Measuring the impact of strategic and tactic allocation for managed futures portfolios

Using design of experiments

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Abstract

The optimal asset allocation is an ever current matter for investment managers. This thesis aims to investigate the impact of risk parity and target volatility on the Sharpe ratio of a portfolio consisting of futures contracts on equity indices and bonds during the period 2000-2018. In addition, this thesis examines on which level - instrument, asset class or total portfolio level - a momentum strategy has the largest effect. This is done by applying design of experiments.

The final result in this thesis is that risk parity and target volatility improve the Sharpe ratio compared to a classic 60/40 capital allocation. Furthermore, utilising momentum strategies is the most beneficial on the asset class level, i.e. to allocate between equity indices and bond futures.
Sammanfattning


Det slutgiltiga resultatet i denna uppsats visar att riskviktning och målrisk förbättrar Sharpe-kvoten jämfört med en klassisk 60/40 kapitalallokering. Vidare är nyttjande av momentumstrategier det mest fördelaktiga på tillgångsklassnivå, det vill säga att allokera mellan aktieindex- och obligationstermins-kontrakt.
Acknowledgements

We would like to express our greatest gratitude to our supervisors at Lynx Asset Management, Tobias Rydén and Ola Backman, for continuously providing insightful and valuable feedback. In addition, we want to thank Despina Xanthopoulou for introducing us to the topic of balanced funds. We are also grateful to Lynx Asset Management for giving us the opportunity to write our thesis.

Lastly, we would like to thank our supervisor at the Royal Institute of Technology, Pierre Nyquist for his advice and encouragement.
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1 Introduction

1.1 Background

To be a successful investor over time, it is vital to have a well-maintained portfolio. The portfolio strategy needs to have an asset allocation which makes the trade-off between risk and return aligned with the preferences of the investor, who at all times would prefer to get greater returns and reduced risk. There exists a great body of literature on portfolio strategies and it is an ever current topic to investigate which portfolio strategies yield the best risk-adjusted returns for investors.

The two last major financial market crashes in 2002 and 2008 have created a need for portfolio strategies which can manage the total portfolio risk, and thus return, in times of financial turbulence. Two examples of strategies which aim to provide good management of the downside risk are risk parity strategies and volatility targeting strategies. The last decade, these strategies have seen a growing body of literature as well as increasing attention from the asset management side [1].

Risk parity strategies are based on risk weighting of asset classes. This is to be compared with traditional portfolio strategies where 60% is invested into equities and 40% into bonds, but since equities have approximately three to four times higher risk than bonds, this allocation yields that about 90% of the total risk is equity-risk [2]. Put differently, traditional portfolio strategies are not diversified at all when it comes to risk. The risk parity strategy instead uses an allocation scheme between the asset classes with aim of balancing exposure and diversifying risk [1]. Risk parity is naturally related to volatility targeting, since the assets are allocated a certain risk in a risk parity portfolio, the total risk of the portfolio is determined as well [3].

Volatility targeting strategies aim to keep the volatility at a predetermined target, which means that the portfolio’s market exposure is tuned so that the volatility of the total portfolio meets the target. This means that in low-volatility environments the market exposure is increased, perhaps with the use of leverage, and in high-volatility times the market exposure is decreased. These strategies rely on the notion of volatility persistence, i.e. that high and low volatility tends to be persistent over periods of time and that current volatility is a good base for prediction of future volatility [4]. Hence, an investor can leverage this notion and possibly avoid drawdowns in her portfolio.

Another strategy to decide how to adjust the total risk of a portfolio is via momentum signals. Momentum investing strategies became popular in the 1990’s, a bit earlier than risk parity and target volatility strategies entered the spotlight. Momentum investing is based on the empirical observation that securities that have performed well tend to, on average, perform well in the future and securities that have underperformed will continue to underperform. The strategy then consists of increasing the portfolio weight allocated to securities that have performed well, and reduce portfolio weight or go short in the securities that have performed poorly [5].

The first studies of momentum investing only investigated single instruments, but in
more recent time, researchers have also found robust momentum on the sector and asset class levels [6]. Investors can thus allocate more or less weight to securities, sectors and asset classes depending on their momentum signal in order to increase the return of their portfolios.

1.2 Research question

Risk parity, target volatility and momentum investing strategies have all grown in popularity both as research subjects and asset management practices in recent time [4]. The possibility of avoiding drawdowns in portfolio value using these strategies is of interest for investors seeking to maximise portfolio gains while keeping the risk as low as possible. Since this is a rather novel research area, there still exists opportunities for exploring the effects of the above mentioned strategies, to conclude if they increase the risk adjusted return of the portfolio. While equity portfolios are rather well explored, there is a gap in the literature for portfolios consisting of futures contracts.

To investigate the impact of risk parity, and thus risk weighting, such portfolios can be compared with the classical mutual fund approach of capital weighting assets. The momentum investing strategies are able to be applied on both risk weighted and capital weighted portfolios.

In addition, previous research investigating the effect of momentum investing strategies on the Sharpe ratio has focused on one level at time, where the levels correspond to instrument, asset class and total portfolio level. However, interaction effects between momentum strategies on different levels are an unexplored area. For an investor wanting to combine momentum strategies on multiple levels it is of importance to know if the benefits of each single strategy are kept when using a combination of the strategies, or if the benefits are cancelled out.

Thus, the purpose of this thesis is to investigate and find empirical evidence of the impact of risk parity and target volatility strategies, together with momentum investing, on the Sharpe ratio on portfolios consisting of futures contracts. Moreover, this thesis also aims to investigate on which level, i.e. instrument, asset class or total portfolio, momentum strategies have the largest impact and whether momentum investing strategies yield any interaction effects when combined.

The research questions this thesis attempts to answer are:

- What effect do risk parity and target volatility strategies, compared with a classical capital weighted allocation, have on the Sharpe ratio of a futures portfolio?
- How do momentum investing strategies on instrument, asset class and total portfolio level impact the Sharpe ratio of a futures portfolio? On which level do momentum investing strategies have the largest impact?
- Which interaction effects between the aforementioned momentum strategies have an impact on the Sharpe ratio of a futures portfolio?
This thesis makes use of design of experiments to construct portfolios using a combination of the above mentioned strategies. Which strategies to use for which portfolio are designed with experimental design. From these portfolios quarterly Sharpe ratios are calculated, and from these observations of the Sharpe ratio, the effect of the different combinations of strategies are calculated and evaluated.

As a complement, to answer the question on which level momentum investing strategies have the largest impact, this thesis also formulates the question as an optimisation task.

1.3 Scope and limitations

The research is limited to investigating portfolios consisting of only futures contracts. The underlying assets of the included futures contracts are equity indices and bonds, and the time period examined is 2000-2018. Furthermore, we have the restraint that the portfolios can only hold long positions in all assets, thus not allowing short selling. The momentum signals used to determine rebalancing are time series momentum. In addition, the portfolios make use of daily rebalancing and transaction costs are disregarded. The risk of the portfolios is measured as volatility.

1.4 Related work

The article "The impact of volatility targeting" states some key features of volatility targeted portfolios. The main benefits of volatility targeting are improvement of the Sharpe ratio of the equity holdings, and reduces the likelihood of extreme returns for all asset classes. This has the important implication that left-tail events tend to be less severe, as this type of event often occurs at times of high volatility, when a volatility targeted portfolio would have a lower exposure [7].

A concern for the usefulness of volatility targeting is that this strategy has been favoured by the decreasing yields and the corresponding higher return of bonds during the past three decades. The same concern can be applied to risk parity strategies, which also increase the exposure to bonds compared with a capital weighted 60/40 portfolio. Hurst et al. however claim that a risk parity strategy might still outperform a classic 60/40 capital weighted asset allocation even during prolonged periods of rising rates [9]. Hogan et al. agree with this in a 2017 paper, where they conclude that risk parity as an investment strategy is not entirely dependent on falling bond yields [8].

However, both papers also bring up the situations where risk parity portfolios are negatively affected by rising rates, this is when rates are rising faster or higher than expected, which could make bond return negative. It is in this type of situation where risk parity portfolios are severely harmed by rising rates.

Dachraoui, in the article "On the optimality of target volatility strategies" finds that target volatility strategies can outperform an underlying index portfolio, but remain suboptimal in regards to the efficient frontier [4]. The author also emphasises the significance of the estimation of volatility, and the rebalancing frequency of the
strategy.

The existence of momentum can be considered to be in conflict with the notion of prices as a random walk and the efficient market hypothesis. Moskowitz et al. find time series momentum in futures contracts on equity indices, commodities and currencies. They note that time series momentum violates the basic random walk as the momentum theory implies that past returns are informative about future return [9]. Jegadeesh and Titman, who investigate stocks and cross-section momentum, suggest that investor expectations are systematically biased which would also be in conflict to the efficient market hypothesis [5]. Thus, to implement momentum investing strategies, one dismisses the effective market hypothesis.

In the article "Fact, Fiction, and Momentum investing" the authors provide evidence against the concern that momentum has too small effect and is too sporadic to be used as an investment strategy. The authors utilise meta-analysis of other research articles on momentum investing and analysis of U.S. equity data between 1927-2013 [10].

Hagani and Dewey, in a 2016 article find statistically and economically significant improved returns by using momentum and value strategies on an asset class level in long-only portfolios [11]. The authors dynamically adjust the asset allocation according to measures of momentum and value, and find superior returns compared to more static allocation strategies. The authors also suggest that momentum on an asset class level might be a large scale anomaly, that might not disappear even if its existence would be widely known. Georgopoulou and Wang also document that time series momentum is consistent and robust across asset classes [6].

