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Fundamental Aspects of Cooperative Interference Management

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Abstract

Today and future wireless networks are facing one of their greatest limiting factors: interference. This is due to the unprecedented increase in the number of connected devices. Therefore, in order to meet the ever increasing demand for data rate and quality of services, more advanced techniques than what we have today are required to deal with interference. This thesis takes a step towards interference management in multiuser wireless systems by means of relaying and cooperation. We study *four* fundamental building blocks in network information theory, propose new coding schemes, and derive limits on the capacity regions.

The *first* problem we consider is the one-sided interference channel with bidirectional and rate-limited receiver cooperation. We propose a coding scheme that tailors two versions of superposition coding with classical relaying protocols. The proposed scheme unifies and recovers previous results for the unidirectional cooperation, yet in simpler forms. Analytical and numerical results confirm the benefits of cooperation and illuminate the ideas behind the coding strategy.

The *second* problem generalizes the first one by allowing the existence of both crossover links in the channel. We propose a coding scheme for this channel by extending noisy network coding to encompass rate-splitting at the encoders. The achievable rate region is shown to be the same as a region achieved by explicit binning. As a corollary, we prove that noisy network coding achieves the capacity region of the Gaussian channel within 1 bit, under strong interference. Our result is among the first to show constant-gap optimality of noisy network coding for a multiple-unicast problem, and to demonstrate equivalence in terms of achievable rates of two different coding approaches for a noisy interference network.

We follow up by introducing a dedicated relay into the interference channel which simultaneously helps both receivers. For this *third* problem, the interference channel with a relay, we propose new coding schemes based on layered codes for long- and short-message quantize-forward techniques. The short-message schemes show improvements in the achievable rates compared to other known coding techniques, especially when the channel is asymmetric, while relaxing the excessive delay issue of the long-message scheme. The analysis also reveals the trade-off between achievable rates, encoding and decoding delays, and complexity.

In the *fourth* problem, we propose a new model for cooperative communication, the interfering relay channels, which consists of two neighboring relay channels inducing interference to each other. Each relay, by utilizing a finite-capacity and noise-free link to its own receiver, helps the receiver decode the desired message. We characterize the exact and approximate capacity region and sum-capacity for various classes of channels. The established results generalize and unify several known results for the relay and interference channels.

The methods and results shown in this thesis aim at providing insight into potential techniques for cooperative interference management in real-world systems.

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Stockholm, October 2013

Contents

List of Figures

Chapter 1

Introduction

Wireless communication is characterized by two main features: the broadcast and the superposition of signals. The former means a signal sent by a node in a network can be heard by its neighboring nodes, who share the same resources such as time and frequency. The latter means signals from different sources are linearly added at a particular destination. These two characteristics create the main challenge in wireless communications: mitigating the additive interference. One obvious strategy to mitigate interference is letting a transmitter increase the transmit power so that the intended receiver can distinguish its desired signal from the undesired signals or interference. This approach, however, has two main drawbacks. First, each transmitter has a power constraint. One cannot increase the transmit power as much as one wants to, not to mention the difficulty in designing the transmitter when the power range is too large. Second, in a multi-node network, increasing the transmit power at a node will increase the interference at other nodes. To draw an analogy, imagine a party where multiple dialogs take place at the same time, if a person raises his voice then the people around him are disturbed and they may react by raising their voices as well. In the worst case this may lead to a situation where everybody shouts but nobody understands anything. That said, the strategy of raising the transmit power is not advisable in a multiuser communication network. Another strategy is to allow the nodes to alternately utilize the whole resource, resulting in what is often called *orthogonal* schemes due to the fact that the communication of different members do not interfere with each other. For example, each transmitter can transmit in one time slot while the other transmitters remain silent, or each transmitter can use a different frequency band. It is easy to see that this strategy works but in a very inefficient way because the whole resource is occupied by only one member while it can possibly be shared among multiple members. Fortunately, the interference is fundamentally different from the thermal noise present in the receivers. Being the signals sent by the transmitters, the interference often has certain structure that can be exploited to combat the interference itself. One of the most natural and efficient way to combat interference is allowing the nodes in a network to help each other to either cancel, reduce, or forward the interference. In so doing each receiver either receives less interference or has more information about the interference to better mitigate it. This general idea of *cooperative interference management*, a special kind of so-called *cooperative communication*, has recently gained great interest from the information theory, communication theory, and signal processing research communities.

The main driving force for cooperative communication comes from the mobile and wireless industry. The second and third generation (2G and 3G) cellular systems heavily rely on a macro-centric network planning process and orthogonal schemes to avoid interference. The base stations have similar configurations and power levels, and are located according to a planned layout to maximize the network coverage and minimize the interference to each other. Each base station serves roughly the same number of mobile terminals and with similar characteristics of data flows. This homogeneous network design, however, is no longer efficient enough to meet the demand of the networks of today and the future. In the world of the so-called *internet of things*, not only more bandwidth but also more advanced techniques are required to catch up with the unprecedented increase in data traffic, the mobility of nodes, and higher quality of services. In this way cooperative communication has started making its way to current and future mobile and wireless systems. For example, in the evolution to the fourth generation (4G) wireless cellular networks, the new concept of *Coordinated Multi-Point transmission and reception* (CoMP) has been widely accepted by both industry and academia [MF11a, LKL+12]. CoMP is the general terminology for techniques that allow multiple points, e.g., base stations and mobile terminals, to jointly transmit/receive and process signals. As a result, CoMP helps realize potential multiantenna gains and allows a better control of intra-cell and inter-cell interference. A proven effect of CoMP is the improved throughput of cell-edge users as well as of the whole system. Similarly, *Heterogeneous Networks* (HetNet) is being considered the key enabler for a flexible and cost-effective method of providing uniform high speed connections to all users. Instead of having only large macro cells as in traditional homogeneous networks, heterogeneous networks also consist of smaller pico and femto cells and the use of low-power relay nodes, as depicted in Fig. 1.1. This brings about numerous benefits: more flexible network deployment, better network coverage, increased data rates, improved cell-edge users experience. In particular, having small cells allows for the *range expansion* in LTE-Advanced systems [Qua11, OS12]. Beside great advantages, mixed networks like HetNet pose very challenging requirements for coordinating different nodes and controlling interference. Efforts have been made by the industry and led to the discussion and standardization of intercell interference coordination (ICIC) and enhanced ICIC techniques by 3GPP [LPGDlR+11]. However these techniques are mainly based on scheduling and power control schemes. Different scenarios for CoMP have also been proposed to include HetNet [LKL+12]. Note that in HetNet and CoMP the role of *backhaul* links is particularly important. These are the links that connect the macro/pico/femto base stations with each other and with the core network. The backhauls can be high-speed optical links, consumer's broadband con-

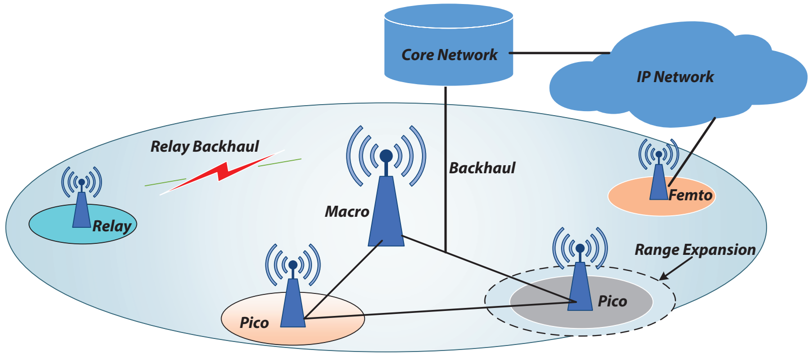

Figure 1.1: Heterogeneous network in LTE-Advanced, adapted from Fig. 2 in [Qua11].

nections, or wireless links like the relay backhaul in Fig. 1.1. The wireless backhaul is also expected to be crucial in mmWave small cells in the 5G networks. Another example is an European Union's project aiming at 5G wireless cellular standards, the *Mobile and Wireless Communications Enablers for the 2020 Information Society* (METIS) [MET]. This project is presently looking at practical implementation of cooperative communications, collaborating base stations, interference coordination, massive MIMO, direct device-to-device[1] (D2D) communication. All of these are related to interference management.

Despite the aforementioned endeavours, a thorough understanding of cooperative interference management is still lacking, especially in the physical layer. This thesis follows a fundamental approach to improve our understanding of the most important problems. We are not looking at practical schemes, instead we employ simple and abstract models of "small pieces" of a general wireless network. In so doing we strive to develop new coding theorems, characterize the information theoretic bounds of cooperation in noisy interference networks. The obtained insight will in turn provide guidelines for the design of coding schemes and protocols for real-world systems.

Fig. 1.2 provides a pictorial summary of the network topologies that are investigated in this thesis.

1.1 Organization and Contributions of the Thesis

We will start in Chapter 2 with some fundamental concepts that are useful for reading the remaining chapters. The reader who is familiar with network information

[1] This is being introduced into LTE Direct of Qualcomm and has been proposed to be part of LTE Standard (3GPPR12).

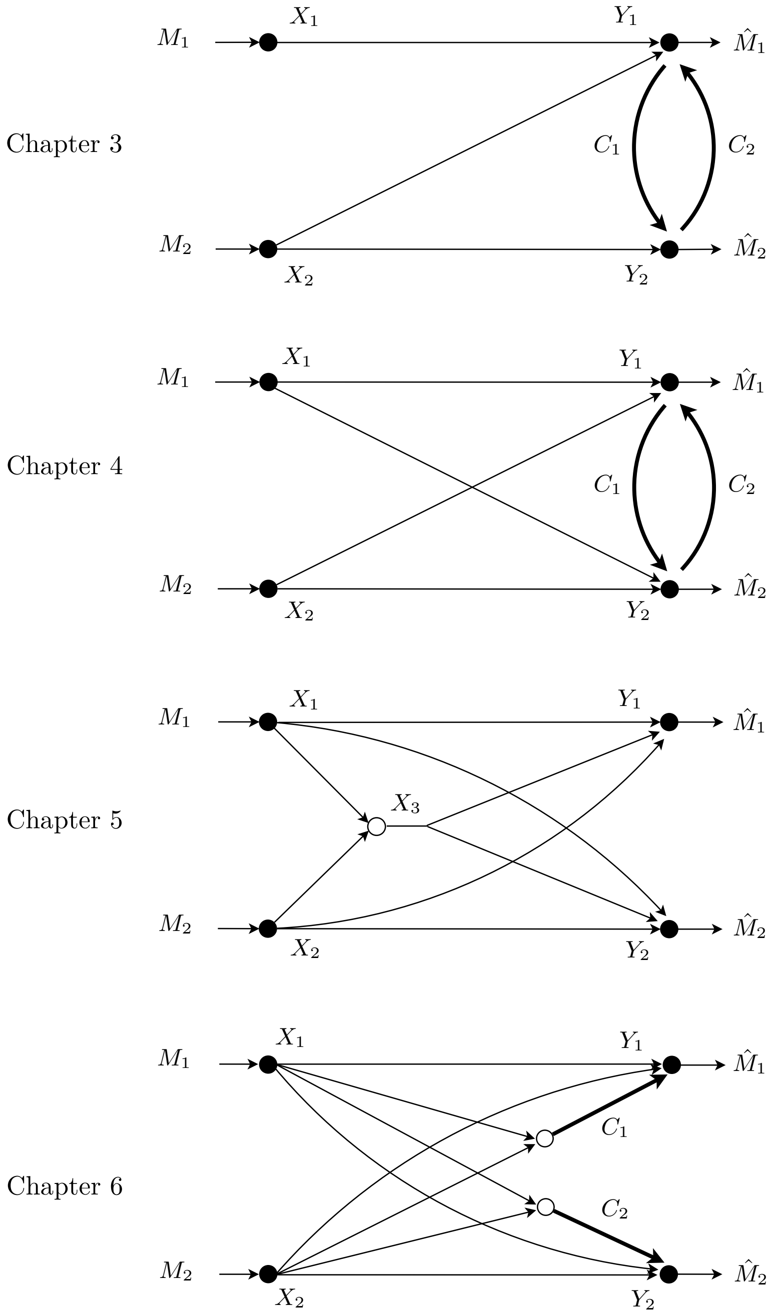

Figure 1.2: Channel models studied in this thesis. Black circles denote senders and receivers, hollow circles denote relays. Bold links with capacity C_1, C_2 are noise-free orthogonal links. M_1 and M_2 are the sent messages, $\hat{M}_1$ and $\hat{M}_2$ are the respective decoded messages.

theory can skip this chapter. Chapter 3 studies an asymmetric form of the interference channel, whose receivers can mutually help each other through dedicated cooperation links. In Chapter 4 we consider the full interference channel with cooperating receivers. Chapter 5 studies the interference channel with an extra relay. Chapter 6 studies the interfering relay channels or the interference channel with two relays, which is closely related to the problems in Chapter 4 and Chapter 5. The content and contributions of each chapter are summarized as follows.

1.1.1 Chapter 2

In chapter 2 we introduce some basic concepts of network information theory as well as fundamental network models, focusing on the interference channel and the relay channel. These models and their coding concepts are the building components of the more complex setups that we study in the subsequent chapters.

1.1.2 Chapter 3

In this chapter we propose a new coding protocol for the one-sided or Z-interference channel with orthogonal cooperation links at the receivers' side. The protocol combines rate-splitting and superposition encoding techniques with the conventional decode-forward and compress-forward strategies. It is shown that such a careful design of codebooks and coding scheme, which is obtained from the intuition of superposition coding, can considerably reduce the complexity of the analysis. Analytical and numerical results show that the proposed scheme, although not universally optimal, can achieve the capacity region or sum-capacity exactly or asymptotically in certain scenarios. Various limits of sum-capacity gain due to cooperation are also explored.

The material in this chapter has been published in part in

- [DOS13b] H. T. Do, T. J. Oechtering, and M. Skoglund, "On Asymmetric Interference Channels with Cooperating Receivers," *IEEE Transactions on Communications,* vol. 61, no. 2, pp. 554-563, Feb. 2013.

Shorter versions also appeared in

- [DOS10b] H. T. Do, T. J. Oechtering, and M. Skoglund, "The Gaussian Z-interference channel with rate-constrained conferencing decoders," in *Proc. of IEEE International Conference on Communications (ICC).* Cape Town, South Africa, May 2010.

- [DOS09] H. T. Do, T. J. Oechtering, and M. Skoglund, "An achievable rate region for the Gaussian Z-interference channel with conferencing," in *Proc. of the 47th Annual Allerton Conference on Communication, Control, and Computing.* Monticello, IL, USA, Sep.-Oct. 2009.

1.1.3 Chapter 4

This chapter studies a more complete version of the model in Chapter 3, in the sense that both receivers are affected by the interference from the undesired transmitter. We propose a new coding protocol for the discrete memoryless channel. The coding scheme is built upon common-private rate-splitting and superposition coding at the transmitters, noisy network coding, and non-unique joint decoding at the receivers. As a case study the general achievable region leads to an inner bound for the Gaussian interference channel whose receivers cooperate through rate-limited orthogonal channels. We show that noisy network coding achieves within 1 bit to the capacity region when the interference is strong, and achieves within 1 bit to the capacity region of the Gaussian compound multiple access channel with conferencing decoders, regardless of channel parameters. To the best of our knowledge the study in this chapter, whose results appeared in [DOS11b], is the first to explore the benefit of noisy network coding for an interference network. Moreover, our result is the first to show that NNC can achieve a constant gap for a multiple unicast channel. We are also the first to show the equivalence between noisy network coding and quantize-bin-forward in terms of achievable rates for a interference network with orthogonal components. Finally, some results in this chapter are instrumental to the analysis in later chapters.

The material in this chapter has been published in part in

- [DOS11b] H. T. Do, T. J. Oechtering, and M. Skoglund, "Noisy Network Coding Approach to the Interference Channel with Receiver Cooperation," in *Proc. of the 49th Annual Allerton Conference on Communication, Control, and Computing.* Monticello, IL, USA, Sep.-Oct. 2011.

1.1.4 Chapter 5

We continue the evolution in network topology by considering an interference channel with a dedicated relay node that can help both receivers at the same time, i.e., the interference channel with a relay. We develop three new coding schemes for this channel model. Our schemes are based on long- and short-message quantize-forward and having layered codes. We show that having layers is a key factor to increase the achievable rates, especially when the channel is asymmetric. Analyzing and comparing the proposed coding schemes reveals different trade-offs between encoding/decoding delays, achievable rates, and complexity. The results in this chapter provide more comprehensive understanding of different coding strategies for interference channel with a relay.

The material in this chapter is based on

- [DOS13c] H. T. Do, T. J. Oechtering, and M. Skoglund, "Layered Coding for the Interference Channel With a Relay," submitted to *IEEE Transactions on Information Theory,* 2013.

Main ideas and shorter versions also appeared in

- [DOS13a] H. T. Do, T. J. Oechtering, and M. Skoglund, "Layered Coding Strategies for the Interference Channel With a Relay," invited talk at *Information Theory and Applications Workshop (ITA).* San Diego, CA, USA, Feb. 2013.

- [DOS12b] H. T. Do, T. J. Oechtering, and M. Skoglund, "A new inner bound for the interference relay channel," in *Proc. of the 46th Annual Conference on Information Sciences and Systems (CISS).* Princeton, NJ, USA, Mar. 2012. (Invited paper).

- [DOS12a] H. T. Do, T. J. Oechtering, and M. Skoglund, "Layered Quantize-Forward for the Two-Way Relay Channel," in *Proc. IEEE International Symposium on Information Theory (ISIT).* Boston, MA, USA, Jul. 2012.

1.1.5 Chapter 6

This chapter explores a model that is highly suitable for heterogeneous wireless access networks. In particular, we introduce and study a model in which two relay channels interfere with each other. Each channel has one relay which is designed to help its own receiver via a noise-free finite-capacity link. The receivers, however, are still interfered by signals from the neighboring transmitters. We name this new model the *interfering relay channels* (IRC). We derive inner and outer bounds for achievable rates and show that they coincide for new discrete memoryless classes of channels. The results generalize and unify several known cases involving interference and relay channels. Next, we study various Gaussian counterparts and characterize the capacity region and sum-capacity for those channels to within a constant gap. Our study extends the optimality of extended hash-forward or quantize-bin-forward to new classes of channels with interference. This has an important engineering implication because such a simple coding technique, where no decoding is required at intermediate nodes, is generally preferable in practical systems.

The material in this chapter is based on

- [DOSV13c] H. T. Do, T. J. Oechtering, M. Skoglund, and M. Vu, "Interfering Relay Channels," submitted to *IEEE Transactions on Information Theory,* 2013.

Shorter versions also appeared or will appear in

- [DOSV13a] H. T. Do, T. J. Oechtering, M. Skoglund, and M. Vu, "Capacity Region of a Class of Interfering Relay Channels," in *Proc. IEEE Information Theory Workshop.* Seville, Spain, Sep. 2013.

- [DOSV13b] H. T. Do, T. J. Oechtering, M. Skoglund, and M. Vu, "Gaussian Interfering Relay Channels," in *Proc. 47th Asilomar Conference on Signals, Systems and Computers.* Pacific Grove, CA, USA, Nov. 2013.

Contributions Outside the Scope of the Thesis

Besides the contributions listed above, the author of this thesis has also contributed to some other topics, which are published in the papers listed below. For the consistency of the thesis structure, these contributions are not covered in the thesis.

- [DOS11a] H. T. Do, T. J. Oechtering, and M. Skoglund, "Bounds on the Capacity Region of the Z Channel," in *Proc. of IEEE Information Theory Workshop (ITW).* Paraty, Brazil, Oct. 2011.
- [DOS10a] H. T. Do, T. J. Oechtering, and M. Skoglund, "Coding for the Z Channel With a Digital Relay Link," in *Proc. of IEEE Information Theory Workshop (ITW).* Dublin, Ireland, Sep. 2010.
- [ODS10a] T. J. Oechtering, H. T. Do, and M. Skoglund, "Achievable Rates for Embedded Bidirectional Relaying in a Cellular Downlink," in *Proc. of IEEE International Conference on Communications (ICC).* Cape Town, South Afri--ca, May 2010.
- [ODS10b] T. J. Oechtering, H. T. Do, and M. Skoglund, "Capacity-Achieving Coding for Cellular Downlink with Bidirectional Communication," in *Proc. of Int. ITG Conference on Source and Channel Coding.* Siegen, Germany, Jan. 2010.

1.2 Copyright Notice

As detailed in Section 1.1, parts of the material presented in this thesis are based on the works co-authored by the thesis's writer, which have been previously published in or submitted to journals and conferences held by or sponsored by the Institute of Electrical and Electronics Engineers (IEEE). IEEE holds the copyright of the published papers and will hold the copyright of the submitted papers if they are accepted. Material is reused in this thesis with permission.

1.3 Notation and Abbreviations

Throughout this thesis we follow the notation of the book by El Gamal and Kim [EK11]. In particular, we use $\mathcal{T}_\epsilon^{(n)}$ to denote the set of $\epsilon-$typical $n-$sequences as defined in [EK11, Chap. 2]. The set of random variables based on which the typical sequences are defined is omitted if it can be straightforwardly deduced from the context. We use $[1:2^r]$ to denote the set $\{1,2,\ldots,2^{\lceil r\rceil}\}$, where $\lceil r\rceil$ is the smallest integer $\geq r$. We define $\mathcal{C}(x)=\frac{1}{2}\log(1+x)$ except in Chapter 4 where $\mathcal{C}(x)=\log(1+x)$. $(x)^+$ denotes $\max(0,x)$ for a real number x. All logarithms in this thesis are to the base 2, unless explicitly specified otherwise. $a:=b$ means a equals b by definition, $X\leftarrow Y$ means "substitute X by Y" or "Y replaces X". In all coding theorems we reserve the random variable Q for the time-sharing variable,

with a finite support set $\mathcal{Q}$ having cardinality $|\mathcal{Q}|$. The cardinality of $\mathcal{Q}$ can generally (but not always) be upper bounded by applying the support lemma [CK81, Lemma 3.4, pp. 310]. Random variables are written in capital letters while their realizations are written in lower-case letters. Superscript ^{n} indicates a vector of length n. As such, X^n denotes random vector $(X_1, X_2, \ldots, X_n)$ and x^n denotes a realization of X^n. For brevity, we also use boldface letters for vectors, e.g, $\mathbf{x}$ is an alternative for x^n. Tx and Rx are used as shorthand notation for transmitter and receiver, respectively. $\mathbf{1}$ denotes a vector whose elements are all 1's.
The next two tables summarize the notation and abbreviations used in the thesis.

Notation

log	Logarithm to the base 2
$\mathsf{P}(E)$	Probability of an event E
$H(X)$	Entropy of discrete random variable X
$H(X,Y)$	Joint entropy of discrete random variables X and Y
$H(Y\|X)$	Conditional entropy of discrete random variable Y given X
$h(X)$	Differential entropy of continuous random variable X
$h(X,Y)$	Joint differential entropy of continuous random variables X and Y
$h(Y\|X)$	Conditional differential entropy of continuous random variable Y given X
$I(X;Y)$	Mutual information between random variables X and Y
I_k	Identity matrix of dimension k
H^T	Transpose of matrix H
$\mathsf{H}^\dagger$	Conjugate transpose of matrix H
$\mathcal{N}(\mu,\sigma^2)$	Gaussian distribution with mean μ and variance σ^2
$\mathcal{CN}(\mu,\sigma^2)$	Circularly symmetric complex Gaussian distribution with mean μ and variance σ^2
$p_X(x)$	Probability mass function of discrete random variable X
$f_X(x)$	Probability density function of continuous random variable X
P_e	Probability of error
$\mathrm{Conv}(\mathcal{S})$	Convex hull of set $\mathcal{S}$
$\|\mathcal{S}\|$	Cardinality of set $\mathcal{S}$
$(x)^+$	$\max(x,0)$
$[i:j]$	Set of integers $\{i, i+1, \ldots, j\}$, $i < j$.

Abbreviations

AWGN	Additive white Gaussian noise
BC	Broadcast channel
CF	Compress-forward
DF	Decode-forward
DMC	Discrete memoryless channel
DMN	Discrete memoryless network
DoF	Degrees of freedom
GCF	Generalized compress-forward
GDoF	Generalized degrees of freedom
GIC-ORC	Gaussian interference channel with orthogonal receiver cooperation
GHF	Generalized hash-forward
IC	Interference channel
ICIC	Intercell interference coordination
IC-IIR	IC with an in-band reception/in-band transmission relay
IC-IOR	IC with an in-band reception/out-of-band transmission relay
ICR	Interference channel with a relay
i.i.d.	Independent and identically distributed
INR	Interference-to-noise ratio
IRC	Interfering relay channels
LNNC	Layered noisy network coding
LTE	Long Term Evolution
LQF	Layered quantize-forward
LQF-FW	Layered quantize-forward with forward sliding window decoding
LQF-BW	Layered quantize-forward with backward decoding
MAC	Multiple-access channel
MIMO	Multiple-input multiple-output
NNC	Noisy network coding
pdf	Probability density function
pmf	Probability mass function
RC	Relay channel
SC	Simultaneous coding
SNR	Signal-to-noise ratio
SPC	Superposition coding
ZIC	Z-interference channel

Chapter 2

Background

This chapter introduces some fundamental concepts and network topologies that serve as a background to study the problems in the subsequent chapters. We will start out with definitions of basic information measures. We then describe the general discrete memoryless network (DMN), move on with an introduction to the interference channel (IC) and the relay channel (RC), and conclude with some extensions of the interference and relay channels.

2.1 Definitions

The two most important concepts in information theory are entropy and mutual information. In the following we define these quantities.

Definition 2.1 (Entropy)**.** Let X be a discrete random variable with finite alphabet $\mathcal{X}$ and a probability mass function (pmf) $p_X(x)$. The *entropy* of X, denoted by $H(X)$, is defined as

$$H(X) = -\sum_{x\in\mathcal{X}} p_X(x) \log p_X(x). \tag{2.1}$$

The unit of $H(X)$ depends on the base of the logarithm. $H(X)$ is measured in *bits* if the logarithm is to the base 2, and is measured in *nats* if the logarithm is to the base e.

Definition 2.2 (Conditional entropy)**.** Let X and Y be two discrete random variables with finite alphabets $\mathcal{X}$ and $\mathcal{Y}$, and a joint pmf $p_{XY}(x, y)$. The conditional entropy of Y given X, denoted by $H(Y|X)$, is defined as

$$H(Y|X) = -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p_{XY}(x, y) \log_{Y|X} p(y|x). \tag{2.2}$$

Definition 2.3 (Mutual information, discrete case)**.** Let X and Y be two discrete random variables with finite alphabets $\mathcal{X}$, $\mathcal{Y}$ and joint pmf $p_{XY}(x, y)$. The mutual information between X and Y, denoted by $I(X;Y)$, is defined as

$$I(X;Y) = \sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p_{XY}(x,y)\log\frac{p_{XY}(x,y)}{p_X(x)p_Y(y)}, \tag{2.3}$$

where $p_X(x)$ and $p_Y(y)$ are the marginal pmf of X and Y, respectively.

The following definitions apply to continuous random variables.

Definition 2.4 (Differential entropy)**.** Let X be a continuous random variable with the probability density function (pdf) $f_X(x)$. The *differential entropy* of X, denoted by $h(X)$, is defined as

$$h(X) = -\int_{\mathcal{S}_X} f_X(x)\log f_X(x)dx, \tag{2.4}$$

where $\mathcal{S}_X$ is the support of X.

Definition 2.5 (Conditional differential entropy)**.** Let X and Y be continuous random variables with joint pdf $f_{XY}(x,y)$. The conditional differential entropy of X given Y, denoted by $h(X|Y)$, is defined as

$$h(X|Y) = -\int_{\mathcal{S}_{XY}} f_{XY}(x,y)\log f_{X|Y}(x|y)dxdy, \tag{2.5}$$

where S_{XY} is the support of (X,Y).

Definition 2.6 (Mutual information, continuous case)**.** Let X and Y be continuous random variables with joint pdf $f_{XY}(x,y)$. The mutual information between X and Y, denoted by $I(X;Y)$, is defined as

$$I(X;Y) = \int_{\mathcal{S}_{XY}} f_{XY}(x,y)\log\frac{f_{XY}(x,y)}{f_X(x)f_Y(y)}dxdy, \tag{2.6}$$

where $\mathcal{S}_{XY}$ is the support of (X,Y), and $f_X(x)$ and $f_Y(y)$ are the marginal pdf's of X and Y, respectively[1].

From the definitions it is clear that

$$I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) \tag{2.7}$$

for discrete random variables, and

$$I(X;Y) = h(X) - h(X|Y) = h(Y) - h(Y|X) \tag{2.8}$$

for continuous random variables.

[1]For general random variables, the mutual information is defined as

$$I(X;Y) = \int_{\mathcal{S}_{XY}} \left(\log\frac{dP_{XY}}{d(P_X\times P_Y)}\right)dP_{XY},$$

where P_{XY}, P_X, and P_Y are the probability measures of (X,Y), X, and Y, respectively. $\frac{dP_{XY}}{d(P_X\times P_Y)}$ denotes the Radon-Nikodym derivative of the joint probability measure P_{XY} with respect to the product measure $P_X\times P_Y$ [Gra09, Yeu08]. Furthermore, we define $H(X) = I(X;X)$.

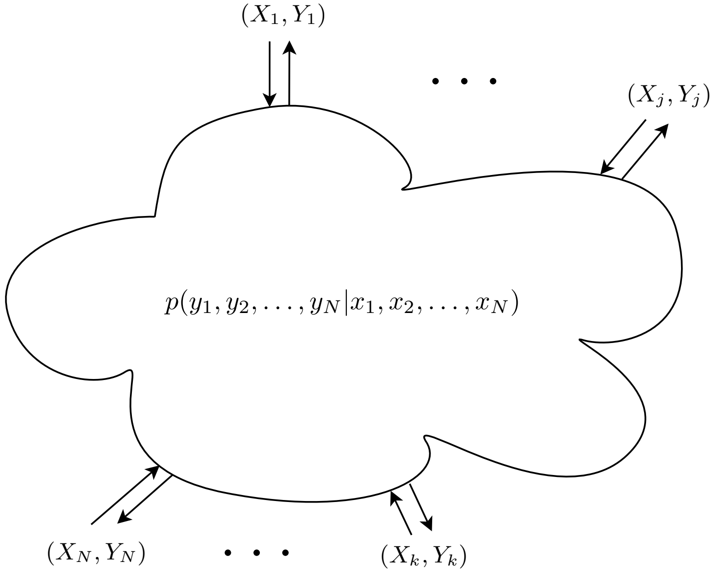

Figure 2.1: Discrete memoryless network.

2.2 Discrete Memoryless Networks

The classical approach of information theory to communications is to characterize real-world scenarios by the use of mathematical models which allow us to gain insight, to establish different limits of communication, and to develop coding schemes that can be transferred back to the original problems. From this point of view, any multiple-node communication problem can be modeled by a graph as depicted in Fig. 2.1. In this N-node network, each node wants to send a *message* to one or many other nodes. Assume node $k \in \{1, 2, \ldots, N\}$ wants to send a message M_k, drawn from a message set $\mathcal{M}_k$, to a set of nodes $\mathcal{D}_k \subseteq \{1, 2, \ldots, N\}$. A random input symbol at node k is denoted by X_k, drawn from a finite input alphabet $\mathcal{X}_k$, and a random output symbol Y_k is distributed over a finite alphabet $\mathcal{Y}_k$. The input-output relation is governed by the conditional probability distribution

$$p_{Y_1, Y_2, \ldots, Y_N | X_1, X_2, \ldots, X_N}(y_1, y_2, \ldots, y_N | x_1, x_2, \ldots, x_N). \tag{2.9}$$

The discrete networks considered in this thesis are *stationary* (or *time-invariant*) and *memoryless*, in the sense that

$$\begin{aligned} &p(y_{1i}, \ldots, y_{Ni} | m_1, \ldots, m_N, x_1^i, \ldots, x_N^i, y_1^{i-1}, \ldots, y_N^{i-1}) \\ &\quad = p_{Y_1, \ldots, Y_N | X_1, \ldots, X_N}(y_{1i}, \ldots, y_{Ni} | x_{1i}, \ldots, x_{Ni}), \end{aligned} \tag{2.10}$$

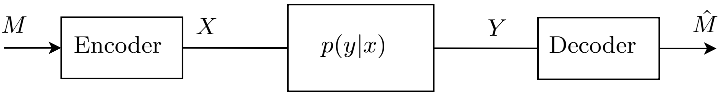

Figure 2.2: Point-to-point discrete memoryless channel.

for any positive integer i, often used to denote the time index. m_k, $k \in \{1, 2, \ldots, N\}$, is a particular message selected from the set $\mathcal{M}_k$. The stationary and memoryless assumptions allow us to focus on the channel conditional probability distribution in (2.9), often shortened as $p(y_1, \ldots, y_N | x_1, \ldots, x_N)$, for our analysis.

A $(2^{nR_1}, 2^{nR_2}, \ldots, 2^{nR_N}, n)$ code for the DMC consists of

- N message sets: $\mathcal{M}_k = \{1, \ldots, 2^{nR_k}\}$, $k \in \{1, 2, \ldots, N\}$,
- a set of encoders where encoder $k \in \{1, 2, \ldots, N\}$ produces an input symbol $x_{ki} \in \mathcal{X}_k$ from a message $m_k \in \mathcal{M}_k$ and a past received sequence $y_k^{i-1} \in \mathcal{Y}_k^{i-1}$ for $i \in \{1, 2, \ldots, n\}$,
- a set of decoders where decoder j, $j \in \mathcal{D}_k$, puts out an estimate $\hat{m}_{jk} \in \mathcal{M}_k$ from its own message $m_j \in \mathcal{M}_j$ and its received n-sequence $y_j^n \in \mathcal{Y}_j^n$.

Assuming that $(M_1, M_2, \ldots, M_N)$ is uniformly distributed over $\mathcal{M}_1 \times \mathcal{M}_2 \times \ldots \times \mathcal{M}_k$, the average error probability of the code $(2^{nR_1}, 2^{nR_2}, \ldots, 2^{nR_N}, n)$ is defined as:

$$P_e^{(n)} = \mathsf{P}\{\hat{M}_{jk} \neq M_k \text{ for some } k \in \{1, 2, \ldots, N\}, j \in \mathcal{D}_k\}. \tag{2.11}$$

A rate vector $(R_1, R_2, \ldots, R_N)$ is said to be *achievable* if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, \ldots, 2^{nR_N}, n)$ codes, indexed by n, such that the average error probability $P_e^{(n)}$ can be made arbitrarily small when n is sufficiently large. It is common to write $P_e^{(n)} \to 0$ as $n \to \infty$. In later chapters of this thesis, we will define the codes under consideration in connection to each problem we study, making the treatment more concrete and easier to follow.

An N-dimensional region $\mathcal{R}$ is called an *achievable rate region* for the DMN if every point in that region is achievable. The *capacity region* $\mathscr{C}$ of a DMN is the closure of the set of achievable rate vectors, and the *sum-capacity* C_{sum} of the DMN is defined as $C_{sum} = \max\{\sum_{k=1}^{N} R_k : (R_1, R_2, \ldots, R_N) \in \mathscr{C}\}$. Due to the time-sharing argument [CT06, Chap. 15], the capacity region of any DMN is a convex set. The fundamental problem of network information theory is to characterize the capacity region of discrete memoryless networks and their continuous counterparts. The common method of doing so is to demonstrate an *inner bound*, i.e., an achievable rate region, and an *outer bound* to the capacity region, and show that the two bounds meet.

A special case of the DMN is the point-to-point channel, depicted in Fig. 2.2. For this channel, achievable rate vectors and the capacity region reduce to scalar

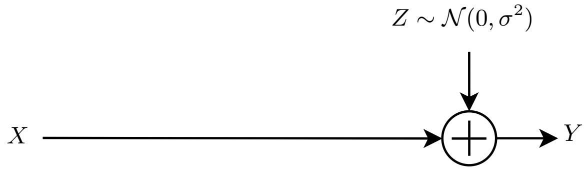

Figure 2.3: Gaussian point-to-point channel.

values, called *achievable rates* and the *capacity*, respectively. The capacity of a discrete memoryless point-to-point channel is the supremum of all achievable rates over that channel. It is well known that the capacity of a discrete memoryless point-to-point channel with channel input X, channel output Y, and having no feedback, is given by [CT06]:

$$C = \max_{p(x)} I(X;Y), \tag{2.12}$$

where $I(X;Y)$ is the mutual information between the random variables X and Y.

The general network topology in Fig. 2.1 is universal in the sense that it can describe any communication network. However, finding the capacity region of such a general network is a prohibitively difficult task. In the network information theory literature, there are a number of realizations of the topology in Fig. 2.1 that are considered fundamental problems. They are the *multiple access channel* (MAC), the *broadcast channel* (BC), the *interference channel* (IC), and the *relay channel* (RC). These four problems have been subject to research for the past some 40 years. Surprisingly, only the capacity region of the MAC has been fully characterized in single-letter form [Ahl71, Lia72]. The capacity regions of the last three problems are generally unknown, except for some special cases. In this thesis we will mainly investigate the extension of the interference channel and the relay channel. Therefore we will briefly introduce fundamental concepts of these channels in the next sections. More thorough treatments can be found in textbooks [CT06, EK11].

2.3 Gaussian Networks

The preceding section introduces the DMN and the coding problem for the DMN. There we assume that the channel input and output alphabets of the DMN are finite sets. In many real-world problems, however, the input and output of a channel do not take discrete values, but rather continuous ones. The most common model that is widely studied in information theory and communication theory is the Gaussian network. In a Gaussian network, the signals sent by a transmitter, after being scaled by the channel gain between the transmitter and the receiver, are corrupted by additive white Gaussian noise (AWGN). In the special case of only one transmitter and one receiver, the network reduces to the Gaussian point-to-point channel depicted Fig. 2.3, which is the counterpart of the discrete memoryless channel in

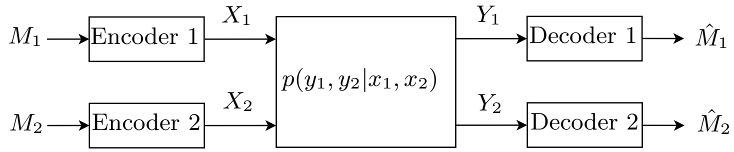

Figure 2.4: Discrete memoryless interference channel.

Fig. 2.2. The capacity of this channel is given by the famous Shannon's formula

$$C = \log(1 + \mathsf{SNR}), \tag{2.13}$$

where SNR is the *signal-to-noise ratio.*

Recall that for the discrete memoryless networks we assume the input and output alphabets are finite sets. All the random variables in coding theorems are therefore discrete random variables and we can define entropy for the variables as in Section 2.1. On the other hand, for continuous channels, the random variables often have densities, and there we use the notion of differential entropy. The entropy and differential entropy have different meanings and properties, especially the entropy is always nonnegative while the differential entropy can be negative. More importantly, a number of proofs in network information theory rely on the concept of *strong typicality* [Ber78, Yeu08] or *robust typicality* [OR01, EK11], which are only well defined for discrete random variables. That said, there are many results that hold for the DMN but not necessarily hold for the Gaussian networks. Luckily, many results regarding achievable rates in (network) information theory are expressed in a manner which involves mutual information, which is the difference between two entropy quantities or two differential entropy quantities, as shown in Section 2.1. This allows us to generalize coding theorems for a DMN to its Gaussian network counterpart. The most common method of doing so is the method of discretization, which is done by first discretizing the channel inputs and outputs of the Gaussian channel, then apply the coding theorem for the resulting discrete channel, and by taking proper limits one can show that the mutual information of the discretized channel will converge to that of the original continuous channel when the quantization gets finer and finer. The details can be found in [Gal68] [EK11, Sec. 3.4.1]. In this thesis we will also apply this technique to several problems, without detailed derivations.

2.4 Interference Channel

The interference channel is a channel consisting of two sender-receiver pairs and each receiver is interested in decoding only the messages sent by its pairing sender. However, due to the nature of the transmission medium, each receiver also receives

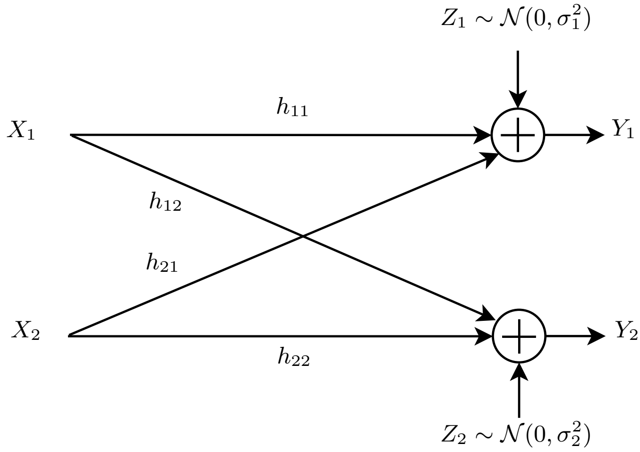

Figure 2.5: Gaussian interference channel. h_{ij}, $i, j \in \{1, 2\}$, denote channel gains; Z_1 and Z_2 denote additive Gaussian noises.

signals from the unpairing sender. The discrete memoryless interference channel is illustrated in Fig. 2.4 and the Gaussian channel in Fig. 2.5.

The interference channel was first studied by Ahlswede in [Ahl74], where the first inner and outer bounds are established. So far the most well-known achievable rate region for the interference channel is due to Han and Kobayashi [HK81]. To achieve this rate region each transmitter employs the so called *common-private rate-splitting* technique [Car78]: Each message is split into two parts, called *common* message and *private* message. The purpose of doing so is to allow each receiver to decode the undesired common message, thereby effectively reduces the interference on its desired messages. Specifically, each receiver in [HK81] jointly decodes its desired common and private messages and the undesired common message coming from the interfering transmitter. The resulting achievable region consists of a dozen of inequalities and bears the polymatroidal structure [HK81]. About 25 years later, Chong et al. [CMGE08] have observed that the common message need not be decoded correctly at the unintended receiver. They also propose a superposition coding scheme based on the scheme of [Ber73], which results in a simplified expression of the Han-Kobayashi rate region. In Chapter 3 we will discuss more on this issue. In 2008 Etkin et al. [ETW08] have shown that the Han-Kobayashi scheme with a smart power allocation strategy achieves the capacity region of the Gaussian interference channel within 1 bit, regardless of channel parameters. In Chapter 6 we will apply the intuition developed by Etkin et al. to the power allocation part of our coding schemes. The capacity region of the interference channel is unknown in general. Some special cases where the capacity regions are known include: Gaussian IC with very strong interference [Car75], Gaussian IC with strong

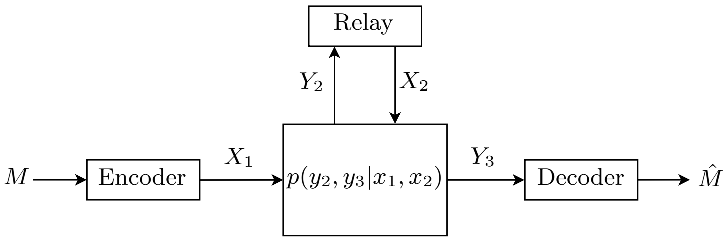

Figure 2.6: Discrete memoryless relay channel.

interference [Sat81, HK81], discrete memoryless IC with strong interference [CE87], to name a few. In all these cases, the Han-Kobayashi coding scheme is shown to be capacity achieving.

There are also works on interference channels with more than two users. For obvious reasons, finding the capacity region of the multiuser IC is very difficult. It has been shown that a direct generalization of the Han-Kobayashi scheme does not achieve the capacity region of the Gaussian interference channel [BPT10]. A more crude approach is to instead look at the degrees of freedom of the channels, that is, focus on characterizing the behavior of sum rates at high signal-to-noise ratio. This research topic has recently created a large body of works focusing on the so called *interference alignment* technique [MAMK08, CJ08]. We do not cover this technique in the current thesis.

It has been long known that obtaining a good outer bound for the capacity region of the interference channel is very difficult. The most well-known approach to obtain an outer bound for the IC is the *genie-aided* technique [EC82, Kra04]. In this technique we assume a genie helps the decoders by giving them certain *side information* such as a clean version or a noisy version of the signal sent by one of the transmitters, or the signal that the other receiver receives. Obviously this side information cannot be harmful because the receivers can always choose not to use it. Therefore, any outer bound on the capacity region of the genie-aided channel is also an outer bound on the capacity region of the original channel. However, to obtain a tight outer bound the genie should be smart to give useful but not too much side information. Moreover, the genie-aided channel should have some nice properties so that the outer bound is computable. The success of having useful and smart genies have been instrumental in obtaining the most important advances in the research of the Gaussian IC [ETW08, SKC09, MK09, AV09], namely the sum-capacity in the noisy interference regime. In this thesis we will make use of this bounding technique to derive outer bounds in Chapter 3 and Chapter 6.

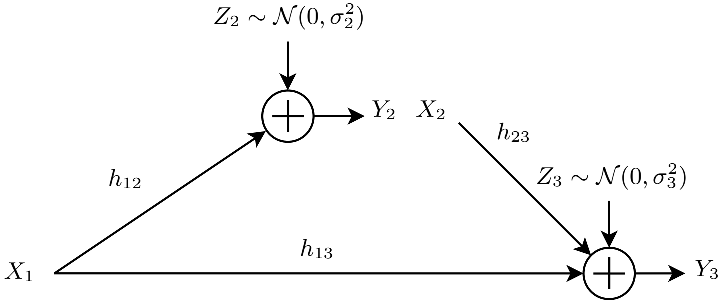

Figure 2.7: Gaussian relay channel. h_{12}, h_{13}, h_{23} denote channel gains; Z_2 and Z_3 denote additive Gaussian noises.

