KTH ROYAL INSTITUTE
OF TECHNOLOGY

Degree Project in Technology

Second Cycle, 30 credits

Optimal expansion for high voltage network

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Paris, France, 2025

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	Examensarbete TRITA-ITM-EX 2025:39 Optimal expansion för högspänningsnätverk Utveckling av ett optimeringsverktyg med Benders dekomposition för att utföra transmissionsutbyggnadsplanering på ett detaljerat nätverk Noémie Aloy	
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Sammanfattning

Utfasningen av fossila bränslen inom industrin medför en ökning av utbyteskapaciteten mellan länder och elektrifieringen av användningsområden, vilket kommer att bidra till att intensifiera energiflödena i transmissionsnätet. Systemansvariga för överföringssystem (TSO:er) genomför expansionsstudier för att identifiera nya nätinvesteringar som kan avlasta de potentiella överbelastningarna. Syftet med denna studie är att utveckla ett optimeringsverktyg för den franska TSO:n RTE för att utföra Transmission Expansion Planning (TEP, på svenska Transmissionsutbyggnadsplaneri) på detaljerade nätverk och ge ramverk för att göra det tillämpligt på storskaliga studier.

Metoden som konstrueras i denna avhandling är ett optimeringsproblem som modellerar funktionen hos ett detaljerat elektriskt nätverk under DC-antaganden, under ett år, med ett tidssteg på 1 timme. Vid varje timme begränsas kraftflödena över nätet av maximala elöverföringar, i N- och N-1-situation (nätet ska klara av förlusten av en linje). För att säkerställa dessa begränsningar modellerar problemet gratis hävstänger (optimering av HVDC-linjernas börvärde och fasskiftstransformatorernas (PST) fas) och kostsamma hävstänger (omfördelning av elproduktion och förlust av last). Modellen kan också investera i nya linjer genom binära variabler, under en given maximal investeringsbudget. Målfunktionen för problemet är att minimera driftkostnaderna, på grund av redispatching och lastförluster. Problemet löses med Benders dekompositionsmetod, som är väl lämpad för flerstegsproblem som detta. Denna metod dekomponerar problemet i ett huvudproblem (investering) och delproblem av en vald storlek (beräkning av driftkostnader).

Verktyget utvecklas först och testas på det akademiska nätverket IEEE RTS-96 (93 noder). Fördelen med Benders dekomposition med stabilisering för att påskynda lösningen demonstreras, dividera med 3 beräkningstiden för det årliga problemet. Betydelsen av att hitta rätt storlek på delproblemen (dvs. antalet timmar som studeras per delproblem) för att göra problemet hanterbart betonas också. Ett 1000 timmar problem kan lösas 2 gånger snabbare med rätt val av storlek på delproblem.

Verktyget testas sedan på det europeiska nätverket (cirka 20 000 noder), som används i en långsiktig expansionsstudie från RTE för det franska högspänningsnätet. Givet den stora storleken på studien skapas mindre testfall, där antalet studerade timmar, mängden investeringskandidater och den maximala budgeten varieras. Endast N-situationer, dvs utan att beakta contingencies, studeras. Dessa testfall kördes framgångsrikt men med högre beräkningstider än förväntat, ibland överstigande 2 timmar för ett problem som var tänkt att ta en timme. Numeriska problem som kan uppstå i detta mer komplexa nätverk bör åtgärdas först, och därefter bör utvecklingen av en metod för att minska huvudproblemet (genom att minska

antalet kandidater eller den maximala budgeten) beaktas. För att ta hänsyn till contingencies och modellera N-1-situationen bör storleken på delproblemen minskas. En nätverksreduktion med en detaljerad representation av enheter och laster är lämplig för vår metod.

Nyckelord

Transmissionsutbyggnadsplanering, Detaljerat nätverk, Optimering, Benders dekomposition, Långsiktiga investeringar

KTH Industrial Engineering and Management	**Master of Science Thesis TRITA-ITM-EX 2025:39**
	Optimal expansion for high voltage network
	Development of an optimization tool using Benders Decomposition to perform Transmission Expansion Planning on a detailed network
	Noémie Aloy

Approved	Examiner	Supervisor
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Abstract

Industry decarbonization comes with an increase in exchange capacities between countries and the electrification of uses, which will contribute to intensifying the power flows on the transmission network. Transmission System Operators (TSOs) conduct expansion studies to identify new network investments that can relieve potential congestion. The objective of this study is to develop an optimization tool for the French TSO RTE to perform Transmission Expansion Planning (TEP) on detailed networks and provide frameworks to make it applicable to large-scale studies.

The method developed in this thesis is an optimization problem that models the functioning of a detailed electrical network under DC assumptions, over a year, with a timestep of 1 hour. At each hour, the power flows across the network are constrained by maximum electricity transmissions, in N and N-1 situations (the network should cope with the loss of one line). To ensure these constraints, the problem models free levers (optimization of HVDC line setpoints and Phase Shift Transformers (PSTs) phase) and costly levers (redispatching of electricity generation and loss of load). The model can also invest in new lines through binary variables, under a given maximum investment budget. The objective function of the problem is to minimize the operation costs, which arise from redispatching and loss of load. The problem is solved using the Benders decomposition method, which is well-suited for multi-stage problems like this one. This method decomposes the problem into a master problem (investment) and subproblems of a chosen size (operation costs computation).

The tool is first developed and tested on the academic network IEEE RTS-96 (93 nodes). The advantage of the stabilization with Benders decomposition to accelerate the resolution is demonstrated, dividing by 3 the computation time for the yearly problem. The importance of finding the right subproblem size (i.e., the number of hours studied per subproblem) to make the problem tractable is also highlighted. A 1000 hours problem can be solved 2 times quicker with the right subproblem's size choice.

The tool is then tested on the European network (around 20 000 nodes), used in an RTE long-term expansion study on the French high-voltage network. Given the large size of the study, smaller test cases are created, varying the number of hours studied, the amount of investment candidates, and the maximum budget. Only N situations, i.e., without considering contingencies, are studied. These test cases ran successfully but with higher computation times than expected, exceeding sometimes 2 hours for only a one hour problem. Numerical issues that can be encountered in this more complex network should be addressed first, and secondly, the development of a method to reduce the master problem size (by reducing the number of

candidates or the maximum budget) should be considered. To account for contingencies and model the N-1 situation, the size of subproblems should be reduced.

Keywords:

Transmission Expansion Planning, Detailed network, Optimization, Benders Decomposition, Long-term investments

Acknowledgements

First of all, I would like to thank Valentin Jarry-Cammas, my industrial supervisor at RTE, for introducing me to the interesting problem of grid expansion and helping me throughout this work with precious feedback and advice.

Thanks also to the members of the team 'Evolution du Réseau' : Olivier Despouys, Anne-Claire Léger, Laurent Cornaggia and Nicolas Lhuillier for all the insights and learnings on electrical networks.

Special thanks to Thomas Bittar and Manuel Ruiz for giving me key understandings on optimization problems and helping with the resolution of the problem, particularly the use of the Benders decomposition tool.

Thanks to my academic supervisor Jagruti Ramsing Thakur for her insights and guidance to conduct this master thesis.

Finally, I am thankful for all the CLER team at RTE RD. The exchanges we had about each other's work were valuable and I spent six enjioyable months.

Paris, April 2025

Nomenclature

Abbreviations

AC Alternative Current

CPU Central Processing Unit

DC Direct Current

ENSTO-E European Network of Transmission System Operators for Electricity

HVDC High-Voltage Direct Current

IEEE Institute of Electrical and Electronics Engineers

LODF Line Outage Distribution Factor

N Situation with all lines working

N-1 Situation with the loss of one line

OPF Optimal Power Flow

PST Phase Shift Transformer

RTE Réseau de Transport d'Electricité

SCOPF Security-Constrained Optimal Power Flow

SDDR Schéma Décennal de Développement du Réseau

TEP Transmission Expansion Planning

TSO Transmission System Operator

TYNDP Ten Year Newtork Development Program

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Chapter 1

Introduction

1.1 Background

The electricity sector is currently facing several challenges associated with climate change which requires industry decarbonization. Firstly, electricity generation needs to be decarbonized by transitioning to carbon dioxide-free methods such as nuclear power or renewable energy sources. In addition, to fully decarbonize the industry, a shift from fossil fuel-based energy sources to electric ones is necessary. This involves for example the development of electric vehicles instead of thermic ones or adopting electric heating systems. While the current overall electricity consumption remains stable, the future will witness the development of new electrical technologies. In conclusion, there is a growing need for an increased supply of green electricity.

The electricity transmission grid is at the heart of the energy transition. The increase in exchange capacities between countries and the electrification of uses will contribute to increase the load on the transmission network. The massive penetration of renewable generation, which is located based on the available resource rather than its proximity to network lines capable of carrying the power flow without any congestion, adds to the challenge. The transmission network requires new investments to be able to absorb the increase in power flows.

This Transmission Expansion Planning (TEP) problem has been receiving increased attention in recent years and several projects have emerged. The Ten Year Network Development Program (TYNDP) aimed to determine an European TEP and is carried out by ENTSO-E (European Network of Transmission System Operators for Electricity, an association of 43 European Transmission System Operators (TSOs). Every two years, a network analysis is conducted to identify bottlenecks and propose investment candidates evaluated under different scenarios for the following ten years [1].

At the national scale, TSOs also conduct TEP. In France, the *Schéma Décennal de*

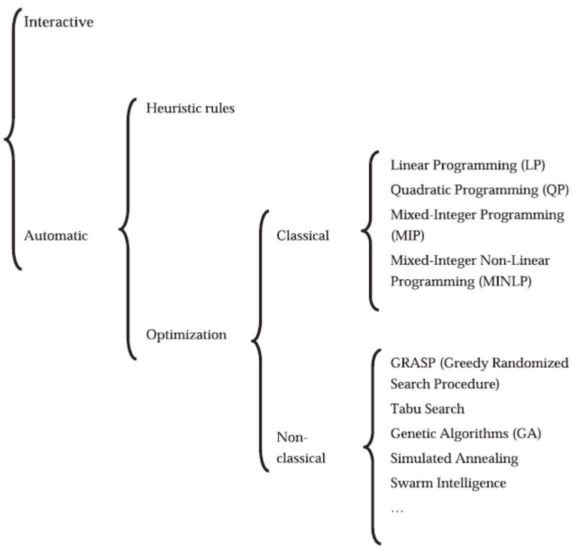


Figure 1.1.1: Solution Approaches to TEP [3]

Développement du Réseau (SDDR, in english *Decennial Grid Development Plan*) [2] is developed by RTE at the request of the government. It provides a proposal for the evolution of the transmission network (lines and substations at 400, 225, 90, and 63 kV), taking into account various scenarios aimed at achieving public objectives and carbon neutrality. In order to make strategic decision-making, Network Studies Department of RTE is conducting studies to determine investment trajectories for the extra-high voltage 400kV network, which constitutes the backbone of the public transmission network. Based on forward-looking electricity demand and production scenarios, RTE's experts evaluate future line congestions and propose investments tailored to meet the future needs. Existing solutions to relieve congestion on power lines include the creation of new AC lines, replacing existing ones with low-expansion cables, or even the development of HVDC lines. Those network expansion studies are currently managed by zone, rather than across the entire high-voltage grid.

However, the optimality and suitability of the traditional interactive approaches, where investment candidates are assessed individually by TSOs, is being questioned [3]. It is expected to be complemented by automatic search methods, mainly driven by optimization techniques (Figure 1.1.1). Yet, the size of the problem also increases as the scale of the projects grows (from regional to national or event continental, such as the European projects above mentioned), and many scenarios need to be modeled to account for long-term uncertainties in demand, renewable resources, and generation. The complexity of a classical MILP TEP problem applied to the French extra-high voltage network (at least a thousand of nodes considered) becomes too important to be solved within a reasonable time.

The objective of this master thesis, carried out within RTE's R&D department, is to develop an optimized TEP framework that could be applied to a study on the French extra-high voltage grid development. The focus is on exploring different modeling or solving approaches to make the problem tractable.

Chapter 2

Literature Review

The TEP optimization problem is a multi-stage problem. The goal is to determine which investments (main problem) will minimize the system's operational costs over a certain time (subproblem, commonly known as the Optimal Power Flow problem (OPF)).

2.1 Optimal Power Flow problem

The OPF is a mathematical optimization process used in power systems to determine the most efficient operating conditions for electricity generation and transmission. It aims to minimize operational costs while ensuring the system meets technical constraints, such as power balance, voltage limits, and line capacity. The OPF helps optimize power flow across a network while maintaining system reliability and stability [4]. The possible actions for the operator, which correspond to the variables of the problem, include setting the points of power flow controlling devices (particularly phase-shifting transformers (PSTs) and HVDC lines), redispatching generation units, and, as a last resort, implementing load shedding. The latter two options incur costs for the operator and are typically minimized [5].

However, the OPF becomes significantly more complex to solve for large networks when contingencies (loss of one or more lines, loss of a generation unit) are considered. The goal is to satisfy the N-1 criterion, a rule defined by the CEER that ensures the power system remains operational even after an outage. The OPF problem then becomes a Security Constrained Optimal Power Flow (SCOPF), which can be preventive (the system must remain operational after the loss of an element without requiring new actions) or corrective (such actions are permitted) [5] [6]. To address this problem more efficiently, several methods exist and have been compared in [6] using two academic networks [1] (24 and 118 nodes). In a preventive SCOPF, the contingency problem can be viewed as a subproblem of the OPF under N conditions (no

[1] An academic network is a simplified network model, designed to represent realistic grid conditions while being computationally manageable. A lot of them are provided by IEEE. They are used by researchers to develop and validate methods.

outage considered). Using this formulation, it is possible to employ a Benders decomposition algorithm, which generates cuts and accelerates the resolution process. Another method of robust optimization involves formulating the problem as a max-min bilevel model to identify the worst contingency for the network. Finally, a third method exploits line outage distribution factors (LODFs), which determine the proportion of power on line k that will be redistributed to another line l if line k fails. This method, also employed in a study involving multiple AC zones connected by HVDC lines [4], appears to be the most promising in terms of computation time when not too many contingencies are studied simultaneously (typically N-1 and N-2 situations).