Brinson argues that more than 90% of portfolio returns are attributable to asset allocation, leaving security selection attributable to little return [12]. Considering this, it would be expected that momentum investing strategies would be the most impactful on the asset class level, and less so on the instrument level.
2 Financial background

2.1 Forward contracts

A forward contract is a financial derivative, where it is specified on which date and at what price the specified quantity of the underlying asset will be traded, where the holder of the forward contract is obliged to make the transaction. In that way, it differs from an option, which also has a specified date, price and quantity, but the holder of the option is not forced to go through with a transaction, and will only do so when it is profitable.

2.2 Futures contracts

A futures contract is similar to a forward contract, as it also is an agreement between two parties to buy or sell an asset at a specified date to a specified price, but the two contract types have some important differences. The most significant differences are that futures contracts are marked-to-market, and that they are traded on an exchange.

Futures contracts can have either physical delivery or cash settlement, like forward contracts, meaning that upon maturity, the seller of the contract gets the agreed spot price and in turn delivers either the physical underlying asset, e.g. oil, or a basket of stocks, or settles with cash. This thesis will use futures contracts with cash settlement for equity indices, and physical delivery for bonds.

As the trading of futures contracts is done on an exchange connected to a clearing house, the buyers and sellers of the futures contracts do not have each other as counterparties, but the clearing house. The clearing house becomes the seller to the buyer, and the buyer to the seller. Thus, the clearing house has a net zero position. The market participants use clearing houses to reduce counterparty risk.

This leaves the clearing house with counterparty risk to both the buyer and the seller of the futures contract. Instead of waiting until the maturity of the contract for the buyer and seller to deliver, the clearinghouse demands them to each day settle profits and losses of the contract. At inception, the buyer and seller are required to post an initial margin. If the futures price increases, the buyer’s margin account is credited with the rise, and the seller’s margin account is reduced with the same amount, i.e. the buyer benefits from increases in the futures price.

If one of the parties would continuously experience daily losses, their margin account may fall below the maintenance margin. The maintenance margin is the amount required by the clearing house to continue to clear the trades of the investor. If the margin account is below the maintenance margin, the clearing house will make a margin call, demanding the investor to post supplementary margin or their positions are closed [13].

A challenge for investigating longer time periods is that the futures contracts as they mature, cease to exist. In order to be able to study an elongated time period
spanning over the life of multiple futures contracts, we utilise rolling of contracts. Rolling is the process of moving from a position that will settle to another contract with maturity at a later point in time. As this thesis will use arithmetic returns, we utilise arithmetic rolling, this is done by simply linking the forward price of the price series with the latest maturity with the previous price series, adding or subtracting some value to the older price series to have the same price as the new price series at the date of rolling, as visualised in Figure 1. This impacts the historical price levels, but does not impact the historic arithmetic return.

Figure 1: Arithmetic rolling of futures contracts

![Arithmetic rolling of futures contracts](image)

The value of a futures contract is the expected spot price at the maturity of the contract, discounted to present value (and, in some cases, less the cost of holding the underlying asset during the period of the contract). Thus, the futures price \( F_0 \) for futures on stock indices is given as:

\[
F_0 = S_0 e^{(r-d)T}
\]

where \( S_0 \) is the underlying spot price, \( r \) the risk-free interest rate, \( d \) the dividend yield rate, and \( T \) is the time to maturity.

The futures price for treasury bonds is slightly more complicated. The seller of the futures contract has the option to deliver any bond from a basket of bonds. The basket of deliverable bonds consists of government bonds with remaining life close to the lifetime of the futures contract, e.g. for the CME Group 2 year U.S. treasury bond futures contract, any U.S. government bonds with remaining life between 1.75 and 5.25 years can be in the basket. The rules of which bonds are included in the deliverable basket is determined by the exchange. This differs from the seller of a futures contract on an equity index, who is only allowed to cash settle that particular index. The seller in this case will choose to deliver the bond that is cheapest. This is called the cheapest-to-deliver bond.
If we assume that we know the cheapest-to-deliver bond, and the delivery date is known, the futures price $F_0$ for futures on treasury bonds is:

$$F_0 = (S_0 - I)e^{rT} \quad (2)$$

where $S_0$ is the spot price of the cheapest-to-deliver bond, $I$ the present value of the coupons during the life of the futures contract, $T$ the time until maturity of the futures contract, and $r$ the risk-free interest rate applicable to a time period of length $T$ [14].

The futures price is paid at the date of maturity. No payment occurs when the contract is initialised, but an initial margin is posted to each party’s margin account. More on margin accounts below.

### 2.3 Sharpe ratio

The Sharpe ratio is a method to examine the return of an investment by comparing it with the risk entailed. The Sharpe ratio, $SR_i$, is defined as:

$$SR_i = \frac{E[X_i - r_0]}{\sigma_i} \quad (3)$$

where $X_i$ is the return of the risky asset, $r_0$ is the risk free return and $\sigma_i$ is the standard deviation/volatility of the risky asset [15].

The Sharpe ratio for a portfolio, $SR_p$, consisting of several risky assets with weights $w_i$ for $i = 1,..,m$ and covariance matrix $\Sigma$, is defined as:

$$SR_p = \frac{E[\sum_{i=1}^{m} w_i X_i - r_0]}{\sqrt{\sum_{i=1}^{m} \sum_{j=1}^{m} w_i \Sigma_{i,j} w_j}} \quad (4)$$

When calculating the Sharpe ratio for portfolios holding exclusively futures contracts, the risk-free return can be disregarded since no payment occurs when the contract is initialised and it is thus possible for the investor to invest the capital at the risk free rate.
3 Mathematical background

3.1 Design of experiments

Design of experiments (DOE) refers to the design of an experiment in order to explain the variation in the response variable by changing input variables. A designed experiment is thus one or many tests where changes are made to the input variables of a system, with the aim of being able to observe and explain the changes in the response variable.

3.1.1 Factorial design

A subset of DOE is factorial design, which is a designed experiment where in all complete replications of the experiment, all possible combinations of factors, i.e. input variables, at all levels of the factors are investigated. Compared to experiments investigating one factor at a time (OFAT experiments), factorial design have several major advantages. Factorial design is more efficient since it can provide more or similar information at the same cost. Also, factorial design has the advantage over OFAT experiments as interaction effects can be discovered.

The main effect of a factor is defined as the change in the response variable produced by a change in the level of the factor. Interaction effects occur when the effect of a factor depends on the level another factor.

The most widely used factorial designs are so called \(2^k\) factorial designs where there are \(k\) factors, each at two levels. Such a design requires \(2 \cdot 2 \cdot 2 \ldots \cdot 2 = 2^k\) observations, thereby the name. The levels in a \(2^k\) factorial design could be quantitative, but also qualitative, e.g. "high" and "low".

These interaction effects are so called higher order effects and when the number of factors used in the experiment increases, the number of interaction effects increase exponentially, e.g. a factorial design with 6 factors each having 2 levels will have 63 degrees of freedom. Of these 63 degrees of freedom, 6 correspond to main effects, and the remaining degrees of freedom correspond to interaction effects.

For example, a factorial design with two factors both having two levels each, e.g. low and high, is called a \(2^2\) factorial design. Here it is obvious that there are 4 possible combinations of factors at each level. Let the factors be called \(A\) and \(B\). Let \(-1\) denote the factor at the low level, and 1 be the factor at the high level. Then Table 1 displays all possible combinations of factors.

After running several experimental runs with \(n\) replicates of each factor combination, the average effect of the factors can be calculated as the change in response produced by a change in the level of each factor, averaged over the levels of the other factors. Let \(ab\) be the outcome of the experiment where both \(A\) and \(B\) are on their high level, \(a\) be the outcome of the experiment when \(A\) is high and \(B\) is low, and \((1)\) be the experiment where both \(A\) and \(B\) are low. Then we have the average effects \(\bar{A}\)
Table 1: Example - Factor and level combinations in experimental design

<table>
<thead>
<tr>
<th>Label</th>
<th>Treatment combination</th>
<th>A</th>
<th>B</th>
</tr>
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<tbody>
<tr>
<td>(1)</td>
<td>A low, B low</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>a</td>
<td>A high, B low</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>b</td>
<td>A low, B high</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>ab</td>
<td>A high, B high</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

of factor $A$ and $\tilde{B}$ for factor $B$:

$$\tilde{A} = y_{A^+} - y_{A^-} = \frac{1}{2n}\left([ab - b] + [a - (1)]\right)$$

$$\tilde{B} = y_{B^+} - y_{B^-} = \frac{1}{2n}\left([ab - a] + [b - (1)]\right)$$

### 3.1.2 Fractional factorial design

When the number of factors increase the number of experiments needed to investigate all possible combinations of factors grows exponentially. For example, if all factors have 2 levels, a complete replicate of an experiment with 8 factors requires 256 trials (and the number of first order effects is only 8). This can be costly and demanding considerable resources with only a modest number of factors to be investigated. A solution to this problem of dimension is fractional factorial design.

Fractional factorial designs use subsets of the full factorial design for the trials in order to keep the costs and resources needed down. Commonly, a half or a quarter fractional factorial design is investigated. These designs are denoted $2^{k-1}$ and $2^{k-2}$ respectively.

Montgomery poses three key ideas that contribute to a successful use of fractional factorial design: the sparsity of effects principle, the projective property, and sequential experimentation [16]. The sparsity of effects refers to the principle that when several factors exist, the system is primarily driven by some of the main effects and lower order interactions. This means that a full factorial design might be redundant and might not add extra information in comparison with a carefully chosen fractional factorial design. The projective property is that fractional factorial designs can be projected into larger designs in the subset of significant factors. Finally, sequential experimentation means that it is possible to combine multiple trials of fractional factorial designs to sequentially assemble a larger design.

