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Optimal process design with simulation-based optimization

The pellet induration process at LKAB

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Abstract

Nowadays, it has become crucial to transform company production processes in order to reduce carbon emissions. Therefore, the company LKAB is working to make the production process of iron ore pellets fossil-free. This thesis project focuses on the pellet induration stage and addresses the mathematical optimization of the process. In particular, the idea is to combine state-of-the-art optimization algorithms with simulation software.

This thesis aims to address the problem of multi-objective optimization within the context of simulation-based scenarios, i.e., the aim is to study Simulation-Based Multi-Objective Optimization Problems.

The primary focus of this thesis is to thoroughly investigate and compare three different algorithms applied to two distinct problem formulations. By doing so, we aim to gain valuable insights into the suitability of different approaches and evaluate the algorithms' performance in achieving the desired objectives.

Keywords

Optimization, Multi-Objective, Simulation-based

Sammanfattning

Nuförtiden har det blivit mycket viktigt att förändra företagens produktionsprocesser för att minska koldioxidutsläppen. Företaget LKAB arbetar för att göra produktionsprocessen av järnmalmspellets fossilfri. Detta examensarbete fokuserar på pelletiseringsfasen och behandlar den matematiska optimeringen av processen. Mer specifikt, tanken är att kombinera toppmoderna optimeringsalgoritmer med simuleringsprogramvara.

Syftet med examensarbetet är att studera problem med flermålsoptimering inom ramen för simuleringsbaserade scenarier, dvs. syftet är att studera ett simuleringsbaserat flermålsoptimeringsproblem.

Det primära fokuset i examnesarbetet är att grundligt undersöka och jämföra tre olika algoritmer som tillämpas på två distinkta problemformuleringar. Genom att göra detta vill vi få värdefull insikt om lämpligheten av de olika strategierna och utvärdera algoritmernas prestanda för att uppnå de önskade målen.

Nyckelord

Optimering, flermålsoptimering, simuleringsbaserad

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Stockholm, June 2023
Giulia Grasso

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Chapter 1

Introduction

In today's society, there is an increased focus on reducing carbon emissions that cause damage to our planet. LKAB is such a company that is striving to become carbon neutral. It is a mining company in northern Sweden that sells iron ore, minerals and special products. This thesis focuses on the production process of pellets and how carbon emissions can be reduced through optimal process design. In particular, the idea is to study optimization algorithms and combine them with software that simulates the pellet induration process.

This thesis concern LKAB's facilities at the Kiruna site and focuses on the pelletizing process in the grate-kiln plants. In particular, the focus is on the pellet induration process. This thesis aims to maximize the bed temperature and oxidation degree at specific locations in the machine while minimizing fuel consumption during the process.

1.1 The pelletizing process

The pelletizing process is complex and requires a series of steps. The process starts with the work of miners extracting iron ore from mines. However, it is not pure, and therefore it is necessary to separate what contains ferrous material from what is merely gangue and, therefore, not useful for our purposes. Once the material is extracted from the mines, it is taken to the sorting plants in Kiruna, where the first separation between the iron-containing material and the gangue takes place.

The material is then distributed to the sorting plants in Kiruna, Svappavaara and Malmberget. Each of these locations has a pelletizing plant that produces

pellets according to market requirements, i.e. each plant produces different types of pellets according to the market for which they are intended. The Kiruna site is the one that provides materials to the markets in Europe, North Africa, the Middle East, South-East Asia and LKAB minerals' customers.

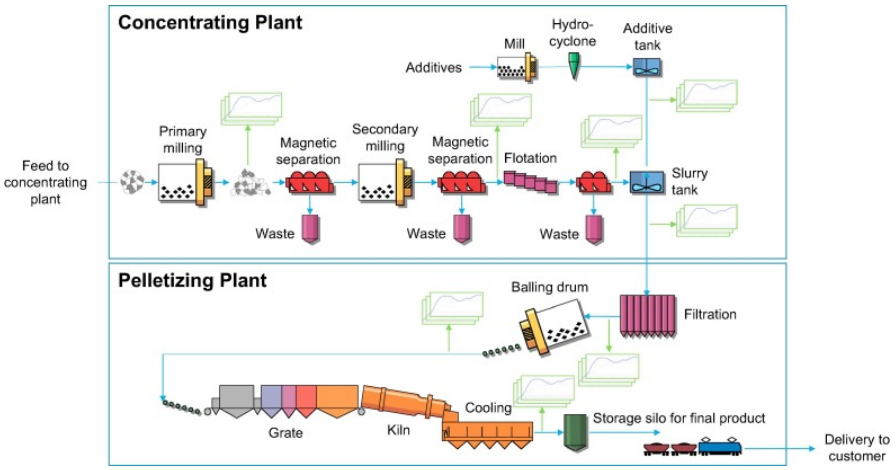


Figure 1.1: Pelletizing Process

In Kiruna, the ore is mined at depth and consists mainly of magnetite. After extraction, it is crushed into smaller fractions and sent to a sorting plant, where it is further reduced in size and subjected to magnetic separation and sieving. The ore then passes through concentration plants (Figure 1.1), where further separation is carried out in which water and additives are mixed with the ore in order to remove the phosphorous content. Once this is achieved, additives such as olivine, quartzite and limestone are added, and the mixture is sent to the pelletizing plant, where the water content is reduced to 8.5-9%. At this point, spheres are created with the material. These spheres are subjected to a control filter to prevent them from being too small or too large. Once this check has been passed, one moves on to the *"warm"* phase of the pellet production process, which is the focus point of this dissertation.

The pellets must be heated to dry out residual water and increase their mechanical strength, which is typically due to sintering. The quality of the finished product depends on dwell time, particle size, additives and the temperature profile during heat treatment. Studies to date show that oxidation of the ore plays a key role in the creation of a good product, so this is also taken into account in this process. In particular, the ultimate goal is to obtain pellet spheres in which the initial magnetite has turned into hematite.

At this point, the process can be divided into three parts: a travelling grate, a rotary kiln and a cooler. In the furnace, the magnetite balls are dried and preheated using the hot air from the cooler. The travelling grate is usually divided into four zones, as displayed in Figure 1.2. In the first two zones, the wet pellets are first dried by heat, in particular first from below, thus updraft drying (UDD), and then by downdraft drying (DDD). After the drying zones, the pellets are transported through the tempered preheating zone (TPH) and the preheating zone (PH).

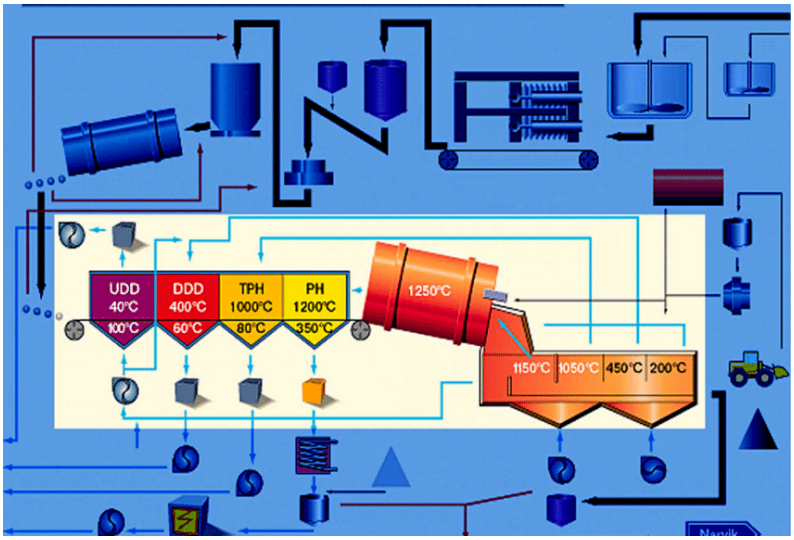


Figure 1.2: Induration Process Plant [1]

The heated pellet bed then enters the rotary kiln. To achieve a sufficiently high process temperature, fuel is used. The rotation of the furnace allows for even heating of the pellets in the process and thus gives a more homogeneous pellet quality. Afterwards, the now synthesised and oxidised pellets enter the cooling zone, where they are slowly cooled by passing through four different zones until they reach room temperature to obtain the finished product.

1.2 Background

The focal point of this master's degree project lies in the exploration of next-generation fossil-free induration processes, addressing the considerable emissions associated with pellet production. The aim is to identify optimal

design criteria by leveraging state-of-the-art optimization algorithms and the pre-existing simulation software developed at LKAB's Kiruna site. By studying a mathematical model, the project aims to minimize fuel usage during the induration process of pellets while concurrently preserving the quality of the final product. In order to ensure that the induration process meets quality and environmental standards, several constraints that must be satisfied were identified, including the range where the temperature in the bed and where the oxidation degree can be feasible in reality.

In formulating this problem, the idea is to study various functions that can be optimized so that an equilibrium can be found to achieve the goal of this thesis, which is to make the pellet production process fossil-free. Examples of functions that can be optimized are:

1. the temperature of the bed at the end of the PH zone,
2. the degree of oxidation at the end of the PH zone,
3. the temperature of the outgoing gas in PH,
4. the amount of fuel used to heat the pellet.

Since there are several objective functions to be optimized, there is a class of optimization problems called *"Multi-Objective Optimization"* whose general formulation is given by:

$$\min_{x\in\Omega} F(x) = \min_{x\in\Omega}(f_1(x)\ ,\ f_2(x)\ ,\ \ldots\ ,\ f_m(x))^T \tag{1.1}$$

where: $F : \Re^n \longrightarrow \Re^m \cup \{+\infty\}^m$ and $\Omega \subseteq \Re^n$ is nonempty.
Furthermore, it should be noted that in this specific case, there may not exist an analytical expression of the functions to be optimized but only an estimate of their value at a given point since simulations have to be performed to obtain these values. For these reasons, the problem to be studied in this thesis is called *"Simulation-Based Optimization"*.

1.2.1 Preliminary Terminology

It is deemed essential to provide the following definitions to aid readers in their understanding and acclimatization to the subject matter.

As explained in [2], *"Black-box optimization"* (BBO) refers to a class of

problems where the inner workings of the objective and constraint functions cannot be effectively exploited. This often arises when evaluating these functions entails, for example, running time-consuming simulations using computational models that are typically inaccessible to users. The term *"Derivative-Free Optimization"* (DFO), on the other hand, encompasses algorithms that solely rely on function values since either the partial derivatives are undefined or not accessible. Although gradient approximations may sometimes be obtained, the considerable effort required to ensure their reliability may not justify the undertaking. In other words, "DFO is the mathematical study of optimization algorithms that do not use derivatives" [3]. An essential difference between DFO and BBO derives from including heuristic methods. BBO methods include heuristic approaches for which convergence analysis and study is usually impossible. Typically, in contrast, DFO methods allow a rigorous study of convergence.

The book [4] introduces the concept of *"Simulation optimization"* (SO), which deals with optimization problems where the objective function cannot be computed precisely and is instead estimated with inherent noise. This framework is sometimes referred to as *"stochastic optimization"*, highlighting the reliance on simulations for estimation. Consequently, direct access to the objective function is unavailable, necessitating the collection of observations from simulation replication outputs that correspond to realizations of random variables.

1.3 Literature Review

Computer simulation is widely used in various fields to study and analyze complex systems and phenomena. As explained [5], simulation-based optimization is very important and versatile, as it allows one to modify and study the performance of a system where there is no direct knowledge of the form and properties of the objective function. The literature on these topics mostly focuses on problems with single-objective function, and the approach is called *"Optimization via Simulation"*. Generally, when we speak of Simulation Optimization, we mean *"the search for those specific settings of the input parameters to a stochastic simulation such that a target objective is minimized"* [6]. Since, in this class of problems, the analytical structure of the objective functions is not available, the calculation of its derivatives is impossible.

For this reason, classical optimization methods are unsuitable and the class of problems known as *"Derivative-free Problems"* was born. Principally, as explained in [7] and [8], these problems are divided into *"direct search methods"*, whose pioneers were Hooke and Jeeves [9], and *"model-based methods"* with Winfield [10]. In addition, [8] also highlights a third class called *"Implicit filtering methods"* which uses the simplex gradient to make an approximation of the current gradient of the objective function and then do a line search along the direction in which the gradient is approximated. However, this class of methods, although it would seem applicable to the problem it proposes to solve, has not been considered and studied in detail due to time constraints for developing this thesis. On the other hand, direct methods are zero-order methods that determine the direction of the search by computing the value of the objective function directly and do not require information on the derivatives. These methods are very simple, and all they do is sample the objective function at a set of points according to a given criterion and decide the next step based only on the information obtained from the sampling. Examples of this class of methods are the Nelder-Mead Simplex method and the Mesh Adaptive Direct Search [11]. However, these, although very simple to implement, are unsuitable for large-scale problems as they would be very slow. On the other hand, model-based methods approximate the objective function by constructing a model of it and minimising the function in a neighbourhood of the current iterate and then finding a point for the next iteration.

In the classification of methods for studying and solving single-target optimization problems, three classes of methods can be identified: Direct-Search Methods, Response Surface Methods, and Heuristic Methods. In particular, within the scope of this project, three different methods will be studied and analyzed, one belonging to each class. *Direct-search methods* (DSM), which are, in general, free from derivative dependencies, employ a finite number of objective function samples in each iteration. Decisions regarding subsequent actions are made exclusively based on these function values, without the need for explicit or implicit derivative approximations or model construction [12]. These types of methods generally work from a current solution and analyze a series of trial points in search of an improvement in the objective function. If there is such an improvement, then the current solution is updated; otherwise, the step size parameter is decremented in order to obtain a new set of trial points to be analyzed. *Response Surface Methodology* (RSM) is an approach that aims, usually,

to fit a polynomial of the appropriate degree to the response of the system of interest, allowing the underlying simulation to be approximated through an input-output relationship. It is usually a sequential procedure in which successive experimental steps are performed to move towards the optimal region. Eventually, a 'final' polynomial is applied, and the optimum is determined using conventional deterministic methods. The advantage of this approach lies in the ability to define a functional form for the surface, allowing the use of derivative-based optimization techniques to further exploit the learned input-output relationships [13], [6]. *Heuristic methods* represent a class of problems for which there is no guarantee that a solution to the proposed problem will be found. Evolutionary Algorithms stand out in this class of problems, among which are *Genetic Algorithms* (GA). They are based on the process of natural selection that mimics biological evolution, i.e. they generate solutions based on biologically inspired operators such as mutation and genetic selection. Genetic algorithms begin by establishing an initial population of randomly generated individuals. Potential solutions to the given problem are represented by each individual in the population. These individuals undergo a process of evolution through successive iterations, known as generations. In each generation, each individual in the population is evaluated with a certain measure of fitness. Subsequently, genetic operators are used to create the population for the next generation. This process continues until a termination condition is met [14].

However, the methods presented so far are methods formulated for optimization problems with a single-objective function. Unfortunately, in the case of problems with more than one objective function, the literature is limited, and now one of the aims of research in this area is to extend the methods that have been studied and validated for single objective functions to the case of more than one objective function. When one speaks of Multi-Objective Optimization problems, one is talking about problems in which there are several objective functions that one would like to optimize simultaneously or would like to find a trade-off between different conflicting objectives. The problem proposed in this project falls precisely into this category. The main difference that needs to be taken into account when moving from single to multi-objective optimization is that in this, you want to obtain a set of points that identify a region in which the different objective functions are optimized by creating an equilibrium situation between them. The solutions obtained are called Pareto Optimal Solutions and represent an equilibrium point between the multiple objective functions so that a case does not arise in which one of

the objective functions is optimized to the detriment of the others [15].
Multi-objective optimization problems are usually categorized according to when the decision maker is able to establish preferences for different objectives [16], [17]. Three types of problems are distinguished in the literature [18]:

- *without preferences*, in which any solution to the problem is acceptable
- *with a priori articulation of preferences*, in which decision markers are required to combine the different objective functions into a single function. In this case, a single non-dominated point is obtained.
- *with a posteriori articulation of preferences*, in which decision-makers are used to determine the best solution from a set of proposed solutions. Globally convergent derivative-free methods and heuristic methods belong to this class of problems.

Chapter 2

Mathematical Model of the problem

2.1 Introduction to multi-objective optimization

As mentioned earlier, the concept of Pareto Optimality is fundamental within multi-objective optimization. For this reason, it is necessary to provide the following definitions before explaining the algorithms used in this project. Thus, the following definitions and theorems will form the basis of the algorithms studied within this project.

Definition 1 (Multi-objective problem) *Let* $F(x) = (f_1(x), \ldots, f_m(x))^T \in \Re^m$ *such that* $f_i : \Re^n \longrightarrow \Re$*, for* $i = 1, \ldots, m$*, and let* $g : \Re^n \longrightarrow \Re^m$*. Then, a* non-linearly constrained multi-objective minimization problem *is:*

$$\min F(x) \quad s.t. \quad g(x) \leq 0, \;\; l \leq x \leq u \tag{2.1}$$

where $l, \; u \; \in \Re^n, \;\; l < u$*. Moreover, the set* $\mathcal{X} = \{x \in \Re^n \;\mid\; l \leq x \leq u\}$ *is compact and* $\mathcal{F} = \{x \in \Re^n \;\mid\; g(x) \leq 0\} \cap \mathcal{X}$ *is defined as the set of feasible solutions.*

Given two vectors $x, y \in \Re^n$, the following conventions hold:

$$\begin{aligned} x = y &\iff x_i = y_i \ \forall \ i = 1, \ldots, n \\ x < y &\iff x_i < y_i \ \forall \ i = 1, \ldots, n \\ x \leq y &\iff x_i \leq y_i \ \forall \ i = 1, \ldots, n \ \ and \ \ x \neq y \\ x \geq y &\iff -x \leq -y \end{aligned}$$

Definition 2 (Pareto dominance) *Consider the problem described in Definition 1. Given two points $x, y \in \mathcal{F}$, x is said to* dominate *y if $F(x) \leq F(y)$*

In other words, a point is non-dominated, also known as Pareto optimal or efficient point, if no other feasible point is better in all objectives and not worse in any objective.
The concept of Pareto Optimality is introduced because when there are several objective functions, it is not always the case that one point is optimal for all objective functions. To this end, it is usual to introduce the concept of optimality at both global and local levels so that it is possible to identify the so-called Pareto front, i.e. a set of non-dominated points that represent the optimal solutions, which may be global or local, of the proposed multi-objective optimization problem.

Definition 3 (Global Pareto Optimality) *Consider the problem described in Definition 1. A point $\bar{x} \in \mathcal{F}$ is a* global Pareto optimizer *for the problem (2.1) if it does not exist a point $y \in \mathcal{F}$ such that $F(y) \leq F(\bar{x})$.*

Definition 4 (Local Pareto Optimality) *Consider the problem described in Definition 1. A point $\bar{x} \in \mathcal{F}$ is a* local Pareto optimizer *for the problem (2.1) if it does not exist a point $y \in \mathcal{F} \cap \mathcal{B}(\bar{x}, \rho)$ such that $F(y) \leq F(\bar{x})$ for some $\rho > 0$, where $\mathcal{B}(\bar{x}, \rho) = \{y \in \Re^n : ||\bar{x} - y|| \leq \rho\}$.*

A very important definition that is often used as an assumption in optimization algorithms is the following:

Definition 5 *A real-valued function $f : \Re^n \longrightarrow \Re$ is said to be* Lipschitz Continuous *if*

$$\exists \, K > 0 \ : \ \forall x \ , \ y \ \in \ \Re^n \ \ ||f(x) - f(y)|| \ \leq \ K \ ||x - y|| \tag{2.2}$$

2.2 Numerical Methods

In the following sections, three algorithms belonging to three different classes of methods will be analyzed in detail. In particular, the following will be considered:

1. A Derivative-Free Multi-Objective Approach (DFMO) (Section 2.2.1), which belongs to the class of Direct-Search Methods
2. A Non-dominated Sorting Genetic Algorithm (NSGA) (Section 2.2.2), belonging to the class of Heuristics Algorithm
3. A method for Multi-Objective Optimization by Radial Basis Function Interpolation in Trust-regions (Morbit) (Section 2.2.3), belonging to the class of Response Surface Methods.

2.2.1 A Direct Search Method: The DFMO Algorithm

The algorithm that will now be described belongs to the class of algorithms with a posteriori preference articulation and can be classified as a Direct Search Method. This algorithm was devised and explained by Liuzzi et al. in [19].

The basic idea is to describe new optimality conditions that are able to take bound constraints into account explicitly and are derived from the sole assumption that the functions are Lipschitz continuous. In order to be able to handle nonlinear constraints, in this approach, the idea is to construct a merit function that penalizes nonlinear inequality constraints for each function $f_i(x)$ so as to solve an optimization problem subject only to bound constraints.

Definition 6 (Penalty Function) *Given problem (2.1), $\epsilon > 0$ and:*

$$Z_j(x;\epsilon) = f_j(x) + \frac{1}{\epsilon}\sum_{i=1}^{m} max\{0, g_i(x)\} \quad \forall j = 1,\dots,n \tag{2.3}$$

Then, the adding term to the objective function is called penalty function

Definition 7 (Penalized bound constrained multi-objective problem) *The problem:*

$$\min Z(x;\epsilon) = \min(Z_1(x;\epsilon),\dots,Z_n(x;\epsilon))^T \quad s.t. \quad x \in \mathcal{X} \tag{2.4}$$

is called the Penalized bound constrained multi-objective problem

Definition 8 (Penalized bound constrained multi-objective problem) *The problem:*

$$\min Z(x;\epsilon) = \min(Z_1(x;\epsilon),\ldots,Z_n(x;\epsilon))^T \quad s.t. \quad x \in \mathcal{X} \tag{2.5}$$

is called the Penalized bound constrained multi-objective problem*, where* $\mathcal{X} = \{x \in \Re^n \mid l \leq x \leq u\}$.

The Derivative-Free Multi-Objective (DFMO) algorithm generates, at each iteration k, a point set L_k, which can be described as follows:

$$L_k = \{(x_i\ ,\ \alpha_i)\ :\ x_i\ \in \mathcal{X}\ ,\ \alpha_i\ >\ 0\ ,\ i\ =\ 1\ ,\ \ldots\ ,\ \mid L_k \mid\} \tag{2.6}$$

where α_i denotes the trial step-size associated with point x_i and $\mathcal{X} = \{x \in \Re^n \mid l \leq x \leq u\}$.

For each point in the set L_k, the algorithm performs a line search along a direction d_k. If d_k gives a sufficient decrease, then a "sufficiently" significant movement along this direction will be performed, and a new set of points will be generated. In other words, if at least one objective function is sufficiently reduced at the new point generated along d_k for each point of L_k, then d_k is said to give a sufficient decrease. In this case, the movement along the direction d_k is performed by a procedure called *Projected Expansion*.

In Algorithm 1 and Algorithm 2, there is the pseudo-code describing the DFMO algorithm and the Projected Expansion procedure, respectively. The Projected Expansion Procedure involves a sufficiently large movement along the d_k direction, leading to the modification of $\tilde{L}$ through the addition and filtration of a set of points. Note that **1** denotes a vector of all ones of dimension n and that the symbol $[x]_{[l\ ,\ u]}$ denotes the projection of a point x onto the set $\mathcal{X}$.

Algorithm 1 Algorithm DFMO [19]

Data. $\gamma > 0, \quad \theta \in [0,1], \quad \epsilon > 0, L_0 = \{(x_i, \alpha_i) \mid x_i \in \mathcal{X}, \alpha_i > 0, i = 1, \ldots, |L_0|\}$ a sequence $\{d_k\} \subset \Re^n$ such that $||d_k|| = 1, \forall k$.

for $k = 0, 1, ...$ **do**
  Set $\tilde{L}_k = L_k$
  for $i = 1, 2, ..., |L_k|$ **do**
    Let (x_i, α_i) be the i-th couple of L_k
    if $(\mathcal{Z}([x_i + \alpha_i d_k]_{[l,u]}; \epsilon) \not\succ \mathcal{Z}(x_j; \epsilon) - \gamma {\alpha_i}^2 1) \; \forall x_j \in \tilde{L}_k$ **then**
      Set $\tilde{L}_k =$ Projected Expansion $(x_i, \alpha_i, d_k, \tilde{L}_k, \gamma)$
    else
      (Failure Step)
      If $(x_i, \alpha_i) \in \tilde{L}_k$, set $\tilde{L}_k = (\tilde{L}_k \setminus \{(x_i, \alpha_i)\}) \cup \{(x_i, \theta\alpha_i)\}$
  Set $L_{k+1} = \tilde{L}_k$

Algorithm 2 Projected Expansion$(y, \hat{\alpha}, p, \tilde{L}, \gamma)$ [19]

Data. $\delta \in (0, 1)$
Step 1. Set $\alpha = \hat{\alpha}$
Step 2. Let $\beta = \alpha/\delta$
Step 3. **if** $(Z([y + \beta p]_{[l,u]}; \epsilon) \not\prec Z([y + \alpha p]_{[l,u]}; \epsilon) - \gamma(\beta^2 - \alpha^2)1)$ **then**
  Set $\tilde{L} = Add\&Filter(\tilde{L}, ([y + \alpha p]_{[l,u]}, \alpha))$
end
Step 4. **if** $(Z([y + \beta p][l, u]; \epsilon) \not\succ Z(x_j; \epsilon) - \gamma\beta^2 1) \; \forall x_j \in \tilde{L}$ **then**
  Set $\alpha = \beta$ and go to Step 2
end
Step 5. Return $\tilde{L}$

In Projected Expansion (Algorithm 2), the $Add\&Filter$ (Algorithm 3) procedure is called, which, given a list of non-dominated pairs L and a pair $(x\,, \alpha)$ that is not dominated by any pair in L, returns a list $\tilde{L}$ of non-dominated pairs from the union between L and the given pair.