2.5 Relay Channel

The relay channel is a network consisting of 3 nodes: one sender, one receiver, and one helper which is called the relay. Fig. 2.6 depicts the discrete memoryless relay channel and Fig. 2.7 depicts the Gaussian channel. The relay channel was first introduced by van der Meulen in [Meu71]. In 1979, Cover and El Gamal published a seminal paper on coding theorems for the relay channel [CE79]. In the paper, the authors proposed two coding schemes which are now widely known as decode-forward (DF) and compress-forward (CF). Using these coding schemes and the cut-set upper bound, the authors established the capacity of several types of channels: the physically degraded relay channel, the reversely degraded relay channel, and the relay channel with feedback. In Chapter 3 we will propose a coding scheme based on a combination of these two classical relaying schemes. The capacity of the semideterministic discrete memoryless relay channel was found by El Gamal and Aref [EA82]. In the last ten years, the research on the relay channel have undergone tremendous developments, and the capacity of several types of relay channels such as the discrete memoryless and Gaussian relay channels with orthogonal sender components [EZ05], a class of semideterministic relay channels [CK07, Kim08], capacity of the primitive relay channel with oblivious relaying [SES10] have been characterized. However, the capacity of the general relay channel is still unknown.

In recent years, the research on relay channels has also extended to relay networks, in which there are multiple relays. Such great efforts have led to new coding schemes for the relay networks, most notably the quantize-map-forward (QMF) [ADT11] and noisy network coding (NNC) [LKEC11] schemes. Especially it has been shown that NNC achieves the capacity region of the Gaussian multicast relay networks within a constant gap [LKEC11].

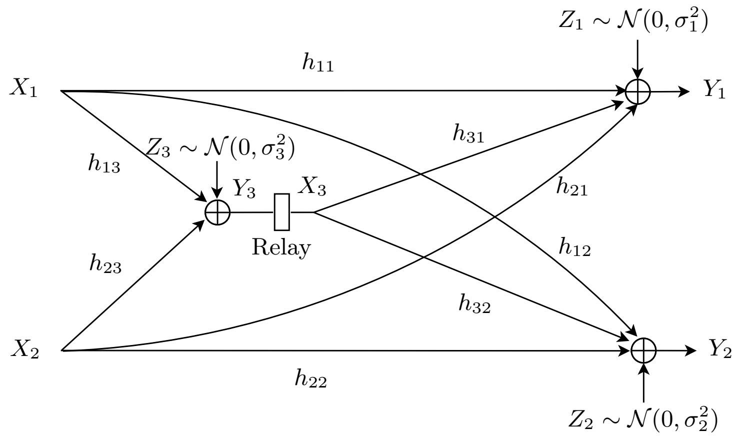

Figure 2.8: Gaussian interference channel with a relay. h_{ij}, $i, j \in \{1, 2, 3\}$, denote channel gains; Z_1, Z_2, and Z_3 denote additive Gaussian noises.

2.6 Interference-Relay Networks

For decades the problems of interference channel and relay channel have been studied separately, mostly focusing on finding the capacity region of each channel. The recent advances of wireless communication have led to the incorporation of relay nodes into wireless networks. Naturally, this has pushed forward the study of using relays not only to increase the network throughput but also to mitigate interference. The early stage of this research direction can be categorized into two main topics: the first one studies interference channels with cooperating transmitters and/or receivers, and the second one studies interference channels with one or many relays. The former topic is marked with the results in [ZY12, PV11b, PV11a, WT11b, WT11a] in which various bounds on different classes of channels are proposed and the capacity region or the sum-capacity were characterized to within constant gap. In Chapter 3 and Chapter 4 we will investigate new coding techniques for two channels of this type. The research on the latter topic was initiated by a paper by Sahin et al. [SE07], where a dedicated relay node is added to the two-user interference channel in order to help the communication of all parties. This channel, depicted in Fig. 2.8, is often called *interference relay channel* or *interference channel with a relay*, and it can be categorized into different classes depending on how the relay uses the channel's resources. In the last few years there have been a large body of works on the interference channel with a relay, which can be found in [SES09, MDG08, MDG09, SSE11a, SES10, SSE11b, SES11, CS12]. We will study the interference channel with a relay in Chapter 5 of this thesis, and in Chapter 6 we will introduce and study a new channel model of this kind, the *interfering relay channels*. In general, the research on interference-relay networks still have many open problems to solve since even the capacity region of each individual channel is unknown. However, the obtained results so far have shown that using relays to mitigate interference is a natural and beneficial approach. The subsequent chapters of this thesis will further illuminate this conclusion.

Chapter 3

Interference Channels With Cooperating Receivers: Asymmetric Case

This chapter studies a model for communications in wireless networks supported by designated cooperation links. In particular, a 2-user Gaussian one-sided interference channel with two rate-limited and orthogonal communication links between the receivers is considered. A communication protocol for the channel is proposed, which combines rate-splitting and two versions of superposition encoding techniques with the conventional decode-forward and compress-forward strategies. It is shown that a careful design of codebooks and coding scheme, which is obtained from the intuition of superposition coding, can reduce the complexity of the strategy and simplify the analysis. Analytical and numerical results show that the proposed scheme, although not universally optimal, can achieve the capacity region or sum-capacity exactly or asymptotically in certain scenarios. Various limits of sum-capacity gain due to cooperation are also discussed.

3.1 Introduction

A one-sided interference channel [Sat81] is a communication model with two pairs of transmitter-receiver, where only one receiver is affected by interference. In this chapter, we extend the model by allowing the two receivers to cooperate through designated cooperation links, which neither interfere with each other nor with the main channel. We call this channel the *Asymmetric Interference Channel with Receiver Cooperation* (AIC-RC).

As a subclass of the classical 2-user interference channels introduced in Chapter 2, the one-sided interference channel has been a subject of study for many years. On the one hand, this channel model is motivated by relevant situations in wireless and wireline networks where the interference/crosstalk to one receiver is blocked or is made negligible by barriers or by distance. One such example is when a mobile terminal in a cellular network is located at the cell edge, as shown in [SSPS09]. Another situation is when there exists a barrier between one transmitter and one receiver

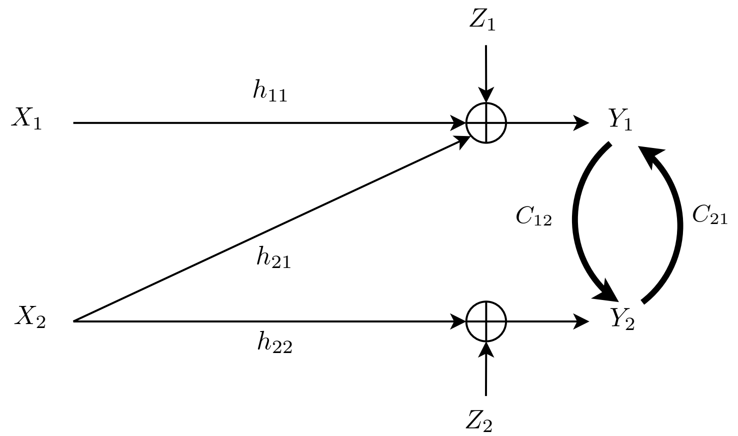

Figure 3.1: Gaussian asymmetric interference channel with orthogonal receiver cooperation. The cooperation links are noise-free with finite capacities C_{12} and C_{21}.

in satellite (line of sight) communication. On the other hand, the existing work on the one-sided interference channel has helped shed light on the general interference channel. This is because removing one interference link leads to an upper bound on the capacity region of the Gaussian interference channel [Kra04] while knowing how to mitigate the remaining interference provides insight into how to construct codes to approach the capacity. Despite its simpler structure compared to the general interference channel, the capacity region of the one-sided interference channel is still unknown in general. Only limited results have been obtained for the capacity region and the sum-capacity or maximum throughput [Sas04]. Recently, sparked by advances in cooperative communication, especially in terms of new results for the relay channel, the study of interference channels with some forms of cooperation has drawn increasing interest. For example, the interference channel with cooperative transmitters has been thoroughly studied in [HM06, NJGM07, PV11b, WT11b], while [HM06, SGP+09, SSPS09, PV11a, WT11a] study the cooperation at the receivers' side. The cooperation can occur either in the same spectral resource with the main channel, namely *in-band* cooperation [HM06, PV11a], or in a spectral resource orthogonal to the main channel, namely *orthogonal* or *out-of-band* cooperation [SGP+09, WT11a].

In the present chapter, we consider the Gaussian one-sided interference channel in which *both* receivers can exchange information over rate-limited cooperation links. We focus on cooperation links modeled as orthogonal spectral resources which are reserved for the purpose of cooperation, i.e., out-of-band cooperation. As motivated in Chapter 1, these links correspond to backhauls in heterogeneous networks (HetNet) and play crucial roles in Coordinated Multipoint transmission and reception (CoMP) techniques. For example, the base stations in HetNet can be connected by high-speed optical backhauls or by microwave channels whose rates are pre-allocated and guaranteed. Another scenario is direct device-to-device communication, in which wireless receiving nodes are close enough to be able to establish reliable cooperation links. From a network perspective, the channel under consideration can be regarded as an element in Wyner's asymmetric cellular model or Wyner's soft-handoff model for multi-cell communications in wireless cellular networks, see [Wyn94, SSPS09, SW11, GMFC09, MF11b, GHH$^+$10] and references therein.

The main results of this chapter are summarized as follows: We propose a communication scheme which is built upon the coding techniques for: i) the interference channel, i.e., rate-splitting technique; ii) the broadcast channel, i.e., superposition coding; and iii) for the relay channel, i.e., compress-forward and decode-forward. It is shown that by careful design of codebooks and encoding/decoding strategies, the analysis of achievable rates can be simplified. Our achievable region recovers previously known results for no cooperation and for unidirectional cooperation as special cases, yet in simpler forms. We present an outer bound to the capacity region of the channel and use it to show the optimality of our coding scheme in a certain scenario as well as to characterize the possible sum-rate gain due to cooperation. We provide analytical and numerical analysis for the achievable sum-rate of our proposed scheme. Throughout the chapter we discuss the intuition behind our proposed scheme and results.

The channel model studied in this chapter is related to those studied in [SSPS09], [WT11a], and [ZY12]. Reference [SSPS09] considers a cascade of many one-sided interference channels with cooperation links as a model for multicell communication, and shows the impact of codebook information, decoding delay, and network planning on the performance of multicell decoding. Unlike [SSPS09], we concentrate on designing a coding scheme for a single channel. Reference [ZY12] considers the Z-interference channel with a *unidirectional* cooperation link between the receivers and proposes coding schemes based on classical compress-forward and decode-forward [CE79]. Our channel model is on the one hand a generalization of, but on the other hand fundamentally different from those of [ZY12], in the sense that the cooperation in our model is inherently *bidirectional.* Therefore, while [ZY12] studies two channels with unidirectional cooperation separately, we focus on designing a *two-round* coding scheme. By doing so, our work not only can recover the achievability results of [ZY12] but also provides new insight into how to efficiently construct codebooks to exploit to the two-way cooperation (Sections 3.3.3, 3.4). [WT11a] studies the general 2-user Gaussian interference channel with limited receiver cooperation, where a *two-round* conferencing scheme based on two-layer superposition coding with fixed power allocation and quantize-bin-forward relaying is shown to obtain an approximate characterization of the capacity region to within a constant gap. In this chapter, we develop a new coding scheme based on simultaneous coding, three-layered superposition coding, and Cover-El Gamal [CE79] decode-forward/compress-forward relaying. To gain more insight into the benefit of cooperation, we perform asymptotic analysis in various "high SNR" scenarios in Sections 3.5.1, 3.5.2, 3.5.3.

The rest of the chapter is structured as follows: Section 3.2 describes the channel model. In Section 3.3 we propose a new coding scheme for the channel. The resulting achievable rate region and an outer bound to the capacity region are presented in Section 3.4. Analysis and numerical examples on the rate region and asymptotic sum-rate are presented in Section 3.5. Section 3.6 summarizes the results and draws some conclusions. For readability, some proofs are relegated to the appendices at the end of the chapter.

3.2 Problem Formulation

3.2.1 Channel Model

Consider a discrete-time memoryless channel as depicted in Fig. 3.1, which is a one-sided interference channel with additive white Gaussian noise (AWGN) at each receiver. At time t, transmitter i puts out an input symbol $x_i[t] \in \mathbb{R}$ to the channel, aiming at the corresponding receiver i, $i \in \{1, 2\}$. Due to the nature of the channel, the signal from Transmitter 2 causes interference to Receiver 1 while Receiver 2 is free of interference. The noise components $z_1[t]$ and $z_2[t]$ at the receivers are realizations of independent real-valued additive white Gaussian random processes with variance 1. h_{11}, h_{21}, and h_{22} denote real-valued channel gains between the transmitters and the receivers, which are assumed to be fixed during each transmission and are known to the transmitters and the receivers. The received signals at Receivers 1 and 2 at time t are given by

$$\begin{aligned} y_1[t] &= h_{11}x_1[t] + h_{21}x_2[t] + z_1[t] \\ y_2[t] &= h_{22}x_2[t] + z_2[t]. \end{aligned}$$

The two receivers can communicate over noiseless cooperation links having finite capacities of C_{12} and C_{21}, and do not interfere with the main channel. The cooperation signal sent by a receiver at time t depends on the signals it has received from the transmitters and from the other receiver up to time $t-1$. As noted in Section 3.1, we call this model the *Asymmetric Interference Channel with Receiver Cooperation* (AIC-RC).

We assume average power constraints at the two transmitters, i.e., $\frac{1}{n}\sum_{t=1}^{n} x_i^2[t] \leq P_i$, $i \in \{1, 2\}$, where n is the codeword length. By scaling the channel gains we can assume without loss of generality that $P_1 = P_2 = 1$ and define the following signal-to-noise ratios (SNR) and interference-to-noise ratio (INR):

$$\mathsf{SNR}_1 := h_{11}^2, \qquad \mathsf{SNR}_2 := h_{22}^2, \qquad \mathsf{INR} := h_{21}^2. \tag{3.1}$$

In this chapter we follow standard definitions of codes, achievable rate, and capacity region as introduced in Chapter 2. The content in Section 3.3 also hinges on the random coding argument presented in textbook [CT06]. Finally, we define $\bar{x} := 1-x$ for $0 \leq x \leq 1$.

3.3 A Coding Scheme

3.3.1 Outline of the Coding Scheme

We define the cooperation protocol as follows: The interfered receiver (receiver Y_1 or Rx1 in Fig. 3.1) partially decodes its desired message, then initiates the cooperation by sending a description of its received signal to the interference-free receiver (Rx2). Rx2 then decodes its desired message with the helping information from Rx1 and sends back some helping information to Rx1. Finally, Rx1 decodes the remaining part of its desired message. One motivation for this protocol is that: Since Rx1 suffers from interference while Rx2 does not, it is natural for Rx1 to start the cooperation. However, the intuition obtained from the current communication protocol can also be applied to the case when Rx2 starts the cooperation.

The encoding and decoding processes, and the cooperation protocol are presented in the following, with the help of Fig. 3.2.

3.3.2 Details of the Coding Scheme

In what follows we present the details of the coding scheme, more intuitive explanations can be found in Section 3.3.3.

Codebook construction. Let m_i, $i \in \{1, 2\}$, denotes the message to be sent from Txi. Tx1 splits its message into two parts: $m_1 = (m_{1c}, m_{1p})$, $m_{1c} \in [1 : 2^{nR_{1c}}]$, $m_{1p} \in [1 : 2^{nR_{1p}}]$, $R_{1c} + R_{1p} = R_1$. Tx2 splits its message into three parts: $m_2 = (m_{2c}, m_{2p}, m_{20})$, $m_{2c} \in [1 : 2^{nR_{2c}}]$, $m_{2p} \in [1 : 2^{nR_{2p}}]$, $m_{20} \in [1 : 2^{nR_{20}}]$. $R_{2c} + R_{2p} + R_{20} = R_2$. We call m_{2c}, m_{2p} the *common* message and *private* message of Tx2. m_{20} is called the *cooperative common*[1] message since it can be decoded by Rx1 with the help from Rx2.

Codebook for Transmitter 1. Randomly and independently generate $2^{nR_{1c}}$ sequences $\mathbf{w}_1(m_{1c})$, $m_{1c} \in [1 : 2^{nR_{1c}}]$, each according to $\prod_{i=1}^{n} p_{W_1}(w_{1i})$, where $W_1 \sim \mathcal{N}(0, P_{1c})$, $P_{1c} = \alpha$ for $0 \leq \alpha \leq 1$. Randomly and independently generate $2^{nR_{1p}}$ sequences $\mathbf{u}_1(m_{1p})$, $m_{1p} \in [1 : 2^{nR_{1p}}]$, each according to $\prod_{i=1}^{n} p_{U_1}(u_{1i})$, where $U_1 \sim \mathcal{N}(0, P_{1p})$, $P_{1p} = 1 - \alpha = \bar{\alpha}$.

Codebook for Transmitter 2. Randomly and independently generate $2^{nR_{2c}}$ sequences $\mathbf{w}_2(m_{2c})$, $m_{2c} \in [1 : 2^{nR_{2c}}]$, each according to $\prod_{i=1}^{n} p_{W_2}(w_{2i})$, $W_2 \sim \mathcal{N}(0, P_{2c})$, $P_{2c} = \beta$. For each $\mathbf{w}_2(m_{2c})$, randomly and independently generate $2^{nR_{20}}$ sequences $\mathbf{v}_2(m_{20}, m_{2c})$, each according to $\prod_{i=1}^{n} p_{V_2|W_2}(v_{2i}|w_{2i})$ where $V_2 = W_2 + S$, $S \sim \mathcal{N}(0, \gamma)$ and independent of everything else. For each $\mathbf{w}_2(m_{2c})$ and $\mathbf{v}_2(m_{20}, m_{2c})$, randomly and independently generate $2^{nR_{2p}}$ sequences $\mathbf{x}_2(m_{2p}, m_{20}, m_{2c})$, each according to the distribution $\prod_{i=1}^{n} p_{X_2|V_2,W_2}(x_{2i}|v_{2i}, w_{2i})$ where $X_2 = V_2 + Q$, $Q \sim \mathcal{N}(0, \eta)$ and independent of everything else. The parameters β, γ, η are chosen to satisfy $0 \leq \beta, \gamma, \eta \leq 1$ and $\beta + \gamma + \eta = 1$.

Partition $2^{nR_{2c}}$ indices $m_{2c} \in [1 : 2^{nR_{2c}}]$ into $2^{n\lambda C_{12}}$ equal-size bins, $0 \leq \lambda \leq 1$ to

[1] This resembles the *cooperative public* message in [PV11a].

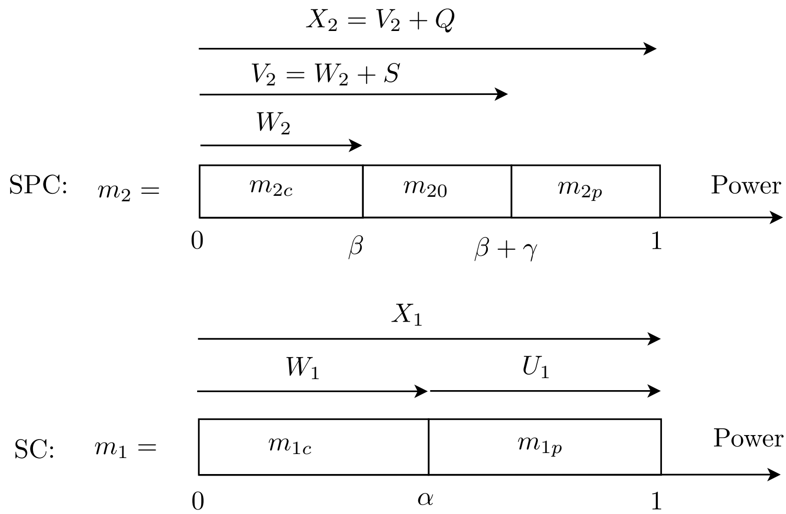

Figure 3.2: Parameters for codebook design at the transmitters. Tx1 constructs an SC code of two components (W_1 and U_1). Tx2 constructs a 3-layer SPC code (W_2, V_2, X_2). α and $1-\alpha$ are powers for common and private parts at Tx1, respectively. β, γ, and $1-\beta-\gamma$ are powers for common, cooperative common, and private parts at Tx2, respectively.

be specified later. Partition $2^{nR_{20}}$ indices $m_{20} \in [1 : 2^{nR_{20}}]$ into $2^{nC_{21}}$ equal-size bins.

Codebook for Receiver 1. Randomly and independently generate $2^{n(\bar{\lambda}C_{12}+I(\bar{Y}_1;\hat{Y}_1))}$ sequences $\hat{\mathbf{y}}_1$ according to $\prod_{i=1}^{n} p_{\hat{Y}_1}(\hat{y}_{1i})$ where the random variables $\bar{Y}_1$ and $\hat{Y}_1$ have the following distributions:

$$\bar{Y}_1 = h_{11}U_1 + h_{21}X_2 + \theta h_{21}W_2 + Z_1$$
$$\hat{Y}_1 = \bar{Y}_1 + Z_q,$$

where $Z_q \sim \mathcal{N}(0, N_q)$, independent of everything else. Variance N_q and parameter $\theta \in \mathbb{R}$ will be specified later. The reason for choosing this structure of $(\bar{Y}_1, \hat{Y}_1)$ is explained in the decoding stage 1 below. Finally, partition $2^{n(\bar{\lambda}C_{12}+I(\bar{Y}_1;\hat{Y}_1))}$ sequences $\hat{\mathbf{y}}_1$ into $2^{n\bar{\lambda}C_{12}}$ equal-size bins.

All codebooks are revealed to all nodes before the transmission and the nodes agree on the transmission strategy.

Encoding. To send a message pair $(m_1, m_2) = ((m_{1p}, m_{1c}), (m_{2p}, m_{20}, m_{2c}))$, Tx1 transmits $\mathbf{x}_1(m_{1p}, m_{1c}) = \mathbf{w}_1(m_{1c}) + \mathbf{u}_1(m_{1p})$, Tx2 transmits $\mathbf{x}_2(m_{2p}, m_{20}, m_{2c})$.

The received signals at the receivers are given by:

$$\begin{aligned}\mathbf{y}_1 &= h_{11}\mathbf{x}_1 + h_{21}\mathbf{x}_2 + \mathbf{z}_1\\ &= h_{11}\mathbf{w}_1 + h_{11}\mathbf{u}_1 + h_{21}\mathbf{x}_2 + \mathbf{z}_1\\ \mathbf{y}_2 &= h_{22}\mathbf{x}_2 + \mathbf{z}_2.\end{aligned}$$

Decoding. The decoding procedure consists of the following 3 stages.
Decoding stage 1: Rx1 decodes the "common" messages jointly. Precisely, Decoder 1 looks for a unique (m_{1c}, m_{2c}) pair such that $(\mathbf{w}_1(m_{1c}), \mathbf{w}_2(m_{2c}))$ is jointly typical[2] with the received sequence $\mathbf{y}_1$. The decoding is therefore equivalent to the decoding of the multiple access channel (MAC) with channel inputs $\mathbf{w}_1$, $\mathbf{w}_2$ and channel output $\mathbf{y}_1$. The achievable rates are given by [CT06, Chapter 15]:

$$R_{1c} \le I(W_1; Y_1|W_2) \tag{3.2}$$
$$R_{2c} \le I(W_2; Y_1|W_1) \tag{3.3}$$
$$R_{1c} + R_{2c} \le I(W_1, W_2; Y_1). \tag{3.4}$$

Upon successfully decoding m_{1c} and m_{2c}, Rx1 subtracts $h_{11}\mathbf{w}_1(m_{1c})$ and adds $\theta h_{21}\mathbf{w}_2$ to $\mathbf{y}_1$, $\theta \in \mathbb{R}$, to obtain

$$\begin{aligned}\bar{\mathbf{y}}_1 &:= \mathbf{y}_1 - h_{11}\mathbf{w}_1 + \theta h_{21}\mathbf{w}_2\\ &= h_{11}\mathbf{u}_1 + h_{21}(\mathbf{x}_2 + \theta\mathbf{w}_2) + \mathbf{z}_1,\end{aligned}$$

and uses a vector quantizer to quantize $\bar{\mathbf{y}}_1$ into $\hat{\mathbf{y}}_1$. Note that here we adopt the technique of introducing the parameter θ of [ZY12] into $\bar{\mathbf{y}}_1$ to take advantage of $\mathbf{w}_2$ as side information when decoding $\mathbf{v}_2$ and $\mathbf{x}_2$ at Rx2.

Next, $\hat{\mathbf{y}}_1$ is conveyed to Rx2 by using the Wyner-Ziv source coding technique [WZ76]. As such, Rx1 uses part of the rate of the cooperation link 1-2, namely $\bar{\lambda}C_{12}$, to convey the corresponding bin index of $\hat{\mathbf{y}}_1$ to Rx2. The necessary condition for that purpose is given by [CE79, Eq. (76b)], namely $\bar{\lambda}C_{12} \ge I(\hat{Y}_1; \bar{Y}_1|Y_2)$, where the mutual information term depends on the quantization distortion N_q. We choose to use the smallest rate required, i.e.,

$$\bar{\lambda}C_{12} = I(\hat{Y}_1; \bar{Y}_1|Y_2), \tag{3.5}$$

to maximally utilize the cooperation link and choose the quantization distortion N_q accordingly. The remaining rate of the cooperation link, namely λC_{12}, is used to send the index of the bin that contains codeword $\mathbf{w}_2$, thereby helping Rx2 decode $\mathbf{w}_2$. This explains the design of codebooks in the codebook construction part.

Decoding stage 2: Upon receiving the bin index of $\hat{\mathbf{y}}_1$ via the cooperation link from Rx1 and using its own received sequence $\mathbf{y}_2$, Rx2 can recover $\hat{\mathbf{y}}_1$ as in classical compress-forward [CE79]. As a result, $\hat{\mathbf{y}}_1$ is utilized as an additional channel output when decoding Rx2's desired messages. Specifically, Decoder 2 looks

[2]We assume the decoders use the joint typicality decoding rule, whose details can be found in textbook [CT06].

for a unique triple (m_{2c}, m_{20}, m_{2p}) such that the corresponding codeword triple $(\mathbf{w}_2(m_{2c}), \mathbf{v}_2(m_{20}, m_{2c}), \mathbf{x}_2(m_{2p}, m_{20}, m_{2c}))$ is jointly typical with $(\mathbf{y}_2, \hat{\mathbf{y}}_1)$ and $\mathbf{w}_2$ lies in the bin whose index is specified by the cooperative message from Rx1. Combining the superposition encoding in [CMGE08] with the decode-forward scheme for the multiple-access relay channel in [SKM04] the following rates are achievable:

$$R_{2p} \leq I(X_2; Y_2, \hat{Y}_1 | W_2, V_2) \tag{3.6}$$

$$R_{2p} + R_{20} \leq I(V_2, X_2; Y_2, \hat{Y}_1 | W_2) \tag{3.7}$$

$$R_{2c} + R_{2p} + R_{20} \leq I(W_2, V_2, X_2; Y_2, \hat{Y}_1) + \lambda C_{12}. \tag{3.8}$$

Intuitively, constraint (3.6) follows from the event where the codewords conveying correct (m_{2c}, m_{20}) and wrong m_{2p} satisfy the decoding rule. Similarly, (3.7) corresponds to correct m_{2c} and wrong (m_{20}, m_{2p}), and (3.8) corresponds to wrong (m_{2c}, m_{20}, m_{2p}). Note that the rate increase by λC_{21} in (3.8) is due to the fact that the decoder can restrict the searching set for codeword $\mathbf{w}_2$ to the bin whose index is sent to Rx2 at the rate of λC_{12}. After correctly decoding (m_{2c}, m_{20}, m_{2p}), Rx2 looks for the bin index of m_{20} and sends this index to Rx1 via the cooperation link C_{21}.

Decoding stage 3: Rx1 decodes m_{1p} with the help from Rx2. Specifically, Rx1 looks for a unique m_{1p} such that $(\mathbf{u}_1(m_{1p}), \mathbf{v}_2)$ is jointly typical with $\mathbf{y}_1$ for some $\mathbf{v}_2$ whose message m_{20} matches the bin index given from Rx2, and given the fact that $\mathbf{w}_1$ and $\mathbf{w}_2$ are already known to Decoder 1 from the first decoding stage. Accordingly, the following rates are achievable [CMGE08, SKM04]:

$$R_{1p} \leq I(U_1; Y_1 | W_1, W_2, V_2) \tag{3.9}$$

$$R_{1p} + R_{20} \leq I(U_1, V_2; Y_1 | W_1, W_2) + C_{21}. \tag{3.10}$$

These constraints can be similarly explained as those in (3.6)–(3.8). Again, the rate enhancement by C_{21} is due to the fact that the decoder can restrict the searching set for the codeword $\mathbf{v}_2$ to the bin whose index is received from Rx2. Also note that since $\mathbf{v}_2$ is not required to be decoded correctly at Rx1, i.e., *non-unique decoding* is employed, there is no constraint on the rate R_{20}.

3.3.3 Comments on the Encoding/Decoding Strategy

Our coding scheme is developed based on a careful combination of *simultaneous coding* (SC) (at Tx1), *superposition coding* (SPC) (at Tx2), and non-unique decoding at the receivers.[3] Both SC and SPC originate from the idea of cod-

[3] By SC we mean that to send messages M_1 and M_2 a transmitter generates one codebook for each M_i using a codebook-generating random variable Q_i, $i = 1, 2$, and Q_1 is independent of Q_2. On the other hand, in SPC, codewords for M_1 are generated first (as cloud centers), and conditioning on each of them the codewords for both M_1 and M_2 are generated (as satellites around the centers). In the current problem, independent random variables W_1 and U_1 are used for SC of m_{1c} and m_{1p} at Tx1 while correlated random variables W_2, V_2, X_2 are used for SPC at Tx2, cf. Fig. 3.2.

ing for broadcast channel by Cover in [Cov72]. Cover himself constructed an SC code [Cov72], while a SPC code was first proposed by Bergman [Ber73] for the degraded broadcast channel. These two schemes achieve the same rate regions for several classes of channels, such as the degraded broadcast channels. Very recently, however, Wang et al. [WcBK13] have shown that SC generally achieves larger rate regions than SPC for broadcast channels, under maximum likelihood decoding. Han and Kobayashi [HK81] used SC for the interference channel, which achieves the best inner bound to date. Chong et al. showed in [CMGE08] that the same rate region as the Han-Kobayashi region [HK81] can be achieved by SPC. As we will see later, the interplay between SC and SPC in our scheme considerably simplifies the decoding procedures as well as the analysis. The method we use to generate the codebooks can be intuitively justified as follows:

- The codebook for message m_1 is generated using SC because it allows Decoder 1 to decode and then subtract the sequence $\mathbf{w}_1$ from the received sequence $\mathbf{y}_1$. Doing so reduces the noise in the quantization signal sent to Rx2.

- The codebook for message m_2 is generated using SPC because it helps reduce the number of constraints for the decoding stages (see Remark 3.1). Regarding the order of encoding for the sub-messages: Since the *common part* m_{2c} is decoded first, it is natural to encode m_{2c} into the coarsest layer (W_2, the cloud centers in SPC). Next, since Decoder 1 is interested in decoding m_{20} only, while it already knows m_{2c}, the cooperative common part m_{20} should be encoded to the second coarsest layer (the inner satellite V_2 in our notation). Finally, the outer most layer is for the private part.

Remark 3.1. One obvious advantage of SPC over SC at Tx2 is that if one used SC decoding, stage 2 would be equivalent to a MAC with 3 inputs, whose capacity region has 7 inequalities [CT06]. Thanks to SPC, the number of inequalities are reduced to 3 as seen in (3.6)–(3.8), which considerably simplifies the Fourier-Motzkin elimination later, cf. Appendix 3.D. In a related work, SC was used in [ZY12] for unidirectional cooperations. It turns out that SPC is also helpful if one uses it for the unidirectional cooperation, cf. Remark 3.2 and Remark 3.3.

3.4 Inner and Outer Bound to the Capacity Region

In this section we state the achievable rate region of the scheme in Section 3.3 and an outer bound to the capacity region of the channel. The proofs are relegated to the appendices.

Theorem 3.4.1. *Let $\mathcal{S}$ be the set of real-valued vectors $\mathbf{s} = (\alpha, \beta, \gamma, \eta, \lambda, \theta)$ such that $0 \le \alpha, \beta, \gamma, \eta, \lambda \le 1$ and $\beta + \gamma + \eta = 1$. Denote by $\mathcal{R}(\mathbf{s})$ the set of nonnegative* *rate pairs* (R_1, R_2) *satisfying the following constraints:*

$$R_1 \leq \mathcal{C}\left(\frac{\alpha\mathsf{SNR}_1}{1+\bar{\alpha}\mathsf{SNR}_1+\bar{\beta}\mathsf{INR}}\right)+\mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1}{1+\eta\mathsf{INR}}\right) \tag{3.11}$$

$$R_2 \leq \mathcal{C}\left(\frac{(1+2\theta\beta+\theta^2\beta)\mathsf{INR}+\theta^2\beta\bar{\beta}\mathsf{SNR}_2\mathsf{INR}}{(1+\mathsf{SNR}_2)(1+N_q+\bar{\alpha}\mathsf{SNR}_1)}\right)+\mathcal{C}\left(\mathsf{SNR}_2\right)+\lambda C_{12} \tag{3.12}$$

$$\begin{aligned} R_2 \leq\ & \mathcal{C}\left(\frac{\beta\mathsf{INR}}{1+\bar{\alpha}\mathsf{SNR}_1+\bar{\beta}\mathsf{INR}}\right)+\mathcal{C}\left(\bar{\beta}\mathsf{SNR}_2\right) \\ & +\mathcal{C}\left(\frac{\bar{\beta}\mathsf{INR}}{(1+\bar{\beta}\mathsf{SNR}_2)(1+N_q+\bar{\alpha}\mathsf{SNR}_1)}\right) \end{aligned} \tag{3.13}$$

$$\begin{aligned} R_2 \leq\ & \mathcal{C}\left(\eta\mathsf{SNR}_2\right)+\mathcal{C}\left(\frac{\beta\mathsf{INR}}{1+\bar{\alpha}\mathsf{SNR}_1+\bar{\beta}\mathsf{INR}}\right)+\mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1+\gamma\mathsf{INR}}{1+\eta\mathsf{INR}}\right)+C_{21} \\ & +\mathcal{C}\left(\frac{\eta\mathsf{INR}}{(1+\eta\mathsf{SNR}_2)(1+N_q+\bar{\alpha}\mathsf{SNR}_1)}\right) \end{aligned} \tag{3.14}$$

$$\begin{aligned} R_1+R_2 \leq\ & \mathcal{C}\left(\eta\mathsf{SNR}_2\right)+\mathcal{C}\left(\frac{\alpha\mathsf{SNR}_1+\beta\mathsf{INR}}{1+\bar{\alpha}\mathsf{SNR}_1+\bar{\beta}\mathsf{INR}}\right)+\mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1+\gamma\mathsf{INR}}{1+\eta\mathsf{INR}}\right)+C_{21} \\ & +\mathcal{C}\left(\frac{\eta\mathsf{INR}}{(1+\eta\mathsf{SNR}_2)(1+N_q+\bar{\alpha}\mathsf{SNR}_1)}\right) \end{aligned} \tag{3.15}$$

$$\begin{aligned} R_1+R_2 \leq\ & \mathcal{C}\left(\frac{\bar{\beta}\mathsf{INR}}{(1+\bar{\beta}\mathsf{SNR}_2)(1+N_q+\bar{\alpha}\mathsf{SNR}_1)}\right)+\mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1}{1+\eta\mathsf{INR}}\right)+\mathcal{C}\left(\bar{\beta}\mathsf{SNR}_2\right) \\ & +\mathcal{C}\left(\frac{\alpha\mathsf{SNR}_1+\beta\mathsf{INR}}{1+\bar{\alpha}\mathsf{SNR}_1+\bar{\beta}\mathsf{INR}}\right), \end{aligned} \tag{3.16}$$

where

$$N_q = \frac{1+\bar{\alpha}\mathsf{SNR}_1}{2^{2\bar{\lambda}C_{12}}-1}+\frac{(1+2\theta\beta+\theta^2\beta+\theta^2\beta\bar{\beta}\mathsf{SNR}_2)\mathsf{INR}}{(2^{2\bar{\lambda}C_{12}}-1)(1+\mathsf{SNR}_2)}. \tag{3.17}$$

The set

$$\mathcal{R} := \mathsf{Co}\left(\bigcup_{\mathbf{s}\in\mathcal{S}}\mathcal{R}(\mathbf{s})\right) \tag{3.18}$$

is achievable for the Gaussian AIC-RC, where $\mathsf{Co}(\cdot)$ *is the convex-hull operator.*

Proof. See Appendix 3.A. □

Theorem 3.4.1 presents an achievable rate region in its most general form, involving multiple parameters. In subsequent sections, we will conduct analysis on some special cases. In each case, the chosen value of each parameter will illuminate the idea behind the coding strategy.

Remark 3.2. We can obtain an achievable rate region for the channel with a *unidirectional* cooperation link from Rx2 to Rx1, call it $\mathcal{R}_{2\to1}(\eta)$, by setting $C_{12} = 0$, $\alpha = 0$, $\beta = 0$, and replacing γ by $\bar{\eta}$ in $\mathcal{R}(\mathbf{s})$. After canceling redundant inequalities we have $\mathcal{R}_{2\to1}(\eta)$ given by the nonnegative pairs (R_1, R_2) satisfying

$$R_1 \le \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\eta\mathsf{INR}}\right) \tag{3.19}$$

$$R_2 \le \mathcal{C}(\mathsf{SNR}_2) \tag{3.20}$$

$$R_1 + R_2 \le \mathcal{C}(\eta\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \bar{\eta}\mathsf{INR}}{1+\eta\mathsf{INR}}\right) + C_{21}, \tag{3.21}$$

where $\eta \in [0,1]$. Note that an achievable region, call it $\mathcal{T}_{2\to1}(\eta)$, using SC was proposed in [ZY12, Eq. (8)] for the same channel configuration. $\mathcal{T}_{2\to1}(\eta)$ is given by the nonnegative pairs (R_1, R_2) satisfying

$$R_1 \le \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\eta\mathsf{INR}}\right) \tag{3.22}$$

$$R_2 \le \mathcal{C}(\mathsf{SNR}_2) \tag{3.23}$$

$$R_2 \le \mathcal{C}(\eta\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\bar{\eta}\mathsf{INR}}{1+\eta\mathsf{INR}}\right) + C_{21} \tag{3.24}$$

$$R_1 + R_2 \le \mathcal{C}(\eta\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \bar{\eta}\mathsf{INR}}{1+\eta\mathsf{INR}}\right) + C_{21}. \tag{3.25}$$

Comparing (3.19)–(3.21) with (3.22)–(3.25) we can easily see that $\mathcal{R}_{2\to1}(\eta) \supseteq \mathcal{T}_{2\to1}(\eta)$ for a fixed power parameter η. The reason for this lies in the advantage of SPC over SC, which can be seen as follows: If one uses SPC instead of SC there will not be the second condition in [ZY12, Eq. (6)]. Furthermore, from Rx2's viewpoint that condition is unnecessary because Rx2 is only interested in the whole rate of m_2. Similarly, the second constraint [ZY12, Eq. (7)] is not needed since Rx1 is not required to decode the common message of user 2 correctly. One would expect that $\bigcup_{0\le\eta\le1}\mathcal{R}_{2\to1}(\eta) \supsetneq \bigcup_{0\le\eta\le1}\mathcal{T}_{2\to1}(\eta)$. However, it turns out that $\bigcup_{0\le\eta\le1}\mathcal{R}_{2\to1}(\eta) = \bigcup_{0\le\eta\le1}\mathcal{T}_{2\to1}(\eta)$, i.e., (3.24) is redundant when taking the union of $\mathcal{T}_{2\to1}(\eta)$ over $\eta \in [0,1]$. To see that, suppose that an achievable rate pair (R_1, R_2) satisfies (3.22), (3.23) , and (3.25), but not (3.24), i.e.,

$$R_2 > \mathcal{C}(\eta\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\bar{\eta}\mathsf{INR}}{1+\eta\mathsf{INR}}\right) + C_{21}. \tag{3.26}$$

Now, from (3.26) and (3.25) we have

$$\begin{aligned} R_1 \le\ & \mathcal{C}(\eta\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \bar{\eta}\mathsf{INR}}{1+\eta\mathsf{INR}}\right) + C_{21} \\ & - \mathcal{C}(\eta\mathsf{SNR}_2) - \mathcal{C}\left(\frac{\bar{\eta}\mathsf{INR}}{1+\eta\mathsf{INR}}\right) - C_{21} \end{aligned}$$

$$= \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\eta\mathsf{INR}}\right), \tag{3.27}$$

which is tighter than (3.22). Combining (3.23) with (3.27) we see that the achievable rate pair (R_1, R_2) must at least satisfy

$$R_1 \leq \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}}\right) \tag{3.28}$$

$$R_2 \leq \mathcal{C}(\mathsf{SNR}_2). \tag{3.29}$$

However, this is achievable by setting $\eta = 1$ in (3.22)–(3.25). Therefore (3.24) is not needed in characterizing $\bigcup_{0\leq\eta\leq1} \mathcal{T}_{2\to1}(\eta)$. This in turn shows that $\bigcup_{0\leq\eta\leq1} \mathcal{R}_{2\to1}(\eta)$ $= \bigcup_{0\leq\eta\leq1} \mathcal{T}_{2\to1}(\eta)$. We note that this result is analogous to the equivalence between the two regions in [CMGE08] and [HK81] for the interference channel, cf. Section 3.3.3.

Remark 3.3. Similarly, we can obtain an achievable rate region $\mathcal{R}_{1\to2}(\eta)$ for the channel with a *unidirectional* cooperation link from Rx1 to Rx2 by setting $\alpha = 1$, $\gamma = 0$, $C_{21} = 0$. We can also show that our achievable region is identical to the achievable rate region for the same channel in [ZY12, Eq. (33)]. Again, note that the second condition in [ZY12, Eq. (70)] is not needed if one uses SPC instead of SC.

Next, we state an outer bound for the capacity region:

Theorem 3.4.2. *The set of nonnegative rate pairs (R_1, R_2) satisfying the following constraints constitute an outer bound for the capacity region of the Gaussian AIC-RC:*

$$R_1 \leq \mathcal{C}(\mathsf{SNR}_1) \tag{3.30}$$

$$R_2 \leq \min\big\{\mathcal{C}(\mathsf{SNR}_2+\mathsf{INR}), \mathcal{C}(\mathsf{SNR}_2)+C_{12}\big\} \tag{3.31}$$

$$\begin{aligned} R_1 + R_2 \leq \min\Big\{&\mathcal{C}(\mathsf{SNR}_1+\mathsf{SNR}_2+\mathsf{INR}+\mathsf{SNR}_1\mathsf{SNR}_2),\\ &\mathcal{C}(\mathsf{SNR}_1+\mathsf{INR}) + \mathcal{C}\left(\frac{\mathsf{SNR}_2}{1+\mathsf{INR}}\right) + C_{21}\Big\}. \end{aligned} \tag{3.32}$$

Proof. The proof follows the bounding techniques for the interference channel, namely cut-set bound and genie-aided bound [ETW08, AV09]. Details are presented in Appendix 3.B. Note that this bound coincides with the outer bound in [WT11a, Lemma 5.1] for the current channel model. □

3.5 Capacity and Sum-rate Analysis

Computing the achievable rate region in Theorem 3.4.1 involves the optimization over all vectors $\mathbf{s} \in \mathcal{S}$, resulting in a non-convex optimization problem. Thus, in order to gain insight into the benefit of cooperation and to quantify the performance of our scheme, we will conduct some exact and asymptotic analysis on the capacity region and sum-capacity of the channel in this section.