One key advantage of the LODF method is that all factors can be calculated once, as they depend only on the network's topology, not its state at a specific time. However, this method as it stands is not compatible with an investment problem involving variable topology, as it depends on the lines being invested in. The LODFs would need to be recalculated at each iteration of the main problem, and this would make the problem non-linear. Therefore, at the industrial level, it is diffcult for TSOs to implement an expansion problem tool based on their existing OPF tools.

2.2 TEP modeling

Several modeling approaches have been adopted to reduce the complexity of the TEP problem while improving solution accuracy [3]. The main hypotheses concern power flow modeling. Indeed, the correct approach for power flow modeling is AC Power Flow, which accurately represents the physical behavior of power systems. AC Power Flow accounts for active and reactive power, voltage magnitudes and angles, and system losses. However, AC Power Flow equations are non-linear and computationally demanding, making them less practical for large-scale optimization problems. For this reason, a simplified and linearized version, DC Power Flow, is often used.

The DC Power Flow model relies on three key assumptions [5] :

1. Line resistances are negligible compared to reactances ($R << X$): As a result, losses are ignored during the calculations and are computed afterward as a correction.
2. Voltage profiles are flat: Variations in voltage magnitudes are not considered, assuming a constant value of $V = 1pu$.
3. Angle differences between neighboring nodes are small: This allows the linearization of trigonometric functions, approximating sin $(\theta_1 - \theta_2) \simeq \theta_1 - \theta_2$.

The first and second assumptions are increasingly valid at higher voltage levels. Since this study focuses on the 400 kV transmission network, they are reasonable. The third assumption is generally acceptable for 400 kV lines with well-distributed power flows, but it may become critical in cases of congestion or contingencies. Keeping these limitations in mind, the DC Power Flow model offers a balance between accuracy and computational efficiency and is therefore often favored in TEP studies [3].

Other modeling assumptions must be made [3]:

- Long-term uncertainties, primarily related to climate conditions and their impact on renewable energy generation, can be addressed using stochastic approaches. Such method involves Monte Carlo sampling over several years to capture variability. However, due to the inherent complexity of TEP, many studies focus on deterministic approaches to simplify the problem.
- To offer a more realistic approach and address the evolving nature of power systems, the TEP problem could be dynamic: it considers multiple planning stages over a long-term horizon and models the sequential decision-making process, optimizing investments at each stage while considering the future impact of current decisions. However, a static TEP, which determines the optimal set of investments for a single planning horizon, is computationally simpler.
- Including market dynamics in the model can bring it closer to real-world conditions. However, their benefits are limited in long-term studies where structural trends matter more than short-term market fluctuations. Consequently, market considerations are not a priority in this context.
- Models could also incorporate environmental impacts and social considerations. However, the primary objective remains cost minimization, which continues to drive most TEP studies.

Different methods have also been studied to solve the TEP optimization problem more efficiently, including Benders decomposition, column generation, or Lagrangian relaxation [3]. Benders decomposition is considered one of the most promising methods. It works by decomposing the problem into a master problem (investment decisions) and subproblems which can be solved in parallel (operational feasibility at each timestep). At each iteration, the investment variables are fixed, and the subproblems are solved to determine the upper bound of the problem. The subproblems then generate Benders cuts, which refine the feasible domain. The master problem is subsequently solved with those new constraints to obtain the lower bound, the current optimal solution, and update the investment variables for the next iteration. The process continues until the gap between the lower and upper bounds falls below a specified tolerance [7].

Many studies have implemented TEP using Benders decomposition on academic networks to demonstrate the effectiveness of the method. In [8], a stochastic static TEP approach is tested on the IEEE 24-bus and IEEE RTS-96 systems. Two Benders approaches (classic and bilinear Benders algorithms) are successfully validated against a standard linear solver. Additionally, the classic Benders method achieves a significant reduction in computation time, solving the problem up to two orders of magnitude faster.

Computation times still remain significant when TEP is applied to large-scale networks. The paper [9] proposes several techniques to accelerate Benders decomposition and tests them on the IEEE 46-bus and IEEE 87-bus systems. These techniques focus either on the master problem, which is slow due to its size and the presence of integer variables, or on optimizing cuts generation. The benchmark identifies the use of semi-relaxed cuts and inexact master problem resolution as promising methods.

2.3 Performance enhancement and scaling-up

Other modeling or solving methods have been tested to enhance the performance of the TEP and try to make the problem tractable on real networks. These methods can focus on either the master computation time or the subproblem computation time.

To reduce the master computation time, one possible solution is to decrease the number of investment variables by reducing the number of candidate lines. This method has been described in the paper [10]. The proposed method consists of iteratively solving a relaxed TEP to identify a set of relevant candidate lines among all possible corridors. It has been tested on a real network with around 2,000 buses, but the TEP was not solved for each hour, only for 20 operating situations. The results show that the method is effective in selecting candidates and reducing computation times. However, the optimal investment solution has not been compared to the solution obtained without reducing the candidate search space.

Acceleration techniques can also focus on the subproblems, which are complex due to the large number of constraints. As stated previously, the N-1 contingency constraints and the large number of studied contingencies significantly slow down the resolution of the OPF and, consequently, the TEP. The paper [11] introduces a dynamic approach where the worst contingency is determined iteratively using DC-OPF with LODFs and added to the TEP problem to identify investment lines that can mitigate this contingency. This process continues until no more contingencies remain. Tested on larger networks (IEEE-118 and IEEE-300), this method accelerates the process while also reducing memory requirements.

The paper [12] goes further by proposing a TEP framework that incorporates sensitivity factors to model line outages for N-1 constraints and the addition of new investment lines. This approach, tested on three academic networks (9, 24 and 118 buses) significantly reduces the number of variables, leading to a considerable decrease in computation time, which does not seem to be affected by the network size.

An other proposed solution when scaling the problem to larger networks is to reduce the number of lines and nodes modeled in the network, resulting in smaller subproblems that can be solved more quickly. Reduction techniques, such as Kron reduction, have the advantage of preserving power flows on the remaining existing lines exactly as in the real network, making them suitable for TEP studies, as shown in [13]. However, in this method, nodes, and particularly generators, are aggregated. In reality, multiple generators with specific costs and power ranges are used for redispatching, but in the model, they are grouped by type, leading to a loss of diversity. As a result, the optimal redispatching differs slightly from that of a detailed network, and the model does not fully capture the exact optimal costs.

Reduction methods have been demonstrated on a larger scale in the e-Highway project, which aims to develop a European transmission network from 2020 to 2050 using optimization techniques [14]. The methodology is based on three main steps: Reduce (nodal-to-zonal reduction based on congested lines), Expand (optimal expansion of the zonal network), and Develop (grid expansion at the nodal level). This approach, tested on the European 400 kV network, effectively reduces the problem size. However, at the zonal level, the links between zones are fictitious, and the optimization results determine capacities to invest in rather than actual transmission lines, leading to a loss of accuracy.

Some research gaps have been identified while conducting this literature review :

- Benders decomposition have been demonstrated on academic networks as a promising technique to solve TEP problem, but some aspects, such as the limits of the subproblems' parallelization, have not been studied or developed.
- The methods that have been proposed to scale-up the problem, apart from network zonal reduction or selection of studied hours, have been only tested on academic network. It is difficult for a TSO to choose one or two promising avenues to develop a tractable method.

Chapter 3

Objective and Scope

3.1 Problem

As seen in the literature review, although several methods and solutions are demonstrated to accelerate the resolution of the TEP optimization problem, the application to large-scale networks remains complex and difficult to solve. The use of network reduction methods, the consideration of fewer time steps or the relaxation of the N-1 criterion deal with excessive computation times in large-scale problems but introduce also more uncertainties, which is why optimized TEP solutions are not yet widely implemented by national TSOs. The research gap lies in the fact that, for now, TEP optimization methods have been solved for academic networks, but no solution has been demonstrated for large-scale problems with less simplifications.

3.2 Research questions

The objective of this thesis is to implement and test an optimization detailed network expansion method, and provide guidance to make it suitable for a TSO's studies.

This thesis shall respond to these following research questions:

- **To what extent Benders is a promising technique to solve a network expansion problem ?**
- **What parameters impact the most the computation time on a large-scale network ?**

3.3 Scope

The scope of this thesis is the following :

- Two networks will be used to develop, test and adapt the TEP model, and answer to the above research questions : an academic network provide by IEEE (RTS-96) and a large-scale study of RTE on high-voltage level network expansion.

- One year (which consists of 8736 hours in RTE's studies) will be considered.
- The timestep of the problem is 1 hour, and each hour is independent while computing operation costs.
- The expansion problem is a single-objective problem, which aims to minimize operation costs under a given investment budget.
- The solving methods which will be tested are : classic MILP solving (branch and bound), Benders without stabilization, Benders with stabilization.
- Both optimization results (network operation, invested lines and minimized objective function) and computation times will be analyzed.

Some assumptions will be made :

- Only one set of timeseries is considered, the model is not stochastic.
- The power flows calculations are made under DC assumptions.

Chapter 4

Methodology

This chapter presents the methodology of the expansion tool developed in this thesis, which is summarized in the Figure 4.0.1. The process consists of several stages. First, the inputs data are translated into AMPL [1] format, an optimization language used to formulate our problems. The initial operation costs and conditions are computed through an Optimal Power Flow model developed in this thesis. The Transmission Expansion Planning problem, also developed in this thesis, is first split into a master problem and subproblems and translated into MPS files (a format used to archive LP and MILP problems), and then solved using a Benders Decomposition algorithm. It has been implemented before the thesis but modified to adapt to the TEP problem. The solution obtained after solving with Benders consists only of the invested lines. Therefore, a second OPF is run with updated network data to determine the new operation costs, which can then be compared to the initial ones.

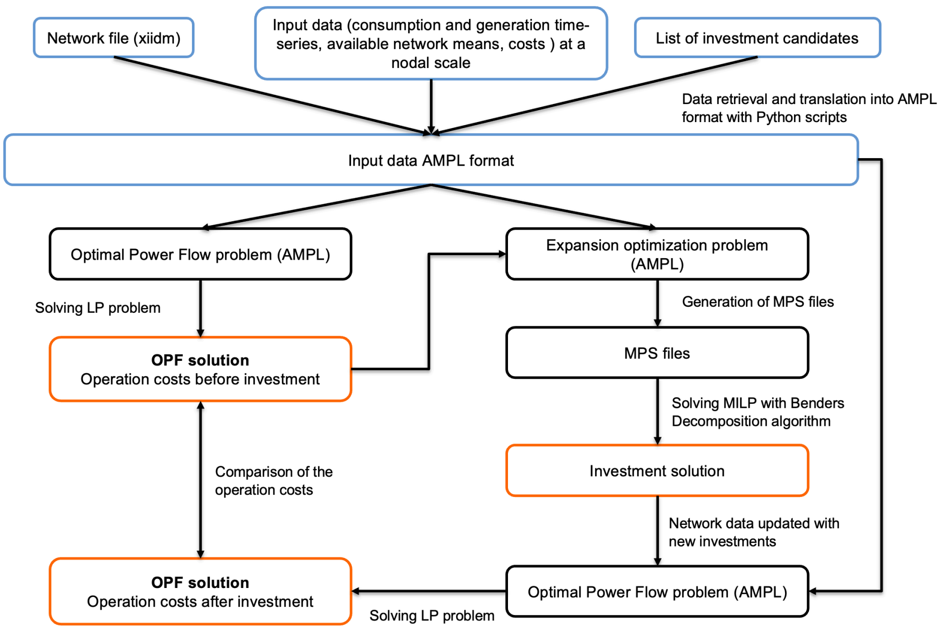


Figure 4.0.1: Methodology overview diagram

[1] AMPL (A Mathematical Programming Language) is a high-level programming language used for optimization modeling [15].

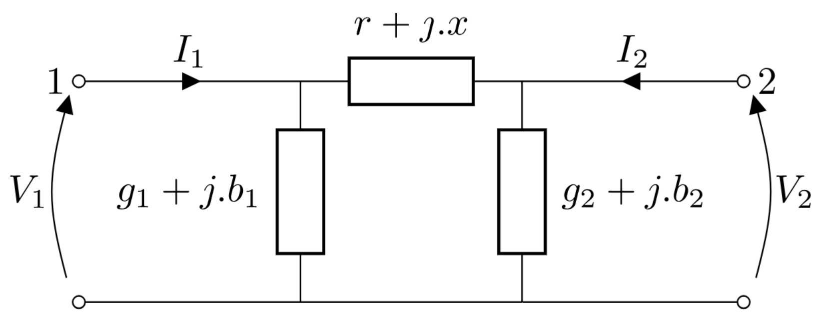


Figure 4.1.1: Electrical model of an AC line

4.1 Input Data

This section presents the network components that will be modeled in our problem. Additionally, it introduces the consumption and production time-series used over a one-year period, capturing variations in demand and operational conditions. These data inputs serve as the foundation for the problem formulation.

4.1.1 Network

In theoretical models, a network can be represented by buses interconnected through branches and other network components. This subsection introduces the key components of the network, details the power flow calculations, and presents the devices utilized to control power flow.

Branches

A branch l connects two buses m and n. It can be an AC transmission line (between two buses of same nominal voltage level) or a transformer (between two different nominal voltage levels within a substation).

An AC line is modeled by its impedance and two shunts (Figure 4.1.1). The impedance can be decomposed into a real part (resistance) and an imaginary part (reactance) : $Z = R + jX(\Omega)$. The resistance accounts for Joule losses during energy transmission along the line. The reactance is positive, indicating that the line is inductive. In power flow computations, the shunt conductance G and susceptance B are often neglected [5].

A two-winding transformer connects two different voltage levels (side 1 and side 2). It works by transferring electrical energy between these voltage levels through magnetic induction. Like an AC line, it is represented in the equations by its resistance, reactance, and the nominal voltage at each side (Figure 4.1.2).

