Bending modes analysis in atmospheric flight for heavy launcher

Author: Valentin POTHIER

KTH supervisor: Ulf RINGERTZ
ArianeGroup supervisor: Emmanuel CHAMBON

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Abstract

This report presents the bending modes study conducted on a heavy launcher. The controller of the launcher takes as inputs the attitude and attitude rate measurements given by the Inertial Measurement Unit (IMU). Since the bending modes generate measurement errors at the IMU location, the study of deformations due to these bending modes is critical to assess the stability of the launcher during the atmospheric flight phase. The goal of this master thesis project is to detect and then select the most excitable bending modes among the large number of modes provided by a detailed structural analysis of the launcher. Only these relevant modes will be later used to generate reduced dynamical models of the launcher in order to efficiently design an appropriate controller. Indeed, considering all the bending modes will dramatically increase the calculation time and will not significantly improve the representativeness of the model at the control law frequency range of interest. To reach this objective, an extended excitability (the maximum of the module of the transfer function between the effective deflection and the considered mode generalized coordinate transported at the IMU location) is defined and computed for each mode. A criterion has been implemented to choose only the relevant modes. The sensitivity study conducted during this master thesis project has shown that with around 20 modes over 200, one can reproduce the dynamic behavior of the complete system.

1. Introduction

Contrary to civil planes, launchers are naturally unstable since the center of mass is behind the center of aerodynamic forces. So designing a control system for a launcher during the atmospheric flight is a very complex control problem. The controlled system needs to meet many design requirements: stability (stabilization of unstable rigid dynamic and bending modes), performance (guidance tracking, structural load minimization) and robustness (physical parameter uncertainties and accommodation to multiple launcher configurations). The controller is designed at a flight instant (maximum pressure instant for example) from linear dynamical models of the launcher (several models are used to take into account the potential variability on parameters such as the payload mass or the trust law). The controller output is the commanded nozzle deflection. The controller inputs are the measured attitude and attitude rate (which are the combination of the rigid-body measurement and an error due to bending modes). To avoid unstable control laws, the linear models used to generate the controller need to represent the launcher as accurately as possible. One of the highly challenging parts is to correctly model the bending modes of the flexible launcher.

The aim of this master thesis project is to detect and then select, among the 200 modes identified during the structural analysis studies, the bending modes which have a significant impact on the dynamics of the launcher over the controller bandwidth. Indeed, considering all the bending modes in the linear dynamical models used for the controller tuning will increase the calculation time and will not significantly improve the representativeness of the model.

2. Dynamic model of a launcher

1. Assumptions

The following assumptions have been taken for the bending modes study:

H1: it is assumed that all the nozzles have the same motion. They are gathered in a fictive nozzle which thrust equals the sum of the thrusts, which mass equals the sum of the masses…

H2: only plane motions are considered (the pitch and yaw axis are studied separately)

H3: there is no interaction between the bending modes

H4: the small angles approximation is used

H5: the sloshing modes are not considered at all (they have a more significant impact on the launcher dynamics during the exo-atmospheric flight phase)

2. Rigid launcher dynamics

A first approximation of the rigid dynamics of a launcher around its center of gravity \( \mathbf{G} \) is given by the equation:
\[ \ddot{\theta}_R = A_6 \dot{\theta}_R + K_1 \beta \]  

(1)

With:

- \( \theta_R \): attitude angle [rad]
- \( \beta \): nozzle deflection angle [rad]
- \( A_6 \): coefficient of aerodynamic efficiency [s\(^{-2}\)]
- \( K_1 \): coefficient of thruster efficiency [s\(^{-2}\)]

**3. Flexible launcher dynamics**

The real motion of the launcher is composed of a rigid motion (described in the previous section) and a flexible motion along the longitudinal axis of the launcher (see figure 2). The total displacement and rotation of the launcher at an abscissa point \( x \) can be written as the sum of the rigid (R) and flexible (F) motions:

\[ \theta(x, t) = \theta_R(x, t) + \theta_F(x, t) \]
\[ z(x, t) = z_R(x, t) + z_F(x, t) \]