Theorem 3.4.1 and Theorem 3.4.2 yield the following lower and upper bound on the sum-capacity of the channel:

Corollary 3.5.1. *The sum-capacity C_{sum} of the Gaussian AIC-RC is bounded by*

$$R_{ac} \leq C_{sum} \leq R_{up} \tag{3.33}$$

where R_{ac} and R_{up} are defined in (3.34) *and* (3.38), *respectively.*

$$R_{ac} := \min\{R_{ac1}, R_{ac2}, R_{ac3}\}, \tag{3.34}$$

where

$$\begin{aligned} R_{ac1} = &\, \mathcal{C}\left(\frac{\alpha\mathsf{SNR}_1 + \beta\mathsf{INR}}{1 + \bar{\alpha}\mathsf{SNR}_1 + \bar{\beta}\mathsf{INR}}\right) + \mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1}{1 + \eta\mathsf{INR}}\right) \\ & + \mathcal{C}\left(\frac{\bar{\beta}\mathsf{INR}}{(1 + \bar{\beta}\mathsf{SNR}_2)(1 + N_q + \bar{\alpha}\mathsf{SNR}_1)}\right) + \mathcal{C}\left(\bar{\beta}\mathsf{SNR}_2\right) \end{aligned} \tag{3.35}$$

$$\begin{aligned} R_{ac2} = &\, \mathcal{C}\left(\frac{\alpha\mathsf{SNR}_1 + \beta\mathsf{INR}}{1 + \bar{\alpha}\mathsf{SNR}_1 + \bar{\beta}\mathsf{INR}}\right) + \mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1 + \gamma\mathsf{INR}}{1 + \eta\mathsf{INR}}\right) \\ & + \mathcal{C}\left(\frac{\eta\mathsf{INR}}{(1 + \eta\mathsf{SNR}_2)(1 + N_q + \bar{\alpha}\mathsf{SNR}_1)}\right) + \mathcal{C}\left(\eta\mathsf{SNR}_2\right) + C_{21} \end{aligned} \tag{3.36}$$

$$\begin{aligned} R_{ac3} = &\, \mathcal{C}\left(\frac{\alpha\mathsf{SNR}_1}{1 + \bar{\alpha}\mathsf{SNR}_1 + \bar{\beta}\mathsf{INR}}\right) + \mathcal{C}\left(\frac{\bar{\alpha}\mathsf{SNR}_1}{1 + \eta\mathsf{INR}}\right) \\ & + \mathcal{C}\left(\frac{\mathsf{INR}}{(1 + \mathsf{SNR}_2)(1 + N_q + \bar{\alpha}\mathsf{SNR}_1)}\right) + \mathcal{C}\left(\mathsf{SNR}_2\right) + \lambda C_{12}. \end{aligned} \tag{3.37}$$

for some $(\alpha, \beta, \gamma, \eta, \lambda, 0) \in \mathcal{S}$ where $\mathcal{S}$ is defined in Theorem 3.4.1.

$$R_{up} := \min\{R_{up1}, R_{up2}, R_{up3}\}, \tag{3.38}$$

where

$$R_{up1} = \mathcal{C}(\mathsf{SNR}_1 + \mathsf{SNR}_2 + \mathsf{INR} + \mathsf{SNR}_1\mathsf{SNR}_2) \tag{3.39}$$

$$R_{up2} = \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}) + \mathcal{C}\left(\frac{\mathsf{SNR}_2}{1 + \mathsf{INR}}\right) + C_{21} \tag{3.40}$$

$$R_{up3} = \mathcal{C}(\mathsf{SNR}_1) + \mathcal{C}(\mathsf{SNR}_2) + C_{12}. \tag{3.41}$$

Proof. R_{ac} is derived from Theorem 3.4.1. For simplicity of analysis we set $\theta = 0$. R_{ac1} and R_{ac2} come from (3.16) and (3.15), respectively. R_{ac3} comes from summing (3.11) and (3.12). The sums of (3.11) and (3.13), and of (3.11) and (3.14) are not active. R_{up1} and R_{up2} come from (3.32), and R_{up3} is obtained by summing (3.30) and the second term in (3.31) (the sum of (3.30) and the first term in (3.31) is redundant due to (3.39)). ∎

Let us first recapitulate the sum-capacity of the channel in Fig. 3.1 without cooperation, i.e., when $C_{12} = C_{21} = 0$.

Lemma 3.1 (Theorem 2 in [Sas04])**.** *The sum-capacity of the Gaussian one-sided interference channel without cooperation is given by*

$$C^0_{sum} = \begin{cases} \mathcal{C}(\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}}\right) & \text{if } 0 \le \mathsf{INR} < \mathsf{SNR}_2, \\ \mathcal{C}(\mathsf{INR} + \mathsf{SNR}_1) & \text{if } \mathsf{SNR}_2 \le \mathsf{INR} < \mathsf{SNR}_2^*, \\ \mathcal{C}(\mathsf{SNR}_1) + \mathcal{C}(\mathsf{SNR}_2) & \text{if } \mathsf{SNR}_2^* \le \mathsf{INR}, \end{cases}$$

where

$$\mathsf{SNR}_2^* := \mathsf{SNR}_2(1 + \mathsf{SNR}_1). \tag{3.42}$$

It can be shown that by setting $C_{12} = C_{21} = 0$, i.e., no cooperation link, and by setting $\alpha, \beta, \gamma, \eta$ in Corollary 3.5.1 at certain values, we can recover the sum-capacity in Lemma 3.1. For example, by choosing $\alpha = 1$, $\beta = \gamma = 0$, $\eta = 1$ we have

$$\begin{aligned} R_{ac1} = R_{ac2} = R_{ac3} &= \mathcal{C}(\mathsf{SNR}_2) + \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1 + \mathsf{INR}}\right) \\ &= C^0_{sum} \text{ if } 0 \le \mathsf{INR} < \mathsf{SNR}_2. \end{aligned} \tag{3.43}$$

This setting is consistent with the conclusions in [Sas04, ETW08, AV09] that, for the interference channel without cooperation, when the interference is weak ($\mathsf{INR} < \mathsf{SNR}_2$), it is sum-rate optimal to set the whole message at Tx2 to be private ($\eta = 1$). Furthermore, treating interference as noise at Rx1 ($\alpha = 1, \beta = 0$) is optimal.

3.5.1 Limit of Sum-capacity Gain Due to Cooperation

From Lemma 3.1 and the outer bound on sum-capacity in Corollary 3.5.1 we can quantify the limit of the sum-capacity gain due to cooperation in the strong interference regime ($\mathsf{INR} \ge \mathsf{SNR}_2$) as follows:

Corollary 3.5.2. *The sum-capacity gain due to cooperation is bounded above as follows*

$$C_{sum} - C^0_{sum} \le \begin{cases} C_{21} + \frac{1}{2} & \text{if } \mathsf{SNR}_2 \le \mathsf{INR} < \mathsf{SNR}_2^* \\ C_{12} & \text{if } \mathsf{SNR}_2^* \le \mathsf{INR}, \end{cases}$$

where SNR_2^* *is defined in* (3.42).

Proof. We have $C_{sum} - C^0_{sum} \le R_{up} - C^0_{sum}$ where R_{up} is defined in (3.38). By comparing R_{up2} in (3.40) and R_{up3} in (3.41) with C^0_{sum} in Lemma 3.1 we complete the proof. ☐

The bounded gains in Corollary 3.5.2 can be roughly explained as follows:

- When $\mathsf{INR} > \mathsf{SNR}_2^*$ the interference caused by Tx2 to Rx1 is so strong that the side information that Rx2 can send to Rx1 via the C_{21} link does not help Rx1 learn more about the interference, and is therefore useless. In that case, the gain in cooperation depends only on the link C_{12}.

- When $\mathsf{SNR}_2 \leq \mathsf{INR} < \mathsf{SNR}_2^*$, the cross link is strong but not too strong. Therefore, both Rx1 and Rx2 can send useful information to each other via cooperation links. Of the two receivers, Rx2 can utilize the cooperation link C_{21} more efficiently (possibly after being helped by Rx1) by giving Rx1 side information about the interference that Tx2 causes to Rx1. Therefore, the sum-rate gain due to Rx2 $\rightarrow$ Rx1 cooperation is upper bounded by C_{21} bits while the rate gain in the other direction is upper bounded by 1/2 bit as derived above. Note that these upper bounds may not be achievable.

Next, we bound the asymptotic sum-capacity gain due to cooperation when the noise powers vanish while the transmit powers and the channel gains remain unchanged, or when the transmit powers go to infinity while the noise powers and the channel gains remain unchanged. This regime is called *high-SNR* or *interference-limited* or *low-noise* regime [HM06, ZY12].

Corollary 3.5.3. *Let us define the* high-SNR regime *to be the asymptotics where* $\mathsf{SNR}_1 \rightarrow \infty, \mathsf{SNR}_2 \rightarrow \infty$ *while the ratios* $\frac{\mathsf{INR}}{\mathsf{SNR}_1}$ *and* $\frac{\mathsf{INR}}{\mathsf{SNR}_2}$ *are finite and kept fixed. For the Gaussian AIC-RC, in the high-SNR regime, weak interference (*$\mathsf{INR} < \mathsf{SNR}_2$*), and finite* C_{21}*, the sum-capacity gain due to cooperation is bounded by:* $C_{sum} - C_{sum}^0 \leq C_{21} + c$*, where* $0 < c < 0.5$ *depends only on the ratio* $\frac{\mathsf{INR}}{\mathsf{SNR}_2}$*.*

Proof. See Appendix 3.C. □

The result in Corollary 3.5.3 can be explained as follows: When the channel is in the high SNR regime, and the interference link is weaker than the direct link, Rx2 can easily decode its desired message, making the cooperation from Rx1 be little help (maximum 0.5 bit, depending on how strong the interference is). In contrast, the interference at Rx1 is very high and Rx1 substantially benefits from the side information from Rx2. As a result, the upper bound on the sum-capacity gain depends on C_{21} but C_{12}. In this case, it is more advisable to invest on link C_{21} rather than on link C_{12}. Corollary 3.5.3 also shows that having link C_{21} with finite rate does not help increase the degrees of freedom or multiplexing gain [ZT03] of the channel. For the asymptotic behavior of the system when C_{12} and C_{21} also grow to infinity, we refer the readers to [WT11a], where the sum-capacity was analyzed in terms of generalized degrees of freedom for symmetric channels.

The bounded gain in sum-capacity is confirmed by numerical examples in Fig. 3.3, and Fig. 3.4. In these examples, we use exhaustive search to find parameters that give the best performance. One can see that in both figures the sum-rate upper bound with cooperation has the same slope as the sum-capacity without cooperation. In Fig. 3.3 and Fig. 3.4 we observe that in mid-SNR regime the achievable

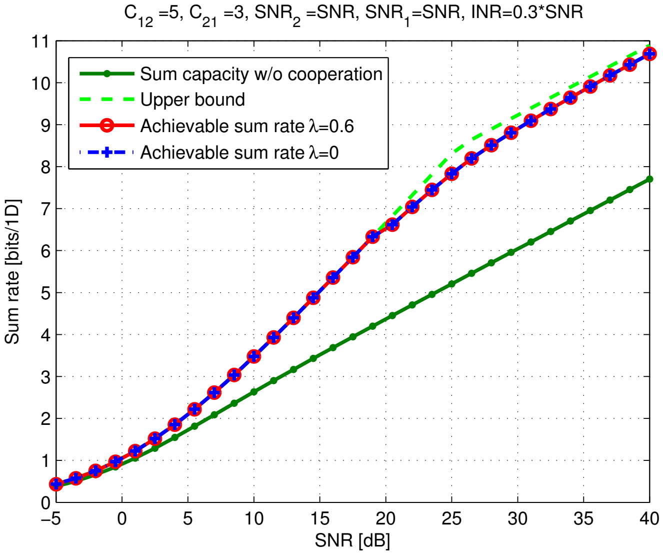

Figure 3.3: Sum-rate bounds for weak interference link ($\mathsf{INR} < \mathsf{SNR}_2$). λ denotes the fraction of the cooperation link C_{12} used for decode-forward.

sum-rates corresponding to different rate allocations on link C_{12} are very close to the upper bound. They maintain constant gaps to the upper bound at high SNR, and significantly better than the sum-capacity without cooperation. This result holds for both weak interference link, i.e., $\mathsf{INR} < \mathsf{SNR}_2$ and strong interference link, i.e., $\mathsf{INR} > \mathsf{SNR}_2$.

3.5.2 Optimality of Unidirectional Cooperation

Capacity Region Optimality

The following theorem shows that the capacity region of the channel can be achieved under special conditions.

Theorem 3.5.1. *If the channel parameters satisfy the following condition:*

$$0 \le C_{12} \le \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}}{(1+\mathsf{SNR}_1)(1+\mathsf{SNR}_2)}\right), \tag{3.44}$$

the capacity region of the Gaussian AIC-RC can be achieved, which is given by the set of all nonnegative rate pairs (R_1, R_2) satisfying

$$R_1 \le \mathcal{C}(\mathsf{SNR}_1) \tag{3.45}$$

$$R_2 \le \mathcal{C}(\mathsf{SNR}_2) + C_{12}. \tag{3.46}$$

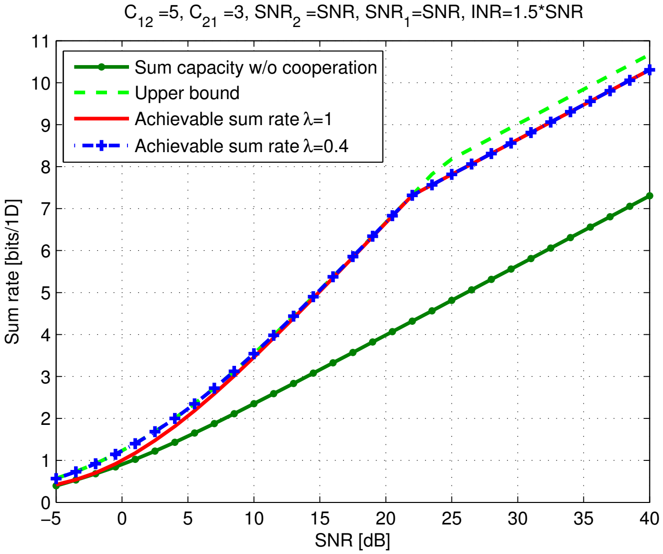

Figure 3.4: Sum-rate bounds for strong interference link ($\mathsf{INR} > \mathsf{SNR}_2$). λ denotes the fraction of the cooperation link C_{12} used for decode-forward.

Proof. Choosing $\lambda = 1, \alpha = 1, \beta = 1, \gamma = \eta = 0$ for $\mathcal{R}(\mathbf{s})$ in Theorem 3.4.1, and removing inactive constraints, we have

$$R_1 \leq \mathcal{C}(\mathsf{SNR}_1) \tag{3.47}$$

$$R_2 \leq \mathcal{C}(\mathsf{SNR}_2) + C_{12} \tag{3.48}$$

$$R_2 \leq \mathcal{C}(\mathsf{INR}) \tag{3.49}$$

$$R_1 + R_2 \leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}), \tag{3.50}$$

where (3.48) follows from (3.12) and the fact that $N_q = \infty$. Under the condition given in (3.44), one can verify that (3.49) is redundant due to (3.48), and (3.50) is redundant due to (3.47) and (3.48). Accordingly, the achievable rate region reduces to the rectangle defined by (3.47) and (3.48), which are the same as (3.45) and (3.46), and meet the outer bound given in Theorem 3.4.2. □

Note that in this case the capacity region is achieved regardless of the value of C_{21}, which means unidirectional cooperation is optimal. The reason for this is that, the interference caused by Tx2 to Rx1 is so strong that Rx1 can fully decode its desired message as well as the interference without rate loss (note that (3.44) implies $\mathsf{INR} > (1 + \mathsf{SNR}_1)\mathsf{SNR}_2$, that is, the channel is in a very strong interference regime). Furthermore, in this case joint decoding and successive decoding at Rx1 are both optimal. To see that, let us set $\alpha = 0, \beta = 1, \lambda = 1$ in Theorem 3.4.1, i.e., Rx1 decodes only $\mathbf{x}_2$ in decoding stage 1. It is not difficult to show that the inner bound will also reduce to the capacity region above. What happens is that after decoding $\mathbf{x}_2$, Rx1 can subtract $\mathbf{x}_2$ from the received sequence and decodes $\mathbf{x}_1$ without interference.

Asymptotic Sum-capacity Optimality

Clearly, the sum-capacity of the channel is also achieved in the aforementioned scenario of Theorem 3.5.1. Let us now perform an asymptotic analysis on the sum-rate for this case. Let SNR_1 be fixed, INR and SNR_2 go to ∞ while their ratio is kept fixed, i.e., $\frac{\mathsf{INR}}{\mathsf{SNR}_2} = k$. This describes the cases when the transmit power at Tx1 is fixed and the transmit power at Tx2 increases to very high values while the channel gains remain unchanged. Theorem 3.5.1 implies that C_{sum} is achieved for any $C_{21} \geq 0$ if

$$\begin{aligned} C_{12} &< \frac{1}{2}\log\left(\frac{k\mathsf{SNR}_2}{(1+\mathsf{SNR}_1)\mathsf{SNR}_2}\right) \\ &= \frac{1}{2}\log\left(\frac{k}{1+\mathsf{SNR}_1}\right), \end{aligned} \tag{3.51}$$

provided that $\frac{k}{1+\mathsf{SNR}_1} \geq 1$. To take an example, consider $k = 10$, $\mathsf{SNR}_1 = 1$ dB, and $C_{12} = 1$ bit, which satisfy (3.51). Numerical examples for $C_{21} = 0.5$ bit and $C_{21} = 8$ bits are given in Fig. 3.5. We see that when SNR_2 gets large enough the condition in (3.44) reduces to (3.51) and the achievable sum-rates converge to the upper bound, regardless of the value of C_{21}. Furthermore, since $\mathsf{SNR}_2^* = \mathsf{SNR}_2(1+\mathsf{SNR}_1) \leq \mathsf{INR}$ and $C_{sum} = R_{up} = R_{up3}$ at high SNR, the gap between the upper and lower curves equal to $C_{12} = 1$ bit at high SNR, as can be seen from Corollary 3.5.2 and Theorem 3.5.1.

Remark 3.4. In order to show a constant gap to the capacity region, [WT11a] uses different coding strategies for different channel parameter regimes, namely the Weak Interference [WT11a, Section V-C], the Mixed Interference [WT11a, Section V-D], and the Strong Interference [WT11a, Section V-E] regimes. Let us consider our channel model when $\mathsf{INR} \geq \mathsf{SNR}_2$, which corresponds to the Mixed Interference regime. For this scenario, the two-round scheme in [WT11a] starts from Rx2, i.e. $2 \to 1 \to 2$, while our scheme starts from Rx1. By [WT11a, Lemma 5.8] their achievable region is within 0.75 bits of the capacity region universally. On the contrary, our scheme can achieve the capacity region under a condition, cf. Theorem 3.5.1. Moreover, we show that under such a condition unidirectional cooperation from Rx1 to Rx2 is enough, which apparently does not apply to their $2 \to 1 \to 2$ scheme. We, however, emphasize that the way [WT11a] chooses the parameters is not necessarily always optimal, but rather to guarantee a constant gap universally over all channel parameters. Optimizing the parameters in [WT11a] is a very challenging task.

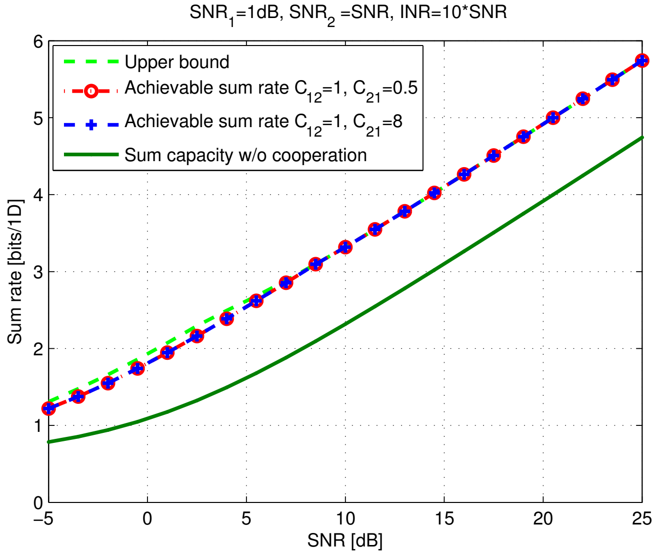

Figure 3.5: Asymptotic sum-rate optimality of unidirectional cooperation. As shown in Section 3.5.2, the value of C_{21} does not affect achievable sum-rates, which converge to the upper bound at high SNR.

3.5.3 Asymptotic Sum-rate When One Cooperation Link Has Infinite Capacity

We now shift our focus to the asymptotic achievable sum-rate when the cooperation links have large capacities. Since the cooperation is bidirectional, if both cooperation links have infinite capacities we can consider the pair (Rx1 and Rx2) as one virtual receiver. Accordingly, the original channel is equivalent to a multiple access channel with two single-antenna senders and one 2-antenna receiver. The sum-capacity can be readily computed and is equal to R_{up1} in (3.39) above. In the following corollary we characterize a sufficient condition on C_{21} so that when C_{12} goes to infinity we achieve the sum-capacity.

Corollary 3.5.4. *Let $R_{ac}(C_{12}, C_{21})$ denote the maximum achievable sum-rate for our coding scheme. We have*

$$\begin{aligned} R_{ac}(\infty, C_{21}) &= R_{ac}(\infty, \infty) \\ &= C_{sum} \\ &= \mathcal{C}(\mathsf{SNR}_1 + \mathsf{SNR}_2 + \mathsf{INR} + \mathsf{SNR}_1\mathsf{SNR}_2) \end{aligned}$$

whenever

$$C_{21} \geq \mathcal{C}\left(\frac{\mathsf{SNR}_2(1+\mathsf{SNR}_1)}{1+\mathsf{SNR}_1+\mathsf{INR}}\right). \tag{3.52}$$

Proof. Let $C_{12} \to \infty$, $0 < \lambda < 1$ in (3.17) we have $N_q = 0$. Furthermore, we set $\alpha = 0$, $\beta = 0$, $\gamma = 1$, $\eta = 0$ in Corollary 3.5.1 to obtain

$$R_{ac1} = \mathcal{C}(\mathsf{SNR}_1 + \mathsf{SNR}_2 + \mathsf{INR} + \mathsf{SNR}_1\mathsf{SNR}_2), \tag{3.53}$$

$$R_{ac2} = \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}) + C_{21}, \tag{3.54}$$

$$R_{ac3} \to \infty \text{ as } C_{12} \to \infty. \tag{3.55}$$

Condition (3.52) implies $R_{ac} = R_{ac1}$, which in turn equals the upper bound R_{up1} in (3.39). Obviously, the sum-capacity C_{sum} is achieved in this case. □

In this case no compression is needed at the first decoding stage at Rx1. One can map directly the received sequence at Rx1 to the cooperative codewords sent on the cooperative link C_{12}. This makes sense since the link C_{12} has infinite capacity. We also observe that the whole message at Rx2 is set to be the *cooperative common* message ($\gamma = 1$), i.e., Rx1 first performs hash-forward [CK07], Rx2 decodes its desired message then performs bin-forward, finally Rx1 decodes its messages with side information from Rx2.

Remark 3.5. A somewhat similar but more difficult problem was posed by Cover in [Cov87] for the primitive relay channel [Kim07], namely finding the critical value C_0 of the noiseless relay link such that $C(C_0)$ first equals $C(\infty)$ where $C(C_0)$ is the supremum of the achievable rates for given C_0. To the best of our knowledge, the problem is still unsolved [Kim07].

3.6 Chapter Conclusion

We proposed a coding scheme for an asymmetric interference channel with orthogonal cooperation links. Our approach is motivated by intuition obtained from simultaneous coding, superposition coding, and non-unique decoding. The resulting achievable rate region is shown to be optimal under certain conditions. The analysis illuminates the intuition behind the properties of the system, providing useful insight into the system design process, e.g., in certain cases it is better to invest more on a particular cooperation link than the other. We also note that what was learned from studying the proposed coding scheme is applicable to the case where the cooperation is initiated at Rx2. To be comprehensive, we remark that there might be some other considerations of the cooperation protocol, e.g., multiple-round conferencing [SSS10]. However, such protocols are either very complicated for the channel model under consideration or do not provide much intuition and are therefore beyond the scope of our study.

Appendices for Chapter 3

3.A Proof of Theorem 3.4.1

Let us define the right hand sides of the 8 inequalities (3.2)–(3.4), (3.6)–(3.8), and (3.9)–(3.10) as $I_1, \dots, I_8$, respectively. Note that by rate-splitting encoding we have $R_1 = R_{1c} + R_{1p}$, $R_2 = R_{2c} + R_{2p} + R_{20}$. By applying the Fourier-Motzkin elimination [DT97] to the aforementioned 8 inequalities (see details in Appendix 3.D), we obtain the following constraints on achievable rate pairs (R_1, R_2):

$$R_1 \le I_1 + I_7 \tag{3.56}$$

$$R_2 \le \min\{I_6, I_2 + I_5, I_2 + I_4 + I_8\} \tag{3.57}$$

$$R_1 + R_2 \le \min\{I_4 + I_3 + I_8, I_5 + I_3 + I_7\}. \tag{3.58}$$

It should be emphasized that thanks to the careful combination of SPC and SC, the number of inequalities during the elimination is minimized. The region $\mathcal{R}(\mathbf{s})$ is obtained by evaluating the constraints in (3.56)–(3.58) with Gaussian input distributions, some details are presented in the sequel.

Recall that $P_{W_1} = \alpha$, $P_{U_1} = \bar{\alpha}$, $P_{W_2} = \beta$, $P_S = \gamma$, $P_Q = \eta$. We first calculate the rate to send bin indices of $\hat{\mathbf{y}}_1$ to the receiver 2, which is given in (3.5):

$$\begin{aligned}\bar{\lambda} C_{12} &= I(\hat{Y}_1; \bar{Y}_1 | Y_2) = h(\hat{Y}_1 | Y_2) - h(\hat{Y}_1 | \bar{Y}_1, Y_2) \\ &= \frac{1}{2} \log\left(2\pi e \sigma^2_{\hat{Y}_1|Y_2}\right) - \frac{1}{2}\log\left(2\pi e N_q\right),\end{aligned} \tag{3.59}$$

where we used $\hat{Y}_1 = \bar{Y}_1 + Z_q$ and the independence of Gaussian random variables. The next step is calculating the conditional variance $\sigma^2_{\hat{Y}_1|Y_2}$. We have:

$$\begin{aligned}\hat{Y}_1 &= h_{11} U_1 + h_{21} X_2 + \theta h_{21} W_2 + Z_1 + Z_q \\ Y_2 &= h_{22} X_2 + Z_2.\end{aligned}$$

Let Σ denote the covariance of random variables, we have:

$$\begin{aligned}\Sigma_{\hat{Y}_1} &= h_{11}^2 P_{U_1} + h_{21}^2 P_2 + (2\theta + \theta^2) h_{21}^2 P_{W_2} + 1 + N_q \\ &= \bar{\alpha}\mathsf{SNR}_1 + (1 + 2\theta\beta + \theta^2\beta)\mathsf{INR} + 1 + N_q \\ \Sigma_{\hat{Y}_1 Y_2} &= h_{21} h_{22} P_2 + \theta h_{21} h_{22} P_{W_2} \\ &= h_{21} h_{22} (1 + \theta\beta) \\ \Sigma_{Y_2} &= h_{22}^2 P_2 + 1 \\ &= \mathsf{SNR}_2 + 1.\end{aligned}$$

As a result,

$$\begin{aligned}\sigma^2_{\hat{Y}_1|Y_2} &= \Sigma_{\hat{Y}_1} - \Sigma_{\hat{Y}_1 Y_2} \Sigma_{Y_2}^{-1} \Sigma_{Y_2 \hat{Y}_1} \\ &= 1 + N_q + \bar{\alpha}\mathsf{SNR}_1 + \frac{(1 + 2\theta\beta + \theta^2\beta)\mathsf{INR} + \theta^2\beta\bar{\beta}\mathsf{SNR}_2\mathsf{INR}}{1 + \mathsf{SNR}_2}.\end{aligned} \tag{3.60}$$

Inserting (3.60) into (3.59) and solve for N_q we obtain N_q as given in (3.17).

Following the same routine and using N_q, we can calculate the mutual information terms $I_1, \dots, I_8$ as follows. Keep in mind that $Y_1 = h_{11}U_1 + h_{11}W_1 + h_{21}W_2 + h_{21}S + h_{21}Q + Z_1$ we have

$$\begin{aligned} I_1 &= I(W_1; Y_1 | W_2) \\ &= h(Y_1|W_2) - h(Y_1|W_2, W_2) \\ &= \frac{1}{2}\log\left(1 + \frac{h_{11}^2 P_{W_1}}{1 + h_{11}^2 P_{U_1} + h_{21}^2(P_S + P_Q)}\right) \\ &= \frac{1}{2}\log\left(1 + \frac{\alpha\mathsf{SNR}_1}{1 + \bar{\alpha}\mathsf{SNR}_1 + (\gamma + \eta)\mathsf{INR}}\right) \\ &= \frac{1}{2}\log\left(1 + \frac{\alpha\mathsf{SNR}_1}{1 + \bar{\alpha}\mathsf{SNR}_1 + \bar{\beta}\mathsf{INR}}\right). \end{aligned} \tag{3.61}$$

In a similar manner, we obtain

$$\begin{aligned} I_2 &= I(W_2; Y_1|W_1) \\ &= \frac{1}{2}\log\left(1 + \frac{\beta\mathsf{INR}}{1 + \bar{\alpha}\mathsf{SNR}_1 + \bar{\beta}\mathsf{INR}}\right) \end{aligned} \tag{3.62}$$

$$\begin{aligned} I_3 &= I(W_1, W_2; Y_1) \\ &= \frac{1}{2}\log\left(1 + \frac{\alpha\mathsf{SNR}_1 + \beta\mathsf{INR}}{1 + \bar{\alpha}\mathsf{SNR}_1 + \bar{\beta}\mathsf{INR}}\right) \end{aligned} \tag{3.63}$$

$$\begin{aligned} I_4 &= I(X_2; Y_2, \hat{Y}_1 | W_2, V_2) \\ &= \frac{1}{2}\log(1 + \eta\mathsf{SNR}_2) + \frac{1}{2}\log\left(1 + \frac{\eta\mathsf{INR}}{(1 + \eta\mathsf{SNR}_2)(1 + N_q + \bar{\alpha}\mathsf{SNR}_1)}\right) \end{aligned} \tag{3.64}$$

$$\begin{aligned} I_5 &= I(V_2, X_2; Y_2, \hat{Y}_1 | W_2) \\ &= \frac{1}{2}\log\left(1 + \bar{\beta}\mathsf{SNR}_2\right) + \frac{1}{2}\log\left(1 + \frac{\bar{\beta}\mathsf{INR}}{(1 + \bar{\beta}\mathsf{SNR}_2)(1 + N_q + \bar{\alpha}\mathsf{SNR}_1)}\right) \end{aligned} \tag{3.65}$$

$$\begin{aligned} I_6 &= I(W_2, V_2, X_2; Y_2, \hat{Y}_1) + \lambda C_{12} \\ &= \frac{1}{2}\log(1 + \mathsf{SNR}_2) + \frac{1}{2}\log\left(1 + \frac{(1 + 2\theta\beta + \theta^2\beta)\mathsf{INR} + \theta^2\beta\bar{\beta}\mathsf{SNR}_2\mathsf{INR}}{(1 + \mathsf{SNR}_2)(1 + N_q + \bar{\alpha}\mathsf{SNR}_1)}\right) + \lambda C_{12} \end{aligned} \tag{3.66}$$

$$\begin{aligned} I_7 &= I(U_1; Y_1 | W_1, W_2, V_2) \\ &= \frac{1}{2}\log\left(1 + \frac{\bar{\alpha}\mathsf{SNR}_1}{\eta\mathsf{INR} + 1}\right) \end{aligned} \tag{3.67}$$

$$\begin{aligned} I_8 &= I(U_1, V_2; Y_1 | W_1, W_2) + C_{21} \\ &= \frac{1}{2}\log\left(1 + \frac{\bar{\alpha}\mathsf{SNR}_1 + \gamma\mathsf{INR}}{\eta\mathsf{INR} + 1}\right) + C_{21}. \end{aligned} \tag{3.68}$$

3.B Proof of Theorem 3.4.2

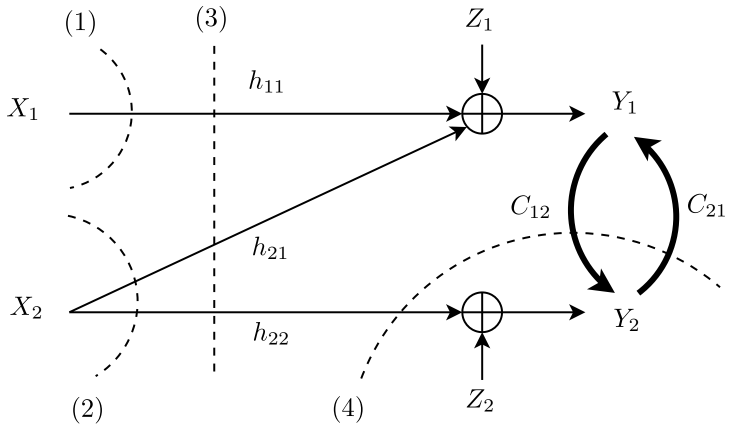

Figure 3.6: Cut sets for the AIC-RC.

The proof follows standard bounding techniques for the interference channel, namely cut-set bound and genie-aided bound [ETW08, AV09]. We first apply the cut-set bound [CT06, Theorem 15.10.1] to our channel model, see Fig. 3.6. As such any achievable rates pair (R_1, R_2) must satisfy the following constraints, where the maximum are taken over joint probability distributions $p(x_1, x_2)$:

- For cut (1): $R_1 \leq \max I(X_1; Y_1|X_2)$.
- For cut (2): $R_2 \leq \max I(X_2; Y_1, Y_2|X_1)$.
- For cut (3): $R_1 + R_2 \leq \max I(X_1, X_2; Y_1, Y_2) = \max h(Y_1, Y_2) - h(Z_1, Z_2)$.
- For cut (4): $R_2 \leq \max I(X_2; Y_2) + C_{12}$.

Note that these bounds can be alternatively derived using Fano's inequality as done in [WT11a]. Using the chain rule of mutual information, the independence of noise terms, and the fact that Gaussian distribution maximizes conditional differential entropy given covariance constraints, see e.g. [AV09], we can show that all mutual information terms in the 4 cuts are maximized by independent Gaussian random variables $X_1 \sim \mathcal{N}(0, P_1)$ and $X_2 \sim \mathcal{N}(0, P_2)$. The resulting bounds are:

$$R_1 \leq \mathcal{C}(\mathsf{SNR}_1) \tag{3.69}$$
$$R_2 \leq \mathcal{C}(\mathsf{SNR}_2 + \mathsf{INR}) \tag{3.70}$$
$$R_1 + R_2 \leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{SNR}_2 + \mathsf{INR} + \mathsf{SNR}_1\mathsf{SNR}_2) \tag{3.71}$$
$$R_2 \leq \mathcal{C}(\mathsf{SNR}_2) + C_{12}. \tag{3.72}$$

We observe that the above bounds do not depend on C_{21}. In the following we derive a genie-aided-type bound on the sum-rate that depends on C_{21}. The idea is to give Y_1^n to Rx2. Let V_{21}^n and V_{12}^n be output sequences of the cooperation links at Rx1 and Rx2, respectively. By Fano's inequality we have

$$\begin{aligned}
&n(R_1+R_2-\epsilon_n)\\
&\le I(X_1^n;Y_1^n,V_{21}^n)+I(X_2^n;Y_2^n,V_{12}^n)\\
&\le I(X_1^n;Y_1^n)+I(X_1^n;V_{21}^n|Y_1^n)+I(X_2^n;Y_2^n,V_{12}^n,Y_1^n)\\
&\le I(X_1^n;Y_1^n)+H(V_{21}^n)+I(X_2^n;Y_2^n,Y_1^n) &(3.73)\\
&\le I(X_1^n;Y_1^n)+nC_{21}+I(X_2^n;Y_2^n,Y_1^n|X_1^n) &(3.74)\\
&=h(Y_1^n)-h(Y_1^n|X_1^n)+nC_{21}+h(Y_2^n,Y_1^n|X_1^n)-h(Y_2^n,Y_1^n|X_1^n,X_2^n)\\
&=h(Y_1^n)+nC_{21}+h(Y_2^n|Y_1^n,X_1^n)-h(Z_1^n,Z_2^n)\\
&=h(Y_1^n)+nC_{21}+h(h_{22}X_2^n+Z_2^n|h_{21}X_2^n+Z_1^n)-2h(Z_1^n), &(3.75)
\end{aligned}$$

where $\epsilon_n \to 0$ as $n\to\infty$, (3.73) is due to Markov chain $V_{12}^n-(Y_1^n,Y_2^n)-X_2^n$ and $I(X_1^n;V_{21}^n|Y_1^n)\le H(V_{21}^n)$. (3.74) follows because X_1^n and X_2^n are independent and that conditioning reduces entropy. It was shown in [AV09] that the right hand side of (3.75) is maximized by i.i.d. Gaussian inputs X_{1i} and X_{2i}, $i=1,\dots,n$. Therefore the conditional variance of $h_{22}X_2+Z_2$ given $h_{21}X_2+Z_1$ can be readily computed, which leads to the following bound

$$\begin{aligned}
&n(R_1+R_2-\epsilon_n)\\
&\le \frac{n}{2}\log\left(2\pi e(h_{11}^2+h_{21}^2+1)\right)+nC_{21}+\frac{n}{2}\log\left(2\pi e\left(1+\frac{h_{22}^2}{h_{21}^2+1}\right)\right)-n\log(2\pi e)\\
&=n\mathcal{C}\left(\mathsf{SNR}_1+\mathsf{INR}\right)+n\mathcal{C}\left(\frac{\mathsf{SNR}_2}{1+\mathsf{INR}}\right)+nC_{21}.
\end{aligned}$$

Divide both sides by n and let $n\to\infty$ we obtain $R_1+R_2\le\mathcal{C}\left(\mathsf{SNR}_1+\mathsf{INR}\right)+\mathcal{C}\left(\frac{\mathsf{SNR}_2}{1+\mathsf{INR}}\right)+C_{21}$. Combining this constraint with (3.69)–(3.72) we complete the proof. □

3.C Proof of Corollary 3.5.3

Recall that we are assuming high SNR while the channel gains are fixed. Thus, let $\mathsf{INR}=k_1\mathsf{SNR}_1=k_2\mathsf{SNR}_2$ for positive real numbers k_1 and k_2. In the limit of high SNR and INR we have:

$$\begin{aligned}
C_{sum}-C_{sum}^0 &\le R_{up2}-C_{sum}^0\\
&=\mathcal{C}(\mathsf{SNR}_1+\mathsf{INR})+\mathcal{C}\left(\frac{1}{k_2}\right)+C_{21}-\mathcal{C}\left(\mathsf{SNR}_2\right)-\mathcal{C}\left(\frac{1}{k_1}\right)\\
&=\frac{1}{2}\log\left(\frac{1+\left(\frac{1}{k_1}+1\right)\mathsf{INR}}{1+\frac{1}{k_2}\mathsf{INR}}\right)+\frac{1}{2}\log\left(\frac{1+\frac{1}{k_2}}{1+\frac{1}{k_1}}\right)+C_{21}
\end{aligned}$$

$$\to \mathcal{C}(k_2) + C_{21} \text{ as } \mathsf{INR} \to \infty$$
$$:= c + C_{21},$$

which depends on the channel SNR only through the ratio between INR and SNR_2. Further, since $\mathsf{INR} < \mathsf{SNR}_2$ we have $k_2 < 1$ and $c < 0.5$. □

3.D Fourier-Motzkin Elimination

Let us define the right hand sides of the 8 inequalities (3.2)–(3.4), (3.6)–(3.8), and (3.9)–(3.10) as $I_1, \dots, I_8$, respectively. We have

$$R_{1c} \le I_1 \tag{3.76}$$
$$R_{2c} \le I_2 \tag{3.77}$$
$$R_{1c} + R_{2c} \le I_3 \tag{3.78}$$
$$R_{2p} \le I_4 \tag{3.79}$$
$$R_{2p} + R_{20} \le I_5 \tag{3.80}$$
$$R_{2c} + R_{20} + R_{2p} \le I_6 \tag{3.81}$$
$$R_{1p} \le I_7 \tag{3.82}$$
$$R_{1p} + R_{20} \le I_8. \tag{3.83}$$

Replacing R_{1p} by $R_1 - R_{1c}$, R_{2p} by $R_2 - R_{2c} - R_{20}$, and imposing the non-negativity on the partial rates we obtain the following inequalities:

$$R_{1c} \le I_1 \tag{3.84}$$
$$R_{2c} \le I_2 \tag{3.85}$$
$$R_{1c} + R_{2c} \le I_3 \tag{3.86}$$
$$R_2 - R_{2c} - R_{20} \le I_4 \tag{3.87}$$
$$R_2 - R_{2c} \le I_5 \tag{3.88}$$
$$R_2 \le I_6 \tag{3.89}$$
$$R_1 - R_{1c} \le I_7 \tag{3.90}$$
$$R_1 - R_{1c} + R_{20} \le I_8 \tag{3.91}$$
$$-R_{1c} \le 0 \tag{3.92}$$
$$-R_{2c} \le 0 \tag{3.93}$$
$$-R_{20} \le 0 \tag{3.94}$$
$$-R_1 + R_{1c} \le 0 \tag{3.95}$$
$$-R_2 + R_{2c} + R_{20} \le 0. \tag{3.96}$$

Step 1: collecting the inequalities that contain R_{20} we have

$$\begin{aligned} R_2 - R_{2c} - R_{20} &\leq I_4 \\ -R_{20} &\leq 0 \\ R_1 - R_{1c} + R_{20} &\leq I_8 \\ -R_2 + R_{2c} + R_{20} &\leq 0. \end{aligned}$$

By summing up each inequality containing R_{20} with each inequality containing $-R_{20}$ we can eliminate R_{20} to obtain

$$R_1 + R_2 - R_{1c} - R_{2c} \leq I_4 + I_8 \tag{3.97}$$
$$0 \leq I_4 \tag{3.98}$$
$$R_1 - R_{1c} \leq I_8 \tag{3.99}$$
$$-R_2 + R_{2c} \leq 0. \tag{3.100}$$

Removing the redundant inequality (3.98) and combining with those inequalities in (3.84)–(3.96) that do not contain R_{20} we have

$$R_{1c} \leq I_1 \tag{3.101}$$
$$R_{2c} \leq I_2 \tag{3.102}$$
$$R_{1c} + R_{2c} \leq I_3 \tag{3.103}$$
$$R_2 - R_{2c} \leq I_5 \tag{3.104}$$
$$R_2 \leq I_6 \tag{3.105}$$
$$R_1 - R_{1c} \leq I_7 \tag{3.106}$$
$$-R_{1c} \leq 0 \tag{3.107}$$
$$-R_{2c} \leq 0 \tag{3.108}$$
$$-R_1 + R_{1c} \leq 0 \tag{3.109}$$
$$R_1 + R_2 - R_{1c} - R_{2c} \leq I_4 + I_8 \tag{3.110}$$
$$R_1 - R_{1c} \leq I_8 \tag{3.111}$$
$$-R_2 + R_{2c} \leq 0. \tag{3.112}$$

Step 2: Following the same procedure, we collect the inequalities that contain R_{2c} and eliminate R_{2c} to obtain

$$R_1 - R_{1c} \leq I_4 + I_8 \tag{3.113}$$
$$R_1 + R_2 - R_{1c} \leq I_2 + I_4 + I_8 \tag{3.114}$$
$$R_1 + R_2 \leq I_3 + I_4 + I_8 \tag{3.115}$$
$$0 \leq I_5 \tag{3.116}$$
$$R_2 \leq I_2 + I_5 \tag{3.117}$$
$$R_2 + R_{1c} \leq I_3 + I_5 \tag{3.118}$$

$$-R_2 \le 0 \tag{3.119}$$
$$0 \le I_2 \tag{3.120}$$
$$R_{1c} \le I_3. \tag{3.121}$$

Removing the redundant inequalities (3.116) and (3.120), and combining with those inequalities in (3.101)–(3.112) that do not contain R_{2c} we have

$$R_{1c} \le I_1 \tag{3.122}$$
$$R_2 \le I_6 \tag{3.123}$$
$$R_1 - R_{1c} \le I_7 \tag{3.124}$$
$$-R_{1c} \le 0 \tag{3.125}$$
$$-R_1 + R_{1c} \le 0 \tag{3.126}$$
$$R_1 - R_{1c} \le I_8 \tag{3.127}$$
$$R_1 - R_{1c} \le I_4 + I_8 \tag{3.128}$$
$$R_1 + R_2 - R_{1c} \le I_2 + I_4 + I_8 \tag{3.129}$$
$$R_1 + R_2 \le I_3 + I_4 + I_8 \tag{3.130}$$
$$R_2 \le I_2 + I_5 \tag{3.131}$$
$$R_2 + R_{1c} \le I_3 + I_5 \tag{3.132}$$
$$-R_2 \le 0 \tag{3.133}$$
$$R_{1c} \le I_3. \tag{3.134}$$

Observe that

- (3.130) is redundant due to (3.127) and the fact that $I_4 \ge 0$.
- (3.127) is redundant due to (3.124) and the fact that $I_7 \le I_8$.
- (3.134) is redundant due to (3.122) and the fact that $I_1 \le I_3$.

