Development of Active Rear Wheel Steering and Evaluation of Steering Feel

SUVANSH KASLIWAL
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Masters Thesis in Vehicle Engineering
TRITA-SCI-GRU 2019:NN
Department of Aeronautical and Vehicle Engineering
KTH Royal Institute of Technology

Cover: Rimac C_TWO

Typeset in \LaTeX
Printed by [Name of printing company]
Stockholm, Sweden 2019
Abstract

In the last few years, with the development of sensor and actuator technology along with increased computation power available on-board of vehicles, the automotive industry is employing more and more mechatronic systems for Advanced Driver Assistance Systems (ADAS) and Autonomous Driving (AD). Driver Assistance Systems are being used to increase safety (eg. Electronic stability program, lane keep assist etc.), reduce drive fatigue (eg. Electronic power steering) and of increasing vehicle performance and handling (eg. torque vectoring).

This thesis explores one such driver assistance system, the Active Rear Wheel steering (ARWS) system, which is capable of increase the stability and handling of the vehicle at high speeds and reduce driver fatigue at very low speeds (such as parking manoeuvre). The thesis starts by discussing the history and present state of art of ARWS systems and the control algorithms used for it. Then, effort is put in to develop tests and objective metrics to evaluate the performance of the system compared to a passive vehicle. These metrics are of importance in situations where subjective driver feedback is either not available at all (such as computer simulations) or when data is needed to back up the driver feedback (inexperienced drivers). These objective metrics can help the design engineer to evaluate and even predict vehicle’s performance during the design and tuning phase.

The thesis then defines how the ARWS system should benefit the handling of the vehicle along with certain undesired behaviour that may arise due to ARWS and should be avoided. This was done based upon feedback from experienced drivers and engineers along with inputs from various literature.

The Sliding Mode Control (SMC) algorithm is chosen for the control of ARWS system due to it relative simplicity and robust performance. The SMC theory is presented and then the controller is developed along with the reference model. The controller is then put in a Software-in-Loop environment with IPG CarMaker and put through various test scenarios for tuning and evaluation purposes. The results suggest that the system improves the vehicle’s handling.

The last part of the thesis looks into the steering feel and its objectification along with how the ARWS system influences the steering feel compared to that of a passive vehicle.
Sammanfattning

Under de senaste åren har bilindustrin använt mer och mer mekatroniska system för avancerade förarstödsystem (ADAS) och autonom körning (AD) med utveckling av sensor- och aktuerteknik tillsammans med ökad beräkningskraft som finns tillgänglig i fordonen. Förarstödsystem används för att öka säkerheten (t.ex. elektroniskt stabilitetsprogram, körfältassistans etc.), minska förarutmattning (t.ex. elektronisk servostyrning) och öka fordonens prestanda och köregenskaper (t.ex. vridmomentvektorering).

Avhandlingen utforskar ett sådant system för förarhjälp, vilket kan öka fordonets stabilitet och hantering vid höga hastigheter och minska förarens utmattning vid mycket låga hastigheter (t.ex. parkeringsmanövrering), ARWS-systemet (Active Rear Wheel steering). Avhandlingen börjar med att diskutera ARWS-systemets historia och nuvarande teknik samt de regleralgoritmer som används för det. Därefter utvecklas test och objektiva mätvärden för att utvärdera systemets prestanda jämfört med ett passivt fordon. Dessa mätvärden är viktiga i situationer där subjektiv föraråterkoppling inte är tillgänglig alls (t.ex. datorsimuleringar) eller när data behövs för kopplingen (örfarna). Dessa objektiva mätvärden kan hjälpa designingenjören att utvärdera och till och med förutsäga fordonets prestanda under design- och utvärderingsfasen.

Avhandlingen definierar hur ARWS-systemet ska gynna hanteringen av fordonet tillsammans med vissa önskade beteenden som kan uppstå på grund av ARWS och som bör undvikas. Detta gjordes baserat på feedback från erfarna förare och ingenjörer tillsammans med ett antal uppslag ur litteratur.


Den sista delen av avhandlingen undersöker styrkänslan och dess objektifiering tillsammans med hur AWRS-systemet påverkar styrkänslan jämfört med hos passiva fordon.
Acknowledgement

First and foremost, I would like to thank Rimac Automobili d.o.o for providing me with such an opportunity and the resources required to complete my thesis. I would like to give my genuine and earnest thanks to my thesis supervisor at Rimac, Mr. Tomislav Šimunić (Senior Vehicle Dynamics Engineer and Chassis team lead, Rimac Automobili) for his constant support and guidance. I truly believe that this project wouldn’t have been possible without his insights and inspiration. He consistently steered me in the right direction and provided me with all of the opportunities to successfully compile this thesis work.

I would like to extend my gratitude towards my colleagues at Rimac for their constructive feedback and support. Especially, Mr. Damiano Cozza for his experienced insights and Mr. Sidharth Malik for letting me bounce hundreds of ideas off him. Their technical and practical expertise helped me at every stage of my thesis. I would also like to thank Mr. Miroslav Zrnčević for providing me with his inputs to various aspects of this thesis through a test driver’s eye. A special thanks goes to Mate Rimac for welcoming me into his company and for some amazing parties.

I would like to express my very sincere gratitude to my academic supervisor Mr. Lars Drugge (Associate professor, Department of Aeronautical and Vehicle engineering, KTH Royal Institute of Technology) for assisting me during the whole duration of my Master’s studies, and in particular for his valuable feedback and suggestions for the objective metrics and steering feel part of this thesis.

This thesis wouldn’t have been complete without the constant support of my parents and friends. I am deeply grateful for their unconditional love and support throughout the years.

And lastly, I am grateful to the dogs at Rimac’s office who proved to be the perfect stress busters, they really made some long days at office easy.
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1 Introduction

This master’s thesis describes the development of an Active Rear Wheel Steering system of a high performance car. The report explains the work process carried out to execute this thesis. Chapter 1 provides a brief background of the thesis along with introduction of the company and the car on which the thesis is based. Literature review and theory behind the various aspects of the thesis is explained in Chapter 2. Chapter 3 details with the test, performance and evaluation metrics used in this thesis to evaluate and differentiate the performance of the developed system from the passive vehicle. This is followed by Chapter 4 and Chapter 5 which explain the theory behind the control algorithm chosen and its implementation respectively. Finally Chapter 6 provides explanation about the simulation and tuning process and discusses results in detail followed by conclusions which are drawn in Chapter 7. The scope for future work is discussed in Chapter 8.

1.1 Motivation and Background

In last few years, with the development of sensor and actuator technology along with increased computation power available on-board of vehicles, the automotive industry is employing more and more mechatronic systems for Advance Driver Assistance Systems (ADAS) and Autonomous Driving (AD). Driver Assistance Systems are mainly being used to increase safety (Eg. Electronic stability program, lane keep assist etc.), reduce drive fatigue (Eg. Electronic Power Steering) and of increasing vehicle performance and handling (Eg. Torque Vectoring).

Rear wheel steering systems were a focus of research and development in the late 80’s and early 90’s, with car makers such as Nissan, Honda, BMW all having some of their passenger cars with their implementation of the rear wheel steering system. Nissan called theirs ‘Super HICAS’, BMW called their ‘AHK’ but they all had comparatively simple control with hydraulic or electro-hydraulic actuators. [1].

But due to the inherent slow speed of the systems these systems weren’t not a success amongst the consumers. This was coupled with legislative shift towards more eco-friendly vehicles across the world which led auto manufacturers to shift their focus and resources towards developing cleaner and more efficient engines and powertrains. This resulted in just a few high end vehicles which were equipped with RWS system.

Now, as the automotive world changes its focus again towards electric vehicles and active safety the Rear Wheel Steering systems are easier to implement and provide a cost-effective way of realising active safety. Further, with better actuators and faster computers the true potential of Active RWS Systems can be harnessed.

Currently the market is witnessing the introduction of RWS systems in high-end vehicles along with passenger cars alike. Audi A8 & Q7, BMW 5, 6, 7 and 8 series, Honda Accord, Mazda RX-7, MX-6, Renault Laguna, Mégane and Talisman, Nissan Skyline GT-R, Porsche 911, 918, Lamborghini Aventador, Centenario, Urus and Huracán Evo, Mercedes-AMG GT R, Ferrari GTC4 Lusso and F12tdf are amongst the vehicles currently on the market with RWS
system as a standard or an optional extra on their vehicles. [2].

The RWS systems have a potential to provide vehicles with better stability at high speeds and more agility at low speeds. These systems can be implemented easily and quickly in the current vehicle architecture by just changing or adding a few parts. They promise a lot of advantages to the driving dynamics of a vehicle and various possibilities to use such a system, these are discussed in section 2.1.

Implementing such a system on a hyper car capable of speeds upto 412 km/h and accelerating from 0 to 100 km/h in 1.85 seconds on a drag strip is very interesting as to study how much of a difference can be achieved by just turning the rear wheels in the right direction by the right amount.

1.1.1 About Rimac Automobili

Rimac Automobili is a Croatian car manufacturer founded in 2009 by Mate Rimac. Headquartered in Sveta Nedejla Croatia. Rimac develops and produces electric hypercars along with drivetrains, battery systems and vehicle infotainment systems.

The company’s first mule was an all electric prototype based upon BMW E30 and nicknamed the 'Green Monster’ which develops 442 kW power and 900 Nm of torque with a 0 - 100 km/h time of 3.3 seconds and a top speed of 280 km/h.

The first car from Rimac which was available for sale was the Concept One. The Concept One is a two-seat high performance all electric car with a power output of 913 kW (1224 hp) from its 4 electric motors driving each wheel. The car can accelerate from 0 to 100 km/h in 2.5 seconds and was acclaimed as world’s first electric sports car and fastest electric car. An exclusive volume of 8 Concept one’s were produced and sold. [35].

The second production car from Rimac is even more powerful than the previous one, the next sub-section discusses the vehicle briefly as all of the work in this thesis is based on that car.
1. Introduction

1.1.2 About Rimac C_Two

The Rimac C_Two (figure 1.1) was unveiled at the 2018 Geneva International Motor Show. The C_Two is a 1,408 kW (1,888 hp), 1950 kg all electric car with a top speed of 412 km/h and a 0-100 km/h time of 1.85 seconds making it one of the fastest accelerating cars ever made. The hyper car is also capable of level 4 autonomy. As the work of this thesis is based on C_Two some of it’s specifications are given in table 1.

![Figure 1.1: Rimac - C_Two](image)

<table>
<thead>
<tr>
<th>Metric</th>
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<td>Curb Weight</td>
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<td>kg</td>
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<tr>
<td>Top Speed</td>
<td>415</td>
<td>km/h</td>
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<tr>
<td>Power</td>
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<td>kW</td>
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<td>Maximum Range (NEDC)</td>
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<tr>
<td>Battery Capacity</td>
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1. Introduction

1.2 Objective

The overall objective of this thesis is to provide results by which comparisons can be presented to visualise the effects of rear wheel steering system on the vehicle and decisions can be made whether such a system would be beneficial for the vehicle.