Algorithm 3 Add&Filter$(L, (x, \alpha_x))$ [19]

Set $\tilde{L} = \{(x, \alpha_x)\}$
for *each* $(y, \alpha_y) \in L$ **do**
  if $Z(x;\epsilon) \nleq Z(y;\epsilon)$ **then**
    $\tilde{L} = \tilde{L} \cup \{(y, \alpha_y)\}$
  end
end
return $\tilde{L}$

As mentioned above, in this algorithm, the nonlinear constraints $g_i(x)$ are handled by the penalty function. In addition, the bound constraints of problem (2.1), i.e. $l \leq x \leq u$ are handled by the projection of the $Z_j(x, \epsilon)$ functions on the interval identified by them, evaluated in the trial point $x+\alpha d$. In particular, the vector of *penalty parameters* ϵ is set as follows, for $i = 1, \ldots, m$:

$$\epsilon_i = \begin{cases} 10^{-3} & if\ max\{0\ ,\ g_i(x_0)\} < 1 \\ 10^{-1} & otherwise \end{cases}$$

Thus, the DFMO algorithm for each pair of L_k compares whether along the direction d_k at least one objective function is improved at a point that is generated along that direction and whether, if so, this point turns out to be Pareto non-dominated. This check determines the new direction along which the search is carried out. In general, if the objective functions are Lipschitz continuous, it has been proven that the algorithm converges to stationary points in the problem. In particular, the following proposition has been given:

Proposition 1 (Proposition 3.16 in [19]) *The Projected Expansion is such that:*

- *the test at Step 4 is satisfied a finite number of times, i.e. the procedure cannot infinitely cycle;*
- *the test at Step 3 is satisfied at least once, i.e. at least a new point is added to the list* $\tilde{L}$

This proposition, therefore, guarantees that the algorithm does not execute at infinity and that there is at least one improvement from the initial set point to the final. However, in the problem studied in this thesis, since there is no analytical expression for the objective functions, it is also impossible to guarantee that the functions are Lipschitz continuous and that the procedure does not execute at infinity. To deal with this problem, a maximum number

of function evaluations was given, which, as soon as it is reached, causes the process to terminate execution.

2.2.2 A Multi-objective Genetic Algorithm: The NSGA-2 Method

2.2.2.1 Terminology

Before presenting the algorithm, it is considered necessary to give the following definitions

Definition 9 (Population) *A* population *refers to a collection or group of individuals representing potential solutions to a given problem. Each individual within the population represents a candidate solution, and the population evolves over successive generations to find an optimal solution. In particular:*

- *The* parent population *refers to the current generation of individuals from which new offspring individuals are created.*
- *The* offspring population *is the newly generated population of individuals created through the genetic operations applied to the parent population.*

Definition 10 (Non-domination rank) *The* non-domination rank *refers to assigning a rank or level of dominance to each individual in the population based on their non-dominated status. It provides a measure of each individual's dominance level in the population.*

Definition 11 (Pareto-Optimal Front) *The* Pareto-optimal front *represents the set of solutions that cannot be improved in any objective without worsening at least one other objective.*

Definition 12 (Crowding Distance) *The* crowding distance *is a measure used to determine the density or spread of solutions within a specific region of the Pareto front. It quantifies the diversity of solutions by considering the distances between neighbouring individuals in multiple objective spaces.*

2.2.2.2 The Approach

The Non-dominated Sorting Genetic Algorithm (NSGA) was one of the first Evolutionary Algorithms to solve multi-objective optimization problems. In this project, the NSGA-2 method will be used, which is an improvement of the NSGA algorithm.

NSGA-2 has been introduced thanks to the work of Deb K. et al. in [20]. The algorithm works in the following way: initially, a population R_t, of size 2N, is formed, which is the union of a parent population P_t, of size N, and offspring population Q_t, of size N. The population R_t is sorted according to the non-dominance criterion, and then sets of non-dominated solutions $\mathcal{F}$ are created. The solutions belonging to $\mathcal{F}_1$ are the best that can be obtained and must therefore be emphasized with respect to all others in the combined population.

At this point, the size of $\mathcal{F}_1$ is looked at; if its size is less than N, then all the members of $\mathcal{F}_1$ will be present in the next population P_{t+1}. The remaining members will be chosen from the next non-dominated fronts in order of their ranking until no other set can be accommodated. Let $\mathcal{F}_l$ denote the final non-dominated set, indicating that it is the last set that can be accommodated without additional sets. This procedure is illustrated in Figure 2.1:

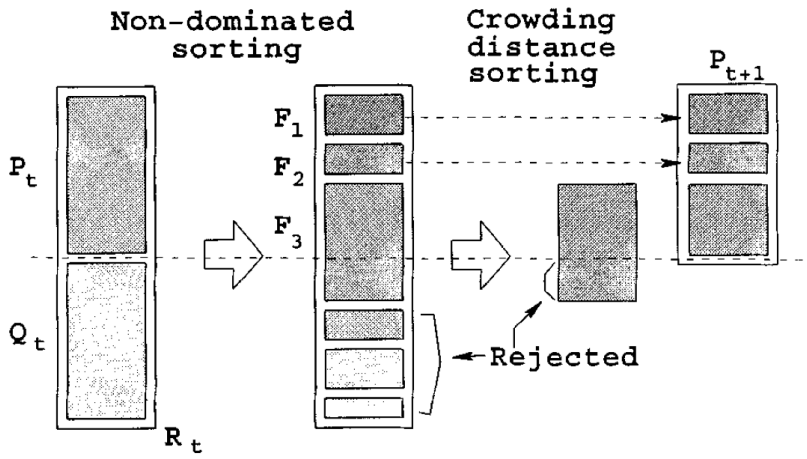


Figure 2.1: NSGA-2 Procedure [20]

In general, the number of solutions in all sets $\mathcal{F}_1$ to $\mathcal{F}_l$ is greater than the size of the population. To choose precisely N members of the population, the solutions of the last set $\mathcal{F}_l$ are ordered with the operator *crowded comparison* in descending order, and the best solutions needed to occupy all places in the population are chosen. The crowded-comparison operator facilitates the selection process throughout the algorithm, ensuring a uniformly distributed Pareto optimal front.

Algorithm 4 Crowding-distance-assignment(I) [20]

$l = |\mathcal{I}|$ // number of solutions in $\mathcal{I}$
for *each* i **do**
  set $|\mathcal{I}|[i]_{distance} = 0$ // initialize distance
end
for *each objective* m **do**
  $\mathcal{I} = sort(\mathcal{I}, m)$ // sort using each objective value
  $\mathcal{I}[1]_{distance} = \mathcal{I}[l]_{distance} = \infty$ // so that boundary points are always selected
end
for $i = 2$ *to* $(l-1)$ **do**
  $\mathcal{I}[i]_{distance} = \mathcal{I}[i]_{distance} + (\mathcal{I}[i+1].m - \mathcal{I}[i-1].m)/(f_m^{max} - f_m^{min})$// for all other points
end

Throughout the algorithm's different stages, the crowded-comparison operator directs the selection process, aiming to achieve a uniformly spread-out Pareto-optimal front. Each individual in the population is characterized by two attributes: non-domination rank and crowding distance. A partial order $\prec_n$ is defined as follows: when comparing two solutions with different non-domination ranks, preference is given to the solution with a lower (better) rank. However, both solutions belong to the same front. In that case, the preference is for the solution located in a less crowded region, where crowded distance is used to estimate the perimeter of the cuboid formed using the nearest neighbours as vertices.

When the sorting procedure has identified an adequate number of fronts to fill P_{t+1} with N members, there is no longer a rationale to continue with the sorting process. The diversity among non-dominated solutions is introduced by implementing the crowding comparison procedure, which is utilized during both tournament selection and the population reduction phase. The tournament selection consists of randomly selecting individuals and having them compete in small tournaments based on their fitness. The

individual with the best performance in each tournament is selected as the parent for the next generation. On the other hand, the population reduction phase ensures a fixed population size by selecting individuals based on their non-domain rank and crowding distance. This process maintains diverse and high-quality solutions while eliminating less desirable individuals. Through the competition based on crowding distance, there is no requirement for an additional niching parameter since it maintains diversity among solutions in the population.

Algorithm 5 presents the description of the main cycle of the procedure performed in NSGA-2.

Algorithm 5 NSGA-2 Main Loop [20]

$R_t = P_t \cup Q_t$ // combine parent and offspring population
F =fast-non-dominated-sort(R_t) // $F = (F_1, ...)$, all non-dominated fronts of R_t
$P_{t+1} = \emptyset$ and $i = 1$

until $|P_{t+1}| + |F_i| \leq N$ // until the parent population is filled
  crowding-distance-assignment(F_i) // calculate crowding-distance in F_i
  $P_{t+1} = P_{t+1} \cup F_i$ // include i^{th} non-dominated front in the parent pop
  $i = i + 1$ // check the next front for inclusion
Sort $(F_i, \prec_n)$ // sort in descending order using $\prec_n$
$P_{t+1} = P_{t+1} \cup F_i[1 : (N - |P_{t+1}|)]$ // choose the first $(N - |P_{t+1}|)$
// elements of F_i
Q_{t+1} = make-new-pop(P_{t+1}) // use selection, crossover and mutation to
// create a new population Q_{t+1}
$t = t + 1$ // increment the generation counter

The approach for handling constraints involves a binary tournament selection method, where two solutions are randomly selected from the population, and the superior solution is chosen. In the presence of constraints, each solution can fall into one of three situations:

1. both solutions are feasible,
2. one solution is feasible while the other is not,
3. both solutions are infeasible.

It is important to give the following definition:

Definition 13 (Constrained Domination [20]) *A solution x is said to* constrained-dominate *a solution y if any of the following conditions is true:*

- *solution x is feasible, and solution y is not*
- *solutions x and y are both infeasible, but solution x has a smaller overall constraint violation*
- *solutions x and y are feasible and solution x dominates solution y*

The application of the constrained-domination principle has a significant influence. Feasible solutions are assigned higher non-domination ranks in comparison to infeasible solutions. The ranking of feasible solutions is determined based on their non-domination level, considering the values of the objective function. However, when dealing with two infeasible solutions, the solution with a minor constraint violation is ranked higher.

2.2.3 A Multi-objective Trust Region Descent Method: The Morbit Method

The third algorithm used within this project is Morbit, which stands for *Multiobjective Optimization by Radial Basis Function Interpolation in Trust-regions*, explained in [21].

2.2.3.1 Terminology

Before going into the details of the algorithm, the following two definitions are given:

Definition 14 (Multi-descent Direction) *A vector* $d \in \mathcal{X} - x$ *is called a* multi-descent direction *for F in x if* $\max_{l=1,\dots,m}\langle \nabla f_l(x*), d\rangle < 0$*, where* $\langle \bullet, \bullet \rangle$ *is the standard inner product on* $\Re^n$ *and* $\nabla f_l(x)$ *denote the gradient of* f_l *at* $x \in \mathcal{X}$*.*

Definition 15 (Multi-objective steepest-descent direction) *Given the problem:*

$$\min_{d \in X-x} \max_{l=1,\dots,m} \langle \nabla f_l(x), d \rangle \quad s.t. \quad ||d||_\infty \leq 1 \tag{2.7}$$

the minimizer d of the problem (2.7) is called the* multi-objective steepest-descent direction.

2.2.3.2 The Approach

The main idea is to iteratively compute multi-descent steps $s^{(t)}$ for each iteration $t \in \mathcal{N}_0$. Since the analytical expression of the objective function and/or its derivatives is not available, the approach is to use surrogate model functions of the form:

$$\tilde{m}^{(t)} : \Re^n \longrightarrow \Re^k, x \longmapsto \tilde{m}^{(t)} = [\tilde{m}_1^{(t)}(x), \ldots, \tilde{m}_k^{(t)}(x)]^T \tag{2.8}$$

Which is built in the trust region around the iterate $x^{(t)}$:

$$B^{(t)} := B(x^{(t)}; \Delta^{(t)}) = \{x \in \mathcal{X} : ||x - x^{(t)}||_\infty \leq \Delta^{(t)}\} \tag{2.9}$$

At this point, the model's steepest descent direction $d_{\tilde{m}}^{(t)}$ can be calculated as the optimum of the surrogate problem given by:

$$\begin{aligned} \omega_{\tilde{m}}^{(t)}(x^{(t)}) : = - \min_{d \in \chi - x} \ & \beta \text{ s.t. } ||d||_\infty \leq 1 \text{ and} \\ & \langle \nabla \tilde{m}_l^{(t)}(x), d \rangle \leq \beta \ \forall \ l = 1, \ldots, m \end{aligned} \tag{2.10}$$

This direction $d_{\tilde{m}}^{(t)}$ does not necessarily have to be a descent direction for the original objective functions f_i. Moreover, given the step-size $\sigma^{(t)} > 0$, the trial point $x_+^{(t)} = x^{(t)} + \sigma^{(t)} d_{\tilde{m}}^{(t)}$ is deemed acceptable if the magnitude of enhancement and the measure $\rho^{(t)}$ of the model exceed a predefined positive threshold ν_+, where $\rho^{(t)}$ can be defined as: $\rho^{(t)} = \frac{f_l(x^{(t)}) - f_l(x_+^{(t)})}{\tilde{m}_l^{(t)}(x^{(t)}) - \tilde{m}_l^{(t)}(x_+^{(t)})}$.

In the case of a positive $\rho^{(t)}$, indicating a reduction in at least one objective function, the subsequent iteration $x_+^{(t)}$ is deemed acceptable if $\rho^{(t)} > \nu_+ > 0$. If $\rho^{(t)}$ reaches a considerable magnitude, i.e. $\rho^{(t)} \geq \nu_{++} > \nu_+ > 0$, the ensuing trust region may exhibit an increased radius $\Delta^{(t+1)} \geq \Delta^{(t)}$. Conversely, when $\rho < \nu_{++}$, the ensuing trust region radius should be diminished, and enhancements to the surrogates should be made.

When these surrogate models are constructed, they are built in such a way as to be fully linear.

Definition 16 (Full Linearity) *Let $\Delta^{ub} > 0$ be given and let $f : \Re \longrightarrow \Re$ be a function that is continuously differentiable in an open domain containing $\mathcal{X}$ and has a Lipschitz continuous gradient on $\mathcal{X}$. A set of model functions $\mathcal{M} = \{\tilde{m} : \Re^n \longrightarrow \Re\} \subseteq C^1(\Re^n, \Re)$ is called a* fully linear *class of models w.r.t. f if the following hold:*

1. *There are positive constants $\epsilon, \dot{\epsilon}$ and $L_{\tilde{m}}$ such that for any given $\Delta \in (0, \Delta^{ub})$ and for any $x \in \mathcal{X}$ there is a model function $\tilde{m} \in \mathcal{M}$ with Lipschitz continuous gradient and corresponding Lipschitz constant bounded by $L_{\tilde{m}}$ and such that*

   - *the error between the gradient of the model and the gradient of the function satisfies*

   $$||\nabla f(\xi) - \nabla \tilde{m}(\xi)|| \leq \dot{\epsilon}\Delta \quad \forall\, \xi \in B(x; \Delta)$$

   - *the error between the model and the function satisfies*

   $$|f(\xi) - \tilde{m}(\xi)| \leq \epsilon\Delta^2 \quad \forall\, \xi \in B(x; \Delta)$$

2. *For this class $\mathcal{M}$ there exists 'model-improvement' algorithm that, in a finite, uniformly bounded (w.r.t. x and Δ) number of steps, can:*

   - *either establish that a given model $\tilde{m} \in \mathcal{M}$ is fully linear on $B(x; \Delta)$ i.e., it satisfies the error bounds in 1,*
   - *or find a model $\bar{m}$ that is fully linear on $B(x; \Delta)$.*

A point $x^* \in \mathcal{X}$ is said *critical* for problem 2.1 if and only if there is no descent direction $d \in \mathcal{X} - x^*$. In particular, all the Pareto Optimal points are critical; thus the procedure described in Algorithm 6 is given:

Algorithm 6 Criticality Routine [21]

Configuration: A backtracking constant $\alpha \in (0, 1), \mu > 0$ from Algorithm TRM;
Input: Current trust region radius $\Delta^{(t)}$, current models $\tilde{m}^{(t)}$;
Output: Fully linear models $\tilde{m}^{(t)}$ and the (possibly shrunken) radius $\Delta^{(t)}$;
Set $\Delta_0 \leftarrow \Delta^{(t)}$
for $j = 1, 2, \ldots$ **do**
  Set radius: $\Delta^{(t)} \leftarrow \alpha^{j-1}\Delta_0$
  Make models $\tilde{m}^{(t)}$ fully linear on $B^{(t)}$ // can change $\omega_{\tilde{m}}^{(t)}(x^{(t)})$
  if $\Delta^{(t)} \leq \mu\omega_{\tilde{m}}^{(t)}(x^{(t)})$ **then**
    Break
  end
end

Then, the general algorithm is given by Algorithm 7:

Algorithm 7 General Trust Region method (TRM) [21]

Configuration: Criticality parameters $\epsilon_{crit} > 0$ and $\mu > \beta > 0$ acceptance parameters $1 > \nu_{++} \geq \nu_{+} \geq 0$, $\nu_{++} \neq 0$, update factors $\gamma_{\uparrow} \geq 1 > \gamma_{\downarrow} \geq \gamma_{\downarrow\downarrow} > 0$, and $\Delta^{ub} > 0$

Input: The initial site $x^{(0)} \in \Re^n$

for $t = 0, 1, ...$ **do**

  if $t > 0$ *and iteration* $(t-1)$ *was model-improving* **then**

    Perform at least one improvement step on $\tilde{m}^{(t-1)}$ and then let $\tilde{m}^{(t)} \leftarrow \tilde{m}^{(t-1)}$

  end

  else

    Construct surrogate models $\tilde{m}^{(t)}$ on $B^{(t)}$

  end

  /* Criticality Step: */

  if $\omega_{\tilde{m}}^{(t)}(x^{(t)}) < \epsilon_{crit}$ *and (*$\tilde{m}^{(t)}$ *not fully linear or* $\Delta^{(t)} > \mu\omega_{\tilde{m}}^{(t)}(x^{(t)})$*)* **then**

    Set $\Delta_*^{(t)} \leftarrow \Delta^{(t)}$

    Call Alg. "Criticality Routine" so that $\tilde{m}^{(t)}$ is fully linear on $B^{(t)}$ with $\Delta^{(t)} \in (0, \mu\omega_{\tilde{m}}^{(t)}(x^{(t)}))$

    Then set $\Delta^{(t)} \leftarrow \min\{\max\{\Delta^{(t)}, \beta\omega_{\tilde{m}}^{(t)}(x^{(t)})\}, \Delta_*^{(t)}\}$

  end

  Compute a suitable descent step $s^{(t)}$

  Set $x_+^{(t)} \leftarrow x^{(t)} + s^{(t)}$, evaluate $f(x_+^{(t)})$ and compute $p^{(t)}$

  /* Perform the following updates: */

  $$x^{(t+1)} \leftarrow \begin{cases} x^{(t)} & \text{if } p^{(t)} < \nu_+ \text{ or } \nu_+ \leq p^{(t)} < \nu_{++} \text{ and } \tilde{m}^{(t)} \text{ is not fully linear} \\ x_+^{(t)} & \text{if } p^{(t)} \geq \nu_{++} \text{ or } \nu_+ \leq p^{(t)} < \nu_{++} \text{ and } \tilde{m}^{(t)} \text{ is fully linear} \end{cases}$$

  $\Delta^{(t+1)} \leftarrow \Delta_+$, where

  $$\Delta_+ = \begin{cases} \Delta^{(t)} & \text{if } p^{(t)} < \nu_{++} \text{ and } \tilde{m}^{(t)} \text{ is not fully linear} \\ \in [\gamma_{\downarrow\downarrow}\Delta^{(t)}, \gamma_{\downarrow}\Delta^{(t)}] & \text{if } p^{(t)} < \nu_{++} \text{ and } \tilde{m}^{(t)} \text{ is fully linear} \\ \in [\Delta^{(t)}, \min\{\gamma_{\uparrow}\Delta^{(t)}, \Delta^{ub}\}] & \text{if } \nu_{++} \leq p^{(t)} \text{ and } \Delta^{(t)} \geq \beta\omega_{\tilde{m}}^{(t)}(x^{(t)}) \\ \min\{\gamma_{\uparrow}\Delta^{(t)}, \Delta^{ub}\} & \text{if } \nu_{++} \leq p^{(t)} \text{ and } \Delta^{(t)} < \beta\omega_{\tilde{m}}^{(t)}(x^{(t)}) \end{cases}$$

end

From the Algorithm 7, the iteration based on $\rho^{(t)}$ can be classified as:

Definition 17 *For given constants* $0 \leq \nu_+ \leq \nu_{++} < 1, \nu_{++} \neq 0$*, the iteration with index* $t \in \mathcal{N}_0$ *is called:*

- successful *if* $\rho^{(t)} \geq \nu_{++}$*. The set of successful indices is* $S = \{t \in \mathcal{N}_0 : \rho^{(t)} \geq \nu_{++}\} \subseteq \mathcal{N}_0$*. The trial point is accepted and the trust region*

radius can be increased.

- model-improving *if* $\rho^{(t)} < \nu_{++}$ *and the models* $\tilde{m}^{(t)} = [\tilde{m}_1^{(t)}(x), \ldots, \tilde{m}_k^{(t)}(x)]^T$ *are not fully linear. In these iterations the trial point is rejected and the trust region radius is not changed.*

- acceptable *if* $\nu_{++} > \rho^{(t)} \geq \nu_+$ *and the models* $\tilde{m}^{(t)}$ *are fully linear. If* $\nu_{++} = \nu_+ \in (0, 1)$*, then there are no acceptable indices. The trial point is accepted but the trust region radius is decreased.*

- unacceptable *otherwise, i.e., if* $\rho^{(t)} < \nu_{++}$ *and* $\tilde{m}^{(t)}$ *are fully linear. The trial point is rejected and the radius decreased.*

The stopping criteria of the algorithm is made by:

- an upper bound $N_{it} \in \mathcal{N}$ on the maximum number of iterations and an upper bound $N_{exp} \in \mathcal{N}$ on the number of expensive objective evaluations.

- The surrogate criticality naturally allows for a stopping test and due to Lemma 11 in [21] the trust region radius can also be used. This is combined with with a relative tolerance test and stop if

$$\Delta^{(t)} \leq \Delta_{min} \;\; OR \;\; (\Delta^{(t)} \leq \Delta_{crit} \;\; AND \;\; \omega(x^{(t)}) \leq \omega_{min})$$

- At a truly critical point the criticality loop Algorithm 1 runs infinitely. The algorithm is stopped after a maximum number $N_{loops} \in \mathcal{N}_0$ of iterations.

- The common relative stopping criteria can also be employed to provoke early stopping:

$$||x^{(t)} - x^{(t+1)}||_\infty \leq \delta_x ||x^{(t)}||_\infty \; and \; ||f(x^{(t)}) - f(x^{(t+1)})||_\infty \leq \delta_f ||f(x^{(t)})||_\infty$$

2.3 Problem Formulation

In order to be able to use the methods presented so far with the problem to be solved, it is necessary to give a formulation of the problem. In this project, two different formulations were made. The first one concerns the most straightforward case that can be considered, in which the variables are subject to bound constraints, i.e. lower and upper bounds related to physical

restrictions. For example, the temperature within each zone physically cannot be smaller than a particular value or larger than another.