Hence we have

$$R_{1c} \le I_1 \tag{3.135}$$
$$R_2 \le I_6 \tag{3.136}$$
$$R_1 - R_{1c} \le I_7 \tag{3.137}$$
$$-R_{1c} \le 0 \tag{3.138}$$
$$-R_1 + R_{1c} \le 0 \tag{3.139}$$
$$R_1 + R_2 - R_{1c} \le I_2 + I_4 + I_8 \tag{3.140}$$
$$R_1 + R_2 \le I_3 + I_4 + I_8 \tag{3.141}$$
$$R_2 \le I_2 + I_5 \tag{3.142}$$
$$R_2 + R_{1c} \le I_3 + I_5 \tag{3.143}$$
$$-R_2 \le 0. \tag{3.144}$$

Step 3: Next, by summing up each inequality containing R_{1c} with each inequality containing $-R_{1c}$ we can eliminate R_{1c} and end up with the following inequalities

$$R_1 + 2R_2 \leq I_2 + I_4 + I_8 + I_3 + I_5 \tag{3.145}$$
$$R_1 + R_2 \leq I_2 + I_4 + I_8 + I_1 \tag{3.146}$$
$$R_2 \leq I_2 + I_4 + I_8 \tag{3.147}$$
$$R_1 + R_2 \leq I_3 + I_5 + I_7 \tag{3.148}$$
$$R_1 \leq I_1 + I_7 \tag{3.149}$$
$$0 \leq I_7 \tag{3.150}$$
$$R_2 \leq I_3 + I_5 \tag{3.151}$$
$$0 \leq I_1 \tag{3.152}$$
$$-R_1 \leq 0. \tag{3.153}$$

Removing redundant inequalities (3.150) and (3.152), and combining with (3.136), (3.141), (3.142), (3.144) we have

$$-R_1 \leq 0 \tag{3.154}$$
$$R_1 \leq I_1 + I_7 \tag{3.155}$$
$$-R_2 \leq 0 \tag{3.156}$$
$$R_2 \leq I_6 \tag{3.157}$$
$$R_2 \leq I_2 + I_5 \tag{3.158}$$
$$R_2 \leq I_3 + I_5 \tag{3.159}$$
$$R_2 \leq I_2 + I_4 + I_8 \tag{3.160}$$
$$R_1 + R_2 \leq I_3 + I_5 + I_7 \tag{3.161}$$
$$R_1 + R_2 \leq I_3 + I_4 + I_8 \tag{3.162}$$
$$R_1 + R_2 \leq I_2 + I_4 + I_8 + I_1 \tag{3.163}$$
$$R_1 + 2R_2 \leq I_2 + I_4 + I_8 + I_3 + I_5. \tag{3.164}$$

Note that

- (3.159) is redundant due to (3.158) and the fact that $I_2 \leq I_3$.
- (3.163) is redundant due to (3.162) and the fact that $I_1 + I_2 \geq I_3$.
- (3.164) is redundant due to (3.158) and (3.162).

Therefore, the final achievable rates are characterized by the following inequalities

$$R_1 \geq 0 \tag{3.165}$$
$$R_2 \geq 0 \tag{3.166}$$

$$R_1 \le I_1 + I_7 \tag{3.167}$$
$$R_2 \le I_6 \tag{3.168}$$
$$R_2 \le I_2 + I_5 \tag{3.169}$$
$$R_2 \le I_2 + I_4 + I_8 \tag{3.170}$$
$$R_1 + R_2 \le I_3 + I_5 + I_7 \tag{3.171}$$
$$R_1 + R_2 \le I_3 + I_4 + I_8. \tag{3.172}$$

Chapter 4

Interference Channels With Cooperating Receivers: Symmetric Case

In Chapter 3 we have explored a mechanism for receiver cooperation in asymmetric interference channels. The coding scheme was built upon classical relaying protocols such as decode-forward and compress-forward. In contrast, the present chapter will focus on studying cooperation in the full interference setup, namely the interference channel with *both* crossover links. A new coding scheme is proposed for the discrete memoryless channel. Similar to Chapter 3, the scheme includes common-private rate-splitting and superposition coding at the transmitters. As opposed to using a combination of CF and DF as in Chapter 3, noisy network coding and non-unique joint decoding are employed at the receivers. As an example, the general achievable region is extended to the Gaussian interference channel whose receivers cooperate through noise-free and finite-capacity orthogonal links. We show that the resulting inner bound is the same as the one-round quantize-bin-forward inner bound established by Wang and Tse in [WT11a]. This result indirectly shows that noisy network coding achieves within 1 bit/s/Hz of the capacity region when the interference is strong, and achieves to within 1 bit/s/Hz of the capacity region of the Gaussian compound multiple access channel with conferencing decoders, regardless of channel parameters. The results in this chapter are among the first in the field demonstrating the approximate optimality of noisy network coding for a multi-unicast problem. Our results are also among the first to show the equivalence, in terms of achievable rates, between noisy network coding and some other coding schemes which are based on explicit binning or hashing.

4.1 Introduction

As introduced in Section 2.6 and demonstrated in Chapter 3, allowing transmitters and/or receivers to cooperate is an important technique to achieve interference mitigation. In the center of this technique are the relaying protocols used by transmitters and/or receivers. Similar to what have been shown in Chapter 3, early works in this topic such as [HM06, NJGM07, MYK07, SGP+09, SSPS09, ZY12] are primarily built on different combinations of the classical decode-forward and compress-forward protocols. Recent advances in the research of relay networks, however, have brought new possibilities. Notably, based on insight obtained from the linear deterministic models, the references [PV11b, PV11a, WT11b, WT11a] have succeeded in characterizing the sum-capacity or capacity region of some Gaussian interference channels with cooperation up to a constant number of bits. In a different approach to network information theory, Lim *et al.* have come up with the concept of *noisy network coding* [LKEC11] (NNC), which combines existing coding techniques in a novel way to yield single-letter characterization of achievable rates in multi-source multi-destination discrete memoryless networks. NNC has been shown to achieve better performance than previously known schemes in certain scenarios [LKEC11], and promises to be a powerful tool in dealing with networks with multiple nodes and multiple messages. As noted in [LKEC11], NNC effectively transforms a multi-node network into an interference channel with some rate penalty. It is therefore natural to develop a new coding strategy that covers both the coding techniques for the traditional interference channel and NNC. In this chapter we realize such a strategy for a fundamental network, the interference channel with receiver cooperation. To quantify the goodness of our coding strategy, we evaluate its performance for a class of Gaussian channels and compare it with the best known result.

The main results of this chapter can be summarized as follows: We generalize the NNC approach to incorporate the traditional coding techniques for the interference channel, namely the common-private rate-splitting [Car78, HK81], and the non-unique decoding of common messages [CMGE08]. To the best of our knowledge our coding scheme produces the first inner bound for the capacity region of the discrete memoryless interference channel (DM-IC) with receiver cooperation. The inner bound enjoys the generality of noisy network coding in dealing with multiple-node networks. Accordingly we expect that the extended coding strategy will be useful for related problems as well. As an example, we use our general region to derive an inner bound on the capacity region of the Gaussian IC with rate-limited conferencing decoders. We then show that this inner bound is identical to the one-round inner bound by Wang and Tse in [WT11a], and it therefore achieves one-bit gap to the capacity region when the interference is strong and one-bit gap to the capacity region of the Gaussian compound multiple access channel with conferencing decoder (CMAC-CD), which was studied in [SGP+09]. We also outline possible extensions of our work.

The rest of the chapter is organized as follows: Section 4.2 introduces the discrete memoryless network that we are working on, Section 4.3 states and proves an inner bound for the capacity region. Section 4.4 studies the Gaussian interference channel whose receivers cooperate by means of rate-limited orthogonal links, where it is shown that the new inner bound is the same as the inner bound achieved by quantize-bin-forward in [WT11a]. Final remarks and open problems are discussed in Section 4.5. Some proofs are gathered in the appendices of this chapter.

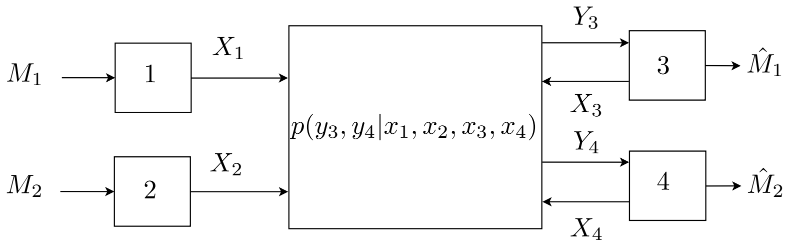

Figure 4.1: Interference channel with receiver cooperation.

Definition

In this chapter we will make use of the notion of the gap between an achievable rate region and an outer bound or the capacity region of certain channels, whose definition is given below.

Definition 4.1 (k-bit gap)**.** Let $\mathcal{S}_1$ and $\mathcal{S}_2$ denote two sets of nonnegative rate pairs (R_1, R_2). The set $\mathcal{S}_1$ is said to be within k bits of the set $\mathcal{S}_2$ if for any $(R_1, R_2) \in \mathcal{S}_2$ we have $((R_1 - k)^+, (R_2 - k)^+) \in \mathcal{S}_1$.

4.2 Problem Formulation

Consider a discrete memoryless (DM) 4-node network as depicted in Fig. 4.1. This network is characterized by a Cartesian product of finite input alphabets $\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3 \times \mathcal{X}_4$, finite output alphabets $\mathcal{Y}_3 \times \mathcal{Y}_4$, and a conditional probability mass function $p(y_3, y_4|x_1, x_2, x_3, x_4)$. Node 1 wants to send a message to node 3 and node 2 wants to send a message to node 4. The messages are denoted by M_1 and M_2, independent of each other and uniformly distributed over their respective message sets $\mathcal{M}_1$ and $\mathcal{M}_2$. Node 3 and node 4 can exchange information to help each other decode their desired messages. A $(2^{nR_1}, 2^{nR_2}, n)$ block code for this network consists of

- message sets $\mathcal{M}_k = [1 : 2^{nR_k}]$, $k \in \{1, 2\}$;
- encoding functions $\Psi_k : \mathcal{M}_k \to \mathcal{X}_k^n$, $k \in \{1, 2\}$, that assign an input sequence X_k^n for message M_k; and encoding functions $\Psi_{k,i} : \mathcal{Y}_k^{i-1} \to \mathcal{X}_k$, $k \in \{3, 4\}$, that assign an input symbol X_{ki}, $i \in [1 : n]$, for Y_k^{i-1}.
- decoding functions $\Phi_k : \mathcal{Y}_k^n \to \mathcal{M}_{k-2}$, $k \in \{3, 4\}$, that put out an estimate $\hat{M}_{k-2}$ for each received sequence Y_k^n.

The average probability of error is defined as

$$P_e^{(n)} = \mathsf{P}\{(\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\}. \tag{4.1}$$

Achievable rate pairs and capacity region of the network are defined as in Section 2.2. We call the network depicted in Fig. 4.1 the *interference channel with receiver cooperation* (IC-RC).

4.3 Inner Bound for the Interference Channel With Receiver Cooperation

In this section we state and prove an achievable rate region for the discrete memoryless interference channel with receiver cooperation. The main ingredients for our coding strategy are: i) common-private rate-splitting [Car78, HK81], ii) noisy network coding [LKEC11] iii) non-uniquely joint decoding of common messages [CMGE08]. First we state the main result.

Theorem 4.3.1. *The set $\mathcal{R}^{IC-RC}$ of nonnegative rate pairs (R_1, R_2) given below is achievable for the discrete memoryless interference channel with receiver cooperation:*

$$R_1 < \min\{I_1 + (I_0 - r_4)^+, I_2\} \tag{4.2}$$

$$R_2 < \min\{I'_1 + (I'_0 - r_3)^+, I'_2\} \tag{4.3}$$

$$R_1 + R_2 < \min\{I_5 + (I_0 - r_4)^+, I_6\} + \min\{I'_3 + (I'_0 - r_3)^+, I'_4\} \tag{4.4}$$

$$R_1 + R_2 < \min\{I_3 + (I_0 - r_4)^+, I_4\} + \min\{I'_5 + (I'_0 - r_3)^+, I'_6\} \tag{4.5}$$

$$R_1 + R_2 < \min\{I_7 + (I_0 - r_4)^+, I_8\} + \min\{I'_7 + (I'_0 - r_3)^+, I'_8\} \tag{4.6}$$

$$\begin{aligned} 2R_1 + R_2 < {} & \min\{I_5 + (I_0 - r_4)^+, I_6\} + \min\{I_3 + (I_0 - r_4)^+, I_4\} \\ & + \min\{I'_7 + (I'_0 - r_3)^+, I'_8\} \end{aligned} \tag{4.7}$$

$$\begin{aligned} R_1 + 2R_2 < {} & \min\{I_7 + (I_0 - r_4)^+, I_8\} + \min\{I'_5 + (I'_0 - r_3)^+, I'_6\} \\ & + \min\{I'_3 + (I'_0 - r_3)^+, I'_4\}, \end{aligned} \tag{4.8}$$

for some distribution of the form

$$\begin{aligned} & p(q)p(u_1, x_1|q)p(u_2, x_2|q)p(x_3|q)p(x_4|q)p(\hat{y}_3|x_3, y_3, q)p(\hat{y}_4|x_4, y_4, q) \\ & \qquad \cdot p(y_3, y_4|x_1, x_2, x_3, x_4), \end{aligned} \tag{4.9}$$

where $p(y_3, y_4|x_1, x_2, x_3, x_4)$ is set by the channel, and

$$r_3 = I(Y_3; \hat{Y}_3|X_3, Q) \tag{4.10}$$

$$r_4 = I(Y_4; \hat{Y}_4|X_4, Q) \tag{4.11}$$

$$I_0 = I(X_4, \hat{Y}_4; X_1, U_2, X_3, Y_3|Q) \tag{4.12}$$

$$I_1 = I(X_1; Y_3|U_2, X_3, Q) \tag{4.13}$$

$$I_2 = I(X_1; \hat{Y}_4, Y_3|U_2, X_3, X_4, Q) \tag{4.14}$$

$$I_3 = I(X_1, U_2; Y_3|X_3, Q) \tag{4.15}$$

$$I_4 = I(X_1, U_2; \hat{Y}_4, Y_3 | X_3, X_4, Q) \tag{4.16}$$
$$I_5 = I(X_1; Y_3 | U_1, U_2, X_3, Q) \tag{4.17}$$
$$I_6 = I(X_1; \hat{Y}_4, Y_3 | U_1, U_2, X_3, X_4, Q) \tag{4.18}$$
$$I_7 = I(X_1, U_2; Y_3 | U_1, X_3, Q) \tag{4.19}$$
$$I_8 = I(X_1, U_2; \hat{Y}_4, Y_3 | U_1, X_3, X_4, Q), \tag{4.20}$$

and I'_k's are obtained from I_k's, $k \in [0:8]$, by swapping indices $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ in the mutual information terms. Q is drawn from some finite alphabet $\mathcal{Q}$.

Remark 4.1. The achievable rate region presented above has a similar form as the new characterization of the Han-Kobayashi region [CMGE08] for the interference channel without cooperation, namely the set of 7 constraints on the rates. It can be readily shown that by setting $X_3 = X_4 = \hat{Y}_3 = \hat{Y}_4 = \emptyset$ the region $\mathcal{R}^{IC-RC}$ reduces to the region in [CMGE08].

Remark 4.2. It is also straightforward to show that if one sets $U_1 = X_1, U_2 = X_2$, i.e., the whole message is set to be the common, one will recover the results in [LKEC11, Theorem 2] applying to the current channel model. On the other hand, if one sets $U_1 = \emptyset, U_2 = \emptyset$, i.e., the whole message is set to be the private, one will recover the results in [LKEC11, Theorem 3] for the current channel model.

The rest of this section is dedicated to the proof of Theorem 4.3.1.

Proof. The transmission of each message takes place in b blocks. We use subscript j to index the blocks. We consider time-sharing random variable $Q =$ const, the proof for general Q can be done by the coded time-sharing technique in [EK11, Sec. 4.5.3]. For a message pair (m_1, m_2) that nodes 1 and 2 want to send, split the messages as follows: $m_1 = (m_{1c}, m_{1p})$, $m_2 = (m_{2c}, m_{2p})$. Fix the distribution

$$p(u_1, x_1)p(u_2, x_2)p(x_3)p(x_4)p(\hat{y}_3|x_3, y_3)p(\hat{y}_4|x_4, y_4).$$

Codebook generation. For each $j \in [1:b]$ and for $k \in \{1,2\}$ randomly and independently generate $2^{nbR_{kc}}$ sequences $\mathbf{u}_{kj}(m_{kc})$, $m_{kc} \in [1:2^{nbR_{kc}}]$, each according to the distribution $\prod_{i=1}^{n} p_{U_k}(u_{k,(j-1)n+i})$. For each $\mathbf{u}_{kj}(m_{kc})$, randomly and conditionally independently generate $2^{nbR_{kp}}$ sequences $\mathbf{x}_{kj}(m_{kp}, m_{kc})$, $m_{kp} \in [1:2^{nbR_{kp}}]$, each according to

$$\prod_{i=1}^{n} p_{X_k|U_k}(x_{k,(j-1)n+i}|u_{k,(j-1)n+i}(m_{kc})).$$

For each $j \in [1:b]$ and for $t \in \{3,4\}$ randomly and independently generate $2^{n\hat{R}_t}$ sequences $\mathbf{x}_{tj}(l_{t,j-1})$, $l_{t,j-1} \in [1:2^{n\hat{R}_t}]$, each according to $\prod_{i=1}^{n} p_{X_t}(x_{t,(j-1)n+i})$; for each $\mathbf{x}_{tj}(l_{t,j-1})$, randomly and conditionally independently generate $2^{n\hat{R}_t}$ sequences $\hat{\mathbf{y}}_{tj}(l_{t,j}|l_{t,j-1})$, $l_{t,j} \in [1:2^{n\hat{R}_t}]$, each according to

$$\prod_{i=1}^{n} p_{\hat{Y}_t|X_t}(\hat{y}_{t,(j-1)n+i}|x_{t,(j-1)n+i}(l_{t,j-1})).$$

The above procedure defines the codebook for block j:

$$\mathcal{C}_j = \{\mathbf{u}_{1j}(m_{1c}), \mathbf{x}_{1j}(m_{1p}, m_{1c}), \mathbf{u}_{2j}(m_{2c}), \mathbf{x}_{2j}(m_{2p}, m_{2c}),\\ \mathbf{x}_{3j}(l_{3,j-1}), \hat{\mathbf{y}}_{3j}(l_{3,j}|l_{3,j-1}), \mathbf{x}_{4j}(l_{4,j-1}), \hat{\mathbf{y}}_{4j}(l_{4,j}|l_{4,j-1})\}.$$

Encoding. To send messages $m_1 = (m_{1c}, m_{1p})$, $m_2 = (m_{2c}, m_{2p})$ in block j node 1 sends $\mathbf{x}_{1j}(m_{1p}, m_{1c})$, node 2 sends $\mathbf{x}_{2j}(m_{2p}, m_{2c})$. Upon receiving $\mathbf{y}_{kj}$ at the end of block j, node $k \in \{3, 4\}$ looks for an index $l_{k,j}$ such that

$$(\hat{\mathbf{y}}_{kj}(l_{kj}|l_{k,j-1}), \mathbf{x}_{kj}(l_{k,j-1}), \mathbf{y}_{kj}) \in \mathcal{T}_{\epsilon'}^{(n)} \tag{4.21}$$

with initialization $l_{k0} := 1$. If there is more than one such index, choose the smallest one. If there is no such index, choose an arbitrary index at random uniformly from $[1 : 2^{n\hat{R}_k}]$. Each node $k \in \{3, 4\}$ transmits $x_{kj}(l_{k,j-1})$ in block j.

Decoding. Let $\epsilon > \epsilon'$. At the end of block b, decoder 3 looks for a unique index pair $(\hat{m}_{1c}, \hat{m}_{1p})$ such that there exist some $\hat{m}_{2c} \in [1 : 2^{nbR_{2c}}]$, $\hat{l}_{4,j} \in [1 : 2^{n\hat{R}_4}]$, $\hat{l}_{3,j} = l_{3,j}$ satisfying

$$\Big(\mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{x}_{1j}(\hat{m}_{1p}, \hat{m}_{1c}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{x}_{3j}(\hat{l}_{3,j-1}), \hat{\mathbf{y}}_{3j}(\hat{l}_{3,j}|\hat{l}_{3,j-1}),\\ \mathbf{x}_{4j}(\hat{l}_{4,j-1}), \hat{\mathbf{y}}_{4j}(\hat{l}_{4,j}|\hat{l}_{4,j-1}), \mathbf{y}_{3j}\Big) \in \mathcal{T}_{\epsilon}^{(n)} \tag{4.22}$$

for all $j \in [1 : b]$.
Similarly for decoder 4.

Error analysis. We perform the analysis for node 3, the analysis for node 4 can be done similarly. Let $M_{1c}, M_{1p}, M_{2c}, M_{2p}$ denote the messages sent by nodes 1 and 2, and let $L_{3,j}$, $L_{4,j}$ denote the indices chosen by node 3 and node 4 in block $j \in [1 : b]$, respectively. The decoder of node 3 makes an error only if one of the following events occurs:

$$\mathcal{E}_{30} = \bigcup_{j=1}^{b} \Big\{ \big(\mathbf{X}_{3j}(L_{3,j-1}), \hat{\mathbf{Y}}_{3j}(l_{3,j}|L_{3,j-1}), \mathbf{Y}_{3j}\big) \notin \mathcal{T}_{\epsilon'}^{(n)} \text{ for all } l_{3,j} \in [1 : 2^{n\hat{R}_3}] \Big\}$$

$$\mathcal{E}_{40} = \bigcup_{j=1}^{b} \Big\{ \big(\mathbf{X}_{4j}(L_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|L_{4,j-1}), \mathbf{Y}_{4j}\big) \notin \mathcal{T}_{\epsilon'}^{(n)} \text{ for all } l_{4,j} \in [1 : 2^{n\hat{R}_4}] \Big\}$$

$$\mathcal{E}_{31} = \Big\{ \big(\mathbf{U}_{1j}(M_{1c}), \mathbf{X}_{1j}(M_{1p}, M_{1c}), \mathbf{U}_{2j}(M_{2c}), \mathbf{X}_{3j}(L_{3,j-1}), \hat{\mathbf{Y}}_{3j}(L_{3,j}|L_{3,j-1}),\\ \mathbf{X}_{4j}(L_{4,j-1}), \hat{\mathbf{Y}}_{4j}(L_{4,j}|L_{4,j-1}), \mathbf{Y}_{3j}\big) \notin \mathcal{T}_{\epsilon}^{(n)}, \text{ for some } j \in [1 : b] \Big\}$$

$$\mathcal{E}_{32} = \Big\{ \big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_{1p}, m_{1c}), \mathbf{U}_{2j}(m_{2c}), \mathbf{X}_{3j}(L_{3,j-1}), \hat{\mathbf{Y}}_{3j}(L_{3,j}|L_{3,j-1}),\\ \mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|l_{4,j-1}), \mathbf{Y}_{3j}\big) \in \mathcal{T}_{\epsilon}^{(n)}, \text{ for all } j \in [1 : b], \text{ for some}$$

$$(m_{1c}, m_{1p}) \neq (M_{1c}, M_{1p}), m_{2c} \in [1 : 2^{nbR_{2c}}], l_{4,j-1}, l_{4,j} \in [1 : 2^{n\hat{R}_4}]\Big\}.$$

The events $\mathcal{E}_{30}$ and $\mathcal{E}_{40}$ mean node 3 or 4 fails to perform the quantization, respectively. The event $\mathcal{E}_{31}$ means the correct codewords are not jointly typical with the received sequence at node 3 for some block $j \in [1 : b]$. The event $\mathcal{E}_{32}$ means some wrong codewords in the codebook of transmitter 1 satisfy the decoding rule at node 3 and therefore can be mistaken as the correct codewords. Hence the error probability can be bounded by

$$\mathsf{P}(\mathcal{E}_3) \leq \mathsf{P}(\mathcal{E}_{30} \cup \mathcal{E}_{40}) + \mathsf{P}(\mathcal{E}_{31} \cap \mathcal{E}_{30}^c \cap \mathcal{E}_{40}^c) + \mathsf{P}(\mathcal{E}_{32}). \tag{4.23}$$

By the covering lemma [EK11, Lemma 3.3] $\mathsf{P}(\mathcal{E}_{30}) \to 0$ and $\mathsf{P}(\mathcal{E}_{40}) \to 0$ as $n \to \infty$ if

$$\hat{R}_3 > I(Y_3; \hat{Y}_3|X_3) \text{ and } \hat{R}_4 > I(Y_4; \hat{Y}_4|X_4). \tag{4.24}$$

By the Markov lemma [EK11, Lemma 12.1] $\mathsf{P}(\mathcal{E}_{31} \cap \mathcal{E}_{30}^c \cap \mathcal{E}_{40}^c) \to 0$ as $n \to \infty$. What remains is bounding $\mathsf{P}(\mathcal{E}_{32})$ in (4.23). Let us define the following event:

$$\begin{aligned}\mathcal{E}_{3,m_{1c},m_{1p},m_{2c}} = \Big\{&\big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_{1p}, m_{1c}), \mathbf{U}_{2j}(m_{2c}), \mathbf{X}_{3j}(L_{3,j-1}),\\ &\hat{\mathbf{Y}}_{3j}(L_{3,j}|L_{3,j-1}), \mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|l_{4,j-1}), \mathbf{Y}_{3j}\big) \in \mathcal{T}_\epsilon^{(n)},\\ &\text{for all } j \in [1 : b], \text{ for some } l_{4,j-1}, l_{4,j} \in [1 : 2^{n\hat{R}_4}]\Big\}.\end{aligned}$$

Due to symmetry of codebooks and for notational brevity we can further assume that $(M_{1c}, M_{1p}, M_{2c}, M_{2p}) = (1, 1, 1, 1)$. Then we can split the event $\mathcal{E}_{32}$ as follows

$$\begin{aligned}\mathcal{E}_{32} &= \bigcup_{(m_{1c},m_{1p})\neq(1,1),m_{2c}} \mathcal{E}_{3,m_{1c},m_{1p},m_{2c}}\\ &= \mathcal{E}_3^1 \cup \mathcal{E}_3^2 \cup \mathcal{E}_3^3 \cup \mathcal{E}_3^4,\end{aligned}$$

where

$$\mathcal{E}_3^1 := \bigcup_{m_{1c}\neq 1, m_{1p}, m_{2c}=1} \mathcal{E}_{3,m_{1c},m_{1p},m_{2c}} \tag{4.25}$$

$$\mathcal{E}_3^2 := \bigcup_{m_{1c}\neq 1, m_{1p}, m_{2c}\neq 1} \mathcal{E}_{3,m_{1c},m_{1p},m_{2c}} \tag{4.26}$$

$$\mathcal{E}_3^3 := \bigcup_{m_{1c}=1, m_{1p}\neq 1, m_{2c}=1} \mathcal{E}_{3,m_{1c},m_{1p},m_{2c}} \tag{4.27}$$

$$\mathcal{E}_3^4 := \bigcup_{m_{1c}=1, m_{1p}\neq 1, m_{2c}\neq 1} \mathcal{E}_{3,m_{1c},m_{1p},m_{2c}}. \tag{4.28}$$

By the union of events bound we have

$$\mathsf{P}(\mathcal{E}_{32}) \leq \sum_{i=1}^{4} \mathsf{P}(\mathcal{E}_3^i). \tag{4.29}$$

Table 4.1: Error events. “x” stands for “$\neq 1$”, “-” stands for “don’t care”.

Case	m_{1c}	m_{1p}	m_{2c}	$l_{3,j-1}$	$l_{4,j-1}$
1.	x	-	1	1	x
2.	x	-	1	1	1
3.	x	-	x	1	x
4.	x	-	x	1	1
5.	1	x	1	1	x
6.	1	x	1	1	1
7.	1	x	x	1	x
8.	1	x	x	1	1

In the sequel we bound $\mathsf{P}(\mathcal{E}_3^1)$, other $\mathsf{P}(\mathcal{E}_3^i)$’s are bounded similarly.
Again, due to the symmetry of codebook generation and for notational simplicity we can further assume that $(\mathbf{L}_1, \ldots, \mathbf{L}_b) = (\mathbf{1}, \ldots, \mathbf{1})$, where $\mathbf{L}_j := (L_{3,j}, L_{4,j})$. For each $j \in [1:b]$, $m_1 = (m_{1c}, m_{1p}) \in [1:2^{nbR_{1c}}] \times [1:2^{nbR_{1p}}]$, $m_{2c} \in [1:2^{nbR_{2c}}]$, $l_{4,j}, l_{4,j-1} \in [1:2^{n\hat{R}_4}]$, and $l_{3,j-1}, l_{3,j} \in [1:2^{n\hat{R}_3}]$, define the event

$$\begin{aligned}\mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^j) :=& \Big\{ \big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_{1p}, m_{1c}), \mathbf{U}_{2j}(m_{2c}), \mathbf{X}_{3j}(1), \\ & \hat{\mathbf{Y}}_{3j}(1|1), \mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|l_{4,j-1}), \mathbf{Y}_{3j}\big) \in \mathcal{T}_\epsilon^{(n)} \Big\}.\end{aligned} \tag{4.30}$$

To support the analysis we form Table 4.1, which shows combinations of indices that result in certain error events.
Bounding $\mathsf{P}(\mathcal{E}_3^1)$. Let us define the set

$$\mathcal{S}_{31} := \{(m_{1c}, m_{1p}, m_{2c}) \in [1:2^{nbR_{1c}}] \times [1:2^{nbR_{1p}}] \times [1:2^{nbR_{2c}}] : m_{1c} \neq 1, m_{2c} = 1\},$$

then

$$\begin{aligned}\mathsf{P}(\mathcal{E}_3^1) &= \mathsf{P}\left(\cup_{\mathcal{S}_{31}} \cup_{l_{4,1}^b} \cap_{j=1}^b \mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^j)\right) \\ &\leq \sum_{\mathcal{S}_{31}} \sum_{l_{4,1}^b} \mathsf{P}(\cap_{j=1}^b \mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^j)) && (4.31) \\ &= \sum_{\mathcal{S}_{31}} \sum_{l_{4,1}^b} \prod_{j=1}^b \mathsf{P}(\mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^j)) && (4.32) \\ &\leq \sum_{\mathcal{S}_{31}} \sum_{l_{4,1}^b} \prod_{j=2}^b \mathsf{P}(\mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^j)), && (4.33)\end{aligned}$$

where (4.31) follows from the union of events bound, (4.32) follows from the fact that codebooks in different blocks are generated independently and the channel is memoryless. Let b_1 and b_2 denote the number of l_4's in the sequence $l_{4,1}^{b-1}$ corresponding to case 1 and case 2 in Table 4.1, respectively. We have

$$\begin{aligned}
&\sum_{l_{4,1}^{b}} \prod_{j=2}^{b} \mathsf{P}(\mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^{j})) \\
&\quad = \sum_{l_{4,b}} \sum_{l_{4,1}^{b-1}} \prod_{j=2}^{b} \mathsf{P}(\mathcal{A}_j(m_1, m_{2c}, l_{4,j-1}^{j})) \\
&\quad \leq \sum_{l_{4,b}} \sum_{l_{4,1}^{b-1}} 2^{-n(b_1(I_1+I_0)+b_2 I_2-(b-1)\delta(\epsilon))} && (4.34)\\
&\quad \leq \sum_{l_{4,b}} \sum_{\substack{b_1,b_2:\\ b_1+b_2=b-1}} \binom{b-1}{b_1} 2^{nb_1\hat{R}_4} 2^{-n(b_1(I_1+I_0)+b_2 I_2-(b-1)\delta(\epsilon))} && (4.35)\\
&\quad = \sum_{\substack{b_1,b_2:\\ b_1+b_2=b-1}} \binom{b-1}{b_1} \sum_{l_{4,b}} 2^{nb_1\hat{R}_4} 2^{-n(b_1(I_1+I_0)+b_2 I_2-(b-1)\delta(\epsilon))} \\
&\quad \leq 2^{b-1} 2^{n\hat{R}_4} 2^{-n(b-1)\left(\min\{(I_1+I_0)-\hat{R}_4, I_2\}-\delta(\epsilon)\right)}, && (4.36)
\end{aligned}$$

where (4.34) and the mutual information terms I_0, I_1, and I_2 (as given in (4.12)–(4.14)) are derived in Appendix 4.A. The binomial coefficient $\binom{b-1}{b_1}$ in (4.35) represents the number of combinations in which b_1 out of $b-1$ positions are filled with $l_4 \neq 1$. Corresponding to each combination we have $(2^{n\hat{R}_4}-1)^b < 2^{nb\hat{R}_4}$ different ways of enumerating the indices. Combining (4.33) and (4.36) we have:

$$\begin{aligned}
\mathsf{P}(\mathcal{E}_3^1) &\leq \sum_{\mathcal{S}_{31}} 2^{b-1} 2^{n\hat{R}_4} 2^{-n(b-1)\left(\min\{(I_1+I_0)-\hat{R}_4, I_2\}-\delta(\epsilon)\right)} \\
&\leq 2^{nb(R_{1c}+R_{1p})} 2^{b-1} 2^{n\hat{R}_4} 2^{-n(b-1)\left(\min\{(I_1+I_0)-\hat{R}_4, I_2\}-\delta(\epsilon)\right)}.
\end{aligned}$$

Therefore, $\mathsf{P}(\mathcal{E}_3^1) \to 0$ as $n \to \infty$ if

$$R_{1c} + R_{1p} < \frac{b-1}{b}\left(\min\{(I_1+I_0)-\hat{R}_4, I_2\} - \delta(\epsilon)\right) - \frac{1}{b}\hat{R}_4,$$

where $\delta(\epsilon) \to 0$ as $\epsilon \to 0$. Accordingly, $P(\mathcal{E}_3^1)$ can be made arbitrarily small as $n \to \infty$ and $b \to \infty$ if

$$R_{1c} + R_{1p} < \min\{I_1 + I_0 - \hat{R}_4, I_2\} - \delta(\epsilon). \tag{4.37}$$

On the other hand, using an alternative method of bounding $P(\mathcal{E}_3^1)$ similar to [WT11a, Eqn. (29)] (details are given in Appendix 4.B) we can show that $P(\mathcal{E}_3^1)$ can be made arbitrarily small as $n \to \infty$ and $b \to \infty$ if

$$R_{1c} + R_{1p} < \min\{I_1, I_2\} - \delta(\epsilon). \tag{4.38}$$

Combining (4.37) and (4.38) we conclude that $P(\mathcal{E}_3^1)$ vanishes as $n \to \infty$ and $b \to \infty$ if

$$R_{1c} + R_{1p} < \min\{I_1 + (I_0 - \hat{R}_4)^+, I_2\} - \delta(\epsilon). \tag{4.39}$$

Similarly we can show that $P(\mathcal{E}_3^2) \to 0$ as $n \to \infty$ and $b \to \infty$ if

$$R_{1c} + R_{1p} + R_{2c} < \min\{I_3 + (I_0 - \hat{R}_4)^+, I_4\} - \delta(\epsilon), \tag{4.40}$$

$P(\mathcal{E}_3^3) \to 0$ as $n \to \infty$ and $b \to \infty$ if

$$R_{1p} < \min\{I_5 + (I_0 - \hat{R}_4)^+, I_6\} - \delta(\epsilon), \tag{4.41}$$

and $P(\mathcal{E}_3^4) \to 0$ as $n \to \infty$ and $b \to \infty$ if

$$R_{1p} + R_{2c} < \min\{I_7 + (I_0 - \hat{R}_4)^+, I_8\} - \delta(\epsilon). \tag{4.42}$$

By i) collecting the constraints (4.39)–(4.42) and (4.24), ii) swapping indices $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ to obtain similar constraints for node 4, iii) applying the union of events bound (cf.(4.29)), iv) applying the Fourier-Motzkin elimination to the resulting constraints, and finally v) removing redundant constraints, we conclude that $\mathsf{P}(\mathcal{E}_3) \to 0$ and $\mathsf{P}(\mathcal{E}_4) \to 0$ as $n \to \infty$ and $b \to \infty$ if conditions in Theorem 4.3.1 are satisfied. The details of the step v) are given in Appendix 4.C. The random coding argument ensures the existence of a good code which achieves the stated rate pairs (R_1, R_2)'s. □

4.4 Gaussian IC With Orthogonal Cooperation Links

In this section we focus on investigating the performance of our coding strategy for a certain class of Gaussian channels. Consider the channel depicted in Fig. 4.2. This is a Gaussian IC with two cooperation links at the receiver side, having finite capacities C_{12} and C_{21}, and not interfering with each other as well as with the Gaussian IC. Note that this model is similar to the channel studied in Chapter 3 in terms of orthogonal cooperation links. The Gaussian IC in in Fig. 4.2 was first studied in [WT11a] where it was called the interference channel with conferencing decoders, and an approximate characterization of the capacity region was found. Such a channel model is highly relevant in practice, for example it can model the uplink of a wireless system where the receiving stations are interconnected via backhaul links, or via dedicated links operating at different frequency bands.

Let us define the channel model in Fig. 4.2 in more detail. The transmitters are nodes 1 and 2, each has an average power constraint

$$\frac{1}{N}\sum_{i=1}^{N} |x_k(i)|^2 \le P_k,\ k \in \{1, 2\}, \tag{4.43}$$

where N is the block length and i is the discrete time index. The channel gains h_{13}, h_{14}, h_{23}, h_{24} are complex values. Noise Z_3 and Z_4 are independent of each other

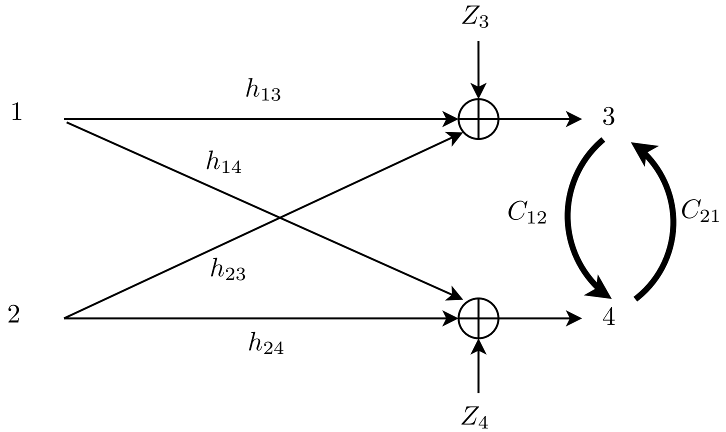

Figure 4.2: Gaussian IC with orthogonal cooperation links (GIC-ORC).

and of the transmitted signals, identically distributed over time with distribution $Z_k \sim \mathcal{CN}(0, N_k)$, $k \in \{3, 4\}$. For the remaining part of this chapter we consider the normalized system where $P_1 = P_2 = N_3 = N_4 = 1$, i.e., all transmit powers and noise variances are normalized to 1. We define the received signal-to-noise ratios (SNR's) and interference-to-noise ratios (INR's) as follows:

$$\mathsf{SNR}_1 := |h_{13}|^2, \mathsf{INR}_2 := |h_{14}|^2,$$
$$\mathsf{SNR}_2 := |h_{24}|^2, \mathsf{INR}_1 := |h_{23}|^2.$$

Since our coding strategy involves the common-private rate-splitting technique, we also define the following quantities

$$\mathsf{SNR}_{1p} := |h_{13}|^2 P_{1p}, \mathsf{INR}_{2p} := |h_{14}|^2 P_{1p}$$
$$\mathsf{SNR}_{2p} := |h_{24}|^2 P_{2p}, \mathsf{INR}_{1p} := |h_{23}|^2 P_{2p},$$

where $0 \leq P_{kp} \leq 1$, $k \in \{1, 2\}$, can be considered as the power used to transmit the private message m_{kp} at node k. More details about the definitions of P_{kp} are presented in Appendix 4.D. For brevity we refer to this channel model as GIC-ORC. The next theorem states an achievable rate region for the GIC-ORC.

Theorem 4.4.1 (Achievable region GIC-ORC)**.** *The following rate region is achievable for the GIC-ORC*

$$\mathcal{R}_2 = \mathsf{Co}\left(\bigcup_{P_{1p}, P_{2p}: 0 \leq P_{1p}, P_{2p} \leq 1} \mathcal{R}_2(P_{1p}, P_{2p}) \right) \tag{4.44}$$

where $\mathcal{R}_2(P_{1p}, P_{2p})$ is the set of nonnegative rate pairs (R_1, R_2) satisfying

$$R_1 < \min\{c_1, c_2\} \tag{4.45}$$
$$R_2 < \min\{c'_1, c'_2\} \tag{4.46}$$
$$R_1 + R_2 < \min\{c_5, c_6\} + \min\{c'_3, c'_4\} \tag{4.47}$$
$$R_1 + R_2 < \min\{c_3, c_4\} + \min\{c'_5, c'_6\} \tag{4.48}$$
$$R_1 + R_2 < \min\{c_7, c_8\} + \min\{c'_7, c'_8\} \tag{4.49}$$
$$2R_1 + R_2 < \min\{c_5, c_6\} + \min\{c_3, c_4\} + \min\{c'_7, c'_8\} \tag{4.50}$$
$$R_1 + 2R_2 < \min\{c_7, c_8\} + \min\{c'_5, c'_6\} + \min\{c'_3, c'_4\}, \tag{4.51}$$

with

$$c_1 = \log\left(\frac{\mathsf{SNR}_1 + \mathsf{INR}_{1p} + 1}{\mathsf{INR}_{1p} + 1}\right) + (C_{21} - \xi)^+ \tag{4.52}$$
$$c_2 = \log\left(\frac{\substack{(1+\sigma_4^2)(1+\mathsf{SNR}_1+\mathsf{INR}_{1p})+\mathsf{INR}_2+\mathsf{SNR}_{2p} \\ +|h_{14}h_{23}-h_{13}h_{24}|^2 P_{2p}}}{(1+\sigma_4^2)(1+\mathsf{INR}_{1p}) + \mathsf{SNR}_{2p}}\right) \tag{4.53}$$
$$c_3 = \log\left(\frac{1 + \mathsf{SNR}_1 + \mathsf{INR}_1}{\mathsf{INR}_{1p} + 1}\right) + (C_{21} - \xi)^+ \tag{4.54}$$
$$c_4 = \log\left(\frac{\substack{(1+\sigma_4^2)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{INR}_2+\mathsf{SNR}_2 \\ +|h_{14}h_{23}-h_{13}h_{24}|^2}}{(1+\sigma_4^2)(1+\mathsf{INR}_{1p}) + \mathsf{SNR}_{2p}}\right) \tag{4.55}$$
$$c_5 = \log\left(\frac{1 + \mathsf{SNR}_{1p} + \mathsf{INR}_{1p}}{\mathsf{INR}_{1p} + 1}\right) + (C_{21} - \xi)^+ \tag{4.56}$$
$$c_6 = \log\left(\frac{\substack{(1+\sigma_4^2)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_{1p})+\mathsf{INR}_{2p}+\mathsf{SNR}_{2p} \\ +|h_{14}h_{23}-h_{13}h_{24}|^2 P_{1p} P_{2p}}}{(1+\sigma_4^2)(1+\mathsf{INR}_{1p}) + \mathsf{SNR}_{2p}}\right) \tag{4.57}$$
$$c_7 = \log\left(\frac{1 + \mathsf{SNR}_{1p} + \mathsf{INR}_1}{\mathsf{INR}_{1p} + 1}\right) + (C_{21} - \xi)^+ \tag{4.58}$$
$$c_8 = \log\left(\frac{\substack{(1+\sigma_4^2)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_1)+\mathsf{INR}_{2p}+\mathsf{SNR}_2 \\ +|h_{14}h_{23}-h_{13}h_{24}|^2 P_{1p}}}{(1+\sigma_4^2)(1+\mathsf{INR}_{1p}) + \mathsf{SNR}_{2p}}\right) \tag{4.59}$$
$$\xi = \log\left(1 + \frac{1}{\sigma_4^2} + \frac{\mathsf{SNR}_{2p}}{(\mathsf{INR}_{1p} + 1)\sigma_4^2}\right), \tag{4.60}$$

and c'_k's are obtained from c_k's, $k \in [1:8]$, by swapping indices $1 \leftrightarrow 2$ and $3 \leftrightarrow 4$ in the expressions of the corresponding c_k's and ξ. σ_3^2, σ_4^2 are nonnegative quantities to be optimized.

Proof. The proof is provided in Appendix 4.D. The idea is to evaluate the constraints in (4.2)–(4.8) with Gaussian inputs, with proper modifications to take into account the orthogonal cooperation links. □

Remark 4.3. In [WT11a] it was shown that a wise choice of σ_4^2 makes the "rate loss" ξ to be bounded above by 1 bit, which helps characterize the capacity region of the channel within a constant gap. We will discuss this in more detail in Chapter 6.

The following theorem states an interesting result about the relation between different coding techniques for the Gaussian interference channel with orthogonal receiver cooperation.

Theorem 4.4.2. *The achievable rate region in Theorem 4.4.1 is the same as the achievable rate region achieved by the one-round quantize-bin-forward in [WT11a, Theorem 5.10].*

Proof. The proof consists of two steps: 1) specializing the rate constraints in (4.39)–(4.42) (before Fourier-Motzkin elimination) to the Gaussian channel with orthogonal cooperation links, and 2) showing that the resulting constraints in step 1 are the same as the constraints at the bottom of [WT11a, pp. 2924]. Details are presented in Appendix 4.E. □

We notice that the authors of [RV11] also find a similar relation between noisy network coding and quantize-bin-forward for relay networks with single source or single destination. As a consequence of Theorem 4.4.2, we have the following corollary.

Corollary 4.4.1. *Noisy network coding achieves within 1 bit/s/Hz of the capacity region of the GIC-ORC in the strong interference regime, i.e.,* $\mathsf{INR}_1 \geq \mathsf{SNR}_2$ *and* $\mathsf{INR}_2 \geq \mathsf{SNR}_1$.

Proof. The proof is the same as the proof of [WT11a, Lemma 5.11]. □

In the strong interference regime the whole message at each transmitter (node 1 and node 2) is set to be the common message, i.e., $U_1 = X_1$, $U_2 = X_2$ and $R_{1p} = R_{2p} = 0$. Therefore, no superposition coding is involved and the rate region recovers [LKEC11, Theorem 2].

Since our achievable rate region is only equivalent to that of the one-round conferencing protocol in [WT11a], while it was shown in [WT11a] that a two-round conferencing protocol is needed to achieve the capacity region within a constant gap in weak and mixed interference regimes, we expect that a more involved use of noisy network coding is needed to achieve a constant gap to the capacity region in those regimes.

