Operation and charging optimization for electric multimodal mobility system

Haoye Chen[a], Giancarlo Herring-Calvo[a], Constantinos Antoniou[b], Wilco Burghout[a], Zhenliang Ma*[a]

[a]*Department of Civil and Architectural Engineering, KTH Royal Institute of Technology, Stockholm, 10044, Sweden*
[b]*Chair of Transportation Systems Engineering, Technical University of Munich, Munich, 80333, Germany*

Abstract

This paper optimizes the vehicle operation and charging for integrated electric mobility-on-demand services and public transit in the multimodal mobility system. The advancement in automatic driving and electric vehicles introduces challenges in operating MoD services jointly with PT services. However, existing literature on optimizing multimodal mobility systems lacks (1) the energy capacity and en-route charging behavior of MoD services, and (2) the temporal dynamics. We propose an generic network flow model to optimize an electric multimodal mobility system by representing the temporal interactions among customers, electric MoD vehicles, and PT services. The model captures (1) the traveling, waiting, and inter-/intra-mode transfers of customers, (2) the routing, rebalancing, waiting, and charging of MoD vehicles, and (3) the schedules and routes of PT services. We perform a case study based on a real dataset from Färingsö island, Stockholm. We compare the existing PT services and the electric multimodal mobility system integrating MoD with PT. The results suggest the integration can overall reduce 11.35% average travel time and 1.90% average travel distance for customers. It also significantly decreases the average initial waiting time (44.59%), maximum travel time (50.00%), and average transfer time (38.20%) for customers. Moreover, the intermodal transfers mainly occur in one location on the island, indicating a minor modification of existing infrastructure for the challenging service coordination problem in the multimodal mobility systems.

Keywords: Multimodal mobility system, Mobility-on-demand services, Public transit, Electric mobility

1. Introduction

Multimodal mobility systems provide travelers with various transport options such as walking, cycling, driving, ridesharing, and Public Transportation (PT). Recent advances in automatic driving and electric vehicles reveal the significant potential of Mobility-on-Demand (MoD) services in the multimodal mobility system.

MoD provides point-to-point services through a vehicle fleet that is normally electric due to economic and environmental reasons. It increases the utilization of vehicles over private cars. A simulation study in Berlin shows one autonomous taxi effectively replaces ten private cars in meeting urban demand Bischoff and Maciejewski (2016). As there are increasingly intensive practices, e.g., Uber, DiDi, and Lyft, the coordination of electric MoD services with other transport modes in the multimodal mobility system becomes critical, especially for PT.

The integration of MoD services with PT can improve the accessibility and attractiveness of PT systems. For customers residing far from PT stations, MoD services can provide first-mile and last-mile connectivity, bridging the gap between their homes and PT hubs. For areas served by low-frequency PT lines, MoD services can act as a complementary solution, reducing waiting times and improving overall service reliability. This integration can also enhance the coverage of PT networks, making it feasible to serve low-density and/or rural areas that may not afford the operational costs of traditional PT services. With the increasing availability of MoD options, public agencies need to cooperate with MoD providers Shaheen and Cohen (2020).

Controlling electric MoD services includes the operation decisions of routing, waiting, rebalancing, and recharging. The fleet picks up customers at or after their requested pick-up times and transports them to their destinations. Idle MoD vehicles may either wait at designated locations or relocate to other areas to better serve incoming requests. Electric vehicles require appropriate charging actions to maintain a sufficient State of Charge (SoC) for operations. The operations of MoD services should consider service quality such as travel time, travel distance, and waiting time. The integration of MoD services with PT also requires low transfer time between different modes.

This article proposes a network flow model that optimizes MoD service operations integrating PT services in the electric multimodal mobility system. The model assumes a centralized platform coordinating MoD operations and customer routes given PT services, aiming to minimize the generalized system total cost. The key contributions are:

- Propose a time-varying optimization model for the integration between PT and electric MoD services, considering the waiting, rebalancing, and charging behaviors of MoD fleets.
- Propose a generic network representation and expansion method that can model multimodal travel modes and their interactions
- Validate the proposed model on a real dataset in Färingsö, Stockholm and derive managerial insights on operating MoD services with public transit services.

Note that we are proving that the model can guarantee the integer solutions for customer and vehicle flows through a pure linear model. To our best knowledge, it is the first linear optimization model with integer solutions for the integration between MoD and PT services. The proof will be added in the full paper.

