Doctoral Thesis in Technology and Health

Network-Agnostic Computational Approaches for Modelling and Validating Evolving Complex Systems

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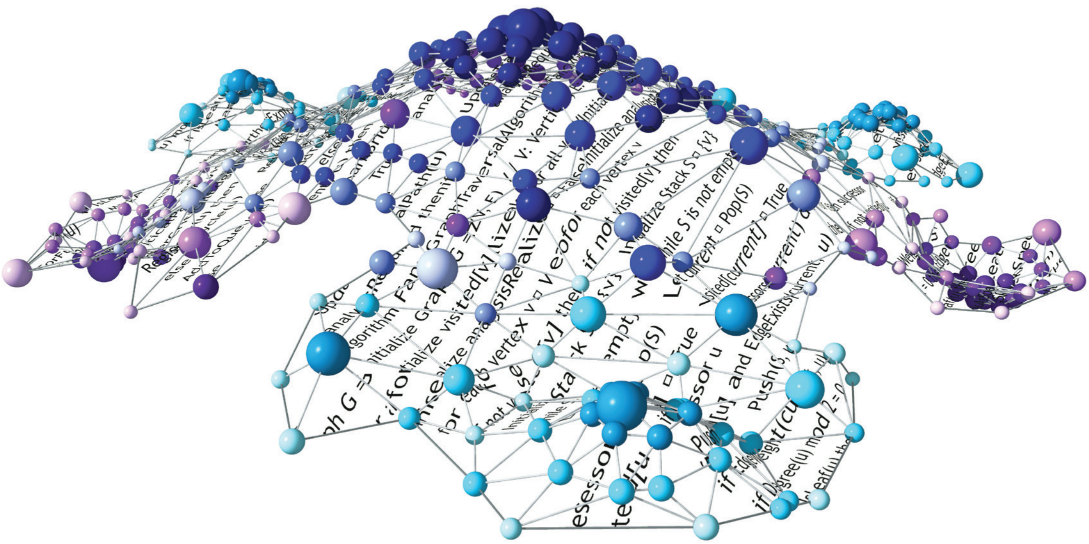
KTH ROYAL INSTITUTE OF TECHNOLOGY



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Academic Dissertation which, with due permission of the KTH Royal Institute of Technology, is submitted for public defence for the Degree of Doctor of Philosophy on Wednesday the 19th February 2025, at 13:00. in T2, Hälsovögen 11C, Huddinge.

Doctoral Thesis in Technology and Health
KTH Royal Institute of Technology
Stockholm, Sweden 2025

Cover page photo: Designed and dedicated by Julie Meghedi Orbelians.

TRITA-CBH-FOU-2024:63
ISBN 978-91-8106-162-8

Printed by: Universitetsservice US-AB, Sweden 2025

Abstract

The dynamic and evolving nature of complex systems influences the behaviour of individual parts and the performance of the whole system, which hinders the flexibility in modelling and applicability of the modelling approaches for various systems. Modelling approaches which are developed for other types of systems have limitations in capturing the behaviour of health systems. This can stem from the dependencies of models on the system structure, type and data structure. From the network perspective, health systems can have various representations, which are defined by each network topology, type and parameters. This makes it challenging to utilize an approach developed to analyse and model systems of other aspects and domains.

The degree of flexibility of an approach depends on its scalability to be able to model a system that grows, its generality to model systems of various types, its adaptability to internal changes, and its flexibility to topology without the need for change. Additionally, flexibility should enable the approach adjustment with new functions and features to facilitate and analyse other aspects of a system easily. Network representation of complex health systems facilitates the application of network algorithms and network simulation methods to enhance the modelling approaches. This requires the adjustment of these algorithms and methods, which can be achieved by tuning, merging, modifying, and gaming procedures. Such adjustments increase the efficiency of a model to analyse different aspects of a system and overcome limitations.

This thesis offers various network-agnostic computational approaches for modelling and validating complex health systems. For this, three different case studies are provided to explore which represent the complex societal, hospital and organisational systems, respectively. The proposed modelling approaches for these systems aim to facilitate the quantification of vertices and edges, classify the vertices' behaviours, promote verification and validation, evaluate the systems' performances, and enhance the link prediction for a more accurate representation of the systems.

The key contribution of this thesis is to offer approaches which can facilitate scalability, generality, adaptability, topological flexibility, and adjustability. This is achieved by adjusting the network algorithms and simulation methods based on the purpose of the modelling. Hence, 1) a gaming simulation approach is proposed

to facilitate the quantification of the edges in a complex societal system, 2) ranking algorithm is merged with path analysis to quantify the vertices and identify the vertices persistency in a complex societal system, 3) ranking algorithm is merged with system dynamic simulation to facilitate the quantification of the vertices as the complex societal system evolve and change, 4) an agent-based network simulation is implemented along with network algorithms to identify the bottlenecks using flow and structural hole algorithms and evaluation the performances of the wards using percolation and perturbation algorithm in a complex hospital system, 5) a modification of flow algorithm is proposed to model the dynamic nonlinear flow of the patients in a complex hospital system, 6) a link prediction approach which merges the path analysis and non_randomness algorithm to identify the missing links in a complex organizational system, and finally 7) a multi-network simulation to evaluate the performances of the organizations in parallel.

The thesis provides two different classifications of the proposed approaches. The first classification indicates how each approach contributes to modelling the system based on the underlying system scale, type, dynamic, topology and model adjustability, and the second classification indicates the proper application of network algorithms and network simulation methods based on the underlying health system type and the purpose of the analysis.

Each approach provides a holistic view of the systems through matrix or network representation to inform about their states and a view of the vertices' dynamic behaviours to evaluate their performances.

Keywords: Network-Agnostic Approaches, Network Simulation, Dynamic Modelling, Network Algorithms, Verification and Validation, Evolving Complex Systems.

Sammanfattning

Den dynamiska och utvecklande karaktären hos komplexa system påverkar beteendet hos enskilda delar och hela systemets prestanda, vilket hindrar flexibiliteten i modellering och tillämpbarheten av modelleringsmetoderna för olika system. Modelleringsmetoder som utvecklats för andra typer av system har begränsningar för att fånga hälsosystemens beteende. Detta kan härröra från modellers beroende av systemstruktur, typ och datastruktur. Ur ett nätverksperspektiv kan hälsosystem ha olika representationer som definieras av varje nätverkstopologi, typ och parametrar. Detta gör det utmanande att använda ett tillvägagångssätt utvecklat för att analysera och modellera system av andra aspekter och domäner.

Graden av flexibilitet för ett tillvägagångssätt beror på dess skalbarhet för att kunna modellera ett system som växer, generalitet till modellsystem av olika typer, anpassningsförmåga till de interna förändringarna och flexibilitet till topologi utan behov av förändring. Dessutom bör flexibilitet möjliggöra tillvägagångssättjustering med nya funktioner och funktioner för att enkelt underlätta och analysera andra aspekter av ett system. Nätverksrepresentation av komplexa hälsosystem underlättar tillämpningen av nätverksalgoritmer och nätverkssimuleringsmetoder för att förbättra modelleringsmetoderna. Detta kräver justering av dessa algoritmer och metoder, vilket kan uppnås genom justering, sammanslagning, modifiering och spelprocedurer. Sådana justeringar ökar effektiviteten hos en modell för att analysera olika aspekter av ett system och övervinna begränsningar.

Denna avhandling erbjuder olika nätverks-agnostiska beräkningsmetoder för modellering och validering av komplexa hälsosystem. För detta tillhandahålls tre olika fallstudier för att utforska vilka som representerar de komplexa samhälleliga, sjukhus- respektive organisationssystemen. De föreslagna modelleringsmetoderna för dessa system syftar till att underlätta kvantifieringen av hörn och kanter, klassificera hörnens beteenden, främja verifieringen och valideringen, utvärdera systemens prestanda och förbättra länkförutsägelsen för mer exakt representation av systemen.

Det viktigaste bidraget för denna avhandling är att erbjuda tillvägagångssätt som kan underlätta skalbarhet, generalitet, anpassningsförmåga, topologisk flexibilitet och justerbarhet. Detta uppnås genom att justera nätverksalgoritmerna

och simuleringsmetoderna utifrån syftet med modelleringen. Därför föreslås 1) en spelsimuleringsmetod för att underlätta kvantifieringen av kanterna i ett komplext samhälleligt system, 2) rankningsalgoritmen slås samman med väganalys för att kvantifiera hörnen och identifiera grenarnas beständighet i ett komplext samhälleligt system, 3) rankning Algoritmen slås samman med systemdynamisk simulering för att underlätta kvantifieringen av hörnen som komplexet samhälleliga system utvecklas och förändras, 4) en agentbaserad nätverkssimulering implementeras tillsammans med nätverksalgoritmer för att identifiera flaskhalsar med hjälp av flödes- och strukturella hålalgoritmer och utvärdering av avdelningarnas prestanda med hjälp av perkolations- och störningsalgoritmer i ett komplext sjukhussystem, 5) en modifiering av flödesalgoritmen föreslås för att modellera det dynamiska olinjära flödet av patienter i ett komplext sjukhussystem, 6) en länkförutsägelsemetod som kombinerar väganalysen och algoritmen för icke-slumpmässighet för att identifiera de saknade länkarna i ett komplext organisationssystem, och slutligen 7) en simulering av flera nätverk för att parallellt utvärdera organisationernas prestationer.

Avhandlingen ger två olika klassificeringar av de föreslagna tillvägagångssätten. Den första klassificeringen anger hur varje tillvägagångssätt bidrar till att modellera systemet baserat på den underliggande systemets skala, typ, dynamik, topologi och modellens justerbarhet; och den andra klassificeringen indikerar korrekt tillämpning av nätverksalgoritmer och nätverkssimuleringsmetoder baserat på den underliggande typen av hälsosystem och syftet med analysen.

Varje tillvägagångssätt ger en holistisk bild av systemen genom matris- eller nätverksrepresentation för att informera om dess tillstånd, och en bild av hörnens dynamiska beteenden för att utvärdera deras prestationer.

Nyckelord: Nätverks-Agnostiska Tillvägagångssätt, Nätverkssimulering, Dynamisk Modellering, Nätverksalgoritmer, Verifiering och Validering, Evolverande Komplexa Nätverk.

List of Papers

The thesis includes the following articles:

I **SocioBalance: A Network-based Simulation Game to Rank Links' Impact Strength in a Complex Social System**
Arsineh Boodaghian Asl, Jayanth Raghothama, Michel Gokan Khan, Adam Darwich, Sebastiaan Meijer
Under review in *Simulation & Gaming (2025).*

II **Using PageRank and Social Network Analysis to Specify Mental Health Factors**
Arsineh Boodaghian Asl, Jayanth Raghothama, Adam Darwich, Sebastiaan Meijer
Published in *Proceedings of the Design Society (2021).*

III **A Hybrid Modeling Approach to Simulate Complex Systems and Classify behaviours**
Arsineh Boodaghian Asl, Jayanth Raghothama, Adam Darwich, Sebastiaan Meijer
Published in *Network Modeling Analysis in Health Informatics and Bioinformatics (2024).*

IV **A Hybrid Approach to Model Hospitals and Evaluate Wards' Performances**
Arsineh Boodaghian Asl, Luca Marzano, Jayanth Raghothama, Adam Darwich, Nina Falk, Patrik Bodeby, Sebastiaan Meijer
Under review in *IEEE Access (2025).*

V **A Dynamic Nonlinear Flow Algorithm to Model Patient Flow**
Arsineh Boodaghian Asl, Jayanth Raghothama, Adam Darwich, Sebastiaan Meijer
Under review in *Scientific Reports (2025).*

VI **A Reference-Based Link Prediction Approach for Complex Networks Using Global and Local Methods**

Arsineh Boodaghian Asl, Jayanth Raghothama, Adam Darwich, Anne-Marie Boström, Susanne Assander, Sebastiaan Meijer
Under review in *Applied Network Science (2025)*.

Other contributions by the author not included in the thesis:

VII **Simulation and Model Validation for Mental Health Factors Using a Multi-Methodology Hybrid Approach**
Arsineh Boodaghian Asl, Jayanth Raghothama, Adam Darwich, Sebastiaan Meijer
Published in *IEEE, Winter Simulation Conference (WSC) (2021)*

VIII **A Data-Driven Discrete Event Simulation Model to Improve Emergency Department Logistics**
Mohamed Nezar Abouraja, Luca Marzano, Jayanth Raghothama, Arsineh Boodaghian Asl, Adam Darwich, Sven Lathvall, Nina Falk, Sebastiaan Meijer
Published in *IEEE, Winter Simulation Conference (WSC) (2022)*

IX **Graph Attention Networks and Deep Q-Learning for Service Mesh Optimization: A Digital Twinning Approach**
Michel Gokan Khan, Javid Taheri, Andreas Kassler, Arsineh Boodaghian Asl
Published in *IEEE, International Conference on Communications (ICC) (2024)*

Authors Contributions in Papers

Authors' Abbreviation for Included Papers in the Thesis

AB: Arsineh Boodaghian Asl; **SM:** Sebastiaan Meijer; **JR:** Jayanth Raghothama; **AD:** Adam Darwich, **MG:** Michel Gokan Khan; **LM:** Luca Marzano; **NF:** Nina Falk; **PB:** Patrik Bodeby; **AMB:** Anne-Marie Boström; **SA:** Susanne Assander.

I **AB** contributed in proposing the approach, developing and validating the simulation game, and writing the manuscript. MG contributed in analysing the data. JR, AD and SM contributed in supervising the project, writing and reviewing the manuscript.

II **AB** contributed in proposing developing and validating the approach, and writing the manuscript. JR, AD and SM contributed in supervising the project, writing and reviewing the manuscript.

III **AB** contributed in proposing, developing and validating the approach, and writing the manuscript. JR, AD and SM contributed in supervising the project, writing and reviewing the manuscript.

IV **AB** contributed in proposing, developing and validating the approach, and writing the manuscript. LM contributed in analysing and validating the data. NF and PB contributed in validating the output. JR, AD and SM contributed in supervising the project, validating the approach, writing and reviewing the manuscript.

V **AB** contributed in proposing, modifying and validating the algorithm, and writing the manuscript. JR, AD and SM contributed in supervising the project, validating the approach, writing and reviewing the manuscript.

VI **AB** proposed the local link prediction approach, validated the networks topologies and developed both the global and local methods for link prediction, and was the principal author of the article. JR and AD contributed to the methodology, and to the text of the article. AD was the principal author of the systems model used in the article. AMB and SA contributed with the data, and validating the mapping between the systems model and the survey data. SM contributed to the methodology, and review of the article.