A physically closely related channel model to the interference channel with receiver cooperation is the compound multiple access channel (MAC) with conferencing decoders studied in [SGP+09]. Thanks to Theorem 4.4.2 we can state the following result regarding the capacity region of the Gaussian version of such a "sibling" channel.

Corollary 4.4.2. *Noisy network coding achieves within 1 bit/s/Hz of the capacity region of the Gaussian compound MAC with conferencing decoders*[1]*, regardless of channel parameters.*

Proof. The proof follows from the proof of [WT11a, Theorem 5.15], namely comparing the achievable rates with the cut-set outer bounds. Note that in this case all constraints in [WT11a, Eq. 25] are needed due to the fact that messages from both transmitters are required to be decoded correctly by both receivers. □

4.5 Chapter Conclusion

In this chapter we develop a new inner bound for the capacity region of the discrete memoryless two-user interference channel with receiver cooperation, and an inner bound for the Gaussian interference channel with orthogonal conference links at the receivers. The latter is shown to be the same as the one-round quantize-bin-forward inner bound specially designed for the channel in [WT11a], which achieves the capacity region within 1 bit when the cross links are strong. To the best of our knowledge, the result in this chapter was the first to show the equivalence of noisy network coding and a scheme based on explicit binning for a multiple-unicast problem, in terms of achievable rates. In the next two chapters this relation will be further examined. We note that our general inner bound can also be extended to other classes of interference channels with receiver cooperation. To take an example, it might be interesting to see how our inner bound performs in comparison with the inner and outer bounds for the Gaussian interference channel with non-orthogonal (in-band) receiver cooperation, which was studied by Prabhakaran and Viswanath in [PV11a]. More work can also be done with respect to the class of channels studied in Section 4.4, for example considering a multi-round conferencing protocol and optimizing the quantization distortion. Finally, it may be interesting to extend our work to more complex network models such as K-user interference network, cascaded interference network [MDFT11].

[1] As defined in [SGP+09].

Appendices for Chapter 4

4.A Proof of (4.34)

In this appendix we prove (4.34) and derive I_0 and I_1. The other terms $I_2, \ldots, I_8$ in Theorem 4.3.1 can be derived similarly. First note that in bounding $\mathsf{P}(\mathcal{E}_3^1)$, which is defined in (4.25), there are two types of events corresponding to cases 1 and 2 in Table 4.1. Now look at case 1 we have that in this case

$$\begin{aligned}
&\Big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_1), \mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4j}|l_{4,j-1})\Big) \\
&\sim \prod_{i=1}^{n} p_{U_1,X_1}(\mathbf{u}_{1,(j-1)n+i}, \mathbf{x}_{1,(j-1)n+i}) p_{\hat{Y}_4,X_4}(\hat{\mathbf{y}}_{4,(j-1)n+i}, \mathbf{x}_{4,(j-1)n+i})
\end{aligned}$$

is independent of

$$\Big(\mathbf{U}_{2j}(1), \mathbf{X}_{3j}(1), \hat{\mathbf{Y}}_3(1|1), \mathbf{Y}_{3j}\Big).$$

Hence, we can upper bound the probability of the event $\mathcal{A}_j(m_1, m_{2c}, l^j_{4,j-1})$, defined in (4.30), as follows:

$$\begin{aligned}
&\mathsf{P}(\mathcal{A}_j(m_1, m_{2c}, l^j_{4,j-1})) \\
&= \sum_{(\mathbf{u}_1,\mathbf{x}_1,\mathbf{u}_2,\mathbf{x}_3,\hat{\mathbf{y}}_3,\mathbf{x}_4,\hat{\mathbf{y}}_4,\mathbf{y}_3)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1, \mathbf{x}_1, \mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{x}_4, \hat{\mathbf{y}}_4, \mathbf{y}_3) \\
&= \sum_{(\mathbf{u}_2,\mathbf{x}_3,\hat{\mathbf{y}}_3,\mathbf{y}_3)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3) \\
&\quad \cdot \sum_{(\mathbf{u}_1,\mathbf{x}_1)\in\mathcal{T}_\epsilon^{(n)}(U_1,X_1|\mathbf{u}_2,\mathbf{x}_3,\hat{\mathbf{y}}_3,\mathbf{y}_3)} p(\mathbf{u}_1, \mathbf{x}_1) \\
&\quad\quad \cdot \sum_{(\mathbf{x}_4,\hat{\mathbf{y}}_4)\in\mathcal{T}_\epsilon^{(n)}\left(X_4,\hat{Y}_4|\mathbf{u}_1,\mathbf{x}_1,\mathbf{u}_2,\mathbf{x}_3,\hat{\mathbf{y}}_3,\mathbf{y}_3\right)} p(\mathbf{x}_4, \hat{\mathbf{y}}_4) \\
&\le \sum_{(\mathbf{u}_2,\mathbf{x}_3,\hat{\mathbf{y}}_3,\mathbf{y}_3)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3) 2^{-n(\kappa-\delta(\epsilon))} \\
&\le 2^{-n(\kappa-\delta(\epsilon))},
\end{aligned}$$

where we used the properties of jointly typical sequence [EK11, Sec. 2.5]:

$$\begin{aligned}
p(\mathbf{u}_1, \mathbf{x}_1) &\le 2^{-n(H(U_1,X_1)-\delta_1(\epsilon))}, \\
|T_\epsilon^{(n)}\left(U_1, X_1|\mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3\right)| &\le 2^{n(H(U_1,X_1|U_2,X_3,\hat{Y}_3,Y_3)+\delta_2(\epsilon))},
\end{aligned}$$

where $\delta(\epsilon)$, $\delta_1(\epsilon)$, $\delta_2(\epsilon)$ go to 0 as $\epsilon \to 0$, to derive κ. κ is given by

$$\begin{aligned}\kappa &= I(U_1, X_1; U_2, X_3, \hat{Y}_3, Y_3) + I(X_4, \hat{Y}_4; U_1, X_1, U_2, X_3, \hat{Y}_3, Y_3)\\ &\stackrel{(a)}{=} I(X_1; Y_3|U_2, X_3) + I(X_4, \hat{Y}_4; X_1, U_2, X_3, Y_3)\\ &= I_1 + I_0,\end{aligned}$$

where we defined $I_1 = I(X_1; Y_3|U_2, X_3)$, $I_0 = I(X_4, \hat{Y}_4; X_1, U_2, X_3, Y_3)$. (a) is due to Markov chains $\hat{Y}_3 - (X_3, Y_3) - (X_4, \hat{Y}_4, X_1, U_2)$ and $U_1 - X_1 - (U_2, X_3, Y_3)$. Other mutual information terms can be derived in the same way, in particular

$$I_2 = I(X_1; \hat{Y}_4, Y_3|U_2, X_3, X_4). \tag{4.61}$$

4.B Alternative Bound for $\mathsf{P}(\mathcal{E}_3^1)$

For each $(m_{1p}, m_{1c}, m_{2c}) \in \mathcal{S}_{31}$ define

$$\begin{aligned}\bar{\mathcal{E}} :=\Big\{ &\big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_{1p}, m_{1c}), \mathbf{U}_{2j}(m_{2c}), \mathbf{X}_{3j}(L_{3,j-1}), \hat{\mathbf{Y}}_{3j}(L_{3,j}|L_{3,j-1}),\\ &\mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|l_{4,j-1}), \mathbf{Y}_{3j}\big) \in \mathcal{T}_\epsilon^{(n)},\\ &\forall j \in [1:b], \text{ for some } l_{4,j-1}, l_{4,j} \in [1:2^{n\hat{R}_4}]\Big\}.\end{aligned}$$

By the union of events bound we have

$$\mathsf{P}(\mathcal{E}_3^1) \le \sum_{\mathcal{S}_{31}} \mathsf{P}(\bar{\mathcal{E}}). \tag{4.62}$$

Now for each fixed $(m_{1p}, m_{1c}, m_{2c}) \in \mathcal{S}_{31}$,

- Let $\Omega_1 \subseteq [2:b]$ denote the set of indices of blocks j's such that in each block there exists some compression index $l_{4,j-1} \neq 1$ satisfying the joint typicality, namely the event $A_j(m_1, m_{2c}, l_{4,j-1}^j)$, which is defined in (4.30), occurs.
- Let $\Omega_2 \subseteq [2:b]$ denote the set of blocks j's such that in each block only the correct compression index $l_{4,j-1} = 1$ satisfying the joint typicality.

Furthermore for each fixed $(m_{1p}, m_{1c}, m_{2c}) \in \mathcal{S}_{31}$ and for each $j \in [2:b]$ we define the following events:

$$\begin{aligned}\mathcal{H}_1(j) :=\Big\{&\exists\, l_{4,j-1} \neq 1 \text{ and some } l_{4,j}, \big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_{1p}, m_{1c}), \mathbf{U}_{2j}(m_{2c}),\\ &\mathbf{X}_{3j}(L_{3,j-1}), \hat{\mathbf{Y}}_{3j}(L_{3,j}|L_{3,j-1}), \mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|l_{4,j-1}), \mathbf{Y}_{3j}\big) \in \mathcal{T}_\epsilon^{(n)}\Big\}\end{aligned}$$

and

$$\begin{aligned}\mathcal{H}_2(j) :=\Big\{&l_{4,j-1} = 1 \text{ and some } l_{4,j}, \big(\mathbf{U}_{1j}(m_{1c}), \mathbf{X}_{1j}(m_{1p}, m_{1c}), \mathbf{U}_{2j}(m_{2c}),\\ &\mathbf{X}_{3j}(L_{3,j-1}), \hat{\mathbf{Y}}_{3j}(L_{3,j}|L_{3,j-1}), \mathbf{X}_{4j}(l_{4,j-1}), \hat{\mathbf{Y}}_{4j}(l_{4,j}|l_{4,j-1}), \mathbf{Y}_{3j}\big) \in \mathcal{T}_\epsilon^{(n)}\Big\}.\end{aligned}$$

Ignoring block 1 where the indices are predefined, and since the codebooks are independently generated and the channel is memoryless we have

$$\mathsf{P}(\bar{\mathcal{E}}) \leq \sum_{\Omega_1 \subseteq [2:b]} \prod_{k \in \Omega_1} \mathsf{P}(\mathcal{H}_1(k)) \prod_{\substack{k \in \Omega_2, \\ \Omega_2 = [2:b] \setminus \Omega_1}} \mathsf{P}(\mathcal{H}_2(k)). \tag{4.63}$$

We now bound $\mathsf{P}(\mathcal{H}_1(k))$ and $\mathsf{P}(\mathcal{H}_2(k))$. Keep in mind that the joint typicality of the sequences below are considered with respect to the fact that $(m_{1p}, m_{1c}, m_{2c}) \in \mathcal{S}_{31}$.

$$\begin{aligned}
\mathsf{P}(\mathcal{H}_1(k)) = & \sum_{(\mathbf{u}_1, \mathbf{x}_1, \mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3) \in \mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1, \mathbf{x}_1, \mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3) \\
& \times \mathsf{P}\Big\{ \exists\, l_{4,k-1} \neq 1 \text{ and some } l_{4,k}, \\
& \big(\mathbf{u}_1, \mathbf{x}_1, \mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3, \mathbf{X}_{4k}(l_{4,k-1}), \hat{\mathbf{Y}}_{4k}(l_{4,k}|l_{4,k-1})\big) \in \mathcal{T}_\epsilon^{(n)} | \\
& \quad \mathbf{U}_{2k} = \mathbf{u}_1, \mathbf{X}_{1k} = \mathbf{x}_1, \mathbf{U}_{2k} = \mathbf{u}_2, \mathbf{X}_{3k} = \mathbf{x}_3, \mathbf{Y}_{3k} = \mathbf{y}_3 \Big\} \\
& \leq \sum_{(\mathbf{u}_1, \mathbf{x}_1, \mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3) \in \mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1, \mathbf{x}_1, \mathbf{u}_2, \mathbf{x}_3, \hat{\mathbf{y}}_3, \mathbf{y}_3) \\
& \leq 2^{-n(I_1 - \delta(\epsilon_1))},
\end{aligned} \tag{4.64}$$

where the last inequality and $I_1 = I(X_1; Y_3 | U_2, X_3)$ are derived by the same technique as in Appendix 4.A. The bounding of $\mathsf{P}(\mathcal{H}_2(k))$ can be done similarly, leading to

$$\mathsf{P}(\mathcal{H}_2(k)) \leq 2^{-n(I_2 - \delta(\epsilon_2))}, \tag{4.65}$$

where I_2 is given in (4.61). Let b_1 and b_2 denote the cardinalities of Ω_1 and Ω_2 in (4.63), respectively. Inserting (4.64) and (4.65) into (4.63), then substituting (4.63) into (4.62) we obtain

$$\begin{aligned}
\mathsf{P}(\mathcal{E}_3^1) &\leq \sum_{\mathcal{S}_{31}} \sum_{\substack{b_1, b_2: \\ b_1 + b_2 = b-1}} \binom{b-1}{b_1} 2^{-n((b_1 I_1 + b_2 I_2) - \delta(\epsilon))} \\
&\leq \sum_{\mathcal{S}_{31}} 2^{b-1} 2^{-n(b-1)(\min\{I_1, I_2\} - \delta(\epsilon))} \\
&\leq 2^{nb(R_{1p} + R_{1c})} 2^{b-1} 2^{-n(b-1)(\min\{I_1, I_2\} - \delta(\epsilon))}.
\end{aligned}$$

Therefore we conclude that with b and n sufficiently large we can make $\mathsf{P}(\mathcal{E}_3^1)$ arbitrarily small if

$$R_{1c} + R_{1p} < \min\{I_1, I_2\} - \delta(\epsilon). \tag{4.66}$$

□

4.C Removing Redundant Constraints

After the Fourier-Motzkin elimination procedure, similarly to Appendix 3.D, and by removing some trivially redundant constraints we obtain the following constraints on R_1, R_2:

$$\begin{aligned}
R_1 &< \min\{I_1 + (I_0 - r_4)^+, I_2\} && (4.67a)\\
R_1 &< \min\{I_5 + (I_0 - r_4)^+, I_6\} + \min\{I_7' + (I_0' - r_3)^+, I_8'\} && (4.67b)\\
R_2 &< \min\{I_1' + (I_0' - r_3)^+, I_2'\} && (4.67c)\\
R_2 &< \min\{I_5' + (I_0' - r_3)^+, I_6'\} + \min\{I_7 + (I_0 - r_4)^+, I_8\} && (4.67d)\\
R_1 + R_2 &< \min\{I_5 + (I_0 - r_4)^+, I_6\} + \min\{I_3' + (I_0' - r_3)^+, I_4'\} && (4.67e)\\
R_1 + R_2 &< \min\{I_5' + (I_0' - r_3)^+, I_6'\} + \min\{I_3 + (I_0 - r_4)^+, I_4\} && (4.67f)\\
R_1 + R_2 &< \min\{I_7 + (I_0 - r_4)^+, I_8\} + \min\{I_7' + (I_0' - r_3)^+, I_8'\} && (4.67g)\\
2R_1 + R_2 &< \min\{I_5 + (I_0 - r_4)^+, I_6\} + \min\{I_3 + (I_0 - r_4)^+, I_4\}\\
&\quad + \min\{I_7' + (I_0' - r_3)^+, I_8'\} && (4.67h)\\
R_1 + 2R_2 &< \min\{I_7 + (I_0 - r_4)^+, I_8\} + \min\{I_5' + (I_0' - r_3)^+, I_6'\}\\
&\quad + \min\{I_3' + (I_0' - r_3)^+, I_4'\} && (4.67i)
\end{aligned}$$

In the following we will prove that (4.67b) and (4.67d) are redundant when we take the union of the above region over all distributions of the form (4.9). Due to symmetry, it suffices to prove the redundancy of (4.67b). First we need a lemma whose proof is straightforward.

Lemma 4.1. *For nonnegative real numbers a, b, c satisfying $b \geq c$, we have*

$$\min\{a, b\} - \min\{a, c\} = \min\{(a - c)^+, b - c\}. \tag{4.68}$$

Now, suppose a rate pair (R_1, R_2) satisfies (4.67) except the constraint in (4.67b), i.e.,

$$R_1 \geq \min\{I_5 + (I_0 - r_4)^+, I_6\} + \min\{I_7' + (I_0' - r_3)^+, I_8'\}. \tag{4.69}$$

From (4.67e) and (4.69) we have

$$\begin{aligned}
R_2 &< \min\{I_3' + (I_0' - r_3)^+, I_4'\} - \min\{I_7' + (I_0' - r_3)^+, I_8'\}\\
&= I_3' - I_7' + \min\{(I_0' - r_3)^+, I_4' - I_3'\} - \min\{(I_0' - r_3)^+, I_8' - I_7'\}. && (4.70)
\end{aligned}$$

Using the fact that $(U_1, X_1), (U_2, X_2), X_3, X_4$ are mutually independent given Q we can easily show that

$$\begin{aligned}
I_8' - I_7' &= I(X_2, U_1; X_3, \hat{Y}_3 | U_2, X_4, Y_4, Q)\\
I_4' - I_3' - (I_8' - I_7') &= I(U_2; X_3, \hat{Y}_3 | X_4, Y_4, Q) \geq 0.
\end{aligned}$$

Therefore, we can apply Lemma 4.1 to (4.70) to get

$$\begin{aligned}
R_2 &< I'_3 - I'_7 + \min\left\{\left((I'_0 - r_3)^+ - (I'_8 - I'_7)\right)^+, I'_4 - I'_3 - (I'_8 - I'_7)\right\} \\
&= I(U_2; Y_4|X_4, Q) + \min\begin{Bmatrix}\left(I'_0 - r_3 - I(X_2, U_1; X_3, \hat{Y}_3|U_2, X_4, Y_4, Q)\right)^+, \\ I(U_2; X_3, \hat{Y}_3|X_4, Y_4, Q)\end{Bmatrix} \\
&= \min\begin{Bmatrix} I(U_2; Y_4|X_4, Q) + \left(I(X_3, \hat{Y}_3; U_2, X_4, Y_4|Q) - r_3\right)^+, \\ I(U_2; X_3, \hat{Y}_3, Y_4|X_4, Q)\end{Bmatrix}. \qquad (4.71)
\end{aligned}$$

From (4.67h) and (4.69) we have

$$R_1 + R_2 < \min\{I_3 + (I_0 - r_4)^+, I_4\}. \qquad (4.72)$$

In summary, if a rate pair (R_1, R_2) violates (4.67b) it must satisfy the following three conditions, which come from (4.67a), (4.71), and (4.72)

$$\begin{aligned}
R_1 &< \min\{I_1 + (I_0 - r_4)^+, I_2\} && (4.73a) \\
R_2 &< \min\begin{Bmatrix} I(U_2; Y_4|X_4, Q) + \left(I(X_3, \hat{Y}_3; U_2, X_4, Y_4|Q) - r_3\right)^+, \\ I(U_2; \hat{Y}_3, Y_4|X_3, X_4, Q)\end{Bmatrix} && (4.73b) \\
R_1 + R_2 &< \min\{I_3 + (I_0 - r_4)^+, I_4\}, && (4.73c)
\end{aligned}$$

and some other constraints (that emerge from the combination of constraints in (4.67) and (4.69)).

Our next step is to show that any rate pair (R_1, R_2) that satisfies (4.73) is achievable by a special case of the joint distribution in (4.9), in which $U_1 = \emptyset$. To see that, let us set $U_1 = \emptyset$ and calculate the mutual information terms in Theorem 4.3.1, now denoted by $\bar{I}_i$ and $\bar{I}'_i$ for $i \in \{0, 1, \ldots, 8\}$. Simple calculations give

$$\begin{aligned}
\bar{I}_0 &= I_0 \\
\bar{I}_5 = \bar{I}_1 &= I_1 \\
\bar{I}_6 = \bar{I}_2 &= I_2 \\
\bar{I}_7 = \bar{I}_3 &= I_3 \\
\bar{I}_8 = \bar{I}_4 &= I_4 \\
\bar{I}'_0 &= I(X_3, \hat{Y}_3; X_2, X_4, Y_4|Q) \\
\bar{I}'_3 = \bar{I}'_1 &= I(X_2; Y_4|X_4, Q) \\
\bar{I}'_4 = \bar{I}'_2 &= I(X_2; \hat{Y}_3, Y_4|X_3, X_4, Q) \\
\bar{I}'_7 = \bar{I}'_5 &= I(X_2; Y_4|U_2, X_4, Q) \\
\bar{I}'_8 = \bar{I}'_6 &= I(X_2; \hat{Y}_3, Y_4|U_2, X_3, X_4, Q).
\end{aligned}$$

As a result, the rate constraints in (4.67) reduce to

$$R_1 < \min\{I_1 + (I_0 - r_4)^+, I_2\} \tag{4.75a}$$

$$\begin{aligned} R_2 &< \min\{\bar{I}_1' + (\bar{I}_0' - r_3)^+, \bar{I}_2'\} \\ &= \min\left\{\begin{matrix} I(X_2; Y_4|X_4, Q) + \big(I(X_3, \hat{Y}_3; X_2, X_4, Y_4|Q) - r_3\big)^+, \\ I(X_2; \hat{Y}_3, Y_4|X_3, X_4, Q) \end{matrix}\right\} \end{aligned} \tag{4.75b}$$

$$R_1 + R_2 < \min\{I_3 + (I_0 - r_4)^+, I_4\} + \min\{\bar{I}_5' + (\bar{I}_0' - r_3)^+, \bar{I}_6'\}. \tag{4.75c}$$

Finally, we notice that the RHS's in (4.75) are greater than or equal to the corresponding RHS's in (4.73) due to Markov chains $U_2 - (X_2, X_4, Q) - Y_4$, $U_2 - (X_2, X_4, Y_4, Q) - (X_3, \hat{Y}_3)$, and $U_2 - (X_2, X_3, X_4, Q) - (\hat{Y}_3, Y_4)$. Hence, any rate pair (R_1, R_2) violating (4.67b) can be achieved by setting $U_1 = \emptyset$ in the joint distribution (4.9). By symmetry, any rate pair (R_1, R_2) violating (4.67d) can be achieved by setting $U_2 = \emptyset$ in the joint distribution. That means we can ignore (4.67b) and (4.67d) when taking the union of the rates in (4.67) over all distributions of the form (4.9). This gives rise to Theorem 4.3.1. □

4.D Proof of Theorem 4.4.1

For this channel we use the codebooks which are generated as in Section 4.3 where the random variables used to generate codebooks have the following distributions:

- For node k, $k \in \{1, 2\}$, $U_k \sim \mathcal{CN}(0, P_{kc})$, and the conditional distribution $p_{X_k|U_k}$ is such that $X_k = U_k + V_k$ where $V_k \sim \mathcal{CN}(0, P_{kp})$ and independent of everything else, $P_{kc} + P_{kp} = P_k$.
- The received signal at node k, $k \in \{3, 4\}$, from nodes 1 and 2 is given by

  $$Y_{ka} = h_{1k} X_1 + h_{2k} X_2 + Z_k, \tag{4.76}$$

  where the subscript a stands for the *analog* part.
- For node k, $k \in \{3, 4\}$, the quantization codebooks are generated by random variables whose distributions satisfy

  $$\hat{Y}_k = Y_{ka} + \hat{Z}_k \tag{4.77}$$

  where $\hat{Z}_k \sim \mathcal{CN}(0, \sigma_k^2)$, independent of everything else.
- With these codebooks we can adapt the general achievable region in Theorem 4.3.1 to the current channel as follows: Let $Y_k = (Y_{ka}, Y_{ko})$, $k \in \{3, 4\}$, where the subscript o stands for *orthogonal* part, which is obtained through the conferencing links. The orthogonal links can be modeled as channels with X_3, X_4 mutually independent of $X_1, X_2, \hat{Y}_3, \hat{Y}_4$, and $I(X_3; Y_{4o}) = C_{12}$, $I(X_4; Y_{3o}) = C_{21}$.

As a result,

$$\begin{aligned} r_3 &= I(Y_3; \hat{Y}_3|X_3) = h(\hat{Y}_3|X_3) - h(\hat{Y}_3|Y_3, X_3) \\ &= h(\hat{Y}_3) - h(\hat{Y}_3|Y_{3a}) \\ &= I(\hat{Y}_3; Y_{3a}). \end{aligned}$$

Similarly, $r_4 = I(\hat{Y}_4; Y_{4a})$. By letting $Q = \emptyset$ and evaluating the mutual information terms with the above codebooks we obtain the rates as stated in Theorem 4.4.1. □

4.E Proof of Theorem 4.4.2

First recall that from (4.39) and (4.24) we have

$$R_{1c} + R_{1p} < \min\{I_1 + (I_0 - r_4)^+, I_2\}, \tag{4.78}$$

where

$$\begin{aligned} r_4 &= I(Y_4; \hat{Y}_4|X_4, Q) \\ I_0 &= I(X_4, \hat{Y}_4; X_1, U_2, X_3, Y_3, Q) \\ I_1 &= I(X_1; Y_3|U_2, X_3, Q) \\ I_2 &= I(X_1; \hat{Y}_4, Y_3|U_2, X_3, X_4, Q) \end{aligned}$$

For the GIC-ORC we follow the procedures presented in Appendix 4.D to calculate the mutual information terms. Accordingly

$$\begin{aligned} r_4 &= I(\hat{Y}_4; Y_{4a}) \\ I_0 &= I(X_4, \hat{Y}_4; X_1, U_2, X_3, Y_3) \\ &= I(X_4; Y_{3o}) + I(\hat{Y}_4; X_1, U_2, Y_{3a}) \\ &= C_{21} + I(\hat{Y}_4; X_1, U_2, Y_{3a}) \\ I_1 &= I(X_1; Y_3|U_2, X_3) \\ &= I(X_1; Y_{3a}|U_2). \end{aligned}$$

Defining $\xi := r_4 - I(\hat{Y}_4; X_1, U_2, Y_{3a})$ (this is the same as ξ in (4.60) as shown below), we have

$$I_0 - r_4 = C_{21} - \xi. \tag{4.79}$$

Now

$$\begin{aligned} \xi &= r_4 - I(\hat{Y}_4; X_1, U_2, Y_{3a}) \\ &= I(\hat{Y}_4; Y_{4a}) - I(\hat{Y}_4; X_1, U_2, Y_{3a}) \\ &= h(\hat{Y}_4|X_1, U_2, Y_{3a}) - h(\hat{Y}_4|Y_{4a}) \end{aligned}$$

$$
\begin{aligned}
&= h(h_{24}V_2 + Z_4 + \hat{Z}_4 | h_{23}V_2 + Z_3) - h(\hat{Z}_4) \\
&= \log\left(1 + \frac{1}{\sigma_4^2} + \frac{\mathsf{SNR}_{2p}}{(\mathsf{INR}_{1p} + 1)\sigma_4^2}\right).
\end{aligned} \tag{4.80}
$$

Similarly we have

$$
\begin{aligned}
I_2 &= I(X_1; \hat{Y}_4, Y_3 | U_2, X_3, X_4) \\
&= I(X_1; \hat{Y}_4, Y_{3a} | U_2).
\end{aligned} \tag{4.81}
$$

Substituting (4.79), (4.80) into (4.78) we obtain

$$
R_{1c} + R_{1p} < \min\left\{ I(X_1; Y_{3a} | U_2) + \left(C_{21} - \xi\right)^+, I(X_1; \hat{Y}_4, Y_{3a} | U_2) \right\}, \tag{4.82}
$$

which is identical to the constraint on $R_{1c} + R_{1p}$ at the bottom of [WT11a, pp. 2924] because ξ in (4.80) is equal to ξ_1 in [WT11a, Eq. (14)].

Similarly we can show that (4.40), (4.41), and (4.42) are identical to the corresponding constraints at the bottom of [WT11a, pp. 2924]. □

Chapter 5

Interference Channels With a Relay

So far we have studied the interference channels where the receivers can also act as relays to help each other decode their desired messages. As pointed out in Section 2.6, there is another form of cooperation for the interference channel, in which a dedicated relay node is employed to concurrently help both the receivers, often called *interference channel with a relay* (ICR). For this type of channels, we ask the following question: How can the relay help the receivers if the channel qualities of the receivers are much different? In this chapter, we seek an answer to the question by studying the impact of layered coding on the achievable rate regions. Specifically, we propose *three* layered schemes for the discrete memoryless ICR: The first approach is inspired by the layered noisy network coding (LNNC) scheme, as proposed by Lim et al. for the two-way relay channel; the second and the third approaches rely on simpler encoding and decoding processes, which we term *layered quantize-forward* (LQF). The performance of the proposed protocols is investigated for two classes of channels with Gaussian noise: the interference channel with in-band relay reception/out-of-band relay transmission and the interference with in-band relay reception/in-band relay transmission. For the former class of channels, it is shown that the achievable rates of the first scheme are the same as the rates achieved by the generalized hash-forward scheme with two-stage binning, thereby indirectly showing that LNNC can achieve a constant gap to the capacity region under certain conditions on the channel parameters. For the latter class of channels, numerical examples show that the achievable sum-rate of the proposed schemes can be significantly improved when compared with other single-layer compress-forward schemes. The obtained results show that having layered codes at the relay can be very beneficial for the ICR. We also identify the trade-off between achievable rates, coding delay, and complexity.

5.1 Introduction

Consider the channel depicted in Fig. 5.1, that is, a two-user interference channel with an extra relay acting as a common helper to assist the communication. Specif-

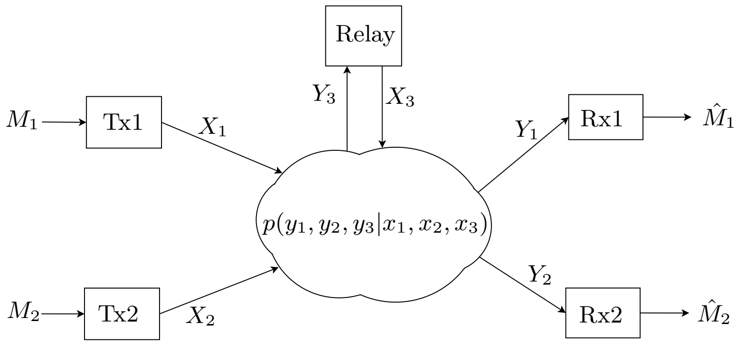

Figure 5.1: Discrete memoryless interference channel with a relay (DM-ICR). Tx stands for transmitter, Rx stands for receiver. Transmitter k wants to send a message M_k to receiver k, $k \in \{1, 2\}$. X_k's denote channel inputs, Y_k's denote channel outputs, $k \in \{1, 2, 3\}$.

ically, transmitter k wants to send a message M_k to receiver k, $k \in \{1, 2\}$. The relay receives signals from both transmitters and conveys the received information, in different forms for different protocols, so as to help the destinations recover their desired messages. As introduced in Chapter 2, the model in Fig. 5.1 is a special case of a general noisy network with multiple unicast traffics, and was first introduced in explicit form by Sahin and Erkip in [SE07]. From a macro point of view, this interference channel with a relay is an important model since it can be considered as a building block for larger multiuser communication networks.

In the Gaussian scenario, depending on how the relay uses the available spectral resources to send and receive signals we can distinguish four types of ICR's as follows [SES09]: in-band relay reception/in-band relay transmission, in-band relay reception/out-of-band relay transmission, out-of-band relay reception/in-band relay transmission, and out-of-band relay reception/out-of-band relay transmission. Substantial previous work has been devoted to analyzing the capacity region of the ICR since its introduction. Early inner and outer bounds are reported in [SE07, MDG08, MDG09, LCTZ+10], and more recent results on the capacity region and generalized of degrees of freedom appear in [TY11, SSE11b, SES11, SSE11a, CS12, TY12, ZY13].

The ICR is an intermixture of several fundamental problems in network information theory such as the interference channel, the relay channel, and the problem of coding with side information. Accordingly, various strategies based on known coding techniques for the constituent problems have been proposed for the ICR. In terms of relaying protocol there are a number of strategies based on amplify-forward [LCTZ+10], decode-forward (DF) [CS12], functional decode-forward [CS12], compress-forward (CF) [TY11] and CF's extensions such as generalized compress-forward (GCF) [TY11, CS12], generalized hash-forward (GHF) [RY10], noisy network coding (NNC) [LKEC11], lattice coding [TY11]. Some of the aforementioned relaying strategies have also been employed for a closely related setup in [GSG+10]. For the Gaussian IC with in-band relay reception/in-band relay transmission, Tian and Yener show in [TY11] that GCF can outperform GHF and approach NNC closely in certain cases. Zhou and Yu show in [ZY13] that a scheme based on GHF can achieve a constant gap to the capacity region of the Gaussian ICR with orthogonal relay links to the receivers, provided that the transmitter-relay links are not infinitely stronger than the interference links. However, the capacity region of the ICR is still far from being established.

In this chapter we look for new insight into the problem by studying the benefit of adapting the relaying protocol to handle potential disparities in channel quality and/or the available side information at the receivers. Specifically, on one hand we want to emphasize the benefit of having multiple layers at the relay, on the other hand we aim at developing novel coding schemes which can possibly achieve the best known performance with reduced delay and complexity. We will show that these two targets can be achieved by the following program, which also represents the main results of the chapter:

- We first develop a coding scheme for the discrete memoryless channel (DM-ICR) which extends the layered noisy network coding (LNNC) protocol, proposed by Lim et al. [LKEC10] for the two-way relay channel, to the ICR.

- Noisy network coding has been shown to achieve the highest rates among variants of CF [LKEC11], but is also known for being the most complex of the CF-oriented schemes. In order to reduce the complexity and delay of our first approach, we proceed to design two new schemes for the DM-ICR, which encode shorter messages and decode in a block-by-block manner. We name these two schemes *layered quantize-forward with forward sliding window decoding (LQF-FW)* and *layered quantize-forward with backward decoding (LQF-BW).*

- We then extend our results to Gaussian channels and show that for the Gaussian IC with in-band relay reception/out-of-band relay transmission (IC-IOR), our first scheme achieves the same rate region as generalized hash-forward with two-stage binning [ZY13], leading to a corollary that LNNC achieves a constant gap to the capacity region of the Gaussian IC-IOR under certain channel parameter conditions. For the Gaussian IC with an in-band relay reception/in-band relay transmission (IC-IIR), numerical examples show that LNNC and LQF-BW universally outperform other compress-forward schemes in terms of achievable sum-rate, while LQF-FW is the runner up when the channel gains to the receivers are asymmetric.

- We compare and reveal the trade-off among different performance metrics of the proposed schemes, namely achievable rates, encoding and decoding delay and complexity.

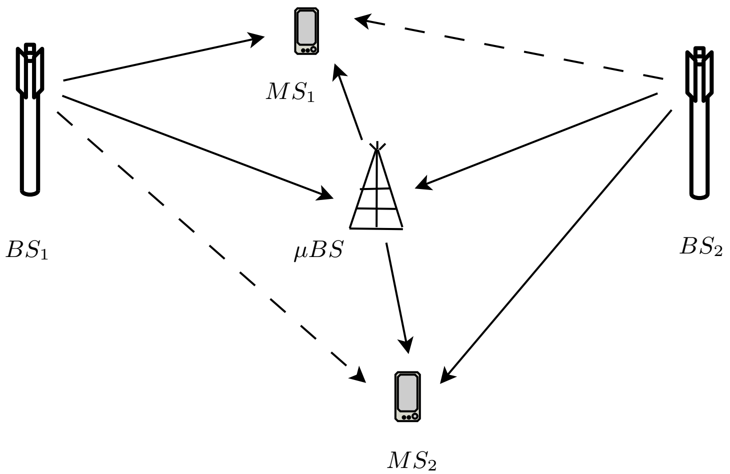

Figure 5.2: Downlink cellular network with a cell-edge micro base station (μBS). In this simplified model, macro base station BS_i talks to its intended mobile station MS_i, $i = 1, 2$, μBS acts as a relay for both cells. The dotted lines illustrates interference from the base stations to the unintended mobile terminals.

- The main components of our schemes include: (i) rate-splitting and superposition coding at the transmitters, (ii) layered compress-forward relaying at the relay, and (iii) joint decoding at the destinations with non-unique decoding of undesired messages. Our achievable regions are first established for the general DM-ICR. One benefit of this generality is that later on it can be adapted to different classes of channel such as Gaussian channels, channels with orthogonal links, as will be detailed in Sections 5.5 and 5.6.

A motivating scenario for the methods we consider is depicted in Fig. 5.2. Here a micro base station is deployed to assist two macro base stations to communicate with their designated mobile nodes. Due to the multi-path nature of the wireless medium and the mobility of the nodes, the quality of the channel as experienced by different mobile nodes is in general quite different, even if they are located at equal distances from the micro base station. Accordingly, it is necessary for the relay to employ a sophisticated coding strategy so that it can maximally help the communications in both cells. Technically, our coding strategies are motivated by the benefits of two-layer relaying for the two-way relay channel as demonstrated in [GTN08] and [LKEC10], as well as the multi-layer coding approach for fading channels (the broadcast approach) by Steiner et al. in [SSS07]. Approaches based on layered quantize-forward like those studied in the current work were first considered in the context of the two-way relay channel in one of our work [DOS12a].

We note that in order to emphasize the benefits of employing multiple-layer quantization at the relay, we restrict the discussion and comparison to different CF-oriented schemes, i.e., we do not explore decode-forward, structured codes, or hybrid digital-analog schemes.

The chapter is organized as follows: Section 5.2 formulates the problem and introduces the preliminaries of the coding schemes. Section 5.3 presents our proposed three inner bounds to the capacity region of the discrete memoryless ICR. Section 5.4 compares the performance as well as the delay and complexity of the three coding schemes. Section 5.5 and 5.6 extend our results to two classes of Gaussian memoryless channels: the IC with in-band relay reception/out-of-band relay transmission and the IC with in-band relay reception/in-band relay transmission, and evaluate the performance of our schemes. Finally, Section 5.7 concludes the chapter. Proofs are given in the appendices.

Terminology

Adopting the classification put forth by Kramer and Hou in [KH11], we use *compress-forward* as a generic term for relaying strategies in which a relay converts its received signal into compressed form, in the lossy sense and with or without binning, and then forwards it to the next hop in the network. *Quantize-forward* refers to the specific sub-class of compress-forward wherein the relay does not employ binning.

5.2 Channel Model and Preliminaries

5.2.1 Channel Model

The discrete memoryless ICR, depicted in Fig. 5.1, consists of 2 sender–receiver pairs whose finite alphabets are denoted by $(\mathcal{X}_k, \mathcal{Y}_k)$, $k \in \{1, 2\}$, a relay with finite input and output alphabets $\mathcal{X}_3$ and $\mathcal{Y}_3$, and a conditional probability mass function (pmf) $p(y_1, y_2, y_3|x_1, x_2, x_3)$. The channel is memoryless and time invariant in the sense that

$$\begin{aligned} p(y_{1i}, y_{2i}, y_{3i}|m_1, m_2, &x_1^i, x_2^i, x_3^i, y_1^{i-1}, y_2^{i-1}, y_3^{i-1}) \\ &= p_{Y_1,Y_2,Y_3|X_1,X_2,X_3}(y_{1i}, y_{2i}, y_{3i}|x_{1i}, x_{2i}, x_{3i}) \end{aligned}$$

for all i. Therefore we can focus on $p_{Y_1,Y_2,Y_3|X_1,X_2,X_3}(y_1, y_2, y_3|x_1, x_2, x_3)$, and we will drop the subscripts in the probability distributions whenever it is clear from the context. A $(2^{nR_1}, 2^{nR_2}, n)$ code, where n denotes the codeword length, for this channel consists of two message sets $\mathcal{M}_k = [1 : 2^{nR_k}]$, two encoding functions $f_k : \mathcal{M}_k \to \mathcal{X}_k^n$, relaying functions $r_i : \mathcal{Y}_3^{i-1} \to \mathcal{X}_3$, $i \in [1 : n]$, and two decoding functions $g_k : \mathcal{Y}_k^n \to \mathcal{M}_k$, $k \in \{1, 2\}$. f_k assigns a sequence X_k^n to each message M_k, r_i maps Y_3^{i-1} to a symbol X_{3i} for all $i \in [1 : n]$, and g_k puts out an estimate $\hat{M}_k$ from a received sequence Y_k^n. The average probability of error is defined as

$$P_e^{(n)} = \mathsf{P}\left\{(\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\right\}. \tag{5.1}$$

Achievable rate pairs and the capacity region of the channel are defined as in Section 2.1.

5.2.2 Long-message Versus Short-message Signaling

In the relay channel literature it is common to consider signaling over b blocks, each consisting of n channel uses, see for example [CE79]. However, there are two main techniques to encode over these b blocks. The first is when the transmitter sends a *different* message in each block, corresponding to the classical approach widely used in many works since the seminal work of [CE79], see [KGG05] for additional references. In the second protocol, which appears in [ADT11] and [LKEC11], the transmitter repeatedly encodes and sends *the same* message over b blocks, with codebooks in different blocks generated independently. Both techniques imply that when b is sufficiently large, certain rates are achievable. A comparison of the performance of these two basic principles for relay networks can be found in [WXar,HK12]. In the present work we adapt the terminology in [KH11] which refers to the first technique as *short-message* encoding and to the second technique as *long-message* encoding. In particular, our first coding scheme uses long-message encoding while the second and third scheme use short-message encoding. More on this topic can be found in Section 5.4.

5.2.3 Overview of the Relaying Protocol

The central ingredients in our coding schemes are the common-private rate-splitting [Car78,HK81] at the transmitters and a two-layer quantization process at the relay, which allows a better adaptation to the disparity of the receivers' channel quality. We study communication over b blocks, each built from n channel uses. Our three coding schemes share the same features at the relay as follows [LKEC10]: For each block, the relay generates two quantization codebooks, whose codewords are denoted by $\tilde{\mathbf{y}}_3$ and $\hat{\mathbf{y}}_3$, respectively. The indices of these quantization codewords are conveyed to the receivers by a superposition channel code [Cov72, Ber73], whose codewords are denoted by $\mathbf{v}$ (the cloud center) and $\mathbf{x}_3$ (the satellite), respectively. This method of constructing codebooks implicitly defines two layers for quantization: a *coarse* layer whose index is conveyed in $\mathbf{v}$ and the *fine* layer whose index is conveyed in $\mathbf{x}_3$. One of the receivers uses both layers in decoding its desired messages, while the other uses only the coarse layer. Fig. 5.3 illustrates how the codebook for block j at the relay is generated. In Fig. 5.3, k_{j-1} denotes the quantization index of the coarse layer while l_{j-1} denotes the quantization index of the fine layer, both indices are sent by the relay in block j, $j \in [1:b]$. Also note that each quantization codeword is indexed by indices related to two consecutive blocks (j and $j+1$), so that each decoder can combine the signals from the transmitters and from the relay, which is one-block delayed, in the decoding process. This method of having "overlapping‘" indices follows the idea of block-Markov encoding and decoding for the relay channel, put forth by Cover and El Gamal in [CE79].

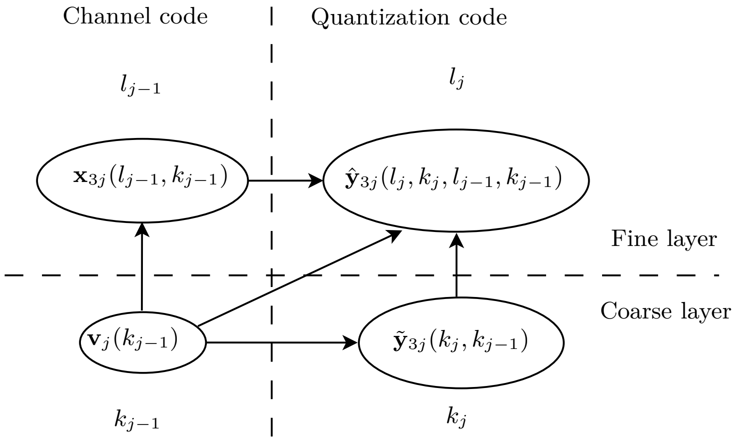

Figure 5.3: Generation of codewords for block j at the relay. $k_j, k_{j-1} \in [1 : 2^{n\tilde{R}}]$; $l_j, l_{j-1} \in [1 : 2^{n\hat{R}}]$. The arrows illustrate how a codeword is conditionally generated given another codeword, e.g., $\tilde{\mathbf{y}}_{3j}(k_j, k_{j-1})$ is generated i.i.d. conditionally on $\mathbf{v}_j(k_{j-1})$.

Codebooks in different blocks are generated independently. At the end of each block, the relay finds two new quantization indices such that the corresponding quantization codewords and the relay's transmitted codewords are jointly typical with the relay's received sequence. This is the quantization or the *covering* process. The relay then sends the newly found quantization indices in the next block via its channel code. Note that unlike in classical compress-forward [CE79], Wyner-Ziv binning is not used here.

5.2.4 Common Distribution and Mutual Information Terms

Before delving into the details of achievable rate regions, let us define a family of joint distributions and some mutual information terms that will appear repeatedly in this chapter.

Definition 5.1 (General distribution)**.** Let $\mathcal{P}$ denote the set of probability distributions $P(\cdot)$ that factor as

$$\begin{aligned}&P(q, u_1, u_2, x_1, x_2, v, x_3, y_1, y_2, y_3, \hat{y}_3, \tilde{y}_3) = \\ &p(q)p(u_1, x_1|q)p(u_2, x_2|q)p(x_3, v|q)p(\hat{y}_3, \tilde{y}_3|v, x_3, y_3, q)p(y_1, y_2, y_3|x_1, x_2, x_3), \quad (5.2)\end{aligned}$$

where $p(y_1, y_2, y_3|x_1, x_2, x_3)$ is set by the channel in Fig. 5.1.