However, this first formulation is far removed from reality, as it needs to consider various factors accompanying the process, such as physical relationships or chemical reactions. For this reason, it was also deemed necessary to provide a second formulation of the problem, in which the energy balance equation for gases is also considered. It should be noted that although the second formulation does not explicitly take chemical reactions into account, this aspect is always considered by the simulation software in both formulations. It should be noted that, in the proposed formulations, only two areas of the process have been considered: TPH and PH. Therefore, in practice, the problem has been scaled down to study the methods and approaches and understand whether this path may be correct.

2.3.1 Bound-Constrained Problem

Let us consider a scenario where certain gas quantities in zone i can be manipulated. Specifically, these quantities include gas temperature, denoted as T^i $[^\circ C]$, oxygen content in the gas, represented by O_2^i $[vol\%]$, and production-specific gas mass flow, denoted as $\hat{m}^i$ $[\cdot]$. The production-specific gas mass flow, $\hat{m}^i$, is defined as the ratio of the gas mass flow rate in zone i, $\dot{m}^i$ $[kg/h]$, to the production rate of the machine, $\dot{P}$ $[kg/h]$. Consequently, the control over gas quantities in the TPH and PH zones is achieved through the variables represented by the vector x, given by:

$$x = \begin{bmatrix} T^{TPH} & \hat{m}^{TPH} & O_2^{TPH} & T^{PH} & \hat{m}^{PH} & O_2^{PH} \end{bmatrix}^T . \tag{2.11}$$

The variables are described in Table 2.1:

Table 2.1: Basic Variables of the Problem

variable	Description	Unit
$\hat{m}^i$	Production specific gas mass flow in zone i.	$[\cdot]$
T^i	Gas temperature in zone i.	$[^\circ C]$
O_2^i	Oxygen concentration of gas in zone i.	$[vol\%]$

To make the problem realistic and due to physical restrictions, it is necessary to define the following bound constraints on the variables of the problem:

$$T^i_{min} \leq T^i \leq T^i_{max}, \tag{2.12}$$
$$\hat{m}^i_{min} \leq \hat{m}^i \leq \hat{m}^i_{max}, \tag{2.13}$$
$$O^i_{2,min} \leq O^i_2 \leq O^i_{2,max}, \tag{2.14}$$
$$i \in \{PH,\ TPH\}. \tag{2.15}$$

To achieve optimal performance in the kiln, having a well-heated bed and a high degree of oxidation is important. A warm process gas at the PH-zone output results in energy loss. The objective is to maximize the average bed temperature, denoted as $\bar{T}^{PH}_{bed}$, and the oxidation degree, denoted as $\bar{O}^{PH}_2$, at the PH-zone output while minimizing the average temperature of the process gas exiting the PH-zone, denoted as $\bar{T}^{PH}_{gas}$. These objectives create a multi-objective function given by:

$$F(x) = \begin{bmatrix} -\bar{T}^{PH}_{bed} & -\bar{O}^{PH}_2 & \bar{T}^{PH}_{gas} \end{bmatrix}, \tag{2.16}$$

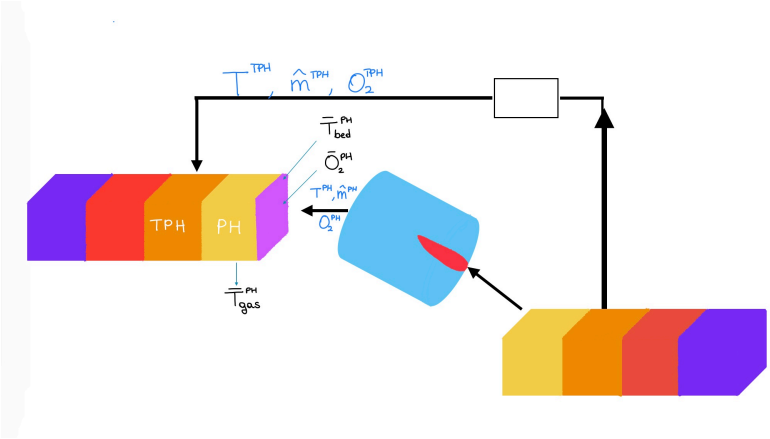
which needs to be minimized. Evaluating the measures defined in equation (2.16) requires post-processing of simulation results. As a result, each evaluation of $F(x)$ requires one simulation.

Combining equations (2.11) and (2.16) gives the black-box bound-constrained multi-objective optimization problem:

$$\begin{array}{ll} \text{minimize} & \begin{bmatrix} -\bar{T}^{PH}_{bed} & -\bar{O}^{PH}_2 & \bar{T}^{PH}_{gas} \end{bmatrix} \\ \text{subject to} & T^i_{min} \leq T^i \leq T^i_{max}, \\ & \hat{m}^i_{min} \leq \hat{m}^i \leq \hat{m}^i_{max}, \\ & O^i_{2,min} \leq O^i_2 \leq O^i_{2,max}, \\ & i \in \{PH,\ TPH\}. \end{array} \tag{2.17}$$

These objective functions have been schematically represented in Figure 2.2. In practice, $\bar{T}^{PH}_{gas}$ is the gas temperature at the exit of the PH zone, while $\bar{O}^{PH}_2$ and $\bar{T}^{PH}_{bed}$ are evaluated at the end of the PH zone.

Figure 2.2: Objective Functions Scheme



The parameters involved in problem 2.17 are describe in Table 2.2:

Table 2.2: Variables of the Bound-Constrained Problem

Parameters	Description	Value
$\hat{m}^i_{min}$	Minimum specific production specific gas mass flow in zone $i \in \{TPH,\ PH\}$.	0.3
$\hat{m}^i_{max}$	Maximum production specific gas mass flow in zone $i \in \{TPH,\ PH\}$.	0.7
T^i_{min}	Minimum gas temperature in zone $i \in \{TPH,\ PH\}$.	850
T^i_{max}	Maximum gas temperature in zone $i \in \{TPH,\ PH\}$.	1250
$O^i_{2,min}$	Minimum oxygen concentration of gas in zone $i \in \{TPH,\ PH\}$.	15
$O^i_{2,max}$	Maximum oxygen concentration of gas in zone $i \in \{TPH,\ PH\}$.	40

In order to better evaluate the presented algorithms, it was deemed necessary to perform simulations on bi-objective optimization problems. In particular, two formulations of the problem (2.17) were chosen in which the two objective functions are contradictory, in the sense that by increasing or decreasing the value of one, it does not happen by default that the other value also undergoes the same variation. The problems can therefore be defined as follows:

$$\begin{aligned} \text{minimize} \quad & \begin{bmatrix} -\bar{T}^{PH}_{bed} & \bar{T}^{PH}_{gas} \end{bmatrix} \\ \text{subject to} \quad & T^i_{min} \leq T^i \leq T^i_{max}, \\ & \hat{m}^i_{min} \leq \hat{m}^i \leq \hat{m}^i_{max}, \\ & O^i_{2,min} \leq O^i_2 \leq O^i_{2,max}, \\ & i \in \{PH,\ TPH\}, \end{aligned} \tag{2.18}$$

and

$$\begin{array}{ll} \text{minimize} & \begin{bmatrix} -\bar{O}_2^{PH} & \bar{T}_{gas}^{PH} \end{bmatrix} \\ \text{subject to} & T_{min}^i \leq T^i \leq T_{max}^i, \\ & \hat{m}_{min}^i \leq \hat{m}^i \leq \hat{m}_{max}^i, \\ & O_{2,min}^i \leq O_2^i \leq O_{2,max}^i, \\ & i \in \{PH,\ TPH\}. \end{array} \tag{2.19}$$

Note that one could also consider the formulation in which there is a study of the correlation between Bed Temperature and Oxidation Degree. However, in a practical sense, that would not give so much interesting information since changing one will also imply a change in the other one. However, for completeness, also this formulation is provided as follows:

$$\begin{array}{ll} \text{minimize} & \begin{bmatrix} -\bar{T}_{bed}^{PH} & -\bar{O}_2^{PH} \end{bmatrix} \\ \text{subject to} & T_{min}^i \leq T^i \leq T_{max}^i, \\ & \hat{m}_{min}^i \leq \hat{m}^i \leq \hat{m}_{max}^i, \\ & O_{2,min}^i \leq O_2^i \leq O_{2,max}^i, \\ & i \in \{PH,\ TPH\}. \end{array} \tag{2.20}$$

2.3.2 Problem with nonlinear constraints

To achieve a more realistic scenario, it is possible to include an energy balance equation for the gas in each zone, along with a bound constraint, to the existing constraints in equation (2.12). The energy balance of zone i relies on the conservation of production-specific gas energy, which defines the energy flow in relation to the production rate, given by:

$$\hat{Q}^i = \frac{\dot{Q}^i}{\dot{P}}. \tag{2.21}$$

Here, $\dot{Q}^i$ $[kJ/h]$ represents the energy flow rate at the input of zone i. Additionally, the energy flow rate at the input of zone i can be estimated as the stored energy associated with the process gas, as described by:

$$\dot{Q}^i = \dot{m}^i c_p \Delta T^i, \quad \Delta T^i = T^i - T_{ref}. \tag{2.22}$$

In equation (2.22), c_p $[kJ/(kg \cdot^\circ C)]$ denotes the specific heat capacity of the process gas. By substituting equation (2.22) into equation (2.21), one obtains:

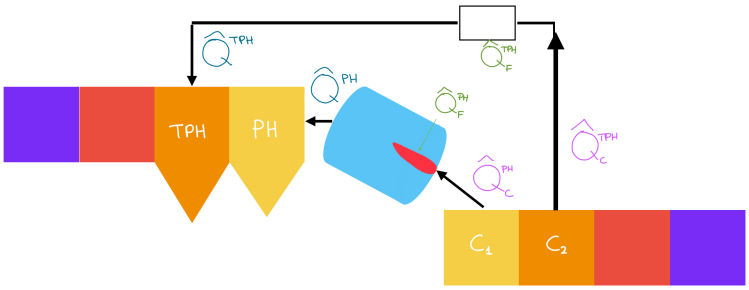
$$\hat{Q}^i = \hat{m}^i c_p \Delta T^i.$$

In an ideal case with no energy losses, the production-specific energy balance of zone i can be expressed as:

$$\hat{Q}^i = \hat{Q}^i_C + \hat{Q}^i_F. \tag{2.23}$$

Here, $\hat{Q}^i_C$ and $\hat{Q}^i_F$ correspond to the production-specific energy from the cooler and the production-specific energy resulting from fuel combustion in zone i, respectively.
Figure 2.3 gives a schematic representation of the production-specific heat capacities within the system:

Figure 2.3: External Energy Scheme



Combining equations (2.21) and (2.23) for zone $i = \{TPH,\ PH\}$ yields the following constraints:

$$\hat{m}^i c_p \Delta T^i - \hat{Q}^i_F - \hat{Q}^i_C = 0, \quad i \in \{TPH,\ PH\},$$

where $\hat{Q}^i_C$ is a parameter, and $\hat{Q}^i_F$ is a variable. In addition to the constraints specified in inequality (2.12), there are further bound constraints imposed by physical limitations on burner capacities.

$$\dot{Q}^i_{F,min} \leq \dot{Q}^i_F \leq \dot{Q}^i_{F,max}, \quad i \in \{TPH,\ PH\}.$$

Consequently, the variables are

$$x = \begin{bmatrix} T^{TPH} & \hat{m}^{TPH} & O_2^{TPH} & \hat{Q}_F^{TPH} & T^{PH} & \hat{m}^{PH} & O_2^{PH} & \hat{Q}_F^{PH} \end{bmatrix}^T. \tag{2.24}$$

Similar to the case with bound constraints, the goal is to maximize the average bed temperature at the output of the PH-zone while minimizing fuel usage.

This leads to the following objective function being minimized:

$$F(x) = \begin{bmatrix} -\bar{T}_{bed}^{PH} & \hat{Q}^{TPH} + \hat{Q}^{PH} \end{bmatrix}. \tag{2.25}$$

Unlike the bound-constrained problem in equation (2.17), only the evaluation of the first component of the objective function requires simulation. In contrast, the second component can be directly determined from x.
However, the resulting problem is a black-box multi-objective problem with nonlinear constraints in the following form:

$$\begin{aligned} \text{minimize} \quad & \begin{bmatrix} -\bar{T}_{bed}^{PH} & \hat{Q}^{TPH} + \hat{Q}^{PH} \end{bmatrix} \\ \text{subject to} \quad & \hat{m}^i c_p \left(T^i - T_{ref}\right) - \hat{Q}_F^i - \hat{Q}_C^i = 0 \\ & T_{min}^i \leq T^i \leq T_{max}^i, \\ & \hat{m}_{min}^i \leq \hat{m}^i \leq \hat{m}_{max}^i, \\ & O_{2,min}^i \leq O_2^i \leq O_{2,max}^i, \\ & \hat{Q}_{F,min}^i \leq \hat{Q}_F^i \leq \hat{Q}_{F,max}^i, \\ & i \in \{PH,\ TPH\}. \end{aligned} \tag{2.26}$$

A comprehensive explanation of the parameters in equation (2.26) along with their corresponding values can be found in Tables 2.1 and 2.2.

Table 2.3: Description Production Specific Energy

Parameters	Description	Value
$\hat{Q}_{F,min}^i$	Minimum production specific energy due to fuel in zone $i \in \{TPH,\ PH\}$.	0.0
$\hat{Q}_{F,max}^{TPH}$	Maximum production specific energy due to fuel in zone TPH-zone.	72.0
$\hat{Q}_{F,max}^{PH}$	Maximum production specific energy due to fuel in zone PH-zone.	288.0

The values of Table 2.3 stem from physical limitations on the burner capacity. In particular, the bounds are due to the power limitations of the burner associated with zone i. E.g., the burner capacity in zone i, $\dot{Q}_{F,max}^i$, $[kJ/h]$, gives the production specific upper bound

$$\hat{Q}_{F,max}^i = \frac{\dot{Q}_{F,max}^i}{\dot{P}}.$$

Maximum burner capacities of $10MW$ for the TPH-zone and $40MW$ for the PH-zone give the values in Table 2.3.

Nonlinear equality constraints often present challenges, and no known algorithms in the existing literature demonstrate the effective handling of such constraints in practice. In theoretical scenarios, it is common to model nonlinear equality constraints as $g(x) = 0$, considering, at the same time, $g(x) \leq 0$ and $g(x) \geq 0$. However, in finite precision arithmetic, this approach may introduce feasibility issues that require careful consideration, depending on the chosen method.

Thus, a relaxed version of the problem (2.26) is solved, where each equality constraint is relaxed to an inequality constraint. This results in a black-box multi-objective problem with nonlinear inequality constraints of the form:

$$\begin{array}{ll} \text{minimize} & \left[-\bar{T}_{bed}^{PH} \quad \hat{Q}^{TPH} + \hat{Q}^{PH}\right] \\ \text{subject to} & \hat{m}^i c_p \left(T^i - T_{ref}\right) - \hat{Q}_F^i - \hat{Q}_C^i \leq 0 \\ & T_{min}^i \leq T^i \leq T_{max}^i, \\ & \hat{m}_{min}^i \leq \hat{m}^i \leq \hat{m}_{max}^i, \\ & O_{2,min}^i \leq O_2^i \leq O_{2,max}^i, \\ & \hat{Q}_{F,min}^i \leq \hat{Q}_F^i \leq \hat{Q}_{F,max}^i, \\ & i \in \{PH,\ TPH\}. \end{array} \tag{2.27}$$

Note that a solution x^* of the problem (2.27) that satisfies the nonlinear equality constraint of (2.26) is also a solution to the problem (2.26).

An alternative formulation can be obtained considering the Oxidation Degree at the end of the PH zone instead of the Bed Temperature at the end of the PH zone:

$$\begin{array}{ll} \text{minimize} & \left[-\bar{O}_2^{PH} \quad \hat{Q}^{TPH} + \hat{Q}^{PH}\right] \\ \text{subject to} & \hat{m}^i c_p \left(T^i - T_{ref}\right) - \hat{Q}_F^i - \hat{Q}_C^i \leq 0 \\ & T_{min}^i \leq T^i \leq T_{max}^i, \\ & \hat{m}_{min}^i \leq \hat{m}^i \leq \hat{m}_{max}^i, \\ & O_{2,min}^i \leq O_2^i \leq O_{2,max}^i, \\ & \hat{Q}_{F,min}^i \leq \hat{Q}_F^i \leq \hat{Q}_{F,max}^i, \\ & i \in \{PH,\ TPH\}. \end{array} \tag{2.28}$$

Chapter 3

Results

3.1 Notes on the implementation

After studying the theoretical aspects of each algorithm, a set of simulations was conducted to assess their suitability for the intended problem. Since most of these algorithms have already been implemented and made publicly available by their authors, it was decided to utilize the existing implementations and make modifications as needed to ensure compatibility with simulation software. This software enables the evaluation of objective function values at specific points, following post-processing procedures. In particular, the simulation software, through an input file, can simulate the pellet production process and generate output files that, after post-processing, give an approximation of the evaluation of function $F(x)$ given a certain x in the input file.

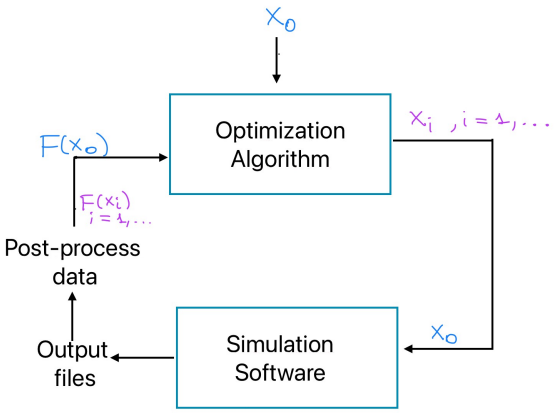
In order to connect the simulation software and the optimization algorithm, the following routine was created and implemented in a Python3 script:

1. Initialize a random x_0 such that $l \leq x_0 \leq u$
2. Call the chosen optimization algorithm with the starting x_0
3. To obtain the evaluation of $F(x_0)$ the simulation software is executed
4. The simulation software generates some output files, and after a post-process, the approximation of $F(x_0)$ is generated
5. The approximated value $F(x_0)$ is given to the optimization algorithm, which uses that value to study the quality of the approximation

6. The optimization algorithm generates a new x_i for $i = 1, ...$
7. This x_i for $i = 1, ...$ is given again as input to the Simulation Software and the process is repeated until stopping criteria are satisfied

This process is also illustrated in Figure 3.1 to clearly understand that.

Figure 3.1: Diagram of the combination process between the optimization algorithm and the simulation software



3.2 Notes on the Algorithms used

In this way, the problem can be tested with different optimization algorithms by changing only Step 2. This 7-step procedure was implemented in a Python3 script.

In order to use the DFMO method (described in Section 2.2.1), a script released by the authors themselves was used. However, the implementation of DFMO was done in Fortran, so it was deemed more suitable to use the Python implementation of the DFMOINT algorithm released by the authors themselves ([22]). This algorithm extends the problem (2.1) where part of the variables are discrete. The approach remains the same, the only difference being that a piece of code is added to handle discrete variables and merge sets containing discrete and continuous variables. In the problem proposed to be solved in this thesis project, all variables are assumed to be continuous, so the DFMOINT algorithm, when called, only uses the part that coincides with DFMO, as it has no discrete variables and does not need to handle them.

In the case of NSGA2 (described in Section 2.2.2), on the other hand, a Python library called *pymoo* was used [23], which makes it possible to solve Multi-objective Optimization Problems.

The Trust Region Method known as Morbit (described in Section 2.2.3) was the final algorithm examined. For its implementation, an existing library was utilized. Specifically, the article's author developed a Julia package that enables the solving of multi-objective problems (`https://github.com/manuelbb-upb/Morbit.jl`, [21]). Since this package is designed for Julia, it became necessary to employ the PyCall library to seamlessly integrate the three-step procedure implemented in Python into a Julia script.

In particular, it is worth noting that Morbit is constantly tested on 50 points in the results presented. This is because NSGA2 and DFMOINT return a set of non-dominated points at the end of the run, whereas Morbit only returns a point belonging to the Pareto Front. For this reason, we chose to run Morbit 50 times in order to obtain 50 distinct points and to be able to compare the performance with the other two methods.

Furthermore, when interpreting the results, it is important to consider that algorithms typically rely on assumptions about the properties of objective functions in order to guarantee convergence and find optimal points. However, due to the nature of the problem and the formulation of the objective functions in this thesis project, it is difficult to prove any specific assumptions. Therefore, the study primarily focuses on exploring and understanding the behaviour of these methods concerning the proposed problem, even without rigorous testing of the underlying hypotheses of the objective functions employed

3.3 Bound Constraint Problem

To evaluate the algorithms presented thus far, the initial objective was to observe their performance on problems incorporating two objective functions, which means that the problem (2.18) and the problem (2.19) where used.

In particular, the first approach was to observe the relationship between Bed Temperature and Oxidation Degree, where the Bed Temperature and Oxidation Degree are referred to the end of the PH zone, which is described by problem the (2.20). In particular, Table 3.1 shows a comparison of

the execution times of each algorithm which gives an initial idea of the performance of the three algorithms with respect to execution times. The table showed that Morbit was, on average slower than the other two algorithms to find one point.

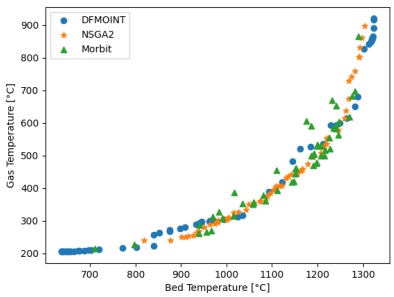
Table 3.1: Comparison of algorithms on problem (2.20) - Bed Temperature vs Oxygen Degree

	DFMOINT	NSGA2	Morbit
CPU time	2h 23m 17s	2h 24m 8s	1h 36m 31s
Number of Points found	218	98	50
Average time for 1 point	40s	88s	116s

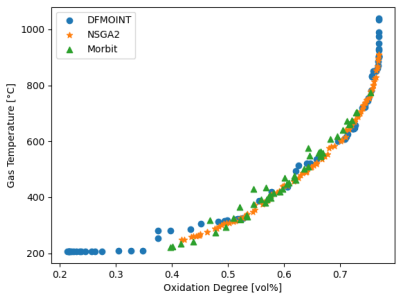
Next, the behaviour of the algorithms was evaluated by considering Bed Temperature with Gas Temperature, which is the problem (2.18), and Oxidation Degree with Gas Temperature, problem (2.19), as objective functions, where the Gas Temperature is the temperature of the outgoing gas of PH zone. Bed Temperature and Oxidation Degree are the same as before. In these cases, since the objective functions are conflicting, the approaches give us more meaningful information regarding what could be best to modify in the current plant to achieve the proposed goal. Furthermore, as can be seen in Figures 3.2a and 3.2b, the three methods give a similar behaviour highlighting almost the same curve in the plane.

Figure 3.2: Bound Constraint Comparison with 2 Objective Functions

(a) Bed Temperature vs Gas Temperature - Problem (2.18)



(b) Oxygen Degree vs Gas Temperature - Problem (2.19)



Again, a comparison of execution times has been made and is described in

Tables 3.2 and 3.3

Table 3.2: Bed Temperature at the end of PH vs Gas Temperature in PH

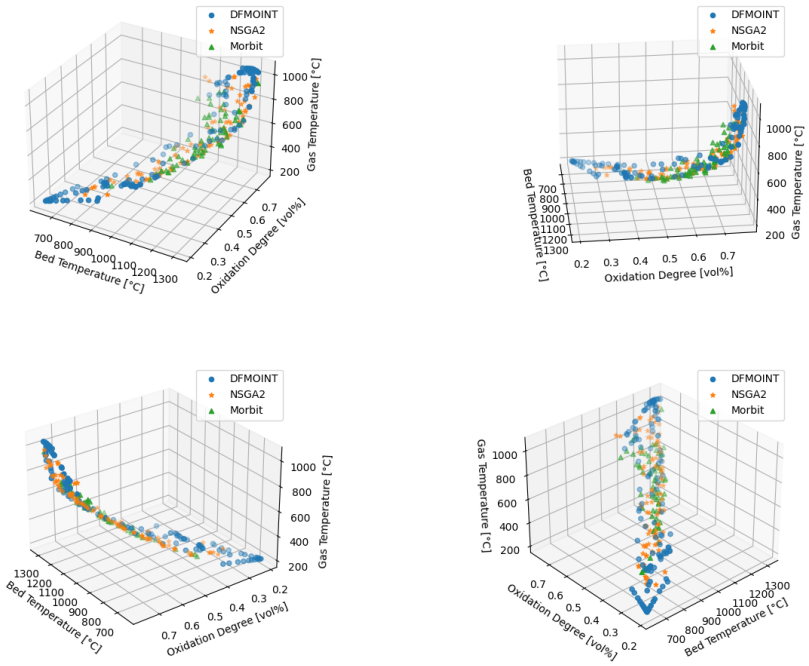
	DFMOINT	NSGA2	Morbit
CPU time	0h 12m 04s	0h 16m 46s	1h 46m 52s
Number of Points found	76	100	50
Average time for 1 point	9s	10s	128s

Table 3.3: Oxidation Degree at the end of PH vs Gas Temperature in PH

	DFMOINT	NSGA2	Morbit
CPU time	2h 13m 14s	2h 12m 52s	1h 54m 24s
Number of Points found	501	98	50
Average time for 1 point	16s	81s	137s

So, this first analysis shows that the DFMOINT algorithm seems to be the fastest in execution time per point found, while Morbit is much slower. At this point, it was also desired to analyze the case where all three objective functions described in equation (2.17) are minimized, shown in Figure 3.3. It was noticed that the algorithms identify approximately the same curve that can be traced back to the Pareto Front. The fact that all three algorithms identify a nearly similar curve suggests that the methods consistently and adequately solve the proposed problem and that this way of approaching the problem is valid.