The flexible motion is usually written as a linear combination of single deformations \( h_i(x) \).
\[ z_F(x, t) = \sum_{i=1}^{N} q_i(t) h_i(x) \]

\[ \theta_F(x, t) = -\sum_{i=1}^{N} q_i(t) h'_i(x) \]

The values \( q_i(t) \) are the generalized coordinates and represent the shape of the mode \( i \) at the instant \( t \). In the following, we will only be interested in the deformations at the thruster and the IMU locations (since the displacements and rotations at the IMU location are “viewed” by the controller). These values will be noted \( h_T \) and \( h_M \). In figure 2, the flexible rotation \( \theta_{M,i} \) and displacement \( h_{M,i} \) at the IMU location for the mode \( i \) are presented.

![Figure 2- Representation of bending modes (for pitch axis)](image)

### 4. Modelling of bending modes

It is common to model the bending modes of a launcher with a second order transfer function. The equation of a free oscillator with a natural angular frequency \( \omega_i \) and damping \( \xi_i \) is given by:

\[ \ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_i^2 q_i = 0 \]
The launcher is subject to the thrust force at the center of gravity of the nozzle. Therefore, the equation of the forced damped oscillator changes and becomes (demonstration is given in [R1]):

\[ \ddot{q}_i + 2\xi_i\omega_i \dot{q}_i + \omega_i^2 q_i = -\omega_i^2 (P_C h_{T,i} \beta_R + M_T L_T h_{T,i} \ddot{r}_R) \]  

(2)

With:

- \( P_C \): controlled thrust force in the plane (OXZ) [N]
- \( M_T \): mass of the nozzle [kg]
- \( L_T \): length between the center of rotation and the center of gravity of the nozzle [m]
- \( \beta_R \): deflection angle performed by the nozzle (it can defer from the commanded deflection due to biases) [rad]

A typical structural analysis delivers two types of bending modes: the “followed” and “unfollowed” modes. The “followed” modes have the highest deformations at the IMU location and have a physical interpretation and so can be followed during the atmospheric flight phase. The “unfollowed” modes have no particular physical interpretation. In all cases, all the followed modes (around 5 modes) will be taken into account to generate the linear models of the launcher since they are the most difficult to control.

3. Launcher flight control system

1. Block diagram

The block diagram of the controlled launcher is given in figure 3. The block diagram is composed of the following parts:

- The rigid launcher
- The \( N \) bending modes
- The Thrust Vector Control (TVC) (i.e. the nozzle)
- The controller
- The measurement unit (IMU)
The “Control team” receives the transfer functions of the TVC and the IMU respectively from the “Thrust Vector Control” and “Navigation” teams. The transfer functions of the rigid and flexible launcher are given by equations (1) and (2). The controller is then tuned with the $H_\infty$ method (not detailed in this report) but widely used during this master thesis project to generate controlled models and perform validations.

2. Controller structure

The controller is composed of two different parts. The first one, the “rigid controller”, contains the gains which stabilize the rigid launcher (in a low frequency range). The second one is a notch filter designed to cut the amplitudes of the bending modes. Without this filter, the launcher would be unstable.

4. Excitability computation

1. Definition

An efficient way to measure the criticality of a mode on the launcher dynamics is to compute its excitability. The mode excitability $Q_i$ is the maximum of the module of the transfer function between the effective deflection ($\beta_R$) and the considered mode generalized coordinate transported at the IMU location ($\theta_F$) or, in other words, this is the maximum of the module of the transfer function between the effective deflection and the measurement error created by the mode $i$. So, the excitability is given by:

$$Q_i = \max_\omega \left| \frac{h'_{M,i}}{\beta_R} q_i(j\omega) \right|$$

The TVC transfer function acts as a low-pass filter. So, the bending modes with a high excitability but at a high frequency will be cut by the TVC. Therefore I decided to compute the excitability of the entire open-loop to take into account this phenomenon. The entire
open-loop is composed of the “rigid controller” $K$, the actuator (TVC) and the flexible launcher dynamics (not including the rigid dynamics). The transfer function formed can be written as:

$$H_i(s) = K \ast TVC(s) \ast L_{F,i}(s)$$

The excitability computation of this transfer function allows detecting the most excitable modes which have to be filtered by the controller in order to ensure the stability of the launcher.