The following is the enumeration of objectives:

- To understand the benefits of rear wheel steering system in a hyper-car
- Identify and develop objective metrics for evaluation of lateral dynamics
- Define desired rear wheel steering action
- Develop a controller which gives desired rear wheel steering action
- Evaluate the effect of RWS through the objective metrics
- Understand steering feel
- Understand the effect of RWS system on steering feel
2 Literature Study

The following section discusses the history and state of the art of the various aspects of this thesis.

2.1 Rear Wheel Steering

In a passive vehicle without either torque vectoring or rear-wheel steering, during cornering the driver generates slip angles in the front tyres by turning the steering wheel whereas the cornering forces in the rear wheels are generated due to rear side slip angles which in-turn are generated as a result of vehicles motion. This creates a delay in generation of forces at the front and rear axle. Further, passive vehicles suffer from yaw oscillations during the transient phase of cornering and there is a phase lag between yaw rate and lateral acceleration which makes them less stable and difficult to control. [3].

In a vehicle equipped with both front and rear wheel steering systems, the rear wheels can be steered either in the same direction as the front wheels (parallel steer) or in opposite direction (counter steer). When the wheels are steered parallel to the front wheels the effective wheelbase of the vehicle increases and hence reducing the yaw moment of the vehicle whereas while counter steering the wheelbase decreases hence the yaw moment of the vehicle increases, refer figure 2.1. Therefore, the rear wheels should be steered parallel or counter based upon the yaw requirements of the vehicle.

![Figure 2.1: Rear wheel steering concept [1]](image)

Usually, when the speed is low and a vehicle needs of manoeuvre a tight turn the rear wheels are counter steered whereas parallel steer is used to stabilise the vehicle at high speeds. So, it is possible to make a very basic control law which makes the vehicle counter steer at low
speeds and parallel steer at high speeds.

Research has been done in the field of rear wheel steering in passenger vehicles since as early as 1984 but the technology used in them was very basic and the whole system was not fast enough to harness the full potential of the system. This was majorly due to limited computational capacity available on-board the vehicle and was coupled with the use of hydraulic or electro-hydraulic actuators which inherently have a slow response.

For example, in 1985 Nissan introduced four wheel steering with the name 'HICAS' (High capacity actively controlled suspension) [4]. Here, the rear wheel steering angle was controlled dynamically based upon vehicle’s velocity and restoring torque of the front wheels. The rear steering were rotated by hydraulic cylinders attached to the wheel carrier. The system only improved the stability of the vehicle as it was limited to parallel rear-wheel steer up to 0.5 degrees and would deactivate under speeds of 30 km/h. Nissan continued to develop the HICAS system and 'Super HICAS’ was featured in Nissan 300ZX and several Infinity models in 1989-1996. The rear wheel steering angle was calculated by the ECU in real time based upon the velocity of the vehicle and hand steering wheel angle. The rear wheels were actuated by servo controlled hydraulic actuators. [1].

In 1992, BMW started offering rear wheel steering as an option in its 8-series models under the abbreviation AHK (Aktive Hinterachskinematik, in German Active rear-axle kinematics, in English). It was an electrohydraulic system which used steering wheel angle and vehicle speed as inputs to calculate the rear wheel steering angle with the aim to modify the amplitude and phase of lateral forces on the rear wheels during high lateral acceleration manoeuvres.[1] [5]

Along with limited on-board computing capabilities and lack of a fast actuators, the decrease in research in RWS Systems was hampered by a shift of industry’s focus towards tackling new emission laws which pushed towards more Eco-friendly vehicles across the world forcing auto manufacturers to shift their focus and resources towards developing cleaner and more efficient engines and powertrains.

More recently, Active rear wheel steering systems are re-entering the market with manufactures like Renault, Nissan, Porsche, Lamborghini, Ferrari etc. offering active rear-wheel steering in many of their models. The developments in control engineering and cheaper and more reliable electrical actuators are a major reason for their comeback.

For example, the Porsche 911 Turbo has a feed forward controller designed to track desired vehicle model’s side slip angle. Upto the speeds of 50 km/h the rear wheels are steered in opposite direction (counter steer) and in same direction (parallel steer) above the speeds of 80 km/h. Between 50 and 80 km/h rear steering direction is constantly changing depending of driving conditions [3].

Some advantages of a rear wheel steering system are as follows,

- Faster response of the vehicle to the steering input
- Increased yaw damping at high speeds
2. Literature Study

- Reduction of side slip angle at the centroid of the vehicle [6]
- Increase in the lateral force margins of the tyres
- Increased yaw rate at low speeds (better manoeuvrability)

The RWS system if not developed properly might lead to some disadvantages as well, such as,

- Actuator delays might lead to driver experiencing lags and undesired actions
- Unfamiliar steering or driving feel experienced by driver

2.2 Rear Wheel Steering Control

In the past few years a lot of research has been done towards control algorithms for rear wheel steering systems. Most of the research is focused towards making the system more and more efficient and robust. Efficient, to utilise benefits of rear wheel steering and robust against any uncertainty during driving such as side winds, reduced friction etc.

Almost all controllers employ a reference following system. The reference is generated by mathematical models which give ‘ideal’ response and the controller tries to make the car follow the ideal response by minimising the error in the chosen state parameters by using the rear wheel steering angle as an control input. For example, the 1995 Nissan Skyline GT-R(R34) the reference model would provide target values for yaw and lateral motions and rear wheel steering angles were calculated to obtain those values. [7].

Some researches have used 6 Degree of Freedom (DoF) models as reference while most of the research uses comparatively simple 2 DoF vehicle model (Bi-cycle model). In [3] [8] [9] [10] [11] [12] [13] [14] all utilise the simple planar 2 DoF model with front steering angle and longitudinal velocity as inputs and body side slip angle and yaw rate as output. The 2DOF model works very close to the real vehicle in the linear range of the tyres but starts deviating at high lateral accelerations.

Some of the research utilises a 3 DoF vehicle models where longitudinal velocity/force is also a state parameter along with body side slip angle and yaw rate [15]. This approach is advantageous when the longitudinal parameters are also needed to be controlled, for example traction control system.

Further, control algorithms of various level of complexities have been employed to get the desired actions. In [8] a Linear Quadratic Control (LQR) strategy whereas [10] and [3] employ a feedback and a feed forward control respectively. The sliding mode control algorithm is used by [11] and [14].

After studying at the various algorithms and judging them on the basis of good control action and ease of implementation, sliding mode control with a 2 DoF reference model was chosen to be used as the controller and reference model for this thesis respectively.
2.3 Steering Feel

The steering system has two main functions, one as an actuator of driver commands and second as a source of sensory feedback to the driver about the vehicle's state, response and driving condition.

Steering feel in very simple words is the sensation the driver feels while steering the vehicle. Steering feel is an essential part of the vehicle - driver interaction which ultimately effects the drivability of the vehicle.

Formally, steering-feel is the complete optical, kinesthetic, and haptic sensations of the driver while steering a car, resulting in a complex, subjective experience. [16].

Most of the drivers seldom drive their vehicle at its dynamic limit, which can be encountered only in some extreme situations or at the track. Majority of driving includes low lateral accelerations which are incurred in daily driving. Therefore, steering feel in this low lateral acceleration range is very important for a good vehicle-driver interaction. Figure 2.2 shows the relative frequencies of lateral acceleration for highway, town and country road driving by a standard driver [16].

![Figure 2.2: Relative frequencies of lateral acceleration experienced for highway, town and country road driving [16]](image)

This lateral acceleration range is known as the ‘on-center’ range and the steering feel experienced in this range is known as on-center feeling. Much work has been dedicated to measure,
understand and optimize this on-center feel to provide an optimum driving pleasure with a balance of safety, comfort and performance.

An overall steering feel should have a distinctive on-center area, a linear area proportional to the steering wheel angle and a transition area up to the limit. The driver should be able to feel distinctively when the steering is at the center and when the vehicle is reaching its limit. Further, it should provide comfort to the driver by adapting steering wheel torque to different driving situations and should provide feedback (usually haptic) to the driver to judge the vehicle’s state and road information.

With virtual simulations, shorter vehicle development phase and lean production strategies drastically reducing the on-track testing time the focus has shifted towards development of quantitative assessments for steering feel [16]. Work has also been done in correlating the subjective steering feel to the developed objective metrics so that the physical testing sessions can be shortened and understood in a deeper and better way [17] [18]. Correlating the subjective assessments with objective metrics helps test engineers to make the subjective assessments which vary in details and accuracy with every driver and their experience, more levelled [21] [22].

Even though a lot of work has been done on steering feel, majority of its evaluation and optimisation still consists of subjective feedback from experienced drivers. This is because it is difficult to correlate subjective feedback to objective metrics and also because of sheer importance of steering feel in the drivability and handling of the vehicle.
3 Performance and Evaluation Metrics

The most important vehicle dynamics evaluation tool is perhaps the subjective driver feedback from physical on-track testing. But, more often at the earlier stage of the design has to rely upon virtual simulations. The performance and evaluation metrics provide the objective feedback to the designer, which is useful for both physical testing and virtual simulations.

In this section some important tests are discussed which are used in the later part of the report to tune the rear wheel steering (RWS) controller and compare it with the passive vehicle.

3.1 Step Steer

In this test the vehicle is driven in a straight line with a pre-decided velocity (usually 100 km/h) followed by a fast step steer input to a predetermined value. The steering wheel angle amplitude is determined by steady state driving on a circle the radius of which gives the preselected steady-state lateral acceleration at the required test speed. The standard steady-state lateral acceleration level is $4 \text{ m/s}^2$. Additional levels of $2 \text{ m/s}^2$ and $6 \text{ m/s}^2$ may be used. In order to keep the steering input short relative to the vehicle response time, the time between 10 % and 90 % of the steering input should not be greater than 0.15 s (figure (3.1)). No change in throttle position shall be made, although speed may decrease. A steering wheel stop may be used for selecting the input angle. [19].

![Step Steer - steering input](image)

Figure 3.1: Step steer - steering input

The following metrics are used:

- **Response time** - defined as the time taken for the vehicle transient response to first reach 90% of the steady state value measure with respect to the reference point. The reference point is defined as the time when the steering wheel input is 50% complete.

- **Overshoot** - calculated as a ratio of the difference of peak value and steady-state value divided by steady-state value.
3. Performance and Evaluation Metrics

- **Peak response time** - the time, measured from the reference point, for a vehicle transient response to reach its peak value.

- **Gain** - calculated as the ratio of transient and steering wheel input at steady state.

- **Lag** - defined as the time difference between when the steering wheel angle and the transient response reach their respective maximum value.

