a constant number and is subject to constant fluctuations. Another deduction noted from graph is that volatility seem to exhibit a stochastic but mean-reverting behaviour. This behaviour can also be found in extracted implied volatilities for options with longer maturities, where for shorter maturities the volatility seem to fluctuate more and as for longer maturities the implied volatility seem to fluctuate less and go towards an equilibrium. Below in Figure 4.4 to further validate this behaviour is the market corroborated implied volatility for the OMXS30 and STOXX50E indexes plotted together with the corresponding volatility rate of change per between maturities as

\[
\frac{IV_{i+1} - IV_i}{T_{i+1} - T_i}
\]  

(4.30)

Where \(IV_i\) is the implied volatility and \(T_i\) the corresponding maturity.
Figure 4.4: Implied Volatility of OMXS30 and STOXX50E

Noted here is that as the maturity of the implied volatility grows larger, the rate of change seem to decrease and thus finds a limiting value. This makes sense, since in options with shorter maturity the market has less events to consider making them harder to evaluate and "weight". While in longer maturities there are more events to consider, thus putting less weight on any individual event during this period.

In general the discussed implied volatility measure is considered a level 2 input variable as it is corroborated from market expectation and not on historical data. The next question then becomes if there exists market data for a specified time period but the financial instrument to be priced exceeds the maturity of the existing data. How much further can volatility be extrapolated without being considered to affect the fair value significantly thus leading to a level 3 classification of the instrument?

This question is rather hard to find a strict answer to as it might be considered subjective and depends on how much the instrument is affected by the volatility. However what might be noted is that implied volatility seems to change less at greater maturities than shorter maturities. This is also used in models such as the Heston model which behaves asymptotically towards the
long run variance in time [12]. This then leads to the deduction that for example extrapolating the four year volatility from two years of market data most likely leads to a more accurate extrapolation then extrapolating the two year volatility from one year of market data.

### 4.4 Interest rates and Discount factors

Future interest rate estimates are crucial in most valuation models and provides estimates for growth in different markets. These estimates are used in most present value techniques by discounting the future value of the expected payoff under the assumption of risk-neutral probabilities.

Rates can be extrapolated via government bonds or treasuries which can be regarded as risk-free and can therefore be used in the framework of the risk-neutral measures. Estimates between maturities can then be interpolated and estimates outside the bound of the data can be extrapolated to retrieve a good estimate of the rate. The relation between the discount factor and continuously compounded interest rate is depicted in equation 4.2. To fit a curve to interpolate or extrapolate rates between data points statistical methods such as least squares and splines are used thus providing estimates for lacking maturities. Forward rates which are rates between future maturities can also be calculated from this data as depicted in equation 4.3.

#### Risk-free rate behaviour

To examine the behaviour of zero rates and how they fluctuate over the time, twelve years of US treasury rates was examined. The absolute rate change between maturities was also investigated i.e. does the one month and three month rate have a higher rate of change (higher fluctuation) than a twenty to thirty year rate. To gain insight of this behaviour, the change between maturities in time was calculated as

\[
\frac{|r_{i+1} - r_i|}{T_{i+1} - T_i}
\]  

(4.31)

This is then plotted in the figure below in a box plot to see how the rate of change of the rates are distributed historically.
Deduced from this behaviour is that historically, for larger maturities the rate changes with smaller increments (since the mean is lower) and also behaves more consistently (since the spread is much narrower) while seemingly reaching an equilibrium since the value seem to tend to zero thus making further extrapolation of longer maturities easier. This is also consistent with the typical behaviour where the curves flattens out for longer maturities[16].

**Forward rates**

Forward rates can be explained as the rate which the market today is prepared to pay at a future date. The predictive behaviour is thus limited and while it’s useful in pricing the present value of a financial instrument might not be particularly representative of future spot rates [17]. Prediction of the future rate should thus be very carefully considered and calculated as they are mainly further extrapolations from a current maturity rate behaviour. The rate stems from the fact of the equivalency of buying a bond with maturity $T$ and rate $r(0, T)$ and buying a bond with maturity $(T - t)$ and rate $r(0, T - t)$ which renews or rolls over into a bond with a future maturity $t$ and a future rate $f(T - t, T)$ which then means that

$$e^{-r(0,T)T} = e^{-r(0,T-t)(T-t)}e^{-f(T-t,T)t}$$

(4.32)

must hold, which also leads to the equation for the forward rate as in equation 4.3.
As mentioned the accuracy of the future rates predictability on the spot rate is considered not to great, this is however not the main issue in the fair value hierarchy. On the other hand it is important that the data used as input in the fair value models agrees with the current markets expectation of the future value.