In the above definition and in the remaining parts of this chapter, random variable U_i represents the common message sent by transmitter i, which can also be decoded by receiver j, $i, j \in \{1, 2\}, i \neq j$. $V, X_3, \tilde{Y}_3, \hat{Y}_3$ represent the codebook generating random variables, as described in Section 5.2.3. Q represents the time-sharing random variable.

Definition 5.2. For a given joint distribution $P \in \mathcal{P}$, define the following mutual information terms:

$$\tilde{r} := I(\tilde{Y}_3; X_3, Y_3 | V, Q) \tag{5.3}$$

$$\hat{r} := I(\hat{Y}_3; Y_3 | X_3, V, \tilde{Y}_3, Q) \tag{5.4}$$

$$a := I(\tilde{Y}_3; U_1, X_2, X_3, Y_2 | V, Q) \tag{5.5}$$

$$b := I(\hat{Y}_3; U_1, X_2, Y_2 | V, X_3, \tilde{Y}_3, Q) \tag{5.6}$$

$$c := I(\tilde{Y}_3; U_2, X_1, Y_1 | V, Q). \tag{5.7}$$

As an interpretation of the above mutual information terms, $\tilde{r}$ and $\hat{r}$ represent the minimum *covering* rates such that the quantization succeeds with high probability at the relay. a, b are the amounts of mutual information that two quantization layers contribute to the achievable rates at decoder 2, assuming that receiver 2 uses both layers for decoding. c is the amount of mutual information that the coarse quantization layer contributes to the achievable rates at decoder 1. The overall rate regions will include the other case, where the roles of the decoders are exchanged.

5.3 Achievability for the Discrete Memoryless ICR

In this section we present three inner bounds for the capacity region of the DM-ICR. We start out with a region based on noisy network coding, proceeding with a region achieved by quantize-forward with forward sliding window decoding and conclude with a region obtained by quantize-forward with backward decoding.

5.3.1 Achievability Based on Layered Noisy Network Coding

Our first inner bound is achieved by a coding scheme which is built upon combining the common-private rate-splitting technique [Car78,HK81] of the interference channel with the recently developed noisy network coding (NNC) technique [LKEC11] for relay networks. In particular, our scheme extends the *layered* NNC for the two-way relay channel proposed by Lim et al. in [LKEC10]. The new ingredients are the rate-splitting part and the non-unique decoding of the undesired messages. This achievable region will also serve as a benchmark for our other achievable regions that follow later.

Definition 5.3. Consider the set $\mathcal{P}$ in Definition 5.1. For a fixed joint distribution $P \in \mathcal{P}$, define $\mathcal{R}_1^{LNNC}(P)$ as the set of rate pairs $(R_1, R_2) = (R_{1c} + R_{1p}, R_{2c} + R_{2p})$

such that

$$R_{1c}, R_{1p}, R_{2c}, R_{2p} \geq 0 \tag{5.8}$$
$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2, V, X_3; Y_2|Q) + a + b - \tilde{r} - \hat{r} \tag{5.9a}$$
$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2, X_3; Y_2, \tilde{Y}_3|V, Q) + b - \hat{r} \tag{5.9b}$$
$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|V, X_3, Q) \tag{5.9c}$$
$$R_{1c} + R_{2p} < I(U_1, X_2, V, X_3; Y_2|U_2, Q) + a + b - \tilde{r} - \hat{r} \tag{5.10a}$$
$$R_{1c} + R_{2p} < I(U_1, X_2, X_3; \tilde{Y}_3, Y_2|V, U_2, Q) + b - \hat{r} \tag{5.10b}$$
$$R_{1c} + R_{2p} < I(U_1, X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_2, V, X_3, Q) \tag{5.10c}$$
$$R_{2c} + R_{2p} < I(X_2, V, X_3; Y_2|U_1, Q) + a + b - \tilde{r} - \hat{r} \tag{5.11a}$$
$$R_{2c} + R_{2p} < I(X_2, X_3; \tilde{Y}_3, Y_2|U_1, V, Q) + b - \hat{r} \tag{5.11b}$$
$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_1, V, X_3, Q) \tag{5.11c}$$
$$R_{2p} < I(X_2, V, X_3; Y_2|U_1, U_2, Q) + a + b - \tilde{r} - \hat{r} \tag{5.12a}$$
$$R_{2p} < I(X_2, X_3; \tilde{Y}_3, Y_2|U_1, U_2, V, Q) + b - \hat{r} \tag{5.12b}$$
$$R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_1, U_2, V, X_3, Q) \tag{5.12c}$$
$$R_{2c} + R_{1c} + R_{1p} < I(U_2, X_1, V; Y_1|Q) + c - \tilde{r} \tag{5.13a}$$
$$R_{2c} + R_{1c} + R_{1p} < I(U_2, X_1; Y_1, \tilde{Y}_3|V, Q) \tag{5.13b}$$
$$R_{2c} + R_{1p} < I(U_2, X_1, V; Y_1|U_1, Q) + c - \tilde{r} \tag{5.14a}$$
$$R_{2c} + R_{1p} < I(U_2, X_1; \tilde{Y}_3, Y_1|V, U_1, Q) \tag{5.14b}$$
$$R_{1c} + R_{1p} < I(X_1, V; Y_1|U_2, Q) + c - \tilde{r} \tag{5.15a}$$
$$R_{1c} + R_{1p} < I(X_1; \tilde{Y}_3, Y_1|U_2, V, Q) \tag{5.15b}$$
$$R_{1p} < I(X_1, V; Y_1|U_2, U_1, Q) + c - \tilde{r} \tag{5.16a}$$
$$R_{1p} < I(X_1; \tilde{Y}_3, Y_1|U_2, U_1, V, Q), \tag{5.16b}$$

where $\tilde{r}, \hat{r}, a, b, c$ are specified in Definition 5.2. Furthermore, define $\mathcal{R}_2^{LNNC}(P)$ by exchanging indices $1 \leftrightarrow 2$ in every term of $\mathcal{R}_1^{LNNC}(P)$.

Theorem 5.3.1 (Inner bound - layered NNC)**.** *For the discrete memoryless interference channel with a relay, any rate pair (R_1, R_2) in the set*

$$\mathcal{R}^{LNNC} = \mathsf{Co}\left(\bigcup_{P \in \mathcal{P}} \mathcal{R}_1^{LNNC}(P) \cup \mathcal{R}_2^{LNNC}(P)\right) \tag{5.17}$$

is achievable.

Proof. The detailed proof is presented in Appendix 5.A. The main idea is as follows. Each encoder i performs rate-splitting of its nbR_i-bit long message, $i \in \{1, 2\}$, and encodes this message b times using b independently generated codebooks, each for one block. The relay's codebook and operation in each block are as described in Section 5.2.3. At the end of the bth block, each receiver jointly decodes its desired message, the common message of the interfering transmitter (non-uniquely), and the quantization indices (non-uniquely), over all b blocks. One of the two receivers will use both quantization layers in decoding while the other one uses only the coarse layer. The error analysis shows that as n and b go to infinity there exists a code with vanishing error probability, so that the rates specified in Definition 5.3 are achievable. □

We can perform Fourier–Motzkin elimination to obtain a region in terms of R_1 and R_2. We keep the region in the current form for the ease of comparing with other achievable regions later.

Remark 5.1. The reader may recognize that the rate region in Definition 5.3 has a similar form to that of the interference channel in [CMGE08]. Namely the rates at decoder i includes 3 types of constraints: on R_{ip}, $R_{ic} + R_{ip}$, $R_{jc} + R_{ip}$, and $R_{ic} + R_{ip} + R_{jc}$, $i, j \in \{1, 2\}, i \neq j$. The difference from that of [CMGE08] is that, due to the participation of the relay, each type of constraint now has more than one inequality. Further, since $\mathcal{R}_1^{LNNC}(P)$ is obtained by assuming that receiver 2 uses two quantization layers for decoding while receiver 1 uses only the coarse layer, each type of constraints at decoder 2 has more inequalities than at decoder 1, e.g., (5.9) versus (5.13). $\mathcal{R}_2^{LNNC}(P)$ is obtained by exchanging the roles of the receivers.

Remark 5.2. By tuning the rate-splitting at the transmitter we can obtain two extremal achievable regions: rate region when the receivers treat interference as noise (by setting $U_i = \emptyset$) and rate region when the receivers decode interference (by setting $U_i = X_i$). Further, by removing one layer of quantization, i.e., removing X_3 and $\hat{Y}_3$, we can recover rate regions achieved by noisy network coding in [LKEC11, Theorem 3] and [LKEC11, Theorem 2] when applying to the ICR, respectively. On the other hand, if we remove all parameters related to the relay we recover the famous Han-Kobayashi rate region [CMGE08] for the two-user interference channel.

Remark 5.3. The above achievable rates are derived from bounding the probabilities of error using the technique of [LKEC11] (noisy network coding). However, as we have seen in Appendix 4.B, one can bound the error probability in an alternative way, which may help improve the achievable rates. For the sake of presentation, we only provide the expressions obtained from NNC-style of error analysis [LKEC11] in Theorem 1. We will partially introduce the alternative characterization in the proof of Theorem 5.5.1.

In Section 5.5 we will indirectly show that the achievable rates in Theorem 5.3.1, when applied to a class of Gaussian channels and under certain conditions, are within a constant gap to the capacity region of the channel. The sum-rate performance for another class of Gaussian channels will be illustrated in Section 5.6.

5.3.2 Layered Quantize-Forward With Sliding Window Decoding

As mentioned in the outlined proof of Theorem 5.3.1, the key techniques used in noisy network coding are: (i) repetitively encoding the same long message over b blocks (by independently generated codebooks), and (ii) jointly decoding over all the blocks. As can be deduced from Section 5.2.2, the size of the message set grows exponentially in b in the case of long-message encoding, while it does not depend on b for short-message encoding. Further, waiting till the end of b blocks and decoding jointly over all blocks cause large delay and complexity. Therefore, our first coding scheme may not be suitable for delay-sensitive applications and/or for low-complexity hardware implementation. In order to reduce the coding delay as well as the complexity, in this section we take one step backward to the classical compress-forward [CE79], by employing short-message encoding and sliding window decoding in a block-by-block fashion. However, unlike [CE79, Theorem 6], the decoders will decode the quantization indices and message indices simultaneously. The detailed proof is presented in Appendix 5.B. Here we state the achievable rates and explain the main ideas.

Definition 5.4. Let $\mathcal{P}_1$ denote a subset of pmf's in $\mathcal{P}$ that satisfies

$$\tilde{r} + \hat{r} < I(V, X_3; Y_2|Q) + a + b \tag{5.18a}$$
$$\hat{r} < I(X_3; Y_2|V, Q) + b \tag{5.18b}$$
$$\tilde{r} < I(V; Y_1|Q) + c, \tag{5.18c}$$

where $a, b, c, \tilde{r}, \hat{r}$ are given in Definition 5.2. For a fixed $P \in \mathcal{P}_1$, define $\mathcal{R}_1^{FW}(P)$ to be the set of rate pairs $(R_1, R_2) = (R_{1c} + R_{1p}, R_{2c} + R_{2p})$ such that

$$R_{1c}, R_{1p}, R_{2c}, R_{2p} \geq 0 \tag{5.19}$$
$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2, V, X_3; Y_2|Q) + a + b - \tilde{r} - \hat{r} \tag{5.20a}$$
$$R_{1c} + R_{2c} + R_{2p} < I(X_3; Y_2|V, Q) + I(U_1, X_2; \tilde{Y}_3, Y_2|V, X_3, Q) + b - \hat{r} \tag{5.20b}$$
$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2; \hat{Y}_3, \tilde{Y}_3, Y_2|V, X_3, Q) \tag{5.20c}$$
$$R_{1c} + R_{2p} < I(V, X_3; Y_2|Q) + I(U_1, X_2; Y_2|U_2, V, X_3, Q) + a + b - \tilde{r} - \hat{r} \tag{5.21a}$$
$$R_{1c} + R_{2p} < I(X_3; Y_2|V, Q) + I(U_1, X_2; \tilde{Y}_3, Y_2|U_2, V, X_3, Q) + b - \hat{r} \tag{5.21b}$$
$$R_{1c} + R_{2p} < I(U_1, X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_2, V, X_3, Q) \tag{5.21c}$$
$$R_{1c} < I(V, X_3; Y_2|Q) + I(U_1; Y_2|X_2, V, X_3, Q) + a + b - \tilde{r} - \hat{r} \tag{5.22a}$$
$$R_{1c} < I(X_3; Y_2|V, Q) + I(U_1; \tilde{Y}_3, Y_2|X_2, V, X_3, Q) + b - \hat{r} \tag{5.22b}$$
$$R_{2c} + R_{2p} < I(V, X_3; Y_2|Q) + I(X_2; Y_2|U_1, V, X_3, Q) + a + b - \tilde{r} - \hat{r} \tag{5.23a}$$

$$R_{2c}+R_{2p} < I(X_3;Y_2|V,Q)+I(X_2;\tilde{Y}_3,Y_2|U_1,V,X_3,Q)+b-\hat{r} \tag{5.23b}$$
$$R_{2c}+R_{2p} < I(X_2;\tilde{Y}_3,\hat{Y}_3,Y_2|U_1,V,X_3,Q) \tag{5.23c}$$
$$R_{2p} < I(V,X_3;Y_2|Q)+I(X_2;Y_2|U_1,U_2,V,X_3,Q)+a+b-\tilde{r}-\hat{r} \tag{5.24a}$$
$$R_{2p} < I(X_3;Y_2|V,Q)+I(X_2;\tilde{Y}_3,Y_2|U_1,U_2,V,X_3,Q)+b-\hat{r} \tag{5.24b}$$
$$R_{2p} < I(X_2;\tilde{Y}_3,\hat{Y}_3,Y_2|U_1,U_2,V,X_3,Q) \tag{5.24c}$$
$$R_{2c}+R_{1c}+R_{1p} < I(U_2,X_1,V;Y_1|Q)+c-\tilde{r} \tag{5.25a}$$
$$R_{2c}+R_{1c}+R_{1p} < I(U_2,X_1;\tilde{Y}_3,Y_1|V,Q) \tag{5.25b}$$
$$R_{2c}+R_{1p} < I(V;Y_1|Q)+I(U_2,X_1;Y_1|U_1,V,Q)+c-\tilde{r} \tag{5.26a}$$
$$R_{2c}+R_{1p} < I(U_2,X_1;\tilde{Y}_3,Y_1|U_1,V,Q) \tag{5.26b}$$
$$R_{2c} < I(V;Y_1|Q)+I(U_2;Y_1|X_1,V,Q)+c-\tilde{r} \tag{5.27}$$
$$R_{1c}+R_{1p} < I(V;Y_1|Q)+I(X_1;Y_1|U_2,V,Q)+c-\tilde{r} \tag{5.28a}$$
$$R_{1c}+R_{1p} < I(X_1;\tilde{Y}_3,Y_1|U_2,V,Q) \tag{5.28b}$$
$$R_{1p} < I(V;Y_1|Q)+I(X_1;Y_1|U_1,U_2,V,Q)+c-\tilde{r} \tag{5.29a}$$
$$R_{1p} < I(X_1;\tilde{Y}_3,Y_1|U_1,U_2,V,Q). \tag{5.29b}$$

Furthermore, define $\mathcal{R}_2^{FW}(P)$ by exchanging indices $1 \leftrightarrow 2$ in every term of $\mathcal{R}_1^{FW}(P)$.

Theorem 5.3.2 (Inner bound, sliding window decoding)**.** *For the discrete memoryless interference channel with a relay, any rate pair (R_1, R_2) in the set*

$$\mathcal{R}^{FW} = \mathsf{Co}\left(\bigcup_{P\in\mathcal{P}_1} \mathcal{R}_1^{FW}(P) \cup \mathcal{R}_2^{FW}(P)\right) \tag{5.30}$$

is achievable.

Proof. The detailed proof is relegated to Appendix 5.B. We outline the scheme here. A long nbR_i-bits message m_i, $i \in \{1,2\}$ is evenly split into b parts, each consisting of nR_i bits. These messages are sent over $b+1$ blocks, each consists of n channel uses. As b gets large the rate loss is negligible. Furthermore, the message in each block is split into a pair of common and private messages. The relay's codebook and operation in each block are as described in Section 5.2.3. At the end of block $j+1$, $j \in [1:b]$, each receiver resolves its desired message by checking *two* joint typicality conditions of two groups of sequences as follows. Group 1 consists of codewords from block j's codebook: the ones that conveys the receiver's desired common and private messages, the one that conveys the common message from the interfering transmitter, the quantization codewords and channel codewords of the relay in block j, and the receiver's received sequence in block j. Group 2 consists of codewords from block $j+1$'s codebook, which includes the channel codewords of the relay in block $j+1$ and the receiver's received sequence in block $j+1$. The indices associated with those sequences must be *unique*, except for the index of the common message of the interfering transmitter (in group 1). Moreover, the quantization indices involving in both typicality checks must be the same, to ensure that the indices found in block j match the ones sent by the relay in block $j+1$. Similarly to the scheme in Section 5.3.1, one of the receivers uses both quantization layers while the other uses only the coarse layer. □

Remark 5.4. The careful reader may notice that the rate constraints in (5.20)–(5.29) have a very similar form to those in (5.9)–(5.16), except for the extra constraints in (5.22) and (5.27). These constraints, together with the conditions in (5.18), appear due to the requirement that the quantization indices are to be decoded uniquely, which is in contrast to the scheme in Section 5.3.1 where this uniqueness is not required. While in the classical 3-node relay channel [CE79] this uniqueness does not incur any loss on the achievable region [KH11, ZV11], it does reduce the achievable rates for the ICR as will be shown in Section 5.4.

5.3.3 Layered Quantize-Forward With Backward Decoding

So far we have presented two coding schemes which differ in the encoding and decoding processes. The first scheme has long encoding and decoding delay while the second results in shorter encoding and decoding delay. We will now describe a coding scheme suitable for applications which are not delay-critical, potentially achieving higher rates than forward sliding window decoding. Let us retain the encoding and relaying processes of the LQF-FW and modify the decoding process by letting the receivers wait until having received $b+1$ blocks, and then start decoding backward block-by-block [Wil82, Ch. 7]. Then the following rate region is achievable.

Definition 5.5. Let $\mathcal{P}_2$ denote a subset of pmf's in $\mathcal{P}$ that satisfies

$$\tilde{r} + \hat{r} < I(V, X_3; Y_2|U_1, X_2, Q) + a + b \tag{5.31a}$$

$$\hat{r} < I(X_3; Y_2, \tilde{Y}_3|U_1, X_2, V, Q) + b \tag{5.31b}$$

$$\tilde{r} < I(V; Y_1|U_2, X_1, Q) + c, \tag{5.31c}$$

where $\tilde{r}, \hat{r}, a, b, c$ are defined in Definition 5.2. For a fixed $P \in \mathcal{P}_2$, define $\mathcal{R}_1^{BW}(P)$ to be the set of rate pairs $(R_1, R_2) = (R_{1c} + R_{1p}, R_{2c} + R_{2p})$ such that

$$R_{1c}, R_{1p}, R_{2c}, R_{2p} \geq 0 \tag{5.32}$$

$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2, V, X_3; Y_2|Q) + a + b - \tilde{r} - \hat{r} \tag{5.33a}$$

$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2, X_3; \tilde{Y}_3, Y_2|V, Q) + b - \hat{r} \tag{5.33b}$$

$$R_{1c} + R_{2c} + R_{2p} < I(U_1, X_2; \hat{Y}_3, \tilde{Y}_3, Y_2|V, X_3, Q) \tag{5.33c}$$

$$R_{1c} + R_{2p} < I(U_1, X_2, V, X_3; Y_2|U_2, Q) + a + b - \tilde{r} - \hat{r} \tag{5.34a}$$

$$R_{1c} + R_{2p} < I(U_1, X_2, X_3; \tilde{Y}_3, Y_2 | U_2, V, Q) + b - \hat{r} \tag{5.34b}$$

$$R_{1c} + R_{2p} < I(U_1, X_2; \tilde{Y}_3, \hat{Y}_3, Y_2 | U_2, V, X_3, Q) \tag{5.34c}$$

$$R_{1c} < I(U_1, V, X_3; Y_2 | X_2, Q) + a + b - \tilde{r} - \hat{r} \tag{5.35a}$$

$$R_{1c} < I(U_1, X_3; \tilde{Y}_3, Y_2 | X_2, V, Q) + b - \hat{r} \tag{5.35b}$$

$$R_{2c} + R_{2p} < I(X_2, V, X_3; Y_2 | U_1, Q) + a + b - \tilde{r} - \hat{r} \tag{5.36a}$$

$$R_{2c} + R_{2p} < I(X_2, X_3; \tilde{Y}_3, Y_2 | U_1, V, Q) + b - \hat{r} \tag{5.36b}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2 | U_1, V, X_3, Q) \tag{5.36c}$$

$$R_{2p} < I(X_2, V, X_3; Y_2 | U_1, U_2, Q) + a + b - \tilde{r} - \hat{r} \tag{5.37a}$$

$$R_{2p} < I(X_2, X_3; \tilde{Y}_3, Y_2 | U_1, U_2, V, Q) + b - \hat{r} \tag{5.37b}$$

$$R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2 | U_1, U_2, V, X_3, Q) \tag{5.37c}$$

$$R_{2c} + R_{1c} + R_{1p} < I(U_2, X_1, V; Y_1 | Q) + c - \tilde{r} \tag{5.38a}$$

$$R_{2c} + R_{1c} + R_{1p} < I(U_2, X_1; \tilde{Y}_3, Y_1 | V, Q) \tag{5.38b}$$

$$R_{2c} + R_{1p} < I(U_2, X_1, V; Y_1 | U_1, Q) + c - \tilde{r} \tag{5.39a}$$

$$R_{2c} + R_{1p} < I(U_2, X_1; \tilde{Y}_3, Y_1 | U_1, V, Q) \tag{5.39b}$$

$$R_{2c} < I(U_2, V; Y_1 | X_1, Q) + c - \tilde{r} \tag{5.40}$$

$$R_{1c} + R_{1p} < I(X_1, V; Y_1 | U_2, Q) + c - \tilde{r} \tag{5.41a}$$

$$R_{1c} + R_{1p} < I(X_1; \tilde{Y}_3, Y_1 | U_2, V, Q) \tag{5.41b}$$

$$R_{1p} < I(X_1, V; Y_1 | U_1, U_2, Q) + c - \tilde{r} \tag{5.42a}$$

$$R_{1p} < I(X_1; \tilde{Y}_3, Y_1 | U_1, U_2, V, Q). \tag{5.42b}$$

Furthermore, define $\mathcal{R}_2^{BW}(P)$ by exchanging indices $1 \leftrightarrow 2$ in every term of $\mathcal{R}_1^{BW}(P)$.

Theorem 5.3.3 (Inner bound - backward decoding). *For the discrete memoryless interference channel with a relay, any rate pair (R_1, R_2) in the set*

$$\mathcal{R}^{BW} = \mathsf{Co}\left(\bigcup_{P \in \mathcal{P}_2} \mathcal{R}_1^{BW}(P) \cup \mathcal{R}_2^{BW}(P) \right) \tag{5.43}$$

is achievable.

Proof. The detailed proof is presented in Appendix 5.C. Here we outline the proof: The codebook generation, encoding and relaying processes are the same as in LQF-FW. The key difference lies in the decoding process. Specifically, after receiving the signals from all $b+1$ blocks, each destination starts decoding backwardly block-by-block. In block j, each receiver resolves its desired message by checking the joint typicality of the following sequences from the codebooks for block j: the codewords conveying the receiver's desired common and private messages, the codeword conveying the common message from the interfering transmitter, the quantization codewords and channel codewords of the relay, and the received sequence of the receiver in block j. The indices associated with those sequences must be *unique*, except for the index of the common message of the interfering transmitter. In the same manner as in LNNC and LQF-FW schemes, one of the receivers uses both quantization layers while the other uses only the coarse layer, resulting in two different regions $\mathcal{R}_1^{BW}$ and $\mathcal{R}_2^{BW}$. □

Remark 5.5. Similarly to Remark 5.4, we notice that the achievable rates by LQF-BW has some extra conditions compared with the achievable rates by LNNC, in (5.35) and (5.40). These extra conditions, together with (5.31), appear due to the unique decodability of the quantization indices in LQF-BW scheme, while in LNNC this uniqueness is not required.

5.4 Comparison of Rate Regions

In this section we compare the three achievable rate regions in Theorem 5.3.1, Theorem 5.3.2, and Theorem 5.3.3. This will help identify the impact of different elements in the coding strategies on the achievable regions. First, it is rather straightforward to see that:

Proposition 5.4.1. *We have*

$$\mathcal{R}^{FW} \subseteq \mathcal{R}^{BW} \subseteq \mathcal{R}^{LNNC}, \tag{5.44}$$

namely the rate regions achieved by layered noisy network coding includes the region achieved by layered quantize-forward with backward decoding, which in turn includes the region achieved by layered quantize-forward with forward sliding window decoding.

Proof. First note that since $(U_1, X_1), (U_2, X_2), (V, X_3)$ are mutually independent, the right hand sides of (5.31) are smaller than those of (5.18). Hence, $\mathcal{P}_1 \subseteq \mathcal{P}_2$. One can also verify that for a fixed distribution, the constraints in (5.33)–(5.42) are looser than the respective constraints in (5.20)–(5.29), i.e., $\mathcal{R}_1^{FW}(P) \subseteq \mathcal{R}_1^{BW}(P)$. Accordingly, the first inclusion in (5.44) is proved. Numerical examples in Section 5.6.1 will show that this inclusion is strict for the Gaussian IC with an in-band relay.

On the other hand, we observe that $\mathcal{R}_1^{BW}(P)$ contains all the rate constraints of $\mathcal{R}_1^{LNNC}(P)$, plus three additional constraints in (5.35), and (5.40). Moreover, $\mathcal{P}_2$ is a subset of $\mathcal{P}$. Therefore, the second inclusion in (5.44) is established. □

Remark 5.6. The first inclusion (5.44) can be explained by the fact that in LQF-FW there are two separate joint typicalities that need to be checked in each decoding step, to make sure that the quantization indices that the decoder is choosing for block j is actually the ones that are sent by the relay in block $j+1$. In contrast, since such indices are known "non-causally" in LQF-BW, only one joint typicality check is needed. This produces looser constraints on the rates, at the price of higher delay.

Table 5.1: Comparison of coding schemes

Scheme	Achievable rate region	Encoding Delay	Decoding delay	Decoding complexity
LNNC	Largest	Linear in b	Exponential in b	Highest
LQF-BW	2nd largest	Does not depend on b	Linear in b	$<$ LNNC
LQF-FW	Smallest	Does not depend on b	Does not depend on b	$=$ LQF-BW

As discussed in the beginning of Section 5.3.2, long-message encoding leads to exponentially larger message sets compared to short-message encoding, which in turn induces heavily increased encoding and decoding complexity since the number of table lookups at the encoder and the number of joint typicality tests at the decoder scale with the size of the message sets. If the number of lookups/tests that can be performed per unit time is fixed, the growth of the size of the message sets will also lead to correspondingly larger encoding and decoding delays. Moreover, from the description of our coding schemes we can see that both LNNC and LQF-BW have much larger decoding delay than LQF-FW due to the fact that the decoders need to wait until the end of b blocks. The decoding complexity of the LQF-BW scheme appears lower than that of LQF-FW since the former requires only one joint typicality test in its decoding rule while the latter requires two joint typicality tests. However, a closer look reveals that the number of codewords that a particular decoder needs to go through during the joint typicality test are the same for both schemes, e.g., (5.60) vs. (5.66). Hence, the two schemes have essentially the same decoding complexity, which is lower than that of LNNC. As a result we have the following Table 5.1, which shows the trade-off of various performance metrics of our proposed schemes.

5.5 Gaussian IC With In-band Relay Reception/Out-of-band Relay Transmission

With the general achievable regions in hand, we now proceed to studying a class of Gaussian channels and show that our first coding scheme, based on rate-splitting and LNNC, performs as well as a scheme which uses explicit binning at the relay, which achieves a constant gap to the capacity region under certain conditions.

Consider the ICR with AWGN noise at the relay and at the two receivers. Suppose that the relay and the receivers are connected by noiseless relay links with rates C_1 and C_2, respectively. Following the classification in [SES09] we call this channel the Gaussian IC with in-band relay reception/out-of-band relay transmission, or Gaussian IC-IOR for brevity. In [ZY13] it was shown that for the Gaussian IC-IOR with a "weak" relay, the GHF coding scheme [RY10] can be extended to achieve the capacity region within a bounded gap, whose value depends on the ratios between the transmitter-relay links and interference links and is constant for a given channel. We now show that for the Gaussian IC-IOR our coding scheme achieves the same rates as in [ZY13].

Theorem 5.5.1. *For the Gaussian IC-IOR the rate region achieved by our first coding scheme (Theorem 5.3.1) is the same as the rate region achieved by an extension of GHF in [ZY13, Theorem 2].*

Proof. The proof consists of the following steps and is presented in detail in Appendix 5.D: i) Introduce an alternative expression of the achievable rates, ii) Describe the quantization method and specialize the achievable rate region in Theorem 5.3.1 to the Gaussian IC-IOR, and show that the resulting rate constraints equal the corresponding ones in [ZY13, Theorem 2]. □

We note that a similar observation has been made in a recent work for the interference channel with a shared out-of-band relay of limited rate [RHZ$^+$13]. As a direct consequence of Theorem 5.5.1 we have the following result:

Corollary 5.5.1. *For the Gaussian IC-IOR, let h_{kr} denote the channel gain between transmitter k and the relay, $k \in \{1,2\}$, and h_{ij}, $i, j \in \{1,2\}, i \neq j$, denotes the channel gain between transmitter i and receiver j. Our coding scheme based on LNNC (Theorem 5.3.1) achieves a constant gap to the capacity region of the Gaussian IC-IOR if*

$$\max\left\{\frac{|h_{1r}|^2}{|h_{12}|^2}, \frac{|h_{2r}|^2}{|h_{21}|^2}\right\} = \rho < \infty. \tag{5.45}$$

Further, the gap is given by

$$\delta = \frac{1}{2}\log\left(2 + \frac{\rho + \sqrt{\rho^2 + 16\rho + 16}}{2}\right). \tag{5.46}$$

Proof. This corollary follows directly from Theorem 5.5.1 above and [ZY13, Theorem 3]. □

Remark 5.7. It has been pointed out in [LKEC11,KH11] that noisy network coding is closely related to the quantize-map-forward (QMF) protocol introduced by Avestimehr et al. [ADT11] for relay networks. However, both these schemes do not employ binning. In contrast, in Theorem 4.4.2 we have shown that NNC achieves the same rates as quantize-bin-forward for the interference channel with orthogonal receiver cooperation. Theorem 5.5.1 further shows that NNC achieves the same rates as GHF, which also uses binning, for the IC-IOR. The reason for the same achievable rates achieved by noisy network coding, quantize-bin-forward, and generalized hash-forward in such channels can be attributed to the implicit binning effect that NNC has on those channels, see for example [RHZ$^+$13]. Theorems 4.4.2 and 5.5.1, therefore, have drawn an interesting connection among different coding strategies for some interference networks with orthogonal components. It also shows that noisy network coding, which is known to be nearly optimal for multicast Gaussian relay networks [LKEC11], can be combined with the rate-splitting technique often used for interference channels, to achieve good performance in interference networks with relays.

Another consequence of Theorem 5.5.1 is that it allows us to do the GDoF analysis for NNC for the Gaussian IC-IOR based on an outer bound established in [ZY13]. The details are the same as in [ZY13, Corollary 2].

5.6 Gaussian IC With an In-Band Relay

Let us now consider the discrete-time memoryless Gaussian IC with an in-band reception/in-band transmission relay. Hereafter we call it IC with an in-band relay or IC-IIR for brevity. The received signals of the Gaussian IC-IIR are given by

$$Y_1 = h_{11}X_1 + h_{21}X_2 + h_{31}X_3 + Z_1 \tag{5.47}$$

$$Y_2 = h_{12}X_1 + h_{22}X_2 + h_{32}X_3 + Z_2 \tag{5.48}$$

$$Y_3 = h_{13}X_1 + h_{23}X_2 + Z_3, \tag{5.49}$$

where Z_1, Z_2, Z_3 are i.i.d.$\sim \mathcal{N}(0,1)$. The channel gains h_{ij}, $i, j \in \{1,2,3\}$, $i \neq j$, are known to all nodes. Transmitter k, $k \in \{1,2\}$, has an average power constraint P_k. The relay has an average power constraint P_3.

In what follows we will apply the achievable schemes in Section 5.3 for the discrete memoryless ICR to the Gaussian channel. Such extension can be fulfilled by the discretization procedure as introduced in Section 2.3. Next, we choose X_i be i.i.d. $\sim \mathcal{N}(0, P_i)$ for $i \in \{1,2\}$, $V \sim N(0, \alpha P_3)$ for $\alpha \in [0,1]$, $X_3 = V + W$ where $W \sim \mathcal{N}(0, (1-\alpha)P_3)$, and X_1, X_2, V, W are mutually independent. Furthermore, let $\hat{Y}_3 = Y_3 + \hat{Z}_3$ and $\tilde{Y}_3 = \hat{Y}_3 + \tilde{Z}_3$ where $\hat{Z}_3 \sim \mathcal{N}(0, \hat{\sigma}^2)$ and $\tilde{Z}_3 \sim \mathcal{N}(0, \tilde{\sigma}^2)$, independent of everything else.

5.6.1 Numerical Examples

In this part we will compare the achievable sum-rate of our coding schemes with other CF-based schemes as summarized in [TY11], namely compress-forward (CF) [CE79], generalized compress-forward (GCF) [TY11], generalized hash-and-forward (GHF) [RY10], noisy network coding (NNC) [LKEC11]. Moreover, the comparison is performed at two extremes: The decoders decode interference and the decoders treat interference as noise. More specifically, in the plots LNNC denotes the rate from Theorem 5.3.1, LQF-FW denotes the rate from Theorem 5.3.2, LQF-BW denotes the rate from Theorem 5.3.3, CF is from [TY11, Theorem 1], GCF is from [TY11, Theorem 2], GHF is from [TY11, Eq.(17)–Eq.(22)], and NNC is from [LKEC11, Theorems 2 and 3]. The expressions for the LNNC, LQF-FW, and LQF-BW rates are given in Appendix 5.E. In these examples the exhaustive search method is used to find the parameters α, $\hat{\sigma}^2$, and $\tilde{\sigma}^2$ that maximize the sum-rates.

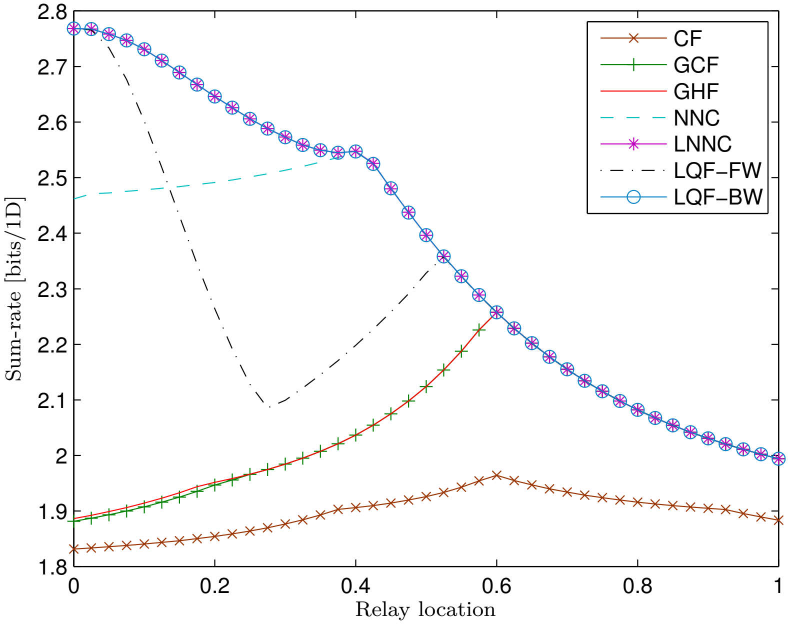

Figure 5.4: Sum-rate for Gaussian ICR, receivers decode interference. $h_{11} = 0.5, h_{12} = 2, h_{21} = 1, h_{22} = 0.5, h_{13} = 1, h_{23} = 2, P_1 = P_2 = P = 10, P_r = \sqrt{P}$.

Fig. 5.4 shows the achievable sum-rates when interference is decoded at each decoder. For illustration we choose the channel parameters following a geometric model as follows: Channel gains between the relay and the receivers are given by $h_{31} = d^{-\gamma}$, $h_{32} = (1-d)^{-\gamma}$, where $d \in (0,1)$ is the location of the relay between receivers 1 and 2, and $\gamma = 3$ is the path loss exponent. Other channel gains are fixed and the underlying interference channel is in the strong interference regime, i.e., $|h_{12}|^2 \geq |h_{11}|^2$ and $|h_{21}|^2 \geq |h_{22}|^2$, as shown in the figure. The LNNC sum-rate is calculated from $\mathcal{R}_2^{LNNC}$ in Theorem 5.3.1, which is in favor of receiver 1 (receiver 1 uses two layers for decoding). We see that when the relay is close to receiver 1, the layered coding schemes outperform those without layered codes. In particular, when d is less than about 0.4, layered NNC outperforms NNC and other schemes. Layered quantize-forward with backward decoding achieves the same sum-rate as layered noisy network coding for all channel gains, demonstrating that we can achieve the LNNC performance by a simpler coding scheme in certain cases. Meanwhile, the forward decoding scheme LQF-FW is the runner up when the relay is in the vicinity of node 1 (d is less than about 0.12). The fact that LQF-FW even outperforms NNC without layering in that case suggests that having layered

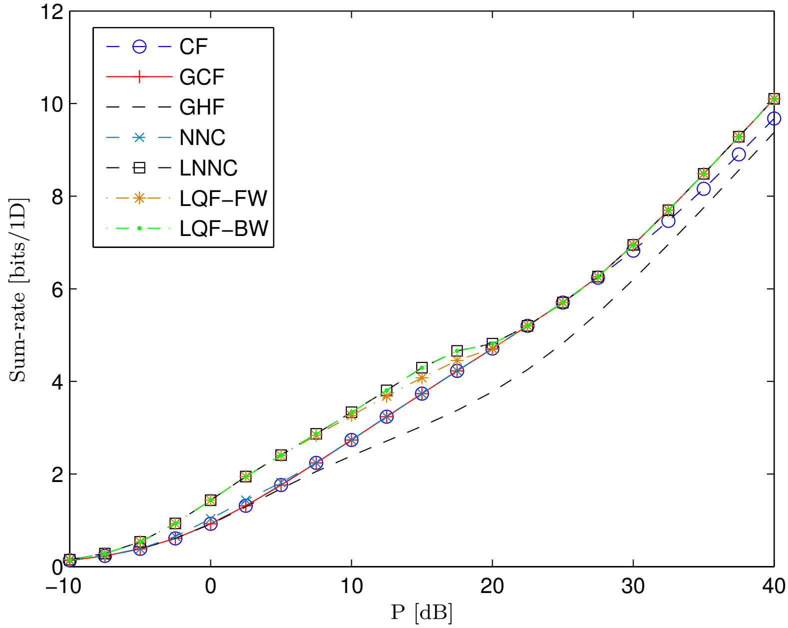

Figure 5.5: Sum-rate for Gaussian ICR, receivers treat interference as noise. $h_{11} = 1, h_{12} = 0.2, h_{21} = 0.5, h_{22} = 1, h_{13} = 0.2, h_{23} = 2, h_{31} = 0.3, h_{32} = 3, P_1 = P_2 = P, P_r = P^2$.

codes can sometimes be more beneficial than the united coding over many blocks. As d gets larger we expect that the disparity in effective channel gains of the receivers diminishes, and the gain due to layered codes reduces. LNNC and LQF-BW gradually converge to NNC. In this case having two-layer quantization is no longer useful in improving the achievable sum-rate. Furthermore, we can see that NNC, LNNC, LQF-FW, and LQF-BW, which are not restricted by the binning process, achieve higher sum-rates than CF, GCF, GHF.

Fig. 5.5 compares achievable sum-rates of the schemes when interference is treated as noise at each decoder. The channel parameters are detailed in the caption and the underlying interference channel is in the weak interference regime. In this plot we create the disparity in the channel quality to the receivers by varying the transmit powers at the transmitters and the relay. The plot shows that when the power of the transmitters and the relay are low or medium, the difference in quality of the overall channels of the two receivers are significant enough so that layered relaying schemes achieve higher sum-rates than the non-layered schemes. In the high SNR regime, however, the difference in channel qualities become negligible and

LNNC, LQF-BW, LQF-FW, NNC, and GCF perform equally well. Similarly to the numerical result in Fig. 5.4, we can also see that the layered scheme with backward decoding achieves the same sum-rate as LNNC in all SNR regimes. The layered scheme with forward sliding window decoding performs equally well as LNNC and the backward decoding scheme for most SNR's, except for certain mid-range SNR, but still better than other non-layered schemes. GHF in this case achieves the least sum-rate.

We also observe that both quantize-forward with forward sliding window decoding and backward decoding achieve higher sum-rate than traditional compress-forward [CE79]. This is in contrast to the classical three-node relay channel, where quantize-forward with sliding window decoding is shown [KH11] to achieve the same rate as classical compress-forward.

With reference to Table 5.1, the above numerical results demonstrate that: When the channel is asymmetric we can rank the importance of different factors in obtaining highest achievable rates as follows: layering > long delay decoding > high decoding complexity.

5.7 Chapter Conclusion

We propose new quantize-forward protocols for the discrete memoryless interference channel with a relay. Our approaches rely on layered coding in order to better adapt to the channel disparity experienced by the receivers. The schemes have different levels of coding delay and complexity, but all show improved performance compared with other non-layered compress-forward schemes when the channels to the receivers are asymmetric. Moreover our schemes can be applied to different classes of ICRs, without resorting to any specific binning scheme. Hence, our proposed protocols provide new general tools for coding for the ICR. The results in this chapter suggest that using layered cods at the relay is very beneficial for the ICR as well as for any network wherein a relay is to serve multiple destinations at the same time. Further, the obtained results also imply that using noisy network coding, when combined with rate-splitting encoding, is sufficient to achieve a bounded gap to the capacity region of some important multi-unicast networks (see also the result in Chapter 4). Finally, our detailed analysis of the different approaches reveals trade-offs between performance, delay and complexity.

As a final remark, the numerical examples in Section 5.6.1 have shown that the LQF-BW scheme achieves the same sum-rate as that of LNNC in some special cases. This suggests that one can further investigate whether they achieve the same rate regions in general. One can consider taking advantage of the short-message nature to improve the performance of LQF-FW and LQF-BW coding schemes. A natural extension would be combining quantize-forward with decode-forward.

Appendices for Chapter 5

5.A Proof of Theorem 5.3.1

In what follows we present the coding scheme that achieves the region $\mathcal{R}_1^{NNC}$ in Theorem 5.3.1, the achievability of $\mathcal{R}_2^{NNC}$ follows by switching the roles of the receivers. The transmission of each message takes place in b blocks. We use subscript j to index the blocks and subscript s to index the transmitters.

Codebook Generation. For a message pair $(m_1, m_2) \in [1 : 2^{nbR_1}] \times [1 : 2^{nbR_2}]$ in the message sets of the two transmitters, split the messages as follows: $m_1 = (m_{1c}, m_{1p})$, $m_2 = (m_{2c}, m_{2p})$, where $m_{sc} \in [1 : 2^{nbR_{sc}}]$, $m_{sp} \in [1 : 2^{nbR_{sp}}]$, and $R_{sc} + R_{sp} = R_s$, $s \in \{1, 2\}$. For brevity we consider $Q = \emptyset$, general Q can be introduced by the coded time-sharing technique [EK11, Sec. 4.5.3].
Fix the distribution $p(u_1, x_1)p(u_2, x_2)p(v, x_3)p(\hat{y}_3, \tilde{y}_3|x_3, y_3, v)$.
Codebook generation for the transmitters. For each $j \in [1 : b]$ and for $s \in \{1, 2\}$: Randomly and independently generate $2^{nbR_{sc}}$ sequences $\mathbf{u}_{sj}(m_{sc})$, $m_{sc} \in [1 : 2^{nbR_{sc}}]$, each according to the distribution $\prod_{i=1}^{n} p_{U_s}(u_{s,(j-1)n+i})$. For each $\mathbf{u}_{sj}(m_{sc})$, randomly and conditionally independently generate $2^{nbR_{sp}}$ sequences $\mathbf{x}_{sj}(m_{sc}, m_{sp})$, $m_{sp} \in [1 : 2^{nbR_{sp}}]$, each according to

$$\prod_{i=1}^{n} p_{X_s|U_s}(x_{s,(j-1)n+i}|u_{s,(j-1)n+i}(m_{sc})).$$

Codebook generation for the relay. For each $j \in [1 : b]$: Randomly and independently generate $2^{n\tilde{R}}$ sequences $\mathbf{v}_j(k_{j-1})$, $k_{j-1} \in [1 : 2^{n\tilde{R}}]$, each according to $\prod_{i=1}^{n} p_V(v_{(j-1)n+i})$. For each k_{j-1}, randomly and conditionally independently generate $2^{n\hat{R}}$ sequences $\mathbf{x}_{3j}(l_{j-1}|k_{j-1})$, $l_{j-1} \in [1 : 2^{n\hat{R}}]$, each according to

$$\prod_{i=1}^{n} p_{X_3|V}(x_{3,(j-1)n+i}|v_{(j-1)n+i}(k_{j-1})).$$

For each $k_{j-1} \in [1 : 2^{n\tilde{R}}]$, randomly and conditionally independently generate $2^{n\tilde{R}}$ sequences $\tilde{\mathbf{y}}_{3j}(k_j|k_{j-1})$, $k_j \in [1 : 2^{n\tilde{R}}]$, each according to

$$\prod_{i=1}^{n} p_{\tilde{Y}_3|V}(\tilde{y}_{3,(j-1)n+i}|v_{(j-1)n+i}(k_{j-1})).$$

For each k_j, k_{j-1} and each l_{j-1}, randomly and conditionally independently generate $2^{n\hat{R}}$ sequences $\hat{\mathbf{y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1})$, $l_j \in [1 : 2^{n\hat{R}}]$, each according to

$$\prod_{i=1}^{n} p_{\hat{Y}_3|\tilde{Y}_3,X_3,V}\big(\hat{y}_{3,(j-1)n+i}|\tilde{y}_{3,(j-1)n+i}(k_j|k_{j-1}), x_{3,(j-1)n+i}(l_{j-1}|k_{j-1}), v_{(j-1)n+i}(k_{j-1})\big).$$

All codebooks are revealed to all nodes before transmission.