The above metrics are calculated for lateral acceleration \( (a_y) \) and yaw rate \( (\dot{\psi}) \) following figure 3.2 and are tabulated in table 2.

### Table 2: Step Steer Metrics

<table>
<thead>
<tr>
<th>Metric</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state yaw rate response gain</td>
<td>( \frac{\dot{\psi}}{\delta H_{ss}} )</td>
<td>s(^{-1})</td>
</tr>
<tr>
<td>Lateral acceleration response time</td>
<td>( T_{ay} )</td>
<td>s</td>
</tr>
<tr>
<td>Yaw rate response time</td>
<td>( T_{\dot{\psi}} )</td>
<td>s</td>
</tr>
<tr>
<td>Lateral acceleration overshoot</td>
<td>( U_{ay} )</td>
<td>-</td>
</tr>
<tr>
<td>Yaw rate overshoot</td>
<td>( U_{\dot{\psi}} )</td>
<td>-</td>
</tr>
<tr>
<td>Lateral acceleration lag</td>
<td>( \Delta_{ay} )</td>
<td>s</td>
</tr>
<tr>
<td>Yaw rate lag</td>
<td>( \Delta_{\dot{\psi}} )</td>
<td>s</td>
</tr>
</tbody>
</table>

Figure 3.2: Step Steer - Steering wheel input and metrics [19]
3. Performance and Evaluation Metrics

3.2 Sine Sweep

The vehicle is driven at test speed (usually 100 km/h) followed by at least 3 periods of sinusoidal steering wheel input with predetermined steering wheel angle and frequency. The frequency is increases in steps from 0.2 to 2.0 Hz. The steering wheel angle is determined to achieve a steady state lateral acceleration level of $4 \text{ m/s}^2$. The steering wheel input is depicted in figure 3.3.

The following metrics are used,

- Lateral acceleration gain - at each frequency
- Yaw rate gain - at each frequency
- Frequency response of lateral acceleration gain and yaw rate gain along with phase delay.

![Figure 3.3: Sinusoidal sweep - steering wheel input](image-url)
3.3 Pulse Steer

Details of the Pulse Steer manoeuvre are enumerated in ISO 7401 [19], the vehicle is driven in straight line at the chosen test speed. Starting from this equilibrium condition a triangular wave form steering wheel input is applied followed by 3 to 5 seconds of neutral steering wheel input. There should be no change in throttle position although the speed may decrease. The triangular pulse should have a width of 0.3 to 0.5 seconds. The amplitude of steering wheel input is determined by steady state circular driving test which gives a pre-selected lateral acceleration at the required test speeds. The standard being $4 \ m/s^2$, higher or lower lateral acceleration values may be used, provided that vehicle remains in the linear range (refer figure 3.4). [19].

![Pulse Steer - SWA](image)

Figure 3.4: Pulse steer - steering wheel input and metrics [19]

The following metrics are used,

- Lateral acceleration
- Yaw rate
- Yaw acceleration
3.4 Single Sine Steer

In this test the vehicle is driven in straight line at test speed and one full sinusoidal steering wheel input is applied with frequency 0.5 Hz and additionally 1.0 Hz (figure 3.5). The amplitude error of the actual waveform compared to the true sine wave shall be less than 5% of the first peak value. No change in throttle position shall be made, although speed may decrease. The test is repeated for both directions. The first peak of lateral acceleration is the desired lateral acceleration the standard being $4 \text{ m/s}^2$. [19].

![Figure 3.5: Single sinusoidal - steering wheel input](image)

The following metrics are used:

- **Lateral Acceleration** - defined as the first peak value of the lateral acceleration time history.
- **Yaw Rate** - defined as the first peak value of the yaw rate time history.
- **Lags** - the time lags between steering wheel angle with lateral acceleration and yaw rate are calculated respectively for the first and second peak by the means of **cross co-relation** of the respective half-waves. [19]
- **Lateral acceleration gain** - calculated as the ratio of lateral acceleration (defined above) to the corresponding steering wheel angle.
- **Yaw rate gain** - calculated as the ratio of yaw rate (defined above) to the corresponding steering wheel angle.
- **Amplification Factor (AMP)** - the ratio of first and second half wave gains of lateral acceleration or yaw rate.
- **Time lag amplification (TLA)** - the ratio of lags of first and second half waves of lateral acceleration or yaw rate gains.

The above metrics are tabulated in table 3.
Table 3: Single sine period metrics

<table>
<thead>
<tr>
<th>Metric</th>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration</td>
<td>$A_y$</td>
<td>m/s²</td>
</tr>
<tr>
<td>Yaw rate</td>
<td>$\psi$</td>
<td>rad/s</td>
</tr>
<tr>
<td>Lateral acceleration gain</td>
<td>$G_{ay}$</td>
<td>g/deg</td>
</tr>
<tr>
<td>Yaw rate gain</td>
<td>$G_\psi$</td>
<td>s⁻¹</td>
</tr>
<tr>
<td>Lateral acceleration lag - peak 1</td>
<td>$\Delta_{ay1}$</td>
<td>s</td>
</tr>
<tr>
<td>Lateral acceleration lag - peak 2</td>
<td>$\Delta_{ay2}$</td>
<td>s</td>
</tr>
<tr>
<td>Yaw rate lag - peak 1</td>
<td>$\Delta_{\psi1}$</td>
<td>s</td>
</tr>
<tr>
<td>Yaw rate lag - peak 2</td>
<td>$\Delta_{\psi2}$</td>
<td>s</td>
</tr>
<tr>
<td>Lateral acceleration TLA</td>
<td>$A_y TLA$</td>
<td>-</td>
</tr>
<tr>
<td>Yaw rate TLA</td>
<td>$\psi TLA$</td>
<td>-</td>
</tr>
<tr>
<td>Lateral acceleration AMP</td>
<td>$A_y AMP$</td>
<td>-</td>
</tr>
<tr>
<td>Yaw rate AMP</td>
<td>$\psi AMP$</td>
<td>-</td>
</tr>
</tbody>
</table>
3.5 Ramp Steer

In this test the vehicle is driven in a straight line at the test speed (usually 100 km/h), after the speed is achieved a steering wheel input is given at a very slow rate between 4 to 10 deg/sec (figure (3.6)). The throttle is kept constant although vehicle speed may decrease. Final steering wheel angle should be chosen such that the vehicle/tyres gets saturated at the chosen final steering wheel angle. Additional speeds of ± 20km/h may be used. Test should be repeated for both left and right steering direction [19].

![Ramp Steer - SWA](image)

Figure 3.6: Ramp steer - steering wheel input [19]

The following metrics are used,

- **Steering gradient** - the ratio of change in steering wheel angle with lateral acceleration, usually read through a plot of SWA v. lateral acceleration

- **Yaw rate gain** - is calculated as the ratio of yaw rate to the corresponding steering wheel angle

- **Steering sensitivity** - is the ratio of lateral acceleration to the steering wheel angle

- **Under-steer gradient** - slope of lateral acceleration vs. steering wheel angle curve

- **Side slip gradient** - slope of lateral acceleration vs. side slip angle curve
3.6 On-Centre Weave Test

The on-center weave test generates characteristic to evaluate the steering feel and steering precision around the central position. The test has been enumerated in ISO 13674 [20].

The weave test is an open-loop driving manoeuvre. The vehicle is driven in a straight line at a pre-decided velocity (test standard is 100 km/h). From the straight line the steering wheel is subjected to an oscillatory sinusoidal input with a frequency of 0.3 Hz ± 10 %. The amplitude of the steering input must be sufficient to produce the required lateral acceleration level. To assume good data is collected at lateral acceleration levels 1 \( m/s^2 \), peak value of 2 \( m/s^2 \) is chosen ensuring that the vehicle is operating outside hysteresis. The test may be performed either by using a steering robot or by an experienced test driver [20].

The following metrics are evaluated-

- Steering-wheel torque vs. steering wheel angle
  - Steering stiffness
  - Steering stiffness at zero steer
  - Steering friction: ordinate dead band
  - Angle hysteresis: abscissa dead band
- Lateral acceleration vs. steering wheel angle
  - Steering sensitivity: average gradient over range \( \pm x \text{ deg} \), where \( x = 20 \) of peak steering angle
  - Minimum steering sensitivity
  - Steering sensitivity at 1 \( m/s^2 \)
  - Lateral acceleration dead band: ordinate dead band
  - Steering hysteresis: area bounded by hysteresis loop and ordinate values 1 \( m/s^2 \), divided by 2 \( m/s^2 \)
- Steering wheel torque vs. lateral acceleration
  - Torque at 0 \( m/s^2 \)
  - Torque at 1 \( m/s^2 \)
  - Lateral acceleration at 0 \( Nm \)
  - Torque gradient at 0 \( m/s^2 \)
  - Torque gradient at 1 \( m/s^2 \)
  - Torque hysteresis: ordinate dead band
  - Lateral acceleration hysteresis: abscissa dead band
- Yaw velocity gain
- Response dead band: abscissa dead band of yaw velocity vs. steering-wheel torque
4 Sliding Mode Control

After reviewing the literature and considering previous experience, sliding mode control was chosen as the main control algorithm. This section is aimed to explain the basics of SMC along with its advantages and drawbacks. The techniques which are relevant to this thesis have been discussed, other techniques and algorithm can be found through the references.

4.1 Theory

Sliding mode control (SMC) is a nonlinear control method that alters the dynamics if a nonlinear systems (such as a vehicle) by applying a discontinuous control signal that forces the system to ‘slide’ along the sliding surface [23].

If the system moves along the Sliding Surface then an ideal sliding motion will take place. Sliding Mode Control technique theoretically ensures exact convergence of error dynamics even in presence of uncertainties and perturbations.

The main work on classical Sliding Mode Control (SMC) theory was done by Prof. Vadim I. Utkin around 1981. In his book 'Sliding Modes in Control Optimisation' [24], Utkin has clearly stated the two step procedure for SMC design,

1. Sliding Surface Design
2. Discontinuous controller design ensuring the sliding motion

Sliding Mode Control Process consists of two basic steps,

1. **The Reaching Phase**: The reaching phase is the part where the describing points start from its initial position (initial condition) and move towards the sliding surface. During this phase, the tracking error cannot be controlled directly and the system response is sensitive to parameter variation and noise. Therefore, it is desirable to shorten or even eliminate the reaching phase. The easiest way to decrease reaching time is to employ a large control input, however this is restricted by actuator dynamics (actuator saturation, lag) and extreme control inputs may cause the system itself to become unstable and may lead to undesirably high chattering.

2. **The Sliding Phase**: After the system has reached the sliding surface it moves only on the desired surface. Here, it is best to ‘stick’ to the sliding surface even in case of disturbances. In case of disturbances, control input should be provided such that the sliding action is always maintained.

