THE TRAVELLER COSTS OF UNPLANNED TRANSPORT NETWORK DISRUPTIONS: AN ACTIVITY-BASED MODELLING APPROACH

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ABSTRACT

In this paper we introduce an activity-based modelling approach to evaluating the traveller costs of transport network disruptions. The model handles several important aspects of such events: delays may be very long in relation to the normal day-to-day fluctuations; the impact of delay may depend on the flexibility to reschedule activities; lack of information and uncertainty about travel conditions may lead to under- or over-adjustment of the daily schedule; delays on more than one trip may restrict the gain from rescheduling activities. We derive properties such as the value of travel time and scheduling costs analytically. Numerical calculations show that the average cost per hour delay increase with the delay duration, so that every additional minute of delay comes with a higher cost. The cost varies depending on adjustment behavior (less adjustment, loosely speaking, giving higher cost) and baseline travel time (longer travel time giving lower cost). The results indicate that existing evaluations of real network disruptions have underestimated the societal costs of the disruptions.

1 Introduction

Disruptions in the road transport system, caused by for example extreme weather, mechanical failures or severe car crashes, can have severe societal consequences. A partial or complete loss of capacity on a road link or across a larger area may lead to delays that spread to the surrounding network through congestion and queues. For individuals, sudden travel time increases can impair the ability to commute to work and take part in other daily activities such as dropping off and picking up children from daycare, doing the grocery shopping, meeting friends, etc. For businesses, negative impacts arise from delayed deliveries and supplies, loss of manpower and customers, increased freight costs, etc.

For many reasons it is desirable to express the impacts of network disruptions in monetary terms. Immediately after an event has occurred, the costs of different restoration schemes can then be compared with their estimated benefits to determine which scheme would be the most efficient. For example, contractors may be given a bonus for every day ahead of normal schedule functionality is restored, as was done after the Northridge earthquake in California 1994 (Wesemann et al.1996) and the I-35W bridge collapse in Minneapolis 2007 (Xie & Levinson 2008). The size of the bonus should reflect the economic losses avoided by the early restoration.

In the planning process, transportation models can be used to identify critical links and areas where the impacts of disruptions would be particularly severe (Jenelius et al. 2006, Taylor et al. 2006, Jenelius & Mattsson 2008, Matisziw & Murray 2009). With impacts valued as economic losses, cost-benefit analysis can be applied to determine if, how much and where resources should be allocated to reduce the likelihood or consequences of potential future disruptions.

Estimations of the costs due to delays have been performed in connection with several major real-world disruptions, e.g., the 1994 Northridge earthquake (Wesemann et al. 1996), the 2007 Minneapolis bridge collapse (Xie & Levinson 2008) and the 2006 landslide in Småröd, Sweden (MSB 2009).[1] In all these studies, the approach was to calculate the delays caused by the disruption using a transportation model system, typically based on static equilibrium traffic assignment. The calculated delays were then multiplied with a standard value of time to obtain a monetary cost. Using this approach, the delay costs of freeway closures due to the 1994 Northridge earthquake were estimated to have exceeded 1.6 million USD per day (Wesemann et al. 1996), the delay costs of the 2007 Minneapolis bridge collapse were estimated to between 71 thousand USD and 220 thousand USD per day (Xie & Levinson 2008), and the delay costs of the 2006 Småröd landslide to about 5.5 million SEK (ca. 750 thousand USD) per day (MSB 2009).

This approach is adopted also in many model-based studies of road network vulnerability (e.g., Jenelius et al. 2006, Taylor et al. 2006, Erath et al. 2009, Jenelius 2009). Since relative comparisons between different disruption scenarios are independent of the value of time (assuming a single value for all users), some studies report the delays directly.

However, there are several reasons to believe that the cost per hour delay due to a significant unplanned transportation network disruption is higher than the ordinary value of time. First, it is well known that delay time is valued higher by travellers than typical travel time (e.g., Wardman 2001, Bates et al. 2001). Empirical estimations of linear schedule delay models, in which costs arise proportionally to the lateness or earliness of arrival in relation to some preferred arrival time, have suggested that delay time is valued approximately three times as high as ordinary travel time (Small 1982, Tseng & Verhoef 2008). In fact, it is likely that the cost often increases faster than linearly with the delay as the time lost from all other possible activities ackumulates.

Second, the unexpected nature of unplanned disruptions means that travellers are not able to adjust their schedules adequately beforehand to compensate for the delays. Also, the disturbance of normal travel conditions and associated uncertainty during the first few days following an event should lead to high costs that diminish as travellers learn and adapt to new routes, departure

[1] In the landslide in Småröd, Sweden in December 2006, 500 meters of the European highway E6 and 200 meters of a nearby railroad were carried away. The highway and railway were restored in February 2007 (MSB 2009).

times, modes and more complex adjustments (Hunt et al. 2002, Cairns et al. 2002, Zhu et al. 2008, He et al. 2009).

Third, it is likely that a disruption induces delays on more than one trip during the day, such as both the morning and the evening commute. If delay occurs only on the morning trip, a flexible work schedule makes it possible to compensate for late arrival to work by working longer in the evening.[2] If delay affects both commute trips, however, this restricts the possibility to make up for late arrival by working later, which should amount to higher costs.

The purpose of this paper is to develop an approach to assessing the traveller costs of unplanned road network disruptions that incorporates the aspects mentioned above. Thus, we do not assume that the cost is necessarily linear in the delay; rather, we formulate a simple activity-based model of the daily travel decisions as the foundation for the analysis (Axhausen & Gärling 1992, Bowman & Ben-Akiva 2001, Ashiru et al. 2004, Timmermans 2005, Ettema et al. 2007). Within this framework, delay costs arise explicitly from the time that is lost from activities that would be more beneficial than the extra time spent travelling.

The basic model formulation as well as parameter estimates are borrowed from Ettema & Timmermans (2003). In their paper the model was mainly used for numerical estimation, and the authors did not fully work out its analytical properties. In particular, they did not properly define the value of travel time savings within the model, which should take into account that saved time in the long run can be allocated to any activity during the day in a way that optimizes utility. Also, although they included time-of-day flexibility of work as a parameter in the model estimation, they did not consider the effect of scheduling flexibility on the optimality conditions, the value of travel time and scheduling costs theoretically. In this paper, we derive these properties analytically for any level of scheduling flexibility, and we demonstrate the relationship between this approach and linear schedule delay models.

With the model we study the impact of exogenous increases in travel time under different types of adjustment behavior, reflecting the different levels of information and certainty that travellers may have about the post-disruption travel conditions (these adjustment profiles are based on empirical evidence from the 2007 Minneapolis bridge collapse). In particular, we investigate how delay costs, in relation to the ordinary value of travel time, vary with delay duration. We also compare the impacts of a disruption affecting both the morning and the evening commute with disruptions affecting only one of the trips, to determine the value of being able to reschedule the remaining day following a long delay.

The paper is organized as follows. In Section 2 the model is formulated and optimality conditions for the daily schedule, as well as analytical expressions for the value of travel time and schedule delay costs, are derived. In Section 3 the characteristics of unplanned network disruptions are discussed, adjustment profiles are defined and delay costs for different adjustment types are identified. Section 4 describes the numerical utility specifications and parameter estimates and

[2] This type of adjustment is implicitly incorporated in studies of the value of travel time *reliability* (e.g., Noland & Small 1995, Bates et al. 2001, Fosgerau & Karlström 2009), which typically only consider delays in the morning commute (however, see Hess et al. 2007). In any case, the kind of stochastic fluctuations in travel time that are often considered in such studies can be assumed to be independent between trips.

Section 5 presents the results from the analysis. The modelling approach, results and possible extensions are discussed and conclusions are drawn in Section 6.

2 Theoretical model

The modeling framework postulates that individuals spend the day taking part in activities and travelling between activities. A daily schedule is a sequence of activities and trips with a specified start time and duration for each activity and trip. The individuals have preferences among the set of feasible schedules which are expressed with a utility function U.

The utility derived from taking part in an activity is assumed to be independent of other activities but to depend in general on both the time of day and on the duration of the participation. The utility gained from spending another unit of time on activity i at time t is expressed in the form of a marginal utility function $u_i(t \mid t_{si})$, where t_{si} is the start time of the activity. Specifically and in accordance with Ettema & Timmermans (2003), we assume that the marginal utility depends on a linear combination of the time of day t and the duration $t - t_{si}$, i.e., $u_i(t - \xi_i t_{si})$, where $\xi_i \in [0,1]$ is a parameter expressing the scheduling flexibility of the activity. Note that $\xi = 0$ means that marginal utility depends only on time of day, while $\xi = 1$ means that marginal utility depends only time since arrival, i.e., activity duration. Generally speaking, time-of-day dependencies arise, e.g., from fixed activity start and end hours and benefits of coordination with others. Duration dependencies arise, e.g., from start-up and fatigue effects.