4.5 Dividends

A dividend is the potential payout a company can provide to shareholders as a distribution of company profit to share owners. The effect of this payout which is usually at a discrete point in time is the stock price dropping. This can be seen as the company giving away a certain amount of the stocks value to it’s investors, thus lowering the price of the stock with an equal amount [6]. The dividend of an asset is thus usually modelled as a steady parameter, but changes in structure to a company can lead significant and abrupt adjustments to this parameter. Such changes can be hard to predict and therefore the question of which level in the fair value hierarchy the input should be classified as arises.

Input models for future dividends only using historical data (e.g. historical dividend payments) which does not take into account current markets expectation should be classified as a level 3 input. There are also models taking advantage of expected growth of the company and compare this together with the historical payout from the company to form an idea of future dividends [18], in such cases where the expected growth of a company is extrapolated from market expectation the classification should be a level 2 input. For some assets there exists dividend futures (mostly for broader equity indexes), extrapolating the future dividend from such futures would be the most reliable method to extrapolate the market expectation of such an input. Hence when such data exists the classification of the input is easier and the input should then be level 2.

4.6 Correlation

As explained in section 3.5 the implied correlation can be estimated using the implied volatilities of plain options and basket options. In theory the correlation factor can be calculated using this method for a portfolio of two assets and thus obtain a implied correlation factor. However in reality such data is often
not available and basket options are often comprised of a multitude of assets, thus specific methods estimating the correlation are used based on the theory presented above. Such methods are for example the Buss and Vilkov’s method or the use of the CBOE implied correlation index and are discussed in the previous thesis "Modeling implied correlation matrices using option prices" [19].

The correlation between assets is a key input parameter in pricing the fair value of financial instruments relying on multiple underlyings. This is because uncorrelated assets will spread the risk more evenly while correlated assets will follow each other thus making it more likely to lose or gain value simultaneously. In the pricing of an autocall as mentioned in section 4.2.8 where the payoff is dependent on the worst performing asset, this would lead to higher payoff barrier (or lower coupon payoffs) for strongly correlated underlyings while the opposite would be the case for more independent underlyings. Thus it’s important to investigate how the value of an instrument varies as the correlation varies.

4.7 Bid/Ask Spread and Market Activity

The main distinction between a classification level 1 and level 2 in the hierarchy is the market activity as stated in the IFRS 13 standard. A financial instrument traded on an active market is to be classified as level 1 within the hierarchy and an instrument traded on an inactive market to be classified as level 2. An active market is defined directly by IFRS 13 as

"A market in which transactions for the asset or liability take place with sufficient frequency and volume to provide pricing information on an ongoing basis." - IFRS 13 [2]

This distinction is also rather unclear and statements such as "sufficient frequency and volume" and "ongoing basis" are key definitions without any clear-cut interpretation.

As the market activity is of major significance to the fair value and a big part of the IFRS 13 regulation, deduction of what signifies an active market and where in the bid/ask spread the most accurate fair value lays is to be investigated. To gain insight on market behavior, the bid/ask spread in relation to the mid price was investigated at. Then daily trade volume in active and inactive assets was compared to see how the distribution behaved and if there
was any observable distinction in relation to the activity.

The bid and ask spread was calculated as

\[ \text{spread} = P_{\text{ask}} - P_{\text{bid}} \]  \hspace{1cm} (4.33)

and mid price then calculated as

\[ P_{\text{mid}} = \frac{P_{\text{bid}} + P_{\text{ask}}}{2}. \]  \hspace{1cm} (4.34)

These values were then used to plot the difference in the last price and mid price to investigate which distribution seems reasonable and to see if the mean difference was distributed around one, which would suggest the best approximation of fair value in the spread to be the mid price. Thus the distribution of

\[ \frac{P_{\text{last}}}{P_{\text{mid}}}. \]  \hspace{1cm} (4.35)

was investigated to see how the mid and last bid is related. This is depicted in figures 4.6, 4.7 and 4.8 below.