The controller should be designed in such a way that it is capable to reach the sliding surface as quickly as possible irrespective of the initial conditions, further, once the sliding surface is
reached the controller should provide enough control inputs to maintain the sliding motion even in case of disturbances. The error dynamics will converge till the system is on the sliding surface.

Advantages of Sliding Mode Control,

- SMC provides theoretical insensitivity with respect to bounded matched uncertainties, that is if the uncertainties are of the same order as that of the system.
- The order of system’s dynamics get reduced once the sliding surface has been reached.
- Finite time convergence to the sliding surface.

Although, SMC has some disadvantages as well, such as

- Chattering - is defined as the fluctuation of the output signals at a very high frequency, this is caused mainly by un-modelled cascade dynamics that increase the system’s relative degree and disturb the ideal sliding mode existing in the system.
- Although the sliding variable converge in finite time, the state variables converge asymptotically only.
- Non-ideal closed-loop performance in presence of parasitic dynamics, discretisation and noises.
- Insensitivity only with respect to matched perturbations

Chattering is one of the major drawbacks of SMC, but it can be successfully reduced or even eliminated by using Second order sliding mode (SOSM). SOSM concept was introduced by Prof. Levant in the 80’s with the ‘twisting algorithm (TA)’ which was designed for a system which has relative two degrees of freedom, the TA collapses the dynamics of the system to the origin thus making it easier to control, but this approach requires the derivative of sliding variable.

In early 90’s, super twisting algorithm was presented and could provide a continuous control signal without using the derivative of sliding variable and was able to maintain a SOSM for first order systems with LIPSCHITZ bounded disturbances. This led to development of the mathematical theory and applications of SOSM algorithms.

Applications of Sliding Mode Control,

- Aircraft Control
- Overhead Cranes
- Vehicle Control
- Industrial processes
The following section explains different approaches to Sliding Mode Control which were used in this thesis.

### 4.2 Equivalent Control Approach

The Equivalent control approach to understand SMC was developed by Utkin [24] and it defines the control actions necessary to maintain the ideal sliding motion on the sliding surface $S$. Consider the following Single-Input Single-Output (SISO) system of order $n$

$$
\dot{x}(t) = f(x, u, t) \quad (1)
$$

where, $x \in \mathbb{R}$ is the state vector and $u \in \mathbb{R}$ is the control signal. Assuming that $f(\cdot)$ is differentiable with respect to $x$ and continuous in time $t$. Now, let $\sigma(x)$, called as the sliding variable, is the output variable and is also differentiable in $x$. The Sliding surface (or manifold) is defined by the restriction,

$$
S = x : \sigma(x) = 0 \quad (2)
$$

For ideal sliding mode to take place on the Sliding Surface (2) the state $x(t)$ should evolve satisfying,

$$
\sigma(x(t)) = 0 \quad (3)
$$

for all $t > t_r$, $t_r$ being some positive finite reaching time.

During the sliding motion the dynamics of the system "collapses" and is forced to move along the sliding surface $S$. This restricted movement implies that the order of the dynamics of the system has been reduced. During the sliding phase,

$$
\dot{\sigma} = \sigma = 0 \quad (4)
$$

therefore,

$$
\dot{\sigma} = \frac{\partial \sigma}{\partial x} \frac{dx}{dt} = \frac{\partial \sigma}{\partial x} f(x, u, t) = 0 \quad (5)
$$

The equivalent control signal $u_{eq}(t)$ can be obtained by satisfying the equality in equation (5).

$$
\frac{\partial \sigma}{\partial x} f(x, u_{eq}, t) = 0 \quad (6)
$$

Once the equivalent control is obtained it can be substituted in equation (1) to obtain a continuous differential equation (7) whose solution for initial conditions $\sigma(x(0)) = 0$ and equation (5) represent the state trajectories in the tangential plane to the sliding surface $S$.

$$
\dot{x} = \frac{\partial \sigma}{\partial x} f(x, u_{eq}, t) \quad (7)
$$
4. Sliding Mode Control

For example, if a SISO system is given by equation (8), assuming that the sliding mode has been reached at time instant $t_r = t_0$ i.e. equation (4) is satisfied for any $t \geq t_r$.

$$\dot{x} = f(x,t) + B(x,t)u$$ (8)

The equivalent control can be given by,

$$\dot{x} = f(x,t) + B(x,t)u_{eq}$$ (9)

$$u_{eq} = -\left\{ \frac{\partial \sigma}{\partial x} B(x,t) \right\}^{-1} \frac{\partial \sigma}{\partial x} f(x,t)$$ (10)

and the dynamics of the system can be expressed as,

$$\dot{x} = \{ I_n - B(x,t)(CB(x,t))^{-1} C \} f(x,t)$$ (11)

where, $C = \frac{\partial \sigma}{\partial x}$

4.3 Reaching Law Approach

In the reaching law approach the dynamics of the sliding surface are directly expressed. [25]. The dynamics of the switching function can be expressed by the differential equation,

$$\dot{S} = -q f(s) - k \text{sgn}(s)$$ (12)

$q, k > 0$

and,

$s f(s) > 0, \forall s \neq 0$

The control law may be obtained directly by the condition $\dot{S} = 0$ for the system.

The thesis utilises a Constant Reaching Law approach, which is given by,

$$\dot{S} = -k \text{sgn}(s)$$ (13)
4.4 Stability of SMC

The stability of the sliding mode controller can be tested by using Lyapunov Function. Lyapunov function is a scalar function defined in the phase space and can be used to prove the stability of an equilibrium point.[26]

The basic idea of Lyapunov stability is to introduce an (abstract) energy measure and show that it does not increase along system trajectories.

The theorem states that, if there exists a continuous function $V(x)$ whose sub-level sets

$$L_v(\alpha) = \{x | V(x) \leq \alpha\}$$

are bounded by every value of $\alpha$ and,

$$\Delta V(x) = V(f(x)) - V(x) \leq 0$$

for all $x$, then all trajectories of equation are bounded and the system can be guaranteed to be asymptotically stable.

Satisfying the Lyapunov’s condition is enough to prove the stability of the system.
5 Rear Wheel Steering : Controller Design

Due to its inherited robustness and relative simplicity, Sliding Mode Control approach has been chosen for the control of the rear wheel steering.

5.1 Reference Model

The reference model is based upon an ideal two degree of freedom single track model (bicycle model), as the controller will mimic this model’s behaviour to keep the driving feel same as a traditional front wheel steered vehicle (so that driver doesn’t feel uncomfortable with active rear wheel steering) the reference model has same steady state yaw rate gain as a front wheel steered car. In order to have no side slip during cornering the side slip angle at CG of the model is zero.

The bicycle model can be represented in state space form as,

\[
\dot{X}_d = A_d X_d + B_d U_d
\]

where, the \( X_d \) is the state vector and \( U_d \) the input vector,

\[
X_d = \begin{bmatrix} \beta_d \\ \dot{\psi}_d \end{bmatrix}
\]

\[
U_d = [\delta_f]
\]

\( A_d \) and \( B_d \) are system and output matrix respectively

\[
A_d = \begin{bmatrix} -1/	au_\beta & 0 \\ 0 & -1/	au_\psi \end{bmatrix}
\]

\[
B_d = \begin{bmatrix} k_\beta/	au_\beta \\ k_\psi/	au_\psi \end{bmatrix}
\]

The yaw rate gain for the above ideal model is given by,

\[
K_{\dot{\psi}_d} = \frac{1}{1 + KV^2} \cdot \left( \frac{V}{L} \right)
\]

Where, \( K \) is the stability factor and is given by,
5. Rear Wheel Steering: Controller Design

\[ K = \frac{m}{L^2} \cdot \left( \frac{a}{2c_f} - \frac{b}{2c_r} \right) \]  \hspace{1cm} (21)

Here, the time lags \( \tau_\beta, \tau_\dot{\psi} \) and the stability K factors are tuned to achieve the desired response.

### 5.2 Vehicle Plant Model

The plant model of the vehicle is a four wheel steered vehicle modelled as a 2-DoF single track vehicle with rear wheel steer. This model should be as close to the actual vehicle as possible to assure, when inverted, provide the exact control input for the rear wheels.

The state space representation is as follows,

The plant system is given by,

\[ \dot{X} = A_p X + B_p U \]  \hspace{1cm} (22)

where, the states \( [X] \) are \( \beta \), the body side slip angle and \( \dot{\psi} \), the yaw rate of the vehicle.

\[ X = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} \]  \hspace{1cm} (23)

With the input matrix, \( U \), consisting of the front and rear steering wheel angles, \( \delta_f \) and \( \delta_r \), respectively

\[ U = \begin{bmatrix} \delta_f \\ \delta_r \end{bmatrix} \]  \hspace{1cm} (24)

\[ A_p = \begin{bmatrix} \frac{2(C_f + C_r)}{mV} & \frac{2(aC_f + bC_r)}{mV^2} - 1 \\ \frac{2(aC_f + bC_r)}{I_z} & \frac{2(a^2C_f + b^2C_r)}{I_zv} \end{bmatrix} \]  \hspace{1cm} (25)

\[ B_p = \begin{bmatrix} \frac{-2C_f}{mV} \\ \frac{-2aC_f}{I_z} \end{bmatrix} \]  \hspace{1cm} (26)

When this model is used in the equivalent control (section 4.2) and when it is "inverted", the input, \( U \), becomes the control output for the linear part of the controller.
5.3 Switching Function

The state error is the difference between the states of reference model and vehicle’s current states.

The error is,

\[ e = X_d - X = \begin{bmatrix} \beta_d - \beta \\ \dot{\psi}_d - \dot{\psi} \end{bmatrix} \]  

(27)

Now, the rate of change of error \( \dot{e} \) is given by

\[ \dot{e} = \dot{X}_d - \dot{X} \]

\[ = A_d X_d + B_d U_d - A_p X - B_p \left[ U + d(x, t) \right] \]

\[ = A_d X_d - A_d X + A_p X - A_p X + B_d U_d - B_p \left[ U + d(x, t) \right] \]

\[ \dot{e} = A_d e + (A_d + A_p) X - A_p X + B_d U_d - B_p \left[ U + d(x, t) \right] \]  

(28)

(29)

The switching function, \( S \), is given by

\[ S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = C \cdot e = \begin{bmatrix} 1 & c_1 \\ 1 & c_2 \end{bmatrix} \cdot \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \]  

(30)

\[ C = \begin{bmatrix} 1 & c_1 \\ 1 & c_2 \end{bmatrix} \]  

(31)

where, \( C \) is a full rank constant matrix. The matrix \( C \) is chosen such that \( CB \) is non-singular. and,

\[ \dot{S} = C \cdot \dot{e} = C \cdot \left[ A_d e + (A_d + A_p) X - A_p X + B_d U_d - B_p \left[ U + d(x, t) \right] \right] \]  

(32)

where, \( c_1 \) and \( c_2 \) are unknown constants and will be determined such that the system is always stable and such that \( CB \) is non-singular.