In power flow calculations, it is common to use a per-unit (p.u.) system. The per-unit values of branch characteristics are computed using the nominal voltage levels of the buses (V_{nom} for the

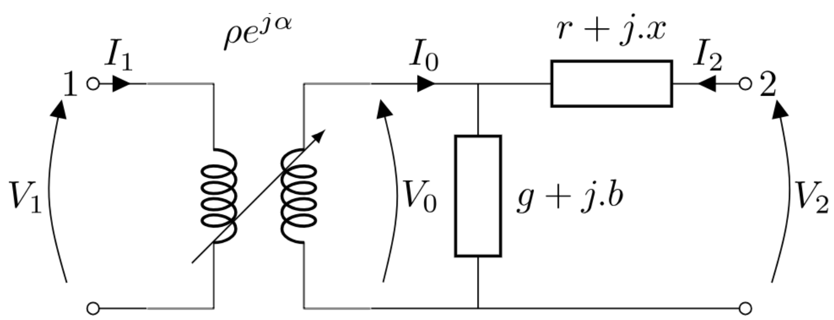


Figure 4.1.2: Electrical model of a two-winding transformater

AC lines and V_{nom2} for the transformers) and a chosen base power P_b (100MW for e.g.) :

$$x = X\frac{P_b}{V_{nom}^2} \tag{4.1}$$

The bus voltages are all set to 1 p.u., and the powers P (line power flow, generator, and load injections) are represented as $p = \frac{P}{P_b}$.

At each bus during grid operation, the voltage can be characterized by its amplitude $|V|$ and phase angle θ, as described by $V = |V|e^{i\theta}$. Under the DC approximations [5], we consider that $R_L \ll X_L$. It simplifies the formulation of the active power flow (the real part of apparent power) through a line l going from bus m to bus n given by :

$$F_l = \frac{|V_m||V_n|}{X_l}\sin\left(\theta_m - \theta_n\right) \tag{4.2}$$

In addition, because the voltage profile is assumed to be flat, $|V| \approx 1p.u$ for each bus. Finally, the DC power flow assumes that the difference of voltage phase angle is small between neighboring nodes, connected by a line, and it gives $\sin\left(\theta_m - \theta_n\right) \approx \theta_m - \theta_n$. It refines the the power flow formulation for each line l to a linear equation, given in $p.u.$:

$$f_l = \frac{1}{x_l}(\theta_m - \theta_n) \tag{4.3}$$

Phase-Shift Transformers

Some transformers can control the flow of active power by regulating the voltage phase angle between the two nodes they connect. These are called Phase-Shift Transformers (PSTs). A PST has a set of tap positions, often centered around 0. Each tap corresponds to a phase shift angle α, which modifies the power flow as follows:

$$f_l = \frac{1}{x_l}(\theta_m - \theta_n + \alpha) \tag{4.4}$$

Additionally, the action of phase shifting results in a change in the transformer reactance. Each tap is also associated with a deviation in reactance $x_{tap}(\alpha)$, given as a percentage. The final equation for the power flow across the PST is:

$$f_l = \frac{1}{x_l(1+\frac{x_{tap}(\alpha)}{100})}(\theta_m - \theta_n + \alpha) \tag{4.5}$$

HVDC lines

High-Voltage DC (HVDC) lines are primarily used to connect islands or different synchronous areas, particularly through offshore links, due to their advantages such as lower transmission losses over long distances and the ability to transmit large amounts of power. Unlike AC lines, the power flow across an HVDC line is controllable within a range, and the voltage phase angles of the two connected buses are decoupled. To transform the power from AC to DC and to AC again, converters are used. The converter functions as a rectifier when converting from AC to DC and as an inverter when converting from DC to AC. There are two main types of converters, each with its own advantages [16]:

- Line Commutated Converters (LCC) : more reliable during faults
- Voltage Source Converters (VSC) : better power control and flexibility to reverse the use of converters (inverter to rectifier and conversely)

Conventionally, the power flow across an HVDC line goes from the rectifier to the inverter. Thus, the power through the rectifier is considered positive, while the power through the inverter is negative. To account for the losses due to the converters, if the power entering the rectifier is p^{VSC}, the power output from the inverter is $p^{VSC} * (1-\lambda)^2$, where λ represents the loss factor of the converters.

Generators and Loads

The power injected into the grid typically comes from various generators connected to the buses. In practice, the target power of a generator G can be controlled if it is a thermal group, a nuclear or an hydroelectric plant, or depends on the resource in case of renewable plants. The power is also limited by a maximum threshold $\overline{G}$ due to the generator's design capacity. Thermal generators also have a minimum threshold $\underline{G}$ above zero when they are turned on.

The power withdrawn from the grid generally comes from loads, which represent the consumption C at the buses.

Batteries may also be installed in a grid, allowing them to capture power from the grid, store it, and reinject it later. The use of batteries will not be modeled in this thesis.

4.1.2 Time series

In RTE's studies, a year is represented by 8736 hours instead of 8760, to account for a whole number of weeks. The following inputs are then given for 8736 time steps.

Production and consumption The model relies on production time series for each generator and consumption time series for each load. These time series are derived froml long-term studies. The simplified process for obtaining them is as follows [2] :

1. Demand curves are estimated at a zonal scale based on various assumptions, scenarios, and forecasting methodologies.

2. Adequacy is performed using the open-source tool Antares Simulator, which is a power system optimization tool [17]. This step determines the allocation of generation to meet demand while considering multiple constraints, such as generation limits and approximate transmission capacities. This adequacy assessment is conducted at a zonal level rather than a nodal level.

3. Network mapping is then carried out using additional internal tools. This step translates the zonal results into a more detailed network representation, assigning a consumption time series to each load and a production time series to each generator. These mapped time series serve as inputs for our TEP problem.

It is important to note that due to the mapping process, certain inconsistencies may occur in the data (e.g., production levels falling below the minimum required generation). I must be corrected beforehand in the problem formulation.

HVDC and PST The network elements that can regulate power flows (HVDC lines and PSTs) are assigned a setpoint at each time step, which they may or may not modified depending on their control mode. The power setpoints for HVDC lines also come from Antares.

4.2 TEP Problem Formulation

The TEP optimization problem implemented for this study is a Mixed-Integer Linear Problem (MILP) multi-stage problem. The optimization language used is AMPL. The main problem determines the optimal combination of investment candidates to minimize the operation costs computed in the subproblem. This section presents the parameters, variables [3], and constraints (equations (C1) to (C20)) used to model the investment candidates, the network facilities, their operation and the consideration of contingencies.

[2]The first two steps were completed prior to this thesis as part of existing studies.
[3]The variables will be written in **bold**.

4.2.1 Main problem : Investment

In the investment problem, a set of new candidate lines is given. These lines can either reinforce existing lines or create new connections between two buses. Each candidate line l has an associated investment cost C_l, which may depend on factors such as line length. To compare the investment costs to the operational costs over one year and determine the gain from the investment in these lines, annualized costs are calculated. The assumption is that the lifespan of an overhead line is 70 years. The annualized cost for each candidate is then given by: $\varsigma_l = \frac{C_l}{8760*70}$.

The binary variable $\boldsymbol{z_l}$ is used to represent the investment decision for candidate l. If the TEP invests in the line, $\boldsymbol{z_l} = 1$; if not, $\boldsymbol{z_l} = 0$.

The investment costs are subject to a constraint based on a maximum budget set by the TSO:

$$\sum_l C_l * \boldsymbol{z_l} \leq C_{max} \tag{C1}$$

4.2.2 Subproblem : Security-Constrained DC OPF

The subproblem is actually composed of as many independent SCOPF subproblems as there are timesteps. This means that the actions taken at a certain time t do not influence the actions at subsequent times. In generation planning optimization, the timesteps are generally not independent since they account for unit commitment and the minimum downtime of thermal generators. However, generation planning is calculated in advance and provided as an input in the present TEP. Additionally, there is no use of storage systems such as batteries to regulate network operations, which makes the problems at each timestep independent.

The network facilities and the equations that model their operation are provided below for each timestep t. Only the main connected component of the network, which may consist of multiple synchronous components connected by HVDC lines, is considered. All parameters are given in $p.u.$.

Branches

Each branch is described by its unique index l and its two terminal buses m and n. Its impedance is x_l. The power flow through the line l is given by the variable $\boldsymbol{f_{l,t}}$, which is computed as below :

$$\boldsymbol{f_{l,t}} = \frac{1}{x_l}(\boldsymbol{\theta_{m,t}} - \boldsymbol{\theta_{n,t}}) \tag{C2}$$

A line can be monitored, meaning that the power flow through it should not exceed a certain limit. The binary parameter γ_l^N indicates whether a line is monitored in N-situation (i.e., when there is no contingency). In that case, the maximum power flow is given by $\overline{f}_l$.

$$|\boldsymbol{f_{l,t}}| \leq \overline{f}_l \quad \text{if } \gamma_l^N = 1 \tag{C3}$$

The power flow through a candidate line l is denoted $\boldsymbol{f_l^{\text{new}}}$. In the model, new lines are always monitored .If the candidate line is installed, the power flow is s subject to the same constraints as existing lines (refer to equations C2 and C1). If the line is not installed, the power flow is equal to 0 and and there are no constraints on the voltage phase angles between the two corresponding buses. This can be expressed as follows:

$$\boldsymbol{f_{l,t}^{\text{new}}} = \begin{cases} \frac{1}{x_l}(\boldsymbol{\theta_{m,t}} - \boldsymbol{\theta_{n,t}}) & \text{if } z_l = 1 \\ 0 & \text{if } z_l = 0 \end{cases} \tag{4.6}$$

$$|\boldsymbol{f_{l,m,n,t}^{\text{new}}}| \leq \overline{f}_{l,t} \tag{4.7}$$

To write these equations correctly in the context of an MILP problem and ensure their proper resolution, a big-M formulation is used. This approach allows the introduction of binary variables without the need for if conditions. The constraints can be rewrite as follow :

$$|\boldsymbol{f_{l,t}^{\text{new}}} - \frac{1}{x_l}(\boldsymbol{\theta_{m,t}} - \boldsymbol{\theta_{n,t}})| \leq M_l(1 - z_l) \tag{C4}$$

$$|\boldsymbol{f_{l,t}^{\text{new}}}| \leq \overline{f}_{l,t} z_l \tag{C5}$$

The parameter M should be chosen sufficiently large to avoid infeasibilities, yet small enough to accelerate the problem's resolution. In constraint C5, the optimal value for M is $\overline{f}_{l,t}$, as it yields the same result as equation 4.7. The choice for the coefficient M_l in constraint 4.6 is less straightforward. If $z_l = 0$, the difference $\theta_{m,t} - \theta_{n,t}$ is unconstrained. M_l should be at least larger than the maximum possible difference in voltage phase angles between buses ($\Delta\theta_{max}$). This value is obtained by performing a pre-investment OPF resolution, and for each line, $M_l = \frac{2\Delta\theta_{max}}{x_l}$ (a factor of 2 is added to account for potential differences between the simple OPF problem and the TEP).

PSTs

In the model, PSTs are treated as branches with additional characteristics, such as a range of phase shift and corrected impedance. These parameters are indexed by a set of PSTs, with the index k representing each PST in the network. For each branch l, the indicator δ_l^{PST} is defined as 0 if the branch is not a PST, and k if it corresponds to a specific PST. The positive variables $\boldsymbol{\alpha_{k,t}^+}$ and $\boldsymbol{\alpha_{k,t}^-}$ represent the phase shift of the PST around the zero position such that $\alpha = \boldsymbol{\alpha_{k,t}^+} - \boldsymbol{\alpha_{k,t}^-}$. In reality, the PST can only take discrete taps between a minimum and maximum phase shift. However, to simplify the resolution of the TEP, integer variables are avoided in the subproblems. Therefore, $\boldsymbol{\alpha_{k,t}^+}$ and $\boldsymbol{\alpha_{k,t}^-}$ are continuous.