The marginal utility derived from travelling, denoted ν, is assumed to be constant.[3] The time required to travel from activity i to $i+1$ depends in general on the departure time t_{di}, $T_i(t_{di})$.

In this paper we consider a daily schedule that consists of three activities and two intermediate trips. Although the model is quite general, we will typically interpret activity 1 as being at home in the morning, activity 2 as being at work during the day and activity 3 as being at home in the evening. Correspondingly, trips 1 and 2 represent the commute from home to work in the morning and back in the evening, respectively.

We assume that the schedule of a day is independent of preceding and subsequent days. This means that we can fix two times $t = 0$ and $t = 1$ that mark the start and end of the day, respectively. There is thus no need to distinguish between time-of-day and duration-dependent utility for activities 1 and 3. For activity 2 we assume that the marginal utility at time t depends on the duration $t - t_{s2}$ as well as the time of day t, where a parameter ξ expresses the scheduling flexibility.

For work trips, late arrival or early departure can have both short-run (e.g., penalties) and long-run negative impacts on the wage, which in turn would affect the available budget for

[3]It is straightforward to generalize this assumption so that utility depends nonlinearly on travel duration, as empirical evidence suggests (e.g., Redmond & Mokhtarian 2001); this issue is further discussed in Section 6. For the present analysis we lack the necessary data to estimate such functions.

consumption. Thus, it is reasonable to interpret U as an indirect utility function, assuming that goods are consumed optimally for any income. The indirect marginal utility of the work activity $u_2(t-\xi t_{s2})$ would then arise both from the direct utility derived from working and the effect on the direct utility from goods consumption through the budget constraint (c.f. Small1982).

In summary, we have the following notation:

t_{d1}	departure time of trip 1 (end time of activity 1).
t_{d2}	departure time of trip 2 (end time of activity 2).
$T_1(t_{d1})$	duration of trip 1 as a function of departure time.
$T_2(t_{d2})$	duration of trip 2 as a function of departure time.
t_{s2}	start time of activity 2 (arrival time of trip 1).
t_{s3}	start time of activity 3 (arrival time of trip 2).
$u_1(t)$	marginal utility of activity 1 at time t.
$u_2(t-\xi t_{s2})$	marginal utility of activity 2 at time t, given arrival at t_{s2} and scheduling flexibility ξ.
$u_3(t)$	marginal utility of activity 3 at time t.
v	marginal utility of travel.

Given that the number and sequence of activities has been fixed, the remaining decision variables for the individual are the durations of the activities (where two durations determine the third) or, equivalently, the departure times of the two trips. The utility associated with the schedule (t_{d1},t_{d2}) is then (c.f. Ettema & Timmermans 2003, Zhang et al. 2005)

$$U(t_{d1},t_{d2}) = \int_0^{t_{d1}} u_1(t)dt + \int_{t_{d1}+T_1(t_{d1})}^{t_{d2}} u_2(t-\xi[t_{d1}+T_1(t_{d1})])dt + \\ + \int_{t_{d2}+T_2(t_{d2})}^{1} u_3(t)dt + v[T_1(t_{d1})+T_2(t_{d2})]. \tag{1}$$

2.1 Optimal schedule

Under normal pre-disruption travel conditions the individual chooses morning and evening departure times in order to maximize its utility. Thus, the individual solves

$$\max_{t_{d1},t_{d2}} \; U(t_{d1},t_{d2}) \tag{2}$$

$$\text{s.t. } 0 \le t_{di} \le 1, \qquad i=1,2, \tag{3}$$

$$t_{d1}+T_1(t_{d1}) \le t_{d2}, \tag{4}$$

$$t_{d2}+T_2(t_{d2}) \le 1. \tag{5}$$

Constraint (3) ensures that both trips are made during the day, constraint (4) ensures that the traveller arrives at activity 2 before departing again, while constraint (5) ensures that the traveller arrives at activity 3 before the end of the day.

The marginal utility functions u_i are assumed to be continuously differentiable, initially increasing (representing a warm-up period) but ultimately decreasing (representing a cool-down period), regardless of arrival time. With realistically formulated utility functions the constraints will be non-binding, so that optimal departure times satisfy $\partial U/\partial t_{d1} = \partial U/\partial t_{d2} = 0$. If we assume that travel times are such that the individual can always depart during the cool-down period and arrive during the warm-up period of each activity, i.e., that $u_1'(t_{d1}) < 0$, $u_2'([1-\xi]t_{s2}) > 0$, $u_2'(t_{d2} - \xi t_{s2}) < 0$ and $u_3'(t_{s3}) > 0$, then the utility function is concave and there is a unique maximum.

To facilitate the analysis of the impacts of exogenous changes in travel time, let us from here on assume that travel time is independent of departure time, i.e, $T_i(t_{di}) = T_i$ for all t_{di}, $i = 1,2$. For any schedule flexibility ξ the optimality conditions require that the departure times simultaneously satisfy

$$u_1(t_{d1}^*) = [1-\xi]u_2([1-\xi]t_{s2}^*) + \xi u_2(t_{d2}^* - \xi t_{s2}^*), \tag{6}$$

$$u_2(t_{d2}^* - \xi t_{s2}^*) = u_3(t_{s3}^*), \tag{7}$$

where $t_{s2}^* = t_{d1}^* + T_1$ and $t_{s3}^* = t_{d2}^* + T_2$ are the arrival times associated with the optimal departure times t_{d1}^* and t_{d2}^*.[4] Note that in general the optimal timing of trip 1 depends on the departure time of trip 2, while the optimal timing of trip 2 depends on the arrival time of trip 1.

If activity 2 is completely fixed ($\xi = 0$) the optimality conditions give that $u_1(t_{d1}^*) = u_2(t_{s2}^*)$ and $u_2(t_{d2}^*) = u_3(t_{s3}^*)$, i.e., the marginal utilities at the start and the finish must be equal (c.f. Ettema & Timmermans 2003, Tseng & Verhoef 2008). In this special case the optimal timing of one trip is independent of the travel time and timing of the other trip.

In the other extreme, if the schedule is completely flexible ($\xi = 1$) optimal departure times will satisfy $u_1(t_{d1}^*) = u_2(t_{d2}^* - t_{s2}^*) = u_3(t_{s3}^*)$. That is, since the departure time from activity 1 and the arrival time to activity 3 are in one-to-one correspondence with the duration of the respective activity, the marginal utility of the duration of each activity must be equal (compare with pure time allocation models, e.g., Jara-Diaz et al. 2008).

Given that the traveller arrives to activity 2 at t_{s2} (not necessarily the optimal arrival time t_{s2}^*), there is an associated optimal departure time from that activity, $t_{d2}^*(t_{s2})$, and an optimal arrival

[4] If travel time is dependent on departure time the optimality conditions must be adjusted accordingly, as shown in the Appendix (see also Tseng & Verhoef 2008).

time to activity 3, $t_{s3}^*(t_{s2})$. It will be useful in the following to calculate the marginal effect on utility of a change in the arrival time t_{s2}, assuming that trip 2 is optimally timed in response to the change. Thus, let $\tilde{U}_2(t_{s2})$ be the utility derived from time t_{s2} to the end of the day under this schedule. We then introduce the "backward optimal" marginal utility function $\tilde{u}_2(t_{s2}) = -d\tilde{U}_2/dt_{s2}$, which is the marginal change in subsequent utility due to an earlier arrival time. The optimality conditions (6)–(7) can thus be summarized as $u_1(t_{d1}^*) = \tilde{u}_2(t_{s2}^*)$.[5] It can be shown that $\tilde{u}_2(t_{s2})$ is given by

$$\begin{aligned}\tilde{u}_2(t_{s2}) &= [1-\xi]u_2([1-\xi]t_{s2}) - \\ &-\xi\left[\frac{u_{3'}(t_{s3}^*(t_{s2}))}{u_{2'}(t_{d2}^*(t_{s2})-\xi t_{s2})-u_{3'}(t_{s3}^*(t_{s2}))}u_2(t_{d2}^*(t_{s2})-\xi t_{s2}) - \right.\\ &\left. -\frac{u_{2'}(t_{d2}^*(t_{s2})-\xi t_{s2})}{u_{2'}(t_{d2}^*(t_{s2})-\xi t_{s2})-u_{3'}(t_{s3}^*(t_{s2}))}u_3(t_{s3}^*(t_{s2}))\right].\end{aligned} \tag{8}$$

Note that $\tilde{u}_2(t_{s2}) = u_2(t_{s2})$ with $\xi = 0$. With $\xi = 1$, arriving earlier or later to activity 2 means that time will be given to or taken from activity 2 and 3 in proportions determined by the relative rates of change of the marginal utility functions.