2. Literature Review

Relatively sparse literature discussed the interaction of MoD services in the multimodal mobility system. They mainly use continuous approximation Calabrò et al. (2021); Rahimi et al. (2018) or simulation Shen et al. (2018); Nguyen-Phuoc et al. (2023); Sieber et al. (2020), with a focus on analysis instead of the joint optimization of MoD services and PT. For example, Sieber et al. (2020) investigated whether MoD services can replace PT in rural areas through simulation cases of four train lines in Switzerland. They showed that, in three out of four cases, automatic vehicle MoD service with unit capacity can reduce both travel times and operational costs. However, the experiments only compare pure PT services with pure MoD services without discussing their integration. Additionally, the charging strategy is not considered in the simulation.

There is limited research on optimizing MoD services in multimodal mobility systems. A network flow model was proposed to jointly optimize autonomous MoD services, PT, walking, and micro-mobility Wollenstein-Betech et al. (2022). Their findings suggest that the integration of autonomous MoD with other transport modes can significantly improve the overall performance. However, their model does not account for time-varying features or the use of electric vehicles. In Salazar et al. (2020), the authors studied the cooperation between AMoD fleets and PT and congestion pricing. They introduced a network flow model for a multimodal system to maximize social welfare, demonstrating that coordinating autonomous MoD services with PT yields significant benefits compared to standalone autonomous MoD services. However, their model does not incorporate electric vehicles or charging behavior.

In Estandia et al. (2021), the authors discussed the interaction between autonomous MoD services and power distribution networks. They proposed an optimization model that coordinates the electric vehicle fleet to fulfill passenger requests while attenuating overloads or voltage drops in the power network. Their model used an expanded network that incorporates three dimensions: locations in the road network, state of charge (SoC), and discrete time steps, but does not account for multimodal mode options.

In summary, most literature use simulation to examine the effects of the interaction between MoD services and PT. Existing optimization models either focus on operating multimodal systems or standalone electric MoD services.

3. Methodology

In this section, we propose a linear, time-varying, and flow-based model capturing the interaction between customers, electric MoD vehicles, and transit vehicles in the multimodal transportation system. It optimizes the MoD vehicles' routes, the customers' transfers between MoD and PT services, and within the transit lines to achieve system optimal total travel time. We assume that (1) MoD vehicles are homogeneous in travel speed, energy capacity, and charging rate, (2) PT schedules and routes are predetermined, and (3) The charging processes for transit vehicles are excluded from the operational model, reflecting the standard practice in which electric transit fleets have scheduled charging after their daily service. Section 3.1 establishes the expanded network as a basis for the operational model. Section 3.2 introduces the fundamental model. 3.3 discusses the route recovery strategy for the solutions.

3.1. Expanded network

Define the mode set $G = \{MoD, PT\}$. Define the planning horizon as T and discretize it into $|T|$ intervals denoted by $T = \{0, 1, ..., |T|\}$. Define the vehicles' State of Charge (SOC) as the set $C = \{0, 1, 2..., n_{soc}\}$ that discretize the full battery capacity into n_{soc} equal intervals. Define the location set as L. $L_c \subseteq L$ represents the charging stations of MoD vehicles. $L_t \subseteq L$ represents the terminal stations of PT services served as the starting and ending locations for bus routes. Set $R = \{(l_1, l_2)|l_1 \in L, l_2 \in L\} \subseteq L \times L$ denotes available road links connecting locations. Road link $(l_1, l_2) \in R$ has associated energy consumption $c_{(l_1,l_2)} \in C$ for MoD vehicles and travel distance $\Delta d_{(l_1,l_2)} > 0$. It also has travel time $t^{(g_1,g_2)}_{(l_1,l_2)} \in T$ for MoD vehicles when $g_1 = g_2 = MoD$, for transit vehicles when $g_1 = g_2 = PT$, for customers' transition when $g_1 \neq g_2$.

We introduce the expanded network $N = (V, E)$ where V and E represent all nodes and edges in the expanded network, respectively. Node $\mathbf{v} = (l_\mathbf{v}, t_\mathbf{v}, c_\mathbf{v}, g_\mathbf{v}) \in V$ is a 4-dimensional tuple, consisting of location $l_\mathbf{v} \in L$, time $t_\mathbf{v} \in T$, state of charge $c_\mathbf{v} \in C$, and travel mode $g_\mathbf{v} \in G$. Due to the assumption that transit vehicles do not require charging during operation, we let $c_\mathbf{v} = 0$ when $g_\mathbf{v} = PT$. For nodes $\mathbf{v}, \mathbf{w} \in V$, an edge $(\mathbf{v}, \mathbf{w}) \in E$, if it exists in a subset of edges that satisfy the conditions outlined in Table 1.