Acknowledgement

I have always enjoyed solving puzzles and still do. Growing up with this interest, I was naturally drawn to subjects that encouraged problem-solving. With the support of my parents, I had the opportunity to attend high school, primarily at the Melkonian Educational Institute in Nicosia, Cyprus, and for a year in Tehran, Iran. Later, I pursued my Bachelor's degree at the State Engineering University of Armenia, followed by a Master's degree at Stockholm University. After spending several years working as a software developer in the industry and conducting side research, I decided it was time for a new challenge and began my PhD journey at KTH Royal Institute of Technology in Stockholm. Now that I have completed my PhD thesis, I would like to thank everyone who supported me throughout this adventure as I prepare for the next adventure.

I would like to thank my main supervisor, Sebastiaan Meijer, and my two co-supervisors, Jayanth Raghothama and Adam Darwich, for always having time for me; answering my emails even on weekends, vacations and nights; finding time to listen and talk to me despite their busy schedules, sharing their perspectives, and also listening to mine. You've been incredibly friendly and caring, giving me unique research opportunities to work on new approaches, experience in teaching and supervision, knowledge in writing funding applications, guidance for non-PhD-related questions, and much more. Every single research project was a unique puzzle for me. I have truly enjoyed our research discussions and collaborations, and I could have never asked for better supervisors: 'What are the odds of having some of the best and most fun supervisors on planet Earth?'

I would like to thank my husband, Michel Gokan Khan, for being my soulmate, research mate, code mate, game mate, travel mate, fun mate, crazy mate, life mate, dream mate, laugh mate, and support mate. You've been one of my biggest supporters, my constant source of strength, and my partner in adventure. I am looking forward to having more adventures with you. Thank you for celebrating my achievements as if they were your own. Your selflessness, humour, and unwavering belief in me have been my foundation throughout this journey. I am so grateful to have you in my life: 'What are the odds of having a husband as crazy as me?'

I would like to thank my amazing parents, Ruben Boodaghian Asl and Armineh Rezooni Matevosian, for spending time with me solving maths problems, play-ing games, letting me reverse-engineer stuff, teaching me PCB assembly to build things, introducing me to programming, reading me astronomy books, teaching me to be an independent woman, providing me with unique opportunities to achieve my goals, and showing interest in my work. Thank you for your selfless support and encouragement in every aspect of my life. One of the best things in life is hearing your voices every day.

I also would extend my heartfelt gratitude to my brothers, Varag and Varand; my in-laws, Soniya Krakousians, David Gokan Khan, Alex, Valentin and Michel's niece, Annabelle; to my grandmother, Nina Boodaghian Asl, and my late aunt Margoosh Boodaghian Asl, the late grandparents Simone Matevosian and Noramair Movsesian and Hayrapet Boodaghian Asl. Special thanks to Janet and John Sören Pettersson at Karlstad University for always supporting me, even way before I start my PhD. During my PhD journey, I had the privilege to meet several amazing people at KTH who supported me in various ways, including Anders Cajander, Maksims Kornevs, Catherine Trask, Mario Romero Vega, Ibrahim Orhan, Nicklas Brandefelt and Peter Sillén. I am immensely thankful to my colleagues at Stockholm University, including Uno Fors, Panagiotis Papapetrou, and Ioanna Miliou. I am also grateful to all my colleagues and friends, including Irini, Merja, Mikaela, Nada, Harsha, Luca, Alina, Meghedi, Sevan, Sahar, Arghavan, Navid, Sara, Nate, Lalageh, Sina, Parsova, as well as relatives and acquaintances, for their invaluable supports.

Stockholm, Sweden 2025
Arsineh Boodaghian Asl

Contents

1 Introduction

1.1 Complex Health Systems Modelling

The purpose of network modelling of health systems is to provide insight into their structure and performance, such as complex societal, hospital or organizational systems. These models represent the elements of the system and their relationships. Through the simulation of these models, we can observe the performance and dynamic behaviour of health systems and identify critical elements (Chen et al., 2017). A challenge is that health systems are complex, having many elements interconnected to other elements (Bhattacharjee and Ray, 2014; Djitog et al., 2018; Zhang et al., 2018). By modelling and simulation only some parts of the system, we will be unable to observe the accurate performance of the system based on the influence of other elements (Rutter et al., 2017; Cassidy et al., 2019; Bhattacharjee and Ray, 2014; Lee Jr, 1973). In parallel, modelling the whole system is time-consuming because it requires us to describe all the elements, their relationships and functionalities, collect lots of data (Siegenfeld and Bar-Yam, 2020; Barjis, 2011; Carter and Busby, 2023) and understand the purpose of the whole underlying system. Furthermore, these systems evolve and change over time, and the rest of the system adapts to this change to maintain the performance (De Haan and Rotmans, 2011; Kudyba, 2018; Aboutaleb and Monsuez, 2015). To overcome these challenges, network-agnostic modelling approaches are necessary to simulate the entire system of any scale, type, change, and topology and be easily adjustable for additional features and functionalities.

Changes in health systems can be due to internal or external influences, foreseen and unforeseen, which may cause performance improvement or perturbations (Ladyman et al., 2013; Martínez-García and Hernández-Lemus, 2013; Tang et al., 2021; Wu et al., 2015). This is due to the dependence of the elements on each other due to continuous interaction (Ladyman et al., 2013; Bhattacharjee and Ray, 2014; Aboueljinane and Frichi, 2022). Through this interaction, elements influence each other (Ladyman et al., 2013). The uncertainty with these changes is whether the effects are temporary or permanent, or which elements in the system cause or affect these changes and to what magnitude (Heino et al., 2021; Batty, 2015; Omrani et al., 2022; Mustafee et al., 2015; Rutter et al., 2017). Furthermore, health systems can be considered complex adaptive systems, (Greenhalgh et al., 2010) and are in continuous change, creating dynamic behaviour in elements where these elements may exhibit performance fluctuations. These fluctuations can be due to the significance of the changes influencing the system or the degree of connectivity causing the elements to have nonlinear behaviour. As a result, each element in a system has a different and unpredictable behaviour pattern (Ladyman et al., 2013; Siegenfeld and Bar-Yam, 2020), making it challenging to identify the long-term changes.

Influence refers to the trigger made to a system which affects the behaviour of some or all elements which is not predictable. These influences can happen externally when a new element joins the system or internally due to the dynamic nature of the system. An influence on a complex health system affects each element differently (Djitog et al., 2018). One of the main reasons is the degree of connectivity (Heino et al., 2021). The higher the degree of connectivity is, the higher might be the nonlinearity and the probability of the perturbation in the performance (Ladyman et al., 2013; Heino et al., 2021). Such behaviours can also relate to the elements' impact dynamic and the direction that influences flows (Ladyman et al., 2013). With a weak degree of connectivity between a pair of elements and uniform impact, one of the elements may hardly exhibit nonlinear behaviour. Such weaknesses in system topology may also cause chaos in one part of the system and no change in the other parts (Ladyman et al., 2013; De Haan and Rotmans, 2011), leading the whole system to eventually collapse. For example, if the link between a society's education level and financial condition is weak, it may lead to unwanted consequences. Another reason is the order in which elements interact (Ladyman et al., 2013). In such cases, elements with significant impact strength (Heino et al., 2021) may have an adverse impact over weaker elements in the neighbourhood where the weak elements may stop performing due to the continuous pressure and affect the rest of the elements in the same order. These elements can be considered crucial to test the system vulnerability (Ladyman et al., 2013) against competing with other elements (Siegenfeld and Bar-Yam, 2020). For example, when critical wards such as the emergency department dispatch the majority of patients to a ward with fewer available resources, significant bottlenecks can occur. The centrality of crucial elements also plays a significant role in influencing a system (Ladyman et al., 2013). Such elements may have a wider impact on the whole system's behaviour, which can be both fruitful and dreadful if the whole system relies on a single element performance (Ladyman et al., 2013). As an example, implementing a health system with a similar purpose or functionality may help the elements to cooperate (Siegenfeld and Bar-Yam, 2020; De Haan and Rotmans, 2011) and reduce the pressure from a single health system, which can cause the system to evolve. Identifying the crucial elements and classifying elements' behaviours can help to control the system performance (De Haan and Rotmans, 2011) and minimize the risk on health systems, where the focus should be on elements' improvement stability and not the significance of the change, which can be misleading.

1.2 Modelling Challenges

Models should be able to identify changes in the system and adapt to those changes (De Haan and Rotmans, 2011; Rutter et al., 2017; Carothers et al., 2017). Difficulties can arise from changes in the impact strength between a pair of elements, changing their interaction or relationship. A model may be incomplete, e.g. missing an element in the modelling process and its influence (Chen et al., 2017). Inability to adapt to changes may provide invalid output (Branlat and Woods, 2010). Yet another modelling challenge is considering the evolving nature of the complex health systems (Carothers et al., 2017; Havlin et al., 2012). This difficulty is due to the unpredictability of where an element will be added to the system, its impact direction, and how it will influence the system. It is difficult to address these challenges without going through the entire modelling process again (De Bruijn and Herder, 2009). Methods are needed to address these modelling challenges.

Models are difficult to reuse because they are specific to the context and problem (Rouse and Zimmerman, 2017; Zhang, 2018). This may be because the models use a specific type of dataset where much of the information might be missing. For example, a model that focuses on the patient flow to uncover the wards with significant bottlenecks may ignore the bottlenecks from the staff's flow perspective. In parallel, to reuse a model, we need to keep it sufficiently detailed and be cautious about how we process this data (Balci et al., 2017; Fujimoto and Loper, 2017). The more detail we add, the harder it becomes to reuse a model (Balci et al., 2017). For a model to be reusable, it should easily enable and disable its functions or add new functions. This can be challenging without ensuring that the model will continue to perform accurately (Rouse and Zimmerman, 2017). Such models should be modular, independent of the system structure and its data.

1.3 Overcoming Modelling Challenges

Complex societal systems keep evolving and affecting how the rest of the elements behave. The elements in such systems can represent the factors affecting society's health. The procedure requires quantification of the elements and links. Ranking algorithms are suitable for quantifying the elements in a system with a network structure. However, links in societal systems have negative and positive values, which require us to tune the algorithm to avoid outliers (Bressan and Peserico, 2010). To quantify the links in complex societal systems, we used gaming simulation. There are several reasons for this. First, the change in the link's value steers the elements' behaviour. Second, by considering human factors (Zhang et al., 2018), we use their experiences to understand the system from their perspective (Bekebrede et al., 2015a,b). Third, such quantification depends on the living environment. Finally, the ranking algorithm quantifies the elements based on the degree of connectivity and the links' values. The data collected from the game and the ranking algorithm are applied to create a hybrid quantification approach for complex societal systems. In parallel, to observe the dynamic behaviour of the elements in a societal system, hybrid modelling approaches allow combining the quantification approach with methods such as path analysis or system dynamic simulation. Path analysis allows us to observe the significance of elements within a path from local and global perspectives when a path is isolated from the whole system. However, by combining the quantification approach with system dynamic simulation, it becomes possible to observe the behaviour variation based on internal and external changes in the elements' ranks.

Hospitals are another type of complex health system. The elements in such systems can represent the wards in a hospital. The behaviour of one ward can influence other wards. Similar to complex societal systems, complex hospital systems also evolve. The data for such systems comes from hospital records. To observe the dynamic behaviour of the wards in a whole hospital, flow algorithm (Williamson, 2019) or agent-based network simulation can be used. To adjust the flow algorithm to capture the dynamic behaviour of the elements, the algorithm is modified to encompass all the necessary parameters and the continuous nonlinear behaviour of the elements through residual graphs. In parallel, agent-based network simulation also allows to model the patient flow throughout the hospital based on different criteria, rules and regulations. Bottlenecks in such systems happen due to structural limitations and flow intensity. Hence, a hybrid approach which uses structural hole (Burt, 1995) and flow algorithm in parallel can uncover the bottlenecks from two different aspects of how hospital pathways are structured and how the patients flow. Furthermore, the application of percolation (Piraveenan et al., 2013) and perturbation algorithms (Yadav and Babu, 2012) allows us to analyse the behaviour variation as a hospital evolves by measuring the divergence of the overflow state in the wards.

The organizational level of a health system can also be viewed as a complex system. The elements in such systems can represent different roles and characteristics within an organization, such as the wellbeing of the individuals. Such systems have similar objectives and elements but may vary in the number of elements and their connectivity. Hence, to define the topology for each one of these systems, both global and local link prediction approaches are implemented (Chen et al., 2020; Aziz et al., 2020; Kumar et al., 2020), where the global method uses Bayesian network approach to predict the overall topologies of the networks, based on survey data, and the local method combines the path analysis and non_randomness algorithm to further enhance the topology by predicting the missing links based on a reference network. By integrating both methods, the approach ensures that the networks' topologies accurately represent the real-world systems (Wang and Le, 2020; Chen et al., 2020). To evaluate the performance of these organizations, a multi-network simulation approach is used to observe and compare their behaviours in parallel.

1.4 Thesis Contribution and Structure

In general network modelling approaches facilitate the quantification, representation, simulation, analysis and validation of the evolving complex systems. This is because it requires to model the whole system and their influences. These approaches are widely used to model and analyse the performance of systems such as telecommunication and transportation for decision-making and optimization. However, health systems have different types and functionalities and deal with different types of problems. Hence, the methods and algorithms being used to model such systems should be adjusted to model the complex health systems accurately.

The contribution of this thesis is to propose network-agnostic modelling approaches which facilitate the flexibility of these models in order to easily adapt to the change in the system, network scale, type, and topology and further be able to adjust the model with other features and functionalities. This is to ensure that the approaches can be applied and integrated to model and analyse other complex health systems, overcome the limitations, and enhance the verification and validation. Therefore, merging, modification, tuning, and gaming approaches are used to adjust the network algorithms and simulation methods in order to facilitate the modelling approaches.

The remaining chapters in this thesis will cover the background, methodology, method, contribution, and the discussion and conclusion. The background chapter discusses the modelling of complex health systems, the application of network algorithms, methods and their limitations. and reflects on the modelling challenges of the complex health systems. Next, the methodology chapter provides information about the research methodology, worldview and questions, and the technical details used for development. Then, the method section describes the network types, provides an overview of the procedures, explains the proposed modelling approaches with corresponding input variables, and the verification and validation. To that end, the thesis reflects on the contribution of the approaches and finalizes the thesis with a discussion and conclusion.

2 Background

2.1 Modelling of Complex Health Systems

Health systems are complex and consist of various interconnected elements, which continuously interact and influence each other and the overall system's behaviour (Djitog et al., 2018; Zhang et al., 2018; Aboueljinane and Frichi, 2022). These systems can be modelled as a network where the elements are the vertices and their connectivities are the edges. Examples of health systems can be a societal system representing elements affecting wellbeing, a system representing elements as wards in a hospital, or a system of primary care, hospitals and social care covering an entire region.