Correspondingly, given that the traveller departs to activity 2 at t_{d2}, there is an associated optimal arrival time to that activity, $t_{s2}^*(t_{d2})$, and an optimal departure time from activity 1, $t_{d1}^*(t_{d2})$. We define $\hat{U}_2(t_{d2})$ as the utility derived from the start of the day to the departure from activity 2 given that trip 1 is timed optimally in response to t_{d2}. We then introduce the "forward optimal" marginal utility function $\hat{u}_2(t_{d2}) = d\hat{U}_2/dt_{d2}$ as the marginal change in preceeding utility due to a later departure time. Using $\hat{u}_2(t_{d2})$ the optimality conditions can be written as $\hat{u}_2(t_{d2}^*) = u_3(t_{s3}^*)$. The expression for $\hat{u}_2(t_{d2})$ is given in the Appendix.

2.2 Value of travel time

An increase in travel time will affect utility not only through the marginal utility of travel, ν, but also through the reduction in activity participation that must occur due to the limited time available in a day (e.g., Jara-Diaz 2000). To capture the full, long-run impact of a change in travel time, it is reasonable to calculate its value given that the individual adjusts its schedule optimally to the change. The full effect on utility of a change in travel time is then dU^*/dT, where U^* is the utility under the optimal schedule and T is a generic travel time.[6] Viewing U as

[5] This can be seen as a dynamic programming approach to optimizing the daily activity schedule, which is straightforwardly extended to more complex schedules (see, e.g., Karlström 2005).

[6] Ettema & Timmermans (2003) define the value of saving time on trip 1 based on the *partial* derivative of U with respect to T_1 holding departure times constant, which does not acknowledge that saved time in the long run can be distributed on all activities. Furthermore, they do not account for the possibility of flexible work hours, nor do they

an indirect utility function within a goods-activities microeconomic framework with constant marginal utility of income λ, the value of travel time is $-dU^*/dT/\lambda$ (c.f. Jara-Diaz 2000).

In general, the impact on utility depends on which trip travel time is changed on, since this determines which activities are most affected. In other words, the value of travel time savings varies between trips. For a general ξ the value of saving time in each trip can be expressed compactly using the backward and forward optimal marginal utility functions $\tilde{u}_2(t_{s2})$ and $\hat{u}_2(t_{d2})$:

$$\frac{dU^*}{dT_1} = \frac{dt^*_{d1}}{dT_1} u_1(t^*_{d1}) - \frac{dt^*_{s2}}{dT_1} \tilde{u}_2(t^*_{s2}) + \nu, \tag{9}$$

$$\frac{dU^*}{dT_2} = \frac{dt^*_{d2}}{dT_2} \hat{u}_2(t^*_{d2}) - \frac{dt^*_{s3}}{dT_2} u_3(t^*_{s3}) + \nu. \tag{10}$$

Expressions for the derivatives dt^*_{d1}/dT_1 etc. are given in the Appendix. It should be noted that the effect of changing the travel time on one trip generally depends on the baseline travel times on both trips from which the change occurs.

If activity 2 is fixed ($\xi = 0$) the optimal timing—and hence the value of saving travel time—of one trip is independent of the travel time and timing of the other trip. For each trip the optimal departure time adjustment depends on the relative steepness of the marginal utilities of the origin and destination activities,

$$\left.\frac{dU^*}{dT_i}\right|_{\xi=0} = \frac{u'_{i+1}(t^*_{si+1})}{u'_i(t^*_{di}) - u'_{i+1}(t^*_{si+1})} u_i(t^*_{di}) - \frac{u'_i(t^*_{di})}{u'_i(t^*_{di}) - u'_{i+1}(t^*_{si+1})} u_{i+1}(t^*_{si+1}) + \nu, \quad i = 1,2. \tag{11}$$

That is, if the marginal utility decreases more steeply at the origin than it increases at the destination, more time will be taken from the latter activity, and vice versa.

If activity 2 is completely flexible ($\xi = 1$) the optimal schedule adjustment and the value of saving travel time will be the same regardless of which trip travel time is saved on. In this special case there is thus a single value of travel time for both trips. Time is taken from all activities in proportions that, again, are determined by the relative steepness of the marginal utility functions,

$$\begin{aligned}\left.\frac{dU^*}{dT}\right|_{\xi=1} &= \frac{u'_2(t^*_{d2} - t^*_{s2}) u'_3(t^*_{s3})}{u'_1(t^*_{d1}) u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1}) u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2}) u'_3(t^*_{s3})} u_1(t^*_{d1}) + \\ &+ \frac{u'_1(t^*_{d1}) u'_3(t^*_{s3})}{u'_1(t^*_{d1}) u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1}) u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2}) u'_3(t^*_{s3})} u_2(t^*_{d2} - t^*_{s2}) - \\ &- \frac{u'_1(t^*_{d1}) u'_2(t^*_{d2} - t^*_{s2})}{u'_1(t^*_{d1}) u'_2(t^*_{d2} - t^*_{s2}) - u'_1(t^*_{d1}) u'_3(t^*_{s3}) - u'_2(t^*_{d2} - t^*_{s2}) u'_3(t^*_{s3})} u_3(t^*_{s3}) + \nu.\end{aligned} \tag{12}$$

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consider the value of travel time for trip 2.

2.3 Schedule delay costs

By definition, making a trip that is not optimally timed causes a reduction in utility. This feature is also captured in the schedule delay model of for example Vickrey (1969) and Small (1982), where costs arise if an individual arrives before or after a preferred arrival time t^*. Specifically, the cost of a trip starting at time t_d and ending at time $t_s = t_d + T$ in that model is, for some parameters α, β and γ,[7]

$$C(t_d, t_s) = \alpha T + \begin{cases} \beta[t^* - t_s] & \text{if } t_s < t^*, \\ \gamma[t_s - t^*] & \text{if } t_s \geq t^*. \end{cases} \tag{13}$$

Tseng & Verhoef (2008) showed that the linear model (13) can be derived as a special case of a model for the timing of the morning commute that is equivalent to the special case $\xi = 0$ of our model (1). They thus considered the work activity to be completely fixed, so that subsequent trips are irrelevant for the timing problem. The authors first identified t^* with the point in time when $u_1(t) = u_2(t)$, i.e., when the marginal utility at home equals the marginal utility at work. If travel were instantaneous, t^* would be the optimal time to travel from home to work. With a positive travel time, the momentary cost of travel at any time t arises from the difference in the marginal utility of travel ν and the largest of $u_1(t)$ and $u_2(t)$, where $u_1(t) > u_2(t)$ for $t < t^*$ and $u_2(t) > u_1(t)$ for $t > t^*$. With constant marginal utility of income λ, Tseng & Verhoef (2008) showed that the total cost of a trip starting at t_{d1} and ending at $t_{s2} = t_{d1} + T_1$ is

$$C(t_{d1}, t_{s2})_{\xi=0} = \frac{1}{\lambda}\int_{t_{d1}}^{t_{s2}} [u_1(t) - \nu]dt + \begin{cases} \dfrac{1}{\lambda}\displaystyle\int_{t_{s2}}^{t^*} [u_1(t) - u_2(t)]dt & \text{if } t_{s2} < t^*, \\ \dfrac{1}{\lambda}\displaystyle\int_{t^*}^{t_{s2}} [u_2(t) - u_1(t)]dt & \text{if } t_{s2} \geq t^*. \end{cases} \tag{14}$$

With activity 2 being arbitrarily flexible (14) no longer reflects the true costs of early or late arrival, as the traveller can adjust its subsequent schedule to account for the arrival time. The formula can be generalized, however, to incorporate this. It is reasonable to calculate the schedule delay cost under the condition that trip 2 is timed optimally given the arrival time of trip 1, which means that $u_2(t)$ should be replaced with the backward optimal marginal utility function $\tilde{u}_2(t)$. Introducing the marginal cost functions $\alpha(t) = [u_1(t) - \nu]/\lambda$, $\beta(t) = [u_1(t) - \tilde{u}_2(t)]/\lambda$ and $\gamma(t) = [\tilde{u}_2(t) - u_1(t)]/\lambda$ gives

[7] This linear specification is frequently used to evaluate the value of travel time reliability, whereby the travel time T is treated as stochastic with known distribution (e.g., Noland & Small 1995, Fosgerau & Karlström 2009).

$$C(t_{d1}, t_{s2}) = \int_{t_{d1}}^{t_{s2}} \alpha(t)dt + \begin{cases} \int_{t_{s2}}^{t^*} \beta(t)dt & \text{if} t_{s2} < t^*, \\ \int_{t^*}^{t_{s2}} \gamma(t)dt & \text{if} t_{s2} \geq t^*. \end{cases} \tag{15}$$

It can be seen that the linear schedule delay model (13) is obtained as a special case of (15) when the three marginal cost functions $\alpha(t)$, $\beta(t)$ and $\gamma(t)$ are constant over time. Thus, if costs are specified to arise due to early or late *arrival* as in the schedule delay model, the cost of travel should be interpreted as the cost of spending time travelling rather than at home. The additional costs of early and late arrival then arise as the costs of time spent at work when being at home would be more beneficial, and time not spent at work when being there would be more beneficial than being at home, respectively (see Tseng & Verhoef 2008).