5.4 Constant Reaching Law

Adopting the constant reaching law, from section 4.3

\[ \dot{S} = -G \cdot sgn(S) \]  

(33)
Where, \( G \) is the matrix of gains,

\[
G = \begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix}
\]  

(34)

and, \( sgn \) is called 'signum function' which is an odd mathematical function that extracts the sign of a real number.

\[
\dot{S} = -\begin{bmatrix} g_1 & 0 \\ 0 & g_2 \end{bmatrix} \cdot \begin{bmatrix} sgn(s_1) \\ sgn(s_2) \end{bmatrix}
\]  

(35)

\[
\dot{S} = -\begin{bmatrix} g_1 sgn(s_1) \\ g_2 sgn(s_2) \end{bmatrix}
\]  

(36)

### 5.5 Sliding Mode Control Law

The control law can be derived by equating equations (32) and (36),

\[
\dot{S} = -G \cdot sgn(S) = C \cdot \begin{bmatrix} A_d e + (A_d + A_p)X + B_d U_d - B_p[U + d(x, t)] \end{bmatrix}
\]  

(37)

Now, assuming that the disturbance \( d(x, t) \) is zero.

\[
U = (C B_p)^{-1} C \cdot \begin{bmatrix} A_d e + (A_d + A_p)X + B_d U_d \end{bmatrix} + (C B_p)^{-1} \cdot G sgn(S)
\]  

(38)

where, \( U \) is the control action necessary to reach and maintain the sliding motion on the sliding surface.

\[
U_{eq} = (C B_p)^{-1} C \cdot \begin{bmatrix} A_d e + (A_d + A_p)X + B_d U_d \end{bmatrix}
\]  

(39)

\[
U_{nl} = (C B_p)^{-1} G sgn(S)
\]  

(40)

This control action \( U \) has two parts, one \( U_{eq} \) is a continuous feedback control and is the nominal equivalent control needed to achieve \( S = 0 \). Whereas, the second part \( U_{nl} \) is the discontinuous non-linear feedback control and is used to deal with system uncertainties such as a sudden side wind gust.

\[
U = U_{eq} + U_{nl}
\]  

(41)
Figure 5.1 shows the schematic block diagram and figure 5.2 shows the actual implementation in simulink of the whole controller, respectively.

Figure 5.1: Rear Wheel Steering Controller Architecture
Figure 5.2: Control system implementation in Simulink
5.6 Stability Analysis and decoupling of the control system

Following section 4.4, let Lyapunov function be defined as,

\[ V_L = \frac{1}{2}(S^T S) \]  

(42)

Taking the time derivative,

\[ \dot{V}_L = (S^T \dot{S}) \]  

(43)

\[ (S^T \dot{S}) = S^T C \cdot \left[ A_d e + (A_d + A_p) X + B_d U_d - B_p [U + d(x, t)] \right] \]  

(44)

that is,

\[ \dot{V}_L = S^T C B_p \left[ U_{nl} + d(x, t) \right] \]  

(45)

where,

\[ C B_p = \begin{bmatrix} -2C_f \left( \frac{1}{mV} + \frac{ac_1}{I_z} \right) & 2C_r \left( \frac{bc_1}{I_z} - \frac{1}{mV} \right) \\ -2C_f \left( \frac{1}{mV} + \frac{ac_2}{I_z} \right) & 2C_r \left( \frac{bc_2}{I_z} - \frac{1}{mV} \right) \end{bmatrix} \]  

(46)

Now, decoupling the switching surface \( s_1 \) and \( s_2 \) in \( C B_p \) and hence making the two states independent of each other,

\[ 2C_r \left( \frac{bc_1}{I_z} - \frac{1}{mV} \right) = -2C_f \left( \frac{1}{mV} + \frac{ac_2}{I_z} \right) \]  

(47)

Obtaining,

\[ c_1 = \frac{I_z}{mVb} \]  

(48)

\[ c_2 = \frac{-I_z}{mVa} \]  

(49)

Substituting these values of \( c_1 \) and \( c_2 \) in equation (46),
\[ CB_p = \begin{bmatrix} \frac{-2C_f L}{m V_b} & 0 \\ 0 & \frac{-2C_r L}{m V_a} \end{bmatrix} \]  

(50)

Equation (45) becomes,

\[ \dot{V}_L = (S^T \dot{S}) = -S^T \begin{bmatrix} \frac{2C_f L}{m V_b} & 0 \\ 0 & \frac{2C_r L}{m V_a} \end{bmatrix} \left[ U_{nl} + d(x, t) \right] \]

(51)

therefore,

\[ \dot{V}_L < 0 \]

\[ \forall U_{nl} \]

Equation (51) is always negative and hence according to Lyapunov’s condition the energy of the system does not increase. Therefore, the above system is stable.

5.6.1 Importance of decoupling

Decoupling the states (equation (50)) essentially means that the gains g1 and g2 (equation (34)) are independent of each other. Tuning g1 will only tune body side slip and tuning g2 will tune yaw rate response of the controlled vehicle.

This decoupling makes tuning the controller comparatively easier and both the gains can be tuned to achieve the required precision of the controlled states.
6 Simulations, Tuning and Results

The following section outlines the simulation and tuning methodology followed by discussion of results.

6.1 Co-Simulation: MatLab/Simulink and IPG CarMaker

The simulation was set-up with a fully defined vehicle model in IPG CarMaker with the control implementation in MatLab/Simulink. The data flow was facilitated by IPG specific blocks provided in Simulink and a master file in MatLab which connects the IPG variables to MatLab/Simulink and vice-versa, all in real time (figure 6.1).

![Figure 6.1: Co-simulation with MatLab/Simulink and IPG CarMaker](image)

IPG CarMaker is an off-the-shelf commercial virtual vehicle simulation software with a detailed vehicle model, intelligent driver model along with highly flexible road and traffic models.

The vehicle model of Rimac C_Two was used in the simulations. The model has fully defined suspension and steering systems with accurate hard-points along with a detailed power-train with actual motor and battery data. The tire data used is the latest data provided by the manufacturer. Aerodynamic data, such as variation of coefficient of drag and down force with speed along with their centres of pressure complete the model.

Complete test manoeuvres (shown in section 3) were defined and developed in IPG CarMaker and IPG’s test manager was used to run these tests in an automated and iterative manner. To isolate the effect of active rear wheel steering system on the vehicle, all other control systems were disconnected from the model during the simulations. Further, same manoeuvres were performed with all control systems switched off so that a comparisons could be drawn.

MatLab/Simulink is also an off-the-shelf software. MatLab offers numerical computing along with dedicated tool-boxes such as digital signal processing tool-box. Whereas, Matlab has a programming interface. Simulink has a graphical programming environment for modelling, simulating and analysing multi-domain dynamical systems.

MatLab was primarily used to post process the data from various tests conducted in the co-simulation environment. Separate sets of codes to post process and generate objective metrics
were written for each manoeuvre. Simulink was used to make the reference and plant models along with the controller itself.

The following terms are frequently used in the following sections,

- Passive vehicle: the vehicle with no control systems active at all.
- Active vehicle: the vehicle with only Rear Wheel Steering System active.

### 6.2 Tuning Methodology

The first step in tuning the controller is to tune the response of the reference model, that is the ideal bicycle model as it is what the controller will be following.

After the reference model has been tuned the controller was tuned using the gains G1 and G2 of the constant reaching law part of the controller, here since the body side slip and yaw rate have been decoupled (Section 5.6) changing gain G1 only influences the body side slip response and changing G2 only influences the yaw rate response.

While tuning the gains (Matrix G) the chattering of the controller should also be addressed and reduced by using the boundary layer approach and increasing the boundary layer (Section 6.2.3).

Here, it is important to understand that increasing the gains too much which lead to more chattering which in turn needs the boundary layer to be wider. The boundary layer acts as a 'dead-zone' for the controller and hence should be kept a small as possible. Therefore, a compromise must be reached between the size of the gains and the switching function.

The equivalent control part of the controller needs no tuning as it is an inverse 2 DoF model of the vehicle, the closer this model is to the real vehicle the better the controller will perform.

### 6.2.1 Tuning the Ideal Bicycle model

Section 5.1 details the ideal bicycle model, here it is needed only to tune the lag time constants of body side slip $\tau_\beta$ and yaw rate $\tau_\psi$, as the rest of the parameters are either inputs or constant to the vehicle.

The yaw rate time constant was tuned to achieve a fast step response without any overshoots or fluctuations keeping in mind that the faster this reference response is, the harder it will be for the controller to follow it and may need to provide high control inputs which might be beyond actuator bounds or may cause high chattering.

The body side slip time constant was kept to zero as zero body side slip is the desired at the vehicle’s centeroid. A low body side slip angle (zero ideally) is characteristic of a stable and a more controllable vehicle [6]. The response are depicted in figure 6.2.
6. Simulations, Tuning and Results

6.2.2 Tuning the Controller

The basic tuning of the controller was done in an iterative manner to produce the desired step steer response. The step steer manoeuvre was chosen for the initial tuning as it is a standard function with a well defined response and metrics. Further, the manoeuvre imitates a very quick transition from straight line driving to turning which is experienced in day to day driving as well.

The vehicle is made to perform a step steer manoeuvre and the gains (G1 and G2) are tuned to have a response as close to the ideal reference model as possible.

6.2.3 Chattering Suppression

The signum function used in the controller extracts out the sign of the error and give the output as positive or negative 1. This decides the direction of steer of the rear wheels, either parallel or counter to that of the steering at the front wheels.

The problem with using such a function is that when the error is fluctuating around zero, the signum function will give signals which will ask the actuators to constantly switch directions which lead to high fluctuations in the final combined control signals will be chattering. This chattering is not good for the dynamics of the vehicle and will generate vibrations which are not good for the driving feel as well. Further, the chattering will take a heavy toll on the actuators, leading to their reduced life and over heating.

To solve this problem of chattering the discontinuous signum function is replaced by a continuous smooth function which works same as a signum function but has a smoother translation defined by $\xi$, the greater the value of $\xi$ smoother will be the slope and smoother will be the

![Figure 6.2: Reference Body side slip and Yaw Rate step response](image)
transition from positive to negative or visa-verse. But, this smoothness will make the response slower and if $\xi$ is increased too much then it will effect the working of the controller. This is because the smoothness ($\xi$) acts as a boundary layer on the sliding surface and the fluctuations within the range of this boundary layer are ignored creating a dead-zone.

The smooth function is given by,

$$\text{sat}(e) = \frac{e}{|e| + \xi}$$  \hspace{1cm} (53)

The value of $\xi$ should always be positive and small. A good initial guess is that $\xi$ should be in the of the order of the fluctuation. If this boundary layer ($\xi$) is too big then the controller will not perform as required as the boundary layer creates a 'dead-zone' in which the controller stops responding.