The price the experimenter will have to pay when designing a fractional factorial design is the confounding of main effects with interaction effects that occurs. For the experimenter this might be acceptable if the experimental design is set according to the sparsity of effects principle, i.e. that the significant main effects are confounded with non-significant interaction effects. The ability to separate main effects from interaction effects is called the resolution of the design. The resolution describes to
which degree the main effects can be aliased, or not confounded, with two-factor interaction effects, three-factor interaction effects, etc [16].

Table 2: Resolution in experimental design

<p>| | |</p>
<table>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>A design with only one run.</td>
</tr>
<tr>
<td>II</td>
<td>Main effects are confounded with other main effects.</td>
</tr>
<tr>
<td>III</td>
<td>Main effects are confounded with two-factor effects, but unconfounded with other main effects.</td>
</tr>
<tr>
<td>IV</td>
<td>Main effects are confounded with three-factor effects, but unconfounded with main and two-factor effects. Two-factor effects might be confounded with other two-factor effects.</td>
</tr>
<tr>
<td>V</td>
<td>Main effects are confounded with four-factor effects, but unconfounded with main, two-factor and three-factor effects. Two-factor effects might be confounded with three-factor effects.</td>
</tr>
</tbody>
</table>

Resolution design III-V are the most important ones. Resolution designs below III are not useful since main effects can not be calculated. Resolution III is a suitable choice of design for initial studies, since it can separate the important single factors from others at a cheap cost. Resolution IV is useful when wanting to understand the system as two-factor interactions are singled out as well. Resolution V designs enable the experimenter to understand even more complex effects than resolution III and IV design and the experimenter could thus possibly develop better predictive models. Designs with a resolution over V are not common in practice, since a lot of resources are put into investigating higher-order interaction effects which is not in line with the sparsity of effects principle. Note that a full factorial design does not have any confounding of effects and the resolution for full factorial design is said to be infinity [16].

3.1.3 Evaluation of design of experiments

Analysis of variance, or ANOVA, is used to compute the sum of squares and F-statistics of the design of experiment data, with the purpose of hypothesis testing. As an example with the given experimental design of factors $A$ and $B$, with $a$ and $b$ denoting the number of levels of each factor and $n$ the number replicates of experimental runs, the ANOVA is given by Table 3

Table 3: Example - ANOVA table in experimental design

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Mean Square</th>
<th>$F_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$SS_A$</td>
<td>$a - 1$</td>
<td>$MS_A = \frac{SS_A}{a - 1}$</td>
<td>$MS_A/MS_E$</td>
</tr>
<tr>
<td>B</td>
<td>$SS_B$</td>
<td>$b - 1$</td>
<td>$MS_B = \frac{SS_B}{b - 1}$</td>
<td>$MS_B/MS_E$</td>
</tr>
<tr>
<td>Interaction AB</td>
<td>$SS_{AB}$</td>
<td>$(a - 1)(b - 1)$</td>
<td>$MS_{AB} = \frac{SS_{AB}}{(a - 1)(b - 1)}$</td>
<td>$MS_{AB}/MS_E$</td>
</tr>
<tr>
<td>Error</td>
<td>$SS_E$</td>
<td>$ab(n - 1)$</td>
<td>$MS_E = \frac{SS_E}{ab(n - 1)}$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$SS_T$</td>
<td>$abn - 1$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The ANOVA summarises how much the variance in the data is explained by the factor effects, and how much is attributable to random error. The null hypothesis, $H_0$, can be rejected at an $\alpha\%$ confidence level if:

$$F_0 > F_{\alpha,k,n-k-1}$$

where the null hypothesis is: $H_0 : All\ individual\ batch\ means\ are\ equal$.

### 3.2 Volatility, returns and portfolio weighting

To be able to answer our research questions, volatilities, returns and portfolio weights must be estimated and calculated. There exists a plethora of methods for estimating volatility and measuring performance of a portfolio. In this chapter, the methods used in this thesis are presented.

#### 3.2.1 Exponential weighted moving average method for estimating volatility and covariance

As volatility is usually not constant over time but instead tends to cluster, a proper estimation of volatility should vary over time and put more weight in recent observations. However, only using the last few days’ observations of volatility would create unstable estimations. A solution to this is an exponential weighted moving average, which puts more weight to more recent observations and as events become distant in time, the less weight they get.

The exponential weighted moving average for estimating variance, $\hat{\sigma}_t^2$, at time $t$ has the following recursive definition:

$$\hat{\sigma}_t^2 = (1 - a)r_t^2 + a\hat{\sigma}_{t-1}^2$$

where $r$ is the arithmetic return and $a \in (0, 1)$ determines the decay of the weights. The first term in the above stated definition, $(1 - a)r_t^2$ determines the intensity of the reaction to market events. The second term $a\hat{\sigma}_{t-1}^2$ determines the persistence in volatility. Consequently, a low $a$ gives a large reaction to yesterday’s market events and low weight to earlier events, when estimating today’s volatility. A common choice of $a$ is $a \in (0.75, 0.99)$.

The covariance is not static over time, and it can also be estimated with an exponential moving average. The estimate of covariance, $\Sigma_t$, at time $t$ is recursively defined as:

$$\hat{\Sigma}_t = (1 - a)r_tr_t^T + a\hat{\Sigma}_{t-1}$$

where $r_t$ is a column vector including returns for all investigated securities at time $t$, and $a$ is the same as in Equation 8.

An assumption made when using an exponential weighted moving average is that the drift component of the securities, i.e. the return, is assumed to be zero. This is reasonable for relatively short time periods. For longer time periods this estimator will tend to overestimate the volatility. Consequently, exponential weighted moving
average is suitable for estimating daily or weekly volatilities and covariance matrices [18].

3.2.2 Garman-Klass method for estimating volatility with Yang-Zhang extension

The classical volatility estimator makes use of closing prices only, but with more information about price movements on the intraday market, it is possible to get more accurate estimates. In 1980, Garman and Klass presented a more sophisticated way to estimate volatility by making use of intraday prices - open, low, high and closing prices. Unlike the exponential weighted moving average method for estimating volatility, the Garman-Klass method does not make the assumption that the securities have no "drift" motion [19].

The Garman-Klass method does not take into consideration the overnight jump between closing and opening prices which leads to an underestimation of the volatility. Yang and Zhang presented in 2000 an extension to the Garman-Klass method which encaptures the overnight jump [20].

The Garman-Klass method for estimating volatility with the Yang-Zhang extension is defined as follows:

\[
\hat{\sigma}^2_t = \frac{1}{N} \sum_{t=1}^{N} \left( (o_t - c_{t-1}) \right)^2 + \frac{1}{2} \left( (h_t - l_t) \right)^2 - (2 \ln(2) - 1) \left( (c_t - o_t) \right)^2
\]

where \( N \) is the number of samples, \( o_t \) and \( c_t \) the open and closing price respectively, and \( h_t \) and \( l_t \) the high and low price respectively.

Note that the above stated definition of the Garman-Klass with Yang-Zhang extension volatility estimator puts equal weight to all observations. It is possible to define the Garman-Klass with Yang-Zhang extension volatility estimator with an exponentially weighted moving average to give recent observations higher importance, see more in previous chapter about exponential weighted moving average.

The Garman-Klass with Yang-Zhang extension method for estimating volatility is defined in the following recursive way with an exponential weighted moving average:

\[
\hat{\sigma}^2_t = a \hat{\sigma}^2_{t-1} + (1 - a) \left( (o_t - c_{t-1}) \right)^2 + \frac{1}{2} \left( (h_t - l_t) \right)^2 - (2 \ln(2) - 1) \left( (c_t - o_t) \right)^2
\]

where \( a \in (0, 1) \) determines the decay of the weights, and the other parameters are the same as above. This is the method used in this study.

In conclusion, the Garman-Klass with Yang-Zhang extension volatility estimator is independent of both the drift and opening jumps of the underlying security’s price movements. Because of these properties, the Garman-Klass method with Yang-Zhang extension is popular for those who seek a more sophisticated method than simpler estimators using closing prices.
3.2.3 Normalised returns and positions

When investigating long time periods of futures contracts, rolling of contracts is needed when the contracts mature. This thesis makes use of arithmetic rolling, which is described in more detail in Chapter 2.2 about futures contracts. With this rolling method, the historical arithmetic return is unchanged and are consequently used for investigating returns. The arithmetic return of an asset at time $t$ is defined as:

$$ R_t = \Delta P_t = P_t - P_{t-1} $$(12)

which is what an investor holding a futures contract receives through the daily mark-to-market procedure.

To calculate the return of several assets, denoted in different currencies, when having arithmetic returns is a challenge. A remedy to this problem is to use normalised returns, where the arithmetic return of an asset, $R_t$, is normalised, by dividing it with the standard deviation of the same asset’s arithmetic return $\sigma_{t-1}$ [4]. The normalised return is defined as follows:

$$ Z_t = \frac{R_t}{\sigma_{t-1}} $$

Note that the standard deviation used in the fraction is estimated one time unit ago. The reason for this is, is for the relationship to be causal. For this thesis, the volatility will be estimated with an exponential weighted moving average of the Garman-Klass method with Yang-Zhang extension.

As previously stated, the benefit of defining returns on this form is that it is a dimensionless quantity as long as the arithmetic returns and the standard deviations are expressed in the same currency. This eliminates the need to take account for the different exchange rates the assets are denominated in.

The normalised return $Z_t$ can be viewed as the amount of standard deviations earned/lost during the period $t$. Consequently, the normalised position size $\gamma_i$ corresponds to the relative risk weight put in the asset $i$, e.g. with the position sizes $\gamma_i = 1/10$ and $\gamma_j = 1/5$, the investor has double the risk in asset $j$ compared with asset $i$. Thus, when modelling with normalised returns, $|\gamma|$ is the standard deviation of the daily return of a position in an asset. The sign of $\gamma_i$ indicates whether the investor goes long or short.