The purpose of modelling is to observe the dynamic behaviour of the complex systems and elements' behaviour patterns based on certain kinds of changes (Cassidy et al., 2019). These changes may cause unpredictable behaviours and perturbations and can even cause catastrophic system failure (Rutter et al., 2017; Tang et al., 2021; Bhattacharjee and Ray, 2014; Wu et al., 2015). Holistic modelling allows us to observe the element's behaviour variation within such complex systems based on various scenarios to ensure the stability, improvement or perturbation and plan ahead (Bhattacharjee and Ray, 2014; Zhang et al., 2018; Rutter et al., 2017). Holistic modelling captures the whole system behaviour and provides realistic results by identifying the potential effect of multiple influences on the dynamic of the system (Rutter et al., 2017; Djitog et al., 2018; Wu et al., 2015; Zhang, 2018). However, such modelling has rarely been implemented due to the system complexity (Günal and Pidd, 2010). To understand the key elements of complex health systems, we need to be able to observe the whole system's performance(Gunal and Pidd, 2005).

One of the challenges to model the whole system is the elements' role diversity. In a complex health system, elements are similar in nature but have diverse roles (Ladyman et al., 2013; Djitog et al., 2018; Wu et al., 2015; De Brún and McAuliffe, 2018). In some systems, elements may serve similar functionalities with various parameters and influences, whereas in other systems, various functionalities have similar parameters. To properly model a complex health system, data is required to quantify all the elements' roles and connectivity, which is also another major challenge in the holistic modelling of health systems. These data are collected from past records that can have missing information, where the system may find it unnecessary to collect all types of data (Zhang, 2018; Carter and Busby, 2023; Djitog et al., 2018). Furthermore, as health systems evolve, models need to be able to adapt to these changes without further modifications and dependency on data (Günal and Pidd, 2010; Batty, 2015) or the role the elements represent. A flexible modelling approach can facilitate the model construction to capture the system behaviour (Günal and Pidd, 2010) and overcome the difficulty of generalizing (Günal and Pidd, 2010; Zhang, 2018; Havlin et al., 2012).

Despite the evolving and dynamic nature of complex health systems, it is crucial to determine boundaries when modelling the whole systems (Günal and Pidd, 2010; Chambers et al., 2012). Reductionism and decomposition approaches are not appropriate for analysing complex systems because focusing only on the effect of key elements and their connectivity can raise invalid output (Aboutaleb and Monsuez, 2015; Wu et al., 2015; Boes et al., 2015; Kar et al., 2022). Decomposition is also not wise, as the selection of certain elements and connectivities and disregarding others can raise validity issues in behaviour, making it difficult to see the overall system behaviour (Boes et al., 2015; Kannampallil et al., 2011). On the contrary, increasing information in modelling can provide a more realistic output but also increases the complexity, which can make it harder to validate a model and can be time-consuming (Günal and Pidd, 2010; Zhang, 2018). Therefore, when limiting the boundaries, we need to take into account whether the impact of these neglected elements on the rest of the system is insignificant (Havlin et al., 2012; Chambers et al., 2012).

2.2 Simulation Methods and Limitations

Discrete-event simulation is one of the primary and widely used methods to model health systems (Barjis, 2011; Zhang et al., 2018). The method is based on a queueing model, allowing information to flow from one element to another. This information can be agents or resources that enter the system based on a certain arrival rate, flow or reside through the links known as queues, take different paths, reside inside elements for a certain amount of time, and then leave the system.

Modelling agents in complex health systems can help to understand their interaction and flow to understand the system performance and uncertainty (Zhang et al., 2018). Such modelling approaches are used when the focus is on the critical elements' behaviour in the system, (Zhang et al., 2018; Cassidy et al., 2019; Zhang, 2018), and are useful when we already know what causes the issue and where the source of the issue is (Zhang, 2018; Mustafee et al., 2015). The limitation of this modelling method is that it provides a limited view of the system and leads the decision-makers' attention to where the problem appears and not from where the problem may originate. Similarly, queue modelling measures the system behaviour based on the information flow according to how the queue is embedded and its disciplines. Such modelling approaches can provide a limited overview of the system performance with less data requirement (Green, 2006).

Modelling agents' movements can also help to understand the environment they interact with, which helps to analyse the system's physical structure (Zhang et al., 2018) rather than the system itself. Unlike discrete-event simulation, where the agents flow between elements through queues, agent-based simulation is where agents are modelled to explore the physical environment and are restricted by behavioural rules and environmental constraints. Agent-based modelling allows understanding the agents' behaviours, distribution and interaction (Zhang et al., 2018). However, merging Agent-based modelling with system dynamic simulation enhances holistic modelling for strategic decision-making and analyse the system from different aspects (Cassidy et al., 2019; Mustafee et al., 2017).

System dynamic simulation models the behaviour of the elements and overall system performance based on changes in the system caused by elements' interaction or evolution (Brailsford et al., 2010). Such methods provide insight for decision-makers from local and global perspectives (Zhang et al., 2018) to identify the element's behaviour patterns and their variation. Unlike discrete-event simulation, which analyses the system performance based on information flow between elements, system dynamic simulation analyses the system performance based on the interaction between elements (Brailsford et al., 2010).

Simulation methods have limitations in capturing the whole system accurately. Hybrid simulation methods increase the modelling efficiency and reduce the limitations (Zhang et al., 2018; Mustafee et al., 2017; Boareto et al., 2022; Brailsford et al., 2019), where each method contributes to achieving a significant outcome and more detailed information about the underlying problem from different aspects (Fakhimi and Probert, 2013; Mustafee et al., 2017; Blanchet and James, 2012). However, merging can be challenging due to the increase in data requirement and difficulty in verifying and validating the model (Wu et al., 2015), and rarely used to model complex health systems (Cassidy et al., 2019).

The challenge with hybrid modelling is that it requires more data and a proper way of merging a method with another method or algorithm (Barjis, 2011) to accurately capture the complexity and the behaviour of elements or agents in the system. In many cases of hybrid modelling, modification and adjustment will be required to embed one method or algorithm into another. Implementing a hybrid model can take time and may even increase the difficulty of validating and the model complexity (Petty, 2018). Hence, knowing how a simulation method and an algorithm functions (Crielaard et al., 2023), what the target system represents, and the underlying problem will help to select, merge and modify the algorithms and methods accordingly (Kannampallil et al., 2011; Howison et al., 2011).

2.3 Network Algorithms for Modelling and Analysis

Network representation of complex health systems makes it easier to identify the elements as vertices and the connectivity as edges (Ladyman et al., 2013; Rathkopf, 2018). This representation allows us to have a holistic view and analysis of the system (De Brún and McAuliffe, 2018; Rathkopf, 2018), such as how densely certain vertices are connected. In parallel, the application of network algorithms to analyse such systems has increased significantly, which allows the identification of the important vertices and edges and the perceive the structure based on the network topology on certain parameters (Tang et al., 2021; Blanchet and James, 2012; De Brún and McAuliffe, 2018; Chambers et al., 2012). Network algorithms' primary development purpose has been to analyse telecommunication and transportation networks for decision-making and systems optimization (Havlin et al., 2012). The aim of these algorithms is to quantify the network vertices or analyse the paths from different perspectives for different purposes (Havlin et al., 2012; Howison et al., 2011; Arul et al., 2023; Wu et al., 2015).

Unlike simulation methods, which require procedural and contextual implementation, network algorithms are ready and available, making it easier to analyse systems quickly (Crielaard et al., 2023). Network algorithms can help to understand how internal and external influences affect a health system (Blanchet and James, 2012; De Brún and McAuliffe, 2018; Wu et al., 2015) because they are sensitive to data or topological changes (Crielaard et al., 2023; Kannampallil et al., 2011; Havlin et al., 2012). However, these algorithms are static and unable to capture the dynamic behaviour of complex health systems (Blanchet and James, 2012; Petty, 2018). The dynamics of a complex system can cause perturbation or improvement in system performance (Kannampallil et al., 2011; Ladyman et al., 2013). Complex systems are not at equilibrium, and the vertex's behaviours are unstable by influencing each other (Havlin et al., 2012; Kannampallil et al., 2011); this makes it difficult to rely on algorithms which cannot capture the dynamic. Some network algorithms have been adjusted to capture dynamic behaviour. However, these adjustments are for certain systems and data requirements, which cannot be reapplied to model how the health systems behave (Fakhimi and Probert, 2013; Havlin et al., 2012).

The problem in using network algorithms is that the way the function is implemented is inconsistent with the underlying complex system (Howison et al., 2011). Applying them to analyse health systems without understanding how these algorithms function can provide invalid output (Crielaard et al., 2023; Howison et al., 2011; Wu et al., 2015). Network algorithms are unable to identify what the system represents except the topology of the network and the position of the vertices unless proper and sufficient data are provided (Howison et al., 2011; Wu et al., 2015). When analysing complex health systems, even the most distant and weakly connected vertex can have a significant role and influence on the rest of the network. The issue is not the boundaries of the system but how the algorithm handles these networks. First, many network methods have embedded path searching and aggregation algorithms that neglect the signed edges or only consider the shortest or longest paths. This can alter the output we intend to observe (Crielaard et al., 2023; Howison et al., 2011). Second, many of these algorithms ignore the continuity of some parameters and reset the network state after each iteration within the algorithm (Howison et al., 2011). Third, some of these algorithms also ignore the edges' directions, which can be problematic when measuring the proper vertex influences (Crielaard et al., 2023) since a vertex with no outgoing edge can influence the rest of the system behaviour through its incoming edges. And finally, fourth, network algorithms may have insufficient input and output parameters to function properly (Ilich, 2009; Crielaard et al., 2023), such as multiple values for vertices and edges. In a complex health system, a vertex may have more than one value to define its behaviour, and the edges may have more than two parameters to define the influence and steer the vertices' behaviours.

Network analysis algorithms are sensitive to the topology of the networks, and these topological properties are handled differently by different network algorithms (Havlin et al., 2012; Anderson, 2002; Howison et al., 2011). Complex health systems encompass all sorts of topological properties (Kannampallil et al., 2011; Havlin et al., 2012) such as vertices with no incoming or outgoing edges, cycles of vertices which link back to each other, and strong and weak ties. Many network algorithms that aim to quantify the important vertices may end up sinking into a vertex with no outgoing edges and identify the vertex as important. Such vertices have no influence on the rest of the system because they lack outgoing edges. Using the results of the analysis for decision-making will have no impact on the health system. Such algorithms should be used as a measurement tool to observe the vertices' behavioural variations, not the vertices' importance. On the contrary, observing any behaviour variation in vertices with no incoming link would be impossible since there is no way to influence them. In parallel, cycles are another issue in algorithms which rely on path searching and aggregation algorithms. Network algorithms can sink into these cycles and endlessly iterate over them. In a complex health system, where information flow begins from one vertex to another, such cycles may prevent the flow from reaching the intended vertices. As a result, this may indicate a bottleneck in the system where, actually, it is the cycle which prevents the flow from proceeding. Lastly, all types of networks have weak and strong ties with the rest of the network. The application of network algorithms to identify these ties depends on the purpose of the system and analysis. In a network which represents a complex societal system where the vertices interact, identifying the strong ties is crucial to making significant influences, whereas in a network which represents a complex hospital system, identifying the weak ties is crucial to understanding the structural bottlenecks (De Brún and McAuliffe, 2018).

2.4 Complex Health Systems Modelling Challenges

Using network algorithms in modelling complex health systems facilitates holistic modelling with the flexibility to adapt, reuse and validate the behaviour of evolving systems. This is because network algorithms are ready to analyse the systems, which helps fill modelling gaps. However, it is necessary to understand how the target complex system, simulation methods and network algorithms function to ensure the model behaviour matches the system behaviour and be able to verify and validate the model and output.

Ranking algorithms facilitate the quantification of such networks (Page et al., 1998; Brin and Page, 1998) and many of them such such PageRank, HITS and SimRank algorithms have sufficient input parameters to tune and adjust the algorithm according to the underlying system. For example, to avoid outlier values, the algorithm's damping factor and the number of iterations can be tuned to avoid such values or the model running endlessly. Furthermore, ranking algorithms can maintain adaptive behaviour due to a change in one part of the network can affect the rest of the network and can also be verified whether the sum of the ranks in the whole network is equal to 1 (Page et al., 1998; Brin and Page, 1998). Such properties allow us to quantify complex systems as they evolve.

Ranking algorithms treat vertices in a complex network equally and quantify them based on their topology, position, and incoming edge values. Path analysis algorithms allow isolating certain edges from the rest of the network to observe the vertex's behaviour stability from different path perspectives. This allows us to ensure that we interpret the outcome correctly and that the identified key vertices are valid. Additionally, these algorithms can be easily merged with other simulation methods, such as agent-based modelling and system dynamic simulation, to quantify the agents or vertices. Such hybrid modelling approaches help to observe the rank variation and classify vertices behaviours (Petty, 2018). Behaviour classification allows further validate the network, which represents the system based on peculiar changes such as missing vertex or edge (Howison et al., 2011). Hence, these approaches can facilitate the holistic modelling of complex health systems based on external influences, such as additional vertices, sub-systems or systems, or internal influences when the relation between two vertices changes.

Like complex societal systems, complex hospital systems also evolve, encounter internal and external influences (Tang et al., 2021; Ladyman et al., 2013; Martínez-García and Hernández-Lemus, 2013), and lack data (Barjis, 2011; Carter and Busby, 2023). In networks where the flow carries network topology and the flow rate, using two different network algorithms, such as structural hole (Burt, 1995) and flow algorithm in parallel, can provide a more valid and reliable outcome for decision-making (Petty, 2018). This further ensures that the algorithms' results are not due to the properties such as cycles and lack of incoming and outgoing edges. However, as discussed, such algorithms are static in nature, and even if there are enhancements in algorithms such as dynamic flow, the enhancements are not intended to capture the dynamic of the complex health systems.

When a complex health system evolves, there can be perturbation or improvement in the overall system's performance. Network algorithms are sensitive in capturing perturbation. However, many network algorithms ignore the behaviour variation in the vertices of a network. Complex health system dynamics can be captured using percolation algorithms (Havlin et al., 2012). The percolation algorithm fills in the existing limitations in centrality algorithms (Piraveenan et al., 2013) to capture the vertices' influence on the system (Crielaard et al., 2023). The divergence of percolation states in vertices of a network allows us to measure the performance of complex health systems.