![Figure 4.6: Distribution around mid for equity HM B, data acquired from Bloomberg 8/4/19](image)
The figures above show that the distribution of the last price seems be distributed around the mid leading to a mean of around one in the figures.

Further the distribution of trade volume was investigated to see if there was any specific distribution linked to an actively vs inactively traded asset. To compare trade volume, large assets which are regarded as actively traded on the Stockholm stock exchange were investigated and compared to assets on the other side of the spectrum regarded as inactive, where trades happen on an irregular basis. The trade volume is presented in the figures below.
In these plotted figures the daily volume distributed over a year was portrayed. The plots are ordered in the sequence of Stock regarded as high in volume to stock with a lower trade volume. In the figure, one can see that for the actively traded stocks they seem to look like something of a log-normal distribution and when looking at the less traded stocks the mean shifts towards the left and the histogram looks more like an exponential distribution decreasing in high-volume days. That is until the last plot with only nine observations (nine days during the last year where the stock was traded) which is hard to draw any conclusion from. The trade volume in the samples were also tested for independence via a Ljung-Box test which test for independence in the time series[20]. The formula for the test is presented below

\[ Q_{LB} = n(n + 2) \sum_{j=1}^{h} \hat{\rho}(j)/(n - j) \]  

(4.36)

and rejects the hypothesis of an independent identically distributed sample with a level of \( Q_{LB} > \chi^2_{1-\alpha}(h) \) where \( \chi^2_{1-\alpha}(h) \) is the \( 1 - \alpha \) quantile of the chi-squared distribution with \( h \) degrees of freedom [20]. The samples were all rejected with a confidence level of more than \( 10^{-4} \) and therefore declared not independent which is synonymous with behaviour in the market were lows and highs in the trade volume seem to happen periodically (e.g. around earnings announcement dates)[21].
4.8 Input Parameter Significance

In this section the significance of input parameters in the presented valuation models are discussed. This is important since in the fair value hierarchy where lower level inputs are used, the distinction between classification levels for the overall financial instrument is also dependent on how much the lower level input affects the overall price of the instrument. Therefore investigation of how much the price varies when altering the input parameters is concluded. The more complex financial instruments which are dependent on multiple input parameters will be more in depth analyzed while simpler instruments will only be considered in context of their explicit formula for valuation.

Interest Rate Instruments

For bonds, floating rate notes, interest rate swaps and currency swaps, the investigate parameter of uncertainty will be the rate used in the valuation model as it is the varying variable in the financial instrument. For long dated maturities of such instruments the rate will be of even more significance because of the compounding behaviour of the rates used in the valuation models. The nature between these instruments differ in which rates used. In bonds where the risk-free rate is used, the accuracy of the input parameters will be easier to extrapolate as the long dated zero rate will not differ much between maturities. For floating rate notes and swaps using a pay variable receive fixed structure the inputs will be of more uncertainty. Such as in the case where the instruments are priced using the three month forwards rate. This is because for a swap with a long maturity where one party pays variable interest rate depending on forward rates, such rates are harder to extrapolate from data. This is because the forward rate does not predict the future spot rate very well [22]. The behaviour of the input of the rates on the instrument will follow the general pricing methodology presented in 4.2. Thus they follow an exponential behaviour for longer maturities. This means that for longer maturities the instrument will be affected more.

Future/Forward contract

For a forward or a future contract as presented in section 4.2.5 the main input parameters will be the zero rate and future payout of dividends for the asset. The impact of these inputs on the instrument will follow the method depicted in section 4.2.5 and therefore exhibit an exponential behaviour in time. This is deduced by investigation of the pricing of the instruments as the inputs will
compound over longer maturities. Thus for longer maturities the inputs will affect the instrument price more significantly.