The normalised return of a portfolio at time $t$ consisting of several assets is:

$$ R_t = \sum_{i=1}^{m} \gamma_i Z_{t, i} $$

where, as before, $\gamma_i$ is the risk weight put in asset $i$ and $Z_{t, i}$ is the normalised return at time $t$ of asset $i$. 

20
3.2.4 Capital weight

With normalised returns, it is straightforward to calculate the risk weights, as $\gamma_i$ for $i = 1, 2, \ldots$ correspond to the relative risk put in each asset, but it is more complicated to retrieve the capital weights. To calculate the normalised capital weights of a futures portfolio, one needs to look at exposures since no price is paid initially when entering a futures contract. The exposure, $e_i$, to market $i$ for a normalised position is calculated as:

$$e_i = \frac{\gamma_i}{\sigma_i} \cdot \text{price of underlying asset}$$  \hspace{1cm} (15)

where $\gamma_i$, as in previous chapter, corresponds to the risk weight and $\sigma_i$ the standard deviation of the arithmetic return of asset $i$. The ratio $\frac{\gamma_i}{\sigma_i}$ corresponds to the number of contracts in the normalised market. The price of the underlying asset is for equities, the spot price, while for bonds, the price of the underlying asset is the market price of the cheapest to deliver reference bond [21].

To retrieve capital weights, one thus needs to set the exposure for each asset so the relative size between them is at the desired level and solve this equation for $\gamma$ as everything else is known.

3.2.5 Constant volatility

The volatility of a portfolio consisting of several risky assets with weights $w_i$ for $i = 1, \ldots, m$ is calculated as:

$$\sigma = \sqrt{w^T \Sigma w}$$  \hspace{1cm} (16)

where $\Sigma$ is the covariance matrix between the assets’ returns [22]. In the context of normalised markets, $w_i$ would be the amount of normalised position sizes and $\Sigma$ the covariance matrix between the normalised returns.

For the portfolio to have a target volatility, one thus needs to modify the weights of the assets, assuming the relative sizes of the assets to be constant. Let $\sigma k$ denote the target volatility, which e.g. could be 10% on a yearly basis. We then get:

$$\sigma k = \sqrt{w^T \Sigma w} \cdot k$$

$$\Rightarrow (\sigma k)^2 = w^T \Sigma w \cdot k^2$$

$$\Rightarrow (\sigma k)^2 = (w^T k) \Sigma (w k)$$  \hspace{1cm} (17)

From this, we note that the new weights, $w_{\text{new}} = k \cdot w_{\text{old}}$, ensure that the portfolio has a constant risk [23].

3.2.6 Risk parity

Risk parity is an approach to investment management where the focus is on the allocation of risk between asset classes or assets, and not the allocation of capital like in the traditional approach. With a risk parity approach, the aim is to better control and diversify the risk of the portfolio. By contrast, in the traditional capital
weighted approach to portfolio management, say e.g a 60% equities and 40% bonds portfolio, 90% of the risk comes from equities and is consequently from a risk perspective not diversified at all [2].

In a risk parity portfolio, bonds, fixed income derivatives and other low volatile products will compromise a larger portion of the invested capital. This will reduce the volatility of the portfolio, but also reduce the expected return since the portion of equities is fairly low. In order for the return to meet the investor’s expectations leverage must be used. For institutional investors, the unlevered portfolio’s expected return would be far lower than the required rate of return [24].

A method for retrieving the weights on the asset class level in a risk weighted portfolio, is to calculate the risk of the bond and equity portfolio separately [2]. Like before, the risk (volatility/standard deviation) is calculated as \( \sigma = \sqrt{w^T \Sigma w} \).

As described in previous chapter about volatility targeting, the volatilities for the bond and equity portfolios can be set to a ratio, and of course levels, which are comfortable for the investor. A ratio of 1 between the bond and equity portfolios’ volatilities corresponds to equal risk contribution from the asset classes. From these target volatilities, the weights for equities and bonds can be calculated, which is described in more detail in previous chapter. A common choice of risk weights is 50% in equities and 50% in bonds which corresponds to that an equal risk contribution from equities and bonds. This approach can be extended to retrieve risk weights on an instrument level as well.

### 3.3 Momentum investing

Strategies that buy stocks that have performed well in the past, and sell stocks that have performed poorly are called momentum strategies. These strategies were popularised in the 1990’s and have proven to generate significant excess returns [5].

The term momentum can have different meanings within finance, the two major divides are time series momentum and cross section momentum. Time series momentum focuses on each security’s past return, where securities’ return in the recent past is a positive predictor of future return. Cross section momentum on the other hand, refers to the phenomenon where securities that have recently outperformed their peers will continue to outperform their peers on average over some period.

Thus, the main difference between time series momentum and cross section momentum is that the first compares each security or asset class with its own past return, and the latter compares each security or asset class return with its peers’ returns. This thesis will use backward looking time series momentum.

In order to determine whether an asset has positive momentum and should receive a larger weight, the average of the last 12 months’ daily return is calculated. The 12 month time series momentum is chosen as it has proven robust in its ability to predict future performance [9]. This average is divided by the standard deviation of the return in order to get nominal variance and this ratio is scaled with the sigmoid
function. The sigmoid function is defined as:

\[ f(x) = \frac{1}{1 + e^{-x}} \]  

(18)

The sigmoid function is used since it has a range between 0 and 1 and is a monotonic increasing function, making the function appropriate for scaling and signaling purposes. This approach is consistent with how momentum strategies are implemented in the industry [25].

Finally, the previous weight of the asset in the portfolio is multiplied with the value from the sigmoid function, which means that an asset that has had a high performance gets increased weight in the portfolio. In order to have the same total portfolio weight as the original portfolio, all of these new portfolio weights are scaled accordingly. This procedure can be done on instrument, asset class and total portfolio levels.

3.4 Optimisation

Optimisation refers to the process of minimising a function by choosing the best possible parameter from a set of candidate choices, with respect to some criteria that the solution has to fulfill.

Convex optimisation problems have the property that a local minimum is also a global minimum. A convex optimisation problem is on the form:

\[
\begin{align*}
\text{minimise} & \quad f_0(x) \\
\text{s.t.} & \quad f_i(x) \leq b_i, \quad i = 1,\ldots,m
\end{align*}
\]

where the functions \( f_0, \ldots, f_m : \mathbb{R}^n \to \mathbb{R} \) are convex.

A subset of convex optimisation is second-order cone programming (SOCP). A second-order cone programming problem is a problem on the form:

\[
\begin{align*}
\text{minimise} & \quad f^T x \\
\text{s.t.} & \quad ||A_i x + b_i||_2 \leq c_i^T x + d_i, \quad i = 1,\ldots,m \\
F x & = g
\end{align*}
\]

where \( x \in \mathbb{R}^n \) is the optimisation variable, \( f \in \mathbb{R}^n \), \( A_i \in \mathbb{R}^{n_i \times n} \), and \( F \in \mathbb{R}^{p \times n} \). A constraint on the form:

\[ ||A x + b||_2 \leq c^T x + d \]  

(19)

where \( A \in \mathbb{R}^{k \times n} \), is called a second-order cone constraint. The second-order cone constraint corresponds to requiring the affine function \( (A x + b, c^T x + d) \) to lie in the second-order cone in \( \mathbb{R}^{k+1} \).

The usefulness of SOCP in this study stems from being able to formulate a minimisation of a linear function, subject to convex quadratic constrains as a SOCP [26].

Since optimisation is not the main focus of this thesis, we refer to the book Convex Optimization, by Boyd and Vanderberghe (2004) for further explanation [27].
4 Methodology

4.1 Data

The data set consists of data from Bloomberg, Global Financial Data, and Data-stream. The datasets collected from these services are price series of the investigated instruments. The instruments investigated are, as previously mentioned, futures contracts of equity indices and bonds. In addition to the collected futures contracts’ price series, the underlying asset of the futures contracts’ price series are collected in order to calculate the exposure of the future contracts, i.e. spot prices of the equity indices and cheapest to deliver reference bonds.

The instruments included in the research have been chosen according to two criteria. Firstly, the instruments must have price series during the investigated time period 2000-2018. Secondly, the instruments have been chosen so that the portfolio has a global exposure in accordance with popular global indices.

In Table 4 and 5, the included instruments are presented.