Figure 3.3: Comparison of the three optimization algorithms considering three objective functions - Problem (2.17)

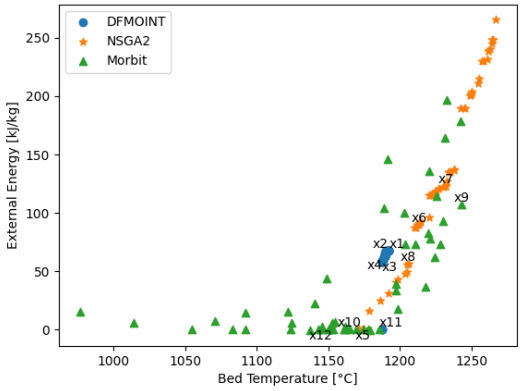


3.4 Nonlinear Constraint Problem

When the problem formulation includes nonlinear constraints, it has been observed that the algorithms have greater difficulty in finding points belonging to the Pareto Front, and this is evidenced by the fact that they turn out to be slower and find fewer points.

At first, the classical case was studied: what happens if our target functions are Bed Temperature and External Energy, which is related to fuel consumption. In particular, the formulation in equation (2.27) was considered. The algorithms perform similarly to the bound-constrained case, except that Morbit requires more time and iterations to converge closer to the same curve. The results are presented in Figure 3.4.

Figure 3.4: Bed Temperature vs External Energy - Problem (2.27)



To better understand the execution time of such algorithms, a comparison of execution times was also sought, similar to the Bound Constraint case. This comparison revealed that the previously observed trend of Morbit being the slowest algorithm still holds. However, NSGA2 appears to exhibit faster performance per point compared to DFMOINT in this case, as indicated in Table 3.4. Additionally, DFMOINT seems to struggle in locating points within a cluster that are dominated by the points generated by NSGA2.

Table 3.4: Comparison of algorithms - Bed Temperature vs External Energy

	DFMOINT	NSGA2	Morbit
CPU time	2h 35m 45s	2h 40m 45s	3h 46m 16s
Number of Points found	56	65	50
Average time for 1 point	167s	148s	272s

As explained in Section 2.3.2, an inequality was considered in formulating the problem instead of equality regarding the function g(x). For this reason, it was deemed necessary to observe the actual values that g(x) takes on the points belonging to the Pareto Front. This evaluation, found in Tables 3.5, 3.6 and 3.7, served to understand that the non-dominated points that the various algorithms find assume values that make the value of the function g(x) almost to zero, and therefore, that, considering inequality instead of equality, does not greatly invalidate the solution of the given problem.

Table 3.5: DFMOINT: Bed Temperature vs External Energy

label	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
x_1	964	0.414	15.68	1250	0.494	40.0	0.0	67.5	-1192	67.5	-0.54	0.16
x_2	964	0.414	15.69	1250	0.494	40.0	0.0	67.3	-1191	67.3	-0.54	0.28
x_3	964	0.413	15.00	1249	0.488	40.0	0.0	59.1	-1187	59.1	-1.45	-0.08
x_4	964	0.414	15.00	1249	0.489	40.0	0.0	60.5	-1188	60.4	-1.24	-0.06

It is important to note that in DFMOINT, certain g(x) values can be positive despite the specified constraint $g(x) \leq 0$. This occurrence is associated with using a penalty function to handle $g(x)$. In such cases, bearing the associated "price" may be advantageous if the decrease in other objectives surpasses the cost, assuming a comprehensive understanding of the situation. By reducing the value of epsilon, the cost of violating the constraint $g(x) \leq 0$ becomes more significant, and it is likely to obtain the value of $g(x)$ closer to 0 when x is a point in the Pareto Front.

Table 3.6: NSGA2: Bed Temperature vs External Energy

label	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
x_5	1239	0.323	16.7	1226	0.453	38.20	0.3	0.03	-1174	0.3	-0.3	-0.7
x_6	1238	0.323	17.1	1226	0.520	37.32	0.2	89.93	-1213	90.2	-0.6	-0.2
x_7	1248	0.338	18.3	1236	0.521	37.310	25.8	97.16	-1232	122.9	-1.2	-0.8
x_8	1233	0.347	19.7	1236	0.468	37.44	31.9	24.5	-1205	56.4	-0.8	-0.2

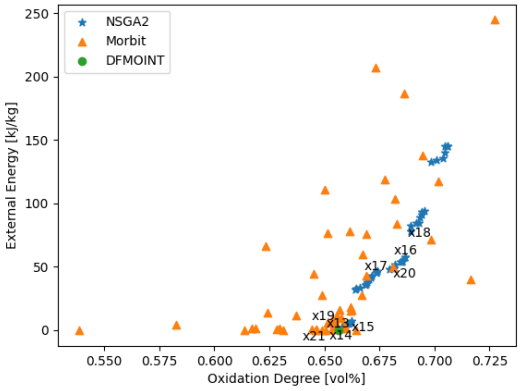
Table 3.7: Morbit: Bed Temperature vs External Energy

label	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
x_9	1248	0.37	15.0	1250	0.470	40.0	72.00	35.17	-1243	107.17	0.20	-0.995
x_{10}	1249	0.32	39.9	1250	0.445	31.7	0.065	0.25	-1164	0.32	0.03	-0.951
x_{11}	1242	0.32	15.83	1249	0.445	39.9	-0.001	-0.00	-1185	-0.00	-0.65	-0.722
x_{12}	1222	0.33	34.19	1238	0.448	29.2	0.132	0.05	-1153	0.18	-3.39	-1.501

The values depicted in Tables 3.5, 3.7, 3.6 are considered important as they can give an insight and general idea under which conditions the function can be optimized. That is to say, what these data allow is a better analysis of what needs to be changed in the current process to obtain optimal values that can help achieve the goal being solved.

After this analysis, observing the behaviour in the case where the objective functions are Oxidation Degree with External Energy was also considered. However, in this case, DFMOINT could not find many solutions to the problem, but only one point, as can be seen from Figure 3.5.

Figure 3.5: Oxidation Degree vs External Energy - Problem (2.28)



However, this single point is on the same curve as the other two algorithms found. Looking at the execution times of the algorithms presented in Table 3.8, the time taken by DFMOINT to find a single point is significantly longer than the other two methods. However, it was not possible to carry out further experiments considering longer timescales to actually describe the algorithm's behaviour in this case and thus come to conclusions due to the tight timeframe for the thesis project.

Table 3.8: Comparison of algorithms - Oxidation vs External Energy

	DFMOINT	NSGA2	Morbit
CPU time	0h 29m 25s	2h 34m 08s	4h 08m 01s
Number of Points found	1	48	50
Average time for 1 point	1766s	193s	298s

Here too, however, the values of g(x), presented in Tables 3.10 and 3.11, were evaluated, which seem to have fairly similar behaviour to what was observed earlier in Tables 3.6 and 3.7

Table 3.9: DFMOINT: Oxidation Degree vs External Energy

label	T^{TPH}	$\dot{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\dot{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
x_{13}	1250	0.3	39.8	1070	0.52	40.0	0.0	0.0	-0.6	0.0	5.68e-14	-0.4

Table 3.10: NSGA2: Oxidation Degree vs External Energy

label	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
x_{14}	1214	0.329	38.65	915	0.608	39.86	0.1	0.07	-0.66	0.3	-0.7	-0.4
x_{15}	1210	0.336	38.67	915	0.607	39.90	7.1	0.10	-0.66	7.2	-0.0	-1.6
x_{16}	1141	0.395	38.62	915	0.608	39.88	57.5	0.25	-0.69	57.8	-1.5	-0.2
x_{17}	1209	0.334	38.95	974	0.608	39.90	4.9	39.91	-0.67	44.8	-0.3	-0.3

Table 3.11: Morbit: Oxidation Degree vs External Energy

label	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
x_{18}	1249	0.37	40.0	850	0.65	40.0	71.7	0.0	-0.69	71.7	0.2	-0.6
x_{19}	1230	0.32	28.6	922	0.61	39.2	0.8	4.6	-0.65	5.4	-3.8	-0.7
x_{20}	1214	0.35	24.1	976	0.58	39.7	35.3	14.3	-0.68	49.6	-0.0	-0.7
x_{21}	1226	0.32	39.8	850	0.65	39.9	0.0	0.0	-0.65	0.1	-6.9	-4.7

Furthermore, it is worth noting that the data that have been collected in Tables 3.5, 3.6, 3.7, 3.9, 3.10, 3.11 are also those highlighted in Figures 3.4, 3.5.

Note that if one is interested in seeing the actual values of all the points shown and analyzed in sections 3.3 and 3.4 and the relative value the objective functions take on at these points, please refer to Appendices A, B, C, D, E and F.

Chapter 4

Conclusions

This thesis project aimed to study the pellet induration process at LKAB company to understand what modifications can be made to the current plant to make the process fossil-free. To achieve this, a multi-objective optimization problem was studied, where the objective function values were derived from the post-processing of data generated by simulation software. Therefore, a simulation-based multi-objective optimization problem was investigated.

This thesis project analyzed three different algorithms found in the literature. Specifically, considering the three method's classes, namely Direct-Search Methods (DSM), Response Surface Methods (RSM), and Heuristic Algorithms, three methods belonging to each of these classes were identified, namely DFMO [19], NSGA-2 [20], and Morbit [21], respectively.

After carefully studying the theory behind these methods and evaluating their compatibility with the problem proposed in this project, it was chosen to use the algorithms and libraries already implemented and publicly available by each author. This approach allowed for a greater focus on the proper formulation of the problem and the study and analysis of the obtained results.

Furthermore, to better study the algorithms, it was decided to give two different formulations of the problem. Initially, the most natural and intuitive formulation of the problem was provided, where the variables are subject to lower and upper limits. This allowed for a better examination of the algorithms' behaviour regarding efficiency and correctness.

Once it was observed that the algorithms exhibited similar behaviour, a more

complex formulation of the problem was introduced, considering a simplified energy balance equation. In this formulation, it was noted that by changing the objective functions of the problem, different behaviours could be observed both in the algorithms and in the solutions themselves.

Finally, this project serves as a foundation for more complex problems, including the entire pellet induration process system. Thanks to this project, it is possible to gain an initial understanding of which approaches are suitable for studying more complex systems.

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Appendix A

Full Output values of Problem (2.20)

This appendix contains the values of the objective functions and variables in the solutions presented in Section 3.3 referred to the problem formulation (2.20).

A.1 DFMOINT Method

Table A.1: Output values for DFMOINT - Problem (2.20)

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
0	-1324.238	-0.759	1250.000	0.700	15.000	1250.000	0.387	38.750
1	-1275.610	-0.769	1250.000	0.700	40.000	1245.000	0.700	40.000
2	-1308.987	-0.767	1230.000	0.700	20.000	1250.000	0.474	40.000
3	-1308.246	-0.768	1250.000	0.700	24.350	1250.000	0.380	40.000
4	-1313.577	-0.767	1250.000	0.700	21.190	1250.000	0.380	40.000
5	-1290.291	-0.769	1250.000	0.700	33.750	1250.000	0.540	40.000
6	-1321.642	-0.764	1245.000	0.700	15.180	1250.000	0.460	40.000
7	-1316.015	-0.766	1250.000	0.700	20.000	1250.000	0.352	39.450
8	-1317.430	-0.765	1240.000	0.700	15.370	1250.000	0.540	40.000
9	-1320.035	-0.765	1250.000	0.700	17.510	1250.000	0.380	40.000
10	-1286.499	-0.769	1250.000	0.700	40.000	1250.000	0.380	40.000
11	-1303.410	-0.768	1250.000	0.700	27.500	1250.000	0.380	39.380
12	-1324.225	-0.760	1250.000	0.700	15.000	1250.000	0.390	39.060

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
13	-1322.401	-0.763	1250.000	0.700	15.000	1250.000	0.474	40.000
14	-1294.987	-0.769	1250.000	0.700	30.000	1250.000	0.540	40.000
15	-1324.077	-0.762	1250.000	0.700	15.000	1250.000	0.390	40.000
16	-1293.882	-0.769	1250.000	0.700	33.750	1250.000	0.380	40.000
17	-1324.109	-0.761	1250.000	0.700	15.000	1250.000	0.380	39.710
18	-1323.222	-0.763	1250.000	0.700	15.000	1250.000	0.446	39.720
19	-1299.085	-0.769	1250.000	0.700	30.000	1250.000	0.380	40.000
20	-1314.353	-0.767	1250.000	0.700	20.000	1250.000	0.446	39.720
21	-1323.174	-0.763	1248.000	0.700	15.090	1250.000	0.420	40.000
22	-1299.720	-0.769	1245.000	0.700	27.690	1250.000	0.460	40.000
23	-1281.271	-0.769	1250.000	0.700	40.000	1250.000	0.620	40.000
24	-1315.855	-0.766	1250.000	0.700	20.000	1250.000	0.352	40.000
25	-1315.730	-0.767	1250.000	0.700	20.000	1250.000	0.380	40.000
26	-1302.966	-0.768	1250.000	0.700	27.510	1250.000	0.380	40.000
27	-1318.964	-0.765	1250.000	0.700	17.510	1248.000	0.380	40.000
28	-1324.095	-0.762	1250.000	0.700	15.000	1250.000	0.380	40.000
29	-1324.233	-0.759	1250.000	0.700	15.000	1250.000	0.390	38.750
30	-1283.662	-0.769	1250.000	0.700	39.420	1245.000	0.448	39.530
31	-1279.152	-0.769	1250.000	0.700	40.000	1250.000	0.700	40.000
32	-1324.220	-0.760	1250.000	0.700	15.000	1250.000	0.390	39.380
33	-1324.129	-0.760	1250.000	0.700	15.000	1250.000	0.380	39.380
34	-1283.602	-0.769	1250.000	0.700	40.000	1250.000	0.540	40.000
35	-1278.097	-0.769	1250.000	0.700	40.000	1248.000	0.680	40.000
36	-1303.128	-0.768	1250.000	0.700	27.500	1250.000	0.380	40.000
37	-1275.948	-0.769	1250.000	0.700	40.000	1245.000	0.694	40.000
38	-1277.362	-0.769	1250.000	0.700	40.000	1248.000	0.700	40.000
39	-1277.403	-0.769	1250.000	0.700	40.000	1248.000	0.700	40.000
40	-1324.234	-0.759	1250.000	0.700	15.000	1250.000	0.390	38.750
41	-1275.906	-0.769	1250.000	0.700	39.690	1245.000	0.700	40.000
42	-1275.776	-0.769	1250.000	0.700	40.000	1245.000	0.698	40.000
43	-1275.700	-0.769	1250.000	0.700	39.960	1245.000	0.700	40.000
44	-1275.776	-0.769	1250.000	0.700	40.000	1245.000	0.696	40.000
45	-1275.650	-0.769	1250.000	0.700	40.000	1245.000	0.700	40.000
46	-1275.649	-0.769	1250.000	0.700	40.000	1245.000	0.700	40.000
47	-1275.628	-0.769	1250.000	0.700	40.000	1245.000	0.700	40.000
48	-1275.625	-0.769	1250.000	0.700	40.000	1245.000	0.700	40.000
49	-1324.237	-0.759	1250.000	0.700	15.000	1250.000	0.387	38.750
50	-1324.236	-0.759	1250.000	0.700	15.000	1250.000	0.387	38.750
51	-1324.236	-0.759	1250.000	0.700	15.000	1250.000	0.387	38.750

A.2 NSGA2 Method

Table A.2: Output values for NSGA2 - Problem (2.20)

id	$\tilde{T}_{bed}^{PH}$	$\tilde{O}_2^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-1291.099	-0.768	1236.000	0.694	30.310	1245.000	0.490	39.120
2	-1269.701	-0.769	1178.000	0.697	32.210	1237.000	0.691	39.320
3	-1313.632	-0.765	1236.000	0.697	16.560	1245.000	0.490	39.140
4	-1287.200	-0.768	1238.000	0.694	30.330	1240.000	0.529	39.120
5	-1313.124	-0.765	1238.000	0.694	17.190	1246.000	0.488	39.120
6	-1287.419	-0.768	1236.000	0.697	29.730	1240.000	0.532	39.140
7	-1299.856	-0.768	1244.000	0.690	25.740	1248.000	0.518	39.030
8	-1315.721	-0.764	1236.000	0.697	15.620	1245.000	0.490	39.140
9	-1294.275	-0.768	1244.000	0.697	29.500	1248.000	0.531	39.030
10	-1309.096	-0.767	1236.000	0.694	19.800	1248.000	0.491	39.020
11	-1292.595	-0.768	1236.000	0.697	29.210	1245.000	0.490	39.150
12	-1310.930	-0.766	1250.000	0.697	19.610	1248.000	0.533	39.140
13	-1282.474	-0.769	1238.000	0.697	34.440	1240.000	0.529	39.140
14	-1317.062	-0.764	1236.000	0.699	15.620	1248.000	0.496	39.110
15	-1252.283	-0.769	1247.000	0.696	33.740	1201.000	0.685	39.210
16	-1296.755	-0.768	1237.000	0.696	25.800	1243.000	0.490	39.030
17	-1286.014	-0.768	1236.000	0.697	30.840	1240.000	0.532	39.180
18	-1302.898	-0.767	1233.000	0.698	20.820	1241.000	0.497	39.110
19	-1314.549	-0.765	1244.000	0.691	17.180	1248.000	0.518	39.030
20	-1281.517	-0.769	1234.000	0.697	34.440	1240.000	0.529	39.140
21	-1280.306	-0.769	1234.000	0.697	35.340	1238.000	0.496	39.090
22	-1305.453	-0.767	1216.000	0.697	18.940	1246.000	0.488	39.120
23	-1236.879	-0.769	1193.000	0.698	32.820	1188.000	0.685	39.320
24	-1319.497	-0.759	1248.000	0.696	15.900	1249.000	0.496	37.350
25	-1190.266	-0.769	1240.000	0.700	33.650	1107.000	0.663	38.580
26	-1317.417	-0.764	1238.000	0.699	15.620	1248.000	0.495	39.110
27	-1284.007	-0.769	1233.000	0.697	30.840	1237.000	0.529	39.310
28	-1293.359	-0.768	1236.000	0.697	29.540	1249.000	0.532	39.140
29	-1311.782	-0.766	1250.000	0.697	19.380	1248.000	0.529	39.140
30	-1282.038	-0.769	1238.000	0.697	34.670	1240.000	0.533	39.140
31	-1295.739	-0.768	1237.000	0.696	26.460	1243.000	0.490	39.020
32	-1295.655	-0.768	1244.000	0.697	29.500	1248.000	0.492	39.030
33	-1300.158	-0.768	1244.000	0.690	25.870	1248.000	0.518	39.040
34	-1314.296	-0.765	1236.000	0.697	16.210	1245.000	0.490	39.140
35	-1309.503	-0.766	1238.000	0.697	19.370	1246.000	0.488	39.120
36	-1291.216	-0.768	1236.000	0.694	30.220	1245.000	0.490	39.120

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
37	-1309.026	-0.767	1236.000	0.694	19.900	1248.000	0.491	39.020
38	-1282.603	-0.769	1236.000	0.697	33.750	1240.000	0.532	39.180
39	-1255.460	-0.769	1247.000	0.696	30.830	1201.000	0.685	39.210
40	-1253.965	-0.769	1247.000	0.696	33.740	1204.000	0.685	39.210

A.3 Morbit Method

Table A.3: Output values for Morbit - Problem (2.20)

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-1319.902	-0.752	1236.236	0.695	15.000	1249.730	0.400	35.142
2	-1228.056	-0.768	1168.881	0.636	35.704	1172.954	0.614	39.339
3	-1303.053	-0.766	1239.348	0.678	17.919	1233.299	0.528	38.685
4	-1234.026	-0.768	1229.804	0.695	21.455	1141.412	0.606	37.346
5	-1268.043	-0.768	1240.586	0.656	38.544	1227.790	0.681	37.352
6	-1273.537	-0.768	1235.989	0.692	33.905	1212.695	0.390	37.506
7	-1279.078	-0.758	1174.147	0.595	27.827	1233.502	0.677	33.212
8	-1310.293	-0.759	1247.259	0.681	19.112	1239.583	0.443	35.121
9	-1317.521	-0.762	1240.705	0.686	16.500	1248.412	0.456	37.641
10	-1275.526	-0.766	1239.452	0.690	39.004	1238.983	0.696	34.179
11	-1272.360	-0.768	1218.659	0.666	32.640	1229.710	0.660	37.047
12	-1324.021	-0.762	1250.000	0.700	15.000	1250.000	0.400	40.000
13	-1299.752	-0.761	1227.594	0.635	17.172	1239.221	0.687	36.719
14	-1282.745	-0.768	1236.925	0.687	23.868	1222.107	0.596	39.254
15	-1306.440	-0.767	1237.159	0.689	22.242	1247.071	0.350	39.198
16	-1287.297	-0.766	1068.157	0.689	16.583	1245.580	0.686	38.726
17	-1269.593	-0.757	1224.848	0.678	36.987	1225.761	0.693	28.706
18	-1300.622	-0.766	1229.492	0.678	18.929	1232.267	0.488	38.266
19	-1261.510	-0.767	1237.266	0.698	25.301	1181.193	0.532	36.649
20	-1233.053	-0.766	1179.277	0.654	33.485	1174.914	0.678	33.850
21	-1239.055	-0.768	1194.555	0.680	29.687	1177.937	0.640	37.105

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
22	-1194.383	-0.769	1224.642	0.714	24.982	1057.423	0.464	40.117
23	-1302.919	-0.762	1225.391	0.601	17.218	1245.748	0.665	37.927
24	-1173.846	-0.769	1193.126	0.663	35.251	1089.076	0.644	38.160
25	-1298.102	-0.765	1212.074	0.661	19.168	1243.556	0.638	38.227
26	-1263.195	-0.768	1221.652	0.634	35.244	1219.274	0.676	37.622
27	-1264.237	-0.768	1238.734	0.680	33.076	1193.847	0.412	37.888
28	-1287.294	-0.769	1250.000	0.700	31.920	1250.000	0.700	40.000
29	-1293.024	-0.767	1231.909	0.687	25.083	1236.471	0.501	38.427
30	-1316.884	-0.764	1250.000	0.700	15.376	1250.000	0.602	40.000
31	-1229.885	-0.769	1193.173	0.695	32.608	1168.265	0.623	37.833
32	-1289.625	-0.767	1218.244	0.611	28.792	1248.438	0.601	39.179
33	-1311.743	-0.764	1250.000	0.700	15.797	1250.000	0.688	40.000
34	-1302.046	-0.763	1247.248	0.670	17.368	1218.696	0.443	36.873
35	-1243.582	-0.764	1169.486	0.637	18.750	1163.768	0.651	35.404
36	-1272.041	-0.768	1229.609	0.691	21.345	1189.455	0.497	38.670
37	-1285.182	-0.768	1241.948	0.684	28.415	1240.180	0.676	39.374
38	-1319.150	-0.761	1249.747	0.692	16.485	1245.506	0.354	38.887
39	-1179.200	-0.769	1250.000	0.700	40.000	987.514	0.300	40.000
40	-1265.792	-0.766	1198.337	0.666	17.105	1175.776	0.484	38.604
41	-1275.421	-0.769	1250.000	0.700	40.000	1244.324	0.700	40.000
42	-1224.548	-0.768	1240.401	0.687	26.457	1127.963	0.573	36.899
43	-1213.920	-0.764	1135.772	0.599	20.359	1131.000	0.697	37.018
44	-1285.666	-0.765	1207.536	0.671	24.151	1236.702	0.677	36.074
45	-1286.613	-0.767	1224.725	0.684	27.635	1229.188	0.387	37.895
46	-1282.971	-0.768	1245.838	0.699	30.955	1222.445	0.412	38.871
47	-1296.151	-0.763	1235.899	0.691	18.773	1232.514	0.651	36.695
48	-1277.807	-0.769	1238.656	0.690	39.246	1247.500	0.693	39.219
49	-1217.889	-0.769	1222.447	0.684	34.926	1130.671	0.491	39.224
50	-1271.278	-0.767	1215.243	0.667	33.568	1220.441	0.452	37.452

Appendix B

Full Output values of Problem (2.18)

This appendix contains the values of the objective functions and variables in the solutions presented in Section 3.3 referred to the problem formulation (2.18).