In Section 4.1 we make use of the relationship with the linear schedule delay model to calibrate the marginal utility of travel, ν.

3 Unplanned disruptions

Empirical evidence tells us that major unplanned transport network disruptions are generally followed by a time—on the order of days or weeks—of uncertainty, learning and adaptation for the travellers. If the network disruption is long-lasting, the traffic eventually approaches a new equilibrium-like state, where travellers have received sufficient information about the new travel conditions and adapted their travel decisions accordingly. Observations are fairly consistent in that the most common responses by individuals are changes in departure time and route choice. To a lesser extent people cancel or consolidate (mainly non-work) trips, whereas people are relatively reluctant to change travel mode (Wesemann et al. 1996, Giuliano & Golob 1998, Hunt et al. 2002, Cairns et al. 2002, Clegg 2007, Zhu et al. 2008).[8]

The delay costs during the first few days of a disruption can be expected to be particularly high, both because the delays themselves are larger (due to, for example, suboptimal route choices) and because the travellers are less able to adapt their schedules to the delays. Immediately following the event, some individuals may be completely uninformed of the disruption when scheduling a trip. Such individuals will depart as normal and, by incurring much longer than expected travel times, arrive late to their destinations.

Soon, however, information about the event will start to spread among travellers through the media, family, friends and other channels (Zhu et al. 2008). Still, even when most travellers have received information about the occurrence of the disruption, there likely remains unusually large uncertainty about the travel conditions for a significant time period. During this period, travellers will plan their travel as best as they can given their predictions of the travel conditions they will

[8]For example, in a survey following the 2007 Minneapolis bridge collapse (used further below), people who stated that they were affected by the bridge collapse responded that they adjusted in the following ways: changed departure time (75.3%), changed route (72.3%), avoided destinations (61.0%), cancelled trips (14.3%), worked from home (9.1%), and changed mode (6.5%) (Zhu et al. 2008).

face. Some travellers may underestimate the induced changes in travel conditions and arrive late to their destinations. Others may overestimate the changes or, given the uncertainty, add a safety margin to the estimated delay in order to reduce the risk of arriving late to work (Noland & Small 1995, Fosgerau & Karlström 2009). If travel conditions turn out to be better than feared, these individuals will arrive early at their destinations.

To investigate whether empirical data supports these hypothesized schedule adjustment types, we use observations from the collapse of the I-35W bridge across the Mississippi River in Minneapolis, Minnesota on 1 August 2007. The collapse resulted in tragic causalties as well as a significant, long-lasting disturbance to the transportation system of the city. A hand-out, mail-in survey was conducted in September 2007 to investigate how travellers were affected by and adjusted to the disruption (Zhu et al. 2008).

Respondents were asked (among other things) about the departure time, travel time, travel mode and route choice in their morning commute during each of four periods: before the bridge collapse, the day after the collapse, the following weeks, and at the time of the survey. Unfortunately for the present analysis, they were not asked about their evening commute. They were also asked whether they have a flexible work schedule or not. Out of a total of 1000 handed out surveys, 141 usable responses were obtained. Here we consider the 79 respondents who stated that they commuted by car before the collapse. Of these, 56 respondents had their pre-collapse schedule affected by more than 5 minutes through changed travel time and/or departure time in at least one post-collapse period; we will refer to this group as ``affected car users".

According to the theoretical model, $dt^*_{d1}/dT_1 < 0$ and $dt^*_{s2}/dT_1 > 0$ for any level of schedule flexibility—i.e., an increase in travel time on trip 1 will move the optimal departure time of that trip earlier and the optimal arrival time later (whether the departure time or the arrival time will change the most depends on the relative steepness of the marginal utility functions). The survey data, however, contains many responses where this principle is violated, in particular the morning after the collapse.

On the first day, 13 (23%) of the affected car users report departing at the same time as before the collapse. Of these, eight (14% of affected car users) report increased travel times, ranging between 17% and 50% of the pre-collapse travel times (two report unchanged and three report reduced travel times). Hence, there is a tendency for under-adjustment of the schedule as compared to the theory. At the same time, 26 (46%) respondents report departing earlier than before the collapse the first day. Of these, nine (16% of affected car users) arrive at work earlier than before the collapse (six report unchanged and eleven report later arrival times). Hence, there is also a tendency for over-adjustment of the schedule.[9] Both these effects, in particular the under-adjustment, seem to decline the following weeks after the collapse.

[9] We found that the share of people who over-adjusted their departure time was significantly (at the $\alpha = 0.05$ level) higher among those with stated fixed work schedule than among those with stated flexible schedule, which suggests that people with fixed schedules are more inclined to add safety margins to travel times in order not to arrive late. We also tested whether household conditions such as the number of small and large children restricted the possibility to depart earlier in the morning, but found no significant effects.

We do not have information about the time-of-day variation of travel times for each respondent before and after the disruption, and hence we cannot in principle rule out that the seemingly suboptimal responses noted above are actually optimal due to changes in the congestion profile. However, we think that a more important explanation for the observed behavior is the lack of reliable information that characterizes the time immediately following a major disruption such as this.

3.1 Delay costs and adjustment profiles

An increase in travel time incurs a utility loss ΔU for the traveller that depends on her information about the delay and the flexibility of her schedule for adjustments. With a highly fixed work schedule, arriving late to work means that much of this time will be lost without possibility of recovery. With a relatively flexible schedule, however, time lost during the morning warm-up period can be somewhat compensated for by a more productive period in the evening and possibly by sacrificing some time at home in the evening. Similarly, with a highly fixed schedule, arriving early to work means that time will be spent unproductively at work compared to the time lost in the morning. With a flexible schedule, work can be carried out about as productively as normally, but productivity will decrease earlier in the evening.

Based on the empirical observations, we formulate a number of stylized adjustment profiles for which we then compare delay costs within our model.

No adjustment. The first type of response that we consider is to not adjust the departure time on any of the two trips when faced with delays. This is often realistic for trip 1 when travellers are completely uninformed of the disruption, but is probably unrealistic for trip 2 when travellers have been exposed to the event. This adjustment profile may therefore represent the upper extreme of an initially uninformed traveller.

No + optimal adjustment. Alternatively, we may assume that a traveller departs as normal in the morning but, being exposed to the disruption on trip 1, is able to gather information and schedule trip 2 optimally given the delay and the arrival time of trip 1. This adjustment profile may represent the lower extreme of an initially uninformed traveller and provides, in combination with the no adjustment profile, a span in which the impact for such a traveller should lie.

Overadjustment. The third type of response is to overestimate the actual delays on both trips, either erroneously or deliberately as a safety margin. The departure times are then optimized with respect to the overestimated travel times and, for trip 2, the arrival time of trip 1. In the analysis we assume, somewhat arbitrarily, that delays are systematically overestimated by 50%. Hence, a traveller with a baseline travel time of 30 minutes and an actual delay of one hour would reschedule her trip assuming a two hour travel time.

Over + optimal adjustment. In correspondence with no + optimal resonse profile, we also consider the case where a traveller overestimates the delay on trip 1 but is able to optimally reschedule trip 2 given the actual delay on that trip and the arrival time of trip 1.

Optimal adjustment. Finally, as a baseline we consider the case where a traveller is able to perfectly predict the travel conditions and adjust her schedule optimally in response to the delay on both trips. If the disruption is long-lasting, this should be the behavior that emerges over time, as travellers learn and adapt to the changed travel conditions.

Figure 1 shows how utility losses due to delays occur during the day with the no adjustment, the overadjustment and the optimal adjustment profiles for a generic set of marginal utility curves—in particular, activity 2 is assumed to be semi-flexible ($\xi = 0.5$). At any time, a loss (shown in red) or gain (shown in green) arises equal to the difference between the current marginal utility and the marginal utility at the same time before the disruption.