The evolving nature of complex organizational systems can influence the network representation of such systems. This can result in health organizations with similar aspects having divergent structures. For network representation of such systems, Bayesian network approaches (Holmes and Jain, 2008; Ankan and Panda, 2015) can predict the topology of these networks based on individual organizational data, and non_randomness (Ying and Wu, 2009) algorithm can increase the validity of the network representation by identifying the missing links in each network topology compared to a reference network.

This requires a thorough analysis of the edges of complex health networks with the consideration of the health aspects the network represents, which limits the application of generic network algorithms. On the other hand, the path analysis of complex health networks depends on the purpose of the analysis and the network type. To determine whether all the paths, the longest path, or a predefined length of a path is required for the analysis, we need to understand if the aim of the analysis is to find the persistent vertices, bottlenecks, or the ill structure of network topology.

As a result, new modelling and analysis approaches which employ network algorithms, can facilitate the holistic modelling and analysis of complex health systems as they change and evolve. However, it requires proper adjustment to avoid invalid results and interpretation and risky decisions (Howison et al., 2011; Havlin et al., 2012). While these approaches enhance the modelling, the accuracy of the network representation of health systems plays a significant role in the validity of the model output.

Various verification and validation techniques can be applied to ensure the accuracy of the network, the model and the output behaviour (Sargent, 2013). The suitability of the techniques depends on the case study, system type, and model purpose. These techniques can be through applying different algorithms in the model, tracing codes programmatically and manually, comparing models, and using domain experts' knowledge. Hence, to accurately model the complex health systems, network topology, model functionality, input and output variables, and their structure should be taken into consideration.

To summarize, first, the application of network algorithms to model and analyse complex health systems has significant limitations. This requires proper selection and adjustment of these algorithms with the consideration of the complex system, system behaviour, and network type. Second, the evolving and changing nature of these systems needs to be captured in the network representation and model. Third, ensure that sufficient input and output variables are used within the model. Finally, fourth, the model has the degree of flexibility to be applied and enhanced for other health systems.

3 Methodology

Three case studies focusing on different complex systems are used to study the health systems. The first case study focuses on complex societal systems, the second case study focuses on complex hospital systems, and the third case study focuses on complex organizational systems. The chapter first covers the research methodology and the worldview of the doctoral study; next, identifies the research questions and the motivation behind it; then, the research methodology procedure through a diagram; and finally, implementation detail for development such as the tools and the library usages.

3.1 Research Methodology and Worldview

The purpose of this PhD study is to offer various network-agnostic computational approaches to model the dynamic behaviour of evolving complex health systems. Each proposed modelling approach consists of either a network simulation method, a network algorithm or a combination thereof. The choices depend on the network type, the underlying research problem, and the target complex health system. The research methodology follows a postpositivism worldview where it offers various approaches to determine where and what influences the outcome and assesses them to understand the real-world systems' performances better. Each method and algorithm aims to handle various underlying issues by partitioning them into small parts. For instance, network algorithms can enable quantification, structural analysis, dynamic analysis, performance analysis, verification and validation, and crucial vertices identification. In contrast, simulation methods or dynamic algorithms can facilitate the modelling of the dynamic behaviour of complex health systems. Such approaches, which follow the postpositivism worldview, also require numerical data to enable precise measurement and observation of the output behaviour. Thus, to identify and assess the significance of causes and effects, the input data should be quantitative or need to be quantified through objective approaches. The aims of these approaches are to:

- Facilitate the quantification of vertices and edges in an evolving complex system through ranking approaches.

- Identify and classify the crucial vertices based on their behaviour variation and stability.
- Enhance the computational verification and validation of the model and the output behaviour.
- Evaluate the performance of the evolving complex system.

To break down the research problem, first, input variable types and data availability are considered. Where available, quantitative input data are adopted. Where data is unavailable, ranking algorithms and gaming simulation are used to overcome the data unavailability. Quantitative data are numerical values which are measured through ranking approaches and stored in files or databases. Next, network algorithms help to model and analyse the dynamics and topologies of these systems to identify the crucial vertices, edges, and sources of causation. Simulation methods or dynamic network algorithms are used to observe the vertices' dynamic behaviour and evaluate the performance change and stability.

3.2 Research Questions

This thesis aims to answer the following research questions:

1. What is the role of network algorithms and simulation methods in the modelling and analysing complex health systems?

   The representation of complex health systems as networks facilitates the application of network algorithms and network simulation methods to model and analyse these systems. This thesis explores the suitability of network algorithms and simulation for different types of health systems and suggests proper adjustment to facilitate dynamic modelling and analysis.

2. What contributions network modelling approaches can make towards the holistic modelling of complex systems?

   Holistic modelling aims to capture the behaviour of all the elements in a system with respect to the complexity and connectivity of its parts. The thesis proposes new network modelling approaches by investigating the functionality and requirements of various complex health systems and the suitability of network algorithms and methods in order to adjust them to model such systems.

3. What are the roles of network algorithms in promoting verification and validation of models and output behaviour?

   The verification and validation of simulation models are challenging. This is due to the uncertainty of the network representation, model structure and input data accuracy. The thesis represents the features and proper application of network algorithms to promote verification and validation.

3.3 Research Methodology

The thesis uses three case studies, one in mental health and wellbeing, one in a hospital and the last in social and home care, to explore the research questions. Figure 3.1 illustrates the research procedure followed to accomplish each case study. The research begins by first formulating research questions and then selecting a case study. Each case study begins by first receiving the network representation of a complex health system and the corresponding data. This allows us to perceive the following:

1. The network type.
2. The required input and output variables to define.
3. The problem required solving.
4. The limitations.

Perceiving this helps to review the relevant literature to identify potential network algorithms and simulation methods. Additionally, the literature review helped to learn about the previous research conducted to solve similar problems and uncover the methodological limitations and the literature gaps.

As mentioned, three different case studies are used to explore network modelling approaches. The first case study aims to model an evolving complex societal system, the second case study aims to model an evolving complex hospital system, and the third case study aims to model an evolving organizational system:

- **Complex societal system:** In this case study, the provided network represents the system describing the mental health and wellbeing of children and young people. Literature was explored to find data on the factors that form the vertices of the network, and because limited data was found, a simulation game was developed to rank the edges in the network. Next, a ranking algorithm is used to quantify the vertices in the network. Merging the ranking algorithm with path analysis helps to identify the behaviour stability whereas merging the ranking algorithm with system dynamic simulation helps to classify the vertices' behaviours.
- **Complex hospital system:** In this case study, the provided network, analysed by Marzano et al. (2024) based on Akademiska University Hospital Data), represents hospital wards and care pathways between the wards. Two modelling approaches are proposed for this case study. The first modelling approach uses agent-based network simulation to understand the wards' behaviours and network algorithms to identify the weaknesses and evaluate the wards' performances. The second modelling approach modifies the flow algorithm to solely model the dynamic nonlinear behaviour of the patient flow without employing a simulation method.

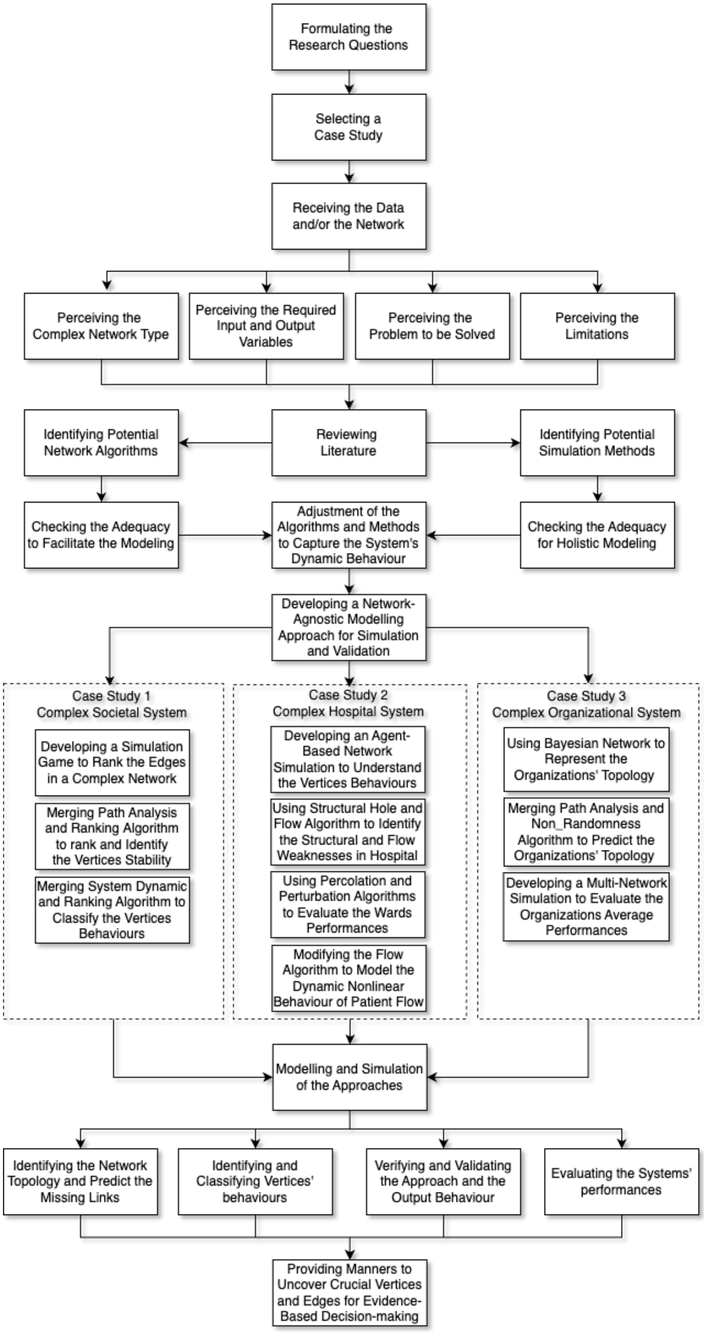

Figure 3.1: Methodology Flow Diagram.

- **Complex organizational system:** In this case study, an approach is proposed to generate networks for each home-care organization. The approach first uses a Bayesian network and then merges the non_randomness algorithm with path analysis to represent these networks by predicting the links. Additionally, a multi-network simulation approach is used to simulate the organizations' networks in parallel and evaluate their average performances.

To that end, the modelling approaches are developed to simulate the networks and classify the vertices' behaviours. This allows us to identify the crucial vertices, evaluate the systems' performances, and verify and validate the model and the output behaviour. To conclude, the proposed approaches provide a manner to uncover the crucial vertices and edges for evidence-based decision-making.

3.4 Implementation

All modelling approaches are developed using Python programming language. Python packages and frameworks are used to facilitate the implementations of models. For instance, Networkx (Hagberg et al., 2008) is an open-source Python package with a wide list of network algorithms. The library allows us to search and explore the source code of these algorithms in order to perceive the application and limitations and further be able to adapt, tune, adjust, and merge them properly with other algorithms and network simulation methods. The package simplifies the construction, manipulation and elimination of the networks. Additionally, queueing-tool (Jordon, 2023) is used to implement the agent-based network simulation for a hospital, which is an open-source Python package, that facilitates the implementation of queues, attributes, and agents and inherits classes for more specific network simulation and analysis purposes. Finally, the Flask micro framework (Grinberg, 2018) is used to implement the SocioBalance web-based simulation game. In parallel, other Python packages are used to enhance the modelling and model validation, which are the Fitter library (Cokelaer, 2023) used to estimate accurate distribution functions, and pgmpy library (Ankan and Textor, 2024) to implement the Bayesian Network approach for network generation from the survey.

4 Method

This chapter presents the approaches proposed and developed during the PhD study. For that, first, the types of networks which are provided or generated are explained. Next, the chapter provides an overview of the approaches' procedures, introduces the input variables and the structures, describes the approach development, and then the necessary steps taken to verify and validate various aspects of the model.

4.1 Network Types

Each case study uses different types of networks. The following points explain the types these networks represent:

- **Interaction network:** In such networks, each vertex has a different role with different impact strength on each other where the edges represent the vertices' interactions. A complex societal system, represented as a network previously developed by Raghothama et al. (2025a,b), is used as a case study. The vertices in this network represent the factors affecting children and young people's mental wellbeing, whereas the edges represent their direct and indirect connections. This network is used as a case study for the approaches proposed in paper I to quantify the edges, in paper II to quantify and identify the most influential vertices, and in paper III to classify the vertices behaviours.

- **Flow network:** In such networks, vertices have similar roles where information and patients flow among vertices through the edges. A complex hospital network, which represents the Akademiska University Hospital in Uppsala (Marzano et al., 2024), is used as a case study. The vertices represent the wards in the hospital, and the edges represent how the patients flow throughout the hospital. This network is used as a case study for the approaches proposed in paper IV using agent-based network simulation and network analysis and V through the modification of the flow algorithm. The aim is to model the patient flow and evaluate the wards' performances.

- **Multiple interaction networks:** Such networks have a similar nature to the interaction network, with the difference that there are replications of these vertices with different topologies and weights. Networks which represent similar complex organizational systems are used as a case study. The vertices in each network represent the wellbeing of the staff and users of each home-care organization. A link prediction approach is proposed in paper VI to generate network topology based on the survey data (Boström et al., 2022) and a reference network (Darwich et al., 2023) to represent various complex organizational systems of the home-care properly.

4.2 Overview

Figure 4.1 illustrates a flowchart to provide a holistic view of the approaches followed for all the case studies. This involves importing input data, network generation, quantification, algorithms and methods adjustment, approach development, and approach simulation and validation.

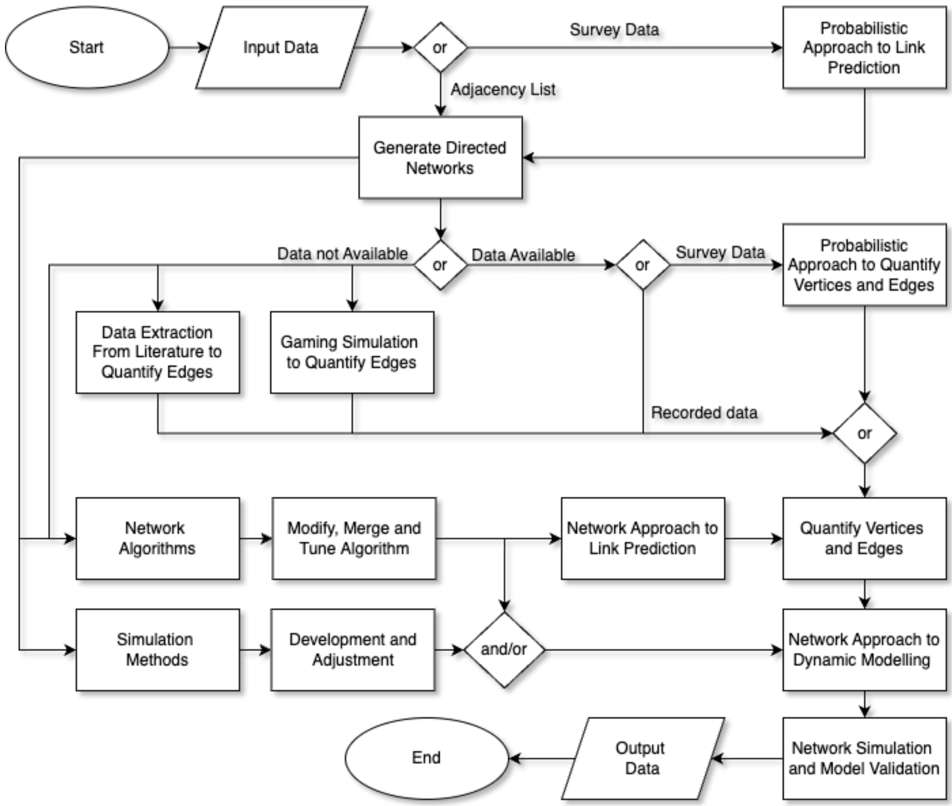

Figure 4.1: Flowchart for all Approaches.