**Total Return Swap**

For the total return swap as presented in section 4.2.6 the parameters of interest when pricing the swap will be dependent on which rates used, if there are multiple re-balancing dates where payments are made and if there is other payouts from the asset (e.g. dividends when there are multiple re-balancing days). The method of valuation as presented in section 4.2.6 will vary somewhat if there are multiple re-balancing days for the instrument or if the return leg of the swap is only paid at maturity where the model using re-balancing days introduces more complexity to the model. Nevertheless due to the pricing of the financial instrument where it can be described as the difference between a bond paying a set rate and the total return of a specified asset, the significant input parameters will also stem from these valuation models.

**Vanilla European Options and Asian Options**

Investigating the significance of the parameters for a European option was done by varying the inputs in Black-Scholes formula by varying the volatility and zero rate to see which parameter seem to affect the price most. The base parameter for the scenario testing was

- Priced for maturities $T = 1, 6$ months and $T = 1, 3, 5$ years
- Volatility parameters $\sigma = 0.2$
- Zero rate continuously compounding $r(0, T) = 0.05\%$
- Yearly dividend of $q = 0\%$
- Investigation was done for two scenarios in for which one was at the money and one deep in the money with $K = S_0/2$
Investigating the significance of the parameters for an Asian option was done by Monte Carlo Simulation, varying the volatility and zero rate to see which parameter seem to affect the price most. The base parameter for the scenario testing was

- Simulated for maturities $T = 30, 60, 90$ days and $T = 1, 2$ years
- Volatility parameters $\sigma = 0.2$
- Zero rate continuously compounding $r(0, T) = 0.05\%$
- Yearly dividend of $q = 0\%$
• Investigation was done for two scenarios in for which one was at the money and one deep in the money with $K = \frac{S_0}{2}$.

Figure 4.12: Parameter investigation at the money Asian Option

Figure 4.13: Parameter investigation in the Money Asian Option

Convergence of the Asian option Monte Carlo simulations are presented below in Figure 4.14 and Figure 4.15.
The convergence of the Monte Carlo simulations were limited by the time it took to perform the simulations but were deemed adequately accurate in the sense of comparing the relative significance of the inputs as convergence of the price is regarded appropriately steady.
Autocallable

As autocalls also have several varying input parameters such as the volatility, zero rate, correlation and dividends, a comparison of how the price is affected by these parameters was simulated. To investigate the input parameter significance of an autocall, the present value of the instrument was calculated using Monte Carlo simulations as presented in equation 4.20 to gain knowledge about how the fair value behaves under different circumstances. The base parameters for the specified scenario testing was:

- Four underlying assets.
- Simulated for autocalls with maturity $T = 1, 2, 3, 4, 5, 6$ years.
- Risk barrier $0.7$ of start value.
- Payoff barrier $1.05$ of start value.
- Correlation parameters $\rho_{i,j} = 0.5$ for $i, j = 1, 2, 3, 4$.
- Volatility parameters $\sigma_i = 0.2$ for $i = 1, 2, 3, 4$.
- Zero rate continuously compounding $r(0, T) = 0.05\%$.
- Yearly dividend of $q = 0\%$.
- The Payoff was increasing by $10\%$ each observation date.
- The observations started from one year in time and every half year thereafter to the end of maturity. the simulation all asset starting values were set to 1 which leaves the return of the asset to the same as the present value of the asset.
Figure 4.16: Parameter investigation autocall

Figure 4.17: Parameter investigation autocall
Convergence of the autocall Monte Carlo simulations are presented in the Figure 4.18 below.

![Figure 4.18: Autocall Monte Carlo convergence](image)

As with the simulations of the Asian options the convergence of the Monte Carlo simulations were limited by the time it took to perform the simulations but were deemed adequately accurate in the sense of comparing the relative significance of the inputs as convergence of the price is regarded appropriately steady.

### 4.9 Constructing a general framework and guidelines from analyzed models

To conclude a result and answer the research questions the methods stated in this section has been regarded and jointly taken into consideration when constructing a general framework for the investigated instruments.