3.2 Trip cancellation

Instead of adjusting trip departure times, a possible way of responding to expected long delays is to cancel the trip completely. Some individuals, for example, may gain more by working from home one day than travelling to work with long delays. In our setting, assuming that activities 1 and 3 represent being at home in the morning and the evening, respectively, this schedule would yield the utility

$$U_h = \int_0^1 \max\{u_1(t), u_3(t)\} dt. \tag{16}$$

Cancelling the trip is optimal if the utility lost from spending time travelling is greater than the utility gained from taking part in the activity itself, so that $U < U_h$.[10] In an uncertain environment, people may also cancel trips if they *expect* the travel times to be longer than they actually would be. In extreme situations some people may have no possible route to reach their destinations during the disruption, in which cases they will be forced to cancel or postpone the trip.

The survey data from the Minneapolis bridge collapse shows that only one of the 56 affected car users cancelled the morning trip to work the day following the event. This suggests that trip cancellation is a relevant adjustment strategy for some individuals, but that other responses such as route and departure time changes are much more dominant, which also previous investigations have shown (Giuliano & Golob 1998, Hunt et al. 2002, Cairns et al. 2002, Zhu et al. 2008).

4 Utility specifications

To obtain numerical values for the delay costs we need to specify functional forms and parameter values for the marginal utility of activities and travel. In this paper we make use of the estimation work presented by Ettema & Timmermans (2003). Hence, we adopt the same specification of the marginal utilities, namely

[10] The option of trip cancellation can be incorporated explicitly in an extended version of our model as a discrete decision variable that preceeds the choices of departure times.

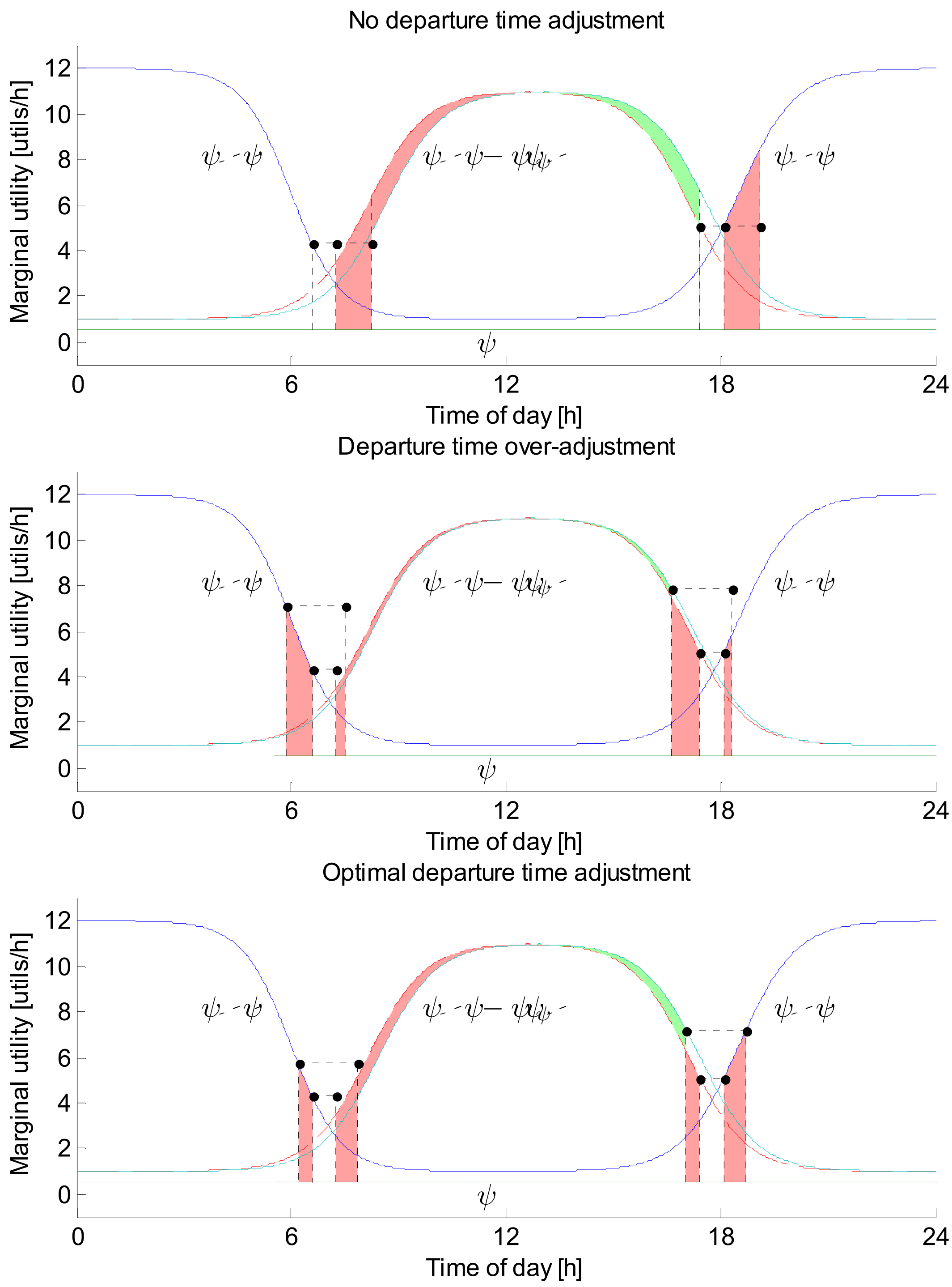

Figure 1: Utility losses due to delay under different schedule adjustment profiles. Semi-flexible scheduling ($\xi = 0.5$). Solid curves show generic marginal activity utilities, dashed curves show pre-disruption optimal marginal utilities, dotted lines mark the intervals spent traveling, red areas show utility losses, green areas show utility gains. Top: No departure time adjustment. Middle: Departure time over-adjustment. Bottom: Optimal departure time adjustment.

$$u_i(t-\xi_i t_{si}) = u_i^0 \phi_i \zeta_i \frac{\exp(-\zeta_i [t-\xi_i t_{si} - \omega_i])}{[1+\exp(-\zeta_i [t-\xi_i t_{si} - \omega_i])]^{\phi_i+1}}, \quad i=1,2,3, \tag{17}$$

where $\xi_1 = \xi_3 = 0$ and $\xi_2 = \xi$. The functional form satisfies our general assumptions of a warm-up period with increasing marginal utility, followed by a cool-down period with decreasing marginal utility. The parameter ω_i determines the location of the marginal utility curve along the time axis, u_i^0 determines the total utility to be gained from the activity, ζ_i determines the concentration around the peak while ϕ_i determines the relative steepness of the warm-up and cool-down periods.

Ettema & Timmermans (2003) estimated the above specification using data on work commute trips from Dutch activity and travel diaries. Due to the nonlinear form of the utility functions, a genetic algorithm optimization method was used in the Maximum Likelihood parameter estimation. The point estimates along with standard errors are shown in Table 1.

Parameter	Activity 1	Activity 2	Activity 3
u^0	9.73 (1.82)	8.63 (1.05)	9.97 (1.40)
ω [h]	3.43 (0.18)	10.4 (0.52)	19.7 (0.073)
ζ [h^{-1}]	1.44 (0.44)	0.96 (0.15)	1.44 (0.16)
ξ	–	0.43 (0.12)	–
ϕ	1	1	1

Table 1: Parameter estimates for the activity marginal utility functions (standard errors in parentheses). Source: Rttema & Timmermans (2003). Where applicable, values have been rescaled from minutes to hours. Note that ϕ was not estimated but fixed to 1.

Figure 2 shows average marginal utility curves across a sample of 100 synthetic "individuals", where each individual is represented by a set of parameter values randomly drawn from normal distributions defined by the means and standard deviations of the parameter estimates reported by Ettema & Ttimmermans (2003). For each set of parameter values, the marginal utility curve of work (activity 2) $u_2(t-\xi t_{s2})$ was calculated assuming a travel time of 40 minutes on each trip and an optimal schedule. Note that these average curves are only for illustration; all calculations below are performed for individual sets of parameters and aggregated only as the last step.

4.1 Calibrating the utility of travel

The study of Ettema & Timmermans (2003) did not include any estimation of the marginal utility of travel ν (when travel time is independent of departure time, as the authors assumed, the optimal schedule is independent of ν). Delay costs, however, depend on the value of ν in relation to the other marginal utilities. To make a rough estimate of ν we have taken advantage of the relationship between our model, the model of Tseng & Verhoef (2008) and the linear schedule delay model that was demonstrated in Section 2.3. The approach was to calibrate ν with the target of reproducing the typical ratio between the late arrival cost and the travel time cost from the schedule delay literature.