The procedure can be divided into the following steps:

- **Network Generation:** For network representation of a complex system, input data should indicate the vertices and their connecting edges. This requires an adjacency list to generate a directed network. Hence, in the case of societal and hospital systems, inputs were adjacency lists to generate a directed network. However, in the case of organizational systems, a probabilistic approach (Bayesian network) is used to identify the links based on the survey data.

- **Network Quantification:** Next, the network requires quantification. This is to understand the vertices behaviours, edges impact, and the system performance. Due to data unavailability, gaming simulation or literature data are used to quantify the edges of a societal network, and a ranking algorithm to quantify the vertices. Whereas to quantify the hospital network, previously recorded and analysed data are used (Marzano et al., 2024). However, to quantify the organizational networks, a probabilistic approach (Conditional Probability Distribution) is used to calculate the vertices' and edges' weights based on the survey data.

- **Network Modelling:** Network algorithms are used for quantification (ranking algorithm), link prediction (to further enhance the organizational networks topologies) and dynamic modelling. To develop the dynamic models, network algorithms are adjusted through modification, merging (with simulation methods) and tuning. In parallel, necessary adjustments are made to simulation methods to model the system properly. Next, to ensure that the networks generated through probabilistic approaches are valid, a link prediction approach is developed to enhance the topologies. Finally, the networks are quantified with the data collected through quantification approaches or previously recorded data.

- **Simulation and Validation:** After accomplishing the aforementioned step, network approaches for dynamic modelling are proposed to simulate and validate complex health systems. Then, network simulation and validation are performed to extract the output for further analysis. The outputs are adjacency lists, which store the vertices' performances per change, and adjacency matrices, which provide a holistic view of the complex health systems.

4.3 Modelling Approaches

To develop the network modelling approaches, necessary input variables are defined to construct the network topology and data structure dynamically within the model. This will provide the freedom to model networks agnostic of the scale, type, topology and data. The first step to model these systems is to create a directed graph to represent their topology. This will help to observe the edges' directions and vertices' degree of connectivity. The second step is to quantify the vertices and edges to perceive the state at which a vertex is and the range for variation in behaviours. Finally, relevant influencing factors are required to properly capture the dynamic behaviour of the system. Table 4.1 represents these input variables structure based on the network type. As a result, the societal, hospital and organizational networks all have only one list of vertices. However, in case of societal and hospital networks, all the vertices $\{V_i\}_{i=1}^n$ and edges $\{E_i\}_{i=1}^m$ which are in the list V and $\{(U_i, V_) : U_i, V_i \in V\}$ are present in the network, whereas in case of organizational network, individual organizations can have various amount of vertices $\{V_i^j\}_{i=1}^{n_j}$ but all derived from one list of vertices $\{V_i\}_{i=1}^n\}$. The presence of a vertex depends on individual organization performance. On the contrary, the way these vertices interact varies, which provides a unique list of edges $\left\{\{E_i^j\}_{i=1}^{m_j}\right\}_{j=1}^k$ to represent each organization $\{V_i\}_{i=1}^n\}$. The data to assign values to these input variables are defined either by the provided adjacency lists or generated based on the survey data.

Table 4.1: Input Variables.

Purpose	Societal	Hospital	Organizational
Vertices	$\{V_i\}_{i=1}^n \subseteq V$		$\{V_i^j\}_{i=1}^{n_j} \subseteq \{V_i\}_{i=1}^n$
Edges	$\{E_i\}_{i=1}^m \subseteq \{(U_i, V_i) : U_i, V_i \in V\}$		$\left\{\{E_i^j\}_{i=1}^{m_j}\right\}_{j=1}^k \subseteq \{(U_i, V_i) : U_i, V_i \in \{V_i\}_{i=1}^n\}$
Vertices Weights	$\{C_i\}_{i=1}^k$	$\{S_{v_i}^l\}_{i=1}^n$, $\{DF_i\}_{i=1}^n$, $\{B_i\}_{i=1}^n$, $\{D_i\}_{i=1}^n$	$\left\{\{P_i^j\}_{i=1}^{n_j}\right\}_{j=1}^k$
Edges Weights	$\{W_i\}_{i=1}^m$	$\{DP_i\}_{i=1}^m$, $\{F_i\}_{i=1}^m$	$\left\{\{W_i^j\}_{i=1}^{m_j}\right\}_{j=1}^k$
Other Factors	α	$\{R_i\}_{i=0}^t$, $\{\{P_{ij}\}_{j=1}^n\}_{i=1}^m$, $\{\{AP_{ij}\}_{j=1}^n\}_{i=1}^m$	I, α

The rest of the variables are the list of vertices' sub-systems, vertices' weights and edges' weights based on the data availability, and other influencing factors based on different modelling purposes. The following subsections will provide more details regarding the implementation of the proposed modelling approaches and the application and computation of these input variables within the models.

Modelling of Societal Network

In the first case study, three separate models are developed. First, a gaming simulation approach, called SocioBalance, is developed to quantify the edges (paper I). The game is developed to quantify the edges in a network by ranking their significant impacts. The objective of the game is for the players to identify the significance of the given vertices based on the given scenario and arrange them based on their direct impacts.

The approach development begins by preparing a database and importing the vertices $\{V_i\}_{i=1}^{n} \subseteq V$, vertices sub-systems $\{C_i\}_{i=1}^{k}$ and edges $\{E_i\}_{i=1}^{m}$ of the mental wellbeing network into the database. Then the vertices are clustered into different sub-systems, where each vertex V_i belongs to a specific sub-system C_k where there is no intersection (eq. 4.1):

$$V_i \in C_j, \quad j = 1, 2, \ldots, k, \quad C_i \cap C_j = \emptyset \quad \text{for } i \neq j \tag{4.1}$$

The ranking of the vertices depends on the arrangement of the predecessors of each vertices. For that, the algorithm checks to ensure that each vertex has at least two incoming edges $d^{-}(v) \geq 2$. Hence, vertices with less than two incoming edges are eliminated (eq. 4.2):

$$V' = d^{-}(V_i) \geq 2, \quad \text{otherwise } V_i \text{ is eliminated} \tag{4.2}$$

Next, to rank the edges based on the players' arrangement, the algorithm represents only the predecessors U' of the selected vertex V_i' (eq. 4.3):

$$U' = \{U_j \in U \mid (U_j, V_i') \in E\} \tag{4.3}$$

Depending on the arrangement of the predecessors U' by each player, a rank R_l is provided as the edges' weights $W(U_l', V_i')$ which directly links the predecessors to the selected vertex. The rank begins from 1 to the number of the predecessors (U') (eq. 4.4):

$$W_{U_l', V_i'} = R_l, \quad l \in \{1, 2, \ldots, |(U')|\} \tag{4.4}$$

After each round of ranking, the algorithm selects the incoming edge with the highest provided rank, as a new vertex to quantify its incoming edges (eq. 4.5):

$$V_i = U_i, \quad \text{if } f(W_{(U_i, V_i')}) = 1 \tag{4.5}$$

The game allows collecting various ranks for each edge based on the number of players and their choices within the game. Next, to quantify the edges, the weighted average approach is used, and the data is stored in $\{W_i\}_{i=1}^{m}$ variable.

After quantifying the edges, in order to observe the vertices' behaviours, in the first approach PageRank is merged with path analysis (paper II). The approach iterates over all the vertices and finds all the connecting directed paths between pair of vertices (V_i, V_j) and constructs a temporary directed graph from it DG' (eq. 4.6):

$$DG' = \bigcup_{V_i, V_j \in V} \text{DirectedPaths}(V_i, V_j) \tag{4.6}$$

This allows to isolate these paths from the rest of the network and then measures the vertices ranks which are located within the DG'. This helps to identify the local most influential vertices (eq. 4.7). To identify the global most influential vertices, the whole DG is used to measure the rank without path analysis (eq. 4.7). This allows us to observe the persistency of the vertices ranks from the global and local network perspectives. This is because, in an evolving complex system, vertices may join the network, which may suppress the popularity of the influential vertices. Damping factor α is tuned accordingly to avoid rank-sink, and PageRank is used to handle negative weights.

$$\text{Local: PageRank}(DG'_{(V_i,V_j)}, \alpha), \quad \text{Global: PageRank}(DG, \alpha) \tag{4.7}$$

In the second approach to model the behaviour variation based on the changes in the edge weights and network scale, we proposed a new approach which merges the PageRank with system dynamic simulation (paper III). The model procedure begins by iterating over the vertices V, identifying its incoming edges, and manipulating the weights based on (eq. 4.8):

$$\forall V_i \in V, \quad W_{U,V_i} \leftarrow W_{U,V_i} \prod_{I=1}^{k} I \tag{4.8}$$

With each value of I, the I_i multiplies by the weights of the incoming edges W_{U_i,V_i}. Then, the model reassesses the ranks in the network (eq. 4.9):

$$\text{PageRank\_numpy}_{I_i}(DG, \alpha) \tag{4.9}$$

After each round of iteration, the model can reset the network to its initial state or continue applying changes to other edges depending on the approach settings. This facilitates the behaviour analysis from multiple vertex perspective. It is noteworthy that both the change in edge weight and network scale affect the vertices' ranks, which results in behaviour variation as the network evolves.

Figure 4.2 is an example which illustrates how the *mental wellbeing* vertex rank varies when only vertices with positive impact influence. The figure compares the behaviour using the data collected from various literature (in blue) with the data collected from the SocioBalance game (in purple). The aim is also to validate the output behaviour, and the data collected from the game.

Modelling of Hospital Network

To model a complex hospital network two approaches are proposed. Each has a limitation and strength. The first approach is to implement an agent-based network simulation (Eden et al., 2021), which models the flow of agents and resources in a network, and a network algorithm to analyse the network. The approach requires embedding queue (Green, 2006) on the edges and defining sufficient parameters. For this, agent-based network simulation is used to model

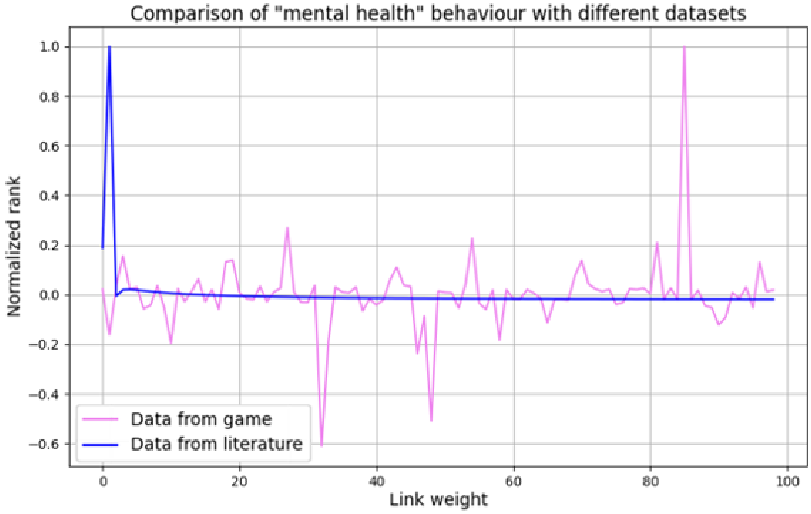

Figure 4.2: The influence of vertices with positive impact on "mental wellbeing" vertex.

the wards' behaviours, and network analysis algorithms are used to identify the wards with persistent bottlenecks and evaluate hospital performance (paper IV).

To develop the agent-based network simulation, first, the edges $\{E_i\}_{i=1}^{m}$ require correct queue disciplines. Queue disciplines help to define the edges' behaviours, such as sharing servers when two edges join, buffers capacity for patients to wait, permission to enter or leave a queue, etc. To embed queue disciplines, we also added temporary edges for the patients to enter and leave the hospital network. For that, the temporary edges E_0 and E_{m-1} are assigned *QueueServer* and *NullQueue* disciplines, respectively. This is because *NullQueue* discipline is responsible for terminating patients' flow, and *QueueServer* discipline allow unlimited patients to enter the hospital and wait in the queue even if the upcoming wards might be at full capacity. The rest of the edges in the hospital network are assigned *LossQueue* disciplines to avoid duplication of the vertex information in case of multiple incoming edges (condition 4.10):

$$\mathrm{Q}(E_i) = \begin{cases} \text{QueueServer}, & \text{if } E_i = E_0, \\ \text{NullQueue}, & \text{if } E_i = E_{m-1}, \\ \text{LossQueue}, & \text{otherwise}. \end{cases} \tag{4.10}$$

After the adjustment of the queue disciplines, input variables are assigned to corresponding vertices $\{V_i\}_{i=1}^{n}$, which represents the wards, and edges $\{E_i\}_{i=1}m$, which represents the care pathways connecting the wards. Some of these inputs are fixed and some vary for each ward and timestep. For instance, the vertices capacities represent the number of beds $\{B_i\}_{i=1}^{n}$, the number of servers represents the number of staff $\{D_i\}_{i=1}^{n}$ and buffers represent the queue size $\{F_i\}_{i=1}^{m}$. However, service times, arrival rates, and patient type vary per patient, ward, or time.