B.1 DFMOINT Method

Table B.1: Output values for DFMOINT from Figure 3.2a

id	$\tilde{T}_{bed}^{PH}$	$\tilde{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
0	-636.645	205.466	850.000	0.300	15.000	850.000	0.300	15.000
1	-1093.279	388.349	1250.000	0.300	35.970	1244.000	0.375	32.360
2	-1121.531	419.489	1249.000	0.300	36.400	1245.000	0.410	33.170
3	-1144.744	483.166	850.000	0.300	15.000	1146.000	0.700	40.000
4	-1212.739	535.039	1250.000	0.300	37.980	1247.000	0.538	36.180
5	-771.310	216.663	850.000	0.300	15.000	1109.000	0.300	15.000
6	-1229.634	592.670	1090.000	0.300	40.000	1219.000	0.700	40.000
7	-1162.118	520.214	1180.000	0.300	34.500	1169.000	0.579	34.050
8	-802.554	219.427	850.000	0.300	15.000	1170.000	0.300	15.000
9	-719.003	212.280	850.000	0.300	15.000	1010.000	0.300	15.000
10	-840.032	223.105	850.000	0.300	15.000	1250.000	0.300	15.000

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
11	-1184.905	526.937	930.000	0.300	40.000	1188.000	0.700	40.000
12	-1248.569	599.947	1250.000	0.300	38.990	1249.000	0.619	38.090
13	-908.658	281.058	1164.000	0.300	20.590	1150.000	0.300	15.000
14	-840.689	258.179	1084.000	0.300	15.590	1070.000	0.300	15.000
15	-933.263	288.699	1167.000	0.300	27.800	1160.000	0.300	21.340
16	-852.137	262.873	1087.000	0.300	22.800	1080.000	0.300	16.340
17	-898.072	275.998	1159.000	0.300	15.000	1130.000	0.300	15.000
18	-874.679	267.852	1089.000	0.300	26.400	1085.000	0.300	23.170
19	-874.933	272.042	1148.000	0.300	15.000	1090.000	0.300	15.000
20	-703.386	210.972	850.000	0.300	15.000	979.300	0.300	15.000
21	-945.233	296.800	1250.000	0.300	15.000	1161.000	0.300	15.000
22	-687.284	209.153	850.000	0.300	15.000	850.000	0.300	35.000
23	-961.442	296.974	1239.000	0.300	15.000	1210.000	0.300	15.000
24	-941.304	292.610	1228.000	0.300	15.000	1170.000	0.300	15.000
25	-696.791	210.007	850.000	0.300	15.000	850.000	0.300	40.000
26	-963.693	299.140	1250.000	0.300	15.000	1205.000	0.300	15.000
27	-664.296	207.363	850.000	0.300	15.000	850.000	0.300	25.000
28	-676.961	208.816	850.000	0.300	15.000	930.000	0.300	15.000
29	-673.401	208.034	850.000	0.300	15.000	850.000	0.300	28.590
30	-657.165	207.170	850.000	0.300	15.000	890.000	0.300	15.000
31	-641.444	205.789	850.000	0.300	15.000	850.000	0.300	16.700
32	-646.096	206.107	850.000	0.300	15.000	850.000	0.300	18.400
33	-647.123	206.332	850.000	0.300	15.000	870.000	0.300	15.000
34	-651.403	206.463	850.000	0.300	15.000	850.000	0.300	20.000
35	-656.192	206.797	850.000	0.300	15.000	850.000	0.300	21.790
36	-652.123	206.780	850.000	0.300	15.000	882.300	0.300	15.000
37	-638.601	205.601	850.000	0.300	15.000	850.000	0.300	15.850
38	-638.034	205.558	850.000	0.300	15.000	850.000	0.300	15.420
39	-1264.087	613.690	930.000	0.380	15.000	1250.000	0.700	40.000
40	-1282.899	650.163	1250.000	0.300	15.000	1250.000	0.700	40.000
41	-1240.819	593.415	930.000	0.380	40.000	1250.000	0.660	40.000
42	-1324.095	919.751	1250.000	0.700	15.000	1250.000	0.380	40.000
43	-1322.137	865.351	1250.000	0.700	15.000	1250.000	0.300	40.000
44	-1323.963	915.639	1250.000	0.700	15.000	1250.000	0.380	37.500
45	-1035.604	316.717	1250.000	0.300	15.000	1247.000	0.300	34.730
46	-1025.333	313.088	1250.000	0.300	15.000	1244.000	0.300	29.450
47	-993.972	305.001	1250.000	0.300	15.000	1238.000	0.300	18.910
48	-972.762	300.534	1250.000	0.300	15.000	1227.000	0.300	15.000
49	-963.757	299.893	1250.000	0.300	15.000	1203.000	0.300	15.000
50	-943.856	296.520	1250.000	0.300	15.000	1157.000	0.300	15.000

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
51	-1288.303	679.421	1250.000	0.324	15.000	1250.000	0.700	39.550
52	-1320.860	860.630	1250.000	0.700	15.000	1249.000	0.300	37.420
53	-1318.730	855.222	1250.000	0.700	15.000	1247.000	0.300	34.840
54	-1312.124	842.795	1250.000	0.700	15.000	1244.000	0.300	29.670
55	-1323.489	890.885	1250.000	0.700	15.000	1250.000	0.339	37.990
56	-1320.342	857.780	1250.000	0.700	15.000	1250.000	0.300	35.970
57	-1316.554	848.701	1250.000	0.700	15.000	1250.000	0.300	31.940
58	-1302.044	826.156	1250.000	0.700	15.000	1250.000	0.300	23.890

B.2 NSGA2 Method

Table B.2: Output values for NSGA2 from Figure 3.2a

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-1215.801	529.259	979.900	0.350	20.530	1249.000	0.589	39.430
2	-818.541	240.382	858.900	0.323	18.760	1031.000	0.311	39.890
3	-1190.296	497.510	1203.000	0.326	19.250	1241.000	0.475	39.840
4	-1002.754	307.791	1053.000	0.325	18.840	1236.000	0.319	38.290
5	-1163.060	453.393	865.900	0.324	18.270	1244.000	0.590	33.560
6	-1108.802	406.061	961.200	0.334	25.690	1238.000	0.462	39.720
7	-981.466	301.905	1053.000	0.325	18.760	1236.000	0.322	25.120
8	-1026.703	327.889	1189.000	0.324	23.410	1234.000	0.300	32.670
9	-997.159	303.688	1063.000	0.323	18.810	1230.000	0.311	39.890
10	-1220.468	552.141	1182.000	0.343	19.210	1236.000	0.530	36.290
11	-937.540	267.832	906.600	0.318	20.550	1249.000	0.329	26.540
12	-1269.044	729.849	1105.000	0.584	16.490	1239.000	0.445	33.910
13	-1291.786	802.328	1160.000	0.613	16.520	1246.000	0.438	37.140
14	-1135.574	435.522	911.200	0.359	16.710	1236.000	0.500	38.110
15	-1292.932	831.531	1164.000	0.586	16.450	1240.000	0.532	37.180
16	-1073.275	359.168	1205.000	0.324	20.530	1249.000	0.324	39.430
17	-1178.627	474.060	865.900	0.350	18.270	1250.000	0.587	34.050
18	-1103.560	395.633	856.900	0.334	19.210	1235.000	0.503	37.280
19	-1219.835	535.516	1175.000	0.302	17.870	1244.000	0.584	32.230
20	-1302.838	897.974	1246.000	0.614	20.320	1239.000	0.507	34.740
21	-1130.842	430.943	911.200	0.359	16.720	1236.000	0.494	38.110

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
22	-1165.932	457.664	865.900	0.326	18.270	1244.000	0.594	33.290
23	-1048.369	350.142	1186.000	0.334	25.690	1238.000	0.312	32.670
24	-1245.435	577.533	979.900	0.350	20.530	1249.000	0.656	38.210
25	-909.627	249.949	906.600	0.317	20.550	1249.000	0.301	27.650
26	-950.924	281.627	891.900	0.343	20.550	1249.000	0.329	26.540
27	-1116.612	407.302	911.200	0.327	16.710	1236.000	0.500	38.100
28	-1005.306	309.668	1050.000	0.325	18.760	1236.000	0.322	38.760
29	-1043.607	333.597	1160.000	0.330	16.520	1246.000	0.308	37.140
30	-1090.550	375.323	865.900	0.324	19.890	1247.000	0.475	39.840
31	-1142.302	443.092	865.900	0.326	18.270	1244.000	0.594	23.650
32	-1223.153	552.306	1183.000	0.343	19.210	1241.000	0.530	35.600
33	-973.738	291.142	906.400	0.350	22.880	1250.000	0.320	39.430
34	-926.441	255.617	871.100	0.323	18.760	1242.000	0.311	38.810
35	-916.266	252.541	866.500	0.321	20.530	1230.000	0.311	38.210
36	-1121.480	407.590	873.100	0.324	20.320	1243.000	0.517	39.600
37	-1259.479	614.346	1175.000	0.303	20.080	1244.000	0.697	31.990
38	-1281.834	758.893	1164.000	0.589	17.460	1246.000	0.438	27.990
39	-1267.951	674.990	1160.000	0.448	16.450	1241.000	0.532	37.190
40	-1275.250	741.913	1191.000	0.467	23.410	1234.000	0.580	32.670
41	-1116.397	406.543	911.200	0.327	16.710	1236.000	0.500	38.110
42	-1189.150	496.441	1197.000	0.326	19.250	1241.000	0.475	39.960
43	-964.013	290.258	961.200	0.323	24.740	1211.000	0.330	39.550
44	-901.127	249.936	855.300	0.326	25.600	1214.000	0.306	38.340
45	-1059.676	354.393	1211.000	0.334	19.980	1238.000	0.312	32.670
46	-1131.180	431.076	911.200	0.359	16.720	1236.000	0.494	38.110
47	-1108.020	400.818	869.600	0.334	19.110	1235.000	0.503	37.280
48	-1086.599	370.198	865.900	0.318	19.900	1247.000	0.475	39.840
49	-1107.159	397.384	1104.000	0.302	16.500	1239.000	0.444	32.230
50	-1297.425	860.799	1177.000	0.584	17.860	1245.000	0.585	33.910
51	-1096.922	384.217	858.000	0.321	16.880	1236.000	0.500	38.110
52	-1207.135	509.010	906.500	0.327	18.270	1247.000	0.642	33.560
53	-876.714	240.399	865.900	0.317	20.550	1212.000	0.303	26.540
54	-999.660	304.022	1063.000	0.323	16.070	1232.000	0.312	39.890
55	-1075.471	361.087	1205.000	0.326	19.810	1249.000	0.324	39.430
56	-981.426	296.889	961.200	0.324	39.630	1249.000	0.330	39.460
57	-1209.948	525.761	1082.000	0.326	16.010	1236.000	0.575	37.180
58	-1261.943	637.087	1186.000	0.348	20.380	1238.000	0.643	32.670
59	-1156.384	451.500	906.600	0.317	20.510	1249.000	0.587	27.270
60	-938.889	268.395	865.900	0.350	18.310	1250.000	0.309	34.440

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
61	-978.381	296.137	961.000	0.324	39.830	1249.000	0.331	37.640
62	-935.089	262.898	872.000	0.325	18.840	1236.000	0.321	38.290
63	-1155.720	445.346	865.900	0.324	18.270	1244.000	0.579	33.610
64	-1015.665	324.809	1100.000	0.318	20.550	1239.000	0.341	26.540
65	-1291.840	802.861	1183.000	0.574	16.410	1241.000	0.475	39.840
66	-1166.588	459.788	865.900	0.332	18.270	1244.000	0.590	33.560

B.3 Morbit Method

Table B.3: Output values for Morbit from Figure 3.2a

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-990.482	308.848	1118.795	0.311	18.202	1193.338	0.321	34.361
2	-1058.159	349.914	1016.683	0.302	16.720	1233.848	0.413	38.062
3	-1270.786	619.107	1152.263	0.302	15.072	1249.989	0.698	39.918
4	-1190.622	503.129	1250.000	0.440	15.000	1250.000	0.300	40.000
5	-1150.574	443.385	1228.294	0.308	17.823	1235.385	0.433	39.416
6	-1246.175	563.761	1083.023	0.301	15.766	1248.670	0.651	39.774
7	-1186.586	591.590	1205.900	0.363	15.900	1167.200	0.621	17.500
8	-1234.924	584.676	1001.360	0.401	17.086	1238.972	0.602	32.703
9	-966.439	271.029	1046.257	0.300	15.000	1250.000	0.300	40.000
10	-983.279	327.146	889.812	0.324	15.000	1148.140	0.432	30.438
11	-969.876	311.982	1017.365	0.352	34.997	1193.174	0.306	33.912
12	-1142.995	419.872	1249.774	0.300	15.013	1249.957	0.410	39.979
13	-1196.269	476.596	850.000	0.300	15.000	1250.000	0.643	40.000
14	-1111.859	392.822	1250.000	0.360	15.000	1250.000	0.300	40.000
15	-1240.148	585.972	974.300	0.378	18.950	1238.413	0.645	35.523
16	-1207.806	529.605	871.682	0.346	15.150	1219.188	0.670	38.723
17	-1191.703	506.457	856.905	0.368	18.529	1229.592	0.620	36.164
18	-1276.226	681.784	1187.861	0.429	18.076	1248.770	0.562	32.002
19	-1081.092	378.558	918.096	0.307	22.266	1217.660	0.485	38.325
20	-1231.735	669.072	1248.345	0.491	26.587	1192.614	0.399	30.620
21	-1190.740	469.576	850.000	0.300	15.000	1250.000	0.634	40.000

id	$\bar{T}_{bed}^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
22	-1109.770	455.966	1024.199	0.334	31.848	1170.583	0.521	32.472
23	-797.824	226.825	877.743	0.300	31.003	1045.279	0.302	36.514
24	-939.244	261.544	859.026	0.306	18.348	1234.884	0.343	38.775
25	-1227.271	521.042	850.946	0.310	15.622	1247.828	0.697	39.022
26	-1150.853	459.939	893.076	0.320	35.537	1224.211	0.594	33.794
27	-1152.172	460.981	852.659	0.353	19.500	1218.909	0.576	37.341
28	-1226.053	554.067	989.369	0.332	17.508	1230.465	0.651	39.265
29	-1015.769	314.746	1127.181	0.307	16.135	1226.956	0.327	36.366
30	-1290.193	866.242	1237.558	0.641	22.157	1248.284	0.469	19.251
31	-1216.330	515.991	1250.000	0.300	15.000	1250.000	0.519	40.000
32	-1086.282	361.881	850.000	0.300	15.000	1250.000	0.491	40.000
33	-1206.931	499.114	886.167	0.305	16.333	1245.000	0.659	36.500
34	-1246.678	604.893	970.960	0.430	17.880	1247.600	0.604	35.171
35	-955.800	265.343	850.000	0.300	15.000	1250.000	0.357	40.000
36	-1148.404	421.883	850.000	0.300	15.000	1250.000	0.570	40.000
37	-1199.436	533.839	1027.779	0.317	16.581	1240.373	0.678	18.250
38	-710.815	214.571	859.093	0.303	17.860	916.077	0.300	27.125
39	-1175.715	605.767	1249.417	0.525	22.335	1145.564	0.315	20.251
40	-1281.709	698.074	1180.859	0.383	18.984	1245.163	0.682	31.098
41	-1035.697	353.287	1144.185	0.338	16.383	1178.520	0.330	38.083
42	-1198.151	529.715	1231.454	0.367	17.246	1221.674	0.444	36.951
43	-1060.390	357.794	854.716	0.305	29.273	1235.452	0.491	30.012
44	-1214.402	500.288	850.000	0.300	15.000	1250.000	0.676	40.000
45	-1197.854	477.907	850.000	0.300	15.000	1250.000	0.645	40.000
46	-940.953	287.313	948.190	0.349	19.728	1185.271	0.311	33.500
47	-1186.933	500.567	1245.682	0.360	19.318	1242.227	0.415	35.250
48	-1240.308	582.977	1126.852	0.325	15.725	1229.423	0.639	34.590
49	-1017.044	386.729	1020.474	0.380	23.687	1112.448	0.383	32.398
50	-1240.437	652.767	1247.158	0.523	16.705	1190.319	0.334	35.397

Appendix C

Full Output values of Problem (2.19)

This appendix contains the values of the objective functions and variables in the solutions presented in Section 3.3 referred to the problem formulation (2.19)

C.1 DFMOINT Method

Table C.1: Output values for DFMOINT from Figure 3.2b

id	$\tilde{O}_2^{PH}$	$\tilde{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
0	-0.769	1038.473	1250.000	0.700	40.000	1210.000	0.700	40.000
1	-0.213	205.466	850.000	0.300	15.000	850.000	0.300	15.000
2	-0.555	388.349	1250.000	0.300	35.970	1244.000	0.375	32.360
3	-0.658	535.039	1250.000	0.300	37.980	1247.000	0.538	36.180
4	-0.696	599.947	1250.000	0.300	38.990	1249.000	0.619	38.090
5	-0.606	435.901	1250.000	0.300	39.660	1210.000	0.424	39.190
6	-0.577	419.489	1249.000	0.300	36.400	1245.000	0.410	33.170
7	-0.348	210.007	850.000	0.300	15.000	850.000	0.300	40.000
8	-0.525	324.605	1250.000	0.300	39.320	1210.000	0.300	38.370
9	-0.374	253.553	1050.000	0.300	22.980	1007.000	0.300	21.180

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
10	-0.620	492.831	1190.000	0.300	33.990	1169.000	0.539	33.090
11	-0.397	281.575	930.000	0.380	20.000	930.000	0.300	26.790
12	-0.433	284.784	930.000	0.380	20.000	930.000	0.300	33.590
13	-0.727	657.005	1250.000	0.300	40.000	1250.000	0.700	40.000
14	-0.739	719.725	1250.000	0.380	37.500	1210.000	0.660	37.500
15	-0.483	313.208	1249.000	0.300	30.970	1164.000	0.300	27.360
16	-0.755	780.468	1250.000	0.469	39.830	1210.000	0.562	39.590
17	-0.769	988.961	1171.000	0.700	38.600	1175.000	0.700	40.000
18	-0.305	208.349	850.000	0.300	15.000	850.000	0.300	30.620
19	-0.326	209.153	850.000	0.300	15.000	850.000	0.300	35.000
20	-0.275	207.363	850.000	0.300	15.000	850.000	0.300	25.000
21	-0.375	279.879	930.000	0.380	20.000	930.000	0.300	23.400
22	-0.769	951.395	1250.000	0.700	40.000	1250.000	0.380	40.000
23	-0.625	512.740	1010.000	0.460	25.000	1010.000	0.460	35.000
24	-0.756	831.055	1010.000	0.660	35.000	1170.000	0.620	35.000
25	-0.493	316.096	1250.000	0.300	26.810	1204.000	0.300	29.450
26	-0.769	1034.840	1250.000	0.700	40.000	1212.000	0.676	39.930
27	-0.513	320.702	1250.000	0.300	33.410	1207.000	0.300	34.730
28	-0.713	625.286	1170.000	0.300	40.000	1234.000	0.700	40.000
29	-0.769	906.542	1250.000	0.700	40.000	1250.000	0.300	40.000
30	-0.647	520.755	1010.000	0.460	25.000	1010.000	0.460	40.000
31	-0.640	520.214	1180.000	0.300	34.500	1169.000	0.579	34.050
32	-0.768	886.596	1032.000	0.700	40.000	1090.000	0.700	40.000
33	-0.749	743.885	1250.000	0.380	37.500	1210.000	0.700	40.000
34	-0.721	645.277	1250.000	0.300	40.000	1210.000	0.700	40.000
35	-0.750	755.038	1250.000	0.380	40.000	1250.000	0.700	40.000
36	-0.744	729.677	1090.000	0.540	30.000	1090.000	0.540	40.000
37	-0.759	850.167	1010.000	0.660	37.500	1180.000	0.660	35.000
38	-0.767	869.739	1021.000	0.700	40.000	1050.000	0.700	40.000
39	-0.246	206.463	850.000	0.300	15.000	850.000	0.300	20.000
40	-0.262	206.938	850.000	0.300	15.000	850.000	0.300	22.810
41	-0.707	607.304	1170.000	0.300	37.910	1174.000	0.700	40.000
42	-0.769	930.949	1096.000	0.700	40.000	1110.000	0.700	40.000
43	-0.257	206.797	850.000	0.300	15.000	850.000	0.300	21.790
44	-0.769	926.490	1093.000	0.700	37.200	1100.000	0.700	40.000
45	-0.756	834.821	1010.000	0.660	37.500	1180.000	0.620	35.000
46	-0.521	323.715	1250.000	0.300	38.640	1210.000	0.300	36.740
47	-0.711	615.781	1170.000	0.300	40.000	1202.000	0.700	40.000
48	-0.766	860.417	1016.000	0.700	39.410	1030.000	0.700	40.000
49	-0.764	851.783	1013.000	0.700	32.200	1020.000	0.700	38.660
50	-0.744	722.773	1250.000	0.380	37.500	1210.000	0.660	40.000
51	-0.229	205.958	850.000	0.300	15.000	850.000	0.300	17.500
52	-0.238	206.198	850.000	0.300	15.000	850.000	0.300	18.900
53	-0.725	648.790	1250.000	0.300	40.000	1221.000	0.700	40.000
54	-0.235	206.107	850.000	0.300	15.000	850.000	0.300	18.400
55	-0.224	205.789	850.000	0.300	15.000	850.000	0.300	16.700
56	-0.750	754.524	1250.000	0.380	37.500	1250.000	0.700	40.000
57	-0.708	610.899	1170.000	0.300	40.000	1186.000	0.700	40.000
58	-0.221	205.713	850.000	0.300	15.000	850.000	0.300	16.250
59	-0.218	205.601	850.000	0.300	15.000	850.000	0.300	15.850

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
60	-0.708	608.541	1170.000	0.300	40.000	1178.000	0.700	40.000
61	-0.724	645.882	1250.000	0.300	40.000	1211.000	0.700	40.000
62	-0.215	205.522	850.000	0.300	15.000	850.000	0.300	15.400
63	-0.217	205.570	850.000	0.300	15.000	850.000	0.300	15.620
64	-0.723	645.469	1250.000	0.300	40.000	1210.000	0.700	40.000
65	-0.722	645.342	1250.000	0.300	40.000	1210.000	0.700	40.000
66	-0.216	205.558	850.000	0.300	15.000	850.000	0.300	15.420
67	-0.526	324.988	1250.000	0.300	38.630	1209.000	0.300	38.880
68	-0.523	324.142	1250.000	0.300	37.260	1207.000	0.300	37.760
69	-0.516	321.748	1249.000	0.300	34.510	1205.000	0.300	35.530
70	-0.499	316.790	1249.000	0.300	29.030	1199.000	0.300	31.060
71	-0.451	305.267	1247.000	0.300	18.060	1189.000	0.300	22.120
72	-0.769	975.069	1250.000	0.700	39.420	1209.000	0.448	39.530
73	-0.768	902.482	1250.000	0.700	38.840	1209.000	0.300	39.060
74	-0.768	900.453	1250.000	0.700	37.690	1208.000	0.300	38.120
75	-0.768	903.201	1250.000	0.700	38.930	1212.000	0.300	39.130