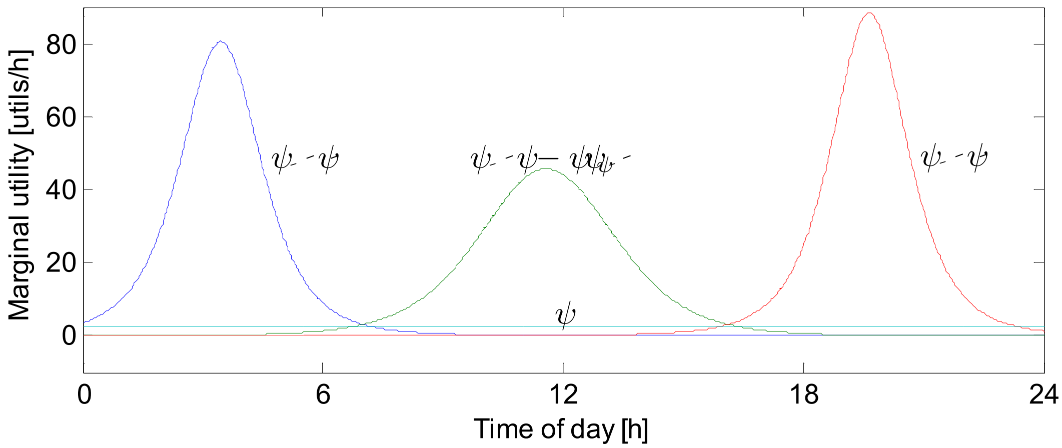

Figure 2: Average marginal utility curves based on random draws from normal distributions defined by the mean and standard deviation of the parameter estimates reported by Ettema & Timmermans (2003) and shown in Table 1.

A set of 100 synthetic individuals was created, each represented by a set of randomly drawn parameter values as described in connection with the average marginal utility curves above. For each individual, $u_1(t)$ and the backward optimal marginal utility function $\tilde{u}_2(t)$ were calculated and the ideal departure time t^* when the two curves intersect was identified; $\tilde{u}_2(t)$ was calculated assuming a travel time of 40 minutes for trip 2. The marginal utility curves were then expressed as functions of $t' = t - t^*$, i.e., time away from t^*, producing 100 realizations of $u_1(t')$ and $\tilde{u}_2(t')$. Finally, average curves for $u_1(t')$ and $\tilde{u}_2(t')$ were calculated by taking the mean value across all individuals for every t'.

Using the average curves for $u_1(t')$ and $\tilde{u}_2(t')$ we calculated optimal departure and arrival times t_{d1}^*, $t_{s2}^* = t_{d1}^* + T_1$ relative to t^* for a few different trip 1 travel times T_1 ranging from 30 to 60 minutes. We used the results from Section 2.3 to obtain the following expressions for the average travel time and late arrival cost coefficients:

$$\bar{\alpha} = \frac{1}{\lambda T_1}\left[\int_{t_{d1}^*}^{t_{s2}^*} u_1(t)dt - \nu T_1\right], \tag{18}$$

$$\bar{\gamma} = \frac{1}{\lambda(t_{s2}^* - t^*)}\int_{t^*}^{t_{s2}^*}[\tilde{u}_2(t) - u_1(t)]dt. \tag{19}$$

From the literature on linear schedule delay models, including Tseng & Verhoef (2008) and Small (1982), we obtained the approximate empirical relationship $\gamma/\alpha \approx 3$. Using this for the ratio γ/α and solving for ν finally gives

$$\nu \approx \frac{1}{T_1}\int_{t_{d1}^*}^{t_{s2}^*} u_1(t)dt - \frac{1}{3(t_{s2}^* - t^*)}\int_{t^*}^{t_{s2}^*}[\tilde{u}_2(t) - u_1(t)]dt. \tag{20}$$

The estimated value of ν was relatively stable between different trip 1 travel times, varying between 2 and 3.5. We found the average value to be $\nu \approx 2.5$, which is the value we used in the subsequent delay cost calculations; it is shown in relation to the other marginal utility curves in Figure 2.

The fact that $\nu > 0$ may justify a few comments. First, the value 0 bears no great significance in this setting since only differences in utility are meaningful; that the marginal activity utilities have a minimum of 0 is only an effect of the specified functional forms. What is relevant is the fact that ν exceed the marginal activity utilities at certain times, suggesting, e.g., that individuals prefer additional travel rather than arriving early to work. This effect has been observed empirically also by Tseng & Verhoef (2008).

5 Results: Traveller costs of disruptions

For a given traveller the effect ΔU of an increase in travel time ΔT depends on the size of ΔT, the distribution of ΔT on trip 1 and 2, the baseline pre-disruption travel time on both trips, and the response in terms of schedule adjustments. The change in utility can be put in relation to the effect of a marginal change in travel time under optimal schedule adjustment, dU^*/dT. We use the ratio

$$R(\Delta T) = \frac{\Delta U/\Delta T}{dU^*/dT} \tag{21}$$

to express how much higher the average delay cost per hour delay is in relation to the ordinary value of travel time savings.

Both dU^*/dT and $\Delta U/\Delta T$ vary between individuals due to different parameter values in the marginal utility functions. As in the calibration of ν, we have calculated the changes in utility across a sample of synthetic individuals, each represented by a set of parameter values randomly drawn from normal distributions defined by the means and standard deviations of the parameter estimates reported by Ettema & Timmermans (2003) and shown in Table 1.

In reality the different parameters are likely correlated, so that for example large values of one parameter are typically associated with small values of another. Unfortunately we lack information about the variance-covariance matrix besides the standard errors. This means that we may overestimate the variation in delay costs between individuals and that some sets of parameter values may be unrealistic. Indeed, the calculations showed that some sets of parameter values resulted in negative values of travel time (since the utility of travel exceeded the simultaneous utility of activities), while others resulted in unrealistic boundary solutions for long delays (e.g., departing from activity 2 immediately after arriving). Such sets of parameter values were not included in the reported results. Furthermore, we consistently excluded the highest 10% and lowest 10% of the calculated relative delay costs to remove the most extreme and potentially unrealistic sets of parameter values. Reported mean values and standard deviations were calculated for these trimmed samples. An initial sample size of 100 synthetic individuals was determined to be sufficient to produce stable results.

Figure 3 shows the average cost per hour delay in relation to the ordinary value of time, $R(\Delta T)$, calculated for delays ranging from 0 to 5 hours, equally distributed on both trips, for the five schedule adjustment profiles described in Section 3.1. The value of travel time is calculated for a marginal change in travel time also equally distributed on both trips, and the baseline travel time is set to 40 minutes on both trips. Table 2 gives the corresponding values for baseline travel times 30 minutes and 50 minutes on both trips, respectively, and also reports the trimmed standard deviations within the random sample.

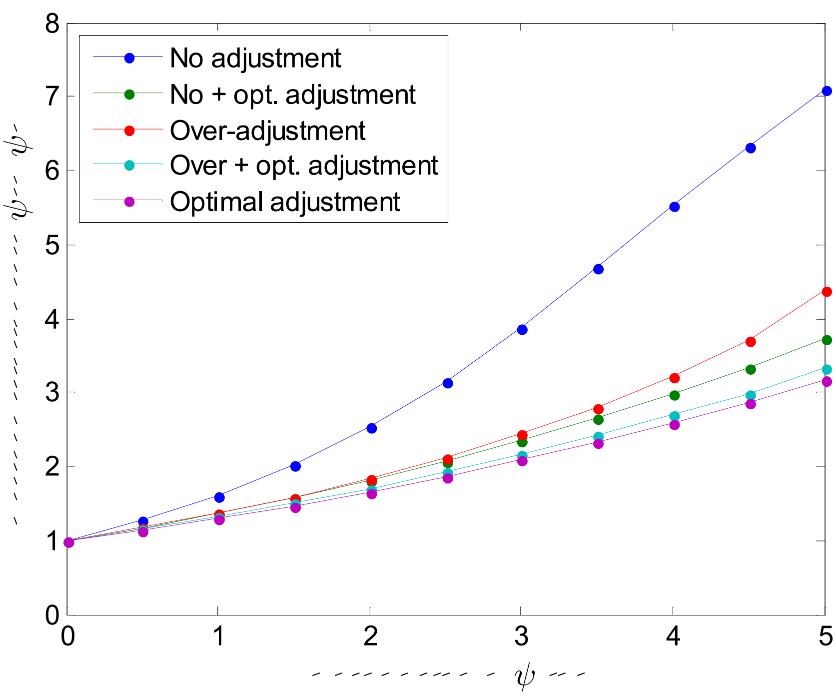

Figure 3: Average delay cost per hour delay in relation to the marginal value of travel time savings, $R(\Delta T)$, with delay evenly distributed on both trips. Baseline travel time is 2×40 minutes.