To assign a proper length of stay per patient for each ward, l number of service times $\{S_{v_i}^{l}\}_{i=1}^{n}$ for the given number of vertices n are used to estimate proper distribution functions. for that, Fitter python package (Cokelaer, 2023). Equation 4.11 represents the distribution function $\{DF_i\}_{i=1}^{n}$ per ward, which dynamically generates a service time $T_{V_i}^{P_j}$ per patient at a given time in specific ward within the condition if the T_{V_i} is within the range of the maximum and minimum service time T collected from the hospital record:

$$T_{V_i}^{P_j} \sim DF_{V_i}, \quad \text{if } T_{V_i}^{P_j} \in [S_{V_i,\text{min}}, S_{V_i,\text{max}}], \tag{4.11}$$

For the model to generate a new patient into the hospital network, a discrete list of arrival rates $\{R_i\}_{i=0}^{t}$ are assigned to the edge E_0 for the amount of given times t. Next, in order for the patients to move throughout the hospital, a list of distribution probabilities $\{DP_i\}_{i=1}^{m}$ are given to each edge. Then, to ensure that the patients visit only the required wards, a class is defined to assign each agent a disease type and identify their specific paths to follow. In this case, the Patient Class is derived from *GreedyAgent* Class. Originally, the *GreedyAgent* class aims to find the shortest path for an agent to follow; however, this is modified to ensure the patient only follows the path based on their disease issue and priority. Equation 4.12 represents the list of edges E' each patient is allowed to visit. For that a matrix of patients pathways $\{\{P_{ij}\}_{j=1}^{n}\}_{i=1}^{m}$ (or $\{\{AP_{ij}\}_{j=1}^{n}\}_{i=1}^{m}$ if the patient is a priority type) is used, which indicates the permission a patient can have to visit a ward based on its disease issue if the cell value is non-zero. Hence, the class updates the list of E' by keeping only the edges that the patient is permitted to follow. Finally, the most suitable edge for the patient to follow is chosen by the given distribution probability.

$$E'_{U_i,V_{\text{successors}}} = \{(U_i, V_i) \mid V_i \in \mathcal{V}_{\text{successors}}(U_i) \text{ and } \mathcal{P}[\text{disease issue}, V_i] \neq 0\} \tag{4.12}$$

Bottlenecks arise due to patients' flow intensity and structural weaknesses such as low degree of connectivity. Therefore, flow and structural hole algorithms are used to identify the wards with persistent bottlenecks. To implement the analysis, the output gathered from the Agent-based network simulation, along with the network, wards capacity and edge distribution probabilities, are used to measure the residual flow and the structural limitations. Finally, the common wards with significant persistent bottlenecks are selected as evidence for scenario analysis.

The second approach to capture the dynamic behaviour of complex hospital systems is to modify and adjust the flow algorithm to observe the dynamic nonlinear behaviour of the vertices. The modification of such algorithms facilitates the modelling and simulation rather than using simulation methods. To observe the dynamic nonlinear behaviour, the algorithm should encompass sufficient input variables and ensure that the output illustrates the vertices' dynamic behaviours. Furthermore, all flow algorithms have embedded path search algorithms, which may ignore the critical properties of the vertices or edges while analysing the system. Hence, network algorithms need to be properly modified to capture the proper behaviour of the vertices in a network (Crielaard et al., 2023; Howison et al., 2011).

This requires proper modification of the flow algorithm to model the patients' flow without the need for a simulation method. Hence, we modified the flow algorithm to model the dynamic nonlinear flow of the patient based on Ford-Fulkerson (Ford and Fulkerson, 1962) and Edmond-Karps (Edmonds and Karp, 1972) flow algorithms. First, a dynamic residual graph is defined to store the highest and the lowest residual flow. This helps to measure the bottlenecks' persistency and severity in each edge and vertex. Algorithm 1 represents the core modification of the flow algorithm to capture the dynamic behaviour of the wards. The algorithm iterates over the list of arrival rates R and the edges E. Within the iteration loop, it measures the proper service time given the unique distribution function DF for the wards (similar to eq. 4.11) for each ward v. Then, the algorithm measures the service rate based on the number of doctors $DG[v][d]$ per ward and the current arrival rate r, flow intensity based on the current arrival rate r and service rate, and the ratio to observe the overflow state of the wards (number of beds $[b]$). Next, for each edge, the algorithm measures the inflow rate based on the current arrival rate r effect on each edge distribution probability $DG[u][v][dp]$, and the effect of the edges' buffer size $[f]$ on the inflow rate. Finally, the algorithm adds the adjusted capacity of the current vertex cap_v with the adjusted capacity of the current edge cap_uv and stores it in a residual graph $staRG[u][v][c]$ as a residual capacity to continue processing the flow. It is worth mentioning that the pseudocode notations, such as v, r, df, etc, refer to the same mathematical notations V_i, R_i, DF_i, etc, respectively.

Additionally, the algorithm is set to use depth_first_search and eliminate the backward flow. This is because patients follow the paths for recovery not the shortest or alternative paths to leave the hospital. The algorithm finds the available paths and identifies the minimum residual capacity of the path. Unlike agent-based network simulation, the algorithm does not require temporary edges and the $source$ and $target$ vertices are assigned to ensure where the patients' flow begins and ends. Finally, the algorithm stores the highest and the lowest residual flow in a directed graph, which allows one to observe the severity, persistency and overflow state, and stores the current flow state at each ward per time in an adjacency list to observe the wards' behaviours. The complete modification of the algorithm is in paper V.

Figure 4.3 illustrates the nonlinear behaviour of selective wards within 24 hours based on the Akademiska hospital data. The outcome indicates that the

Algorithm 1: Dynamic Nonlinear Flow Algorithm

foreach $r \in R$ **do**
  foreach $(u, v) \in E$ **do**
    while *!* **(*min***$(DG[v][L_v]) \leq sTime\_v\_r \leq$ ***max***$(DG[v][L_v])$*)* **do**
      $sTime\_v \leftarrow DG[v][df].rvs(DG[v][df_{\mathrm{arg}}])$
    $service\_rate\_v \leftarrow \frac{DG[v][d]}{sTime\_v}$
    $flow\_intensity\_v \leftarrow \frac{r}{service\_rate\_v}$
    $ratio\_v \leftarrow \frac{flow\_intensity\_v}{(1-flow\_intensity\_v)}$
    $cap\_v \leftarrow ratio\_v \times DG[v][b]$
    $inflow\_uv \leftarrow r \times [u][v][dp]$
    $cap\_uv \leftarrow inflow\_uv \times DG[u][v][f]$
    $staRG[u][v][c] \leftarrow cap\_v + cap\_uv$
  while $True$ **do**
    $path \leftarrow$ **DepthFirstSearch**$(StaRG, source, target)$
    if $!path$ **then**
      break
    $res\_cap \leftarrow$ $\mathbf{min}_{e \in path}(res\_cap, StaRG[e][c])$
    foreach $e \in path$ **do**
      $StaRG[e][c] \leftarrow StaRG[e][c] - res\_cap$

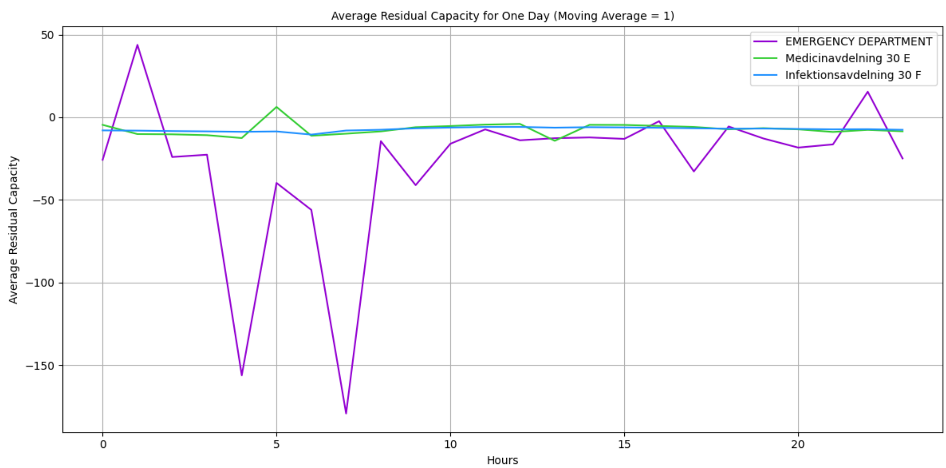

Figure 4.3: The nonlinear behaviour of Emergency Department, Medicinavdelning and Infektionsavdelning in one day.

Emergency Department has significant perturbation, *Medicinavdelning* has moderate perturbation, and *Infektionsavdelning* has steady behaviour.

Modelling of Organizational Networks

In the third case study, a link prediction approach is developed for accurate network representation of various organizations using both global and local methods (paper VI). The global method uses Bayesian network approach to construct a network for each organization using survey data, and assign corresponding vertices in $\{V_i^j\}_{i=1}^{n_j} \subseteq \{V_i\}_{i=1}^{n}$ where all the networks use the common V to access to the list of vertices, and the corresponding edges in $\left\{\{E_i^j\}_{i=1}^{m_j}\right\}_{j=1}^{k} \subseteq \{(U_i, V_i) : U_i, V_i \in \{V_i\}_{i=1}^{n}\}$ where each network has a distinct set of edges. Additionally, conditional probability distribution is assign weights to the vertices $\left\{\{P_i^j\}_{i=1}^{n_j}\right\}_{j=1}^{k}$ and edges $\left\{\{W_i^j\}_{i=1}^{m_j}\right\}_{j=1}^{k}$. Next, the local method merges the non_randomness algorithm with path analysis to further enhance the prediction of the missing links in each organization network if certain conditions are satisfied (eq. 4.13). In this case, the conditions to add a link to the organization network are: 1) If for the identified edge, equivalent path length exists in the reference network DG_{ref}; 2) If the identified edge increases the non_randomness score; and 3) If the sum of the weights in a given path $W_{U_i \to \cdots \to V_i,,ref}$ does not exceed a certain threshold $W_{\max}$.

$$DG \cup \{E_i = (U_i, V_i)\}, \text{ if: } \begin{cases} 1 : \text{Path length } K : U_i \to \cdots \to V_i \text{ in } DG_{ref}, \\ 2 : \text{non\_randomness}(G + \{E_i\}) > \text{non\_randomness}(G), \\ 3 : \sum W_{U_i \to \cdots \to V_i, ref} \leq W_{\max}. \end{cases} \tag{4.13}$$

Next, a multi-network simulation approach is used to simulate the networks of similar organizations in parallel with the following equation 4.14:

$$\forall V_i \in V, \quad P_i(t+1) = P_i(t) + \alpha \times \sum_{j \in \text{Predecessors}(i)} \left(P_j(t) \times W_{(U_j, V_i)}\right) \tag{4.14}$$

The simulation begins by first iterating over the vertices $\{V_i\}_{i=1}^{n}$ of all the networks in parallel. Based on the edge $\left\{\{E_i^j\}_{i=1}^{m_j}\right\}_{j=1}^{k}$ availability, the model measures the influence of the incoming edges weights W_{U_j, V_i} and the predecessor vertices weights P_j at a given time, and sums up these values. Then, the damping factor α effect is measured and updates the current vertex weight P_i. This procedure continues for the given time interval to evaluate the organizations' performances at the same time with the same influence.

Figure 4.4 illustrates the performance of selective organizations. The multi-network simulation output behaviour indicates that the diverse topologies can result in diverse performances among these organizations. This indicates that lack of certain vertices (elements) or edges (interactions) within this organiza-

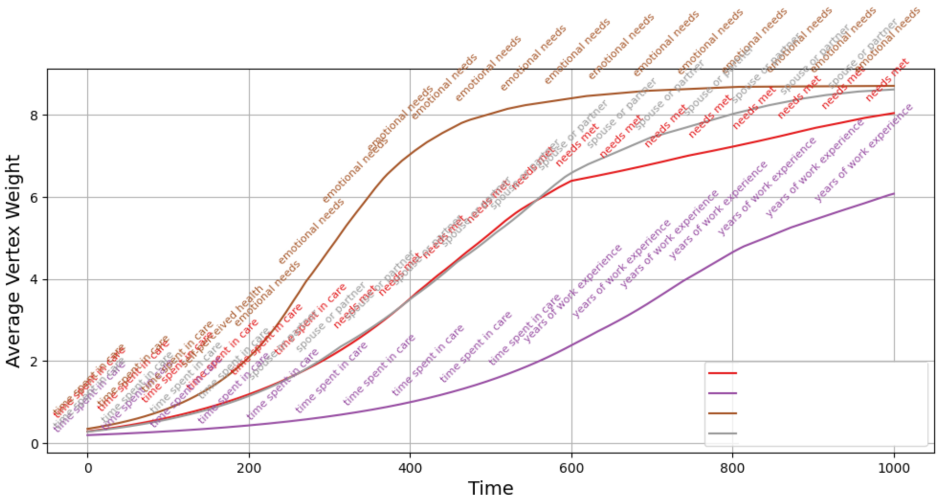

Figure 4.4: The performances of selective organizations based on their significant diversity of the topology, (red: unit R, purple: entire network irrespective of specific unit, brown: unit Q, gray: unit S).

tions affect their overall performances. Additionally, the labels in Figure 4.4 represent the vertices which causes the drop in each organization's performance at the give time.

4.4 Verification and Validation

One of the challenges in modelling complex health systems is the verification and validation of the network topology, dynamic models and output behaviour. This depends on the input data, and sufficient variables to model and influence the system and proper development and adjustment of the computational model. The following approaches (Sargent, 2013) are performed to ensure the verification and validation of the models:

- **Verification:** Ranking algorithms such as PageRank have features to verify the vertices' ranks accuracy in a network. The first feature is that the sum of the ranks should be equal to 1 after each change, and the second feature is that the ranks should be within a specific range. Otherwise, it is known as a rank-sink. To ensure there is no outlier value, the procedure is implemented in papers I, II and III. The procedure helped to tune the damping factor of the PageRank properly for each approach. In general, for all the approaches proposed in this thesis, the defined variables are extracted at various steps to ensure they are accurately adapted to the changes within the equation. For example, to ensure the accuracy of the modified flow algorithm in paper V, we extracted and visualized the paths with the flow to uncover the reason for the blockage.