Table 4: Investigated futures - Equity indices

<table>
<thead>
<tr>
<th>Instrument name</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>aex</td>
<td>Netherlands</td>
</tr>
<tr>
<td>cac</td>
<td>France</td>
</tr>
<tr>
<td>canada60</td>
<td>Canada</td>
</tr>
<tr>
<td>dax</td>
<td>Germany</td>
</tr>
<tr>
<td>dow</td>
<td>USA</td>
</tr>
<tr>
<td>estoxx</td>
<td>Europe</td>
</tr>
<tr>
<td>ftse</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>hangseng</td>
<td>Hong Kong</td>
</tr>
<tr>
<td>mib</td>
<td>Italy</td>
</tr>
<tr>
<td>nasdaq</td>
<td>USA</td>
</tr>
<tr>
<td>nikkeiOSE</td>
<td>Japan</td>
</tr>
<tr>
<td>nikkeiSIM</td>
<td>Japan</td>
</tr>
<tr>
<td>omx</td>
<td>Sweden</td>
</tr>
<tr>
<td>russel</td>
<td>USA</td>
</tr>
<tr>
<td>sp</td>
<td>USA</td>
</tr>
<tr>
<td>taiwan</td>
<td>Taiwan</td>
</tr>
<tr>
<td>topix</td>
<td>Japan</td>
</tr>
<tr>
<td>singapore</td>
<td>Singapore</td>
</tr>
<tr>
<td>jse</td>
<td>South Africa</td>
</tr>
<tr>
<td>kospi</td>
<td>Korea</td>
</tr>
</tbody>
</table>
Table 5: Investigated futures - Bonds

<table>
<thead>
<tr>
<th>Instrument name</th>
<th>Region</th>
</tr>
</thead>
<tbody>
<tr>
<td>aus10y</td>
<td>Australia</td>
</tr>
<tr>
<td>aus3y</td>
<td>Australia</td>
</tr>
<tr>
<td>jgb</td>
<td>Japan</td>
</tr>
<tr>
<td>ktb3y</td>
<td>Korea</td>
</tr>
<tr>
<td>bobl</td>
<td>Germany</td>
</tr>
<tr>
<td>bund</td>
<td>Germany</td>
</tr>
<tr>
<td>schatz</td>
<td>Germany</td>
</tr>
<tr>
<td>10ynote</td>
<td>USA</td>
</tr>
<tr>
<td>2ynote</td>
<td>USA</td>
</tr>
<tr>
<td>5ynote</td>
<td>USA</td>
</tr>
<tr>
<td>tbond</td>
<td>USA</td>
</tr>
<tr>
<td>gilt</td>
<td>United Kingdom</td>
</tr>
<tr>
<td>cggb10y</td>
<td>Canada</td>
</tr>
</tbody>
</table>

4.2 Constructing portfolios

For this study, two separate experiments have been designed. One experiment where risk weighted portfolios are studied and one experiment where capital weighted portfolios are studied. For these portfolios, Sharpe ratios on a quarterly basis are calculated. With the time period of 19 years, one gets 76 observations of the quarterly Sharpe ratio for each portfolio.

For both experiments the following factors are investigated:

A: Momentum strategy on an instrument level

B: Momentum strategy on an asset class level

C: Momentum strategy on a total portfolio level

The momentum strategy used on each level is a time series momentum. Each factor is a two-level factor and the corresponding "high" (1) and "low" (-1) level of each factor for the two experiments is defined in the following way:

Table 6: Factors and levels for capital weighted portfolios

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Dynamic allocation on instrument level</td>
<td>Static allocation on instrument level</td>
</tr>
<tr>
<td>B</td>
<td>Dynamic allocation on asset class level</td>
<td>Static allocation on asset class level</td>
</tr>
<tr>
<td>C</td>
<td>Dynamic exposure target</td>
<td>Static exposure target</td>
</tr>
</tbody>
</table>
Table 7: Factors and levels for risk weighted portfolios

<table>
<thead>
<tr>
<th>1</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Dynamic allocation on instrument level</td>
</tr>
<tr>
<td>B</td>
<td>Dynamic allocation on asset class level</td>
</tr>
<tr>
<td>C</td>
<td>Dynamic risk target</td>
</tr>
</tbody>
</table>

4.2.1 Instrument level

When constructing the portfolios, first the target weights on the instrument level are set. For the equity indices the target weights are set according to the country weights of Morgan Stanley Capital International All Countries World index (MSCI ACWI), and for bonds the according to the country weights of J.P. Morgan Global Bond index (JPM Global Bond). These weights are used as the relative risk weights and capital weights on the instrument level respectively in the portfolios studied in this thesis. As the investigated instruments do not cover all countries represented in the MSCI ACWI and JPM Global Bond indices, the countries that are unaccounted for amongst the investigated instruments are excluded in this study. Additionally, some instruments are listed in the same country, e.g. SP500 and Dow Jones, and have an equal weight between them, be it risk weight or capital weight.

With these static instrument weights determined, the momentum strategy is applied to those portfolios containing factor A, making the weighting dynamic on instrument level. For each asset, the average daily return over the last 12 months are calculated and used as input in the sigmoid function. The returned signal from the sigmoid function ranges from $[0, 1]$ and is multiplied with the static target weight. When all weights have been multiplied with corresponding momentum signal, the total portfolio weight is scaled to have the same total weight as the original one. The momentum strategy used on instrument level is further described in Chapter 3.3. Examples of momentum signals for the equity index ftse and bond gilt are presented in Figure 2.
4.2.2 Asset class level

Moving on to the asset class level, the target relation between the asset classes is to be determined. For the risk weighted portfolios the weights are allocated, on the asset class level, so that 60% of the risk is equity risk and 40% is bond risk. While the weights on the asset class level for the capital weighted portfolio, are allocated so that the equity exposure is 60% and bond exposure is 40%. The weighting on the asset class level depends on whether the portfolio has dynamic allocation on the instrument level or not, i.e. factor $A$. E.g. if the portfolio includes factor $A$, the equity and bond combinations for the asset class level is calculated based on that allocation.

For the portfolios containing factor $B$, making the allocation dynamic on asset class level, a momentum strategy is implemented in the same manner as on the instrument level, and further described in Chapter 3.3.

In Figure 3, the momentum signals on asset class level for bonds and equities with static risk weighted allocation on instrument level are presented.
4.2.3 Total portfolio level

Finally, the total risk or exposure is determined. In the capital weighted portfolios, an exposure target is set to 1, which means that all capital is invested and that the portfolios are unlevered. By contrast, in the risk weighted portfolio, a risk target of 8% in annualised volatility is set in order to ensure comparable returns with the capital weighted portfolios. 8% corresponds to the average annual volatility of a 60/40 index. In other words, the risk weighted portfolios may make use of leverage. In addition, short positions are not allowed in either of the experiments.

Based on these portfolios with a static total risk/exposure, a momentum strategy is included for the portfolios including the high level of factor $C$. Yet again, in this final step of the portfolio construction procedure, the total risk/exposure and momentum signal is calculated on the actual portfolios. That is, if the portfolio has dynamic allocation on all three levels, i.e. the portfolio ABC, those allocations and the return they yield are what the momentum signal is calculated on.

The momentum strategy on the total portfolio corresponds, in the risk weighted experiment, to having a dynamic risk target, i.e. when the momentum signal for the total portfolio is strong the risk target is increased above 8% and vice versa when there is a weak momentum signal.

As the momentum signal returned is in the range of [0, 1] the signal must be adjusted
and centered around 1, in order to be used as a constant that is multiplied with the target risk. With the aim of centering the momentum signal around 1, the dynamic target risk at time $t$ is calculated as: $\text{target risk} = 0.08 \cdot (0.5 + \text{momentum signal}_t)$.

In the capital weighted experiment, the total exposure of the portfolio is adjusted according to the momentum signal on the portfolio level. In this experiment, compared with the risk weighted, the exposure cannot exceed 1, that is the portfolio cannot use leverage. A strong signal is thus ignored, while a weak momentum signal will decrease the total exposure of the portfolio. A signal over or equal to 0.5 is considered to be strong, which corresponds to the case when the average monthly return over the last 12 month is non-negative. Consequently a weak signal is when the signal is below 0.5, i.e. when the average monthly return over the last 12 month is negative.

The aim is to have a target exposure which can avoid the large negative returns during financial crises but yet be able to ride the wave in times of high returns. This leads to the following formula, centered around 1, for retrieving the dynamic target exposure at time $t$: $\text{exposure} = 0.5 + \text{momentum signal}_t$.

In Figure 4, momentum signals for the risk weighted portfolio with dynamic risk target, portfolio C, is presented as an example on how the momentum signal is used. The illustrated portfolio has static weighting on instrument and asset class level.

Figure 4: Example of momentum signals on total level for portfolio C
4.2.4 Design scheme

As three factors are studied, it is possible to create two full factorial designs, since $2 \cdot 2^3 = 16$ portfolios are feasible to construct. The following design matrix is constructed:

Table 8: Design matrix for portfolios

<table>
<thead>
<tr>
<th>Notation</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>A</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>B</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>AB</td>
<td>1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>(1)</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>AC</td>
<td>1</td>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>BC</td>
<td>-1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

In Table 17 and 18 in the appendix, a more detailed overview of the constructed portfolios is presented.

4.2.5 Evaluation

From the created portfolios, quarterly Sharpe ratios are calculated. A time period of 19 years yields 76 quarterly observations for each portfolio, and thus a total of 608 observations of the Sharpe ratio for the 8 portfolios constructed in each of the two experiments.

In each set-up, i.e. with either risk weighted or capital weighted portfolios, the effects of the different strategies are calculated and the residuals from the factorial designs are analysed. Lastly, paired t-tests are performed between the Sharpe ratios of the risk weighted and capital weighted portfolios to detect if there is a significant difference in performance between risk and capital weighting, which would have implications for the conclusions about how risk parity and volatility targeting impacts the Sharpe ratio.

4.3 Optimisation set-up

To conclude on which level an allocation strategy would be the most impactful if applied perfectly, a deterministic optimisation approach is used. Assuming that everything that will happen in the future is known, it is possible to adjust portfolio holdings accordingly. Three different set-ups are constructed, where it is possible to adjust either the total risk, the risk weighting of the asset classes or the risk weighting of each single instrument. Each of these portfolio optimisations are based on the risk weighted portfolio with static allocation on all levels, i.e. portfolio (1). Thus, the same time period is investigated for this optimisation problem as in the previous experiment, i.e. 2000-2018.
For every set-up, an optimisation problem is formulated. The objective is to maximise returns. To make the problem related to the real world, some constraints are formulated.