As can be seen from Figure 3, the average relative delay cost increases faster than linearly with the delay for all adjustment profiles, expect for a slight saturation for large delays with the no adjustment type. By construction the delay cost is lowest for the optimal adjustment profile, since optimizing the schedule against the delays means minimizing the delay costs. For this adjustment profile the cost per hour delay is about two times the ordinary value of travel time when the total delay is three hours (1.5 h on each trip) and about three times that value when the total delay reaches five hours.

The largest delay costs occur for the no adjustment profile, i.e., travellers who retain their pre-disruption departure times on both trips. With no schedule adjustment the relative cost per hour delay is about four for a three hour total delay and about five for a four hour total delay, which is about twice the cost of the optimal adjustment profile. The cost for the no + optimal adjustment type—i.e., travellers who retain their pre-disruption departure time on trip 1 but optimally reschedule trip 2—is considerably lower than the pure no adjustment profile, which shows that the impacts for initially uninformed travellers can cover a wide range. The large difference also shows the great value of receiving information and being able to reschedule during the day; in fact, this gain is considerably greater than the additional gain from being able to schedule both trips, compared to only the evening trip, optimally.

The costs for the two over-adjustment profiles are restricted to a smaller interval, but should not be interpreted as upper and lower bounds for travellers who over-estimate delays, since it is of course possible to overestimate delays by more or less than 50% which was assumed here.

Delay ΔT	Adj. trip1+2 No	No+opt.	Over	Over+opt.	Optimal
Base. $T = 1.0$ h					
1.0 h	1.65 (0.27)	1.40 (0.12)	1.40 (0.13)	1.35 (0.11)	1.33 (0.10)
2.0 h	2.67 (0.78)	1.89 (0.29)	1.92 (0.33)	1.79 (0.26)	1.73 (0.24)
3.0 h	4.17 (1.67)	2.50 (0.52)	2.59 (0.60)	2.32 (0.46)	2.21 (0.41)
4.0 h	6.03 (2.90)	3.24 (0.82)	3.45 (0.98)	2.93 (0.72)	2.78 (0.64)
5.0 h	7.80 (4.10)	4.08 (1.20)	4.74 (1.52)	3.60 (1.01)	3.44 (0.93)
Base. $T = 1.67$ h					
1.0 h	1.57 (0.21)	1.34 (0.09)	1.35 (0.11)	1.30 (0.08)	1.28 (0.08)
2.0 h	2.45 (0.63)	1.75 (0.22)	1.79 (0.26)	1.66 (0.20)	1.62 (0.18)
3.0 h	3.72 (1.33)	2.25 (0.39)	2.35 (0.48)	2.09 (0.35)	2.01 (0.32)
4.0 h	5.23 (2.23)	2.84 (0.62)	3.09 (0.78)	2.58 (0.53)	2.48 (0.49)
5.0 h	6.59 (3.06)	3.52 (0.91)	4.18 (1.28)	3.14 (0.72)	3.02 (0.71)

Table 2: Average delay cost per hour delay in relation to the marginal value of travel time savings, $R(\Delta T)$. Results are shown for two baseline daily travel times, 1.0 h (2×30 min) and 1.67 h (2×50 min), and five adjustment types. Travel times and delays are distributed equally on both trips. Numbers represent trimmed mean values across 100 random draws, standard deviations are shown in parentheses.

Table 2 shows that delay costs to a certain degree depend on the baseline travel time from which the delays occur. Specifically, longer baseline travel times mean that delay costs are lower. Note that this is an effect of the shape of the marginal utility functions only as travel time is treated as exogenous—we have not taken into account that individuals with different utility curves may choose different baseline travel times according to their preferences. Table 2 also shows that the variation in cost between travellers, as measured by the standard deviation, is considerable for the no adjustment profile, where all delay will affect the destination activity regardless of preferences, but quite moderate for the other adjustment profiles, which all contain some element of individual schedule optimization.

5.1 Influence of delay distribution

We have compared the delay costs for the main scenario above in which delay is evenly distributed on both trips with those under two other possible disruption scenarios. The first is a significant but brief disruption that only affects the travel time on trip 1; the second scenario is a disruption that occurs in the middle of the day and only affects the travel time on trip 2. The purpose of the comparison is to investigate the interplay between the increasing marginal delay cost that was found above (which should mean that delay concentrated to one trip has a higher cost) and the possibility to make up for time lost in the morning by rescheduling the remaining day (which should mean that delay has a higher cost if concentrated to trip 2 than to trip 1).

For the main scenario with evenly distributed delay we consider the three basic adjustment profiles: no adjustment, over-adjustment and optimal adjustment of both trips. For the first alternative scenario with delay concentrated to trip 1 we compare each of these profiles with the same type of adjustment on trip 1 (no, over- and optimal adjustment); for trip 2 we assume that all travellers are informed that the disruption is over and are able to adjust the departure time optimally given the arrival time to work.

Correspondingly, for the second alternative scenario with delay concentrated to trip 2 we compare each of the three adjustment profiles with the same type of adjustment on trip 2 (no, over- and optimal adjustment given delay and arrival time to work); we assume that all travellers are uninformed of the disruption to occur in the evening when they depart in the morning and make no adjustment of the departure time for trip 1.

Table 3 shows the delay cost in each of the two alternative scenarios in relation to the cost of the same total delay in the main scenario. It can be seen that the ratio is well below 1 and even below 0.5 for all adjustment profiles when delay is concentrated to trip 1. Thus, a one hour delay in the morning, say, is about half as costly as a 30 minutes delay in the morning plus a 30 minutes delay in the evening.

	Trip 1 only			Trip 2 only		
Delay ΔT	No+opt.	Over+opt.	Opt.+opt.	No+no	No+over	No+opt.
0.5 h	0.45 (0.12)	0.47 (0.13)	0.47 (0.13)	1.97 (0.24)	1.70 (0.17)	1.61 (0.15)
1.0 h	0.41 (0.11)	0.46 (0.12)	0.46 (0.12)	2.52 (0.43)	1.90 (0.22)	1.71 (0.18)
1.5 h	0.39 (0.11)	0.46 (0.12)	0.46 (0.12)	3.05 (0.64)	2.10 (0.27)	1.80 (0.20)
2.0 h	0.36 (0.10)	0.46 (0.11)	0.46 (0.11)	3.45 (0.79)	2.32 (0.34)	1.90 (0.23)
2.5 h	0.34 (0.10)	0.45 (0.10)	0.46 (0.10)	3.50 (0.80)	2.53 (0.40)	1.99 (0.25)

Table 3: Delay cost with delay only on one trip, in relation to with the same delay distributed equally on both trips. Results are shown for the baseline daily travel time 1.33 h (2×40 min), and three adjustment types. Numbers represent mean values across 100 random draws, standard deviations are shown in parentheses.

When delay occurs only on trip 2, on the other hand, the ratio is considerably larger than 1 for all adjustment types, often larger than 2. The ratio increases as the level of adjustment decreases from optimal to no adjustment. For example, a one hour delay in the evening is roughly twice as costly as a 30 minutes delay in the morning plus a 30 minutes delay in the evening, with some variation depending on adjustment profile.

The results show that the possibility to adjust the subsequent schedule in response to delay has a significant impact on the cost. When delay affects only the morning trip, the evening trip can be rescheduled appropriately (to the extent that scheduling flexibility allows), but when it affects only the evening trip, no more schedule adjustments can be made to make up for the delay that day.

6 Discussion and conclusion

The main point of this paper has been to introduce activity-based modelling as a viable approach to evaluating the traveller costs of transport network disruptions. The modelling framework is able to handle several important aspects of such events: delays may be very long in relation to the normal day-to-day fluctuations; the impact of delay may depend on the flexibility to reschedule activities; lack of information and uncertainty about travel conditions may lead to under- or over-adjustment of the daily schedule in response to the delay; delays on more than one trip may restrict the gain from rescheduling activities.

The numerical calculations show that the average cost per hour delay increase with the delay duration, so that every additional minute of delay comes with a higher cost. The cost varies depending on adjustment behavior (less adjustment, loosely speaking, giving higher cost) and baseline travel time (longer travel time giving lower cost). The results also show that a disruption affecting only the morning commute is less costly than if the same total delay is equally distributed on both trips, which in turn is less costly than a disruption affecting only the evening commute. This reflects the benefit of scheduling flexibility and being able to adjust the remaining schedule in response to earlier events.