- **Model Validity:** Algorithm selection and adjustment affect the output accuracy. One of the main problems is the networks with signed edges. The network we used to study the complex societal system had negative weights. Such networks are common in presenting complex societal systems. However, this prevents the application of a default ranking algorithm for such systems. Hence, to prevent this, the pagerank_numpy version of the ranking algorithm is used, which allows networks with negative edges, and the output can be both negative and positive depending on the changes in the vertex. Additionally, the algorithm version is not sufficient for the validity. To avoid outlier values emerging from negative weights and also increase in the edge weights, the damping factor is further tuned to observe proper output behaviour. For a hospital system, to identify the bottlenecks in a complex hospital network, the preflow_push (Goldberg and Tarjan, 1988) flow algorithm is used, which has the lowest running complexity along with constrain (structural hole) algorithm to identify the structural weaknesses (paper IV). The parallel application of both algorithms helped to ensure that the identified bottlenecks were caused by both the wards' connectivity and the flow of the patient. On the contrary, to modify the flow algorithm for modelling the dynamic nonlinear behaviour of the hospital wards (paper V), Ford-Fulkerson (Ford and Fulkerson, 1962) and Edmonds-Karp (Edmonds and Karp, 1972) algorithms are used. This is due to the algorithm's simple structure, which makes it easy to modify. Furthermore, the modification allowed us to solely simulate the system without complicating it with a simulation method. Lastly, algorithm selection has been based on their functionality as well, such as conditional blocks, iterations, data types and manipulations, and sufficient input and output parameters. An example can be the usage of a depth_first_search instead of a breadth_first_search to model the patient flow using a flow algorithm. From the complex societal network perspective, adverse and negative scenario analysis allows us to see the positive and negative influences on the network, which further promotes the validity of the network topology and the model as a data collection approach.

- **Operational Validity:** To ensure that a model operates accurately, different scenarios can be used to simulate and compare the results' variation. In complex societal network simulation, two different datasets are used to quantify the links. The first dataset contains correlation coefficients collected from various literature, whereas the second dataset contains correlation coefficients collected using gaming simulation (paper I). Next, in the simulation of complex hospital systems using a dynamic nonlinear flow algorithm (V), two different distribution probabilities are used for patient flow. The first dataset contains distribution probabilities of patients from Akademiska University Hospital, and the second dataset contains equal distribution probabilities based on the number of outgoing edges from each ward. This allows us to observe the reason behind path blockage. Furthermore, a proper distribution function is assigned to each ward to generate the service time per patient using the Fitter python package (Cokelaer, 2023). To ensure the results are realistic, the generated data are compared to the length of stay (LOS) and also restricted with the maximum and minimum service time from the data. Furthermore, the high perturbation in the ranks can also be due to the high variability of the edges' weights, which requires tuning the damping factor until proper out behaviour is achieved.

- **Decision Validity:** Issues raised from a complex system can have different reasons, such as system structure or the rate of flow. Hence, to increase the evidence for decision-making in a hospital, first, two algorithms are taken into consideration: a structural hole algorithm and a flow algorithm. This allows us to uncover the common bottlenecks in a hospital from both structural and flow perspectives. Second, the persistency of the vertices' behaviours is taken into consideration if the system performance improves and stays steady.

- **Structural validity:** The accuracy of a network topology representing a complex system can affect model output behaviour. Therefore, the data collected from the SocioBalance game is tested through scenarios which help to observe the positive and negative impacts on the vertices' behaviours and also compared to the data collected from the literature for the same purpose. Additionally, in paper VI, the non_randomness (Ying and Wu, 2009) algorithm is used to ensure that the predicted links accurately enhance the network topology of the organizations.

- **Data Validity:** Input data accuracy specifically for complex societal systems is challenging. It requires understanding the human experiences through the lens of the society. Thus, a simulation game was developed to quantify the links in a complex societal system by adding the human factor to the model (paper I). Such a data collection approach enhances the modelling validity since it engages the player in various societal situations and can also focus on a specific geographical location. Similarly, the approach can be used for the validation of complex organizational systems where the players are an organization's staff and users.

- **Face Validity:** Professionals and stakeholders within a field or organization have the knowledge and expertise about a system's performance. Involving them throughout the development procedure, helped to validate the model based on its output behaviour. Hence, the model output from the agent-based network simulation was presented to the hospital managers to evaluate the accuracy.

5 Contributions

The chapter highlights the contributions of modelling approaches, the application of network algorithms, the output structure, and the roles these approaches provide.

5.1 Modelling Approaches' Contributions

The proposed approaches provide easier ways to model different complex systems irrespective of the changes in network topology, scale, type, and values, which also facilitate the adjustability to enhance functionality. Table 5.1 displays the list of contributions each modelling approach provides.

In this table scalability refers to the approach's ability to accommodate an evolving network; generality refers to the approach's reusability for any type of network; adaptability refers to the approach's ability to adapt the changes dynamically; topological flexibility refers to the approach being able to model networks with any topology; Finally, adjustability refers how the approach can be enhanced to increase its efficiency or functionality.

The proposed modelling approaches in this thesis are all scalable to one evolving network except the multi-network simulation approach which can model multiple networks in parallel of various scales. This is because the model replicates the network vertices, assigns different edges to each replicated network and eliminates the isolated vertices (paper VI). Furthermore, when applying agent-based network simulation, the addition of a new ward requires defining which patient types should visit there (paper IV). To reflect, when changing the scale of a network, approaches or data are required to quantify the additional vertices and edges.

The hybrid approaches, which are based on ranking algorithms, can be easily applied to quantify and simulate various types of networks (papers II and III). This facilitates the approach generality to model different types of complex networks. This is because the ranking algorithm quantifies the vertices based on their connectivity, network scale, and the weights of the incoming edges. Therefore, these modelling approaches can be easily applied to other types of networks, such as biological or economical. Similarly, the SocioBalance simulation game is capable of ranking the edges in any complex societal system. However, it requires

Table 5.1: Contributions of the Proposed Modelling Approaches.

	Scalability	**Generality**	**Adaptability**	**Topological Flexibility**	**Adjustability**
Hybrid Approach (Ranking Algorithm Merged with Path Analysis)	Scalable to accommodate evolving networks	Generalizable across all network types	Vertices ranks dynamically adapt to the changes in the network scale	Topologically flexible	Can be replaced by algorithms other than ranking
Hybrid Approach (Ranking Algorithm Merged with System Dynamic Simulation)	Scalable to accommodate evolving networks	Generalizable across all network types	Vertices ranks dynamically adapt to the changes in the incoming edges' weights	Topologically flexible, optimal with at least one incoming edge per vertex	Can be replaced by algorithms other than ranking
SocioBalance: Edge Ranking Simulation Game	Scalable to accommodate evolving networks	Generalizable across societal networks, adjustable to other network	N/A	Topologically flexible, optimal with at least two incoming edges per vertex	Can be adjusted to be used for other network types
Agent-Based Network Simulation	Scalable to accommodate evolving networks, requires to define the patients' types to visit the new ward	Generalizable to flow-based hospital networks	Wards' and pathways' occupancy dynamically adapt to the changes in the inflow and outflow of the patient	Topologically flexible, optimal with a single outgoing edge from the source vertex and single incoming edge toward the sink vertex	Can be extended by inheriting additional classes for resource flow
Dynamic Nonlinear Flow Algorithm	Scalable to accommodate evolving networks	Generalizable to flow-based hospital networks	Wards' and pathways' occupancy intensity dynamically adapt to the changes in the inflow and outflow of the patient	Topologically flexible	Can be extended by routing patient flow based on disease types or multi-commodity
Link Prediction Approach (Non_Randomness Algorithm Merged with Path Analysis)	Scalable to accommodate evolving networks	Generalizable to non-flow type networks	Dynamically adapts to the changes in topology	Topologically flexible, optimal with paths of minimum two edges	Can be extended to consider the cold start of a vertex
Multi-Network Simulation	Scalable to accommodate 1-to-many evolving networks of various scales	Generalizable across multiple non-flow network types	Vertices behaviours dynamically adapt to the changes in the incoming edges' weights	Topologically flexible to networks of varying topology	Can be enhanced to consider the influence of similar organizations within a shared environment

slight adjustment of the aspects in the game to be applicable to other types of networks (paper I). On the contrary, the approaches which model the dynamic of the complex hospital systems can only be used to model hospitals. This is because agent-based network simulation inherits a class to model the patients' flow based on their disease (paper IV) types, and the dynamic nonlinear flow algorithm is modified to specifically model the patient flow in hospitals (paper V). Moreover, the link prediction approach and multi-network simulation are suitable for non-flow type networks (paper VI).

Internal influences are defined as the changes in the network topology, edges' weights and their impact on the vertices' behaviours. Such influences are captured in the hybrid approaches, link prediction approach, and multi-network simulation to define the networks' proper topologies and to perceive the vertices' behaviours. Another type of internal influence is defined for flow-type networks where the vertices' behaviours are influenced based on the inflow and outflow rate. Such influences are captured in agent-based network simulation and dynamic nonlinear flow algorithm, which measure the patient inflow rate based on the arrival rates and the outflow rate based on each ward's service time (length of stay) and the number of servers in each ward. As a result, these proposed approaches can dynamically adapt to the changes in the network based on certain conditions. However, the SocioBalance simulation game is proposed to collect data and quantify the edges in complex networks. While the game elements within the game adapt to the progress the player makes, the collected data does not dynamically influence the model.

Topological flexibility refers to the sensitivity of an approach to the structure of a network, such as the way the edges are arranged and connected to the vertices. The modelling approaches proposed here are all topologically flexible and handle exceptional cases. For instance, the hybrid approaches are optimal when each vertex has at least one incoming edge. This is because the model searches for the incoming edges to manipulate the weights in order to influence the network. As a result, the model is unable to capture the influence of the vertices with no incoming edges. In the case of SocioBalance, the game functionality is based on the ranking of the edge with the highest impact. Hence, the model requires at least two incoming edges to compare. Therefore, the game eliminates the vertices where there are less than two incoming edges. Thus, a network where each vertex has only one incoming edge would not be suitable for this game. In the case of complex hospital systems, both modelling approaches are topologically flexible. However, agent-based network simulation is more optimal when there is a single outgoing edge from the source vertex and a single incoming edge towards the sink vertex. This is because the model requires an edge to generate the patients and an edge to eliminate the patients. Thus, it will prevent the other edges from receiving patients. Finally, the link prediction approach is optimal when a path consists of at least two edges, and the multi-network simulation is flexible to simulate networks of different topologies in parallel.

The proposed approaches are also easily adjustable to model other features and functionalities of the same complex systems. For instance, the ranking algorithm can be replaced with other network algorithms to measure the vertices' behaviours based on criteria other than their connectivity and the weights of the edges. SocioBalance can be adjusted to study other types of networks, such as organizational or economical systems. For hospital systems, the agent-based network simulation can inherit other classes to model the resource flows, such as staff, and the dynamic nonlinear flow algorithm can be extended to consider the patient disease type and multi-commodity. Finally, the link prediction approach can be extended to consider the cold start of the vertex, and the multi-network simulation approach can be enhanced to consider the effect of similar organizations on each other within a shared environment.

5.2 Classification of Algorithms and Methods

Modelling a complex health system requires understanding the functionality and structure of each complex health system. This is because complex health systems function differently than other types of complex systems, and these functionalities can be easily missed by many algorithms and methods. This requires the proper adoption and adjustment of the network algorithms and simulation methods. This section classifies the algorithms and methods which are considered suitable to model and analyse the case studies. The choices are based on their suitability to model and analyse various aspects of the complex societal, hospital and organizational systems.

Table 5.2 classifies the network algorithms and simulation methods based on various analysis purposes for health systems. In societal systems the vertices behaviours are steered by the changes in edges' weights. To analyse the vertices in such systems, the network algorithm should consider negative weighted edges. The ranking algorithm is suitable for analysing these vertices and has sufficient input variables to tune the damping factor and the number of iterations to avoid outlier values. To analyse the edges in this system, a gaming simulation approach is developed to allow the ranking of the incoming edges. This is because edges' weights depend on the network type and context. Therefore, there is no network algorithm to rank the edges. Ranking algorithms can be suitable for complex organizational systems as well. However, the ranks will be solely for individual networks and will not be affected by a change in similar organizational networks. Furthermore, to analyse the vertices of a hospital system, a percolation algorithm is used. The percolation algorithm considers the vertices' weights and states, which helps to analyse the network based on the wards' capacities and overflow states. However, in this type of network, the edges indicate the flow paths and not the impact strength on the neighbouring vertices. Thus, a static flow algorithm can provide information about the bottleneck that emerged due to the flow, where it considers the sum of the vertices and edges' weights. This is because hospital wards' capacities are the main cause of the overflow than the edge weights.

Table 5.2: Classifications of network algorithms and simulation methods to analyse various aspects of the complex health systems.

	Societal Systems	**Hospital Systems**	**Organizational Systems**
Vertices Analysis	Ranking algorithms which considers the edges' negative weights and directions are suitable to identify the most influential vertices affecting societal wellbeing	Percolation algorithm which considers vertices' weights to analyse the wards relative impact on a hospital based on its overflow state	
Edges Analysis	SocioBalance simulation game which considers the edge direction can be used to perceive the impact strength of the vertices in a societal system	Static flow algorithm which considers the edges' and vertices' weights to measure the possibility of bottlenecks	
Topology Analysis	Non_randomness algorithm which considers the edge weights to validate the topology and ensure the network properly represent the real world system	Structural hole (constraint) algorithm which considers the edges' weights to perceive the hospital pathways connectivity weaknesses	Non_randomness algorithm which considers the edge weights to validate the topology and ensure each network represents individual organizations structure accurately
Path Analysis	Algorithms considering all the paths connecting two vertices help to perceive the vertices influential persistency	Depth_First_Search algorithm is more suitable to identify the source of bottlenecks in hospitals since bottlenecks can also emerge from less priority wards	Algorithms considering the shortest paths of at least 2-edges length between vertices help to identify the missing links
Networks interrelation Analysis		Perturbation algorithm to evaluate the performance of a hospitals before and after the structural changes	Comparison with a reference network or against Barabasi-Alberts null network to evaluate the network structural accuracy
Dynamic Analysis	System dynamic simulation	Dynamic nonlinear flow algorithm modified specifically or Agent-based network simulation	System dynamic simulation

The topology analysis provides information about the degree of connectivity, structure, strengths and weaknesses of the network. Complex societal and organizational networks require a certain degree of structure to ensure they represent the system properly. Hence, the non_randomness algorithm is used to validate the topology and ensure the network representation is, to some extent, structured. Similarly, hospital structural weaknesses can trigger bottlenecks in the hospital where the number of incoming edges of a vertex is more than the number of outgoing edges, or the vertices are weekly tied. However, this can be affected by the distribution of the patients throughout the network. Hence, the structural hole algorithm should consider the distribution probability as an edges' weights.

Path analysis algorithms are widely used to analyse the indirect impact or the distance of the vertices. In a complex societal system, the indirect impacts can be influenced through multiple connecting paths. To analyse these systems, path analysis algorithms should consider all these paths irrespective of the length. This can help to isolate these paths and measure the vertices' stability without the influence of other parts of the network. In parallel, to analyse the hospital paths, depth_first_search algorithms can be more suitable. This is because the algorithm returns longer paths to uncover bottlenecks, which can be located somewhere deep in the graph paths and might be considered less critical. Finally, to predict the links in complex organizational systems, considering the paths with the shortest length can be more suitable to identify the small details. It is worth mentioning that due to the similarity in network types for societal and organizational, the same path analysis algorithms can be used to uncover the vertices and predict the links.