Table 1: Conditions for the existence of an edge between two points in the expended network

Edge set	$(l_\mathbf{v}, l_\mathbf{w})$	$t_\mathbf{w} - t_\mathbf{v}$	$c_\mathbf{w} - c_\mathbf{v}$	$(g_\mathbf{v}, g_\mathbf{w})$
E^{MoD}_{travel}	$\in R$	$t^{(g_\mathbf{v},g_\mathbf{w})}_{(l_\mathbf{v},l_\mathbf{w})}$	$-c^{(g_\mathbf{v},g_\mathbf{w})}_{(l_\mathbf{v},l_\mathbf{w})}$	(MoD, MoD)
E^{MoD}_{wait}	$l_\mathbf{v} = l_\mathbf{w} \in L_c$	1	0	(MoD, MoD)
E^{MoD}_{charge}	$l_\mathbf{v} = l_\mathbf{w} \in L_c$	1	$\delta_{c_\mathbf{v}}$	(MoD, MoD)
E^{PT}_{travel}	$\in R$	$t^{(g_\mathbf{v},g_\mathbf{w})}_{(l_\mathbf{v},l_\mathbf{w})}$	0	(PT, PT)
$E^{customer}_{transition}$	$l_\mathbf{v} = l_\mathbf{w} \in L$	0	$c_\mathbf{w}\|c_\mathbf{v}$	$g_\mathbf{v} \neq g_\mathbf{w}$
$E^{customer}_{wait}$	$l_\mathbf{v} = l_\mathbf{w} \in L$	1	0	(PT, PT)

In Table 1, $(l_\mathbf{v}, l_\mathbf{w})$ are the locations of two nodes $\mathbf{v}$ and $\mathbf{w}$. $(l_\mathbf{v}, l_\mathbf{w}) \in R$ means there is a road link connecting two locations. $t_\mathbf{w} - t_\mathbf{v}$ is the traversed time step from $\mathbf{v}$ to $\mathbf{w}$. $c_{\mathbf{w}} - c_{\mathbf{v}}$ is the consumed or charged energy from $\mathbf{v}$ to $\mathbf{w}$. $(g_{\mathbf{v}}, g_{\mathbf{w}})$ are the travel modes of two nodes.

$E^{MoD}_{travel}, E^{MoD}_{wait}$, and E^{MoD}_{charge} are subsets of E, denoting the feasible edges for MoD vehicles' traveling, waiting, and charging, respectively, in the expanded network. E^{PT}_{travel} is the subset of E, representing the feasible edges for transit vehicles' traveling in road links according to given schedules. $E^{customer}_{transition}$ denotes customers transfer between MoD services and PT. $E^{customer}_{wait}$ represents that customers wait for MoD or PT services. Without losing generality, we assume customers wait only at nodes belonging to PT in the expanded network. Waiting for MoD services can be represented by waiting at PT nodes and an edge belonging to $E^{customer}_{transition}$. Note that customers can get off MoD or transit vehicles and wait for other vehicles, enabling transfers within MoD or PT services. In general, edge $(\mathbf{v}, \mathbf{w}) \in E = E^{MoD}_{travel} \cup E^{MoD}_{wait} \cup E^{MoD}_{charge} \cup E^{PT}_{travel} \cup E^{customer}_{transition} \cup E^{customer}_{wait}$ means the status transition from $\mathbf{v}$ to $\mathbf{w}$ is feasible for customers, MoD vehicles, or transit vehicles.

For simplicity, we define the sets $E_1 = E^{MoD}_{travel} \cup E^{PT}_{travel} \cup E^{customer}_{transition} \cup E^{customer}_{wait}, E_2 = E^{MoD}_{travel} \cup E^{MoD}_{wait} \cup E^{MoD}_{charge}$, and $E_3 = E^{PT}_{travel}$. They are the movement ranges of customers, MoD vehicles, and transit vehicles within the expanded network E as shown in Table 2. Define sub-expanded networks as $N_1 = (V_1, E_1), N_2 = (V_2, E_2)$ and $N_3 = (V_3, E_3)$ where V_1, V_2 and V_3 are the nodes composing E_1, E_2 and E_3, respectively.

Table 2: Feasible movement ranges for customers, MoD vehicles, and transit vehicles

	E1	**E2**	**E3**
Participator	Customers	MoD vehicles	Transit vehicles
E^{MoD}_{travel}	✓	✓	
E^{MoD}_{wait}		✓	
E^{MoD}_{charge}		✓	
E^{PT}_{travel}	✓		✓
$E^{customer}_{transition}$	✓		
$E^{customer}_{wait}$	✓		

3.2. Fundamental model

We define the travel demand by the set of requests M. For each request $m = (v_m, w_m, t_m, \lambda_m) \in M$, $v_m \in L$ is the request's origin location, $w_m \in L$ is the request's destination location, $t_m \in T$ is the requested pick-up time step, λ_m is the customer rate. All requests are parameters in the model, assumed known, and deterministic.