Figure 6.3 shows how the signum and smooth function differ graphically,

![Graph showing difference between signum function (solid line) and smooth function (dotted line)](image)

Figure 6.3: Difference between signum function (solid line) and smooth function (dotted line)

The value of $\xi$ was tuned to get the desired response with no chattering.
6.3 Step Steer

In step steer the vehicle transitions from straight line to steady-state conditions, a certain response time is required for the vehicle to achieve this steady state and usually leads to overshoot behaviour.

The Step Steer manoeuvre was performed as stated in section 3 for the lateral acceleration values of 0.4 and 0.8 g. The various metrics are tabulated in tables 4 and 5. The lateral acceleration time histories can be seen in figure 6.4 with blue and red colours for the passive and active vehicle respectively.

It can be observed that for both lateral acceleration values the vehicle with active rear wheel steering (active RWS) has a faster response compared to the passive one. This is attributed to the fact that slip forces are generated at both front and rear tyres at the same time in the active vehicle as opposed to the slip forces only generating at the front and taking time to translate to the rear of the vehicle in the passive vehicle. It can be notice that this difference in response time between the two vehicles is reduced for higher lateral accelerations.

Further, there is an overshoot in lateral acceleration for the passive vehicle whereas it is absent in the active vehicle. Here, in the passive vehicle the driver would first feel increasing lateral acceleration with the steering wheel input and when the steering input is constant he/she will feel a reduction in lateral acceleration until the steady state is reached whereas the above mentioned overshoot is absent for the active vehicle and driver will just feel a smooth transition to steady state. The $A_y$ overshoot values reach 2% and 3.32 % of the maximum for the passive and zero for active vehicle for 0.4 and 0.8 g lateral accelerations, respectively.

Both the vehicles can be seen that they reach the steady state at almost the same time, the passive one overshoots and then settles to a steady state value whereas the active vehicle reaches a certain value and then gradually reaches steady state. The active vehicle is perceived to have a faster response as the lateral acceleration achieves 90% of its steady state value before the response is slowed down by the controller to reach the steady state in a stable manner. Here, it can be noticed that the two parts of the controller are working, the first initial rising part has more contribution from the non-linear part as there is a disturbance of the vehicle from straight line, as the vehicle starts settling towards steady state the non-linear part reduces and equivalent control is seen as the prominent controller.
6. Simulations, Tuning and Results

The yaw rate response of the step steer manoeuvres at 0.4 and 0.8g are presented in figure 6.5. There is almost no lag between the yaw rate response for passive and active vehicle, but overshoot in yaw rate can be seen for both of the passive vehicle responses with the overshoot getting worse from 7.14% to 10.71% with increase in lateral acceleration from 0.4 to 0.8g. The active vehicle has no overshoots and settles very quickly compared to the passive vehicle. The RWS system shows a 100% reduction in yaw rate overshoot.

The effect on the yaw rate response is magnified when studying the yaw acceleration (figure 6.6). The yaw acceleration curves show the gradient of yaw rate, whenever the curve crosses the x-axis the direction of this gradient changes. This change in direction is most noticeable and unpleasant for the driver. This x-axis crossing is very evident for the passive vehicle and it has more fluctuations with increase in lateral acceleration.
Figure 6.6: Step steer - yaw acceleration response

Figure 6.7 shows the front wheel steer angles for the passive and active vehicle along with the rear wheel steer response. The active vehicle has to be steered more to achieve the same lateral acceleration value as the passive one. When the rear wheels are steered in parallel to the front ones, a small lag in rear wheel steering response can be seen. The RWS response is a bit fluctuating before it reaches the steady state but these fluctuations are of minute order and probably will not be felt by the driver.

Figure 6.7: Step steer - steer angles response
### Table 4: Step steer metrics - 0.4g

<table>
<thead>
<tr>
<th>Metric</th>
<th>Symbol</th>
<th>Passive</th>
<th>Active</th>
<th>Unit</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration rise time</td>
<td>$T_{ay}$</td>
<td>0.22</td>
<td>0.11</td>
<td>s</td>
<td>- 50%</td>
</tr>
<tr>
<td>Yaw rate rise time</td>
<td>$T_\psi$</td>
<td>0.12</td>
<td>0.13</td>
<td>s</td>
<td>+ 8%</td>
</tr>
<tr>
<td>Lateral acceleration overshoot</td>
<td>$U_{ay}$</td>
<td>2.04</td>
<td>0</td>
<td>%</td>
<td>- 100%</td>
</tr>
<tr>
<td>Yaw rate overshoot</td>
<td>$U_\psi$</td>
<td>7.14</td>
<td>0</td>
<td>%</td>
<td>- 100%</td>
</tr>
<tr>
<td>Lateral acceleration lag</td>
<td>$\Delta_{ay}$</td>
<td>0.12</td>
<td>0.01</td>
<td>s</td>
<td>- 91.6%</td>
</tr>
<tr>
<td>Yaw rate lag</td>
<td>$\Delta_\psi$</td>
<td>0.02</td>
<td>0.03</td>
<td>s</td>
<td>+ 50%</td>
</tr>
</tbody>
</table>

### Table 5: Step steer metrics - 0.8g

<table>
<thead>
<tr>
<th>Metric</th>
<th>Symbol</th>
<th>Passive</th>
<th>Active</th>
<th>Unit</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration rise time</td>
<td>$T_{ay}$</td>
<td>0.22</td>
<td>0.1</td>
<td>s</td>
<td>- 54.5%</td>
</tr>
<tr>
<td>Yaw rate rise time</td>
<td>$T_\psi$</td>
<td>0.11</td>
<td>0.12</td>
<td>s</td>
<td>+ 9%</td>
</tr>
<tr>
<td>Lateral acceleration overshoot</td>
<td>$U_{ay}$</td>
<td>3.32</td>
<td>0</td>
<td>%</td>
<td>- 100%</td>
</tr>
<tr>
<td>Yaw rate overshoot</td>
<td>$U_\psi$</td>
<td>10.71</td>
<td>3.57</td>
<td>%</td>
<td>- 66.67%</td>
</tr>
<tr>
<td>Lateral acceleration lag</td>
<td>$\Delta_{ay}$</td>
<td>0.12</td>
<td>0</td>
<td>s</td>
<td>-100%</td>
</tr>
<tr>
<td>Yaw rate lag</td>
<td>$\Delta_\psi$</td>
<td>0.01</td>
<td>0.02</td>
<td>s</td>
<td>-100%</td>
</tr>
</tbody>
</table>
6.4 Sine Sweep

The sine sweep manoeuvre makes the vehicle to go through all the steering wheel frequencies which it would go through in normal on road and on track driving. The sine sweep manoeuvre is performed as discussed in section 3. The amplitude-frequency response plots reflect the capability of a vehicle to follow steering inputs at different frequencies.

Figure 6.8 shows the bode plot yaw rate gain and its phase delay with the steering wheel actuation frequency. It can be clearly seen that the yaw rate gain for passive vehicle has a higher magnitude for all frequencies compared to the active vehicle. This is of course because driver needs to steer more to achieve the same lateral acceleration as for the passive vehicle. Further, the yaw rate gain for the passive vehicle, first slowly increases with increasing frequency, reaches its peak at 1.5 Hz (resonance frequency) then starts reducing at a faster rate. Whereas, for the active vehicle, the peak is at 0 Hz and then it gradually decreases with increase in steering frequency. The gradient of this curve is much flatter compared to the same of the passive vehicle which is very desirable from the view point of steer-ability and predictability of the vehicle, especially at high speeds [27].

Now, analysing the phase delay plots, both the vehicles have about the same phase delays from 0 to 1 Hz range with both having the ideal delay of zero degrees at 0 Hz but then with the increase in frequency the delays also increase, with delays of passive vehicle increasing at a higher rate. The maximum delay is of about 90 degrees (5 Hz) and about 60 degrees (5 Hz) for the passive and active vehicles respectively.

Figure 6.8: Sine sweep - yaw rate response
Figure 6.9 shows the lateral acceleration gain for the same manoeuvre. The steady state lateral acceleration gain (gain at 0 Hz) for the passive vehicle is higher than that of the active vehicle due to the reasons explained previously. The lateral acceleration gain for the passive vehicle starts with a high value of 4.92 $m/deg.s^2$ at 0 Hz and follows a half cosine wave like form to end at a low of 0.068 $m/deg.s^2$ at 3.5 Hz and stays in its vicinity until 5 Hz. This leads to very low gains at high frequency inputs and high gains at low frequency inputs. This essentially means that the passive vehicle is very responsive for low frequency inputs and otherwise for higher frequencies and the driver needs to get used to this vehicle behaviour. On the other hand, the lateral acceleration gain curve is almost flat for the active vehicle with a difference of only 0.64 $m/deg.s^2$ from 0 to 5 Hz. This will lead to a very predictable response for the vehicle with active rear wheel steering.

A similar trend is seen in the phase delay curves of the active vehicle where the delay increases with increase in frequency with a maximum delay of about 30 degrees, whereas for the passive vehicle the delay starts from 0 at 0 Hz, increases until it reaches the maximum value of about 77 degrees at 2.5 Hz and then reduces to almost zero at 5 Hz. This reduction of phase delay might be attributed to the very low gains at same frequencies. Both the vehicles have an ideal delay of zero at 0 Hz.

![Bode Diagram](image)

**Figure 6.9: Sine sweep - lateral acceleration response**

Tables 6 and 7 show the lateral acceleration and yaw rate gains for specific frequencies.
Table 6: Sine sweep : Lateral acceleration gains

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4.92</td>
<td>2.83</td>
</tr>
<tr>
<td>1</td>
<td>4.41</td>
<td>2.77</td>
</tr>
<tr>
<td>2</td>
<td>2.2</td>
<td>2.63</td>
</tr>
<tr>
<td>3</td>
<td>0.96</td>
<td>2.50</td>
</tr>
<tr>
<td>4</td>
<td>0.66</td>
<td>2.32</td>
</tr>
<tr>
<td>5</td>
<td>0.77</td>
<td>2.19</td>
</tr>
</tbody>
</table>

Table 7: Sine sweep : Yaw rate gains

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Passive</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.148</td>
<td>0.087</td>
</tr>
<tr>
<td>1</td>
<td>0.161</td>
<td>0.084</td>
</tr>
<tr>
<td>2</td>
<td>0.157</td>
<td>0.077</td>
</tr>
<tr>
<td>3</td>
<td>0.122</td>
<td>0.068</td>
</tr>
<tr>
<td>4</td>
<td>0.095</td>
<td>0.058</td>
</tr>
<tr>
<td>5</td>
<td>0.078</td>
<td>0.053</td>
</tr>
</tbody>
</table>

6.4.1 Pole-Zero Analysis

The transfer function of the system can be estimated from the sine sweep data presented in figures 6.8 and 6.9. The poles and zeros of this transfer function can be calculated and provide useful insights into the systems response.