Encoding. Let m_1, m_2 be the messages to be sent. In block j: Node 1 sends $\mathbf{x}_{1j}(m_{1c}, m_{1p})$, node 2 sends $\mathbf{x}_{2j}(m_{2c}, m_{2p})$. Upon receiving $\mathbf{y}_{3j}$ at the end of block j, the relay looks for an index pair (k_j, l_j) such that

$$\big(\hat{\mathbf{y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1}), \tilde{\mathbf{y}}_{3j}(k_j|k_{j-1}), \mathbf{x}_{3j}(l_{j-1}|k_{j-1}), \mathbf{v}_j(k_{j-1}), \mathbf{y}_{3j}\big) \in \mathcal{T}_{\epsilon'}^{(n)},$$

where $(l_0, k_0) = (1, 1)$ by convention. If there are more than one such index pair, choose one of them uniformly at random. If there is no such index pair, choose an arbitrary pair at random. The relay transmits $\mathbf{x}_{3(j+1)}(l_j|k_j)$ in block $j+1$.

Decoding. Let $\epsilon > \epsilon'$. At the end of block b, receiver 1 looks for a unique index pair $(\hat{m}_{1c}, \hat{m}_{1p})$ such that

$$\big(\mathbf{x}_{1j}(\hat{m}_{1c}, \hat{m}_{1p}), \mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{v}_j(\hat{k}_{j-1}), \tilde{\mathbf{y}}_{3j}(\hat{k}_j|\hat{k}_{j-1}), \mathbf{y}_{1j}\big) \in \mathcal{T}_{\epsilon}^{(n)} \quad \text{for all } j \in [1:b],$$

for some $\hat{m}_{2c} \in [1 : 2^{nbR_{2c}}]$, some $\hat{k}_{j-1}, \hat{k}_j \in [1 : 2^{n\tilde{R}}]$. Receiver 2 looks for a unique index pair $(\hat{m}_{2c}, \hat{m}_{2p})$ such that

$$\begin{aligned}&\big(\mathbf{x}_{2j}(\hat{m}_{2c}, \hat{m}_{2p}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{v}_j(\hat{k}_{j-1}), \mathbf{x}_{3j}(\hat{l}_{j-1}|\hat{k}_{j-1}), \tilde{\mathbf{y}}_{3j}(\hat{k}_j|\hat{k}_{j-1}),\\ &\quad \hat{\mathbf{y}}_{3j}(\hat{l}_j|\hat{k}_j, \hat{l}_{j-1}, \hat{k}_{j-1}), \mathbf{y}_{2j}\big) \in \mathcal{T}_{\epsilon}^{(n)} \text{ for all } j \in [1:b],\end{aligned}$$

for some $\hat{m}_{1c} \in [1 : 2^{nbR_{1c}}]$, some $\hat{k}_{j-1}, \hat{k}_j \in [1 : 2^{n\tilde{R}}]$, and some $\hat{l}_{j-1}, \hat{l}_j \in [1 : 2^{n\hat{R}}]$.

Error analysis. Let $(M_{1c}, M_{1p}, M_{2c}, M_{2p})$ denote the message indices sent by transmitters 1 and 2. (L_j, K_j) denote the indices chosen by the relay at block j, $j \in [1 : b]$. Let k^b denote the sequence $k_1, k_2, \ldots, k_b$, and similarly for l^b. We define the following events

$$\begin{aligned}
\mathcal{E}_0 &:= \bigcup_{j=1}^{b} \big\{ \big(\mathbf{V}_j(K_{j-1}), \mathbf{X}_{3j}(L_{j-1}|K_{j-1}), \tilde{\mathbf{Y}}_{3j}(k_j|K_{j-1}), \hat{\mathbf{Y}}_{3j}(l_j|k_j, L_{j-1}, K_{j-1}),\\
&\qquad\qquad \mathbf{Y}_{3j}\big) \notin \mathcal{T}_{\epsilon'}^{(n)} \text{ for all } (l_j, k_j)\big\},\\
\mathcal{E}_{10} &:= \big\{ \big(\mathbf{X}_{1j}(M_{1c}, M_{1p}), \mathbf{U}_{1j}(M_{1c}), \mathbf{U}_{2j}(M_{2c}), \mathbf{V}_j(K_{j-1}), \tilde{\mathbf{Y}}_{3j}(K_j|K_{j-1}), \mathbf{Y}_{1j}\big)\\
&\qquad \notin \mathcal{T}_{\epsilon}^{(n)} \text{ for some } j \in [1:b]\big\}\\
\mathcal{E}_{11} &:= \big\{ \big(\mathbf{X}_{1j}(m_{1c}, m_{1p}), \mathbf{U}_{1j}(m_{1c}), \mathbf{U}_{2j}(m_{2c}), \mathbf{V}_j(k_{j-1}), \tilde{\mathbf{Y}}_{3j}(k_j|k_{j-1}), \mathbf{Y}_{1j}\big) \in \mathcal{T}_{\epsilon}^{(n)}\\
&\qquad \text{for all } j \in [1:b] \text{ for some } (m_{1c}, m_{1p}) \neq (M_{1c}, M_{1p}), m_{2c} \in [1 : 2^{nbR_{2c}}], k^b\big\}\\
\mathcal{E}_{20} &:= \big\{ \big(\mathbf{X}_{2j}(M_{2c}, M_{2p}), \mathbf{U}_{2j}(M_{2c}), \mathbf{U}_{1j}(M_{1c}), \mathbf{V}_j(K_{j-1}), \mathbf{X}_{3j}(L_{j-1}|K_{j-1}),\\
&\qquad \tilde{\mathbf{Y}}_{3j}(K_j|K_{j-1}), \hat{\mathbf{Y}}_{3j}(L_j|K_j, L_{j-1}, K_{j-1}), \mathbf{Y}_{2j}\big) \notin \mathcal{T}_{\epsilon}^{(n)} \text{ for some } j \in [1:b]\big\}
\end{aligned}$$

$$\begin{aligned}\mathcal{E}_{21} := \Big\{ & \big(\mathbf{X}_{2j}(m_{2c}, m_{2p}), \mathbf{U}_{2j}(m_{2c}), \mathbf{U}_{1j}(m_{1c}), \mathbf{V}_j(k_{j-1}), \mathbf{X}_{3j}(l_{j-1}|k_{j-1}),\\ & \tilde{\mathbf{Y}}_{3j}(k_j|k_{j-1}), \hat{\mathbf{Y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1}), \mathbf{Y}_{2j}\big) \in \mathcal{T}_\epsilon^{(n)} \text{ for all } j \in [1:b]\\ & \text{for some } (m_{2c}, m_{2p}) \neq (M_{2c}, M_{2p}), m_{1c} \in [1:2^{nbR_{1c}}], k^b, l^b\Big\}.\end{aligned}$$

Receiver 1 makes a decoding error only if one of the events $\mathcal{E}_0$, $\mathcal{E}_{10}$, $\mathcal{E}_{11}$ occurs. Receiver 2 makes a decoding error only if one of the events $\mathcal{E}_0$, $\mathcal{E}_{20}$, $\mathcal{E}_{21}$ occurs. Hence, we can bound the probability of error at the receivers as:

$$\begin{aligned}\mathsf{P}(\mathcal{E}_1) &\leq \mathsf{P}(\mathcal{E}_0) + \mathsf{P}(\mathcal{E}_0^c \cap \mathcal{E}_{10}) + \mathsf{P}(\mathcal{E}_{11})\\ \mathsf{P}(\mathcal{E}_2) &\leq \mathsf{P}(\mathcal{E}_0) + \mathsf{P}(\mathcal{E}_0^c \cap \mathcal{E}_{20}) + \mathsf{P}(\mathcal{E}_{21}).\end{aligned}$$

By [LKEC10, Lemma 1], which uses the same techniques as in [EK11, Appendix 8A], we have $\mathsf{P}(\mathcal{E}_0) \to 0$ as $n \to \infty$ if

$$\tilde{R} > I(\tilde{Y}_3; X_3, Y_3|V) - \delta(\epsilon') \tag{5.50a}$$

$$\tilde{R} + \hat{R} > I(\tilde{Y}_3; X_3, Y_3|V) + I(\hat{Y}_3; Y_3|X_3, V, \tilde{Y}_3) - \delta(\epsilon') \tag{5.50b}$$

where $\delta(\epsilon') \to 0$ as $\epsilon' \to 0$.
By the conditional typicality lemma [EK11, Sec. 2.5] we have $\mathsf{P}(\mathcal{E}_0^c \cap \mathcal{E}_{10}) \to 1$ and $\mathsf{P}(\mathcal{E}_0^c \cap \mathcal{E}_{20}) \to 0$ as $n \to \infty$. Next we bound $\mathsf{P}(\mathcal{E}_{21})$. The bounding of $\mathsf{P}(\mathcal{E}_{11})$ follows similarly. Let us define the following event:

$$\begin{aligned}\mathcal{E}_{2,m_{2c},m_{2p},m_{1c}} := \Big\{ & \big(\mathbf{X}_{2j}(m_{2c}, m_{2p}), \mathbf{U}_{2j}(m_{2c}), \mathbf{U}_{1j}(m_{1c}), \mathbf{V}_j(k_{j-1}), \mathbf{X}_{3j}(l_{j-1}|k_{j-1}),\\ & \tilde{\mathbf{Y}}_{3j}(k_j|k_{j-1}), \hat{\mathbf{Y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1}), \mathbf{Y}_{2j}\big) \in \mathcal{T}_\epsilon^{(n)}\\ & \text{for all } j \in [1:b] \text{ and for some } k^b, l^b\Big\}\end{aligned}$$

Assume without loss of generality that $(M_{1c}, M_{1p}, M_{2c}, M_{2p}) = (1,1,1,1)$. We can split the event $\mathcal{E}_{21}$ as follows:

$$\begin{aligned}\mathcal{E}_{21} &= \bigcup_{(m_{2c}, m_{2p}) \neq (1,1), m_{1c}} \mathcal{E}_{2,m_{2c},m_{2p},m_{1c}}\\ &= \mathcal{E}_2^1 \cup \mathcal{E}_2^2 \cup \mathcal{E}_2^3 \cup \mathcal{E}_2^4,\end{aligned}$$

where

$$\begin{aligned}\mathcal{E}_2^1 &:= \cup_{m_{2c} \neq 1, m_{2p}, m_{1c} = 1} \mathcal{E}_{2,m_{2c},m_{2p},m_{1c}}\\ \mathcal{E}_2^2 &:= \cup_{m_{2c} \neq 1, m_{2p}, m_{1c} \neq 1} \mathcal{E}_{2,m_{2c},m_{2p},m_{1c}}\\ \mathcal{E}_2^3 &:= \cup_{m_{2c} = 1, m_{2p} \neq 1, m_{1c} = 1} \mathcal{E}_{2,m_{2c},m_{2p},m_{1c}}\\ \mathcal{E}_2^4 &:= \cup_{m_{2c} = 1, m_{2p} \neq 1, m_{1c} \neq 1} \mathcal{E}_{2,m_{2c},m_{2p},m_{1c}}.\end{aligned}$$

By the union bound we have

$$\mathsf{P}(\mathcal{E}_{21}) \leq \sum_{i=1}^{4} \mathsf{P}(\mathcal{E}_2^i). \tag{5.51}$$

In what follows we bound $\mathsf{P}(\mathcal{E}_2^1)$, other $\mathsf{P}(\mathcal{E}_2^i)$'s are bounded similarly.
Again, due to the symmetry of codebook generation, for error analysis we can assume $(K_1, \ldots, K_b) = (1, \ldots, 1)$ and $(L_1, \ldots, L_b) = (1, \ldots, 1)$, cf. [EK11, Appendix 11A]. For each $j \in [1:b]$, $m_2 = (m_{2c}, m_{2p}) \in [1 : 2^{nbR_{2c}}] \times [1 : 2^{nbR_{2p}}]$, $m_{1c} \in [1 : 2^{nbR_{1c}}]$, $k_j, k_{j-1} \in [1 : 2^{n\tilde{R}}]$, and $l_{j-1}, l_j \in [1 : 2^{n\hat{R}}]$, define an event

$$\begin{aligned}\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j) := \big\{\big(&\mathbf{X}_{2j}(m_{2c}, m_{2p}), \mathbf{U}_{2j}(m_{2c}), \mathbf{U}_{1j}(m_{1c}), \mathbf{V}_j(k_{j-1}), \\ &\mathbf{X}_{3j}(l_{j-1}|k_{j-1}), \tilde{\mathbf{Y}}_{3j}(k_j|k_{j-1}), \hat{\mathbf{Y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1}), \mathbf{Y}_{2j}\big) \in \mathcal{T}_\epsilon^{(n)}\big\}.\end{aligned} \tag{5.52}$$

To support the analysis we form Table 5.2, which shows different combinations of the message and quantization indices.
Bounding $\mathsf{P}(\mathcal{E}_2^1)$*:*

$$\begin{aligned}\mathsf{P}(\mathcal{E}_2^1) &= \mathsf{P}\Big(\bigcup_{\substack{m_{2c}\neq 1, m_{2p},\\ m_{1c}=1}} \bigcup_{l^b, k^b} \bigcap_{j=1}^{b} \mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j)\Big) \\ &\leq \sum_{\substack{m_{2c}\neq 1, m_{2p},\\ m_{1c}=1}} \sum_{l^b, k^b} \mathsf{P}\Big(\bigcap_{j=1}^{b} \mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j)\Big) \\ &\overset{(a_1)}{=} \sum_{\substack{m_{2c}\neq 1, m_{2p},\\ m_{1c}=1}} \sum_{l^b, k^b} \prod_{j=1}^{b} \mathsf{P}\big(\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j)\big) \\ &\leq \sum_{\substack{m_{2c}\neq 1, m_{2p},\\ m_{1c}=1}} \sum_{l^b, k^b} \prod_{j=2}^{b} \mathsf{P}\big(\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j)\big),\end{aligned} \tag{5.53}$$

where (a_1) is due to the independence of codebooks across blocks and the fact that channel is memoryless. Let b_1, b_2, b_3, and b_4 denote the number of appearances of $(\neq 1, \neq 1)$, $(\neq 1, 1)$, $(1, \neq 1)$, and $(1, 1)$ in the sequence of index pairs $(k_j, l_j)_{j=1}^{b-1}$, respectively. We have

$$\begin{aligned}&\sum_{k^b, l^b} \prod_{j=2}^{b} \mathsf{P}(\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j)) \\ &= \sum_{k_b, l_b} \sum_{k^{b-1}, l^{b-1}} \prod_{j=2}^{b} \mathsf{P}(\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j))\end{aligned}$$

Table 5.2: Combinations of $(m_{2c}, m_{2p}, m_{1c}, k_{j-1}, l_{j-1})$. The symbol $*$ corresponds to an index not equal to 1, the symbol $-$ corresponds to an index whose value does not matter. Dotted lines separate cases inducing the same joint pmf of the corresponding sequences.

Case	m_{2c}	m_{2p}	m_{1c}	k_{j-1}	l_{j-1}
1.	$*$	$-$	1	$*$	$*$
2.	$*$	$-$	1	$*$	1
3.	$*$	$-$	1	1	$*$
4.	$*$	$-$	1	1	1
5.	$*$	$-$	$*$	$*$	$*$
6.	$*$	$-$	$*$	$*$	1
7.	$*$	$-$	$*$	1	$*$
8.	$*$	$-$	$*$	1	1
9.	1	$*$	1	$*$	$*$
10.	1	$*$	1	$*$	1
11.	1	$*$	1	1	$*$
12.	1	$*$	1	1	1
13.	1	$*$	$*$	$*$	$*$
14.	1	$*$	$*$	$*$	1
15.	1	$*$	$*$	1	$*$
16.	1	$*$	$*$	1	1

$$
\begin{aligned}
&\overset{(a_2)}{\le} \sum_{k_b, l_b} \sum_{k^{b-1}, l^{b-1}} 2^{-n(\sum_{i=1}^4 b_i I_i - (b-1)\delta(\epsilon))} \\
&= \sum_{k_b, l_b} \sum_{\{b_1^4 : \sum_{i=1}^4 b_i = b-1\}} \binom{b-1}{b_1, \dots, b_4} 2^{nb_1(\hat{R}+\tilde{R})} 2^{nb_2\tilde{R}} 2^{nb_3\hat{R}} 2^{-n(\sum_{i=1}^4 b_i I_i - (b-1)\delta(\epsilon))} \\
&= \sum_{\{b_1^4 : \sum_{i=1}^4 b_i = b-1\}} \binom{b-1}{b_1, \dots, b_4} \sum_{k_b, l_b} 2^{nb_1(\hat{R}+\tilde{R})} 2^{nb_2\tilde{R}} 2^{nb_3\hat{R}} 2^{-n(\sum_{i=1}^4 b_i I_i - (b-1)\delta(\epsilon))} \\
&\le 4^{b-1} 2^{n(\tilde{R}+\hat{R})} 2^{-n(b-1)\left(\min\{I_1 - \hat{R} - \tilde{R}, I_2 - \tilde{R}, I_3 - \hat{R}, I_4\} - \delta(\epsilon)\right)},
\end{aligned} \tag{5.54}
$$

where in (a_2) we use 4 upper bounds on $\mathsf{P}(\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j))$, corresponding to Case 1 to Case 4 in Table 5.2. I_i, $i = [1:4]$ are mutual information terms whose values are as follows:

$$
\begin{aligned}
I_1 &= I_2 = I(X_2, V, X_3; Y_2|U_1) + I(\tilde{Y}_3; U_1, Y_2, X_2, X_3|V) + I(\hat{Y}_3; U_1, Y_2, X_2|V, X_3, \tilde{Y}_3)\\
I_3 &= I(X_2, X_3; \tilde{Y}_3, Y_2|U_1, V) + I(\hat{Y}_3; U_1, Y_2, X_2|V, X_3, \tilde{Y}_3)\\
I_4 &= I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_1, V, X_3).
\end{aligned}
$$

The detailed upper bound for Case 1 and derivation of I_1 are presented in Appendix 5.A.1. The multinomial coefficient $\binom{b-1}{b_1,\dots,b_4}$ has a similar meaning as the binomial coefficient in (4.35), which was explained accordingly.
From (5.53) and (5.54), and the fact that $I_1 = I_2$ we have

$$\mathsf{P}(\mathcal{E}_2^1) \leq 2^{nb(R_{2c}+R_{2p})}4^{b-1}2^{n(\tilde{R}+\hat{R})}2^{-n(b-1)\left(\min\{I_1-\hat{R}-\tilde{R}, I_3-\hat{R}, I_4\}-\delta(\epsilon)\right)}. \tag{5.55}$$

Therefore $\mathsf{P}(\mathcal{E}_2^1) \to 0$ as $n \to \infty$ and $b \to \infty$ if

$$R_{2c} + R_{2p} < \min\{I_1 - \hat{R} - \tilde{R}, I_3 - \hat{R}, I_4\} - \delta(\epsilon). \tag{5.56}$$

Combining with the lower bounds on $\tilde{R}$ and $\hat{R}$ in (5.50) we obtain the rate constraints in (5.11).

In the same way we can derive the sufficient conditions for $\mathsf{P}(\mathcal{E}_2^2)$, $\mathsf{P}(\mathcal{E}_2^3)$, $\mathsf{P}(\mathcal{E}_2^4)$, and therefore $\mathsf{P}(\mathcal{E}_{21})$, to vanish as n and $b \to \infty$. These conditions correspond to the constraints on $R_{2c} + R_{2p} + R_{1c}$ in (5.9), $R_{2p} + R_{1c}$ in (5.10), and R_{2p} in (5.12). As a result, $\mathsf{P}(\mathcal{E}_2) \to 0$ as $b, n \to \infty$ if (5.9)–(5.12) are satisfied.

By swapping user indices $(1 \leftrightarrow 2)$, ignoring l_{j-1}, l_j, and substituting $\mathbf{X}_{3j} \leftarrow \mathbf{V}_j$, $\hat{\mathbf{Y}}_{3j} \leftarrow \tilde{\mathbf{Y}}_{3j}$ and $\hat{R} = 0$, we can show that $\mathsf{P}(\mathcal{E}_1) \to 0$ as $b, n \to \infty$ provided that the rate constraints in (5.13)–(5.16) are satisfied. □

5.A.1 Bounding $\mathsf{P}(\mathcal{A}_j(m_2, m_{1c}, t_{j-1}^j, l_{j-1}^j))$ for Case 1 in Table 5.2.

Here we bound $\mathsf{P}(\mathcal{A}_j(m_2, m_{1c}, t_{j-1}^j, l_{j-1}^j))$ for Case 1 in Table 5.2, which leads to (5.54). For Case 1:

$$
\begin{aligned}
&\big(\mathbf{X}_{2j}(m_{2c}, m_{2p}), \mathbf{U}_{2j}(m_{2c}), \mathbf{V}_j(k_{j-1}), \mathbf{X}_{3j}(l_{j-1}|k_{j-1}), \tilde{\mathbf{Y}}_{3j}(k_j|k_{j-1}),\\
&\qquad \hat{\mathbf{Y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1})\big)\\
&\sim \prod_{i=1}^{n} p_{X_2,U_2}(\mathbf{x}_{2,(j-1)n+i}, \mathbf{u}_{2,(j-1)n+i})\\
&\qquad \cdot p_{X_3,V}(\mathbf{x}_{3,(j-1)n+i}, \mathbf{v}_{(j-1)n+i}) p_{\tilde{Y}_3|V}(\tilde{\mathbf{y}}_{3,(j-1)n+i}|\mathbf{v}_{(j-1)n+i})\\
&\qquad \cdot p_{\hat{Y}_3|\tilde{Y}_3,X_3,V}(\hat{\mathbf{y}}_{3,(j-1)n+i}|\tilde{\mathbf{y}}_{3,(j-1)n+i}, \mathbf{x}_{3,(j-1)n+i}, \mathbf{v}_{(j-1)n+i})
\end{aligned}
$$

is independent of $(\mathbf{U}_{1j}(m_{1c}), \mathbf{Y}_{2j})$ (given $K_{j-1} = K_j = L_{j-1} = L_j = 1$). Hence, we have

$$
\begin{aligned}
&\mathsf{P}\big(\mathcal{A}_j(m_2, m_{1c}, k_{j-1}^j, l_{j-1}^j)\big)\\
&\quad = \sum_{(\mathbf{u}_1,\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)\\
&\quad = \sum_{(\mathbf{u}_1,\mathbf{y}_2)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1,\mathbf{y}_2) \sum_{(\mathbf{u}_2,\mathbf{x}_2)\in\mathcal{T}_\epsilon^{(n)}(U_2,X_2|\mathbf{u}_1,\mathbf{y}_2)} p(\mathbf{x}_2,\mathbf{u}_2)\\
&\qquad \cdot \sum_{(\mathbf{v},\mathbf{x}_3)\in\mathcal{T}_\epsilon^{(n)}(V,X_3|\mathbf{u}_1,\mathbf{y}_2,\mathbf{x}_2,\mathbf{u}_2)} p(\mathbf{v},\mathbf{x}_3)\\
&\qquad \cdot \sum_{\tilde{\mathbf{y}}_3\in\mathcal{T}_\epsilon^{(n)}(\tilde{Y}_3|\mathbf{u}_1,\mathbf{y}_2,\mathbf{x}_2,\mathbf{u}_2,\mathbf{v},\mathbf{x}_3)} p(\tilde{\mathbf{y}}_3|\mathbf{v})\\
&\qquad \cdot \sum_{\hat{\mathbf{y}}_3\in\mathcal{T}_\epsilon^{(n)}(\hat{Y}_3|\mathbf{u}_1,\mathbf{y}_2,\mathbf{x}_2,\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)} p(\hat{\mathbf{y}}_3|\tilde{\mathbf{y}}_3,\mathbf{x}_3,\mathbf{v})\\
&\quad \overset{(a_3)}{\le} \sum_{(\mathbf{u}_1,\mathbf{y}_2)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1,\mathbf{y}_2) 2^{n(H(U_2,X_2|U_1,Y_2)+\epsilon_1)} 2^{-n(H(U_2,X_2)-\epsilon_2)}\\
&\qquad \cdot 2^{n(H(V,X_3|U_2,X_2,U_1,Y_2)+\epsilon_3)} 2^{-n(H(V,X_3)-\epsilon_4)}\\
&\qquad \cdot 2^{n(H(\tilde{Y}_3|V,X_3,U_2,X_2,U_1,Y_2)+\epsilon_5)} 2^{-n(H(\tilde{Y}_3|V)-\epsilon_6)}\\
&\qquad \cdot 2^{n(H(\hat{Y}_3|\tilde{Y}_3,V,X_3,U_2,X_2,U_1,Y_2)+\epsilon_7)} 2^{-n(H(\hat{Y}_3|V,X_3,\tilde{Y}_3)-\epsilon_8)}\\
&\quad \le \sum_{(\mathbf{u}_1,\mathbf{y}_2)\in\mathcal{T}_\epsilon^{(n)}} p(\mathbf{u}_1,\mathbf{y}_2) 2^{-n(I_1-\delta(\epsilon))}\\
&\quad \le 2^{-n(I_1-\delta(\epsilon))}
\end{aligned}
$$

where in (a_3) we use properties of jointly typical sequences [EK11, Sec. 2.5] and I_1 is given below.

$$
\begin{aligned}
I_1 &= H(U_2, X_2) - H(U_2, X_2|U_1, Y_2) + H(V, X_3) - H(V, X_3|U_2, X_2, U_1, Y_2)\\
&\quad + H(\tilde{Y}_3|V) - H(\tilde{Y}_3|V, X_3, U_2, X_2, U_1, Y_2) + H(\hat{Y}_3|V, X_3, \tilde{Y}_3)\\
&\quad - H(\hat{Y}_3|\tilde{Y}_3, V, X_3, U_2, X_2, U_1, Y_2)\\
&= I(U_2, X_2; U_1, Y_2) + I(V, X_3; U_2, X_2, U_1, Y_2) + I(\tilde{Y}_3; U_1, Y_2, X_2, X_3|V)\\
&\qquad + I(\hat{Y}_3; U_1, Y_2, X_2|V, X_3, \tilde{Y}_3)\\
&= I(U_2, X_2; Y_2|U_1) + I(V, X_3; Y_2|U_2, X_2, U_1) + I(\tilde{Y}_3; U_1, Y_2, X_2, X_3|V)\\
&\qquad + I(\hat{Y}_3; U_1, Y_2, X_2|V, X_3, \tilde{Y}_3)\\
&= I(X_2, V, X_3; Y_2|U_1) + I(\tilde{Y}_3; U_1, Y_2, X_2, X_3|V) + I(\hat{Y}_3; U_1, Y_2, X_2|V, X_3, \tilde{Y}_3).
\end{aligned}
$$

Similarly we can upper bound $\mathsf{P}(\mathcal{A}_j(m_2, m_{1c}, t_{j-1}^j, l_{j-1}^j))$ for cases 2–4 in Table 5.2 and obtaining other mutual information terms.

5.B Proof of Theorem 5.3.2

In what follows we present the coding scheme that achieves the region $\mathcal{R}_1^{FW}$ in Theorem 5.3.2, the achievability of $\mathcal{R}_2^{FW}$ follows by switching the roles of the receivers. The proof uses standard techniques as can be found in [EK11], but here the code at the relay has one an extra layer. The transmission of each message takes place in $b+1$ blocks. We use subscript j to index the blocks and subscript s to index the transmitters.

Codebook generation. For $s \in \{1,2\}$: Split a long message of nbR_s bits into b equal parts, each of nR_s bits, index them as m_{sj}, $j = [1:b]$. For each block $j \in [1:b]$, split the message sent by transmitter s in that block as follows: $m_{sj} = (m_{scj}, m_{spj})$, where $m_{scj} \in [1:2^{nR_{sc}}]$, $m_{spj} \in [1:2^{nR_{sp}}]$, $R_{sc} + R_{sp} = R_s$. For brevity we consider $Q = \emptyset$, general Q can be introduced by the coded time-sharing technique [EK11, Sec. 4.5.3].
Fix the distribution $p(u_1, x_1)p(u_2, x_2)p(v, x_3)p(\hat{y}_3, \tilde{y}_3|v, x_3, y_3)$.
Codebook generation for the transmitters. For each $j \in [1:b]$ and $s \in \{1,2\}$: Randomly and independently generate $2^{nR_{sc}}$ sequences $\mathbf{u}_{sj}(m_{scj})$, $m_{scj} \in [1:2^{nR_{sc}}]$, each according to $\prod_{i=1}^n p_{U_s}(u_{sj,i})$. For each $\mathbf{u}_{sj}(m_{scj})$, randomly and conditionally independently generate $2^{nR_{sp}}$ sequences $\mathbf{x}_{sj}(m_{scj}, m_{spj})$, $m_{spj} \in [1:2^{nR_{sp}}]$, each according to

$$\prod_{i=1}^n p_{X_s|U_s}(x_{sj,i}|u_{sj,i}(m_{scj})).$$

Codebook generation for the relay. For each $j \in [1:b]$: Randomly and independently generate $2^{n\tilde{R}}$ sequences $\mathbf{v}_j(k_{j-1})$, $k_{j-1} \in [1:2^{n\tilde{R}}]$, each according to $\prod_{i=1}^n p_V(v_{j,i})$. For each $k_{j-1} \in [1:2^{n\tilde{R}}]$, randomly and conditionally independently generate $2^{n\hat{R}}$ sequences $\mathbf{x}_{3j}(l_{j-1}|k_{j-1})$, $l_{j-1} \in [1:2^{n\hat{R}}]$, each according to

$$\prod_{i=1}^n p_{X_3|V}(x_{3j,i}|v_{j,i}(k_{j-1})).$$

For each $k_{j-1} \in [1:2^{n\tilde{R}}]$, randomly and conditionally independently generate $2^{n\tilde{R}}$ sequences $\tilde{\mathbf{y}}_{3j}(k_j|k_{j-1})$, $k_j \in [1:2^{n\tilde{R}}]$, each according to

$$\prod_{i=1}^n p_{\tilde{Y}_3|V}(\tilde{y}_{3j,i}|v_{j,i}(k_{j-1})).$$

For each $(k_j, l_{j-1}, k_{j-1}) \in [1:2^{n\tilde{R}}] \times [1:2^{n\hat{R}}] \times [1:2^{n\tilde{R}}]$, randomly and conditionally independently generate $2^{n\hat{R}}$ sequences $\hat{\mathbf{y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1})$, $l_j \in [1:2^{n\hat{R}}]$, each according to

$$\prod_{i=1}^{n} p_{\hat{Y}_3|\tilde{Y}_3,X_3,V}(\hat{y}_{3j,i}|\tilde{y}_{3j,i}(k_j|k_{j-1}), x_{3j,i}(l_{j-1}|k_{j-1}), v_{j,i}(k_{j-1})) \tag{5.57}$$

All codebooks are revealed to all nodes before transmission.

Encoding. Let m_{1j}, m_{2j} be the messages to be sent in block j. In block j: Tx1 sends $\mathbf{x}_{1j}(m_{1cj}, m_{1pj})$, Tx2 sends $\mathbf{x}_{2j}(m_{2cj}, m_{2pj})$. Upon receiving $\mathbf{y}_{3j}$ at the end of block j, the relay looks for an index pair (l_j, k_j) such that

$$(\hat{\mathbf{y}}_{3j}(l_j|k_j, l_{j-1}, k_{j-1}), \tilde{\mathbf{y}}_{3j}(k_j|k_{j-1}), \mathbf{x}_{3j}(l_{j-1}|k_{j-1}), \mathbf{v}_j(k_{j-1}), \mathbf{y}_{3j}) \in \mathcal{T}_{\epsilon'}^{(n)} \tag{5.58}$$

where $(l_0, k_0) = (1, 1)$ by convention. If there are more than one such index pair, choose one of them uniformly at random. If there is no such index pair, choose an arbitrary pair at random. The relay transmits $\mathbf{x}_{3,j+1}(l_j|k_j)$ in block $j+1$.

Decoding. Let $\epsilon > \epsilon'$. Assume that Rx1 has decoded k_{j-1} correctly from the previous decoding step. At the end of block $j+1$, knowing both $\mathbf{y}_{1j}$ and $\mathbf{y}_{1(j+1)}$, Rx1 looks for a unique index triple $(\hat{m}_{1c}, \hat{m}_{1p}, \hat{k}_j)$ such that

$$\Big(\mathbf{x}_{1j}(\hat{m}_{1c}, \hat{m}_{1p}), \mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{v}_j(k_{j-1}), \tilde{\mathbf{y}}_{3j}(\hat{k}_j|k_{j-1}), \mathbf{y}_{1j}\Big) \in \mathcal{T}_{\epsilon}^{(n)}, \tag{5.59a}$$

for some $\hat{m}_{2c} \in [1 : 2^{nR_{2c}}]$, *and*

$$\Big(\mathbf{v}_{j+1}(\hat{k}_j), \mathbf{y}_{1(j+1)}\Big) \in \mathcal{T}_{\epsilon}^{(n)}. \tag{5.59b}$$

Then decoder 1 declares $(m_{1cj}, m_{1pj}) = (\hat{m}_{1c}, \hat{m}_{1p})$. Otherwise it declares an error.

Assume that Rx2 has decoded k_{j-1} and l_{j-1} correctly from the previous decoding step. At the end of block $j+1$, knowing both $\mathbf{y}_{2j}$ and $\mathbf{y}_{2(j+1)}$, Rx2 looks for a unique index tuple $(\hat{m}_{2c}, \hat{m}_{2p}, \hat{k}_j, \hat{l}_j)$ such that

$$\begin{aligned}\Big(&\mathbf{x}_{2j}(\hat{m}_{2c}, \hat{m}_{2p}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{v}_j(k_{j-1}), \mathbf{x}_{3j}(l_{j-1}|k_{j-1}), \tilde{\mathbf{y}}_{3j}(\hat{k}_j|k_{j-1}),\\ &\hat{\mathbf{y}}_{3j}(\hat{l}_j|\hat{k}_j, l_{j-1}, k_{j-1}), \mathbf{y}_{2j}\Big) \in \mathcal{T}_{\epsilon}^{(n)}\end{aligned} \tag{5.60a}$$

for some $\hat{m}_{1c} \in [1 : 2^{nR_{1c}}]$, *and*

$$\Big(\mathbf{v}_{j+1}(\hat{k}_j), \mathbf{x}_{3(j+1)}(\hat{l}_j|\hat{k}_j), \mathbf{y}_{2(j+1)}\Big) \in \mathcal{T}_{\epsilon}^{(n)}. \tag{5.60b}$$

Then decoder 2 declares $(m_{2cj}, m_{2pj}) = (\hat{m}_{2c}, \hat{m}_{2p})$. Otherwise it declares an error. *The encoding and decoding processes are illustrated in Table 5.3.*

Table 5.3: Encoding and sliding window decoding.

Block	1	2	...	$b+1$
X_1	$\mathbf{u}_{11}(m_{1c1}), \mathbf{x}_{11}(m_{1c1}, m_{1p1})$	$\mathbf{u}_{12}(m_{1c2}), \mathbf{x}_{12}(m_{1c2}, m_{1p2})$	...	$\mathbf{u}_{1(b+1)}(1), \mathbf{x}_{1(b+1)}(1, 1)$
X_2	$\mathbf{u}_{21}(m_{2c1}), \mathbf{x}_{21}(m_{2c1}, m_{2p1})$	$\mathbf{u}_{22}(m_{2c2}), \mathbf{x}_{22}(m_{2c2}, m_{2p2})$	...	$\mathbf{u}_{2(b+1)}(1), \mathbf{x}_{2(b+1)}(1, 1)$
Y_3	$\tilde{\mathbf{y}}_{31}(k_1 \mid 1)$, $\hat{\mathbf{y}}_{31}(l_1 \mid k_1, 1, 1)$	$\tilde{\mathbf{y}}_{32}(k_2 \mid k_1)$, $\hat{\mathbf{y}}_{32}(l_2 \mid k_2, l_1, k_1)$	...	$\tilde{\mathbf{y}}_{3(b+1)}(k_{b+1} \mid k_b)$, $\hat{\mathbf{y}}_{3(b+1)}(l_{b+1} \mid k_{b+1}, l_b, k_b)$
X_3	$\mathbf{v}_1(1)$, $\mathbf{x}_{31}(1 \mid 1)$	$\mathbf{v}_2(k_1)$, $\mathbf{x}_{32}(l_1 \mid k_1)$	...	$\mathbf{v}_{b+1}(k_b)$, $\mathbf{x}_{3(b+1)}(l_b \mid k_b)$
Y_1	$\emptyset$	$\hat{m}_{1c1}, \hat{m}_{1p1}$	...	$\hat{m}_{1cb}, \hat{m}_{1pb}$
Y_2	$\emptyset$	$\hat{m}_{2c1}, \hat{m}_{2p1}$	...	$\hat{m}_{2cb}, \hat{m}_{2pb}$

Error analysis. Here we do error analysis for Rx2. Due to the symmetry of codebook generation, sub-codebooks with respect to (l_j, k_j) have the same distribution. Therefore, for the error analysis we can assume $(L_j, K_j) = \mathbf{1}$ is the index pair chosen by the relay in block j, for all $j \in [1:b]$, cf. [EK11, Appendix 11A]. We also assume $(M_{1c}, M_{1p}, M_{2c}, M_{2p}) = \mathbf{1}$ is the message tuple sent by the transmitters in block j. Let us define the following events:

$$\mathcal{E}_0 := \bigcup_{j=1}^{b} \big\{ \big(\mathbf{V}_j(1), \mathbf{X}_{3j}(1|1), \tilde{\mathbf{Y}}_{3j}(k_j|1), \hat{\mathbf{Y}}_{3j}(l_j|k_j,1,1), \mathbf{Y}_{3j}\big) \notin \mathcal{T}_{\epsilon'}^{(n)} \text{ for all } (l_j,k_j) \big\},$$

$$\begin{aligned}\mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l, k) := \big\{ \big(&\mathbf{X}_{2j}(m_{2c}, m_{2p}), \mathbf{U}_{2j}(m_{2c}), \mathbf{U}_{1j}(m_{1c}), \mathbf{V}_j(1), \mathbf{X}_{3j}(1|1),\\ &\tilde{\mathbf{Y}}_{3j}(k|1), \hat{\mathbf{Y}}_{3j}(l|k,1,1), \mathbf{Y}_{2j}\big) \in \mathcal{T}_{\epsilon}^{(n)} \big\}\\ \mathcal{E}_{2,j+1}(l,k) := \big\{ \big(&\mathbf{v}_{j+1}(k), \mathbf{x}_{3(j+1)}(l|k), \mathbf{y}_{2(j+1)}\big) \in \mathcal{T}_{\epsilon}^{(n)} \big\}.\end{aligned}$$

Error probabilities of decoding at Rx2 at the end of block $j+1$ is bounded by:

$$\begin{aligned}\mathsf{P}(\mathcal{E}_2) \le \mathsf{P}(\mathcal{E}_0) &+ \mathsf{P}(\mathcal{E}_{2,j}^c(1,1,1,1,1) \cup \mathcal{E}_{2,j+1}^c(1,1)|\mathcal{E}_0^c)\\ &+ \mathsf{P}\Big(\bigcup_{(m_{2c},m_{2p},m_{1c},l,k)\neq(1,1,1,1,1)} \mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l, k) \cap \mathcal{E}_{2,j+1}(l,k) \Big).\end{aligned} \tag{5.61}$$

Similarly to Appendix 5.A we have $\mathsf{P}(\mathcal{E}_0) \to 0$ as $n \to \infty$ if the conditions in (5.50) are satisfied, namely

$$\tilde{R} > I(\tilde{Y}_3; X_3, Y_3|V) - \delta(\epsilon') \tag{5.62a}$$

$$\tilde{R} + \hat{R} > I(\tilde{Y}_3; X_3, Y_3|V) + I(\hat{Y}_3; Y_3|X_3, V, \tilde{Y}_3) - \delta(\epsilon') \tag{5.62b}$$

and $\mathsf{P}\big(\mathcal{E}_{2,j}^c(1,1,1,1,1) \cup \mathcal{E}_{2,j+1}^c(1,1)|\mathcal{E}_0^c\big) \to 0$ as $n \to \infty$. Next we bound the last term in (5.61), denote it by $\mathsf{P}(\mathcal{E}_{2,j,(j+1)})$. To support the error analysis we form Table 5.4, where each case corresponds to a decoding error event.

Let $\mathcal{A}_j := \mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l_j, k_j) \cap \mathcal{E}_{2,j+1}(l_j, k_j)$. By the union of events bound we have

$$\mathsf{P}(\mathcal{E}_{2,j,(j+1)}) \le \sum_{i=1}^{16} \mathsf{P}(E_{2i}),$$

where $E_{2i} := \cup_{(m_{2c},m_{2p},m_{1c},l_j,k_j)=\mathcal{I}_i} \mathcal{A}_j$, and $\mathcal{I}_i$ is the index tuple corresponding to Case i in Table 5.4, e.g, $I_5 = (\neq 1, 1, \neq 1, 1, \neq 1)$. We can bound $\mathsf{P}(E_{21})$, which is corresponding to Case 1 in Table 5.4, as follows:

$$\begin{aligned}\mathsf{P}(E_{21}) &= \mathsf{P}\left(\cup_{m_{1c}\neq1, m_{2c}\neq1, m_{2p}, k_j\neq1, l_j} \mathcal{A}_j \right)\\ &\overset{(a)}{\le} \sum_{m_{1c}\neq1, m_{2c}\neq1, m_{2p}, k_j\neq1, l_j} \mathsf{P}\big(\mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l_j, k_j) \cap \mathcal{E}_{2,j+1}(l_j, k_j)\big)\end{aligned}$$

$$\overset{(b)}{=} \sum_{m_{1c}\neq 1, m_{2c}\neq 1, m_{2p}, k_j\neq 1, l_j} \mathsf{P}(\mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l_j, k_j))\mathsf{P}(\mathcal{E}_{2,j+1}(l_j, k_j))$$
$$\leq 2^{n(R_{1c}+R_{2c}+R_{2p}+\hat{R}+\tilde{R})}\mathsf{P}(\mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l_j, k_j))\mathsf{P}(\mathcal{E}_{2,j+1}(l_j, k_j)), \tag{5.63}$$

where (a) comes from the union of events bound, (b) due to the fact that the codebooks in different blocks are generated independently and channel is memoryless. Applying the same bounding technique as in Appendix 5.A.1, and noticing the joint pmf of the sequences corresponding to Case 1 in Table 5.4, we can show that

$$\mathsf{P}(\mathcal{E}_{2,j}(m_{2c}, m_{2p}, m_{1c}, l_j, k_j)) \leq 2^{-n(I_{1.1}-\delta(\epsilon))} \text{ and } \mathsf{P}(\mathcal{E}_{2,j+1}(l_j, k_j)) \leq 2^{-n(I_{1.2}-\delta(\epsilon))}$$

where $\delta(\epsilon) \to 0$ as $\epsilon \to 0$, and

$$\begin{aligned} I_{1.1} &= I(U_1, U_2, X_2; Y_2|V, X_3) + I(\tilde{Y}_3; U_1, U_2, X_2, X_3, Y_2|V) \\ &\quad + I(\hat{Y}_3; U_1, U_2, X_2, Y_2|V, X_3, \tilde{Y}_3) \\ I_{1.2} &= I(V, X_3; Y_2). \end{aligned}$$

Insert these results to (5.63) we conclude that $\mathsf{P}(E_{21}) \to 0$ as $n \to \infty$ if

$$\begin{aligned} R_{1c} + R_{2c} + R_{2p} + \hat{R} + \tilde{R} &\leq I_{1.1} + I_{1.2} \\ &= I(U_1, U_2, X_2, V, X_3; Y_2) + I(\tilde{Y}_3; U_1, U_2, X_2, X_3, Y_2|V) \\ &\quad + I(\hat{Y}_3; U_1, U_2, X_2, Y_2|V, X_3, \tilde{Y}_3). \end{aligned}$$

Combining this condition with the conditions in (5.62), and adding the time-sharing random variable Q, we conclude $\mathsf{P}(E_{21}) \to 0$ as $n \to \infty$ if (5.20a) is satisfied. Following the same procedure we can show that $\mathsf{P}(E_{2i}) \to 0$ as $n \to \infty$, for $i = [2 : 16]$ if the conditions in (5.18.a.b) and (5.20.b)–(5.24) are satisfied, as specified in the last column of Table 5.4. Note that the conditions in (5.22) and (5.18.a.b) are due to the fact that Decoder 2 needs to decode the quantization indices uniquely, which is in contrast to the non-unique decoding rule of our first coding scheme (based on noisy network coding). Finally, by the union bound, we conclude that the decoding error probability at Rx2 vanishes as $n \to \infty$ if (5.18.a.b) and (5.20)–(5.24) are satisfied.