Customer flow: To model the movement flows of the customer, we define $M_d = \{m | m \in M, w_m = d\}$ as the subset of requests targeting to destination d and set $D = \{\cup_m^M w_m\}$ as the customer destinations. We adopt the bundled-flow representation for customers Bar-Gera (2002). Variable $f_1(d, \mathbf{u}, \mathbf{v})$ denotes the customer flow from $\mathbf{u}$ to $\mathbf{v}$ of all requests targeting to destination $d \in D$. Variable $\lambda^{\text{in}}_{m,(0,PT)}$ denotes the customer rate departing with charge level 0 by mode PT, where $m \in M$. Variable $\lambda^{\text{out}}_{m,t,(0,PT)}$ denotes the customer rate arriving at the destination w_m at time t with charge level 0 by mode PT, where $m \in M$, $t \in T$. Without losing generality, we model customers departing and arriving only at PT nodes in the expanded network with the help of $E^{customer}_{transition}$ to represent the departure and arrival through MoD services.

$$\begin{aligned}
&\sum_{\mathbf{u}:(\mathbf{u},\mathbf{v})\in E_1} f_1(d, \mathbf{u}, \mathbf{v}) + \sum_{m \in M_d} 1_{w_m = d} 1_{l_{\mathbf{v}} = v_m} 1_{t_{\mathbf{v}} = t_m} \lambda^{\text{in}}_{m,(c_{\mathbf{v}}, g_{\mathbf{v}})} \\
&= \sum_{\mathbf{w}:(\mathbf{v},\mathbf{w})\in E_1} f_1(d, \mathbf{v}, \mathbf{w}) + \sum_{m \in M_d} 1_{w_m = d} 1_{v_{\mathbf{v}} = w_m} \lambda^{\text{out}}_{m,t_{\mathbf{v}},(c_{\mathbf{v}}, g_{\mathbf{v}})}, \\
&\qquad \forall \mathbf{v} \in V_1, \forall d \in D \qquad (1)
\end{aligned}$$

$$\lambda^{\text{in}}_{m,(0,PT)} = \lambda_m, \quad \sum_{t \in T} \lambda^{\text{out}}_{m,t,(0,PT)} = \lambda_m, \quad \forall m \in M. \qquad (2)$$

Constraint (1) represents the conservation of customer flows within the feasible movement range E_1 in the expanded network. $1_{x=y}$ is the indicator function. Constraint (2) means that the customer rates of sources and sinks in the flow conservation equal the total demand of requests.

Rebalancing flow: We define $f_2(\mathbf{u}, \mathbf{v})$ as the rebalancing flows of MoD service from $\mathbf{u}$ to $\mathbf{v}$. Parameter $N^{MoD}_I(\mathbf{v})$ denotes the initial MoD vehicles by making $N^{MoD}_I = 0$ when $t_{\mathbf{v}} \neq 0$. Parameter $N^{MoD}_F(\mathbf{v})$ denotes the final

MoD vehicles by making $N_F^{MoD}(\mathbf{v}) = 0$ when $t_\mathbf{v} \neq |T|$.

$$\sum_{\mathbf{u}:(\mathbf{u},\mathbf{v})\in E_2} f_2(\mathbf{u},\mathbf{v}) + \sum_{m\in M} 1_{v_\mathbf{v}=w_m}\lambda^{\text{out}}_{m,t_\mathbf{v},(c_\mathbf{v},g_\mathbf{v})} + N_I^{MoD}(\mathbf{v}) + \sum_{d\in D} f_1(d,\mathbf{v},(l_\mathbf{v},t_\mathbf{v},0,PT)) =$$
$$\sum_{\mathbf{w}:(\mathbf{v},\mathbf{w})\in E_2} f_2(\mathbf{v},\mathbf{w}) + \sum_{m\in M} 1_{v_\mathbf{v}=v_m} 1_{t_\mathbf{v}=t_m}\lambda^{\text{in}}_{m,(c_\mathbf{v},g_\mathbf{v})} + N_F^{MoD}(\mathbf{v}) + \sum_{d\in D} f_1(d,(l_\mathbf{v},t_\mathbf{v},0,PT),\mathbf{v}),$$
$$\forall \mathbf{v} \in V_2 \tag{3}$$

Constraint (3) represents the conservation of MoD rebalancing flows. The left side includes the sources of rebalancing while the right side are the sinks of rebalancing. Specifically, in the source side, the second term is the customer flow arriving at their destinations. The third term is the initial vehicles. The last term represents the customer flow transferring from MoD services to PT services. On the sink side, the second term is the customer flow departing at their origins. The third term is the vehicles in the final time step. The last term represents the customer flow transferring from PT services to MoD services.