According to control theory, if the system has real poles in the left-half of the s-plane then the system will have an exponentially decaying component in its homogeneous response. The poles far from the origin in the left-half plane correspond to components that decay rapidly whereas poles near to the origin correspond to slowly decaying components. Further, poles at the origin means that it has a component that is constant in amplitude and defined by the initial conditions.

A complex conjugate pole pair in the left-half of the s-plane a will have an oscillating component and any poles on the right hand side of the s-plane will either have an exceptionally increasing response (poles on positive real axis) or have an increasing oscillating component (complex conjugate poles on the right hand side of the s-plane). Therefore, poles on the left-hand side of the s-plane dictate stability and poles on the right hand side dictates instability [31] [32]. Figure 6.10 summarises the above.
Figure 6.10: Pole placement and stability

Figure 6.11 shows a pole-zero map of the transfer functions for the yaw rate to the front steer angle of the passive and active vehicle. This system has 3 poles and 1 zero similar to a 2DoF bicycle model.

Figure 6.11: Sine Sweep - Yaw rate response - Pole zero map. Poles (x) and zeros (o)
It can be seen that the passive vehicle has a pair of complex conjugate poles and one pole on the negative real axis whereas the active vehicle has two poles on the negative real axis and the remaining one at the origin. This implies that both the active and passive systems have poles on the left hand and hence will be stable. But the passive vehicle will have an oscillating but damped component while, the active vehicle will have a response with an exceptionally decaying component. And an exceptionally decaying response is perceived as more stable over an oscillating response as it reaches steady state faster and has no fluctuations.

Similarly, Figure 6.12 shows a pole-zero map of the transfer functions for the lateral acceleration to the front steer angle of the passive and active vehicle. The system has 3 poles and 3 zeros similar to the 2DoF bicycle model. It can be noticed here as well that the passive vehicle has a pair of complex conjugate poles and one pole on the negative real axis, whereas two poles of the active vehicle have poles on the negative real axis with one pole on the origin. This implies that the active vehicle will be more stable and less oscillatory.
6.5 Pulse Steer

The pulse steer manoeuvre is performed as stated in section 3. This manoeuvre mimics a sudden disturbance to the system such as a side wind gust. The vehicle is made to go through many frequencies in fraction of a second. Here, it is interesting to note how the vehicle settles back to its initial states after the said disturbance.

Figure 6.13a shows the lateral acceleration response of the two vehicles. It can be clearly seen that both vehicles reach same peak lateral acceleration but the active vehicle reaches the peak faster than the passive one. Further, it settles back to the steady state very quickly and with no overshoots (fluctuations), whereas, for the passive vehicle the lateral acceleration reduces slowly to steady state with both fluctuations and overshoots.

Figure 6.13b shows the yaw rate response of the two vehicles. It can be noticed that both the vehicles reach their peak yaw rate at the same time but the yaw rate of the active vehicle is almost half of that of the passive vehicle which is attributed to higher yaw dampening provided by the parallel steer of the RWS system. Further, the RWS system settles back faster to initial states but also eliminates any overshoots.

![Pulse Steer - Lateral Acceleration](image1)

(a) Lateral acceleration response

![Pulse Steer - Yaw Rate](image2)

(b) Yaw rate response

Figure 6.13: Pulse steer - response

Figure 6.14b depicts the rear wheel steering response to the pulse disturbance, along with parallel steer configuration. It can be noticed how the RWS controller fluctuates its control output to follow the reference and reduce any overshoots.
6. Simulations, Tuning and Results

(a) Yaw acceleration response

(b) Front and rear steer angles

Figure 6.14: Pulse steer
6.6 Single Sine Steer

The single sine steer test is a close approximation of a lane change, therefore is important to access this everyday driving scenario [28].

The single sine steer test describes the transient drivability of the vehicle by means of a sinusoidal steering angle input. The steering angle input and the vehicle response of this test are based upon double lane changes but is fundamentally different from the severe lane change manoeuvres described in [29] (commonly known as ISO Double lane change) as the former is an open loop manoeuvre whereas the latter is a closed loop manoeuvre where the driver needs to follow a certain path at high speeds which result in high lateral accelerations. Simply put, the single sine steer mimics double lane changes in day to day driving while the ISO Double lane change mimics extreme situations such as obstacle avoidance.

Figure (6.15a) shows the lateral acceleration response. It can be clearly seen that the passive vehicle lags behind to that of active one, whereas, there is no lag between the yaw rate response, figure (6.15b). It is also interesting to note that the passive vehicle shows some overshoot for both lateral acceleration and yaw rate while returning back to straight line driving.

The cross correlation of first and second half waves shows that the lateral acceleration lags for active vehicle reduces by 62.5 % and 55.55 % respectively when compared to the passive one, this is again attributed to slip angles generated at the rear wheels due to the rear wheel steering. Whereas, the time lag amplification (TLA) in second half wave is increased by 33 % for the active vehicle compared to 12.5 % for the passive. This may be due to the 37.9 % decrease in lateral acceleration gain in the active vehicle as the driver now need to use a higher steering value to reach the same lateral acceleration as passive vehicle.

![Image](image.png)

**Figure 6.15: Single sine steer**

The yaw rate cross correlation shows that there is no change in yaw rate lag for the first peaks

46
of both vehicles but the lag increases by 20% for the second half of the active vehicle. This increase in second half wave lag is peculiar and is only 1 millisecond, this can also be attributed to decrease in yaw rate gain for the active vehicle (-33.2%) which also results in 20% time lag amplification (TLA) in yaw rate.

Table 8: Single sine steer

<table>
<thead>
<tr>
<th>Metric</th>
<th>Symbol</th>
<th>Unit</th>
<th>Passive</th>
<th>Active</th>
<th>Percentage Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral acceleration</td>
<td>(A_y)</td>
<td>m/s²</td>
<td>3.78</td>
<td>3.79</td>
<td>0</td>
</tr>
<tr>
<td>Yaw rate</td>
<td>(\dot{\psi})</td>
<td>rad/s</td>
<td>0.14</td>
<td>0.138</td>
<td>0</td>
</tr>
<tr>
<td>Lateral acceleration gain</td>
<td>(G_{ay})</td>
<td></td>
<td>0.029</td>
<td>0.018</td>
<td>-37.9</td>
</tr>
<tr>
<td>Yaw rate gain</td>
<td>(G_{\dot{\psi}})</td>
<td></td>
<td>0.592</td>
<td>0.395</td>
<td>-33.2</td>
</tr>
<tr>
<td>Lat acceleration lag - peak 1</td>
<td>(\Delta_{ay1})</td>
<td>s</td>
<td>0.08</td>
<td>0.03</td>
<td>-62.5</td>
</tr>
<tr>
<td>Lat acceleration lag - peak 2</td>
<td>(\Delta_{ay2})</td>
<td>s</td>
<td>0.09</td>
<td>0.04</td>
<td>-55.55</td>
</tr>
<tr>
<td>Yaw rate lag - peak 1</td>
<td>(\Delta_{\dot{\psi}1})</td>
<td>s</td>
<td>0.05</td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>Yaw rate lag - peak 2</td>
<td>(\Delta_{\dot{\psi}1})</td>
<td>s</td>
<td>0.05</td>
<td>0.06</td>
<td>+20</td>
</tr>
<tr>
<td>Lateral acceleration TLA</td>
<td>(A_y^{TLA})</td>
<td></td>
<td>1.125</td>
<td>1.33</td>
<td>+18.2</td>
</tr>
<tr>
<td>Yaw rate TLA</td>
<td>(\dot{\psi}^{TLA})</td>
<td></td>
<td>1</td>
<td>1.2</td>
<td>+20</td>
</tr>
</tbody>
</table>

TLA: Time Lag Amplification
6. Simulations, Tuning and Results

6.7 Ramp Steer

The slowly increasing steering input of ramp steer manoeuvre takes the vehicle through its quasi static states. The vehicle behaviour can be evaluated at every point assuming it is at steady state at those points. This manoeuvre helps to identify the linear operating region of the vehicle and it’s transition point to the non-linear region along with its physical limits.

Figure 6.16 shows the amount of steering wheel angle the driver needs to input to achieve certain lateral acceleration values. It can be seen that the driver needs to input more steering wheel angle in the active vehicle to achieve the same lateral accelerations as that of the passive vehicle. This attributed to the fact that the radius of turn increases when rear wheels are steered parallel to the front wheels, therefore the active vehicle has a more understeering behaviour than that of the passive vehicle. Further, both the vehicles have the same physical limit of lateral acceleration.

![Steering Wheel Angle v. Lateral Acceleration](image)

Figure 6.16: Ramp steer - lateral acceleration response

Figure 6.17 shows the vehicle’s side slip response with increasing lateral acceleration. Both the passive and active vehicle start off at zero side slip (straight line driving), the passive vehicle shows a proportional increase in side slip with increase in lateral acceleration, whereas, the active vehicle stays in the neighbourhood of zero side slip angle as the controller is designed to follow reference of zero side slip angle, at very high lateral accelerations the side slip curve flattens out while its magnitude keeps on increasing for the passive vehicle. It is interesting to note that the direction of the side slip produced in active and passive vehicle are opposite in direction.
Figure 6.17: Ramp steer - side slip angle v. lateral acceleration response

Figure 6.18 shows how the rear wheel steering angle varies with lateral acceleration. The RWS angle increases linearly with lateral acceleration until 60% of maximum lateral acceleration and then increases at a faster rate. This shows the working of the controller as it is dependent on the errors of yaw rate and body side slip, which increase at high lateral accelerations.

Figure 6.18: Ramp steer - rear wheel steering response
6.8 **On-Centre Weave Test**

The on-centre weave test is performed as stated in section 3.6. The following figures compare the characteristics of on-centre steering feel for the passive and active vehicle.

Figure 6.19 shows the variation of steering wheel torque with steering angle for values up to 0.2g of lateral acceleration. It can be seen that both the active and passive vehicles require almost the same amount of steering wheel torque to achieve the same lateral acceleration although the required steering angle increases. Steering stiffness is defined as the slope of the curves over a range of 10% of peak steering wheel angle. It can be noticed from figure 6.19 and table 9 that the steering stiffness felt by the driver reduces by 14.21% when the RWS system is active.

Further, a increase of 28.12% in steering friction can be seen for the active system when compared to the passive one. This means that the driver will feel the steering to be 'heavier' around the on-centre area and will find it easier to locate the centre position of the steering wheel.