The control mode of the PST k is defined by μ_k^{PST}. If μ_k^{PST} = 1, the PST can be optimized and the phase shift stays within the bounds. If not, its phase shift is fixed at each time step t to a given value $\alpha_{k,t}^0$. This introduces new constraints:

$$\underline{\alpha_k} \leq \boldsymbol{\alpha_{k,t}^+} - \boldsymbol{\alpha_{k,t}^-} \leq \overline{\alpha_k} \tag{C5}$$

$$\boldsymbol{\alpha_{k,t}^+} - \boldsymbol{\alpha_{k,t}^-} = \alpha_{k,t}^0 \qquad \text{if } \mu_k^{PST} \neq 1 \tag{C6}$$

As presented in subsection 4.1.1, the reactance of the PST is also impacted when phase shifting. However,$x_{k,t}^{PST}$ does not vary linearly with α and the formula for power flow computation becomes non-linear. For modeling purposes, this value is then fixed, depending only on the control mode of the PST:

- If $\mu_k^{PST} = 1$, the PST impedance is equal to the impedance when the phase shift is zero: $x_{k,t}^{PST} = x_{k,t}^{PST}(0)$
- If $\mu_k^{PST} \neq 1$, the PST impedance is equal to the impedance at the initial phase shift: $x_{k,t}^{PST} = x_{k,t}^{PST}(\alpha_{k,t}^0)$

The constraint on power flow calculation through PSTs can then be written as follows:

$$\boldsymbol{f_{l,t}} = \frac{1}{x_{k,t}^{PST}}(\theta_{m,t} - \theta_{n,t} + \boldsymbol{\alpha_{k,t}^+} - \boldsymbol{\alpha_{k,t}^-}) \qquad \text{if } \delta_l^{PST} = k \geq 1 \tag{C7}$$

HVDC lines

Each HVDC line is indexed by its identifier k and is connected to buses n_1 and n_2 through two VSC converters v_1 and v_2. At each timestep t, a target power is given for the HVDC line: $p_{k,t}^{HVDC,0}$. It is possible to adjust this power through the HVDC by modifying it with two positive variables $\boldsymbol{\Delta p_{k,t}^{HVDC,+}}$ and $\boldsymbol{\Delta p_{k,t}^{HVDC,-}}$, such that the new power through the line is $p_{k,t}^{HVDC,0} + \boldsymbol{\Delta p_{k,t}^{HVDC,+}} - \boldsymbol{\Delta p_{k,t}^{HVDC,-}}$. Like a PST, an HVDC line has two operating modes. If the parameter $\mu_k^{HVDC} = 1$, the HVDC power can be optimized. Otherwise, the variables are set to zero.

$$\boldsymbol{\Delta p_{k,t}^{HVDC,+}} = 0 \qquad \text{if } \mu_k^{HVDC} \neq 1 \tag{C8}$$

$$\boldsymbol{\Delta p_{k,t}^{HVDC,-}} = 0 \qquad \text{if } \mu_k^{HVDC} \neq 1 \tag{C9}$$

The power through the two converters is $\boldsymbol{p_{v1}^{VSC}}$ and $\boldsymbol{p_{v2}^{VSC}}$. To simplify the model, the loss factor λ is assumed to be zero for all converters. The chosen convention also specifies that the power mentioned for the HVDC line corresponds to the power flowing through the converter v_1. This imposes the following conditions:

$$\boldsymbol{p_{v1,t}^{VSC}} = p_{k,t}^{HVDC,0} + \boldsymbol{\Delta p_{k,t}^{HVDC,+}} - \boldsymbol{\Delta p_{k,t}^{HVDC,-}} \tag{C10}$$

$$\boldsymbol{p_{v1,t}^{VSC}} = -\boldsymbol{p_{v2,t}^{VSC}} \tag{C11}$$

The power through each converter should remain within a specified range:

$$\underline{p_v^{VSC}} \leq \boldsymbol{p_{v,t}^{VSC}} \leq \overline{p_v^{VSC}} \tag{C12}$$

Generators

Each generator (also called unit) is indexed by its identifier k and is connected to its bus n. A target generated power $g_{k,t}^0$ is given. In case of congestion on certain lines, the operator can perform redispatching between different production units. Each generator thus has two positive variables, $\boldsymbol{g_{k,t}^+}$ and $\boldsymbol{g_{k,t}^-}$, which correspond to positive and negative redispatching, respectively. The parameter μ_k^{UNIT} describes the operating mode of the generator, as not all units are available for redispatching. If it is equal to 1, the unit is available. Otherwise:

$$\boldsymbol{g_{k,t}^+} = 0 \qquad \text{if } \mu_k^{UNIT} \neq 1 \tag{C13}$$

$$\boldsymbol{g_{k,t}^-} = 0 \qquad \text{if } \mu_k^{UNIT} \neq 1 \tag{C14}$$

Each generator has a minimum power $\underline{g_k}$ and maximum power $\overline{g_k}$, depending on the specific characteristics of the unit. The new generation after redispatching should stay within these ranges. However, thermal units have two operating setpoints: they can either be turned on with $g_{k,n,t}^0 \in [\underline{g_k}, \overline{g_k}]$ ($\underline{g_k} > 0$) or shut down, with $g_{k,n,t}^0 = 0$. In reality, redispatching should be performed on running generators; otherwise, startup costs and constraints must be considered.

To simplify this in the model, the plants are always considered to be turned on if $\mu_k^{UNIT} = 1$, and the minimum power is reevaluated if $g^0_{k,n,t} < \underline{g_k}$.

$$\min\left(\underline{g_k}, g^0_{k,t}\right) \leq g^0_{k,t} + \boldsymbol{g^+_{k,t}} - \boldsymbol{g^-_{k,t}} \leq \overline{g_k} \tag{C15}$$

Each generator has an upward cost $\varsigma_k^{UNIT,+}$ and a downward cost $\varsigma_k^{UNIT,-}$ associated with redispatching. This cost is initially expressed in €/MWh, and divided in the model by the power base to adapt to the per-unit system.

- For thermal or nuclear generators, redispatching costs are equal to the marginal costs. Positive redispatching requires more fuel ($\varsigma_k^{UNIT,+} = c_{marg}$), while negative redispatching saves fuel ($\varsigma_k^{UNIT,-} = -c_{marg}$).
- For renewable generators (solar, wind, or hydro), the marginal costs are zero. The cost associated with negative redispatching is $\varsigma_k^{UNIT,-} = 0$. However, positive redispatching is considered impossible since the resource cannot be increased to produce more. To prevent positive redispatching, the cost is set to a high value: $\varsigma_k^{UNIT,+} = 99999$.

The resulting operating costs for each unit are $\varsigma_k^{UNIT,+} * \boldsymbol{g^+_{k,t}} + \varsigma_k^{UNIT,-} * \boldsymbol{g^-_{k,t}}$.

Loads

Each load k connected to a bus n represents a specific consumption $c^0_{k,t}$. If network congestion cannot be relieved through HVDC lines, PSTs, or production redispatching, the final recourse is to reduce consumption, resulting in a loss of load, denoted as $\boldsymbol{\Delta c^-_{k,t}}$. This loss is always positive, but the remaining load must not fall below a minimum limit $\underline{c_k}$, ensuring :

$$c^0_{k,t} - \boldsymbol{\Delta c^-_{k,t}} \geq \underline{c_k} \tag{C16}$$

Since loss of load is the least desirable outcome, it is associated with a high cost ς_k^{LOAD} (set at 13000 €/MWh in this study). The cost of the loss of load is given by $\boldsymbol{\Delta c^-_{k,t}} * \varsigma_k^{LOAD}$.

Furthermore, the loss of load must remain balanced with production, ensuring:

$$\boldsymbol{g^+_{k,t}} - \boldsymbol{g^-_{k,t}} + \boldsymbol{\Delta c^-_{k,t}} = 0 \tag{C17}$$

Power Balance

All power flows across lines and power injections are interconnected through a power balance constraint at each bus in the network. The convention used in the model is as follows: any power entering the node is considered negative, while any power leaving the node is considered positive. This results in the following constraint for each bus b :

$$\begin{aligned}&\sum_{m=b} \boldsymbol{f_{l,t}} - \sum_{n=b} \boldsymbol{f_{l,t}} \\ &+ \sum_{m=b} \boldsymbol{f_{l,t}^{\text{new}}} - \sum_{n=b} \boldsymbol{f_{l,t}^{\text{new}}} \\ &+ \sum_{n=b} \boldsymbol{p_{v,t}^{\text{VSC}}} \\ &- \sum_{n=b} \left[g_{k,t}^{0} + \boldsymbol{\Delta g_{k,t}^{+}} - \boldsymbol{\Delta g_{k,t}^{-}}\right] \\ &+ \sum_{n=b} \left[c_{k,t}^{0} - \boldsymbol{\Delta c_{k,t}^{-}}\right] \\ &= 0\end{aligned} \tag{C18}$$

For each synchronous area of the network, a slack bus is selected. The primary function of the slack bus is to accommodate any initial imbalance between load and supply ($\Delta P_{LS} = \sum c_{k,t}^{0} - \sum g_{k,t}^{0}$), This net difference can be incorporated into the power balance equation of the slack bus. Slack buses are typically chosen from significant and controllable units within the synchronous area, such as a thermal or nuclear plant. For each slack bus b_s, the constraint C18 becomes :

$$\begin{aligned}&\sum_{m=b} \boldsymbol{f_{l,t}} - \sum_{n=b} \boldsymbol{f_{l,t}} \\ &+ \sum_{m=b} \boldsymbol{f_{l,t}^{\text{new}}} - \sum_{n=b} \boldsymbol{f_{l,t}^{\text{new}}} \\ &+ \sum_{n=b} \boldsymbol{p_{v,t}^{\text{VSC}}} \\ &- \sum_{n=b} \left[g_{k,t}^{0} + \boldsymbol{\Delta g_{k,t}^{+}} - \boldsymbol{\Delta g_{k,t}^{-}}\right] \\ &+ \sum_{n=b} \left[c_{k,t}^{0} - \boldsymbol{\Delta c_{k,t}^{-}}\right] \\ &= \Delta P_{LS}\end{aligned} \tag{C19}$$

In addition, slack buses b_s are used as a reference to compute voltage phase angle. Their voltage phase angles are then set to 0 :

$$\boldsymbol{\theta_{b_s,t}} = 0 \tag{C20}$$

Contingencies

The subproblem models contingencies to ensure compliance with the N-1 criterion: the network must remain operational even with the loss of a line and without any further actions. Ideally, this criterion would be checked for every line, but TSOs typically select only a subset of lines, considered the most critical in the network, to verify the N-1 criterion. Incidents that break the connectivity of the network (i.e., splitting it into two separate networks) are not considered, as they cause a supply-demand imbalance in each new area and would require corrective actions to restore the situation. Similarly, incidents that break the synchronicity of the network (i.e., splitting one synchronous area into two, connected only by an HVDC line) are also not considered, since the power flow through the HVDC line is predetermined and cannot adjust without operator intervention to reconfigure the flows and ensure power balance at each node.

A set of contingencies indexed by i is provided as input. $i = 0$ represents the normal (N) situation, while $i > 0$ represents an N-1 situation where line $l = i$ is out of service.

In the OPF problem without investment, Line Outage Distribution Factors (LODFs) are calculated using the network topology (which nodes are connected) and the line impedances. Details of the calculation can be found in [18]. The LODFs are represented by the parameter ρ_l^i, which indicates the redistribution of power on line l during contingency i, where line i is out of service. The new power flow in N-1 conditions on line l is given by:

$$\boldsymbol{f_{l,t}^{N-1,i}} = \boldsymbol{f_{l,t}^{N}} + \rho_l^i \cdot \boldsymbol{f_{i,t}^{N}} \tag{4.8}$$

If the line l is monitored in N-1 situation ($\gamma_l^{N-1} = 1$), a new constraint is added :

$$|\boldsymbol{f_{l,t}^{N-1,i}}| \leq \overline{f_{l,t}^{N-1}} \qquad \text{if } \gamma_l^{N-1} = 1 \tag{4.9}$$

In the TEP problem, the network topology is expected to change as new lines are added. Consequently, the LODFs cannot be computed prior to the optimization and depend on the investment variables $\boldsymbol{z_l}$. Therefore, the following variables, that define the network state, must be indexed with the contingencies: $\boldsymbol{f_{l,t,i}}$, $\boldsymbol{f_{l,t,i}^{\text{new}}}$, $\boldsymbol{\theta_{n,t,i}}$.

The constraints that compute the power flow across lines (C2, C3, C4, C5, C7) are thus modified and applicable in both the N and each N-1 contingency situation. This therefore adds many more variables and constraints than the LODF formulation.

The other variables related to the operation of PSTs, HVDC lines, generators, or loads are not indexed by i. Indeed, only preventive operations are modeled. Therefore, a single mode of operation is calculated at each time step, which must ensure the network functions properly in both N and N-1 situations.

4.2.3 Objective function

The subproblem's objective function comprises the costs of the operations that help alleviate network congestion and meet the constraints outlined in section 4.2.2. These operational costs include redispatching costs and loss of load costs. However, because of the mapping, the generation planning provided as input to our problem can be suboptimal (e.g., a cheaper unit is not generating when it could), and the solver may attempt to re-optimize this planning through dispatching, even when there is no congestion to relieve. This artificial redispatching is unnecessary and introduces additional negative costs that do not reflect actual network operation. To prevent this, a gap cost ς^{Δ} is introduced in the objective function, ensuring that any redispatching incurs a minimal positive cost. This gap cost must be at least equal to the maximum marginal cost of all generators.

Additionally, while the use of PSTs and HVDC lines incurs no direct costs, small penalties ς^{PST} and ς^{HVDC} are added to the objective function. These penalties help avoid equivalent solutions and prevent unnecessary increases in computation time.

In the literature, the objective function of a TEP model aims to minimize the sum of investment costs, adjusted to the study period, and operating costs over the same period. In this thesis, the objective function will only consider operating costs. As explained above, the use of a gap cost is necessary to avoid artificial re-dispatching opportunities. If the objective function includes both operating and investment costs, it could be minimized by opting for cheaper investments which lead to a reduction of the objective function but with higher operation costs. Therefore, this model treats investment costs as a constraint (e.g., capped at X euros) rather than incorporating them into the objective function. This approach enables the identification of the configuration with the lowest OPEX within a given CAPEX budget, aligning more closely with the methodology used by TSOs in their studies.

However, to avoid equivalent investment solutions that could slow down the resolution of the problem, each line is assigned a unique weight ς_l^{NEW} which is added to the objective function. These weights are significantly lower than the operational costs.

The objective function of the problem is then

$$\min \sum_{t} \Bigg\{ \sum \left[\boldsymbol{\Delta c^{-}_{k,t}} \cdot (\varsigma_k^{\text{LOAD}} + \varsigma^{\Delta}) \right] \\ + \sum \left[\boldsymbol{\Delta g^{+}_{k,t}} \cdot (\varsigma_k^{\text{UNIT},+} + \varsigma^{\Delta}) + \boldsymbol{\Delta g^{-}_{k,t}} \cdot (\varsigma_k^{\text{UNIT},-} + \varsigma^{\Delta}) \right] \\ + \sum \left[(\boldsymbol{\Delta p^{\text{HVDC,-}}_{k,t}} + \boldsymbol{\Delta p^{\text{HVDC,+}}_{k,t}}) \cdot \varsigma^{\text{HVDC}} \right] \\ + \sum \left[(\boldsymbol{\alpha^{+}_{k,t}} + \boldsymbol{\alpha^{-}_{k,t}}) \cdot \varsigma^{\text{PST}} \right] \\ + \sum \left[\boldsymbol{z_l} \cdot \varsigma_l^{\text{NEW}} \right] \Bigg\} \tag{Obj. Fun.}$$

4.3 TEP Problem Solving

In this master's thesis, two solving methods are tested and compared: a classical MILP solving method and one implementing Benders decomposition. The *XPRESS* solver [19] was used either to solve the entire problem in the case of the first method or to independently solve the master problem and the subproblem in the case of the second method.