The expanded network N enforces MOD vehicle energy constraints through systematic edge pruning in E_{travel}^{MoD} to ensure $\{(\mathbf{v},\mathbf{w}) \in E_{travel}^{MoD} \mid c_\mathbf{w} > 0\}$

Transit services: Edge set E_3 contains all schedules and routes for buses. An edge $(\mathbf{v},\mathbf{w}) \in E_3$, if $(l_\mathbf{v}, l_\mathbf{w})$ are two adjacent stops, $t_\mathbf{w}$ and $t_\mathbf{v}$ are the arrival time for the two stops, $c_\mathbf{w} = c_\mathbf{v} = 0$, and $g_\mathbf{v} = g_\mathbf{w} = PT$.

Consider an example of an existing bus line with 10 stops. The bus departs every 15 minutes starting at 6:00 AM. Below are some edges that represent this example bus line in E_3:

- The first two edges for the transit vehicle starting from 6:00 am: ((Stop 1, 6:00, 0, PT), (Stop 2, 6:03, 0, PT)) and ((Stop 2, 6:03, 0, PT), (Stop 3, 6:10, 0, PT)).
- The first two edges for the transit vehicle starting from 6:15 am: ((Stop 1, 6:15, 0, PT), (Stop 2, 6:18, 0, PT)) and ((Stop 2, 6:18, 0, PT), (Stop 3, 6:25, 0, PT)).
- The first two edges for the transit vehicle starting from 6:00 am in the opposite direction: ((Stop 10, 6:00, 0, PT), (Stop 9, 6:05, 0, PT)) and ((Stop 9, 6:05, 0, PT), (Stop 8, 6:10, 0, PT)).

For irregular bus frequency, E_3 can include the corresponding departure time at the start stop. For short-turn bus services implemented during specific operational periods, E_3 only contains locations along the truncated route segment, reflecting a service pattern in which buses operate on a partial route rather than an end-to-end coverage. Therefore, E_3 can represent all bus services from pre-defined schedules and routes.

Edge $(\mathbf{u}, \mathbf{v}) \in E^{PT}_{travel}$ is associated with a capacity $Q(\mathbf{u}, \mathbf{v})$. It is the aggregate capacity for all buses that run between the same adjacent stops during concurrent time intervals, derived from GTFS data.

$$\sum_{d \in D} f_1(d, \mathbf{u}, \mathbf{v}) \leq Q(\mathbf{u}, \mathbf{v}) \qquad \forall (\mathbf{u}, \mathbf{v}) \in E_3 \tag{4}$$

Constraint (4) represents that customer flows cannot exceed the capacity of PT service.

3.3. Objective

The goal is to minimize the total general cost to customers in the multimodal system, including travel time, waiting time, transfer actions, and travel distance. We assume that all passengers have identical perceptions and define the generation cost as:

$$\begin{aligned} J(f_1, f_2) = & \beta_t \sum_{d \in D, (\mathbf{v}, \mathbf{w}) \in E^{PT}_{travel} \cup E^{MoD}_{travel}} (t_{\mathbf{w}} - t_{\mathbf{v}}) f_1(d, \mathbf{v}, \mathbf{w}) + \\ & \beta_w \sum_{d \in D, (\mathbf{v}, \mathbf{w}) \in E^{customer}_{wait}} (t_{\mathbf{w}} - t_{\mathbf{v}}) f_1(d, \mathbf{v}, \mathbf{w}) + \\ & \beta_d \sum_{d \in D, (\mathbf{v}, \mathbf{w}) \in E_1} \Delta d_{(l_{\mathbf{v}}, l_{\mathbf{w}})} f_1(d, \mathbf{v}, \mathbf{w}) + \qquad (5) \\ & \beta_r \sum_{(\mathbf{v}, \mathbf{w}) \in E^{MoD}_{travel}} \Delta d_{(l_{\mathbf{v}}, l_{\mathbf{w}})} f_2(\mathbf{v}, \mathbf{w}), \qquad (6) \end{aligned}$$

where β_t denotes the value of travel time for passengers using PT or MoD services. β_w represents the value of waiting time that includes both the initial waiting time (before boarding) and the transfer waiting time (including intra-PT-line transfers and intermodal transfers between PT and MoD services). β_d is the unit value of travel distance for customers. β_r denotes the unit value of rebalancing distance for MoD vehicles

The proposed operational model can be summarized as:

$$\begin{aligned} min \quad & J(f1, f2) \\ s.t. \quad & Eq.(1),\ Eq.(2),\ Eq.(3),\ Eq.(4) \end{aligned} \tag{7}$$