The analysis of the interrelation between two similar types of systems can help to evaluate the performance improvement or the influence caused on each other within an environment. For instance, the perturbation algorithm can help to analyse the changes in the system by edge and vertices removal until the optimal condition is met. This can be suitable in a hospital system analysis where hospital care pathway restructuring or adding a new ward is required. In the case of complex organizational systems, the interrelation analysis can validate the networks' topologies either by comparing them to a reference network or against a Barabasi-Albert null network. This is because Barabasi-Albert network is created to test heterogeneous networks randomness. Similarly, this can be applied to analyse the structure of the complex societal system.

Finally, to analyse the dynamic of the system, system dynamic simulation can be more suitable for interaction networks, whereas dynamic nonlinear flow algorithm or agent-based network simulation can be more suitable for flow-type networks. System dynamic simulation makes it easier to embed network algorithms to either quantify or measure the interrelation of the networks. Dynamic nonlinear flow algorithms facilitate the simulation of the flow networks without the implementation of simulation methods. Lastly, agent-based network simulation facilitates the modularity of the modelling of the patient or resource flows and interactions through class inheritance. However, it is challenging to embed network algorithms within the model due to its structure. This requires iterating over the output of the model and using it as an input for the algorithms.

In general, to model and analyse complex health systems, shortest path finding algorithms may not be suitable due to neglecting the crucial vertices in the system. The network algorithms should also consider the direction of the edges to identify the direction of the impact or flow. Furthermore, identifying if the algorithm can handle and consider the impact of negative weights on the dynamic of the system. Apart from the edges' weights, vertices' weights play a crucial role in health system performances. As a result, to accurately analyse the health systems, network algorithms should consider both vertices and edges' weights if possible; otherwise, at least one of these weights should be taken into consideration.

5.3 Outputs Structures and Purposes

The modelling approaches proposed in this thesis aim to provide a holistic view of the system to identify the critical vertices and paths which significantly affect the system and further the individual vertices' behaviours to understand how internal and external influences affect individual vertices' behaviours. This will help to understand the fluctuations and the stability of different parts of the system. The following provides an overview of the output structures and Contributions:

- **Outcome of holistic view**: Depending on the case study, the output is either a matrix (2D lists) or network representation (adjacency list containing edges).
  - **Vertices with most influential and persistent effect on mental wellbeing**: The outcome is based on the comparison and divergence of the local and global vertices' ranks. The persistent vertices are the ones which preserve their importance, and the influential ones are the vertices with significant rank and impact on the network. This helps to understand which vertices can have a vast influence on the whole system.
  - **Wards with severe and persistent overflow**: The outcome is based on the lowest and the highest flow in each ward. This helps to identify the wards with overflow and the root causes of the overflow. Additionally, the outcome provides an overview of the severity and persistency of the flow in a hospital ward.
  - **Organizations with diverse network topology**: This outcome is based on the network representation of each organization. The network topology of each organization helps to understand the performance diversity of similar health organizations within a shared area.
- **Outcome of dynamic behaviour**: The output is an adjacency list containing the vertices' values such as weights, time, or steps that weight changed.
  - **Vertices with adverse and beneficial impact on wellbeing**: The outcome is based on the influence of one or many vertices on other ver-

tices. This helps to identify which vertices in a network can cause significant changes and whether the impacts are beneficial or adverse.

– **Wards with significant perturbation and improvement in performances**: There are two outcomes for this. The first outcome helps to evaluate the wards' performances based on the variability of the wards' service times and patients' arrival rates. The second outcome helps to evaluate the wards' performances after shifting patients flow with specific disease categories toward a new ward.

– **Organizations with diverse behaviour and adverse vertices**: The outcome is based on an influence into all the organizations' networks at the same time in order to observe their performances in parallel. This helps to compare their performances and identify the vertices which cause significant adverse impacts on each organization's performance.

5.4 Reflection on Contribution

While these algorithms simplify the modelling, they also offer additional information regarding weaknesses, strengths, and performance on different aspects of the health system. When adopting network algorithms to model complex health systems, the output of the analysis is not independent of the rest of the system's representation and performance. For example, some of the vertices' popularities (ranks) shifted as the mental wellbeing network scale changed or was based on internal influences. In the case of hospital simulation, the severity of the bottlenecks (structural and flow algorithm) in some wards was reduced when a new ward was introduced to the hospital. Additionally, organizations' representations begin to have a more diverse topology when the link prediction approach ensures that links are not randomly added to the network. This is due to the nature of network algorithms considering the position of vertices and edges within a system.

Parallel to these algorithms, approaches were required to analyse the dynamics of the whole health system. System dynamic simulation was suitable to analyse health systems where the vertices interact. Since they leverage the underlying system structure, which can be observed either through data or designed through expert knowledge, they are easier to implement. Additionally, agent-based network simulations are more suitable for complex health systems such as hospitals, which represent a flow network.

The representation and modelling of these systems as networks helped to understand the relationship of the different parts and how these parts affect each other. For example, in a complex societal system, the proposed models (paper I, II, III) help to understand which vertices can make a significant in the society's wellbeing by considering the effect of all the elements in the system. in complex hospital system, the proposed models (IV, V) help to understand which wards are the root causes of the bottlenecks irrespective of their importance by considering all the wards in the hospital. Finally, in complex organizational systems, the proposed model (paper VI) helps to understand how diverse similar health organizations' structures and performances can be through network representation and multi-network simulation. As a result, with network algorithms considering the position of vertices and edges, influences on one distant vertex can appear on another vertex which is under observation.

However, verification and validation of holistic models are more challenging. They grow large and complex. Dealing with complex health systems, the accuracy of these models is crucial in order to prevent risky decisions. During the study of complex health systems, network algorithms helped to validate the network topology, output behaviour, vertices and edges weights, and evidence for more accurate decision-making. For example, the non_randomness algorithm ensures that the topology of the networks accurately represents real-world systems. Moreover, the ranking algorithm ensures that there is no rank-sink while ranking vertices. Furthermore, the use of structural hole (constraint) and flow algorithm in parallel, increased the evidence regarding the existence of bottlenecks in a ward.

6 Discussion and Conclusion

The thesis offers various network-agnostic approaches to facilitate modelling and model validation for complex health systems. The aim of developing such approaches was to overcome the modelling challenges, which in turn enables scalability, generality, adaptability and topological flexibility. Furthermore, to ensure that the proposed approaches were easily adjustable for integration and extension with more features and functionalities. These models facilitate the quantification, identify crucial vertices, classify the behaviours, evaluate the performances, and promote the verification and validation. These are facilitated through the proper adoption and adjustment of network algorithms and network simulation methods.

6.1 Reflection on Modelling Approaches

Modelling complex systems is challenging due to the data requirement, irregular and complex connectivity, and nonlinear behaviour variability. Network simulation methods and algorithms facilitated the representation, modelling and analysis of these systems. However, adjustments were required to these methods and algorithms in order to properly capture the performance of complex health systems. Therefore, for proper adjustment, the algorithms were tuned, merged with simulation methods, modified based on the health system dynamic, and game to engage players in understanding how health systems perform from their perspectives.

Quantifying a complex health system where the vertices represent various aspects of wellbeing and the edges of the impact strength on each other is data-dependent. A quick approach is to implement ranking algorithms to quantify the vertices. The challenge with this approach is that due to network topology and the negative edge values, proper ranking algorithms and tuning are required to model the vertices' behaviours accurately. Additionally, tuning the damping factor of the algorithm prevents the rank-sink and the outlier values, which also depend on the edges' impact strength. In a complex societal system, the edges' impact strengths depend on how different vertices impact each other. When dealing with a societal system, the involvement of individuals in a game representing various aspects of society can help in perceiving the significance of the impact. As a result, a simulation game called SocioBalance was developed, which engaged players to rank the vertices connecting incoming edges per vertex based on how they perceive and prioritise the impacts. The data collected from the game was used in the hybrid approach proposed in paper III where PageRank was merged with system dynamic simulation, which helped to validate the SocioBalance gaming simulation and the hybrid approach.

Merging simulation methods and network algorithms overcame the limitations and utilises the strengths of modelling. This promoted behaviour analysis, critical vertex identification, performance evaluation, verification and validation. Combining the ranking algorithm with path analysis or system dynamic simulation helped to perceive how the vertices' ranks vary under different circumstances. This allowed us to identify the vertices with a significant change in their behaviour or their importance stability. Additionally, ranking algorithms have a feature which adapts to changes in the network. This was validated by ensuring that the sum of the ranks is equal to 1 when a network value or structure changes. Furthermore, adapting flow and structural hole algorithms allowed us to uncover the bottlenecks that emerged from the hospital's structural and patient flow perspectives. Furthermore, the wards' performances were evaluated using the percolation and perturbation algorithms.

Network algorithms have the limitations to capture the dynamic behaviour of the network vertices. This requires merging or modification of these algorithms, depending on the nature of the problem and the network type. In the case of a complex societal system, PageRank was embedded within the path analysis algorithm and system dynamic simulation to observe the rank changes. Merging not only helped to capture the dynamic behaviour but also helped to validate the network topology and the output behaviour. For instance, in the case of complex organizational systems, the non_randomness algorithm was embedded within the path analysis algorithm to validate the network representation of the health organizations and ensure that the links are predicted accurately to enhance the topologies. However, in the case of complex hospital systems, modifying the flow algorithm helped to capture the nonlinear dynamic behaviour of the wards in the hospital based on the patient flow. This is performed by iterating over the patients' arrival rate, measuring the service rate and flow intensity, and calculating the residual capacity. Additionally, instead of the breadth_first_search algorithm, the depth_first_search algorithm is used, and the backward flow is eliminated. This is because the patients within a hospital network are required to follow the wards based on the required treatment, and there is no short or alternate path for recovery. Hence, to use network algorithms for complex health systems, we need to understand how the system functions in order to apply and adjust them for modelling and analysis. Nonetheless, modifying network algorithms can provide a ready approach for faster modelling and analysis of complex health systems.

6.2 Challenges and Limitation

In general, there are three challenges in applying network algorithms to model complex health systems: first is the static nature of the algorithms, second is the interpretability issue, and third is the underlying network type and the target system. Complex health systems have dynamic nonlinear behaviours; this requires the modification of these algorithms or embedding them within a simulation method. In parallel, the output of these algorithms varies significantly depending on how each algorithm handles the input data. For instance, there are different versions of the ranking algorithm; each developed for different purposes, e.g. one considers the directions of the edges, and another considers only the weights or positive weights. Lastly, the network type and target system affect the accuracy of the algorithm's output, such as if the algorithm is intended to model the flow of the cars in a transportation network, the packets in a telecommunication network, or the patients in a hospital network. As a result, flow algorithms vary significantly, handling the path searching and augmentation differently. In this case, the flow algorithm was modified to specifically handle the patient flow in a complex hospital system by considering the arrival rate and service time variability over time, where the rest of the variables were fixed. To further advance this algorithm, the patient disease categories and wards functionality need to be implemented.

One of the limitations of network modelling approaches is the data requirements to perceive the network structure and quantify the vertices and edges. In the case of complex societal network modelling, ranking algorithms and gaming simulation approaches were utilised to quantify the vertices and edges. The limitations of using a ranking algorithm within a system dynamic simulation require the vertices to have incoming edges in order to observe the impact on the rest of the network. In parallel, the challenge with such algorithms is that they require adjusting suitable damping factors to avoid invalid behaviours. On the contrary, the limitation of the SocioBalance simulation game is that a vertex has sufficient incoming edges. This is because there should be at least two incoming edges per vertex so that the players are able to rank them, which prevents quantifying all the edges in a network. One way to solve these limitations for the ranking algorithm and SocioBalance is to add a temporary edge and vertices or a manual quantification system.

Indicating the number of staff per ward in a hospital is challenging. This is because hospital records systems are for the patients, and the staff continuously visit various wards due to demand. In this context, the proposed modelling approaches lack the number of staff. Hence, the model assumes that each ward has an equal number of staff as the beds. Another way to overcome this limitation is to assign a distribution function to each ward to handle the staff numbers variability, which still requires data from the hospital to indicate the range of variability. Another challenge with the hospital's modelling was the difficulty of embedding algorithms within agent-based network modelling. This forced us to use the output from the simulation as an input for network algorithms to measure the bottlenecks and evaluate the wards' performances. On the contrary, the modification of the flow algorithm from static to dynamic made it possible to model the patient flow without the utilisation of a simulation method to find the bottlenecks over time. One thing which was commonly observed was the extreme outlier values, which were identified and removed.

Finally, models' dependencies on external resources and compatibility require continuous updates and adjustments. For instance, many of these models depend on libraries such as Networkx or queueing_tools to perform. These libraries continuously maintain to solve an issue in the library or enhance them with new functionality, such as adding a SharedServer class, which is suitable to handle multiple flows toward one ward. In parallel, many algorithms and tools deprecate from these libraries, such PageRank_numpy, which was suitable to handle negative edges in a network or does not exist due to the runtime complexity.

6.3 Applications and Future Direction

The modelling approaches proposed in this thesis can be applied to model other types of complex systems. For instance, approaches which are based on the ranking algorithms can be applied to model the dynamic of complex economical systems for less risky investment based on how different economical factors' ranks vary or the dynamic of complex environmental systems for more sustainable decision-making. Additionally, SocioBalance can also be used by policymakers to foresee and prevent adverse impacts on societal, economical and environmental systems by just importing the network representation of these systems into the database.

Both approaches, which are developed to model the dynamic of the complex hospital system, can be applied to model other hospitals as well. This can be achieved by substituting the data for the vertices, edges, weights and patients' pathways and disease issues with the data from a specific hospital.

Lastly, the multi-network modelling approach can be applied to analyse the parallel behaviour of organizations with similar goals and their interrelations, such as all the pharmaceutical organizations established in the same region. This can help individual organizations change their structure to improve performance or prevent perturbation.

To conclude, the future direction of such modelling should focus on the adoption and adjustment of the network algorithms. This will help to prevent complicating the models and the implementation and will automate the procedure. Even though network algorithms are a great benefit to model complex systems, the adoption and the adjustment should be done cautiously. This requires understanding the target complex system, the type, and how it operates. Through network algorithm adoption, the approaches will have the readiness feature to be quickly applied for modelling and analysing the dynamics of various systems.

Furthermore, such approaches simplify the difficulty of capturing the dynamic of complex systems' behaviours and give them a certain degree of flexibility since the algorithms can handle some of the functionalities of the system through proper adjustment.

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