### 4.10 Limitations

This framework is constructed in a general setting for which considerations to take into account when classifying a financial instrument. This yields that
under significant anomalies in market behaviour (e.g. affecting market corroborated inputs such that examined models are no longer valid) these general guidelines might not be applicable and further research needs to be performed.
Chapter 5

Results

The results in this thesis are suggestions for ways to classify the discussed financial instruments and where the line between classification levels can be drawn. The rules are deduced from the investigated behaviour of the inputs and significance to their specific instrument as presented in the methodology section. The rules will given for the same general decision chart as presented in the begin of section 4.1 also presented below again for convenience of the reader.

![General decision chart for classification according to the fair value hierarchy](image)

Figure 5.1: General decision chart for classification according to the fair value hierarchy
5.1 Market Activity

Market activity as discussed in Section 4.7 seem to follow a skewed distribution with a heavier right tail (high trade volume), as market activity decreases the skew further moves left and resembles an exponentially decreasing distribution. The trade volume data was found to not be independent which means that periods with higher and lower market activity seem to exist which is analogous with general assumptions about market behaviour of low and high trade periods in the market.

Thus the Suggested rule for an active market is

- An active market for the asset is such that the mean daily traded volume on the market for the previous month is greater or equal to 5% of the owned equity and the traded last price is at least within the last three trading days.

The Suggested rule for which general price in the bid/ask spread to value the asset as

- The mid price.

5.2 Input Characteristics

Zero Rates

- Asymptotic Behaviour in time.

Forward Rates

- Reflects the rate which the market today values the price at the future date.
- Bad predictor of future spot rate.

Implied Volatility

- Asymptotic Behaviour in time.
- Fluctuates to a greater extent in shorter maturities.
Implied Correlation

- Derived from Implied volatility from multiple instrument thus leading to higher complexity and more assumptions.

Dividends

- Often assumed having a constant growth rate.

5.3 Financial Instrument Characteristics and Rules

European Option

- Volatility most significant parameter.

- Price changes less in time (i.e. price difference between option with 1 year and 3 year maturity is greater than option with 3 year and 5 year maturity).

Suggested classification rule (level 2 vs level 3)

- Extrapolation of volatility allowed for maturity where market data covers at least $2/3$ of period (e.g. extrapolating the third year volatility from 2 years of market data) and market data available for 1.5 year or more, else classify as level 3.

Asian Option

- Volatility most significant parameter. Price changes less in time (i.e. price difference between option with 1 year and 3 year maturity is greater than option with 3 year and 5 year maturity).

- Volatility has lower effect on price than for European Options and is less susceptible to near maturity price changes.

Suggested classification rule (level 2 vs level 3)

- Extrapolation of volatility allowed for maturity where market data covers at least $2/3$ of period (e.g. extrapolating the third year volatility from 2 years of market data) and market data available for 1.5 year or more, else classify as level 3.
Autocall

- Volatility and correlation most significant parameters.
- Price decreases with higher volatility.
- Greater correlation leads to higher price. Price changes less in time (i.e. price difference between option with 1 year and 3 year maturity is greater than option with 3 year and 5 year maturity).

Suggested classification rule (level 2 vs level 3)

- Correlation data confidence interval leads to a price variation greater than 5% classifies as a level 3 financial instrument.

Forward/Futures

- Commonly priced using the risk neutral measure and expected future price is dependent on the dividends and risk-free rate.

Suggested classification rule (level 2 vs level 3)

- Dividend estimates extrapolated using only historical data for maturities longer than 1 year classifies as overall level 3 financial instrument.

Interest Rate Instruments

Including Swaps, Bonds and Floating rate notes

- Prized using present value techniques.
- Swaps dependent on floating coupons will have an extra dimension of complexity considering forward rates are bad predictors of future spot rates.

Suggested classification rule (level 2 vs level 3)

- Instruments including varying coupon payments on floating rate (e.g. 3-month LIBOR) allows for extrapolation of forward rate data when observable market data is at least 70% of the instruments maturity (i.e. if observable market data exists for less than 70% of the instrument maturity, classify as level 3 financial instrument).
Total Returns Swap

- Prized using present value technique

- Total return leg is priced depending on the use of several re-balancing days

- The set rate leg can be priced using fixed or floating rate

Suggested classification rule (level 2 vs level 3)

- For a total returns swap which includes varying coupon payments (for the rate paying leg) on floating rate (e.g. 3-month LIBOR) allows for extrapolation of forward rate data when observable market data is at least 70% of the instruments maturity.