3.4. Route recovery

The bundled-flow representation of the customer flow f_1 allows faster computation speed but makes the solutions less tractable to provide explicit routes. To recover the detailed routes for customers in the expanded network, define the imbalance $\phi_d(\mathbf{v})$ at node $\mathbf{v}$ for all customer flows targeted at destination d:

$$\phi_d(\mathbf{v}) = \sum_{\mathbf{w}:(\mathbf{v},\mathbf{w})\in E_1} f_1(d,\mathbf{v},\mathbf{w}) - \sum_{\mathbf{u}:(\mathbf{u},\mathbf{v})\in E_1} f_1(d,\mathbf{u},\mathbf{v}), \quad \forall d \in D \tag{8}$$

Define source set $S(d) = \{\mathbf{v}|\phi_d(\mathbf{v}) > 0\}$ and sink set $S'(d) = \{\mathbf{v}|\phi_d(\mathbf{v}) < 0\}$ for every destination $d \in D$. The route recovery problem $\mathbf{P}_d$ for destination d is as follows:

$$min \sum_{\mathbf{s}\in S,\mathbf{s}'\in S',(\mathbf{u},\mathbf{v})\in E_1} (t_\mathbf{v} - t_\mathbf{u})x_{\mathbf{s},\mathbf{s}',\mathbf{u},\mathbf{v}} \tag{9}$$

$$\sum_{\mathbf{s}\in S,\mathbf{s}'\in S'} x_{\mathbf{s},\mathbf{s}',\mathbf{v},\mathbf{w}} = f_1(d,\mathbf{v},\mathbf{w}) \quad \forall(\mathbf{v},\mathbf{w}) \in E_1 \tag{10}$$

$$\sum_{\mathbf{u}:(\mathbf{u},\mathbf{v})} x_{\mathbf{s},\mathbf{s}',\mathbf{u},\mathbf{v}} = \sum_{\mathbf{w}:(\mathbf{v},\mathbf{w})} x_{\mathbf{s},\mathbf{s}',\mathbf{v},\mathbf{w}} \quad \forall \mathbf{s} \in S, \mathbf{s}' \in S', \mathbf{v} \in V_1, \mathbf{s} \neq \mathbf{v} \neq \mathbf{s}', \tag{11}$$

where $x_{\mathbf{s},\mathbf{s}',\mathbf{v},\mathbf{w}}$ is the variable denoting customer flow on edge $(\mathbf{v},\mathbf{w}) \in E_1$ starts from source $\mathbf{s} \in S$ to sink $\mathbf{s}' \in S'$. Objective (9) aims to obtain the routes with the lowest total travel time. Constraint (10) ensures $\mathbf{x}$ can constitute the obtained f_1. Constraint (11) considers the flow conservation between one pair of source and sink.

Note that this problem can be solved in parallel by destination d for f_1, ensuring an efficient computation.

4. Case study

This section conducts an experiment based on a real case study in Stockholm to validate the proposed model and analyze the integration of MoD and PT. We benchmark existing PT services against an optimized multimodal framework that strategically combines fixed-route PT services with MoD options.

4.1. Experiment details

The experimental transportation network was developed through existing road infrastructure and transit routes in Färingsö, Stockholm, to create a realistic environment. Fig. 1 illustrates the network with 16 nodes and 5 transit routes. The existing schedules and operation ranges (e.g., short-turns in the peak hour) for bus lines are obtained from GTFS data. We assume there are 10 MoD vehicles which are parameterized with a speed of 30 km/h, an energy consumption of 200 Wh/km, and a charging rate of 60 kW. Each vehicle has a 12.5 kWh battery, discretized into 25 SOC increments.