4.3.1 Classic solving method

To solve the entire TEP, which is a linear optimization problem with binary variables, the solver applies the branch and bound algorithm. It first relaxes the integer variables and resolves the linear problem with continuous variables. Once the relaxed problem is solved, an integer solution is searched. The algorithm works by systematically exploring subsets of the solution space, dividing the problem into smaller subproblems (branching), and calculating bounds for these subproblems to eliminate non-promising regions (bounding). The process continues recursively until an optimal integer solution is found.

4.3.2 Benders decomposition

As discussed in the literature review, Benders' technique [20] is used to solve multi-stage problems more efficiently. Since this aligns with the structure of the TEP problem, it can be effectively applied here. The Benders algorithm used have been developed prior to the thesis by RTE and have been adapted to the TEP problem during the tests on the academic network.

Principle

The principle of Benders decomposition, illustrated in Figure 4.3.1, involves the following steps:

Problem decomposition : The problem is split into two parts: the master problem (investment decisions) which is a MILP and the subproblems (SCOPF) which are formulated as LPs. The subproblems are independent and can be solved in parallel, depending of the number of CPUs used. The size and number of the subproblems is chosen before the beginning of the process. An optimality gap ε is defined.

Iteration 1 :

- The master problem is solved with only the constraints related to investment variables (e.g., budget constraints). The objective function from this solution provides underestimated operational costs due to the lack of detailed constraints. This value becomes the lower bound (LB), shown in red in the graph.
- The investment solution z_1 obtained from the master problem is sent to the subproblems.
- The subproblems are solved with z fixed at z_1. The resulting objective function value is feasible but not optimal, becoming the upper bound (UB), depicted in white on the graph.
- The difference between UB and LB is evaluated.
- The subproblem solutions generate cuts, which are constraints linking operational costs to investment variables, added to the master problem. Each subproblem produces a corresponding cut.

Iteration 2 :

- The master problem is resolved with the new constraints (cuts). This provides a new LB for operational costs and a new investment solution z_2.
- The subproblems are solved with z fixed at z_2. The objective function value now becomes the new UB, and the solution generates additional cuts for the master problem.
- The difference between the new UB and LB is calculated. If it is below the defined optimality gap ε, the process stops; otherwise, it continues.

. . .

Iteration i :

- The master problem is solved with all accumulated cuts obtained with the previous iterations. If the objective function exceeds the previous LB, it becomes the new LB. The investment solution is z_i.
- The subproblems are solved with z fixed at z_i. If the objective function is below the previous UB, it becomes the new UB. New cuts are generated.

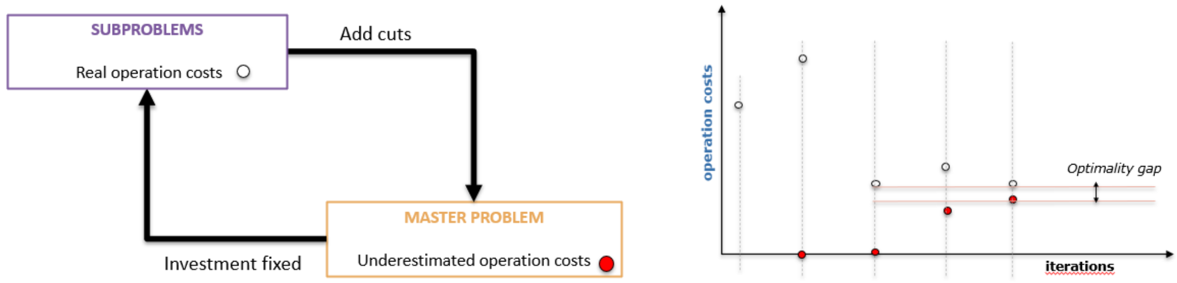


Figure 4.3.1: Benders decomposition principle

- If the difference between UB and LB is less than ε, the process stops, and the investment solution z_i is returned as the final result.

This iterative process continues until the gap between UB and LB falls below the predefined optimality threshold, ensuring a near-optimal solution is found.

Subproblem size

As explained in Benders decomposition principle, the size of the subproblems (the number of hours studied in each), is chosen by the user. When working over a year, the problem can be structured as a single subproblem of 8736 hours, 8736 subproblems of 1 hour, or any combination in between. At each iteration, each subproblem is solved independently and generates a cut that is added to the master problem.

The solving times for subproblems of different sizes on the academic network are depicted in the Figure 4.3.2. It appears that computation time is not linear with the number of hours. Therefore, minimizing the number of hours per subproblem helps linearize and reduce their computational time.

However, increasing the number of subproblems leads to more constraints in the master problem, significantly increasing its solving time. Consequently, the size of the subproblems must be chosen wisely to balance computational efficiency. In the Results section, the size of the subproblems will be specified for each test.

Acceleration technique : stabilization

As explained in the literature review, additional techniques exist to accelerate the Benders algorithm. In this thesis, the stabilization method will be tested and applied.

With stabilization, the first iterations of the Benders algorithm are modified as follows :

- The master is solved, yielding a solution z_i.
- The investment solution that has given the best UB up to this iteration is denoted $\overline{z}$

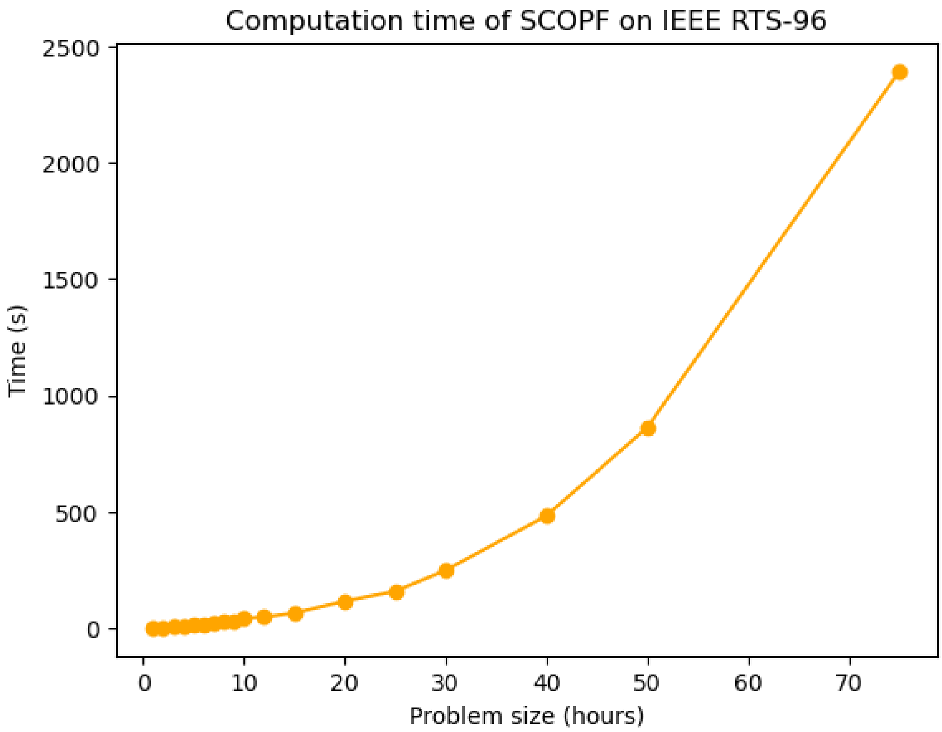


Figure 4.3.2: Computation time of subproblem for the IEEE RTS-96

- The subproblems are solved using a combination of these two solutions : $z = \alpha z_i + (1 - \alpha)\overline{z}$ where α is called the separation parameter.
- If the objective function value is lower than the current best UB, it becomes the new UB. Cuts are added to the master problem.
- Once the difference between the best UB and the LB is smaller than a relaxed gap ε_{stab}, the stabilization stops and the Benders algorithm resumes in its standard form, solving the subproblems with z_i.

In this thesis, the separation parameter α is set to 0.5.

4.4 Case study

This section will present the two case studies of the thesis. One academic case called, based on the IEEE RTS-96 network, and one real study on french high-voltage level network, called THT400, will be used to develop and test the model.

4.4.1 Networks

In this thesis, the TEP model will be developed and tested on two networks:

- The IEEE RTS-96 (Reliability Test System) [21] to which 2 Phase Shifts Transformers

(PSTs) and an HVDC line were added (following the study [22]) (Figure 4.4.1).

- The European network grid, called THT400 in the study, used in a RTE long-term study on the development of high-voltage (including all 400 kV and some 225 kV lines).

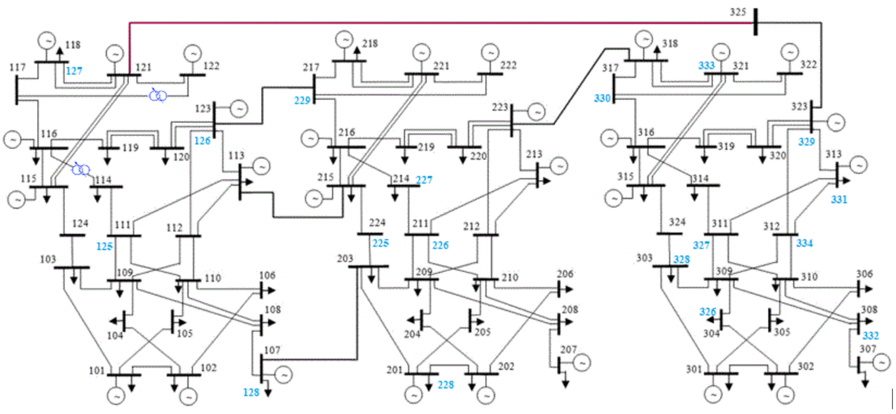


Figure 4.4.1: IEEE96-RTS 96

The investment candidates for the THT400 network are provided by RTE’ s Network Studies Department and consist of new lines. The set of investment candidates for the IEEE RTS-96 was constructed after performing a load flow analysis with optimization of PSTs and HVDC setpoints, determining which lines are overloaded and identifying where it might be beneficial to add new lines. The characteristics of both networks and studies are summarized in the Table 4.4.1.

Network	**IEEE RTS-96**	**THT 400**
Buses	93	21 972
Branches (AC lines + Transformers)	139	31 057
PSTs	2 (2 optimized)	296 (25 optimized)
HVDC lines	1 (1 optimized)	58 (2 optimized)
Generators	198	26 736
Loads	60	15 449
Incidents considered for N-1 criterion	65	390
Monitored lines	67	454
Synchronous areas	1	7
Investment candidates	14	88

Table 4.4.1: Characteristics of the main connected component of the two networks: IEEE RTS-96 and THT 400.

4.4.2 Selection of test cases for THT400

The size of the problem for the large-scale case is gradually increased to test the model and assess computation times. Four parameters are varied: the number of hours studied, the number of candidate lines, the maximum budget, and the number of modeled contingencies.

Hours studied

An OPF is performed on the 100 first hours (Figure 4.4.2, where the scale of costs is removed for confidentiality issues) to determine relevant timesteps to study. The hours with high operation costs are chosen for the intial tests. The different hour ranges studied are : **1 hour (hour 15), 10 hours (hours 15 to 24), 50 hours (hours 0 to 49), 100 hours (hours 0 to 99).**

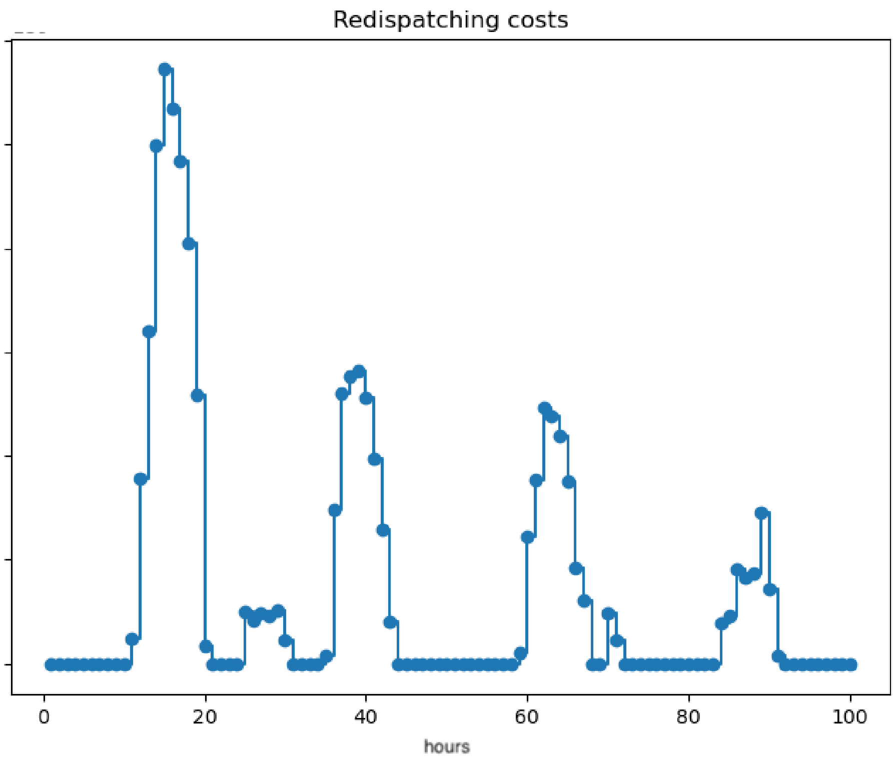


Figure 4.4.2: Operation costs computed by the OPF

Candidate lines

The total number of preselected candidate lines in the THT400 study is 88. For the initial tests, this number varies. Three batch sizes are tested: **22, 44, and 88.** Since the investment cost of each line differs, randomly creating batches could result in imbalanced groups (a batch with cheaper lines could invest in more lines and likely require longer computation times). Candidates are then grouped into batches of 22, maintaining an equal distribution of lines with similar costs.