Figure 6.19: On-centre weave - steering wheel angle vs. steering wheel torque

Figure (6.20) shows the variation of steering wheel angle with lateral acceleration. Here, the active vehicle needs to steer more to achieve the same lateral acceleration as the passive vehicle. This is consistent with the reduced lateral acceleration gain with rear wheels steered parallel.
The yaw rate as a function of steering wheel angle curve (figure 6.21) provides the yaw rate response gain of the vehicle. The active vehicle has a significant less (-17%) yaw rate response gain as compared to the passive car. Again, this is a result of parallel steer of the rear wheels. But, analysing the yaw rate with steering wheel torque curve (figure 6.22), it can be inferred that both the active and the passive vehicles give a similar torque feel yet the active vehicle has a higher (+34.46%) response dead-band compared to the passive vehicle.
Figure 6.22 shows the lateral acceleration variation with steering wheel torque. The lateral acceleration at 0 Nm is an indicator for returnability, basically, at this lateral acceleration the steering wheel, if released, will ‘stick’ as the torque is 0 Nm. This value is 6% more for the active vehicle implying that it has a slightly better returnability and a slightly ‘heavier’ feel to it when compared to the passive vehicle.

Steering wheel torque at $1\text{ m/s}^2$ is a measure of steering effort. When compared, the steering effort for the active vehicle increases by 4% and 5% while steering towards and away from the centre, respectively. While the steering wheel torque at $0\text{ m/s}^2$ indicates the coulomb friction in the steering system, which increases by 6.17 % for the active vehicle.

Further, the torque gradient at $1\text{ m/s}^2$ is an indicator of ‘road feel’. It represents the handling of the vehicle during highway driving which includes nominally straight-line driving and the negotiation of large radius bends at high speeds but low lateral accelerations.

From table 9, it can be seen that there is not much of a difference between the passive and active vehicle for lateral acceleration vs. steering wheel torque curve.
6. Simulations, Tuning and Results

Figure 6.23: On-centre weave - lateral acceleration vs. steering wheel torque

Table 9: On-centre weave test

<table>
<thead>
<tr>
<th>Metric</th>
<th>Unit</th>
<th>Passive</th>
<th>Active</th>
<th>Percentage change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering wheel angle vs. steering wheel torque</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steering stiffness</td>
<td>Nm/deg</td>
<td>0.1055</td>
<td>0.0905</td>
<td>- 14.21</td>
</tr>
<tr>
<td>Angle hysteresis</td>
<td>Nm</td>
<td>0.929</td>
<td>1.383</td>
<td>+ 48.86</td>
</tr>
<tr>
<td>Steering friction</td>
<td>Nm</td>
<td>0.096</td>
<td>0.123</td>
<td>+ 28.12</td>
</tr>
<tr>
<td>Lateral acceleration vs. steering wheel torque</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lateral acceleration at 0 Nm</td>
<td>m/s²</td>
<td>+0.33</td>
<td>+0.35</td>
<td>+ 6.0</td>
</tr>
<tr>
<td>Torque at 0m/s²</td>
<td>Nm</td>
<td>0.178</td>
<td>0.189</td>
<td>+ 6.17</td>
</tr>
<tr>
<td>Torque at 1m/s²</td>
<td>Nm</td>
<td>0.519</td>
<td>0.540</td>
<td>+ 4.04</td>
</tr>
<tr>
<td>Torque at −1m/s²</td>
<td>Nm</td>
<td>-0.433</td>
<td>-0.456</td>
<td>+ 5.3</td>
</tr>
<tr>
<td>Torque gradient at 1m/s²</td>
<td>m/s²</td>
<td>0.3695</td>
<td>0.38</td>
<td>+ 2.84</td>
</tr>
<tr>
<td>Torque hysteresis</td>
<td>m/s²</td>
<td>0.6691</td>
<td>0.7009</td>
<td>+ 4.75</td>
</tr>
<tr>
<td>Lateral acceleration hysteresis</td>
<td>Nm</td>
<td>0.249</td>
<td>0.270</td>
<td>+ 8.43</td>
</tr>
<tr>
<td>Yaw rate vs. steering wheel angle</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yaw rate response gain</td>
<td>s⁻¹</td>
<td>0.01</td>
<td>0.0083</td>
<td>- 17.0</td>
</tr>
<tr>
<td>Yaw rate vs. steering wheel torque</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Response dead-band</td>
<td>Nm</td>
<td>0.177</td>
<td>0.238</td>
<td>+ 34.46</td>
</tr>
</tbody>
</table>
7 Conclusion

This section summarises the results discussed in section 6 as attributes which are influenced by the RWS system.

Overall, the active rear wheel steering system improves the response of the vehicle even though it has a very small actuation area (less than 3 degrees of rear wheel steer). The active vehicle is perceived to be more agile, stable and predictable but has a more under-steering characteristic (Figure 6.16).

One of the major drawbacks of the RWS system is that at higher speeds when the rear wheels are steered parallel to the front wheels, the effective turning radius increases and therefore the driver needs to provide more steering wheel input. Hence, the active vehicle has lower yaw rate and lateral acceleration gains. One solution to this problem can be to decrease the steering ratio of the active vehicle.

Further, the sliding mode algorithm chosen for the rear wheel steering controller is simple to implement and robust. Once the gains were set according to the desired step steer response, there was a little need to tune the controller further. The robustness of the controller can be seen by how the controller controls the vehicle in case of an external disturbance and helps it to come back to steady state (Section 6.5).

As the RWS system mainly effects the lateral driving dynamics of the vehicle along with steering feel, terms which are regularly used to define driving dynamics such as agility, stability and predictability along with effects of external disturbances are discussed in detail in the following subsections.

7.1 Agility

Agility of a vehicle is defined as its ability to move quickly and easily when given a command by the driver. The RWS system makes the vehicle overall response faster and the vehicle should feel more sporty and agile to drive. The major improvement is seen in the lateral acceleration response, the active vehicle is about 50% quicker than the passive one when transitioning from straight line driving to steady state conditions (Tables 4 and 5).

Further, the active vehicle has a faster response when transitioning to straight line driving as well, this can be seen from figure 6.13a. During lane change the vehicle responds 55 to 60% quicker to the driver’s steering input, this is shown by the lateral acceleration lags calculated for a single sine steer test (Table 8).
7.2 Stability

The functioning of the RWS system is best seen in terms of transitional stability. Transitional stability of a vehicle can be defined as its ability to transition from one steady state to another, such as straight line driving to cornering, without any undesired effects like overshoots and oscillations.

The transitional stability provided by the active RWS system can be seen by the reduction or even elimination of overshoots when the vehicle transitions. Transition of the vehicle from straight line driving to cornering is emulated by the step steer manoeuvre, tables 4 and 5 show that there is a 100% reduction of overshoots for the lateral acceleration for moderate and high lateral accelerations (0.4g and 0.8g respectively), whereas for the yaw rate response the active system provides a 100% reduction of overshoots for moderate lateral accelerations and a 66% reduction for high lateral accelerations. The driver can expect the vehicle to follow a curvature without feeling the need to reduce the steering input to control the overshoot.

Further, stability can also be evaluated through the pole-zero maps of the vehicles (Figures 6.11 and 6.12). The pole-zero maps of both lateral acceleration and yaw rate gain show that both active and passive vehicles are in the stable, left hand side of the s-plane, but the passive vehicle has complex conjugate poles which make the system (hence the vehicle) to exhibit an oscillatory (damped) behaviour before settling. Whereas, the active vehicle has no complex conjugate poles and all of it poles are either at the origin or on the negative real axis, meaning the active vehicle will settle without any oscillations.

7.3 Predictability

A vehicle is said to be predictable when it behaves the way as expected by the driver at all speeds and operating points. Of course, the vehicles response will change with frequency of steering input and speed but this variation should not be abrupt and unpredictable.

Predictability can be assessed by studying the frequency response curves of the lateral acceleration and yaw rate gains (figure 6.8 and 6.9) it can be seen that the active vehicle has a very linear response, the magnitude of gain reduces and phase lags increase in a very linear fashion whereas the same curves for the passive vehicle has peaks (resonant frequency) and high degree of non-linearity which is natural for a passive vehicle.

7.4 Effect of external disturbances

Disturbances are a common occurrence in day to day or on track driving, these disturbances can be caused by a side wind gust, a patch of uneven road/ice or potholes and in worst cases due to impact from another vehicle or the environment.

The pulse steer manoeuvre gives an insight of how disturbances like these influence the vehicle. As the driver would like to continue the manoeuvre which was being performed even
after the disturbance, the ease of returnability is an important parameter to consider when considering the effect of disturbances.

Figures 6.13a and 6.13b depicting the lateral acceleration and yaw rate response for pulse steer manoeuvre respectively, show that while returning from instability (i.e. pulse input) the passive vehicle overshoots before settling back to straight line driving whereas the overshoots are absent in case of the active vehicle. This absence of overshoots points out that the rear wheel steering system is more stable in case of external disturbances.

7.5 Steering Feel

The most important curve to evaluate the steering feel of a vehicle is the steering wheel angle versus steering wheel torque curve (Figure 6.19). The on-centre steering feel of the active vehicle is different from that of the passive vehicle as it has a lower steering stiffness along with higher steering friction and hysteresis. While having a higher steering friction along with reduced steering stiffness means that it will be easier to locate the centre position of the steering wheel.

When comparing the curves of lateral acceleration and yaw rate with steering wheel torque there is a minute difference between both the vehicles. But, the major difference is noticed while analysing the curves with steering wheel angle. This is because of the reduced yaw rate and lateral acceleration gains with rear wheels steered parallel to the front ones. Therefore, it can be said that the active vehicle would have a closer steering feel to the passive vehicle if it had a lower steering ratio in the steering gearbox.
8 Future Work

The findings from the work carried out in this thesis highlight the potential improvement in the performance of a vehicle with an active rear wheel steering system working along with a conventional front wheel steering.

Due to the fact that some simplifications were made in this thesis and limited availability of time along with conclusions drawn from the results, potential future work can be highlighted and is listed below.

- **Actuator Dynamics**
  The effect of actuator dynamics was not considered in this thesis but is very important to be accounted for and studied as the actuators might introduce some delays and disturbances into the system.

- **Hardware in Loop testing**
  Hardware in loop (HIL) testing was beyond the scope of this thesis. Therefore, after incorporating actuator dynamics into the system the controller should be tested in a HIL environment. This will bring out any shortcomings of the system before it is incorporated in an actual vehicle.

- **Variation of Steering ratio**
  The steering ratio should be varied and its affect on the yaw rate and lateral acceleration gains should be studied. This should be done with an aim to match the driving feel of the active vehicle to the passive one.

- **Varying reference model**
  The reference model has parameters such as the understeer gradient and time lags of lateral acceleration and yaw rate. It will be interesting to study how the controller performs when these are varied.

- **In-depth study of steering feel**
  Steering feel evaluation in this thesis was brief and limited to on-centre steering feel. It is important and interesting to compare and evaluate steering feel over the whole operating range of the vehicle.

- **With other control systems**
  The aim of this thesis was to evaluate and compare the effect of rear wheel steering system only, but it will be interesting to study how the developed system works along with the other control systems of the vehicle such as the torque vectoring system.
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