The optimisation on the portfolio level is formulated as follows in Equation (20):

\[
\begin{align*}
\text{minimise} \quad & -a^T u \\
\text{s.t.} \quad & |a_i - a_{i-1}| \leq 0.05, \\
& 0.6 \leq a_i \leq 1.4 \\
& (a_i - a_{i-1})^T G_i (a_i - a_{i-1}) \leq \text{tolerance} \quad \forall \ i = 2,...,N
\end{align*}
\]

where \(a \in \mathbb{R}^{N \times 1}\) is the optimisation parameter, \(u \in \mathbb{R}^{N \times 1}\) the return vector of the portfolio (1), \(G_i\) the variance of the return of portfolio (1) at day \(i\) which is a constant and corresponds to an annualised risk of 8\%, and \(N\) the number of days in the sample.

As aforementioned, to make the optimisation problem somewhat related to real conditions, constraints are set such. The first constraint corresponds to not allowing the optimisation to change the allocation by more than 5\% in one day in each instrument compared to portfolio (1). In addition, constraints are set such that the total weights of the portfolio is between 0.6 and 1.4, compared to the portfolio (1). The final constraint is that the change in daily variance must be less than a certain tolerance level. The logic behind including this constrained change in daily variance is to create more reasonable results, as portfolio managers seldom make drastic changes in risk allocation from day to day.

The problem in Equation (20) is solved on a sequential quarterly basis, starting with the first quarter in the investigated period. The reason for solving this on a quarterly basis is because of computational efficiency. The optimal solution from the previous quarter is used as starting value for the following quarter. For the first quarter, a constraint of \(a_1 = 1\) is used.

For the optimisation problems on asset class and instrument level, the problem is defined as Equation (21) below:

\[
\begin{align*}
\text{minimise} \quad & -\sum_{i=1}^{N} a_i^T u_i \\
\text{s.t.} \quad & |a_{ij} - a_{i-1,j}| \leq 0.05, \\
& 0.6 \leq a_{ij} \leq 1.4 \\
& (a_i - a_{i-1})^T G_i (a_i - a_{i-1}) \leq \text{tolerance} \quad \forall \ i = 2,...,N, \\
& \quad j = 1,...,M
\end{align*}
\]

where \(a_{ij}\) is the optimisation parameter for asset class/instrument \(j\) on day \(i\), \(u_{ij}\) is the corresponding daily normalised return on day \(i\) for asset \(j\), \(G_i \in \mathbb{R}^{[k]}\) the covariance matrix of returns for all assets on day \(i\). \(N\) is the number of days in the sample and \(M\) the number of assets.
For each optimisation setup, 11 optimisations are computed, each with different tolerance levels of the change in daily variance to see on which level a difference in the daily change of variance has the largest effect. The tolerance in change in daily variance ranges from $10^{-8}$ to $10^{-3}$. To solve the optimisation problems, the toolbox Yalmip is used [28].

The meticulous reader might have noticed that the third constraint in Equations 20 and 21 does not equal the daily change in variance, which would be defined as

$$
\sigma^2_i - \sigma^2_{i-1} = a_i^T C_i a_i - a_{i-1}^T C_{i-1} a_{i-1}
$$  (22)

Since Equation 22 does not form a convex optimisation constraint, we define the average of the two covariance matrices, $C_i$ and $C_{i-1}$, as $G_i$ and use the triangle inequality to get a convex optimisation problem. We assume that the covariance between to adjacent days will not change drastically.

The reverse triangle inequality gives the following:

$$
\left| \sqrt{a_i^T G_i a_i} - \sqrt{a_{i-1}^T G_{i-1} a_{i-1}} \right| \leq \sqrt{(a_i - a_{i-1})^T G_i (a_i - a_{i-1})} \leq \sqrt{\text{tolerance}} \quad (23)
$$

The right hand side in the inequality in Equation 23 has the properties of being a convex constraint and is used as an upper limit for how much the daily variance can be changed. The following constraint on the change in daily variance is then retrieved:

$$
\sqrt{(a_i - a_{i-1})^T G_i (a_i - a_{i-1})} \leq \sqrt{\text{tolerance}} \quad (24)
$$

$$
\Rightarrow (a_i - a_{i-1})^T G_i (a_i - a_{i-1}) \leq \text{tolerance}
$$
5 Results

Initially, the return, risk and exposure of the risk and capital weighted portfolios are analysed. Furthermore, by calculating quarterly Sharpe ratios for the risk and capital weighted portfolios, an evaluation of the influence of the factors - dynamic instrument allocation, dynamic asset allocation and dynamic risk/exposure target - on the Sharpe ratio is studied. A comparison between the risk and capital weighted portfolios is also performed. Since bonds have enjoyed a period of decreasing rates and high returns, a scenario analysis is performed to study how robust the results are. Lastly, the results of the optimisation set-up are presented.

5.1 Risk weighted portfolios

The cumulative returns of the eight portfolios are shown in Figure 5. The ranking of the portfolios’ performance remains relatively stable, where the largest gap in performance is between 2002-2011. Looking closer to the period 2001-2004 in Figure 6 it is clear that the portfolios with dynamic risk allocation between asset classes are superior. This outperformance of portfolios with this factor against the other portfolios remains but becomes less salient during the end of the investigated period. The two portfolios with the highest cumulative returns are B and BC, i.e. the portfolio where only allocation between asset classes is dynamic, and the portfolio where the allocation between asset classes and the total risk level are both dynamic.

Figure 5: Cumulative returns of risk weighted portfolios
In Figure 7 the risk and underlying exposure for each portfolio is shown. The exposure is calculated by using the underlying equity indices for the equity portion and the cheapest to deliver bond for the bond exposure. Due to equities being much more volatile, bonds are leveraged in all portfolios in order to account for 40% of the portfolio volatility. Thus, the portfolio is leveraged and the exposure is above 1. Note that the risk is stable on 8% for the portfolios who do not include factor $C$, dynamic risk target. In the portfolios including factor $C$, we see that the risk is scaled down in 2008-2009 which were the turbulent times around the global financial crisis.
Figure 7: Risk and exposure levels for risk weighted portfolios

(a) ABC  
(b) A  
(c) B  
(d) C  
(e) (1)  
(f) AB
Looking at Figure 8 (a) we again see that factor $B$ seems to have the most positive effect on the return, which is in line with the cumulative return plot. However, this result is not significant on the widely used 95% confidence level, according to the ANOVA in Table 9. Looking at Figure 8 (b), the two-level interaction effects, we note that the effect seem to have a very small impact, and yet again, none of these effects are significant on the 95% confidence level either.

Table 9: ANOVA for risk weighted portfolios

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.17</td>
<td>0.17</td>
<td>0.16</td>
<td>0.6856</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.27</td>
<td>0.27</td>
<td>0.26</td>
<td>0.6074</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.16</td>
<td>0.16</td>
<td>0.15</td>
<td>0.6942</td>
</tr>
<tr>
<td>A*B</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9764</td>
</tr>
<tr>
<td>A*C</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9560</td>
</tr>
<tr>
<td>B*C</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9997</td>
</tr>
<tr>
<td>A<em>B</em>C</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9996</td>
</tr>
<tr>
<td>Residuals</td>
<td>600</td>
<td>613.48</td>
<td>1.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 8: Factor effects on the risk weighted portfolios

(a) Main effects

(b) Two-level interaction effects
Table 10: Factor effects on the risk weighted portfolios

<table>
<thead>
<tr>
<th>Factor</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.0166</td>
</tr>
<tr>
<td>B</td>
<td>0.0211</td>
</tr>
<tr>
<td>C</td>
<td>-0.0161</td>
</tr>
<tr>
<td>A*B</td>
<td>-0.0012</td>
</tr>
<tr>
<td>A*C</td>
<td>0.0023</td>
</tr>
<tr>
<td>B*C</td>
<td>0.0000</td>
</tr>
<tr>
<td>A<em>B</em>C</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Figure 9: Residual analysis of the risk weighted portfolios

Finally, the residuals are studied. In the QQ-plot, the left tail seems to be slightly lighter than the normal distribution. The histogram and the Box plot do not seem to be in conflict with the normality assumption. In the ”Prediction vs Residual” plot in Figure 9, the pattern is repeating itself for every 76:th observation. This is because each of the 8 portfolios has 76 observations and they are ordered one after another, and since the portfolios are not too different the Sharpe ratios will not differ too much from the others at the same time point.
5.2 Capital weighted portfolios

In Figure 10, the cumulative returns of the capital weighted portfolios are presented. During the financial recessions in 2002 and 2008, we note that the portfolios using target exposure (factor $C$) have a higher return than the portfolios not having target exposure. Interesting to note is that the portfolio performing best during this time period investigated is the portfolio only containing a high level of $C$, i.e. the portfolio with dynamic exposure target.

Figure 10: Cumulative returns of capital weighted portfolios
In Figure 11, a closer view of the period 2007-2010 is shown. During this period, it is evident that portfolios including factor B manage to avoid the largest drawdowns. Furthermore, the portfolio with the worst performance during the period is portfolio A, which only includes a momentum strategy on the instrument level. This indicates that the momentum estimate was not particularly accurate on the instrument level during this period.

For the capital weighted portfolios, the volatility is calculated, like for the risk weighted portfolios, with an exponential weighted moving average with $a = 0.99$. From Figure 12, we note that the risk of the capital weighted portfolios mainly consists of equity risk while the bond risk is fairly low. Furthermore, the risk for all portfolios are the highest in late 2008/early 2009, which corresponds to the recession that occurred. The annualised volatility for portfolio (1) spiked to 60% during this time.