Maximum budget

The maximum allocated budget directly affects the computation time of the problem, as it allows for the construction of more lines and increases the number of possible line combinations. The initial tests are conducted with a small budget, which is then doubled. The different budget levels are: **200, 400, 800 1600 M €**.

Incidents modeled

As presented in the section 2.1, the number of contingencies is expected to be one of the most limiting factors in making the problem tractable, as it significantly increases the number of constraints in the subproblem. The initial tests are conducted in N situation (ie without any contingencies). However, introducing contingencies and additional constraints could also eliminate some equivalent investment solutions, potentially slowing down the resolution of the master problem. Problems with **3 and 5 contingencies** are solved. The contingencies are selected using the following procedure:

- An OPF is run.
- The 30 lines that exceed their threshold most frequently in N-1 situations are retained.
- The contingencies associated with these threshold violations are identified.
- The most frequently occurring contingencies are selected.

Chapter 5

Results and Analysis

5.1 Resolution on the academic network

This section presents the results of developing and testing the TEP model on the IEEE RTS-96 academic network. First, the results obtained with the solving methods described in the methodology are compared, as well as the computation times. Then, the optimization tool is used on the yearly test case, and the obtained results are presented.

In this section, the candidate lines are identified by their index (from 1 to 14).

5.1.1 Comparison of solving methods

Classic VS Benders

In this section, we compare the performance of the classical solving method and the Benders decomposition method in solving the TEP model. The comparison is performed across a range of different problem sizes, from 1 hour to 20 hours, for two allocated budgets: 200M€ and 550M€. The size of Benders' subproblems is set to one hour. The relative optimality gap ϵ is 10^{-6}. The results are presented in the table 5.1.1.

Table 5.1.1: Computation times and results of the TEP

Size (hours)	Budget (M€)	Classic time (s)	Benders time (s)	Invested lines	Objective function (€)
1	200	13.7	5.2	1/6	80 891
1	550	29.4	34.4	1/3/7/14	44 918
2	200	32.9	10.5	1/6	168 461
2	550	85.5	54.4	1/3/7/14	79 188
5	200	239.7	23.5	14	421 060
5	550	188.2	122.9	1/7/13/14	173 874
10	200	2604	37	14	597 155
10	550	4183	132	1/7/9/14	200 793
20	200	23 364	65	14	949 983
20	550	Out of time	299	1/7/9/14	544 047

It is important to first note that both solving methods yield the same results in terms of invested lines and objective function values, confirming their correctness.

Regarding computation time, Benders' method becomes more effective starting from a problem size of 10 hours for both budgets. While computation times increase rapidly as the problem size grows (already exceeding 6 hours for a 20-hours problem with the classical method), Benders' method remains much more manageable, increasing in an almost linear fashion. The 20-hour problem is solved in 65 seconds, which is 350 times faster than with the classical method.

This significant acceleration was expected, as a result of two key advantages of the Benders decomposition technique: parallelization of subproblem solving and a more efficient search for optimal candidates through the addition of cuts.

Stabilization

The stabilization technique, combined with the Benders decomposition method, has been tested on slightly larger problems. For these tests, the optimality gap remains set at 10^{-6}, but only the highest budget (550M€) is considered. The 500-hour problem is solved using 25 CPUs, while the 8736-hours problem is solved using 40 CPUs. The computation times and the number of iterations for each test are presented in table 5.1.2.

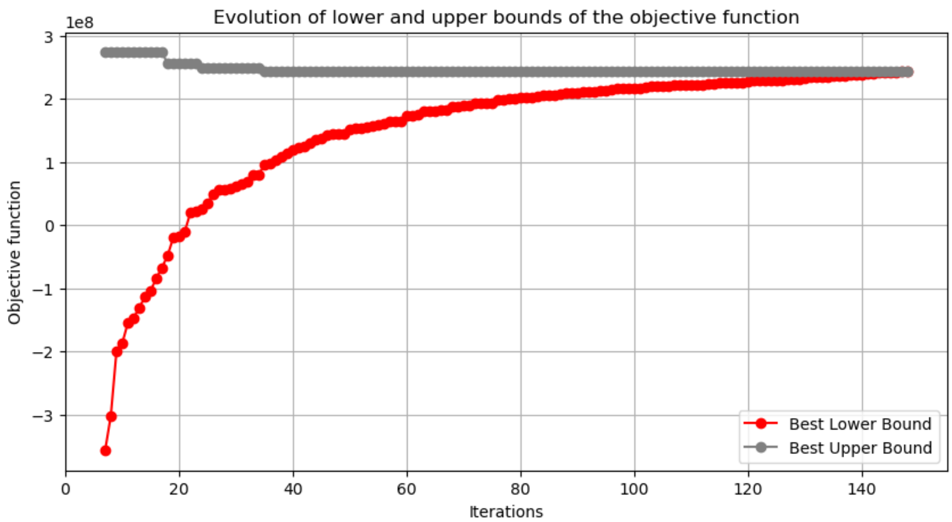
Table 5.1.2: Computation times with and without stabilization

Size (hours)	Benders method		Benders with stabilization	
	Time (s)	Number of iterations	Time (s)	Number of iterations
10	184	69	115	55 (7 relaxed)
50	809	52	489	37 (8 relaxed)
500	3 731 (25 CPU)	72	747 (25 CPU)	44 (7 relaxed)
8736	67 613 (40 CPU)	148	20 627 (40 CPU)	89 (8 relaxed)

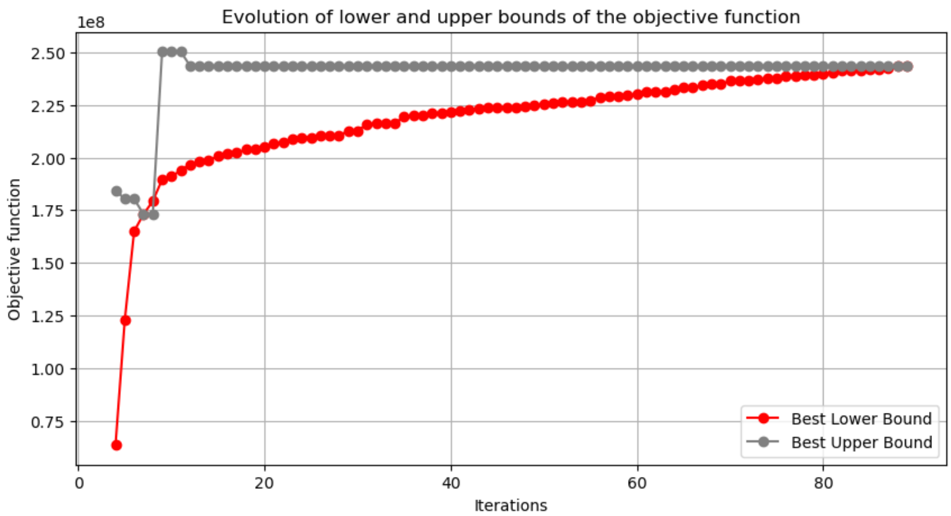
While computation times are similar for the 10-hours problem, stabilization provides a notable acceleration for larger problems, primarily because the number of iterations is reduced. For the largest problem, computation time is divided by three. It is also worth noting that the number of relaxed iterations does not seem to depend on the problem size.

The two graphs in Figure 5.1.1 show the evolution of the lower and upper bounds throughout the solving process for the 8735 hours problem, until the difference falls below the optimality gap. The upper bound remains relatively constant, while the lower bound takes some time to increase.

The stabilization process and its effect on accelerating the solving process are clearly visible in Figure 5.2.3c. During the initial iterations, the solution is relaxed, allowing the upper bound to be lower than it should be. Once the upper and lower bounds converge, the investment variables



(a) Without stabilization



(b) With stabilization

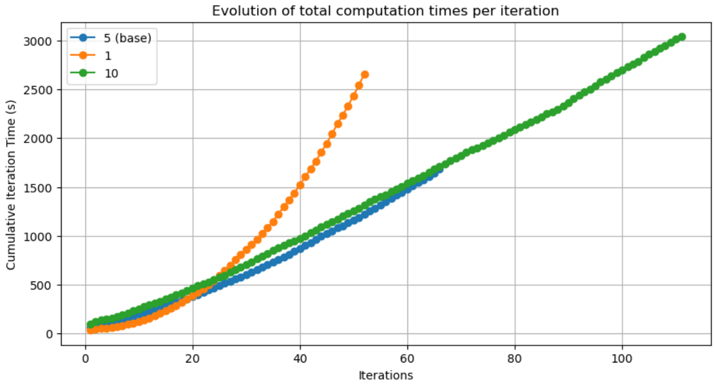
Figure 5.1.1: Evolution of the lower and upper bounds (8736 hours)

become binary again, causing the upper bound to increase. However, the gap is already small, as compared to the case without stabilization. This results in fewer iterations and less computation time to reach the optimality gap.

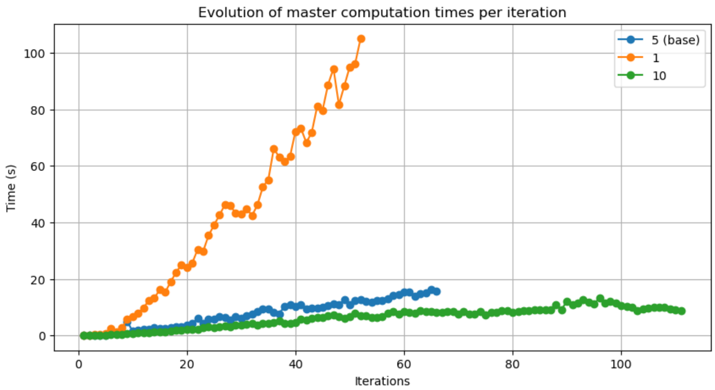
Subproblems' size

Three sizes of subproblems have been tested on a 1000 hours problem : 1 hour, 5 hours and 10 hours. The computation times are depicted in Figure 5.1.2.

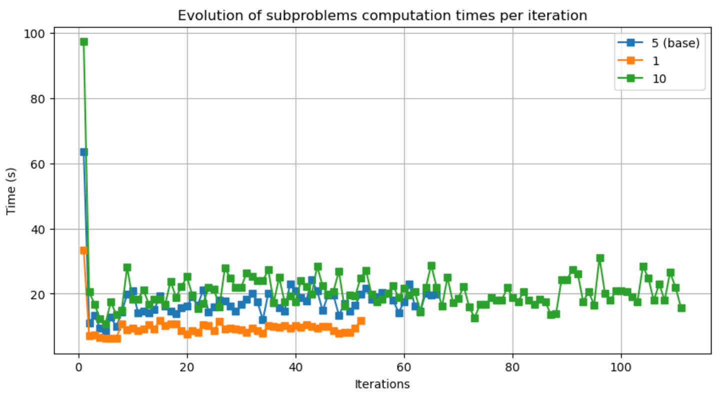
As expected, the computation time of the subproblems is lower when they are smaller. Additionally, the problem requires fewer iterations to converge. Since the subproblems are smaller, there are more of them in the overall problem, leading to more Benders cuts being added at each iteration. However, more cuts also make the master problem longer to solve, as clearly illustrated in subfigure (b): the master computation time increases much faster with 1-hour subproblems than in the other cases. This confirms that carefully choosing the size of the subproblems is crucial.



(a) Total iteration time



(b) Master time



(c) Subproblems time

Figure 5.1.2: Evolution of computation time per iteration depending of the suproblems' size

5.1.2 Resolution over the year

Given the benefits of these methods, Benders' decomposition with stabilization is applied. We choose to use subproblems of 10 hours, as it seems to be a good compromise. Based on the times presented in the methodology section, the resolution of each time step remains feasible (approximately 40 seconds), and the total number of subproblems (874) will not add too much constraints. The problem is solved using 40 CPUs and the optimality gap is still 10^{-6}. This subsection firstly presents the investment results, as they could be obtained by the operator running this optimization problem, and secondly the breakdown of computation times for this case.

Investment results

First, an OPF is run over the year using the initial network. The operational costs, which correspond to redispatching costs (with no loss of load required), are obtained. The total yearly costs amount to approximately 26M€. The details of these costs per hour are shown in the blue graph in Figure 5.1.3. The hourly costs are not constant but the peaks are distributed throughout the year.

Next, the TEP problem is solved, resulting in the choice of five new lines to invest into: 1, 3, 5, 7, and 9. Another OPF is then performed with the updated network, as shown in Figure 5.1.4, leading to new operational costs of 9.6M€, detailed in the orange graph in Figure 5.1.3.

The total investment cost is 550M€, which can be converted into an annualized cost of 7.9M€. The net yearly gain is therefore 8.5M€.

Computation times

The optimization problem was solved in 20 627 seconds and 89 iterations. The computation time for each iteration is shown in Figure 5.1.5, with the breakdown between the time spent solving the master problem and the time spent solving the subproblems.

At the beginning of the process, most of the iteration time is spent solving subproblems. The first iteration takes a significant amount of time, but this decreases in subsequent iterations due to the use of a warm start. This technique allows the solver to reuse information from previous iterations rather than starting from scratch each time, improving efficiency. The computation time for subproblems then remains relatively constant throughout the rest of the optimization.

The master problem, on the other hand, is initially solved very quickly since no cuts have been added yet. Gradually, as more constraints are introduced, the master problem takes longer to solve. Its computation time increases linearly, in proportion to the number of subproblems.

This graph highlights once again the need to find a balance between the time spent solving the subproblems and the master problem.