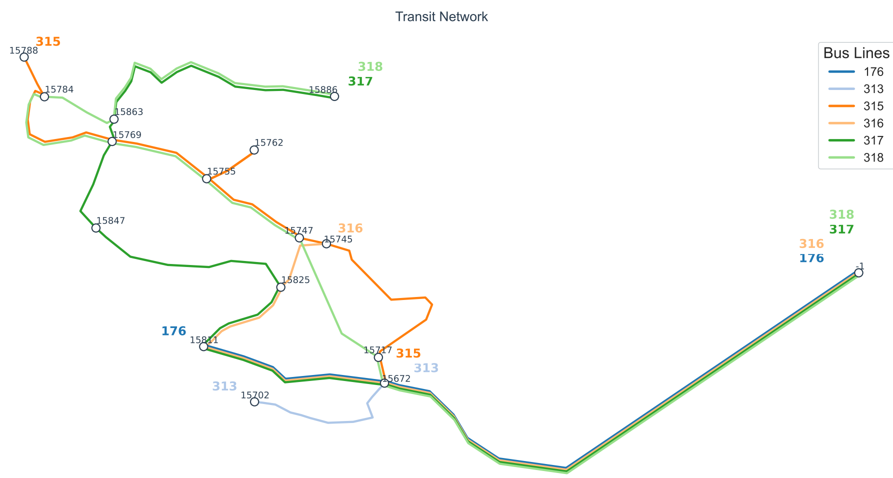

Figure 1: The experiment network and bus lines

The experiment range starts from 6:00 am to 12:50 pm, divided into 5-min intervals. We consider 201 requests with 628 passengers from 6:00 am to 12:00 pm, which are sampled from the real travel demand distribution over three months. Fig. 2 shows the experimental requests, with green and red arcs indicating origins and destinations, respectively. Most of the customers hope to visit the urban area on the island or the city center in Stockholm. We conducted an experiment with existing bus services and another experiment with 10 MoD vehicles and the same bus services. The objective has $\beta_w = 3$ kr/min for customer waiting time, $\beta_t = 2$ kr/min for customer travel time, $\beta_d = 3$ kr/km for customer travel distance, and $\beta_r = 1$ kr/km for rebalancing distance. The problem is solved using a Gurobi 10.0.3 solver, executed on a laptop with an Intel(R) i9-12900H CPU with 32 GB RAM under a Windows operating system.

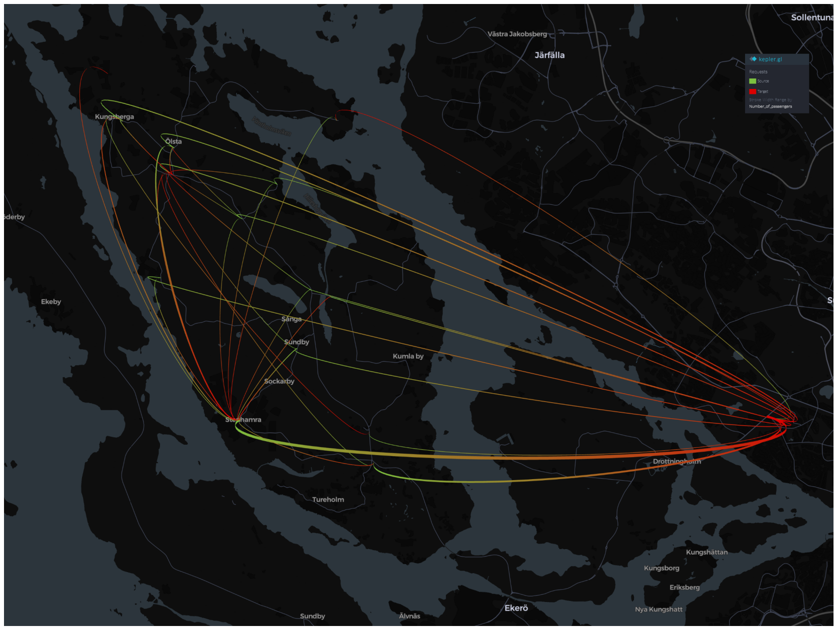

Figure 2: The travel request distribution

4.2. Results

Tab. 3 shows overall metrics for the experiment with existing bus lines and the one with integrated MoD and bus services. Ave./Max. travel time is the average/maximum time for a passenger traveling from its origin to destination. Ave. waiting time for a passenger includes the initial waiting time, the transfer times within the PT or MoD services, and the transfer time between PT and MoD services. Ave./Max. initial waiting time refers to the duration between the arrival of the passenger at a stop and the first pickup by a service vehicle. Ave./Max. transfer time presents the average transfer period within or between PT/MoD services. Ave. travel distance displays the length of an average passenger's trip.

In general, the integration between MoD and PT reduces the average travel time and distance by 11.35% and 1.90% for a passenger trip, compared to the existing bus services, respectively. It significantly reduces the maximum travel time, average waiting time, and average initial waiting time by half or nearly 50%, indicating the MoD services effectively bridge time gaps between PT services. The maximum initial waiting time decreases by 14.29%, mitigating the worst response time to customer requests. The intro-

Table 3: The comparison between PT and the integration of PT/MoD for customer metrics

	PT	PT and MoD	Improve
Ave. travel time	18.42 min/pas.	16.33 min/pas.	11.35%
Max. travel time	120 min	60 min	50.00%
Ave. waiting time	3.51 min/pas.	1.92 min/pas.	45.30%
Ave. initial waiting time	3.14 min/pas.	1.74 min/pas.	44.59%
Max. initial waiting time	35 min	30 min	14.29%
Ave. transfer time	9.79 min/transfer	6.05 min/transfer	38.20%
Max. transfer time	25 min	15 min	40.00%
Ave. travel distance	10.01 km/pas.	9.82 km/pas.	1.90%

Ave. = Average
Max. = Maximum
pas. = passenger

duction of MoD services also substantially reduces the average transfer time (38.20%) and maximum transfer time (40.00%) between two trip legs in the multimodal system.