The total exposure level for the portfolios with target exposure (factor C) has lowered exposure during the economically difficult times around 2002 and 2008 when the momentum signals indicate poor performance. During these times, the portfolios with dynamic asset class allocation tend to have a higher exposure to bonds and lower exposure to equities.
Figure 12: Risk and exposure levels for capital weighted portfolios

(a) ABC

(b) A

(c) B

(d) C

(e) (1)

(f) AB
Turning the attention to analysing the impact of the factors on the Sharpe ratios with the ANOVA table presented in Table 11. From the ANOVA table we see that factor $B$, dynamic asset allocation, has the largest effect on the Sharpe ratio and that factor $C$, dynamic target exposure, has the second largest effect. However, none of these factors are close to significant on the typical 95% confidence level. Factor $A$, dynamic allocation on instrument level, and the interaction effects do not have an impact on the Sharpe ratio.

Table 11: ANOVA for capital weighted portfolios

<table>
<thead>
<tr>
<th>Factor</th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1</td>
<td>0.11</td>
<td>0.11</td>
<td>0.09</td>
<td>0.7593</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.25</td>
<td>0.25</td>
<td>0.22</td>
<td>0.6376</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.8162</td>
</tr>
<tr>
<td>A*B</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.9270</td>
</tr>
<tr>
<td>A*C</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.9262</td>
</tr>
<tr>
<td>B*C</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9703</td>
</tr>
<tr>
<td>A<em>B</em>C</td>
<td>1</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.9098</td>
</tr>
<tr>
<td>Residuals</td>
<td>600</td>
<td>686.24</td>
<td>1.14</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The magnitude and sign of the effects are presented in Figure 13 and Table 12. The sign of all the main effects are negative and as previously presented factor $B$ has the largest impact on the Sharpe ratio. The interaction effects have a negligible impact.
Figure 13: Factor effects on the capital weighted portfolios

(a) Main effects

(b) Two-level interaction effects
Table 12: Factor effects on the capital weighted portfolios

<table>
<thead>
<tr>
<th>Factor Effect</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-0.013297</td>
</tr>
<tr>
<td>B</td>
<td>-0.020441</td>
</tr>
<tr>
<td>C</td>
<td>-0.010085</td>
</tr>
<tr>
<td>A*B</td>
<td>0.003974</td>
</tr>
<tr>
<td>A*C</td>
<td>-0.004017</td>
</tr>
<tr>
<td>B*C</td>
<td>-0.001617</td>
</tr>
<tr>
<td>A<em>B</em>C</td>
<td>-0.004917</td>
</tr>
</tbody>
</table>

Figure 14: Residual analysis of the capital weighted portfolios

The model assumptions are tested by studying several residual graphs; QQ-plot, Box plot, histogram and a "Prediction vs Residual" run order plot, which can be found in Figure 14. No serious violation of the normality assumption is detected. The residuals are centered around zero and show no time dependent patterns. As for the risk weighted portfolios, in the "Prediction vs Residual" plot, the quarterly Sharpe ratios have the same recurring pattern. In addition, the predicted Sharpe ratios are similar for all of the portfolios since the portfolios are rather very much
5.3 Comparison

To answer the question on what impact risk parity and target volatility strategies, compared with a classical capital weighted allocation, have on the Sharpe ratio of a futures portfolio, paired t-tests are performed. This is to see if the paired observations come from the same population, by studying the difference between the paired observations.

In Table 13, the results of the eight t-tests are presented. For each test, the differences between the capital weighted portfolio’s Sharpe ratios and the corresponding risk weighted portfolio’s Sharpe ratios are tested. The null hypothesis, $H_0$, is for every test that there is no difference in Sharpe ratio between the portfolios.

Table 13 shows that there is on a 75% confidence level for all portfolios a difference between the populations. For all portfolios containing factor $B$, the difference is on a 90% confidence level.

The difference in mean reveals that risk weighted portfolios are favorable to investors. On average, the risk weighted portfolios get between 0.12-0.24 higher Sharpe ratios than the capital weighted ones.
Table 13: Paired T-tests between risk and capital weighted portfolios’ Sharpe ratio

<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>df</th>
<th>p</th>
<th>Mean of differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ABC</td>
<td>-1.900</td>
<td>75</td>
<td>0.061</td>
<td>0.213</td>
</tr>
<tr>
<td>(b) A</td>
<td>-1.386</td>
<td>75</td>
<td>0.170</td>
<td>0.139</td>
</tr>
<tr>
<td>(c) B</td>
<td>-2.029</td>
<td>75</td>
<td>0.046</td>
<td>0.238</td>
</tr>
<tr>
<td>(d) C</td>
<td>-1.180</td>
<td>75</td>
<td>0.242</td>
<td>0.120</td>
</tr>
<tr>
<td>(e) (1)</td>
<td>-1.379</td>
<td>75</td>
<td>0.172</td>
<td>0.138</td>
</tr>
<tr>
<td>(f) AB</td>
<td>-1.827</td>
<td>75</td>
<td>0.072</td>
<td>0.199</td>
</tr>
<tr>
<td>(g) AC</td>
<td>-1.243</td>
<td>75</td>
<td>0.218</td>
<td>0.127</td>
</tr>
<tr>
<td>(h) BC</td>
<td>-1.759</td>
<td>75</td>
<td>0.083</td>
<td>0.207</td>
</tr>
</tbody>
</table>

5.4 Scenario analysis

A possible reason for factor B to be dominant in the experiments above could be that bonds as an asset class have enjoyed a period of decreasing rates and high returns during the examined time period, making them superior to equities. Another
A scenario which is interesting to investigate is if factor $B$ would still have the largest impact of all factors if both asset classes had the same total return during the period. To create this scenario we simply set the total cumulative normalised return of bond futures portfolio weighted according to JPM Global Bond Index equal to the total cumulative normalised return of an equity index futures portfolio weighted according to MSCI ACWI index for the period 2000-2018.

Running the same experiments as in our original scenario, with these adjusted bond returns, produces analogous results. Factor $B$ still has the greatest impact on Sharpe, seen in the main effects plots in Figure 15 and 16. The Sharpe overall is slightly lower because of the new lower returns for the bond futures.

It is interesting to note that the Sharpe ratio of the capital weighted portfolios seems to be more severely harmed than that of the risk weighted portfolios. Looking at the ANOVA result in Table 15, the factor $B$ is also more significant for capital weighted portfolios in this setting with adjusted bond returns than in the original experiment.

Figure 15: Factor effects on the risk weighted portfolios - adjusted bond returns
Figure 16: Factor effects on the capital weighted portfolios - adjusted bond returns

Table 14: ANOVA Risk weighted - adjusted bond returns

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
<td>0.6724</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>0.27</td>
<td>0.27</td>
<td>0.26</td>
<td>0.6108</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
<td>0.6901</td>
</tr>
<tr>
<td>A*B</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9728</td>
</tr>
<tr>
<td>A*C</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9606</td>
</tr>
<tr>
<td>B*C</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9997</td>
</tr>
<tr>
<td>A<em>B</em>C</td>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9998</td>
</tr>
<tr>
<td>Residuals</td>
<td>600</td>
<td>614.12</td>
<td>1.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 15: ANOVA Capital weighted - adjusted bond returns

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum Sq</th>
<th>Mean Sq</th>
<th>F value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.05</td>
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</tr>
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<td>B</td>
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<td>0.69</td>
<td>0.69</td>
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<td>0.4447</td>
</tr>
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<td>C</td>
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<td>0.07</td>
<td>0.06</td>
<td>0.8129</td>
</tr>
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<td>0.02</td>
<td>0.02</td>
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</tr>
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<td>0.01</td>
<td>0.9413</td>
</tr>
<tr>
<td>B*C</td>
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<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.9738</td>
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<td>0.01</td>
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<td>0.9249</td>
</tr>
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<td>Residuals</td>
<td>600</td>
<td>711.86</td>
<td>1.19</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Studying Table 16, the risk weighted portfolios again have a higher mean Sharpe
ratio than the capital weighted portfolios. The tests also have a higher significance level than previously as well when compared to Table 13.

Table 16: Paired T-tests between risk and capital weighted portfolios’ with adjusted bond returns Sharpe ratio

<table>
<thead>
<tr>
<th></th>
<th>t</th>
<th>df</th>
<th>p</th>
<th>Mean of differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) ABC</td>
<td>-2.044</td>
<td>75</td>
<td>0.044</td>
<td>0.256</td>
</tr>
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<td>(b) A</td>
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5.5 Optimisation

When conducting the optimisation, 11 portfolios on each level are optimised, each with different constraints. The constraint which is relaxed is the allowed change in daily variance, where the tolerance goes from $10^{-6}$ to $10^{-3}$.

Figure 17: Plot of Sharpe ratios against tolerance in daily change of risk

![Plot of Sharpe ratios against tolerance in daily change of risk](image)

Note that Figure 17’s x-axis denotes risk, i.e. the square root of the variance, the lowest tolerance of $10^{-6}$ thus corresponds to a tolerance in daily change of risk of $10^{-3}$.

Viewing the results, we observe that for low tolerances in change of daily risks, optimising on the different levels yield similar Sharpe ratios. Optimising on the instrument level soon becomes the best alternative. This is in line with expectations as the possibility to change weighting between all 33 instruments provides more freedom and the ability to avoid drawdowns increases with the amount of options. However, optimising on the portfolio and asset class levels remain close in terms of Sharpe ratio even as the tolerance in daily change of risk increases.

Ideally, we would like to isolate the effect of changing allocation on only one level, i.e. instrument, asset class, or total portfolio level, at a time. This is unfortunately not possible in this optimisation setting as the constraint needed in that case would cause the set of solutions to be infeasible.
In this setting, as the optimisation is conducted on for example the asset class level, the total portfolio risk level may also be altered. Thus not isolating the effect of only changing the allocation between asset classes while keeping the other aspects of the portfolio constant. As a remedy for this issue, and to increase comparability, after the optimisation is conducted adjustments are made such that the portfolio risk is constant for the optimisations on asset class and instrument level. For the optimisation on instrument level, the 60/40 risk allocation between equity indices and bond futures is also restored after optimising. These adjustments do not have to be done for the optimisation on the total portfolio level as both the allocation between asset classes and instruments is preserved in that problem setting. However, as this version of the problem changes the total risk of the portfolios, an additional constraint is set such that the average annualised risk is below 8%.