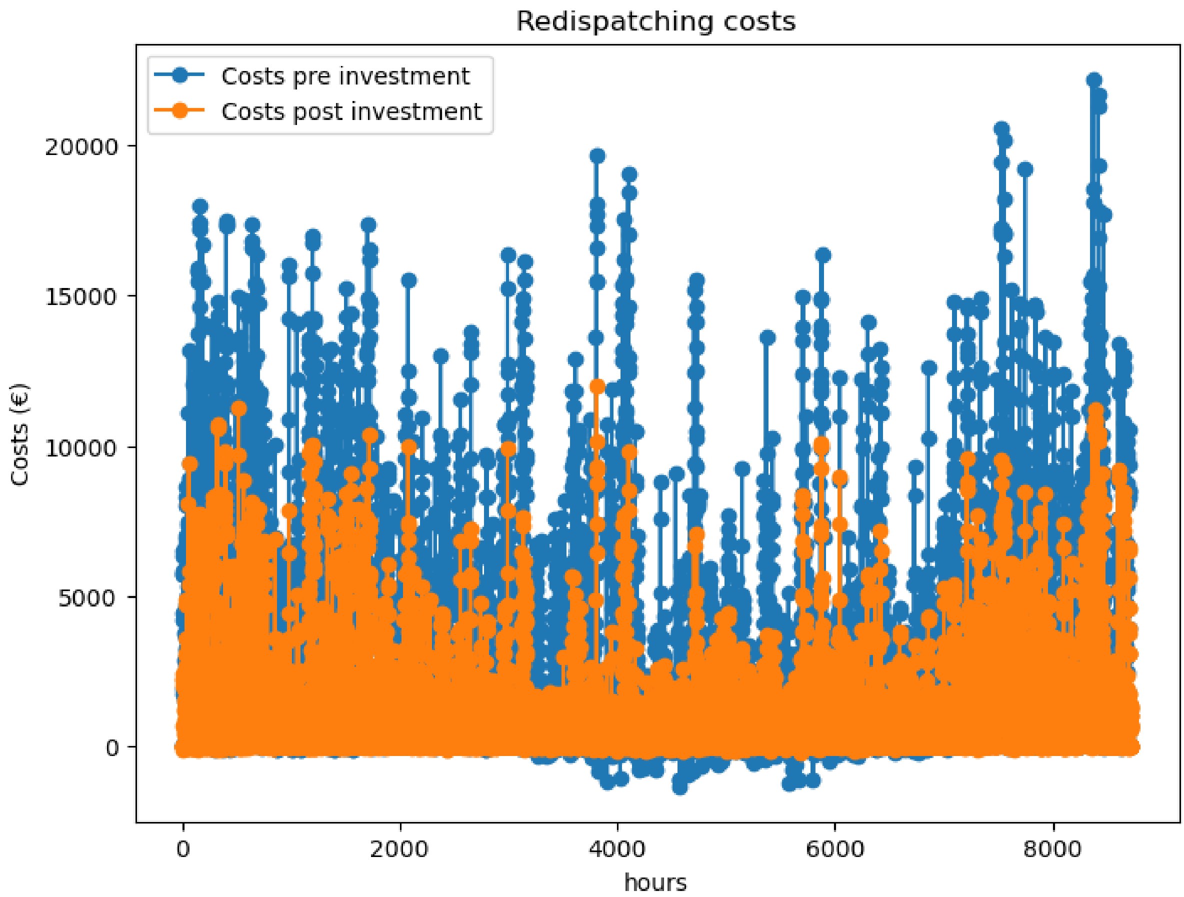


Figure 5.1.3: Operation costs before and after investment

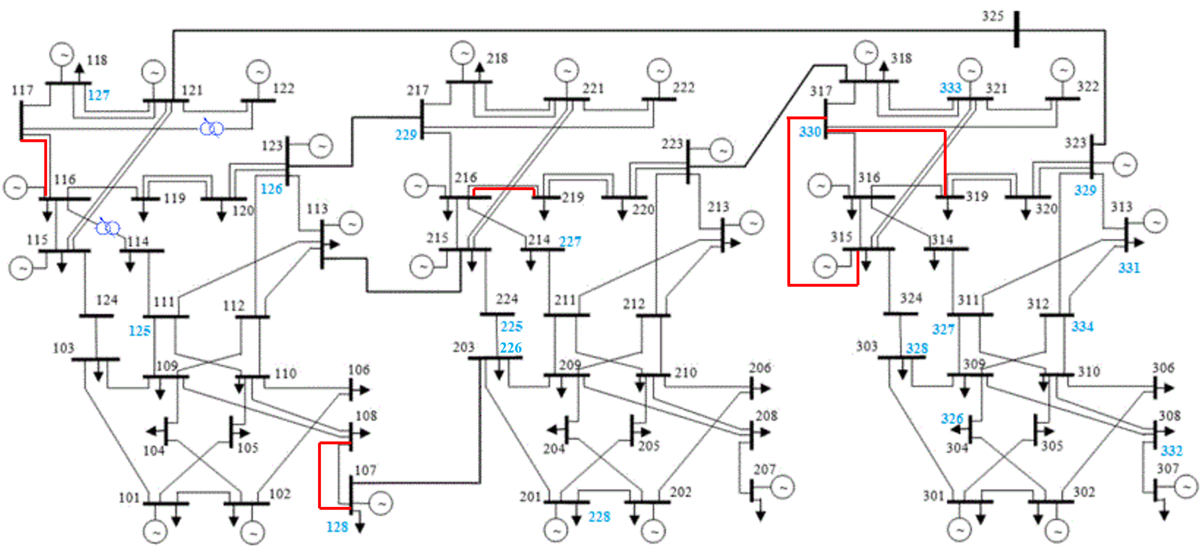


Figure 5.1.4: IEEE RTS-96 network with invested lines (in red)

5.2 Feasibility on the large-scale network

The problem is now tested on the real-case study THT400 from RTE.

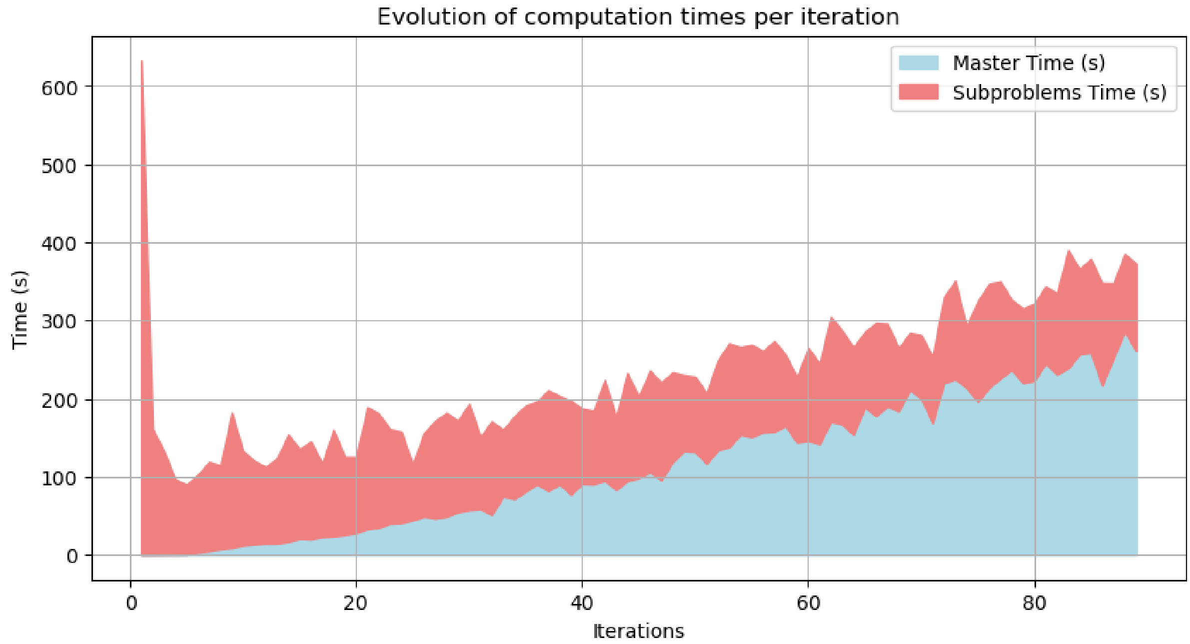


Figure 5.1.5: Computation time per iteration

As expected, the subproblems took too long to solve with all incidents modeled, as each one requires recomputing power flows across the entire network (composed of 30 000 branches).

The size of the problem for the large-scale case is gradually increased to test the model, without considiring contingencies. The resolution of the smaller test case (1 hour, 22 candidates, 200M€ budget and no incidents) is first tested without stabilization. The problem is untractable because the lower bound of the objective function increases too slowly at each iteration, and the gap can not fall below the optimality gap. The resolution is then tested with stabilization, and converges. The following tests are then conducted with stabilization.

The computation times for the different case tests are presented in the Table 5.2.1.

At first glance, it appears that computation times are extremely high, making the problem difficult to solve over a full year. The case with all candidates, a budget of only 400 M€ (which allows for the investment in just two lines), and a single hour takes 7197 seconds (around 2 hours, without any contingencies) (Figure 5.2.1). The number of iterations is also very high (749 in this case). The high number of iterations required for convergence could explain why computation times are so long.

However, some cases have fewer iterations but still high computation times. For example, with 22 and 44 candidates, when the budget increases from 200 to 400 M€, computation times grow much more than the number of iterations. When looking at the breakdown between master and subproblem computation times for these cases, we see that the master problem times remain

Size (hours)	Budget (M€)	Candidates	Incidents	Computation time (s)	Iterations
1	200	22	0	212s	14
1	400	22	0	1296s	34
1	800	22	0	909s	32
1	1600	22	0	2048s	474
1	200	44	0	128s	42
1	400	44	0	1632s	79
1	200	88	0	1721s	120
1	400	88	0	7197s	749
10	400	22	0	1159s	25
50	400	22	0	2949s	35
10	400	44	0	1721s	61
100	400	44	0	43614s	214
1	400	44	3	2070s	146
1	400	44	5	32831s	544
1	200	44	3	483s	58
1	200	44	5	4758s	119

Table 5.2.1: Computation times of the TEP on THT400 case tests

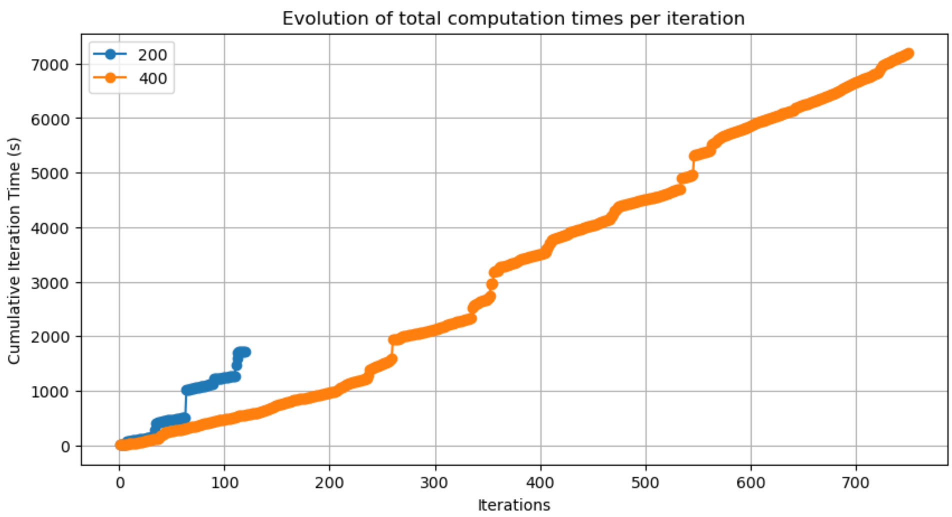
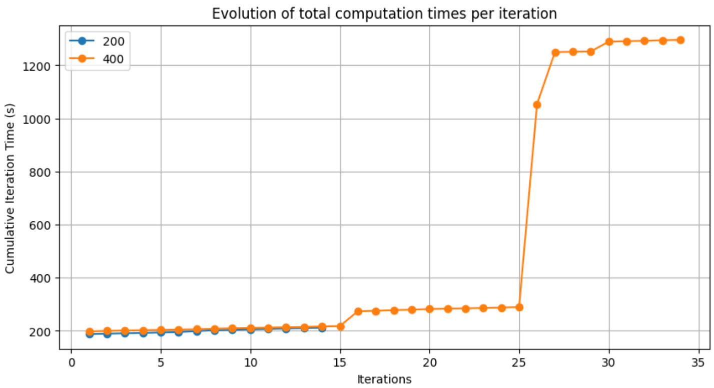


Figure 5.2.1: Computation time per iteration for different budgets (1 hour, 88 candidates)

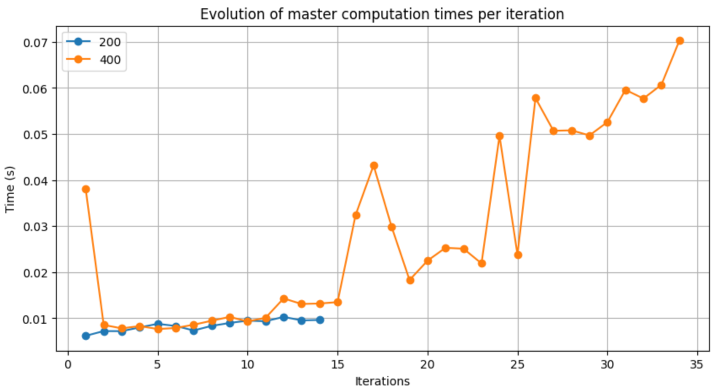
fairly reasonable. The subproblem computation time for 1 hour is usually below 10 seconds, averaging around 4 seconds. However, in some iterations, it is unexpectedly long (Figure 5.2.2).

In the total computation time per iteration graph, there is a spike at iteration 25, where the subproblem takes 800 seconds to solve, significantly impacting the total computation time, which was 300 seconds until then.