The fact that the delay costs are higher than the ordinary value of time suggests that existing evaluations of real network disruptions have underestimated the societal costs of the disruptions. It also indicates that actions taken to reduce the impacts of a disruption, such as reducing the time until the network capacity is restored or providing up-to-date and relevant information, could have greater economic benefits than previously thought.

The calculations suggest that the cost per hour delay for, say, a total delay of three hours distributed equally on both trips, is in the range of two to four times the ordinary value of travel time. The precise values, of course, rely heavily on the estimated parameter values of the model. Some issues can be raised, however, regarding how a model such as this is estimated.

In this case ettematimmermans2003 estimated a single set of parameter values for all individuals in a sample of travel and activity diaries. This means, for example, that the time-of-day location parameters ω_i of all activities are assumed equal for all individuals. If these parameters actually differ between individuals (e.g., if some individuals are required to be at work earlier than others), this variation may instead be captured by other parameters, such as lower values for the steepness parameters ζ_i. This, in turn, could mean that we underestimate the loss of utility that is caused by late arrival or early departure. Even though we take the variation in parameter values into account and randomly draw sets of parameter values, we cannot fully compensate for this potential problem.

For future applications of models of this kind, therefore, there seems to be a need to develop the estimation procedure (an opinion also expressed by Zhang et al. (2005)). One possible way to go would be to allow parameters to vary among individuals.

The model employed here is quite simple, involving only three activities, two trips and two decision variables. It is in principle straightforward to extend the model to include more activites and trips as well as day-to-day dependencies. Although the rapidly increasing number of decision variables would make the analysis increasingly challenging, the essential features would remain the same: delays will give less time to take part in activities, which leads to costs that depend on scheduling flexibility, adjustment response, etc. It is reasonable to assume that more complex schedules, involving more scheduling constraints, will lead to higher costs in the event of large delays.

In our analysis travel time is held completely exogenous. In an extended framework one could also include other adjustment dimensions such as route, destination and mode choice. An interesting topic for future work is to integrate our cost model with a model of the dynamic traffic evolution following a disruption, that would provide the travel times as input to the cost model. In a fully integrated model these costs would, in turn, affect traveller's decisions the following days, which give rise to new costs, etc.

Besides travellers' delay costs, there may be many other impacts that contribute to the total societal costs of transport network disruptions. For travellers, the increased time spent travelling may cause discomfort that is independent of whether time is lost from other activities. This could be represented by a marginal utility of travel the decreases beyond the normal travel time. For firms, employees arriving late, delayed deliveries, cancelled business meetings, etc. lead to productivity losses. There are also costs associated with not being able to reach societal services such as emergency health care as fast as normally. Analyzing these other costs are (perhaps quite disparate) topics for further research.

Acknowledgments

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Appendix

Optimality conditions when travel times depend on departure times (to ensure that departing later cannot make a traveller arrive earlier—i.e., the FIFO principle—we require that $T_i'(t) > -1$ (e.g., Noland & Small 1995):

$$u_1(t_{d1}^*) - \nu = \left[1 + T_1'(t_{d1}^*)\right][[1-\xi]u_2([1-\xi]t_{s2}^*) + \xi u_2(t_{d2} - \xi t_{s2}^*) - \nu], \tag{22}$$

$$u_2(t_{d2}^* - \xi t_{s2}) - \nu = \left[1 + T_2'(t_{d2}^*)\right][u_3(t_{s3}^*) - \nu]. \tag{23}$$

Backward and forward optimal marginal utility functions $\tilde{u}_2(t_{s2})$ and $\hat{u}_2(t_{d2})$:

First, $\tilde{U}_2(t_{s2})$ is defined as

$$\tilde{U}_2(t_{s2}) = \int_{t_{s2}}^{t^*_{d2}(t_{s2})} u_2(t-\xi t_{s2})dt + \int_{t^*_{s3}(t_{s2})}^{1} u_3(t)dt + vT_2, \tag{24}$$

and we obtain $\tilde{u}_2(t_{s2}) = -d\tilde{U}_2/dt_{s2}$ as

$$\begin{aligned}\tilde{u}_2(t_{s2}) &= [1-\xi]u_2([1-\xi]t_{s2}) + \\ &+ \xi\Big[u_{2'}(t^*_{d2}(t_{s2})-\xi t_{s2})\cdot u_3(t^*_{s3}(t_{s2})) - u_{3'}(t^*_{s3}(t_{s2}))\cdot u_2(t^*_{d2}(t_{s2})-\xi t_{s2})\Big]/ \\ &\quad [u_{2'}(t^*_{d2}(t_{s2})-\xi t_{s2}) - u_{3'}(t^*_{s3}(t_{s2}))].\end{aligned} \tag{25}$$

Second, $\hat{U}_2(t_{d2})$ is defined as

$$\hat{U}_2(t_{d2}) = \int_0^{t^*_{d1}(t_{d2})} u_1(t)dt + \int_{t^*_{s2}(t_{d2})}^{t_{d2}} u_2(t-\xi t^*_{s2}(t_{d2}))dt + vT_1, \tag{26}$$

and we obtain $\hat{u}_2(t_{d2}) = d\hat{U}_2/dt_{d2}$ as

$$\begin{aligned}\hat{u}_2(t_{d2}) &= [\xi u_{2'}(t_{d2}-\xi t^*_{s2}(t_{d2}))\cdot[u_1(t^*_{d1}(t_{d2})) - [1-\xi]u_2([1-\xi]t^*_{s2}(t_{d2}))] + \\ &+ \Big[u_{1'}(t^*_{d1}(t_{d2})) - [1-\xi]^2 u_{2'}([1-\xi]t^*_{s2}(t_{d2}))\Big]\cdot u_2(t_{d2}-\xi t^*_{s2}(t_{d2}))]/ \\ &[u_{1'}(t^*_{d1}(t_{d2})) - [1-\xi]^2 u_{2'}([1-\xi]t^*_{s2}(t_{d2})) + \xi^2 u_{2'}(t_{d2}-\xi t^*_{s2}(t_{d2}))].\end{aligned} \tag{27}$$

Value of travel time for a general ξ:

For brevity, let $u_1 = u_1(t_{d1})$, $u_2 = u_2(t_{d2}-\xi t_{s2})$, $v_2 = u_2([1-\xi]t_{s2})$, $\tilde{u}_2 = \tilde{u}_2(t_{s2})$, $\hat{u}_2 = \hat{u}_2(t_{d2})$ and $u_3 = u_3(t_{s3})$. The marginal change in utility due to a change in the travel time of trip 1 can be written as

$$\frac{dU^*}{dT_1} = \frac{dt^*_{d1}}{dT_1}u_1 - \frac{dt^*_{s2}}{dT_1}\tilde{u}_2 + v, \tag{28}$$

where

$$\frac{dt^*_{d1}}{dT_1} = \frac{[1-\xi]^2 v_{2'}[u_{2'}-u_{3'}] + \xi^2 u_{2'}u_{3'}}{(u_{1'}-[1-\xi]^2 v_{2'})[u_{2'}-u_{3'}] - \xi^2 u_{2'}u_{3'}} \in (-1,0), \tag{29}$$

$$\frac{dt^*_{s2}}{dT_1} = \frac{u_{1'}[u_{2'}-u_{3'}]}{(u_{1'}-[1-\xi]^2 v_{2'})[u_{2'}-u_{3'}] - \xi^2 u_{2'}u_{3'}} \in (0,1). \tag{30}$$

Similarly for trip 2,

$$\frac{dU^*}{dT_2} = \frac{dt^*_{d2}}{dT_2}\hat{u}_2 - \frac{dt^*_{s3}}{dT_2}u_3 + \nu, \tag{31}$$

where

$$\frac{dt^*_{d2}}{dT_2} = \frac{(u_{1'} - [1-\xi]^2 v_{2'} + \xi^2 u_{2'})u_{3'}}{(u_{1'} - [1-\xi]^2 v_{2'})[u_{2'} - u_{3'}] - \xi^2 u_{2'} u_{3'}} \in (-1,0), \tag{32}$$

$$\frac{dt^*_{s3}}{dT_2} = \frac{(u_{1'} - [1-\xi]^2 v_{2'})u_{2'}}{(u_{1'} - [1-\xi]^2 v_{2'})[u_{2'} - u_{3'}] - \xi^2 u_{2'} u_{3'}} \in (0,1). \tag{33}$$

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