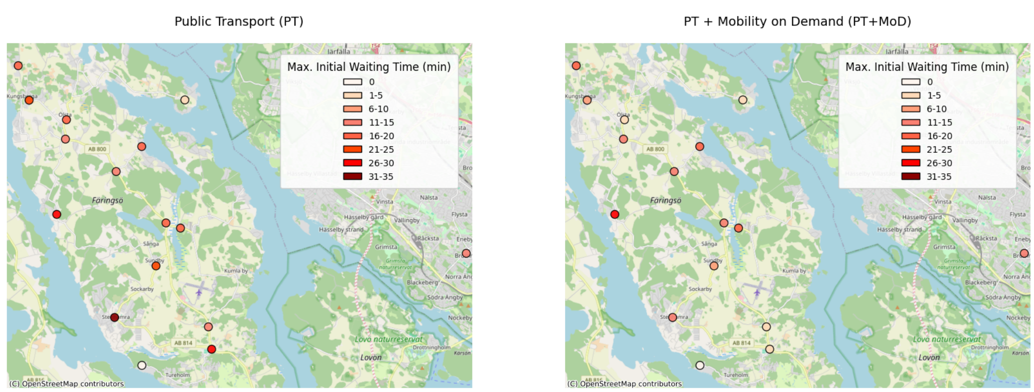

Figure 3: The maximum initial waiting time

Fig. 3 displays the maximum initial waiting time at each stop. It shows that the integration reduces this metric in most stops, especially on the south and north sides of the island, improving the spatial equality. Moreover, it also reduces the worst case for the urban area on the island.

Fig. 4 illustrates the number of intermodal transfers in the integration. The majority of transfers between the MoD and PT services occur at the

Figure 4: The number of transfers between MoD and PT services

bottleneck stop to the city center. In contrast, only minimal transfers occur at three stops, while no transfers are observed at the remaining stops. This pattern highlights the centralized demand flows toward the urban core, even with MoD integration. It also suggests small modifications of the existing infrastructure to coordinate intermodal transfers in the multimodal system.

Table 4: The benefits and costs of the introduction of MoD services

	MoD	MoD+PT	Improvement
Total customer distance (km)	6286.067	6169.416	116.651
Total customer time (min)	11570	10255	1315
MoD vehicle distance (km)	\	1207.086 (37.63%)	-1207.086

Tab. 4 displays the benefits and costs from the introduction of MoD services to the existing PT services. In general, the integration reduces total customer travel distance by 116.651 km and total customer travel time by 1315 min. However, the operation of the MoD fleet also causes an extra vehicle distance of 1207.086 km 37.63% of which is the rebalancing distance carrying no passengers.

5. Conclusion

This paper proposed a network flow model that optimizes the integration between MoD and PT services in a multimodal mobility system. The model is able to capture the temporal interactions among customers, MoD vehicles, and existing PT services. Moreover, it considers the energy consumption, capacity, and charging of electric MoD vehicles. We experiment with a real-world case study on Färingsö island on realistic data. The results suggest that, with 10 MoD vehicles, the integration can improve 11.35% average customer travel time and 1.90% average customer travel distance in general compared to existing PT services. It also significantly improves (over 38%) the maximum travel time, average waiting time, average initial waiting time, and average/maximum transfer time for customers. The integration also improves the spatial equality of mobility services on the island. The intermodal transfers mainly occur in limited locations, suggesting that only minor modifications of existing infrastructure are needed.

The proposed model is able to tackle the routing, rebalancing, waiting, and charging of common MoD services (e.g., taxi and e-scooters) and integrate them with PT services (e.g., buses and subway lines) in a multimodal mobility system. The model provides a generic framework for the integration of MoD and PT services through building the expanded network according to MoD vehicle movement patterns and existing PT schedules.

The proposed model is based on an expanded network, which has growing complexity with the network's size. The pure linear structure of the model suggests an interesting future direction in exploring its efficient computation. For the modeling, the proposed model leaves discussions of ride-sharing services for the MoD vehicles to pick up more than one request at the same time for future research. This paper reveals the benefits of integrated MoD and PT services in the multimodal system given existing PT schedules. Future research may focus on reshaping the PT services under the MoD integration.

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