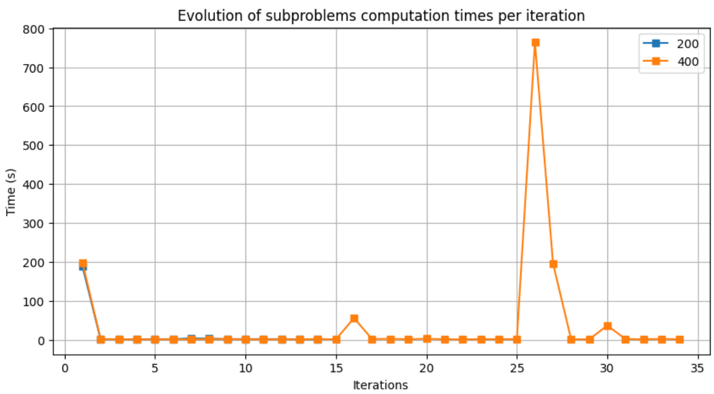
The same happens in the case where the number of candidates doubles (44 to 88) under a budget of 200 M€(Figure 5.2.3), where 8 spikes are observed. Without these spikes, the resolution time would be much lower (around 400s instead of 1721s).



(a) Total iteration time



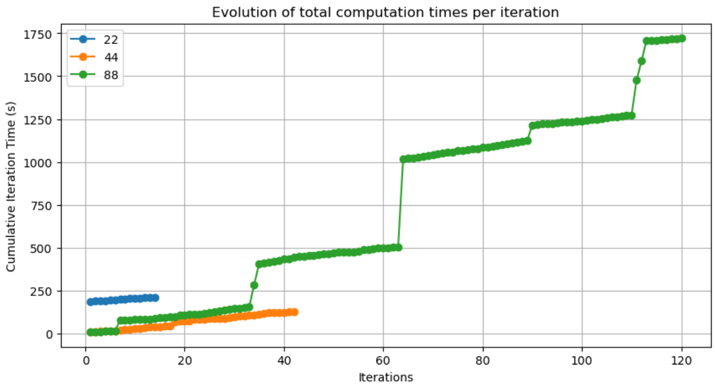
(b) Master time



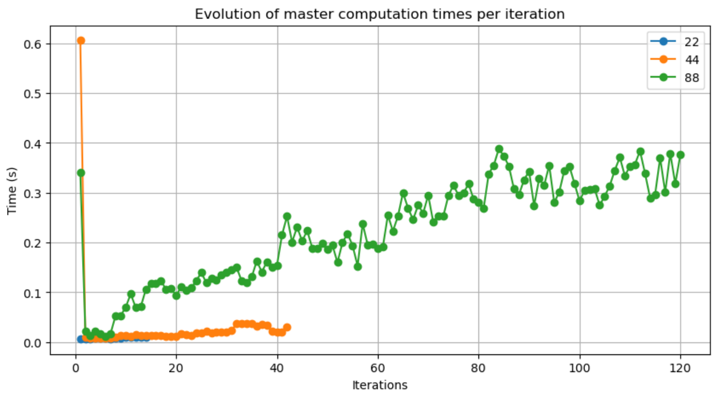
(c) Subproblems time

Figure 5.2.2: Computation time per iteration for different budgets (1 hour, 22 candidates)

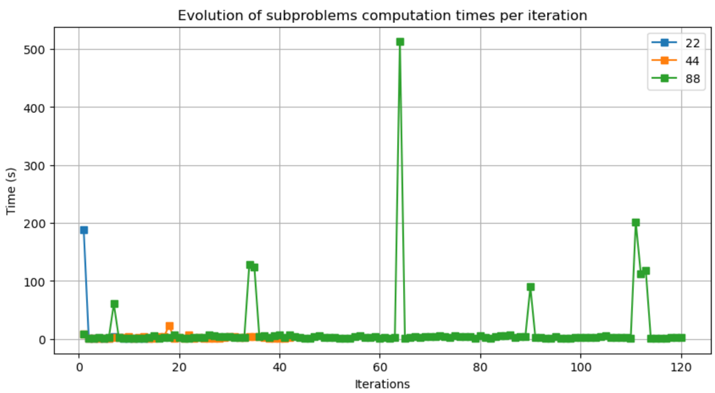
These large differences in subproblem computation times are difficult to predict and could be due to numerical issues in the optimization problem.



(a) Total iteration time



(b) Master time



(c) Subproblems time

Figure 5.2.3: Computation time per iteration for different number of candidates (1 hour, 200 M€budget)

5.3 Discussion

The case studied on the IEEE network has clearly demonstrated that Benders' decomposition is a promising method for solving the transmission expansion problem, due to its master-subproblem structure. One key advantage highlighted in this study is the possibility of parallelizing the resolution of subproblems. As seen in the methodology section, subproblem computation times increase significantly as their size grows. However, it is crucial to choose an appropriate subproblem size. Over-parallelization (e.g., choosing 1 hour subproblems) can cause the master problem's computation time to explode. This must be carefully considered when applying Benders decomposition to large-scale networks. To determine the optimal size of the subproblem, an OPF should be run first over different cases to identify acceptable sizes.

Another major benefit of Benders decomposition is the availability of various acceleration techniques. In this thesis, the stabilization method was tested and showed significant improvements in solving the problem by reducing the number of iterations. On the large-scale problem, it was necessary to use it to make the solution converge. Other acceleration techniques could also be explored, such as Benders by batch, which avoids solving all subproblems at each iteration to reduce subproblem computation time, or aggregation, which limits the number of cuts added at each iteration to reduce master problem computation time. These approaches could provide interesting avenues for further improvement, especially the second one, which will address the issue of over-parallelization presented above.

In N situation, the problem is still not tractable at this stage, because of numerical issues that affects the resolution of the subproblems. It is then more difficult to interpret the computation times results and highlight the most impactful parameters. However, by looking at the number of iterations and the master computation time evolution, it appears thanthe maximum budget and the number of candidates had a greater impact than expected, even though they were similar to those in the academic case. For example, doubling the budget led to a five times increase in computation time, as more iterations were required. This impact on the number of iterations increases also with the number of incidents.

The most impactful parameters of our problem remains the size of the subproblems (number of lines and nodes modeled along with the number of incidents), since the OPF remains not tractable in N-1 situation. RTE expansion studies model the entire European network but focus primarily on the French grid (specifically the 400 kV and 225 kV high-voltage levels). This means that while the monitored lines and those considered for N-1 contingencies are fewer than the total number of branches, all generators are still used for redispatching. A method of reducing the network could help reduce computation times by keeping only the relevant buses and branches for the study. However, unlike other studies that have applied network reduction [14], the groups would not be aggregated. This would ensure that each generator remains distinct and that redispatching remains as close to reality as possible.

To complete it, an alternative modeling approach for incidents could be considered. As presented in the literature review, one method involves using sensitivity factors and power injections to model incidents instead of recalculating power flows for each scenario [12]. The key advantage of this method is its ability to represent the loss of multiple lines, making it well-suited for the expansion problem, where the topology changes. Additionally, it could reduce the problem size by decreasing the number of variables, leading to more efficient computations.

These preliminary results nevertheless show that the number of candidates and the budget both have an important influence on the resolution, without necessarily succeeding in identifying a trend or finding the most impactful one. Reducing the number of candidates could be then beneficial to solve the problem without contingencies, as it decreases the number of iterations. However, beyond a certain point, the impact becomes minimal, and the risk of losing optimal candidates increases. A more effective approach could be to focus on the maximum budget. An iterative process (first selecting a smaller number of lines to identify the most promising ones, then removing them from the candidate set) could be a solution. However, if the chosen budget is less than the investment cost of certain lines, they will automatically be excluded. Introducing a new constraint to the master problem to limit the maximum number of lines invested could help mitigate this issue. It remains to be seen whether this process leads to a suboptimal solution, as tests on the academic network have shown that the combination of invested lines can vary significantly depending on the budget (see Table 5.1.1).

5.4 Sustainability Assessment

An optimized TEP method supports industrial decarbonization and reduces greenhouse gas emissions, contributing to environmental integrity. It ensures that no more transmission lines are built than necessary, which limits unnecessary land use and minimizes environmental disruption. By optimizing electricity transportation, it helps maintain a balance between resource use and environmental protection.

From the intra-generational equity point of view, optimized TEP reduces operational costs and investment expenses, making electricity infrastructure development more cost-efficient, which can benefit all members of the present generation. By limiting unnecessary construction, it helps reduce the social impact on communities affected by infrastructure projects. The method can be integrated with battery storage and HVDC lines, ensuring fairer distribution of benefits within the present generation by improving grid access and reliability.

Concerning the inter-generational equity aspect, optimized TEP ensures sustainable investment that does not place excessive financial burdens on future generations. It allows for equitable distribution of resources between present and future generations by ensuring that the energy infrastructure remains effective and efficient over time. By supporting the decarbonization of industries, it helps preserve the environmental conditions necessary for future generations to meet their needs.

Finally, this section will explore how the use of an optimized tool by a TSO to conduct Transmission Expansion Planning contributes to achieving Sustainable Developments Goals

(SDG) [23]. SDGs are a set of 17 global objectives defined by the United Nations to address poverty, inequalities and climate change while preserving environment by 2030.

- **SDG 7, Affordable and clean energy :** The development of the grid in every country is essential to ensure the transportation of an increasing electricity demand. The use of an optimized tool supports investment decisions, and minimizes costs. It can also facilitate and speed up the process in smaller countries.e
- **SDG 9, Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation :** It strengthens the grid by reducing congestion and enable clean energy access to industries. The use of optimization techniques to perform TEP aligns with a comitment to improve energy solutions.

Chapter 6

Conclusion

For reference, the objective of this thesis was to develop an optimized Transmission Expansion Planning method, to provide guidance for the development of a tool suitable for RTE studies.

The methodology developed in this thesis has been successfully demonstrated on the academic network IEEE RTS-96, returning optimal investments and operation costs for a yearly problem. The comparison of computation times between a classic branch and bound solving approach and the Benders' decomposition technique on this academic network demonstrated that the latest is a promising method for TEP, particularly due to its ability to parallelize subproblem resolution. However, careful selection of subproblem size is crucial, as excessive parallelization can drastically increase computation time and makes the problem intractable. To determine optimal subproblem sizes, an OPF should be conducted beforehand. Cuts aggregation could also be an interesting enhancement approach to implement in the Benders' algorithm. Still regarding solving and Benders, the stabilization has shown potential as an accelerating technique in improving convergence and reducing iterations, and even proved to be necessary at a large scale.

The methodology applied on the large-scale study still presents challenges. With contingencies consideration, the problem is intractable for one hour, demonstrating the need to consider differently the incidents in the model. Network reduction techniques and an alternative incident modeling using sensitivity factors could improve tractability, but they should be implemented in such a way it still models precisely the network operation. In N situation, the problem has converged for small test cases but computation time remain too long to be scalable to a year, due to long subproblem resolution times and numerical issues. These issues have not been fixed for the moment, and the results are more difficult to interpret. The maximum budget and the number of candidates seem to have more importance than expected on the number of iterations. While reducing candidate lines can help, an iterative approach adjusting the budget and constraining the number of invested lines may be more effective. However, this raises concerns about potential suboptimal solutions, as investment decisions depend significantly on budget allocation.

The validity of the model has been conducted by comparing the results obtained with our method without investment and with the internal OPF tool used for long-term studies IMAGRID, on the IEEE network case, which gave identical results. However, since no other detailed network expansion model exists at RTE, it was not possible to validate the investment results. The validity of the Benders decomposition method to solve our problem has been conducted by comparing the investment solution and value of objective function to the ones given by the solving with XPRESS.

The method is reliable on the IEEE network and the large-scale network, since a lot of tests have been conducted many times and gave the same results each time.

The method is replicable for the IEEE study, since the network and the time series can be obtained on demand from the authors. The language AMPL can be used with a free academic license, and the Benders algorithm used to solve the problem is open-source, part of the Antares-Xpansion tool [17]. However, it is not fully replicable for the THT400 study, because the time series come from long-term studies and are not public.

Finally, the method is not yet scalable, since it has not been demonstrated on a large-scale network, but avenues for corrections and enhancement are targeted for future work.

In terms of limitations, the accuracy of the results depends on the quality and precision of the input data. While the electrical data describing the network can be obtained with high accuracy, the same is not true for generation and consumption data. These are only estimates of future conditions, meaning that a probabilistic approach with different scenarios would be more precise. However, this study does not address that aspect, as considering multiple input data variants for generators and loads would make the problem more complex. That is why a deterministic approach was chosen as a first step in developing the method.

Additionally, generation and consumption time series are estimated at a zonal scale, and detailed time series are then obtained through a mapping process. This method may introduce errors and lead to a suboptimal adequacy solution.

As explained in the methodology section, the objective function does not directly represent operational costs. An offset is added to avoid redispatching when the adequacy of input data is not optimal. However, in some cases, the optimal solution for the objective function does not correspond to the lowest operational costs. Indeed, to minimize the objective function, the model sometimes reduces the volume of redispatching (thus minimizing the term offset × redispatched power) but ends up selecting more expensive units for redispatching (which increases the term marginal cost × redispatched power). As a result, the objective function value is lower, but the actual operational costs are higher.

The goal of this study was to develop an optimal network expansion method based on current grid expansion tools. Since the offset-based objective function is already implemented in

IMAGRID, it was used in this model as well. However, in future developments, a better objective function that more accurately reflects operational costs will be necessary. Another limitation concerns the solutions obtained from grid expansion studies. With an interactive search approach, users can test and compare multiple solutions, identify similar alternatives, and select the most suitable option while considering factors that may not be explicitly modeled (geographical constraints, social acceptance, investment costs). However, with an optimization network expansion method, only one solution is returned to the user, making it harder to explore other potential alternatives that could also be valuable.

Many ways are considered to continue this work. First, the problem will be solved over a one-year period, without contingencies. The model and Benders algorithm used to solve it will be improved to eliminate numerical issues that slow down some subproblem resolutions. The objective is to test different maximum budget levels to fully understand their impact on computation time.

Next, the network will be reduced using a Kron reduction and integrated into a modified model. This approach has been validated on the academic network, which models the lines and nodes of an equivalent network while preserving generators, loads, HVDCs, and PSTs. This technique aims to reduce the problem size by excluding foreign or low-voltage lines and nodes that are not central to the study. As a result, the subproblems should be faster to solve.

Finally, the method using linear sensitivity factors will be tested first on the academic network and then on the THT400 study, allowing for the integration of contingencies.

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