Inspecting Figure 18 where the risks on asset class and instrument level have been adjusted after the conducted optimisation, we see that the superiority of optimising on the instrument level has diminished. Furthermore, we note that optimal tactic allocation on portfolio level gives a higher Sharpe ratio than on instrument level.

Figure 18: Plot of adjusted Sharpe ratios against tolerance in daily change of risk

Note that the x-axes in Figure 17 and 18 are the tolerances in daily change of risk which correspond to the optimisation constraint, and it is not necessary that it is the same as the realised daily change of risk. To view how the different portfolios in the optimisation settings utilise their different constraints, the realised annual risk turnover is calculated. In Figure 19 and 20 the x-axes correspond to the realised
annualised risk turnover.

Figure 19: Plot of Sharpe ratios against annualised risk turnover

![Plot of Sharpe ratios against annualised risk turnover](image1)

Figure 20: Plot of adjusted Sharpe ratios against annualised risk turnover

![Plot of adjusted Sharpe ratios against annualised risk turnover](image2)

From Figure 19 and 20, we note that the portfolios optimising on the instrument level utilise the tolerance in change of daily variance the most, as these portfolios have the highest turnover. Moreover, Figure 19 and 20 reveal that the portfolios optimised on the asset class level have the highest Sharpe ratio in relation to the annualised risk turnover. An adjustment is made such that the portfolios optimised on instruments and asset classes have the same daily volatility. The portfolios opti-
mised on the instrument level are also adjusted such that the 60/40 risk allocation between asset classes is constant. After these adjustments, instrument level optimisation has the lowest Sharpe ratio in relation to the annualised risk turnover, as seen in Figure 20.
6 Discussion

6.1 Portfolio discussion

Returning to the results for the risk weighted portfolios in Chapter 5.1, factor \( B \), allocation on the asset class level, has the largest impact on Sharpe ratio. The other factors, \( A \) and \( C \), have a negative impact on the Sharpe ratio. None of the effects are significant on a 95% confidence level. This lack of significance is not surprising, as arbitrage theory and the efficient market hypothesis would induce that the past should not be able to predict the future, and thus should momentum strategies based on past returns not yield statistically significant improvements in return [29]. In addition, financial markets are often noisy and considered to be random walks, which would also imply that statistical significance is seldom present. These results with a low statistical significance should however not be rendered meaningless.

The capital weighted portfolios on the other hand have the highest cumulative return when factor \( C \), dynamic exposure target, is included. However, when inspecting Figure 13, we see that this factor has a negative impact on the Sharpe ratio. This implies that the higher cumulative return comes at a cost of higher risk, lowering the Sharpe ratio. The effects of the capital weighted portfolios are as for the risk weighted not significant on a 95% confidence level.

From Figure 7 we note that the exposure levels for the risk weighted portfolios are fairly large. Some risk weighted portfolios (C, AB, AC, BC) are on some occasions leveraged as high as 14 times. For an investor, leverage of this magnitude can be uncomfortable, but leverage is needed in order to maintain a target volatility of the portfolio which in turn is set to ensure a return comparable to an unlevered capital weighted portfolio.

Comparing the results of the risk and capital weighted portfolios, it would be reasonable to expect that the factor effects would have the same sign and order of magnitude. The order of magnitude is similar, with factor \( B \) being far largest for both experiment settings. The sign of the effect of factor \( B \) however differs. This might result from when the momentum signal indicates to increase the weight of bonds, the risk weighted portfolios usually lever up as they buy more bond to maintain the desired total risk level. While the capital weighted portfolios rather decrease the allocation in equities to increase the weight in bonds.

For both the risk weighted and capital weighted portfolios, the interaction effects are very small in comparison with the main order effects. The interaction effects are not close to being as significant as the main order effects. This is in line with Montgomery’s sparsity of effects property [16].

The comparison of the Sharpe ratios of the risk and capital weighted portfolios, performed by doing paired t-tests, shows that the Sharpe ratios of all risk weighted portfolios are superior to the capital weighted ones. The reason for the better performance of risk weighted portfolios on the Sharpe ratio could be because of the excellent bond returns caused by decreasing rates during the investigated time pe-
period. As it is unreasonable to assume that interest rates will continuously decrease, an analysis is performed to study if the result is unaltered in a scenario where the return of bonds are not superior to that of equity indices.

The results from the scenario analysis, where the bonds’ return is adjusted to have the same return as that of equity indices, show that factor $B$ has the largest effect for both the capital and risk weighting, as in the original experiment. In addition, the results from the scenario analysis confirm that risk weighted portfolios yield higher Sharpe ratio than the capital weighted ones. From these results of the scenario analysis, it is reasonable to conclude that the results from the original study seem robust even if bonds would experience a period of lower returns.

6.2 Optimisation discussion

Turning to the optimisation problem where we assume that we had complete information about future returns and studying on which level allocation strategies have the largest effect. In reality, these portfolios created with optimisation are of course impossible to realise as we cannot prophesy the future. However, it still offers some insight into what a portfolio manager should focus on.

Optimising on the instrument level results in the highest Sharpe ratio, for larger tolerances in change in daily variance. Furthermore, optimising on asset class level gives higher Sharpe ratio than on total portfolio level when relaxing the constraint on change in daily variance. However, the task for a portfolio manager to research and keep updated on an instrument level is more demanding than focusing on only the allocation between asset classes. The pay-off when having future information about 33 instruments does not give a result 33 times better when having only future information on total portfolio level. Thus, in the real world, it might still be optimal to focus on the higher levels of portfolio management, asset classes and total risk exposure, where the chances of getting it right might be higher.

When looking at the Sharpe ratios in relation to the annualised risk turnover, allocating on the asset class level is superior, followed by allocating on the total portfolio level. This also supports that allocating on the asset class level is the best choice in reality, as the high turnover in risk for the portfolios optimised on the instrument level would have significantly higher transaction costs.

The data used for the optimisation problem is the same as for the experimental design, and one could thus suspect that the results would be fairly aligned. The results from the optimisation set-up adjusted to isolate the effects of allocating on different levels also show that the largest impact on the Sharpe ratio is on asset class level, confirming the results in the experimental design set-up.

6.3 Possible sources of error

The assumptions and restrictions in this study reduce the applicability of the results to be generalised and may have lead to results that are not able to answer the research questions properly.
Typical errors when using design of experiments are human errors such as mistakes in data entry and mistakes in the design of the experiment itself [16]. These error sources are believed to be minimal in this study as all data is provided by financial data services which are believed to have high quality of data. If none of the factors investigated would have an impact on the Sharpe ratio, it could indicate that the experiment was incorrectly designed with irrelevant factors. Assuming that the factors included are indeed relevant, the order of cause and effect is clear in the experiments and would not cause an error.

Studying the past and finding relationships in historic data do not guarantee that the same relationships will continue to exist in the future. This study includes a rather long time period of 19 years. Since a long time period includes many events and different market conditions, this increases the chance of the results being accurate. However, since bonds as an asset class have had such strong performance during the investigated period, it is not certain that the long period is enough to create robust results.

The conclusions whether momentum strategies have an impact, and the magnitude of the impact depends on the momentum strategy used. The conclusions in this thesis rely on the rather naive measure of momentum as a 12 month time series momentum. Some other momentum strategy could create different results.

6.4 Future work

For researchers and investment managers interested to study this area, the authors of this thesis recommend including different momentum strategies and including other time periods than studied in this thesis to omit the possible sources of error discussed in Chapter 6.3. For example cross-section momentum could be used instead of time series momentum, and a time period with increasing interest rates and low bond returns could be included in the data.

Furthermore, an alternative research design to design of experiments could be utilised and a different measure than Sharpe ratio could be used to evaluate the results. Looking away from futures contracts, this study could be replicated using different financial assets.
7 Conclusion

Risk parity and target volatility strategies have a positive effect on the Sharpe ratio of a futures portfolio, compared to a standard capital weighted 60/40 allocation. This is in part due to the role of leverage that comes with risk parity and targeting volatility strategies. In addition, these results are consistent even in a period where the return of bonds have been reduced.

The answer to the second part of the research question, how momentum investing strategies on instrument, asset class, and total portfolio level impact the Sharpe ratio of a futures portfolio is somewhat conflicting. In the two design of experiments settings, dynamic allocation between asset classes has the largest effect. These effects are however different in direction, where risk weighted portfolios benefit and capital weighted portfolios are harmed by including this factor. In the complementary optimisation setting, dynamic allocation on asset class level has the most positive impact on Sharpe ratio, in relation to turnover, confirming the results from the experimental designs.

Interaction effects between momentum strategies have negligible effects, both for capital and risk weighted portfolios.

From these results we conclude that for portfolio managers it is most reasonable to focus their efforts on the asset class level. Since such efforts both generate a higher Sharpe ratio and since it requires less research effort to get the right allocation between two asset classes than between more than ten times as many instruments.
References


## Appendix

Table 17: Constructed risk weighted portfolios

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Table 18: Constructed capital weighted portfolios

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Figure 21: QQ-plots of the Sharpe ratios from the risk weighted portfolios
Figure 22: QQ-plots of the Sharpe ratios from the capital weighted portfolios
Figure 23: ACF-plots of the Sharpe ratios from the risk weighted portfolios
Figure 24: ACF-plots of the Sharpe ratios from the capital weighted portfolios