- For a total returns swap which includes several re-balancing days the forward rate (for the total return paying leg) the instrument allows for extrapolation of forward rate data when observable market data is at least 70% of the instruments maturity.
Chapter 6
Discussion and Motivation of Result

The results of this thesis are guidelines for how to follow the IFRS 13 fair value hierarchy. The investigated inputs were investigated through a literature study and further supported by examples from market data. Financial instruments were investigated by literature study of their pricing models and investigated regarding how particular inputs affect the price of the instruments.

In the examination and generalizing of input behaviour, one needs to regard limitations of the generalization which can be that market observations for different assets will not necessarily behave consistent across the market and anomalies will in reality occur. The regulation is set up as a way to standardize the reporting of financial instruments and make it easier to interpret the level of accuracy of the reported fair value and is thus only an indication stating that this instrument is valued with more or less observable market data and not necessarily stating the reliability of the said data. Using valuation models to approximate a fair value will after all, always introduce complexity which skews the price somewhat.

The levels in the hierarchy is also set up in such a way that the accuracy within each level will vary. A level 1 classified fair value instrument will have a varying accuracy depending on activity and the bid/ask spread for the asset in the market. A level 2 financial instrument will vary in accuracy depending on the financial instrument, the complexity of the inputs used and how many which are necessary to price the instrument. A level 3 classification will vary depending on the degree to which the unobservable inputs used are estimated.
Ultimately the conclusion of appropriate rules limiting the level of the financial instrument was based on which the most unreliable significant input to the instrument was deemed. This was in the sense to market availability of the inputs and the significance to the instrument.

The drawn conclusions about input behaviour are mainly based on previous research and then supported via examples using real market data to verify and further illustrate the behaviour. The results from this thesis are thus guidelines for where to draw lines between the levels which are more clear cut and interpretable than those given in the official regulation.
Chapter 7

Conclusions

The IFRS 13 framework and fair value hierarchy is a loosely defined framework which allows for the institutes to themselves classify their own assets. In this thesis valuation models of financial instruments and their respective inputs were investigated to make it easier to classify financial instruments by introducing a clear-cut line depending on the availability in observable market data for the instrument.

The conclusion of this thesis was then presented as suggested guidelines for how to classify the financial instruments within the fair value hierarchy as presented in the Result section.
Chapter 8
Reflections and Further Research

In the framework general data and theory has been researched. The behaviour of the discussed input variables will certainly vary depending on the underlying equity which means that there will be inputs which behaves differently than discussed in this thesis. This leaves room for more in-depth behavioural analysis of the discussed inputs. For further research going in to more detail about how much room for interpretation there is within each level could be looked at and how specific market scenarios affect the classification could also be looked at. Further it could be of interest to investigate how the accuracy within the levels are projected i.e. how much difference is there in price accuracy between the levels and if there is an increased risk between financial instruments with different classifications.

Regarding dividends as discussed in Section 4.5 market data indicating future dividend is not always available for specified maturities, in the existence of dividend futures there could be a way of extrapolating the future dividend payout making it a level 2 input, but basing the dividend on historical prices should classify the input as a level 3 input which could affect the overall financial instrument classification. Because the dividend input is present in a large number of financial instruments this could lead to a misrepresentation for these instruments. Thus for inputs which are "semi-corroborated" using market future expectation and historical data, it could be interesting to look at where to draw a line if such an input should be classified as a level 2 or 3 input depending on how much the input is corroborated by historical versus market expectation data.

Regarding market activity it would be interesting to further research the
impact on a market when larger buys or sell-offs happen, how the market has the potential handle such events and what the implication is on the price of the asset.

Something not discussed in this thesis is credit risk which indicates the risk of a counterparty defaulting on the debt and loss of income from the instrument take place[6]. Such risks need to be considered within the framework and will affect the overall instrument value, such risks are often approximated using credit value adjustments which measures the difference between the risk-free rate and the entity’s rate. Further research might entail the affect of such inputs on financial instruments and in which degree such observable data is available.
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Appendix A

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