C.2 NSGA2 Method

Table C.2: Output values for NSGA2 from Figure 3.2b

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-0.768	910.250	1181.000	0.652	37.770	1079.000	0.573	39.410
2	-0.417	249.560	862.200	0.324	22.780	1043.000	0.324	38.130
3	-0.689	584.067	1081.000	0.378	19.890	1206.000	0.576	39.000
4	-0.709	616.693	1225.000	0.328	17.500	1237.000	0.613	39.930
5	-0.626	469.543	1211.000	0.403	18.860	1154.000	0.339	39.760
6	-0.678	554.705	1190.000	0.322	23.550	1145.000	0.584	39.580
7	-0.548	371.682	972.500	0.316	26.150	1138.000	0.454	39.830
8	-0.434	258.017	906.600	0.320	17.260	1071.000	0.322	38.340
9	-0.561	374.681	1188.000	0.325	34.650	1154.000	0.348	39.760
10	-0.700	600.504	1185.000	0.328	16.500	1234.000	0.613	39.830
11	-0.422	249.590	862.200	0.324	17.190	1038.000	0.324	39.950
12	-0.476	287.648	1050.000	0.311	20.380	1103.000	0.321	37.940
13	-0.672	545.193	1118.000	0.328	15.920	1157.000	0.599	39.930
14	-0.709	639.968	1056.000	0.485	16.230	1103.000	0.541	39.220
15	-0.450	268.969	909.000	0.327	17.260	1155.000	0.322	37.700
16	-0.590	422.153	1071.000	0.327	26.530	1149.000	0.459	39.830

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
17	-0.617	455.948	1195.000	0.401	18.860	1154.000	0.335	39.760
18	-0.764	854.516	1207.000	0.629	25.160	960.000	0.496	37.700
19	-0.577	409.312	1071.000	0.327	26.530	1067.000	0.459	39.830
20	-0.462	276.903	962.900	0.323	17.730	1080.000	0.324	39.950
21	-0.522	324.408	1113.000	0.309	26.220	1212.000	0.335	39.910
22	-0.598	438.495	1233.000	0.401	20.480	916.200	0.320	36.950
23	-0.657	524.157	1080.000	0.327	19.890	1206.000	0.576	39.000
24	-0.534	339.252	1234.000	0.310	16.750	1080.000	0.326	39.950
25	-0.680	576.855	972.200	0.504	24.360	1196.000	0.463	39.940
26	-0.514	316.429	1190.000	0.306	15.670	1081.000	0.319	39.820
27	-0.629	480.260	1065.000	0.307	26.180	1115.000	0.579	39.830
28	-0.462	276.478	963.300	0.324	17.730	1080.000	0.324	39.950
29	-0.764	829.381	1171.000	0.630	38.160	883.800	0.496	39.060
30	-0.667	535.882	1187.000	0.328	16.500	1228.000	0.529	39.930
31	-0.544	347.896	1182.000	0.328	37.780	1207.000	0.310	39.960
32	-0.503	307.620	1069.000	0.321	16.440	1150.000	0.326	39.950
33	-0.766	868.515	1170.000	0.645	34.500	1145.000	0.464	39.840
34	-0.633	485.879	1182.000	0.309	37.540	1071.000	0.532	39.800
35	-0.483	295.953	1168.000	0.306	15.670	876.700	0.319	39.360
36	-0.587	417.416	1062.000	0.321	34.650	1145.000	0.464	39.840
37	-0.767	889.058	1184.000	0.664	29.440	1050.000	0.495	37.950
38	-0.753	767.683	1110.000	0.514	35.210	1165.000	0.608	39.910
39	-0.606	442.071	1056.000	0.328	16.500	1234.000	0.475	39.410
40	-0.737	708.252	1179.000	0.486	19.970	957.000	0.583	39.350
41	-0.740	718.000	1116.000	0.533	35.210	1213.000	0.464	39.930
42	-0.703	606.211	1118.000	0.380	15.920	1157.000	0.599	39.930
43	-0.548	355.361	1234.000	0.324	17.140	1080.000	0.326	39.950
44	-0.575	403.027	1249.000	0.356	24.850	1057.000	0.328	36.810
45	-0.488	301.807	978.000	0.343	27.210	1120.000	0.325	39.430
46	-0.725	669.324	1177.000	0.461	16.400	1065.000	0.531	39.350
47	-0.572	393.897	1214.000	0.355	25.040	1079.000	0.328	38.510
48	-0.510	312.905	1064.000	0.309	26.150	1244.000	0.335	39.480
49	-0.529	330.399	1187.000	0.308	16.580	1231.000	0.322	39.410
50	-0.497	307.319	1065.000	0.306	26.150	1210.000	0.337	37.120
51	-0.648	507.191	1188.000	0.328	16.450	1085.000	0.530	39.930
52	-0.761	824.308	1165.000	0.630	38.160	883.800	0.496	37.750
53	-0.527	326.608	1182.000	0.309	37.620	1207.000	0.312	39.800
54	-0.438	261.433	903.200	0.324	18.140	1080.000	0.324	38.530
55	-0.683	579.768	1180.000	0.349	24.730	1080.000	0.601	38.490
56	-0.599	441.206	1062.000	0.321	34.650	1145.000	0.497	38.490
57	-0.642	501.672	1186.000	0.464	16.330	986.100	0.315	39.350
58	-0.532	337.729	1200.000	0.324	17.140	1080.000	0.316	39.950
59	-0.720	657.777	1177.000	0.464	16.400	986.100	0.531	39.350
60	-0.657	520.020	1171.000	0.307	26.180	1118.000	0.579	39.830
61	-0.569	386.893	1181.000	0.356	36.780	1079.000	0.328	39.920
62	-0.766	861.490	1180.000	0.649	26.380	1067.000	0.459	39.830
63	-0.480	288.986	1052.000	0.310	16.750	1080.000	0.326	39.880
64	-0.717	653.787	1177.000	0.461	16.400	986.100	0.531	39.350
65	-0.735	700.824	1205.000	0.461	16.520	1139.000	0.532	39.350

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
66	-0.443	261.507	903.200	0.324	17.140	1080.000	0.324	39.950
67	-0.752	755.182	1243.000	0.466	25.820	1045.000	0.606	39.980
68	-0.639	488.808	1200.000	0.328	24.570	1207.000	0.466	39.960
69	-0.741	730.157	1117.000	0.514	35.230	1177.000	0.531	38.460
70	-0.759	800.431	1045.000	0.652	37.510	1081.000	0.521	39.840
71	-0.744	736.557	1054.000	0.649	21.950	1200.000	0.392	39.900
72	-0.492	306.570	978.000	0.348	27.210	1120.000	0.325	39.430
73	-0.768	896.135	1184.000	0.666	37.650	986.700	0.532	37.960
74	-0.649	508.718	1188.000	0.328	17.330	1085.000	0.531	39.930
75	-0.745	738.389	1175.000	0.520	24.680	863.600	0.607	39.870
76	-0.716	643.274	1208.000	0.378	23.690	1204.000	0.576	39.000
77	-0.740	727.337	968.100	0.650	37.490	1196.000	0.463	39.940
78	-0.727	678.126	1177.000	0.461	26.970	1065.000	0.531	39.350
79	-0.754	777.552	1121.000	0.513	33.640	1181.000	0.614	39.330
80	-0.640	491.045	1187.000	0.328	16.500	1234.000	0.475	39.410
81	-0.768	907.935	1181.000	0.649	37.790	1079.000	0.574	39.400
82	-0.641	501.627	1207.000	0.351	25.160	963.700	0.493	39.890
83	-0.759	798.371	1045.000	0.649	37.540	1081.000	0.522	39.830
84	-0.751	753.621	1246.000	0.465	25.820	1040.000	0.606	39.980
85	-0.747	749.379	1117.000	0.518	34.770	1079.000	0.599	38.460
86	-0.446	263.216	906.600	0.320	17.260	1153.000	0.322	38.340
87	-0.652	514.707	1188.000	0.328	33.210	1085.000	0.530	39.940
88	-0.760	813.143	1165.000	0.628	29.120	867.700	0.495	37.730
89	-0.747	747.615	1109.000	0.520	37.030	1078.000	0.601	38.480
90	-0.564	383.390	973.700	0.309	26.150	1244.000	0.457	39.940
91	-0.731	688.213	1116.000	0.514	35.210	1165.000	0.466	39.910
92	-0.732	694.817	977.900	0.533	24.710	1192.000	0.591	39.940
93	-0.571	389.325	1184.000	0.356	37.750	1080.000	0.328	39.590
94	-0.449	263.927	918.900	0.318	18.860	1116.000	0.323	39.750
95	-0.758	788.654	1165.000	0.630	38.160	883.800	0.398	39.360
96	-0.755	782.275	1047.000	0.649	21.570	1049.000	0.522	39.490

C.3 Morbit Method

Table C.3: Output values for Morbit from Figure 3.2b

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-0.695	617.033	1160.118	0.483	29.193	990.483	0.433	36.578
2	-0.635	500.940	1168.701	0.464	24.028	1110.317	0.311	36.262

id	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
3	-0.662	560.139	1238.869	0.484	22.162	893.580	0.322	34.212
4	-0.575	408.238	992.212	0.446	29.586	1205.567	0.320	38.674
5	-0.665	550.958	1216.105	0.410	32.806	978.180	0.434	37.649
6	-0.397	222.427	850.000	0.300	40.000	1053.321	0.300	40.000
7	-0.618	476.003	1184.813	0.431	23.386	1161.385	0.320	35.324
8	-0.495	292.284	1090.092	0.306	38.654	1243.301	0.301	39.575
9	-0.605	447.493	1105.953	0.440	33.626	1112.523	0.312	38.008
10	-0.438	241.964	895.139	0.300	40.000	1250.000	0.300	40.000
11	-0.569	391.733	1209.022	0.359	38.013	1204.744	0.309	36.488
12	-0.545	429.574	980.724	0.431	22.615	1169.182	0.397	27.307
13	-0.608	451.471	1136.199	0.315	27.136	1201.911	0.474	37.610
14	-0.715	660.167	966.678	0.470	35.402	1171.205	0.650	37.093
15	-0.415	233.467	851.141	0.307	29.205	1162.393	0.305	38.777
16	-0.558	392.336	1125.613	0.347	26.254	1153.961	0.373	35.032
17	-0.720	675.657	972.501	0.541	33.066	1195.736	0.549	37.416
18	-0.401	223.932	850.000	0.300	40.000	1084.716	0.300	40.000
19	-0.665	562.233	1211.557	0.402	37.871	1067.983	0.452	34.891
20	-0.643	575.225	994.696	0.469	35.346	1046.305	0.553	30.166
21	-0.546	375.473	1082.019	0.342	39.003	1183.209	0.369	35.016
22	-0.567	434.593	984.456	0.442	31.213	1189.579	0.371	30.972
23	-0.581	414.611	1129.495	0.352	22.161	1181.432	0.386	37.213
24	-0.716	663.421	897.996	0.544	28.753	1178.243	0.618	38.125
25	-0.597	428.918	1197.723	0.389	37.291	1226.794	0.308	37.863
26	-0.592	420.799	1238.058	0.304	17.062	1215.278	0.416	38.265
27	-0.754	774.647	1158.925	0.656	36.193	958.214	0.321	38.403
28	-0.620	462.477	1219.832	0.362	32.346	1181.029	0.378	38.941
29	-0.704	640.088	1061.230	0.584	26.566	1116.950	0.368	36.805
30	-0.601	469.255	970.839	0.355	28.252	1006.134	0.589	37.655
31	-0.575	397.669	1208.937	0.352	39.167	1201.768	0.324	37.814
32	-0.522	321.289	1145.435	0.319	37.825	1225.228	0.303	39.731
33	-0.712	671.580	1088.161	0.537	31.037	1015.161	0.483	34.990
34	-0.727	701.568	1187.456	0.467	33.886	1001.220	0.579	35.468
35	-0.532	336.205	1224.550	0.307	29.876	1243.197	0.312	37.699
36	-0.645	548.753	1108.181	0.443	27.165	1153.739	0.434	31.873
37	-0.477	274.000	1038.692	0.300	40.000	1250.000	0.300	40.000
38	-0.643	509.877	1009.064	0.332	27.644	1182.859	0.594	37.640
39	-0.665	546.146	1130.782	0.344	21.519	1145.743	0.571	37.334
40	-0.510	324.456	1023.464	0.342	30.920	1142.214	0.335	38.267
41	-0.618	471.658	1095.358	0.431	27.605	1183.079	0.356	37.700
42	-0.661	558.280	1011.000	0.480	37.000	1187.000	0.440	36.000
43	-0.521	364.665	1043.692	0.408	26.240	1178.754	0.300	30.319
44	-0.729	703.759	1160.178	0.553	33.712	897.000	0.440	36.407
45	-0.566	380.259	1213.138	0.325	18.035	1233.598	0.346	38.875
46	-0.669	557.523	1143.403	0.371	31.291	1108.877	0.538	37.132
47	-0.534	330.666	1250.000	0.303	40.000	1250.000	0.300	40.000
48	-0.682	608.404	1117.476	0.504	25.551	1098.896	0.411	33.709
49	-0.573	401.165	1178.439	0.381	22.731	1140.775	0.314	36.576
50	-0.468	316.991	888.760	0.326	37.725	1106.542	0.414	32.200

Appendix D

Full Output values of Problem (2.17)

This appendix contains the values of the objective functions and variables in the solutions presented in Section 3.3 referred to the problem formulation (2.17).

D.1 DFMOINT Method

Table D.1: Output values for DFMOINT from Figure 3.3

id	$\tilde{T}_{bed}^{PH}$	$\tilde{O}_2^{PH}$	$\tilde{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
0	-636.645	-0.213	205.466	850	0.300	15.000	850	0.300	15.000
1	-1279.152	-0.769	1047.576	1250	0.700	40.000	1250	0.700	40.000
2	-647.123	-0.217	206.332	850	0.300	15.000	870	0.300	15.000
3	-657.165	-0.222	207.170	850	0.300	15.000	890	0.300	15.000
4	-676.961	-0.230	208.816	850	0.300	15.000	930	0.300	15.000
5	-719.003	-0.249	212.280	850	0.300	15.000	1010	0.300	15.000
6	-802.554	-0.284	219.427	850	0.300	15.000	1170	0.300	15.000
7	-840.032	-0.293	223.105	850	0.300	15.000	1250	0.300	15.000
8	-839.773	-0.300	240.003	850	0.300	15.000	1170	0.340	15.000
9	-651.403	-0.246	206.463	850	0.300	15.000	850	0.300	20.000
10	-664.296	-0.275	207.363	850	0.300	15.000	850	0.300	25.000
11	-687.284	-0.326	209.153	850	0.300	15.000	850	0.300	35.000
12	-696.791	-0.348	210.007	850	0.300	15.000	850	0.300	40.000
13	-1106.842	-0.620	473.789	850	0.300	40.000	1108	0.700	40.000

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
14	-1144.744	-0.631	483.166	850	0.300	15.000	1146	0.700	40.000
15	-652.123	-0.219	206.780	850	0.300	15.000	882	0.300	15.000
16	-703.386	-0.242	210.972	850	0.300	15.000	979	0.300	15.000
17	-771.310	-0.272	216.663	850	0.300	15.000	1109	0.300	15.000
18	-638.034	-0.216	205.558	850	0.300	15.000	850	0.300	15.420
19	-638.601	-0.218	205.601	850	0.300	15.000	850	0.300	15.850
20	-641.444	-0.224	205.789	850	0.300	15.000	850	0.300	16.700
21	-646.096	-0.235	206.107	850	0.300	15.000	850	0.300	18.400
22	-656.192	-0.257	206.797	850	0.300	15.000	850	0.300	21.790
23	-673.401	-0.295	208.034	850	0.300	15.000	850	0.300	28.590
24	-1277.816	-0.769	1042.207	1240	0.700	40.000	1250	0.700	40.000
25	-1276.349	-0.769	1037.196	1230	0.700	40.000	1250	0.700	40.000
26	-1273.939	-0.769	1027.128	1210	0.700	40.000	1250	0.700	40.000
27	-1269.723	-0.769	1006.227	1170	0.700	40.000	1250	0.700	40.000
28	-1259.311	-0.765	859.711	930	0.700	40.000	1250	0.700	40.000
29	-1259.843	-0.757	801.986	850	0.700	40.000	1250	0.700	40.000
30	-1261.735	-0.762	835.197	930	0.660	40.000	1250	0.700	40.000
31	-1283.062	-0.750	755.038	1250	0.380	40.000	1250	0.700	40.000
32	-1272.005	-0.727	657.005	1250	0.300	40.000	1250	0.700	40.000
33	-1284.241	-0.750	754.524	1250	0.380	37.500	1250	0.700	40.000
34	-1289.546	-0.768	1046.444	1250	0.700	30.000	1250	0.700	40.000
35	-1303.651	-0.767	1042.186	1250	0.700	20.000	1250	0.700	40.000
36	-1312.474	-0.764	1036.769	1250	0.700	15.000	1250	0.700	40.000
37	-1296.722	-0.767	1040.279	1250	0.700	20.000	1240	0.700	40.000
38	-1265.778	-0.769	1043.024	1250	0.700	40.000	1230	0.700	40.000
39	-1251.812	-0.769	1038.462	1250	0.700	40.000	1210	0.700	40.000
40	-1253.749	-0.769	1031.251	1250	0.700	40.000	1210	0.660	40.000
41	-1281.271	-0.769	1030.971	1250	0.700	40.000	1250	0.620	40.000
42	-1283.602	-0.769	1010.526	1250	0.700	40.000	1250	0.540	40.000
43	-1286.499	-0.769	951.395	1250	0.700	40.000	1250	0.380	40.000
44	-1285.839	-0.769	906.542	1250	0.700	40.000	1250	0.300	40.000
45	-1288.101	-0.768	949.471	1250	0.700	40.000	1250	0.380	37.500
46	-1121.531	-0.577	419.489	1249	0.300	36.400	1245	0.410	33.170
47	-1010.545	-0.483	316.288	1247	0.300	32.800	1240	0.300	26.340
48	-972.758	-0.410	304.541	1244	0.300	25.590	1230	0.300	15.000
49	-961.442	-0.391	296.974	1239	0.300	15.000	1210	0.300	15.000
50	-941.304	-0.391	292.610	1228	0.300	15.000	1170	0.300	15.000
51	-897.639	-0.381	284.436	1206	0.300	15.000	1090	0.300	15.000
52	-1261.562	-0.712	628.998	1250	0.300	39.500	1249	0.659	39.050
53	-1248.569	-0.696	599.947	1250	0.300	38.990	1249	0.619	38.090
54	-1212.739	-0.658	535.039	1250	0.300	37.980	1247	0.538	36.180
55	-1093.279	-0.555	388.349	1250	0.300	35.970	1244	0.375	32.360
56	-1007.387	-0.476	315.759	1250	0.300	31.940	1239	0.300	24.720
57	-973.805	-0.409	305.259	1250	0.300	23.880	1228	0.300	15.000
58	-963.693	-0.393	299.140	1250	0.300	15.000	1205	0.300	15.000
59	-945.233	-0.395	296.800	1250	0.300	15.000	1161	0.300	15.000
60	-905.883	-0.391	292.269	1250	0.300	15.000	1071	0.300	15.000
61	-820.336	-0.370	283.192	1250	0.300	15.000	893	0.300	15.000
62	-1289.799	-0.766	980.614	1250	0.700	37.090	1246	0.466	34.480
63	-1291.724	-0.754	883.690	1250	0.700	34.180	1242	0.300	28.960

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
64	-1289.250	-0.767	976.422	1250	0.700	40.000	1250	0.450	34.110
65	-1288.506	-0.755	890.219	1250	0.700	40.000	1250	0.300	28.220
66	-1268.360	-0.769	1043.899	1250	0.700	40.000	1234	0.700	40.000
67	-1256.872	-0.769	1040.214	1250	0.700	40.000	1218	0.700	40.000
68	-1234.794	-0.769	1032.837	1250	0.700	40.000	1185	0.700	40.000
69	-1061.101	-0.647	520.755	1010	0.460	25.000	1010	0.460	40.000
70	-1113.890	-0.764	851.783	1013	0.700	32.200	1020	0.700	38.660
71	-1112.078	-0.766	860.417	1016	0.700	39.410	1030	0.700	40.000
72	-1125.127	-0.767	869.739	1021	0.700	40.000	1050	0.700	40.000
73	-1151.787	-0.768	886.596	1032	0.700	40.000	1090	0.700	40.000
74	-816.840	-0.374	253.553	1050	0.300	22.980	1007	0.300	21.180
75	-819.293	-0.361	259.287	1089	0.300	20.970	1004	0.300	17.360
76	-839.734	-0.363	272.930	1168	0.300	16.940	998	0.300	15.000
77	-1131.067	-0.718	654.654	1010	0.460	15.000	1028	0.700	40.000
78	-1222.308	-0.736	693.947	1010	0.460	15.000	1158	0.700	40.000
79	-958.521	-0.520	387.933	1010	0.460	25.000	1010	0.300	28.400
80	-979.843	-0.565	398.769	1010	0.460	25.000	1010	0.300	38.590
81	-1022.450	-0.443	350.449	930	0.380	20.000	1250	0.380	20.000
82	-888.405	-0.498	336.404	930	0.380	20.000	930	0.380	40.000
83	-750.661	-0.310	245.464	939	0.300	19.500	929	0.339	19.050
84	-727.419	-0.296	228.427	949	0.300	18.990	928	0.300	18.090
85	-767.495	-0.318	252.306	1088	0.300	15.000	918	0.300	15.000
86	-826.951	-0.371	283.613	1247	0.300	15.000	907	0.300	15.000
87	-817.443	-0.369	282.891	1250	0.300	15.000	885	0.300	15.000
88	-984.314	-0.579	447.562	930	0.300	31.650	946	0.700	40.000
89	-995.786	-0.587	453.317	930	0.300	40.000	962	0.700	40.000
90	-1024.161	-0.600	463.557	930	0.300	40.000	994	0.700	40.000
91	-1184.905	-0.662	526.937	930	0.300	40.000	1188	0.700	40.000
92	-1245.344	-0.702	605.971	930	0.380	15.000	1226	0.700	40.000
93	-799.596	-0.375	279.879	930	0.380	20.000	930	0.300	23.400
94	-810.484	-0.397	281.575	930	0.380	20.000	930	0.300	26.790
95	-827.128	-0.433	284.784	930	0.380	20.000	930	0.300	33.590
96	-1172.660	-0.744	729.677	1090	0.540	30.000	1090	0.540	40.000
97	-1163.698	-0.769	926.490	1093	0.700	37.200	1100	0.700	40.000
98	-1168.419	-0.769	930.949	1096	0.700	40.000	1110	0.700	40.000
99	-1010.554	-0.519	384.500	1110	0.300	28.990	1089	0.459	28.090
100	-902.353	-0.429	285.052	1169	0.300	25.970	1084	0.300	22.360
101	-908.303	-0.401	296.357	1248	0.300	21.940	1079	0.300	15.000
102	-903.533	-0.390	292.023	1250	0.300	15.000	1068	0.300	15.000
103	-829.748	-0.373	284.164	1250	0.300	15.000	911	0.300	15.000
104	-1163.664	-0.765	870.151	1011	0.700	34.030	1096	0.700	37.640
105	-1160.977	-0.761	820.315	931	0.700	38.060	1101	0.700	40.000
106	-1141.127	-0.675	554.413	1090	0.300	35.820	1098	0.700	40.000
107	-1145.506	-0.676	557.478	1090	0.300	40.000	1106	0.700	40.000
108	-1158.437	-0.681	562.649	1090	0.300	40.000	1122	0.700	40.000
109	-1183.880	-0.689	572.929	1090	0.300	40.000	1154	0.700	40.000
110	-1229.634	-0.698	592.670	1090	0.300	40.000	1219	0.700	40.000
111	-1169.509	-0.751	790.360	1090	0.540	30.000	1082	0.700	35.890
112	-1161.788	-0.758	793.575	1090	0.540	30.000	1074	0.700	40.000
113	-1150.631	-0.758	789.232	1090	0.540	30.000	1058	0.700	40.000