Note: the transfer function of the flexible mode $L_{F,i}(s)$ is defined by (see equation (2)):

$$L_{F,i} = h^t_{M,i} \frac{q_i}{\rho_R}$$

2. Computation

The software developed during this master thesis project is dedicated to the computation of the modes excitability. The transfer function $H(s)$ is formed from the launcher database (thrust law, displacements at the nozzle and IMU locations, damping coefficients, natural frequencies...). The peak of the Bode diagram of the transfer function $H_i(s)$ is computed (and also the frequency where this peak takes place). It has been decided to select only the unfollowed modes with a peak frequency in the controller bandwidth. Beyond this limit, the resonances due to bending modes are filtered by the controller (it acts as a low-pass filter). The $N$ most excitable unfollowed modes (i.e. the $N$ unfollowed modes with the highest peaks) are stored.

The excitability results are displayed figure 4.

![Figure 4 - Results of the excitability computation at one instant for $N = 10$ unfollowed modes. One point represents the maximum value of the module of the transfer function $H_i(s)$ and the frequency where it takes place](image)
One can see that the followed modes have the lowest peak frequencies but the largest excitabilities. So they are the most critical in terms of stability (the peaks are in the controller bandwidth and create large deformation at the IMU location). Since the unfollowed modes excitabilities are lower and appear at higher frequencies, they have less impact on the launcher stability. This observation explains why the followed modes are systematically considered by the Control team to generate linear models.

A sensitivity study has been conducted to find a suitable number of unfollowed modes N to save. N has to be as small as possible (to reduce computation time for models generation or simulations) but big enough to fit the real behavior of the launcher.

### 3. Sensitivity study

In order to validate the selection of the most excitable unfollowed modes, some simulations are performed. The frequency responses (Bode diagram of the open-loop) of the entire system (rigid controller + TVC + rigid launcher + all the followed modes + N unfollowed modes + delays) are compared for various values of N. The values of N are 0 (only the followed modes), 5, 10, 15 and 200 (all the unfollowed modes). The results are displayed in figure 5.

**Note:** Since the requirements for bending modes are specified on the gain of the open-loop, it is not necessary to try to fit the phase diagram.

![Figure 5 - Bode diagram of the open-loop transfer function. Blue: N=0, Orange: N=5, Yellow: N=10, Purple: N=15, Green: N=200](image)

With a number of unfollowed modes $N = 15$, the reduced model (in purple) fits the peaks of the complete model (200 unfollowed modes in green) over the controller bandwidth. The algorithm developed detects only the peaks, not the lowest points of the Bode diagram. That
explains why the reduced model (with N unfollowed modes) over-estimates the amplitude of the Bode diagram between two peaks.

5. Results and discussion

The main objective of this study was to detect and then select the most excitable bending modes of a launcher. This would enable to generate dynamical models with the lowest number of modes as possible and so simplify the controller tuning while being representative of the flexible dynamics over the controller bandwidth. This goal has been achieved by computing the extended excitability of each mode (i.e. the excitability of the whole open-loop). Only the unfollowed modes with the highest excitabilities and with a peak gain in the controller bandwidth are selected. A sensitivity study has shown that the behavior of the complete system can be reproduced (in the controller bandwidth) with around 20 modes (5 followed and 15 unfollowed modes).

Further studies should focus on the maximum excitabilities that can be handled by the controller (while keeping the same performance). The excitability depends directly on the damping coefficient. This problem can be solved by using the co-design method (or integrated design method): one can define the damping coefficient as a variable of an optimization problem and try to minimize this. This would enable to reduce the stiffness and so the mass of the launcher. This study has been conducted during the last months of this internship but is not presented in this report.

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