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
114	-1128.914	-0.756	780.480	1090	0.540	30.000	1025	0.700	40.000
115	-1084.756	-0.752	762.855	1090	0.540	30.000	960	0.700	40.000
116	-1219.080	-0.769	988.961	1171	0.700	38.600	1175	0.700	40.000
117	-1222.120	-0.769	991.202	1173	0.700	40.000	1180	0.700	40.000
118	-1229.278	-0.769	994.620	1176	0.700	40.000	1190	0.700	40.000
119	-1243.088	-0.769	1002.166	1181	0.700	40.000	1210	0.700	40.000
120	-1271.732	-0.769	1018.044	1192	0.700	40.000	1250	0.700	40.000
121	-1274.466	-0.769	1029.337	1214	0.700	40.000	1250	0.700	40.000
122	-1162.118	-0.640	520.214	1180	0.300	34.500	1169	0.579	34.050
123	-980.234	-0.483	313.208	1249	0.300	30.970	1164	0.300	27.360
124	-959.875	-0.442	306.298	1250	0.300	26.940	1159	0.300	19.720
125	-939.904	-0.401	298.666	1250	0.300	18.880	1148	0.300	15.000
126	-929.831	-0.394	294.979	1250	0.300	15.000	1125	0.300	15.000
127	-909.068	-0.391	292.646	1250	0.300	15.000	1081	0.300	15.000
128	-1296.785	-0.766	937.942	1170	0.620	15.000	1244	0.700	40.000

D.2 NSGA2 Method

Table D.2: Output values for NSGA2 from Figure 3.3

id	T_{bed}^{PH}	O_2^{PH}	T_{gas}^{PH}	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-776.494	-0.296	240.830	872	0.320	19.400	1033	0.322	16.540
2	-1192.094	-0.769	941.267	1245	0.671	33.200	1073	0.493	38.500
3	-1302.621	-0.766	976.391	1245	0.658	19.990	1242	0.588	38.840
4	-759.862	-0.329	229.267	908	0.309	15.050	976	0.300	23.380
5	-1302.763	-0.761	832.431	1247	0.658	21.160	1242	0.307	38.840
6	-835.547	-0.414	247.735	868	0.320	18.220	1049	0.322	37.320
7	-781.754	-0.355	244.960	916	0.328	16.140	968	0.304	25.260
8	-865.255	-0.329	249.236	868	0.320	18.220	1200	0.322	16.680
9	-944.954	-0.435	272.315	938	0.320	17.670	1237	0.322	29.720
10	-999.416	-0.504	307.935	1085	0.322	28.150	1239	0.306	38.030
11	-811.551	-0.421	250.123	917	0.328	16.020	968	0.305	39.540
12	-1270.844	-0.725	666.007	1229	0.408	20.180	1245	0.530	39.410
13	-959.664	-0.469	277.295	938	0.320	17.330	1236	0.322	39.010
14	-1267.551	-0.728	688.762	1138	0.465	21.620	1247	0.530	38.350
15	-1013.896	-0.513	319.931	1085	0.325	27.610	1239	0.319	37.890
16	-908.390	-0.483	291.856	1079	0.318	34.870	1067	0.303	38.470

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
17	-974.545	-0.537	364.764	1032	0.383	19.310	1043	0.347	38.020
18	-1212.387	-0.660	538.267	1162	0.324	16.500	1227	0.556	37.180
19	-1031.569	-0.579	404.038	1161	0.324	32.780	1080	0.407	39.950
20	-1054.265	-0.602	450.166	1032	0.411	20.350	1071	0.413	38.920
21	-1173.565	-0.732	695.910	1131	0.460	19.820	1079	0.618	38.460
22	-1225.516	-0.673	545.085	1229	0.319	20.180	1245	0.530	39.700
23	-1116.218	-0.588	415.443	1194	0.379	16.650	1236	0.320	38.320
24	-774.825	-0.342	243.617	913	0.328	16.020	968	0.305	23.380
25	-885.013	-0.447	261.149	914	0.320	25.920	1119	0.316	39.790
26	-1209.598	-0.712	636.565	1212	0.408	17.300	1149	0.530	39.230
27	-874.390	-0.455	275.797	961	0.320	19.400	1033	0.333	39.130
28	-990.522	-0.523	327.775	1167	0.320	16.500	1120	0.320	39.580
29	-1139.435	-0.620	456.387	1245	0.379	36.080	1243	0.327	39.890
30	-809.563	-0.416	249.714	917	0.328	16.020	968	0.305	38.250
31	-1072.021	-0.560	374.752	1200	0.350	16.500	1207	0.315	38.190
32	-1039.417	-0.534	344.272	1167	0.325	33.830	1235	0.317	36.710
33	-1236.563	-0.679	567.399	1162	0.324	15.800	1240	0.589	38.250
34	-1296.614	-0.765	934.241	1194	0.649	16.560	1236	0.584	38.320
35	-1197.202	-0.768	947.263	1245	0.658	19.990	1073	0.588	38.460
36	-1257.550	-0.754	787.830	1167	0.649	33.830	1235	0.317	37.910
37	-1195.417	-0.656	527.165	1071	0.322	20.040	1211	0.591	38.030
38	-1122.348	-0.635	490.152	1072	0.397	26.520	1156	0.439	39.860
39	-965.927	-0.465	280.703	955	0.320	20.150	1247	0.322	35.680
40	-1231.254	-0.759	820.067	1061	0.613	35.230	1191	0.576	38.450
41	-1187.125	-0.764	852.075	1117	0.631	16.710	1079	0.619	38.460
42	-1104.939	-0.553	398.136	854	0.324	16.500	1227	0.523	34.760
43	-1051.819	-0.767	882.380	1194	0.653	33.310	875	0.587	37.940
44	-911.289	-0.477	287.307	1079	0.318	20.500	1067	0.303	37.920
45	-1170.389	-0.639	495.272	1072	0.397	17.260	1232	0.439	39.860
46	-1088.487	-0.561	382.953	1057	0.336	21.620	1247	0.392	38.270
47	-1146.669	-0.702	616.888	1117	0.382	35.230	1081	0.636	38.460
48	-1224.059	-0.678	552.983	1130	0.318	20.180	1232	0.592	39.700
49	-1287.427	-0.746	769.511	1136	0.457	15.870	1240	0.691	37.630
50	-1171.697	-0.618	507.221	1245	0.375	36.080	1242	0.399	29.700
51	-1097.629	-0.569	386.513	1205	0.348	15.800	1240	0.326	38.250
52	-1204.093	-0.685	604.616	1132	0.468	17.590	1183	0.443	35.680
53	-1199.452	-0.649	517.189	1162	0.324	16.500	1227	0.529	37.180
54	-1196.014	-0.687	589.568	957	0.513	33.530	1229	0.467	39.950
55	-1245.288	-0.750	759.802	1051	0.658	30.860	1240	0.397	39.340
56	-1276.503	-0.761	864.991	1179	0.613	16.690	1205	0.536	37.190
57	-922.347	-0.449	263.695	921	0.303	30.400	1202	0.329	37.050
58	-1024.723	-0.606	451.290	1133	0.333	34.990	1006	0.483	39.950
59	-1136.288	-0.594	432.482	1075	0.306	23.780	1237	0.485	37.270
60	-972.415	-0.516	319.892	1141	0.320	26.160	1119	0.314	39.780
61	-1272.389	-0.741	717.915	1130	0.464	20.180	1238	0.582	39.660
62	-996.062	-0.500	304.068	1085	0.318	27.710	1240	0.306	37.920
63	-1160.689	-0.641	511.885	1061	0.416	26.230	1212	0.440	37.230
64	-1216.390	-0.683	578.597	1188	0.410	32.670	1241	0.440	37.980
65	-1199.850	-0.705	631.018	1137	0.468	20.220	1155	0.469	38.130

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
66	-778.931	-0.350	244.664	916	0.328	16.140	968	0.304	24.580
67	-1246.731	-0.697	607.620	1194	0.379	16.650	1236	0.533	38.320
68	-1081.740	-0.622	473.774	1074	0.397	17.260	1079	0.439	39.860
69	-992.724	-0.489	296.013	1059	0.325	16.020	1247	0.302	37.190
70	-982.241	-0.535	359.317	940	0.414	32.620	1117	0.333	39.840
71	-1254.652	-0.720	648.143	1245	0.379	36.080	1243	0.539	39.890
72	-1263.388	-0.767	906.082	1116	0.656	35.590	1238	0.583	39.660
73	-1225.379	-0.752	773.825	1061	0.613	35.210	1191	0.485	38.450
74	-981.016	-0.463	295.733	1029	0.320	17.730	1237	0.322	29.720
75	-1152.905	-0.612	451.290	947	0.326	16.710	1229	0.541	39.950
76	-1271.727	-0.720	648.226	1245	0.379	15.290	1243	0.552	39.840
77	-1254.885	-0.737	710.820	1075	0.509	34.130	1245	0.530	39.350
78	-1126.175	-0.684	575.348	1106	0.382	35.230	1072	0.576	39.680
79	-1223.906	-0.705	636.379	1041	0.491	33.830	1235	0.487	38.050
80	-919.872	-0.480	289.846	1079	0.320	20.500	1080	0.303	37.920
81	-1151.118	-0.733	706.727	1036	0.517	34.560	1071	0.623	37.970
82	-1271.708	-0.741	731.046	1129	0.460	19.820	1231	0.618	38.460
83	-1040.411	-0.575	398.132	1180	0.324	15.380	1080	0.401	39.850
84	-1074.635	-0.766	862.041	1194	0.653	33.310	875	0.491	37.960
85	-1048.423	-0.531	334.791	1194	0.327	16.880	1248	0.300	38.420
86	-883.238	-0.445	260.266	914	0.320	26.160	1119	0.316	39.780
87	-1119.423	-0.631	479.015	1194	0.320	16.500	1121	0.495	39.630
88	-986.409	-0.494	301.438	1059	0.325	37.840	1247	0.302	36.630
89	-1044.804	-0.531	335.017	1194	0.327	16.650	1236	0.301	38.420
90	-931.712	-0.478	294.961	914	0.322	26.160	1119	0.366	39.780
91	-1135.541	-0.697	610.393	1117	0.381	35.230	1067	0.636	38.460
92	-1099.962	-0.571	390.447	1200	0.351	15.800	1240	0.329	38.250
93	-1079.923	-0.597	443.881	887	0.342	35.110	1120	0.573	39.580
94	-1275.202	-0.766	910.907	1194	0.649	33.830	1236	0.491	37.960
95	-1164.897	-0.764	850.183	1117	0.614	35.740	1079	0.619	38.460
96	-1017.087	-0.598	439.273	1132	0.324	34.990	1006	0.483	39.950
97	-1251.737	-0.766	892.303	1179	0.612	36.450	1205	0.564	38.850
98	-1021.186	-0.482	329.943	947	0.321	16.710	1228	0.401	29.270

D.3 Morbit Method

Table D.3: Output values for Morbit from Figure 3.3

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
1	-1268.733	-0.710	640.707	1233.981	0.343	15.728	1239.078	0.621	37.233
2	-1107.810	-0.726	724.990	1077.229	0.661	33.910	961.281	0.364	32.178
3	-819.150	-0.397	231.877	851.118	0.308	37.500	1091.802	0.308	35.346
4	-1189.000	-0.715	669.936	1020.113	0.530	34.200	1152.645	0.522	36.012
5	-1104.249	-0.617	466.199	1227.894	0.416	33.316	1149.352	0.302	36.113
6	-1057.926	-0.551	358.294	1232.694	0.305	31.816	1220.601	0.337	38.981
7	-1250.992	-0.726	727.341	1186.256	0.448	17.275	1184.409	0.623	32.609
8	-1181.269	-0.668	533.102	1250.000	0.300	40.000	1188.506	0.544	40.000
9	-1133.250	-0.753	804.550	1182.597	0.658	28.101	930.337	0.367	34.256
10	-1137.307	-0.631	486.794	1158.520	0.348	19.107	1159.541	0.469	37.975
11	-1026.862	-0.547	403.719	1100.850	0.379	32.615	1109.980	0.362	29.905
12	-1223.841	-0.677	550.950	1250.000	0.300	40.000	1250.000	0.550	40.000
13	-1039.846	-0.731	734.827	1176.000	0.550	39.000	850.000	0.510	33.000
14	-1214.470	-0.648	502.002	850.000	0.310	15.000	1250.000	0.665	40.000
15	-1041.486	-0.545	349.462	1218.464	0.319	37.896	1217.583	0.312	39.200
16	-1155.208	-0.756	819.710	981.716	0.675	34.962	1080.397	0.675	35.810
17	-804.732	-0.396	225.621	863.464	0.302	27.312	1059.712	0.302	37.955
18	-1160.093	-0.727	700.628	941.167	0.576	31.195	1092.021	0.627	37.288
19	-1245.305	-0.730	721.093	1094.853	0.635	18.145	1209.570	0.368	35.034
20	-1036.530	-0.519	323.879	1053.878	0.301	15.528	1248.404	0.366	39.635
21	-1119.939	-0.595	414.457	1250.000	0.371	24.644	1250.000	0.300	40.000
22	-1112.612	-0.589	418.866	1180.159	0.404	15.732	1227.044	0.300	37.773
23	-1315.780	-0.760	953.522	1247.043	0.686	16.054	1242.608	0.484	36.833
24	-1242.385	-0.709	664.725	861.192	0.561	33.350	1239.670	0.619	35.529
25	-1055.407	-0.568	414.976	1078.132	0.376	36.045	1151.353	0.379	33.654
26	-1016.007	-0.589	442.807	1025.785	0.405	22.946	1008.895	0.428	37.740
27	-1239.273	-0.739	714.339	1246.217	0.373	21.918	1170.977	0.697	38.048
28	-1105.123	-0.687	647.806	1040.293	0.580	26.351	1003.138	0.446	30.804
29	-1152.791	-0.650	535.431	1096.898	0.403	26.012	1157.628	0.479	35.364
30	-1003.660	-0.517	319.472	1158.011	0.302	21.757	1182.556	0.328	38.943

id	$\bar{T}_{bed}^{PH}$	$\bar{O}_2^{PH}$	$\bar{T}_{gas}^{PH}$	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}
31	-1089.433	-0.745	745.976	1120.308	0.563	24.971	934.400	0.559	38.121
32	-1208.295	-0.685	584.186	982.754	0.407	34.703	1213.209	0.592	38.112
33	-1099.864	-0.569	381.335	1213.232	0.301	16.581	1240.271	0.380	39.849
34	-1119.730	-0.684	599.726	908.547	0.671	36.858	1131.306	0.345	38.188
35	-1004.338	-0.518	317.812	1179.677	0.305	21.136	1184.464	0.314	39.301
36	-1075.090	-0.568	394.416	1235.473	0.325	34.957	1194.115	0.353	35.849
37	-1131.881	-0.609	439.351	1235.752	0.383	19.769	1224.523	0.319	39.483
38	-981.605	-0.471	282.346	1106.427	0.300	15.000	1250.000	0.300	33.514
39	-1152.110	-0.622	460.089	1244.581	0.304	39.511	1235.836	0.444	39.642
40	-1130.159	-0.621	465.315	1242.548	0.337	32.669	1186.682	0.409	38.330
41	-1240.452	-0.720	682.324	1230.000	0.550	38.000	1240.000	0.300	36.000
42	-894.969	-0.435	243.724	910.460	0.304	21.969	1198.326	0.301	39.435
43	-1214.786	-0.645	517.065	854.667	0.327	18.085	1239.500	0.677	36.500
44	-1125.412	-0.632	515.199	941.615	0.599	23.403	1214.626	0.308	35.847
45	-1166.533	-0.736	736.875	1211.741	0.486	21.117	1044.333	0.577	34.451
46	-1279.558	-0.714	687.326	1060.363	0.415	15.474	1245.265	0.693	33.378
47	-1110.951	-0.712	646.407	1016.285	0.478	19.655	1005.600	0.648	39.224
48	-1155.895	-0.597	466.268	863.507	0.340	26.240	1222.191	0.596	33.644
49	-1217.048	-0.679	580.303	887.758	0.433	27.124	1221.216	0.625	37.075
50	-1184.597	-0.725	681.203	980.955	0.700	39.956	1213.633	0.311	39.080
51	-1199.489	-0.650	509.310	974.745	0.309	37.738	1243.021	0.617	39.136
52	-1112.737	-0.602	441.010	1194.435	0.311	29.651	1182.046	0.444	37.747
53	-1117.591	-0.700	641.965	1044.241	0.478	28.992	1029.000	0.594	35.766
54	-1185.935	-0.684	587.022	1132.342	0.392	27.268	1161.053	0.538	36.916
55	-1258.826	-0.709	638.900	900.787	0.441	15.177	1238.717	0.692	38.458
56	-1113.196	-0.590	410.089	1233.276	0.369	24.797	1246.579	0.304	39.398
57	-1166.858	-0.655	521.467	1166.602	0.327	37.185	1184.665	0.527	38.514
58	-1152.309	-0.673	585.110	1091.231	0.442	35.597	1128.623	0.489	34.446
59	-1154.394	-0.675	570.875	1133.429	0.470	35.143	1149.619	0.384	37.571
60	-1149.196	-0.602	430.664	1063.696	0.300	15.067	1249.581	0.492	40.272
61	-1190.840	-0.682	566.966	1250.000	0.466	40.000	1223.116	0.300	40.000
62	-1056.947	-0.530	344.939	1248.752	0.304	22.333	1240.535	0.324	34.423
63	-1037.416	-0.522	316.746	1236.779	0.300	15.000	1247.083	0.300	40.000
64	-1114.126	-0.612	453.298	1221.975	0.310	24.698	1155.224	0.457	37.996
65	-992.177	-0.497	306.055	1064.360	0.324	18.101	1212.779	0.316	37.089
66	-1056.745	-0.636	492.015	1249.489	0.430	38.212	987.302	0.302	38.127
67	-1175.062	-0.700	642.354	1168.847	0.487	34.337	1122.546	0.427	34.188
68	-1023.431	-0.523	326.568	1162.746	0.314	26.439	1225.151	0.314	38.509
69	-1203.921	-0.682	611.625	1000.980	0.408	28.384	1177.205	0.647	33.030
70	-832.377	-0.402	229.209	851.561	0.305	34.207	1127.779	0.302	36.872

Appendix E

Full Output values of Problem (2.27)

This appendix contains the values of the objective functions and variables in the solutions presented in Section 3.4 referred to the problem formulation (2.27).

E.1 DFMOINT Method

Table E.1: Output values for DFMOINT from Figure 3.4

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
0	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
1	1250	0.32	15.00	1250	0.45	39.97	0.00	0.00	-1187	0.00	0.00	0.01
2	964	0.41	15.00	1249	0.49	39.99	0.00	58.45	-1187	58.45	-1.45	0.29
3	1250	0.32	15.00	1250	0.45	39.97	0.00	0.00	-1187	0.00	0.00	0.01
4	964	0.41	15.84	1249	0.49	39.99	0.00	67.34	-1190	67.34	-0.54	-2.56
5	964	0.41	15.69	1250	0.49	40.00	0.00	67.38	-1191	67.38	-0.54	0.28
6	964	0.41	15.33	1250	0.49	40.00	0.00	63.02	-1189	63.02	-1.24	0.11
7	964	0.41	15.47	1250	0.49	39.99	0.00	61.74	-1188	61.74	-1.24	-0.81
8	964	0.41	15.86	1250	0.49	39.99	0.00	67.30	-1190	67.30	-0.54	-1.97
9	964	0.41	15.84	1249	0.49	39.99	0.00	66.09	-1189	66.09	-0.54	-1.31
10	964	0.41	15.00	1250	0.49	40.00	0.00	60.71	-1188	60.71	-1.24	0.22
11	964	0.41	15.69	1250	0.49	39.99	0.00	63.12	-1189	63.12	-1.24	0.00
12	964	0.41	15.69	1250	0.49	40.00	0.00	67.47	-1192	67.47	-0.54	0.19
13	964	0.41	15.00	1249	0.49	40.00	0.00	59.10	-1187	59.10	-1.45	-0.09
14	964	0.41	15.00	1249	0.49	40.00	0.00	60.45	-1188	60.45	-1.24	-0.06
15	964	0.41	15.69	1250	0.49	40.00	0.00	63.02	-1189	63.02	-1.24	0.11
16	964	0.41	15.00	1249	0.49	39.99	0.00	59.73	-1187	59.73	-1.45	0.24
17	964	0.41	15.00	1249	0.49	40.00	0.04	59.23	-1187	59.27	-1.49	0.19
18	964	0.41	15.00	1249	0.49	40.00	0.00	59.75	-1187	59.75	-1.45	0.22
19	964	0.41	15.69	1250	0.49	40.00	0.00	63.08	-1189	63.08	-1.24	0.04
20	964	0.41	15.00	1249	0.49	40.00	0.00	59.10	-1187	59.10	-1.45	0.32
21	964	0.41	15.00	1249	0.49	40.00	0.00	60.08	-1188	60.08	-1.66	0.17
22	964	0.41	15.69	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
23	964	0.41	15.33	1250	0.49	39.85	0.00	63.04	-1189	63.04	-1.24	0.09
24	964	0.41	15.69	1250	0.49	40.00	0.00	67.38	-1191	67.38	-0.54	0.28
25	964	0.41	15.00	1249	0.49	39.99	0.00	58.46	-1187	58.46	-1.45	0.28
26	964	0.41	15.00	1249	0.49	40.00	0.00	59.10	-1187	59.11	-1.45	0.32
27	964	0.41	15.69	1250	0.49	39.99	0.00	63.12	-1189	63.12	-1.24	0.00

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
28	964	0.41	15.00	1249	0.49	39.99	0.00	58.45	-1187	58.45	-1.45	0.29
29	964	0.41	15.69	1250	0.49	40.00	0.00	67.38	-1191	67.38	-0.54	0.28
30	964	0.41	15.69	1250	0.49	40.00	0.00	63.03	-1189	63.03	-1.24	0.10
31	964	0.41	15.69	1250	0.49	40.00	0.00	63.03	-1189	63.03	-1.24	0.10
32	964	0.41	15.69	1250	0.49	40.00	0.00	67.38	-1191	67.38	-0.54	0.28
33	964	0.41	15.69	1250	0.49	40.00	0.00	67.38	-1191	67.38	-0.54	0.28
34	964	0.41	15.69	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
35	964	0.41	15.00	1249	0.49	40.00	0.00	59.10	-1187	59.11	-1.45	0.32
36	964	0.41	15.69	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
37	964	0.41	15.69	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
38	964	0.41	15.69	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
39	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
40	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
41	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
42	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
43	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
44	964	0.41	15.00	1249	0.49	39.99	0.00	58.45	-1187	58.45	-1.45	0.29
45	964	0.41	15.69	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
46	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
47	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
48	964	0.41	15.00	1249	0.49	39.99	0.00	58.45	-1187	58.45	-1.45	0.29
49	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
50	964	0.41	15.00	1249	0.49	39.99	0.00	58.45	-1187	58.45	-1.45	0.29
51	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
52	964	0.41	15.68	1250	0.49	40.00	0.00	67.50	-1192	67.50	-0.54	0.16
53	1250	0.32	15.00	1250	0.45	39.97	0.00	0.00	-1187	0.00	0.00	0.01
54	1250	0.32	15.00	1250	0.45	39.97	0.00	0.00	-1187	0.00	0.00	0.01
55	964	0.41	15.00	1249	0.49	39.99	0.00	58.45	-1187	58.45	-1.45	0.29

E.2 NSGA2 Method

Table E.2: Output values for NSGA2 from Figure 3.4

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
0	1248	0.32	17.03	1226	0.52	37.23	2.01	89.80	-1213	91.81	-2.44	-1.61
1	1238	0.32	17.14	1226	0.52	37.32	0.29	89.93	-1213	90.22	-0.70	-0.25
2	1237	0.34	19.27	1237	0.52	37.29	30.05	97.27	-1232	127.32	-1.56	-1.16
3	1248	0.34	18.33	1236	0.52	37.31	25.80	97.16	-1232	122.96	-1.24	-0.81
4	1233	0.35	19.55	1237	0.52	37.53	38.15	97.21	-1233	135.35	-2.22	-0.97
5	1248	0.33	17.00	1226	0.52	37.23	27.65	87.65	-1221	115.30	-9.96	-0.67
6	1239	0.32	16.76	1226	0.45	38.20	0.32	0.03	-1174	0.35	-0.37	-0.71
7	1249	0.34	18.31	1236	0.52	37.31	25.72	97.16	-1231	122.89	-2.99	-0.81
8	1234	0.35	16.60	1226	0.59	37.56	39.73	190.50	-1257	230.27	-1.25	-1.70
9	1233	0.32	19.47	1237	0.60	37.54	1.01	199.60	-1248	200.65	-3.33	-0.21
10	1239	0.32	16.80	1226	0.45	38.17	0.03	0.03	-1173	0.06	-0.09	-1.65
11	1249	0.34	16.25	1226	0.52	37.35	27.95	89.93	-1224	117.87	-5.77	-1.60
12	1249	0.34	15.82	1226	0.52	37.35	27.64	90.56	-1224	118.19	-5.60	-2.37
13	1248	0.34	18.33	1236	0.52	37.31	26.11	97.16	-1232	123.27	-1.42	-0.67
14	1233	0.35	19.72	1236	0.47	37.44	31.88	24.50	-1205	56.37	-0.84	-0.21
15	1250	0.32	16.43	1225	0.59	37.18	1.21	188.80	-1245	190.03	-0.80	-0.65
16	1231	0.34	16.34	1236	0.60	37.23	28.62	210.20	-1261	238.84	-6.74	-1.41
17	1240	0.32	16.12	1226	0.45	38.18	0.50	0.03	-1174	0.53	-0.20	-0.71
18	1248	0.33	19.49	1226	0.52	37.23	27.81	87.29	-1220	115.10	-8.47	-0.31
19	1235	0.35	18.41	1236	0.60	37.22	39.48	209.00	-1264	248.50	-0.34	-0.48
20	1233	0.35	17.23	1237	0.60	37.19	39.50	205.30	-1263	244.77	-2.76	-0.20
21	1240	0.35	16.15	1237	0.45	37.58	38.83	9.25	-1203	48.09	-0.07	-3.36
22	1249	0.34	18.31	1236	0.52	37.31	25.72	96.86	-1229	122.59	-3.27	-0.51
23	1248	0.32	17.16	1226	0.52	37.50	2.33	89.81	-1214	92.14	-1.25	-1.62
24	1249	0.33	17.00	1226	0.52	37.23	28.02	87.65	-1222	115.67	-8.31	-0.67
25	1233	0.35	16.42	1237	0.60	37.36	40.49	199.60	-1262	240.13	-4.84	-0.21
26	1234	0.35	19.47	1236	0.60	37.54	31.79	199.60	-1260	231.43	-0.23	-0.87
27	1238	0.32	20.03	1225	0.52	37.46	0.22	87.19	-1210	87.41	-0.22	-0.78
28	1232	0.35	19.15	1237	0.52	37.53	38.15	97.27	-1235	135.41	-2.61	-1.03
29	1243	0.32	16.41	1237	0.52	38.27	1.90	93.93	-1220	95.83	-1.09	-1.77
30	1243	0.34	19.29	1226	0.52	37.19	30.74	87.64	-1224	118.38	-1.48	-0.80
31	1240	0.35	18.30	1237	0.52	37.29	40.34	97.27	-1238	137.61	-1.71	-1.16
32	1250	0.32	16.43	1225	0.59	37.10	1.17	188.80	-1245	189.99	-0.76	-0.65
33	1239	0.32	16.76	1226	0.45	38.28	0.36	0.03	-1174	0.39	-0.41	-0.71
34	1250	0.32	19.45	1236	0.59	37.10	1.17	201.30	-1250	202.52	-0.76	-5.97

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
35	1249	0.32	16.33	1236	0.60	37.57	2.11	209.00	-1254	211.16	-3.98	-0.48
36	1240	0.35	19.15	1237	0.52	37.53	39.46	97.27	-1237	136.73	-0.83	-1.03
37	1237	0.32	17.14	1226	0.52	37.32	0.29	89.93	-1211	90.22	-1.05	-0.25
38	1237	0.34	16.52	1226	0.52	37.50	30.74	90.24	-1226	120.98	-3.20	-0.43
39	1249	0.34	15.82	1226	0.52	38.29	27.95	90.56	-1225	118.51	-4.95	-4.66
40	1249	0.35	16.49	1236	0.60	37.19	42.19	223.20	-1267	265.39	-1.32	-14.68
41	1235	0.35	18.39	1236	0.60	37.22	39.49	209.00	-1265	248.51	-0.35	-0.48
42	1232	0.35	19.18	1237	0.52	37.53	38.14	97.27	-1233	135.41	-2.60	-1.03
43	1239	0.32	16.76	1226	0.45	38.20	0.32	0.03	-1174	0.35	-0.37	-0.71
44	1236	0.32	16.79	1226	0.45	38.17	0.02	0.03	-1171	0.05	-1.14	-4.49
45	1248	0.33	19.36	1226	0.52	37.23	27.81	87.32	-1220	115.13	-8.47	-0.34
46	1240	0.35	19.47	1236	0.60	37.54	38.75	199.60	-1261	238.38	-4.90	-0.87
47	1249	0.34	18.31	1236	0.52	37.31	25.72	96.86	-1230	122.59	-3.54	-0.51
48	1234	0.32	19.47	1237	0.59	37.54	1.04	199.60	-1249	200.67	-3.00	-4.02
49	1249	0.33	16.87	1225	0.52	38.10	27.79	87.65	-1222	115.44	-9.73	-1.24
50	1234	0.34	19.46	1236	0.60	37.21	30.79	199.60	-1258	230.43	-4.93	-0.87
51	1237	0.32	17.29	1226	0.52	37.31	0.24	87.19	-1211	87.43	-0.46	-0.21
52	1245	0.32	16.42	1237	0.60	37.36	4.08	199.60	-1250	203.71	-6.11	-0.35
53	1236	0.32	16.79	1226	0.45	38.17	0.02	0.03	-1171	0.05	-1.14	-4.49
54	1233	0.35	19.59	1236	0.47	37.44	31.40	24.50	-1205	55.90	-0.36	-0.21
55	1236	0.35	18.39	1236	0.60	37.22	39.11	209.00	-1264	248.13	-1.21	-0.48
56	1249	0.32	16.33	1236	0.60	37.57	5.98	209.00	-1255	215.03	-6.06	-1.16
57	1250	0.32	16.43	1224	0.59	37.09	1.10	188.80	-1242	189.92	-0.69	-1.71
58	1240	0.35	16.15	1236	0.45	37.58	38.83	10.66	-1204	49.49	-0.07	-4.86
59	1250	0.32	19.45	1236	0.60	37.32	0.24	201.30	-1249	201.59	0.17	-3.93
60	1243	0.32	19.70	1237	0.47	37.28	0.23	24.59	-1186	24.83	-0.92	-1.69
61	1199	0.34	19.65	1226	0.46	38.12	11.91	4.24	-1178	16.15	-1.37	-0.47
62	1240	0.35	19.00	1238	0.45	37.53	42.15	0.82	-1198	42.97	-6.52	-1.37
63	1240	0.35	18.28	1237	0.45	37.32	39.41	1.02	-1196	40.43	-0.78	-6.97
64	1235	0.35	16.71	1225	0.45	38.28	31.40	0.02	-1192	31.42	-1.63	-1.87

E.3 Morbit Method

Table E.3: Output values for Morbit from Figure 3.4

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
0	1249	0.32	39.94	1250	0.43	39.94	0.17	0.00	-1169	0.17	0.02	-15.56
1	1249	0.32	27.00	1249	0.44	39.95	-0.38	-0.36	-1178	-0.74	-1.73	-3.46
2	1248	0.37	15.03	1250	0.47	40.00	72.00	35.18	-1243	107.18	0.21	-0.99
3	1243	0.37	39.93	1243	0.48	38.63	72.00	42.56	-1225	114.56	-5.43	-2.92
4	1244	0.36	24.79	1243	0.49	32.45	37.97	24.41	-1224	62.38	19.38	28.76
5	1249	0.31	40.00	1216	0.45	15.00	0.00	0.00	-1092	0.00	-15.87	-7.12
6	1149	0.35	27.17	1244	0.56	37.74	13.38	183.19	-1232	196.57	-16.74	-24.94
7	1250	0.30	15.00	1250	0.42	40.00	-0.00	0.00	-1151	0.00	-27.50	-30.69
8	1085	0.42	29.63	1249	0.48	39.94	35.99	0.61	-1217	36.60	28.64	46.27
9	1182	0.37	19.22	1245	0.46	38.47	47.55	25.39	-1210	72.94	-3.33	-5.49
10	1190	0.33	17.94	1246	0.43	39.80	0.41	1.99	-1161	2.40	-13.23	-19.44
11	1181	0.32	16.07	1231	0.49	38.55	5.21	98.66	-1189	103.87	-22.95	-41.46
12	1243	0.34	15.27	1249	0.44	39.88	32.43	1.53	-1196	33.96	-11.18	-6.18
13	1249	0.33	15.00	1249	0.45	40.00	17.52	0.00	-1198	17.52	0.00	0.00
14	1169	0.35	17.36	1246	0.47	39.62	5.22	33.83	-1197	39.05	0.49	1.37
15	1042	0.30	40.00	1250	0.43	40.00	-0.00	-0.00	-1082	-0.00	-95.94	-26.62
16	1250	0.31	40.00	1250	0.44	28.35	-0.00	-0.00	-1142	-0.00	-19.15	-7.48
17	1246	0.33	15.00	1246	0.51	39.81	11.65	81.59	-1230	93.24	1.00	2.95
18	1248	0.31	37.20	1228	0.44	39.40	7.03	36.55	-1148	43.58	-22.65	-59.74
19	1182	0.38	22.77	1249	0.52	36.24	60.49	117.75	-1242	178.24	-12.77	-9.51
20	1249	0.32	39.98	1250	0.44	31.71	0.06	0.26	-1164	0.32	0.04	-0.95
21	1227	0.38	15.00	1233	0.47	40.00	70.14	94.20	-1231	164.33	-0.00	-62.64
22	1189	0.30	15.00	1250	0.44	40.00	0.00	-0.00	-1149	-0.00	-47.54	-0.72
23	1242	0.32	15.83	1249	0.44	40.00	-0.00	-0.00	-1185	-0.00	-0.65	-0.72
24	1222	0.32	34.19	1238	0.45	29.17	0.13	0.05	-1153	0.18	-3.39	-1.50

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{T}_{bed}^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
25	1083	0.41	17.86	1246	0.49	39.53	72.00	63.69	-1220	135.69	-21.47	-8.94
26	1238	0.32	38.10	1160	0.47	31.15	2.45	3.55	-1124	6.00	-1.32	-18.83
27	1250	0.30	15.00	1250	0.44	39.33	-0.00	-0.00	-1160	-0.00	-27.50	-13.20
28	1196	0.33	18.33	1193	0.46	40.00	-0.00	6.84	-1154	6.84	-6.62	-10.55
29	1250	0.37	40.00	1250	0.45	40.00	72.00	10.40	-1219	82.40	2.94	-10.03
30	1250	0.32	15.00	1250	0.44	34.28	-0.00	0.00	-1174	-0.00	-5.33	-1.52
31	1250	0.30	24.10	1250	0.42	40.00	-0.00	-0.00	-1145	-0.00	-26.73	-35.74
32	1247	0.32	15.00	1247	0.50	39.84	0.45	77.40	-1221	77.85	0.25	0.73
33	914	0.43	40.02	1249	0.44	39.98	-0.09	-0.09	-1137	-0.18	-5.95	-3.86
34	1170	0.34	21.38	1242	0.42	32.08	1.39	1.40	-1145	2.79	0.76	-37.53
35	1196	0.39	22.07	1157	0.51	37.78	61.73	84.70	-1191	146.43	6.02	-47.06
36	1237	0.38	15.00	1247	0.45	39.98	71.88	1.46	-1228	73.34	0.01	0.02
37	1237	0.35	35.42	1249	0.46	33.41	46.29	26.62	-1203	72.91	-7.32	-1.85
38	1250	0.32	15.00	1250	0.41	40.00	-0.00	0.00	-1161	0.00	-6.07	-41.48
39	1190	0.33	19.19	1072	0.51	32.00	4.75	9.62	-1092	14.36	-11.49	-14.24
40	930	0.43	15.48	1155	0.42	15.72	1.69	4.26	-1013	5.95	1.63	-82.59
41	905	0.34	39.35	1184	0.44	16.98	7.45	8.17	-976	15.61	-113.43	-51.52
42	1054	0.38	20.88	1246	0.50	39.13	11.44	89.02	-1202	100.45	-10.03	-9.72
43	1250	0.30	40.00	1250	0.39	40.00	0.01	0.01	-1123	0.02	-21.85	-72.91
44	971	0.38	25.76	1223	0.46	35.84	9.24	6.21	-1121	15.45	-44.90	-3.18
45	1250	0.30	15.00	1250	0.38	15.72	-0.00	0.00	-1054	-0.00	-27.50	-85.92
46	1228	0.32	28.38	1220	0.44	38.69	2.81	2.81	-1152	5.62	-11.97	-21.94
47	1210	0.30	15.92	1243	0.42	16.11	0.83	6.32	-1070	7.15	-38.65	-48.72
48	1249	0.31	18.30	1249	0.45	40.00	0.00	0.51	-1177	0.51	-12.05	-0.00
49	1221	0.30	19.18	1214	0.45	38.87	5.65	17.08	-1140	22.72	-37.58	-34.04

Appendix F

Full Output values of Problem (2.28)

This appendix contains the values of the objective functions and variables in the solutions presented in Section 3.4 referred to the problem formulation (2.28).

F.1 DFMOINT Method

Table F.1: Output values for DFMOINT from Figure 3.5

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
0	1250	0.32	39.81	1070	0.52	40.00	0.00	0.00	0	0.00	0.00	-0.43

F.2 NSGA2 Method

Table F.2: Output values for NSGA2 from Figure 3.5

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
0	1138	0.40	38.54	957	0.66	39.90	54.57	90.49	0	145.06	-0.11	-2.74
1	1214	0.33	38.65	915	0.61	39.86	0.19	0.07	0	0.26	-0.71	-0.42
2	1131	0.39	34.53	957	0.66	39.94	48.86	84.07	0	132.93	-0.80	-4.89
3	1210	0.34	38.67	915	0.61	39.90	7.10	0.10	0	7.20	-0.02	-1.56
4	1205	0.33	37.57	963	0.61	39.71	0.51	31.58	0	32.08	-4.95	-1.54
5	1141	0.40	38.62	915	0.61	39.88	57.51	0.25	0	57.76	-1.49	-0.26
6	1124	0.39	37.29	913	0.61	39.85	48.04	0.09	0	48.13	-0.40	-1.71
7	1125	0.39	38.50	957	0.66	39.93	50.33	84.07	0	134.40	-1.89	-1.77
8	1205	0.34	38.49	974	0.61	39.93	6.83	39.97	0	46.80	-1.59	-0.38
9	1137	0.40	38.60	956	0.66	39.94	54.89	80.59	0	135.48	-0.86	-2.83
10	1205	0.33	38.91	957	0.61	39.88	1.03	32.26	0	33.29	-0.16	-4.36
11	1137	0.39	38.53	960	0.66	39.93	54.22	85.62	0	139.84	-1.32	-1.00

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
12	1137	0.39	38.62	961	0.61	39.93	54.66	34.53	0	89.19	-0.88	-1.35
13	1137	0.39	38.54	960	0.61	39.92	53.98	30.31	0	84.29	-0.08	-1.51
14	1209	0.33	38.95	974	0.61	39.90	4.93	39.91	0	44.84	-0.35	-0.31
15	1123	0.40	38.63	961	0.61	39.94	49.13	33.03	0	82.16	-0.20	-3.66
16	1131	0.40	37.92	915	0.61	39.90	54.06	0.02	0	54.08	-0.40	-0.07
17	1125	0.40	39.22	918	0.61	39.86	49.30	2.39	0	51.69	-0.24	-0.03
18	1137	0.39	38.54	960	0.66	39.93	54.22	90.54	0	144.76	-1.32	-5.92
19	1141	0.40	38.62	915	0.61	39.88	57.51	0.25	0	57.76	-1.49	-0.36
20	1131	0.40	37.90	915	0.61	39.90	54.26	0.02	0	54.28	-0.48	-0.27
21	1199	0.33	38.95	957	0.61	39.93	1.03	31.30	0	32.33	-2.23	-3.20
22	1196	0.34	38.49	974	0.61	39.93	2.59	39.97	0	42.56	-0.68	-0.36
23	1205	0.34	39.02	965	0.61	39.82	6.02	35.38	0	41.40	-0.39	-2.24
24	1210	0.33	38.82	957	0.61	39.93	5.14	35.05	0	40.18	-0.58	-7.19
25	1210	0.33	38.67	915	0.61	39.88	4.93	0.08	0	5.01	0.02	-0.29
26	1205	0.34	38.48	957	0.61	39.95	5.33	31.30	0	36.63	-0.09	-2.62
27	1210	0.33	38.67	914	0.61	39.93	4.93	0.08	0	5.01	0.02	-0.56
28	1133	0.39	34.82	913	0.61	39.85	48.99	0.10	0	49.09	-0.19	-1.72
29	1205	0.34	38.48	962	0.61	39.95	5.33	34.44	0	39.77	-0.09	-2.61
30	1124	0.39	37.38	913	0.61	39.85	48.41	0.28	0	48.68	-0.77	-1.90
31	1205	0.33	37.54	963	0.61	39.93	2.41	35.06	0	37.47	-1.41	-2.88
32	1208	0.33	38.67	912	0.61	39.93	4.43	0.08	0	4.51	-0.22	-2.30
33	1138	0.40	34.79	957	0.66	39.99	54.57	90.49	0	145.06	-0.11	-3.37
34	1137	0.39	38.54	960	0.61	39.93	54.27	38.83	0	93.09	-0.49	-6.23
35	1205	0.33	38.64	915	0.61	39.82	1.83	0.27	0	2.10	-0.31	-1.59
36	1210	0.33	38.93	974	0.61	39.94	4.93	40.94	0	45.88	0.02	-2.52
37	1137	0.39	38.54	960	0.61	39.93	54.27	36.68	0	90.94	-0.49	-4.08
38	1206	0.33	38.65	915	0.61	39.83	1.79	0.10	0	1.89	-0.03	-0.52
39	1210	0.33	38.82	915	0.61	39.93	5.38	0.08	0	5.46	-0.17	-0.43
40	1131	0.40	37.90	915	0.61	39.90	54.07	0.02	0	54.09	-0.29	-0.27
41	1198	0.34	38.75	913	0.61	39.93	3.08	0.28	0	3.36	-0.43	-1.87
42	1198	0.34	38.79	915	0.61	39.93	3.08	0.09	0	3.17	-0.56	-0.13
43	1123	0.39	38.46	960	0.61	39.86	48.02	30.18	0	78.20	-0.20	-1.32
44	1137	0.40	38.54	958	0.62	39.93	55.09	38.89	0	93.98	-1.06	-1.64
45	1137	0.39	38.80	960	0.61	39.93	54.01	30.27	0	84.28	-0.11	-0.31
46	1210	0.33	38.57	957	0.61	39.93	5.14	30.48	0	35.61	-0.58	-3.57
47	1206	0.33	38.65	915	0.61	39.93	1.79	0.57	0	2.36	-0.03	-0.99

F.3 Morbit Method

Table F.3: Output values for Morbit from Figure 3.5

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
0	1097	0.38	36.09	1184	0.50	39.36	29.54	46.49	0	76.03	-7.42	-0.52
1	1225	0.32	24.33	1078	0.50	39.27	3.12	8.21	0	11.33	-13.04	-24.06
2	1249	0.37	40.00	850	0.65	40.00	71.72	0.00	0	71.72	0.20	-0.60
3	1188	0.30	40.00	850	0.63	40.00	0.00	-0.00	0	-0.00	-47.75	-20.78
4	1134	0.36	28.51	1101	0.59	36.11	71.56	135.54	0	207.10	-61.68	-32.60
5	1214	0.33	27.87	992	0.56	39.18	2.15	5.79	0	7.93	-1.30	-9.09
6	1230	0.32	28.62	922	0.61	39.20	0.82	4.61	0	5.43	-3.82	-0.70
7	1230	0.32	21.80	889	0.62	40.01	0.92	0.78	0	1.70	-2.29	-7.96
8	1219	0.33	33.40	995	0.57	40.00	7.18	8.88	0	16.06	-5.12	0.00
9	1115	0.36	33.82	972	0.59	36.61	1.68	42.50	0	44.18	0.25	-20.53
10	1183	0.36	24.67	1067	0.53	39.38	43.72	15.85	0	59.57	-16.69	-9.00
11	1250	0.37	40.00	923	0.70	40.00	39.95	0.00	0	39.95	26.64	98.91
12	1186	0.34	32.05	963	0.57	39.98	0.00	0.71	0	0.71	-0.81	-6.89
13	1225	0.33	38.71	1024	0.56	40.00	0.19	17.95	0	18.14	-1.29	0.26
14	1243	0.36	36.93	913	0.68	39.70	64.36	52.81	0	117.17	-17.33	18.22
15	1230	0.37	27.42	1087	0.66	39.91	57.10	187.71	0	244.80	1.31	-4.26
16	1214	0.36	24.09	976	0.58	39.74	35.27	14.30	0	49.56	-0.00	-0.66
17	1130	0.40	32.32	1138	0.53	39.67	71.28	66.56	0	137.84	-11.64	-14.22
18	1249	0.32	40.00	894	0.62	40.00	0.01	0.01	0	0.02	6.24	-0.56
19	1081	0.38	31.86	1147	0.55	38.96	31.35	87.58	0	118.93	-16.53	-0.35
20	1228	0.32	22.45	951	0.57	39.73	0.18	0.45	0	0.63	-2.94	-20.46
21	1161	0.34	36.70	885	0.63	40.00	2.38	3.63	0	6.01	-5.23	0.08
22	1085	0.43	31.13	875	0.64	40.00	69.72	116.78	0	186.49	1.58	-114.92
23	1238	0.32	39.37	857	0.62	39.28	0.24	0.00	0	0.24	-4.17	-29.99
24	1221	0.34	17.82	876	0.65	39.91	15.35	12.37	0	27.72	-0.20	-0.22

id	T^{TPH}	$\hat{m}^{TPH}$	O_2^{TPH}	T^{PH}	$\hat{m}^{PH}$	O_2^{PH}	$\hat{Q}_F^{TPH}$	$\hat{Q}_F^{PH}$	$\bar{O}_2^{PH}$	$\sum_i \hat{Q}^i$	$g_{TPH}(x)$	$g_{PH}(x)$
25	1250	0.31	15.02	1239	0.42	40.20	0.73	0.73	0	1.45	-7.86	-36.08
26	1220	0.33	27.34	975	0.56	39.56	6.99	4.66	0	11.65	-3.70	-11.75
27	1207	0.35	29.04	958	0.60	38.49	24.66	18.31	0	42.97	-5.88	0.15
28	1249	0.32	15.00	1004	0.55	40.00	0.00	-0.00	0	0.00	-6.56	-4.04
29	1239	0.33	28.09	1032	0.54	37.06	12.51	15.11	0	27.62	-3.47	-18.90
30	1006	0.34	39.92	1046	0.58	40.00	6.24	60.21	0	66.45	-71.53	0.00
31	1237	0.33	40.00	1017	0.55	39.91	2.38	0.20	0	2.57	0.03	-0.33
32	1004	0.39	39.75	866	0.63	39.70	0.05	0.21	0	0.26	-3.73	-11.56
33	1244	0.32	23.01	927	0.60	40.00	6.28	2.85	0	9.13	-6.50	0.12
34	1060	0.43	38.44	1010	0.57	39.09	56.13	47.46	0	103.59	-0.07	-27.79
35	1221	0.32	17.88	1215	0.51	37.86	11.07	99.49	0	110.57	-24.78	-26.27
36	1226	0.32	39.84	850	0.65	39.97	0.04	0.02	0	0.06	-6.93	-4.73
37	1130	0.40	35.08	968	0.59	38.93	50.85	32.96	0	83.82	2.86	-13.87
38	1169	0.35	22.32	951	0.63	37.02	24.24	53.49	0	77.73	-8.44	-7.16
39	1250	0.31	40.02	987	0.55	39.86	-0.22	-0.23	0	-0.45	-10.10	-16.18
40	1248	0.32	39.29	932	0.60	39.99	0.00	1.19	0	1.19	-2.05	0.24
41	1138	0.30	40.00	850	0.46	40.00	-0.00	-0.00	0	-0.00	-64.36	-177.67
42	1241	0.33	39.96	850	0.64	39.84	15.33	0.00	0	15.33	-0.65	-9.15
43	1222	0.35	37.31	1038	0.52	35.47	48.68	27.78	0	76.46	-19.42	-44.74
44	877	0.45	36.86	1210	0.47	39.98	0.00	13.90	0	13.90	-2.85	0.01
45	1241	0.31	39.33	867	0.60	35.75	0.93	0.02	0	0.94	-12.71	-44.02
46	1249	0.33	15.98	857	0.64	39.02	15.72	0.52	0	16.24	-1.02	-11.65
47	1066	0.37	40.00	1035	0.50	39.85	0.00	0.00	0	0.00	-0.50	-41.46
48	852	0.47	20.98	968	0.55	36.47	3.21	0.94	0	4.15	-4.73	-26.03
49	1199	0.34	15.03	1233	0.44	39.44	0.83	0.27	0	1.11	2.00	-13.69