By swapping user indices $(1 \leftrightarrow 2)$, ignoring l_{j-1}, l_j, replacing $\mathbf{X}_{3j}$ by $\mathbf{V}_j$, $\hat{\mathbf{Y}}_{3j}$ by $\tilde{\mathbf{Y}}_{3j}$, and setting $\hat{R} = 0$, it can be shown that the probability of decoding error at Rx1 goes to 0 as $n \to \infty$ provided that the rate constraints in (5.18.c) and (5.25)–(5.29) are satisfied. □

5.C Proof of Theorem 5.3.3

Codebook Generation and Encoding. Similar to the quantize-forward with sliding window decoding scheme, the nodes use $b + 1$ blocks to send b different

Table 5.4: The joint pmfs induced by different $(m_{1c}, m_{2c}, m_{2p}, k_j, l_j)$ tuples. The symbol $*$ corresponds to an index not equal to 1, and the symbol $-$ corresponds to an index whose value does not matter.

Case	m_{1c}	m_{2c}	m_{2p}	k_j	l_j	joint pmf in block $j+1$	joint pmf in block j	Induced constraints
1	$*$	$*$	$-$	$*$	$-$	$p(\mathbf{v},\mathbf{x}_3)p(\mathbf{y}_2)$	$p(\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{u}_2,\mathbf{x}_2)p(\tilde{\mathbf{y}}_3\vert\mathbf{v})p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.20a)
2	$*$	$*$	$-$	1	$*$	$p(\mathbf{v},\mathbf{y}_2)p(\mathbf{x}_3\vert\mathbf{v})$	$p(\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{u}_2,\mathbf{x}_2)p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.20b)
3	$*$	$*$	$-$	1	1	$p(\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)$	$p(\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{u}_2,\mathbf{x}_2)$	(5.20c)
4	$*$	1	$*$	$*$	$-$	$p(\mathbf{v},\mathbf{x}_3)p(\mathbf{y}_2)$	$p(\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_2\vert\mathbf{u}_2)p(\tilde{\mathbf{y}}_3\vert\mathbf{v})p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.21a)
5	$*$	1	$*$	1	$*$	$p(\mathbf{v},\mathbf{y}_2)p(\mathbf{x}_3\vert\mathbf{v})$	$p(\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_2\vert\mathbf{u}_2)p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.21b)
6	$*$	1	$*$	1	1	$p(\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)$	$p(\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_2\vert\mathbf{u}_2)$	(5.21c)
7	$*$	1	1	$*$	$-$	$p(\mathbf{v},\mathbf{x}_3)p(\mathbf{y}_2)$	$p(\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\tilde{\mathbf{y}}_3\vert\mathbf{v})p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.22a)
8	$*$	1	1	1	$*$	$p(\mathbf{v},\mathbf{y}_2)p(\mathbf{x}_3\vert\mathbf{v})$	$p(\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.22b)
9	1	$*$	$-$	$*$	$-$	$p(\mathbf{v},\mathbf{x}_3)p(\mathbf{y}_2)$	$p(\mathbf{u}_1,\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)p(\mathbf{u}_2,\mathbf{x}_2)p(\tilde{\mathbf{y}}_3\vert\mathbf{v})p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.23a)
10	1	$*$	$-$	1	$*$	$p(\mathbf{v},\mathbf{y}_2)p(\mathbf{x}_3\vert\mathbf{v})$	$p(\mathbf{u}_1,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_2,\mathbf{x}_2)p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.23b)
11	1	$*$	$-$	1	1	$p(\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)$	$p(\mathbf{u}_1,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_2,\mathbf{x}_2)$	(5.23c)
12	1	1	$*$	$*$	$-$	$p(\mathbf{v},\mathbf{x}_3)p(\mathbf{y}_2)$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)p(\mathbf{x}_2\vert\mathbf{u}_2)p(\tilde{\mathbf{y}}_3\vert\mathbf{v})p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.24a)
13	1	1	$*$	1	$*$	$p(\mathbf{v},\mathbf{y}_2)p(\mathbf{x}_3\vert\mathbf{v})$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{x}_2\vert\mathbf{u}_2)p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.24b)
14	1	1	$*$	1	1	$p(\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{x}_2\vert\mathbf{u}_2)$	(5.24c)
15	1	1	1	$*$	$-$	$p(\mathbf{v},\mathbf{x}_3)p(\mathbf{y}_2)$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\mathbf{x}_3,\mathbf{y}_2)p(\tilde{\mathbf{y}}_3\vert\mathbf{v})p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.18a)
16	1	1	1	1	$*$	$p(\mathbf{v},\mathbf{y}_2)p(\mathbf{x}_3\vert\mathbf{v})$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\hat{\mathbf{y}}_3\vert\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.18b)

messages. The codebook generation and encoding processes are the same as that of Appendix 5.B. However, the backward decoding scheme requires an extra step to initialize the decoding process at block $b+1$ as follows [CMG07]: For each codeword $\mathbf{x}_3(l|k)$, $(l,k) \in [1:2^{n\hat{R}}] \times [1:2^{n\tilde{R}}]$, generate a sequence $\mathbf{y}_3$ according to the conditional distribution

$$(\mathbf{y}_3|\mathbf{x}_1(1), \mathbf{x}_2(1), \mathbf{x}_3(l|k)) \sim \prod_{i=1}^{n} p(y_{3i}|x_{1i}(1,1), x_{2i}(1,1), x_{3i}(l|k))$$

Next, for the same $\mathbf{x}_3(l|k)$ and $\mathbf{y}_3$, choose a pair of indices (l^*, k^*) such that

$$(\hat{\mathbf{y}}_3(l^*|k^*, l, k), \tilde{\mathbf{y}}_3(k^*|k), \mathbf{x}_3(l|k), \mathbf{v}(k), \mathbf{y}_3) \in \mathcal{T}_{\epsilon'}^{(n)} \tag{5.64}$$

and swap the labeling $(l^*, k^*) \leftrightarrow (1,1)$ for $\hat{\mathbf{y}}_3(l^*|k^*, l, k)$ and $\tilde{\mathbf{y}}_3(k^*|k)$ if such a (l^*, k^*) can be found. Do nothing if no (l^*, k^*) is found. As before, such (l^*, k^*) exists with high probability for sufficiently large n if the conditions in (5.50) are satisfied.

Decoding. Assume that Decoder 1 has decoded k_j correctly in block $j+1$. For block j Decoder 1 looks for a unique index triple $(\hat{m}_{1c}, \hat{m}_{1p}, \hat{k}_{j-1})$ such that

$$\left(\mathbf{x}_{1j}(\hat{m}_{1p}, \hat{m}_{1c}), \mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{v}_j(\hat{k}_{j-1}), \tilde{\mathbf{y}}_{3j}(k_j|\hat{k}_{j-1}), \mathbf{y}_{1j}\right) \in \mathcal{T}_{\epsilon}^{(n)}, \tag{5.65}$$

for some $\hat{m}_{2c} \in [1:2^{nR_{2c}}]$. Then decoder 1 declares $(m_{1cj}, m_{1pj}) = (\hat{m}_{1c}, \hat{m}_{1p})$. Otherwise it declares an error.

Assume that Decoder 2 has decoded l_j, k_j correctly in block $j+1$. For block j Decoder 2 looks for a unique index pair $(\hat{m}_{2c}, \hat{m}_{2p}, \hat{k}_{j-1}, \hat{l}_{j-1})$ such that

$$\begin{aligned}\Big(&\mathbf{x}_{2j}(\hat{m}_{2p}, \hat{m}_{2c}), \mathbf{u}_{2j}(\hat{m}_{2c}), \mathbf{u}_{1j}(\hat{m}_{1c}), \mathbf{v}_j(\hat{k}_{j-1}), \mathbf{x}_{3j}(\hat{l}_{j-1}|\hat{k}_{j-1}), \tilde{\mathbf{y}}_{3j}(k_j|\hat{k}_{j-1}),\\ &\hat{\mathbf{y}}_{3j}(l_j|k_j, \hat{l}_{j-1}, \hat{k}_{j-1}), \mathbf{y}_{2j}\Big) \in \mathcal{T}_{\epsilon}^{(n)}\end{aligned} \tag{5.66}$$

for some $\hat{m}_{1c} \in [1:2^{nR_{1c}}]$. Then Decoder 2 declares $(m_{2cj}, m_{2pj}) = (\hat{m}_{2c}, \hat{m}_{2p})$. Otherwise it declares an error. Then encoding and decoding processes are illustrated in Table 5.5.

Error Analysis. The error analysis follows the same procedure as error analysis for sliding window decoding in Appendix 5.B. The only difference compared with Appendix 5.B is that each decoder is now required to check only one joint typicality condition. We provide the list of error events at Decoder 2 in Table 5.6. It can be shown that the error event corresponding to Case 1 in Table 5.6 has a vanishing probability as $n \to \infty$ if the condition in (5.33.a) is met. The other cases are similarly analyzed, leading to the rate constraints in Definition 5.5. ☐

Table 5.5: Encoding and backward decoding.

Block	1	...	b	$b+1$
X_1	$\mathbf{u}_{11}(m_{1c1}), \mathbf{x}_{11}(m_{1c1}, m_{1p1})$	...	$\mathbf{u}_{1b}(m_{1cb}), \mathbf{x}_{1b}(m_{1cb}, m_{1pb})$	$\mathbf{u}_{1(b+1)}(1), \mathbf{x}_{1(b+1)}(1,1)$
X_2	$\mathbf{u}_{21}(m_{2c1}), \mathbf{x}_{21}(m_{2c1}, m_{2p1})$	...	$\mathbf{u}_{2b}(m_{2cb}), \mathbf{x}_{2b}(m_{2cb}, m_{2pb})$	$\mathbf{u}_{2(b+1)}(1), \mathbf{x}_{2(b+1)}(1,1)$
Y_3	$\tilde{\mathbf{y}}_{31}(k_1\|1)$, $\hat{\mathbf{y}}_{31}(l_1\|k_1, 1, 1)$	...	$\tilde{\mathbf{y}}_{3b}(k_b\|k_{b-1})$, $\hat{\mathbf{y}}_{3b}(l_b\|k_b, l_{b-1}, k_{b-1})$	$\tilde{\mathbf{y}}_{3(b+1)}(1\|k_b)$, $\hat{\mathbf{y}}_{3(b+1)}(1\|1, l_b, k_b)$
X_3	$\mathbf{v}_1(1)$, $\mathbf{x}_{31}(1\|1)$	...	$\mathbf{v}_b(k_{b-1})$, $\mathbf{x}_{3b}(l_{b-1}\|k_{b-1})$	$\mathbf{v}_{b+1}(k_b)$, $\mathbf{x}_{3(b+1)}(l_b\|k_b)$
Y_1	$\hat{m}_{1c1}, \hat{m}_{1p1}$	...	$\leftarrow \hat{k}_{b-1}, \hat{m}_{1cb}, \hat{m}_{1pb}$	$\leftarrow \hat{k}_b$
Y_2	$\hat{m}_{2c1}, \hat{m}_{2p1}$	...	$\leftarrow \hat{l}_{b-1}, \hat{k}_{b-1}, \hat{m}_{2cb}, \hat{m}_{2pb}$	$\leftarrow \hat{l}_b, \hat{k}_b$

Table 5.6: The joint pmfs induced by different $(m_{1c}, m_{2c}, m_{2p}, k_{j-1}, l_{j-1})$ tuples. The symbol $*$ corresponds to an index not equal to 1, and the symbol $-$ corresponds to an index whose value does not matter.

Case	m_{1c}	m_{2c}	m_{2p}	k_{j-1}	l_{j-1}	joint pmf of sequences	Induced constraints
1	$*$	$*$	$-$	$*$	$-$	$p(\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{u}_2,\mathbf{x}_2)p(\mathbf{v},\mathbf{x}_3)p(\tilde{\mathbf{y}}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.33a)
2	$*$	$*$	$-$	1	$*$	$p(\mathbf{v},\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{u}_2,\mathbf{x}_2)p(\mathbf{x}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.33b)
3	$*$	$*$	$-$	1	1	$p(\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{u}_2,\mathbf{x}_2)$	(5.33c)
4	$*$	1	$*$	$*$	$-$	$p(\mathbf{u}_2,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_2\mid\mathbf{u}_2)p(\mathbf{v},\mathbf{x}_3)p(\tilde{\mathbf{y}}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.34a)
5	$*$	1	$*$	1	$*$	$p(\mathbf{u}_2,\mathbf{v},\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_2\mid\mathbf{u}_2)p(\mathbf{x}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.34b)
6	$*$	1	$*$	1	1	$p(\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_2\mid\mathbf{u}_2)$	(5.34c)
7	$*$	1	1	$*$	$-$	$p(\mathbf{u}_2,\mathbf{x}_2,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{v},\mathbf{x}_3)p(\tilde{\mathbf{y}}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.35a)
8	$*$	1	1	1	$*$	$p(\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_1)p(\mathbf{x}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.35b)
9	1	$*$	$-$	$*$	$-$	$p(\mathbf{u}_1,\mathbf{y}_2)p(\mathbf{u}_2,\mathbf{x}_2)p(\mathbf{v},\mathbf{x}_3)p(\tilde{\mathbf{y}}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.36a)
10	1	$*$	$-$	1	$*$	$p(\mathbf{u}_1,\mathbf{v},\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_2,\mathbf{x}_2)p(\mathbf{x}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.36b)
11	1	$*$	$-$	1	1	$p(\mathbf{u}_1,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{u}_2,\mathbf{x}_2)$	(5.36c)
12	1	1	$*$	$*$	$-$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{y}_2)p(\mathbf{x}_2\mid\mathbf{u}_2)p(\mathbf{v},\mathbf{x}_3)p(\tilde{\mathbf{y}}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.37a)
13	1	1	$*$	1	$*$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{v},\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{x}_2\mid\mathbf{u}_2)p(\mathbf{x}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.37b)
14	1	1	$*$	1	1	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3,\hat{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{x}_2\mid\mathbf{u}_2)$	(5.37c)
15	1	1	1	$*$	$-$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{x}_2,\mathbf{y}_2)p(\mathbf{v},\mathbf{x}_3)p(\tilde{\mathbf{y}}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.31a)
16	1	1	1	1	$*$	$p(\mathbf{u}_1,\mathbf{u}_2,\mathbf{x}_2,\mathbf{v},\tilde{\mathbf{y}}_3,\mathbf{y}_2)p(\mathbf{x}_3\mid\mathbf{v})p(\hat{\mathbf{y}}_3\mid\mathbf{v},\mathbf{x}_3,\tilde{\mathbf{y}}_3)$	(5.31b)

5.D Proof of Theorem 5.5.1

5.D.1 An Alternative Form of the Achievable Region

Recall that in Appendix 5.A we show that the probability of an error event at Rx2 vanishes as n goes to infinity as long as the conditions in (5.11) are satisfied, which can be rewritten as follows (using the chain rule):

$$R_{2c} + R_{2p} < I(X_2; Y_2|U_1, Q) + I(V, X_3; Y_2|U_1, X_2, Q) + a + b - \tilde{r} - \hat{r} \tag{5.67}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, Y_2|U_1, V, Q) + I(X_3; \tilde{Y}_3, Y_2|U_1, V, X_2, Q) + b - \hat{r} \tag{5.68}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_1, V, X_3, Q). \tag{5.69}$$

On the other hand, by an alternative method of bounding the error probability as shown in Appendix 4.B, one can show that the probability of decoding error at Rx2 vanishes as n goes to infinity if

$$R_{2c} + R_{2p} < I(X_2; Y_2|U_1, Q) \tag{5.70}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, Y_2|U_1, V, Q) \tag{5.71}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_1, V, X_3, Q). \tag{5.72}$$

Combining (5.67) with (5.70), (5.68) with (5.71), and using (5.69) (or (5.72)) we conclude that the error probability at Rx2 vanishes as $n \to \infty$ if

$$R_{2c} + R_{2p} < I(X_2; Y_2|U_1, Q) + \big(I(V, X_3; Y_2|U_1, X_2, Q) + a + b - \tilde{r} - \hat{r}\big)^+ \tag{5.73}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, Y_2|U_1, V, Q) + \big(I(X_3; \tilde{Y}_3, Y_2|U_1, V, X_2, Q) + b - \hat{r}\big)^+ \tag{5.74}$$

$$R_{2c} + R_{2p} < I(X_2; \tilde{Y}_3, \hat{Y}_3, Y_2|U_1, V, X_3, Q). \tag{5.75}$$

Similar conclusions can be made on other constraints, cf. [DOS12b, Definition 1][1]. However, it suffices to focus on the constraints on $R_{2c}+R_{2p}$ in the rest of the proof.

5.D.2 Extending Theorem 5.3.1 to the Gaussian IC-IOR

We use the same techniques as in [DOS11b] to extend the result in Theorem 5.3.1 to the Gaussian IC-IOR. Specifically, X_1, X_2 are Gaussian distributed and the quantization codebook generating variables are chosen as follows: $\hat{Y}_3 = Y_3 + \hat{Z}_3$, $\tilde{Y}_3 = \hat{Y}_3 + \tilde{Z}_3$, where $\hat{Z}_3 \sim N(0, \hat{\sigma}^2)$, $\tilde{Z}_3 \sim N(0, \tilde{\sigma}^2)$, independent of each other and of everything else. Due to the nature of the channel, in the same way as done in Appendix 4.D we can replace Y_i by (Y_{ia}, Y_{id}), $i = 1, 2$, where subscript a stands for the analog part and d stands for the noiseless "digital" part. Note that in this case we have a Markov chain

$$\tilde{Y}_3 - \hat{Y}_3 - Y_3 - (U_1, X_2, Y_{2a}), \tag{5.76}$$

[1] There were typos in the [DOS12b, Definition 1], where g in the terms related to J_i' should read h.

and $I(V;Y_{1d}) = C_1$, $I(X_3;Y_{2d}) = C_2$.

Inserting the above random variables to (5.73)–(5.75) and making use of (5.76) we obtain:

$$R_{2c} + R_{2p} < I(X_2;Y_{2a}|U_1,Q) + \left(\Delta R - I(\tilde{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\right)^+ \tag{5.77}$$

$$R_{2c} + R_{2p} < I(X_2;Y_{2a},\tilde{Y}_3|U_1,Q) + \left(\Delta R\right)^+ \tag{5.78}$$

$$R_{2c} + R_{2p} < I(X_2;Y_{2a}|U_1,Q) + I(X_2;\hat{Y}_3|U_1,Y_{2a},Q), \tag{5.79}$$

where $\Delta R := C_2 + I(\hat{Y}_3;U_1,X_2,Y_{2a}|\tilde{Y}_3,Q) - I(\hat{Y}_3;Y_3|\tilde{Y}_3,Q)$. One can easily verify that the right hand side (RHS) of (5.77) is smaller than RHS of (5.78), therefore (5.78) is redundant. Next,

$$\begin{aligned}
\Delta R - &I(\tilde{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\\
&= C_2 + I(\hat{Y}_3;U_1,X_2,Y_{2a}|\tilde{Y}_3,Q) - I(\hat{Y}_3;Y_3|\tilde{Y}_3,Q) - I(\tilde{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\\
&= C_2 - H(\hat{Y}_3|U_1,X_2,Y_{2a},\tilde{Y}_3,Q) + H(\hat{Y}_3|Y_3,\tilde{Y}_3,Q) - I(\tilde{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\\
&\overset{(a)}{=} C_2 - I(\hat{Y}_3;Y_3|U_1,X_2,Y_{2a},\tilde{Y}_3,Q) - I(\tilde{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\\
&= C_2 - I(\hat{Y}_3,\tilde{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\\
&\overset{(b)}{=} C_2 - I(\hat{Y}_3;Y_3|U_1,X_2,Y_{2a},Q),
\end{aligned} \tag{5.80}$$

where we use (5.76) in (a) and (b). Inserting (5.80) into (5.77) and combine the result with (5.79) we have

$$R_{2c} + R_{2p} < I(X_2;Y_{2a}|U_1,Q) + \min\left\{\begin{matrix} I(X_2;\hat{Y}_3|U_1,Y_{2a},Q), \\ \left(C_2 - I(\hat{Y}_3;Y_3|U_1,X_2,Y_{2a},Q)\right)^+ \end{matrix}\right\}. \tag{5.81}$$

By the following substitutions of the random variables: $Y_{2a} \leftarrow Y_2$, $U_1 \leftarrow W_1$, $\hat{Y}_3 \leftarrow \hat{Y}_{2r}$, $Y_3 \leftarrow Y_r$ we can see that (5.81) is identical to the constraint on $R_{2c}+R_{2p}$ in [ZY13, Eq. (51)]. In the same way we can show that the remaining rate constraints in Definition 5.3 and the corresponding rate constraints in [ZY13, Eqs. (46)–(53)] are the same. Note that we also use the following substitutions of variables: $Y_{1a} \leftarrow Y_1$, $U_2 \leftarrow W_2$, $\tilde{Y}_3 \leftarrow \hat{Y}_{1r}$. Further, our rate region and that of [ZY13] are optimized over the same joint distribution of random variables. □

5.E Achievable Rates for Gaussian channels

In this section we present the achievable rates for the Gaussian channel given in Section 5.6. We follow the procedure described in Section 5.6 to derive these rates. For brevity we define and use the following quantities in the expressions of the achievable rates:

$$\begin{aligned}
&\mathsf{SNR}_1 := h_{11}^2P_1,\ \mathsf{INR}_1 := h_{21}^2P_2,\ \mathsf{SNR}_{r1} := h_{13}^2P_1,\ \mathsf{HNR}_1 := h_{31}^2P_3\\
&\mathsf{SNR}_2 := h_{22}^2P_2,\ \mathsf{INR}_2 := h_{12}^2P_1,\ \mathsf{SNR}_{r2} := h_{23}^2P_2,\ \mathsf{HNR}_2 := h_{32}^2P_3.
\end{aligned}$$

5.E.1 Sliding Window Decoding, Treat Interference as Noise

Starting from Definition 5.4, we set $R_{kc} = 0$, $R_{kp} = R_k$, $U_k = \emptyset$ for $k \in \{1, 2\}$, i.e., setting the whole message at each transmitter to be private, to obtain an achievable region for the discrete memoryless channel. Applying the procedure described in Section 5.6 to the resulting rate region we obtain the following result.

For the Gaussian ICR, the following nonnegative rate pairs (R_1, R_2) can be achieved by LQF-FW and treating interference as noise at the receivers:

$$
\begin{aligned}
R_2 &< \mathcal{C}\left(\frac{\mathsf{SNR}_2 + \mathsf{HNR}_2}{1 + \mathsf{INR}_2}\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2} + \frac{\mathsf{SNR}_{r1}}{\hat{\sigma}^2(1 + \mathsf{INR}_2)}\right) \\
R_2 &< \mathcal{C}\left(\frac{(\tilde{\sigma}^2(1 + \mathsf{INR}_2) + (1 + \tilde{\sigma}^2 + \hat{\sigma}^2)(\mathsf{SNR}_2 + \bar{\alpha}\mathsf{HNR}_2)}{(1 + \hat{\sigma}^2)(1 + \mathsf{INR}_2) + \mathsf{SNR}_{r1}}\right. \\
&\qquad\left. + \frac{\bar{\alpha}\mathsf{HNR}_2\mathsf{SNR}_{r1} + (1 + \mathsf{INR}_2 + \mathsf{SNR}_2 + \bar{\alpha}\mathsf{HNR}_2)\big(\mathsf{SNR}_{r2} + (h_{13}h_{22} - h_{12}h_{23})^2 P_1 P_2\big)}{\big((1 + \hat{\sigma}^2)(1 + \mathsf{INR}_2) + \mathsf{SNR}_{r1}\big)(1 + \mathsf{INR}_2 + \mathsf{SNR}_2)}\right) \\
&\qquad - \mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2}\right) \\
R_2 &< \mathcal{C}\left(\frac{(1 + \hat{\sigma}^2)\mathsf{SNR}_2 + \mathsf{SNR}_{r2} + (h_{13}h_{22} - h_{12}h_{23})^2 P_1 P_2}{(1 + \hat{\sigma}^2)(1 + \mathsf{INR}_2) + \mathsf{SNR}_{r1}}\right) \\
R_1 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \alpha\mathsf{HNR}_1}{1 + \mathsf{INR}_1 + \bar{\alpha}\mathsf{HNR}_1}\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2 + \tilde{\sigma}^2} + \frac{(1 + \bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r2}}{(\hat{\sigma}^2 + \tilde{\sigma}^2)(1 + \mathsf{INR}_1 + \bar{\alpha}\mathsf{HNR}_1)}\right) \\
R_1 &< \mathcal{C}\left(\frac{(1 + \bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r1} + (1 + \hat{\sigma}^2 + \tilde{\sigma}^2)\mathsf{SNR}_1 + (h_{13}h_{21} - h_{11}h_{23})^2 P_1 P_2}{(1 + \bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r2} + (1 + \hat{\sigma}^2 + \tilde{\sigma}^2)(1 + \mathsf{INR}_1 + \bar{\alpha}\mathsf{HNR}_1)}\right)
\end{aligned}
$$

for some $\alpha \in [0, 1]$ and nonnegative $\hat{\sigma}^2$ and $\tilde{\sigma}^2$ such that:

$$
\begin{aligned}
\hat{\sigma}^2 + \tilde{\sigma}^2 &> \frac{\big(K + \mathsf{SNR}_{r2}(1 + \bar{\alpha}\mathsf{HNR}_1)\big)\big(K + \mathsf{SNR}_1\big)}{\alpha\mathsf{HNR}_1 K} \\
K &= 1 + \mathsf{INR}_1 + \bar{\alpha}\mathsf{HNR}_1 \\
\frac{1 + \mathsf{INR}_2 + \frac{1 + \mathsf{INR}_2 + \mathsf{SNR}_{r1}}{\hat{\sigma}^2}}{1 + \mathsf{INR}_2 + \frac{1 + \mathsf{INR}_2 + \mathsf{SNR}_{r1}}{\hat{\sigma}^2 + \tilde{\sigma}^2}} &< 1 + \frac{\bar{\alpha}\mathsf{HNR}_2}{1 + \mathsf{INR}_2 + \mathsf{SNR}_2} \\
\hat{\sigma}^2 &> \frac{(1 + \mathsf{INR}_2 + \mathsf{SNR}_{r1})(1 + \mathsf{INR}_2 + \mathsf{SNR}_2)}{(1 + \mathsf{INR}_2)\mathsf{HNR}_2}.
\end{aligned}
$$

5.E.2 Sliding Window Decoding, Decode Interference

On the other hand, by setting $R_{kp} = 0$, $R_{kc} = R_k$, $U_k = X_k$ for $k \in \{1, 2\}$ in Definition 5.4, i.e., setting the whole message at each transmitter to be common, and applying the procedure described in Section 5.6, we obtain the following achievable rate region.

For the Gaussian ICR, the following nonnegative rate pairs (R_1, R_2) can be achieved by LQF-FW and decoding interference at the receivers:

$$
\begin{aligned}
R_1 &< \mathcal{C}\left(\frac{\alpha\mathsf{HNR}_1}{1+\mathsf{SNR}_1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1}\right)+\mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}\right)-\mathcal{C}\left(\frac{1}{\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}+\frac{\mathsf{SNR}_{r1}}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1 &< \mathcal{C}\left(\frac{\mathsf{HNR}_2}{1+\mathsf{SNR}_2+\mathsf{INR}_2}\right)+\mathcal{C}\left(\mathsf{INR}_2\right)-\mathcal{C}\left(\frac{1}{\hat{\sigma}^2}\right)\\
R_1 &< \mathcal{C}\left(\frac{\bar{\alpha}\mathsf{HNR}_2}{1+\mathsf{SNR}_2+\mathsf{INR}_2}\right)+\mathcal{C}\left(\mathsf{INR}_2+\frac{\mathsf{SNR}_{r1}}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
&\quad-\mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)\\
R_2 &< \mathcal{C}\left(\frac{\mathsf{HNR}_2}{1+\mathsf{SNR}_2+\mathsf{INR}_2}\right)+\mathcal{C}\left(\mathsf{SNR}_2\right)-\mathcal{C}\left(\frac{1}{\hat{\sigma}^2}\right)\\
R_2 &< \mathcal{C}\left(\frac{\bar{\alpha}\mathsf{HNR}_2}{1+\mathsf{SNR}_2+\mathsf{INR}_2}\right)+\mathcal{C}\left(\mathsf{SNR}_2+\frac{\mathsf{SNR}_{r2}}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
&\quad-\mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)\\
R_2 &< \mathcal{C}\left(\mathsf{SNR}_2+\frac{\mathsf{SNR}_{r2}}{1+\hat{\sigma}^2}\right)\\
R_2 &< \mathcal{C}\left(\frac{\alpha\mathsf{HNR}_1}{1+\mathsf{SNR}_1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1}\right)+\mathcal{C}\left(\frac{\mathsf{INR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}\right)-\mathcal{C}\left(\frac{1}{\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1+R_2 &< \mathcal{C}\left(\mathsf{SNR}_2+\mathsf{INR}_2+\mathsf{HNR}_2\right)-\mathcal{C}\left(\frac{1}{\hat{\sigma}^2}\right)\\
R_1+R_2 &< \mathcal{C}\left(\mathsf{INR}_2+\mathsf{SNR}_2+\bar{\alpha}\mathsf{HNR}_2+\frac{\substack{(\mathsf{SNR}_{r1}+\mathsf{SNR}_{r2})(1+\bar{\alpha}\mathsf{HNR}_2)\\+(h_{13}h_{22}-h_{12}h_{23})^2P_1P_2}}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
&\quad-\mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)\\
R_1+R_2 &< \mathcal{C}\left(\mathsf{INR}_2+\mathsf{SNR}_2+\frac{\mathsf{SNR}_{r1}+\mathsf{SNR}_{r2}+(h_{13}h_{22}-h_{12}h_{23})^2P_1P_2}{1+\hat{\sigma}^2}\right)\\
R_1+R_2 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1+\mathsf{INR}_1+\alpha\mathsf{HNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}\right)-\mathcal{C}\left(\frac{1}{\hat{\sigma}^2+\tilde{\sigma}^2}\right)
\end{aligned}
$$

$$R_1 + R_2 < \mathcal{C}\left(\frac{\substack{(\mathsf{SNR}_1+\mathsf{INR}_1)(1+\hat{\sigma}^2+\tilde{\sigma}^2)\\ +(\mathsf{SNR}_{r1}+\mathsf{SNR}_{r2})(1+\bar{\alpha}\mathsf{HNR}_1)+(h_{13}h_{21}-h_{23}h_{11})^2P_1P_2}}{(1+\bar{\alpha}\mathsf{HNR}_1)(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)$$

for some $\alpha \in [0,1]$ *and nonnegative* $\hat{\sigma}^2, \tilde{\sigma}^2$ *such that*

$$\begin{aligned}
\hat{\sigma}^2 + \tilde{\sigma}^2 &> \frac{1+\mathsf{SNR}_1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1}{\alpha\mathsf{HNR}_1}\\
\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)} &< \frac{\bar{\alpha}\mathsf{HNR}_2}{1+\mathsf{SNR}_2+\mathsf{INR}_2}\\
\hat{\sigma}^2 &> \frac{1+\mathsf{SNR}_2+\mathsf{INR}_2}{\mathsf{HNR}_2}.
\end{aligned}$$

5.E.3 Gaussian ICR - Backward Decoding, Treat Interference As Noise

Similar to the sliding window decoding case, by setting $R_{kc} = 0$, $R_{kp} = R_k$, $U_k = \emptyset$ for $k \in \{1,2\}$ in Definition 5.5 and applying the procedure described in Section 5.6, we obtain the following achievable rate region.

For the Gaussian ICR, the following nonnegative rate pairs (R_1, R_2) *can be achieved by LQF-BW and treating interference as noise at the receivers:*

$$\begin{aligned}
R_2 &< \mathcal{C}\left(\frac{\mathsf{SNR}_2+\mathsf{HNR}_2}{1+\mathsf{INR}_2}\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2}+\frac{\mathsf{SNR}_{r1}}{\hat{\sigma}^2(1+\mathsf{INR}_2)}\right)\\
R_2 &< \mathcal{C}\left(\frac{\substack{\tilde{\sigma}^2(1+\mathsf{INR}_2)+(1+\hat{\sigma}^2+\tilde{\sigma}^2)(\mathsf{SNR}_2+\bar{\alpha}\mathsf{HNR}_2)\\ +\bar{\alpha}\mathsf{HNR}_2(\mathsf{SNR}_{r1}+\mathsf{SNR}_{r2})+\mathsf{SNR}_{r2}+(h_{13}h_{22}-h_{12}h_{23})^2P_1P_2}}{(1+\hat{\sigma}^2)(1+\mathsf{INR}_2)+\mathsf{SNR}_{r1}}\right) - \mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2}\right)\\
R_2 &< \mathcal{C}\left(\frac{(1+\hat{\sigma}^2)\mathsf{SNR}_2+\mathsf{SNR}_{r2}+(h_{13}h_{22}-h_{12}h_{23})^2P_1P_2}{(1+\hat{\sigma}^2)(1+\mathsf{INR}_2)+\mathsf{SNR}_{r1}}\right)\\
R_1 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1+\alpha\mathsf{HNR}_1}{1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1}\right) - \mathcal{C}\left(\frac{1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1+(1+\bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r2}}{(\hat{\sigma}^2+\tilde{\sigma}^2)(1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1)}\right)\\
R_1 &< \mathcal{C}\left(\frac{(1+\bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r1}+(1+\hat{\sigma}^2+\tilde{\sigma}^2)\mathsf{SNR}_1+(h_{13}h_{21}-h_{23}h_{11})^2P_1P_2}{(1+\bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r2}+(1+\hat{\sigma}^2+\tilde{\sigma}^2)(1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1)}\right)
\end{aligned}$$

for some $\alpha \in [0,1]$ *and nonnegative* $\hat{\sigma}^2, \tilde{\sigma}^2$ *such that*

$$\begin{aligned}
\hat{\sigma}^2 + \tilde{\sigma}^2 &> \frac{1+\mathsf{INR}_1+\bar{\alpha}\mathsf{HNR}_1+(1+\bar{\alpha}\mathsf{HNR}_1)\mathsf{SNR}_{r2}}{\alpha\mathsf{HNR}_1}\\
\bar{\alpha}\mathsf{HNR}_2 &> \frac{1+\mathsf{INR}_2+\mathsf{SNR}_{r1}}{\hat{\sigma}^2} - \frac{1+\mathsf{INR}_2+\mathsf{SNR}_{r1}+\bar{\alpha}\mathsf{HNR}_2(1+\mathsf{SNR}_{r1})}{\hat{\sigma}^2+\tilde{\sigma}^2}\\
\hat{\sigma}^2 &> \frac{1+\mathsf{INR}_2+\mathsf{SNR}_{r1}}{\mathsf{HNR}_2}.
\end{aligned}$$

5.E.4 Gaussian ICR - Backward Decoding, Decode Interference

By setting $R_{kp} = 0$, $R_{kc} = R_k$, $U_k = X_k$ for $k \in \{1, 2\}$ in Definition 5.5 and applying the procedure described in Section 5.6, we obtain the following achievable rate region.

For the Gaussian ICR, the following nonnegative rate pairs (R_1, R_2) can be achieved by LQF-BW and decoding interference at the receivers:

$$
\begin{aligned}
R_1 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \alpha\mathsf{HNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1} + \frac{\mathsf{SNR}_{r1}}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1 &< \mathcal{C}(\mathsf{INR}_2 + \mathsf{HNR}_2) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2}\right)\\
R_1 &< \mathcal{C}\left(\mathsf{INR}_2 + \bar{\alpha}\mathsf{HNR}_2 + \frac{\mathsf{SNR}_{r1}(1+\bar{\alpha}\mathsf{HNR}_2)}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right) - \mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)\\
R_2 &< \mathcal{C}(\mathsf{SNR}_2 + \mathsf{HNR}_2) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2}\right)\\
R_2 &< \mathcal{C}\left(\mathsf{SNR}_2 + \bar{\alpha}\mathsf{HNR}_2 + \frac{\mathsf{SNR}_{r2}(1+\bar{\alpha}\mathsf{HNR}_2)}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right) - \mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)\\
R_2 &< \mathcal{C}\left(\mathsf{SNR}_2 + \frac{\mathsf{SNR}_{r2}}{1+\hat{\sigma}^2}\right)\\
R_2 &< \mathcal{C}\left(\frac{\mathsf{INR}_1 + \alpha\mathsf{HNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1 + R_2 &< \mathcal{C}\left(\mathsf{SNR}_2 + \mathsf{INR}_2 + \mathsf{HNR}_2\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2}\right)\\
R_1 + R_2 &< \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{SNR}_2 + \bar{\alpha}\mathsf{HNR}_2 + \frac{\substack{(\mathsf{SNR}_{r1}+\mathsf{SNR}_{r2})(1+\bar{\alpha}\mathsf{HNR}_2)\\ +(h_{13}h_{22}-h_{12}h_{23})^2P_1P_2}}{1+\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
&\quad - \mathcal{C}\left(\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)\\
R_1 + R_2 &< \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{SNR}_2 + \frac{\mathsf{SNR}_{r1} + \mathsf{SNR}_{r2} + (h_{13}h_{22} - h_{12}h_{23})^2P_1P_2}{1+\hat{\sigma}^2}\right)\\
R_1 + R_2 &< \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \mathsf{INR}_1 + \alpha\mathsf{HNR}_1}{1+\bar{\alpha}\mathsf{HNR}_1}\right) - \mathcal{C}\left(\frac{1}{\hat{\sigma}^2+\tilde{\sigma}^2}\right)\\
R_1 + R_2 &< \mathcal{C}\left(\frac{\substack{(\mathsf{SNR}_1+\mathsf{INR}_1)(1+\hat{\sigma}^2+\tilde{\sigma}^2)+(\mathsf{SNR}_{r1}+\mathsf{SNR}_{r2})(1+\bar{\alpha}\mathsf{HNR}_1)\\ +(h_{13}h_{21}-h_{23}h_{11})^2P_1P_2}}{(1+\bar{\alpha}\mathsf{HNR}_1)(1+\hat{\sigma}^2+\tilde{\sigma}^2)}\right)
\end{aligned}
$$

for some $\alpha \in [0,1]$ *and nonnegative* $\hat{\sigma}^2, \tilde{\sigma}^2$ *such that*

$$\begin{aligned}
\hat{\sigma}^2 + \tilde{\sigma}^2 &> \frac{1+\bar{\alpha}\mathsf{HNR}_1}{\alpha\mathsf{HNR}_1} \\
\frac{\tilde{\sigma}^2}{\hat{\sigma}^2(1+\hat{\sigma}^2+\tilde{\sigma}^2)} &< \bar{\alpha}\mathsf{HNR}_2 \\
\hat{\sigma}^2 &> \frac{1}{\mathsf{HNR}_2}.
\end{aligned}$$

Chapter 6

Interfering Relay Channels

In previous chapters, we have studied some mechanisms for managing interference in interference networks with dedicated or non-dedicated relays. These channel models have also been defined and studied in a number of previous works in the literature. In this chapter we introduce and study a new channel model in which two relay channels interfere with each other. Each relay in the channel is connected to its intended receiver through a rate-limited and noise-free link, forming a suitable model for the backhaul links in heterogeneous wireless access networks, e.g., Fig. 1.1. Inner and outer bounds for achievable rates are derived and shown to be tight for new discrete memoryless classes, which generalize and unify several known cases involving interference and relay channels. Capacity region and sum-capacity for multiple Gaussian scenarios are also characterized to within a constant gap. The optimality of bin(or hash)-forward and its extensions for certain relay-interference networks found in this chapter has important engineering implications for the design of future wireless networks.

6.1 Introduction

As introduced in Chapter 2, two of the fundamental building blocks of network information theory are the interference channel (IC) and the relay channel (RC). Despite the fact that the capacity of each channel is still unknown, the use of relaying in interference management has drawn considerable attention in research and in wireless network design, as can be seen in the previous chapters and references therein. In this chapter, we introduce a new channel model which accounts for a particular interaction between interference and relay channels, and establish the capacity region and approximate capacity region for several classes of these channels.

Consider a network as illustrated in Fig. 6.1. In this model a relay channel consisting of transmitter Tx_1, relay R_1, and receiver Rx_1 interferes with a neighboring relay channel consisting of transmitter Tx_2, relay R_2, and receiver Rx_2. Specifically, the signal sent by Tx_i is received by R_j and Rx_j, for $i, j \in \{1, 2\}$, $i \neq j$. Further, each

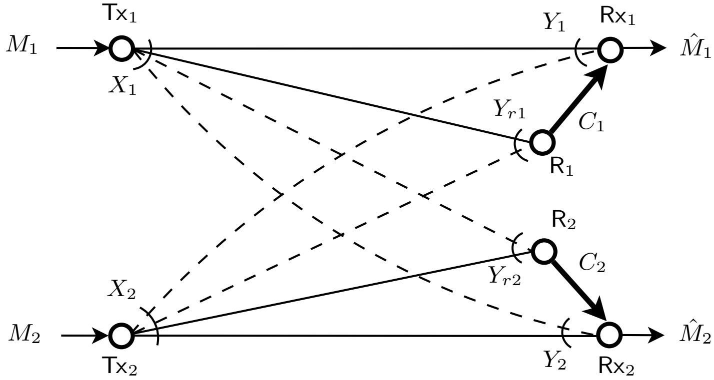

Figure 6.1: Interfering relay channels. Dashed lines depict interference. M_i and $\hat{M}_i$, $i \in \{1, 2\}$, denote a message and its estimate, respectively.

relay R_i is connected to receiver Rx_i via a noiseless link of finite capacity C_i. From the wireless communication perspective, this channel model is highly relevant for heterogeneous networks, which is an important trend in current and future wireless architectures. For example, Tx_i, R_i, and Rx_i can respectively act as a mobile, a pico base station, and the macro base station in a single cell of a cellular network. The pico base station and the macro base station are connected over a finite-rate backhaul link, cf. Fig. 1.1. Naturally, the communication in neighboring cells interfere with each other.

From the information theory perspective, the channel in Fig. 6.1 is a generalization of both the interference and relay channel models. One interesting feature of the channel under consideration is that although each relay only connects to its intended receiver, the function of each relay still includes relaying the intended signal and forwarding the unintended signal (the interference). In this sense, it is similar to the function of the relay in the interference channel with one relay as studied in Chapter 5. Among the prior works, e.g., [MDG08, SES09, TY11, SES11, ZY13, RHZ+13], our setup is most related to the work on the interference channel (IC) with a degraded broadcasting relay [ZY13]. Specifically, if R_1 coincides with R_2 in Fig. 6.1 we recover the channel studied in [ZY13]. It is also clear that one can interpret the channel in Fig. 6.1 as one type of interference channel with two relays. In this sense our channel is closely related to the interference channel with limited receiver cooperation studied in Chapter 4 and in [WT11a], where each receiver also acts as a relay. Specifically, in the special case when R_i coincides with receiver Rx_j, $i, j \in \{1, 2\}$, $i \neq j$, we recover the channels in Fig. 4.2 and in [WT11a]. In subsequent sections we will further comment on the relationship between our work and those of [ZY13] and [WT11a]. Yet from the relay channel viewpoint, the channel in Fig. 6.1 is a generalization of the primitive relay channel, studied by Cover and

Kim in [CK07]. The key difference from the channel in [CK07] is the presence of interference, which certainly affects the coding strategies for the channel. In order to emphasize the fact that each relay in our channel is meant to help only one receiver, we call the network in Fig. 6.1 the *interfering relay channels* (IRC) rather than interference channel with two relays.

In this chapter we study different classes of discrete memoryless and Gaussian IRC. The main results are summarized as follows.

- We propose an inner bound for the capacity region of the discrete memoryless IRC. The coding technique is based on rate-splitting at the transmitters and quantize-bin-forward [WT11a] or extended hash[1]-forward [Kim07].

- We characterize the capacity region of a class of discrete memoryless channels, namely the semideterministic IRC.

- We derive an outer bound for the Gaussian IRC and use it to show constant-bit gap results for multiple scenarios: When the interference is *strong*, we characterize the capacity region of the real-valued Gaussian IRC to within 1/2 bit. For the Gaussian IRC with *high-capacity relay-receiver links*, we characterize the capacity region to within log(7) bits. For other regimes of channel parameters, we show that the inner bound and outer bound are within a bounded gap as long as the interference that each transmitter induces at the neighboring relay is not unboundedly stronger than the interference induced at the neighboring receiver. Moreover, for the Gaussian IRC in the so called *weak interference-relay regime*, we characterize the sum-capacity to within 1 bit.

- We also study a closely related channel model, the compound multiple access relay channel. We characterize capacity region of a class of the semideterministic compound multiple access relay channel and the capacity region of a class of Gaussian compound multiple access relay channels to within 1/2 bit.

The achievable scheme in this chapter is a combination of common-private rate-splitting [Car78, HK81] and quantize-bin-forward [WT11a]. As noted earlier, related problems have also been studied by combining rate-splitting with generalized hash-forward [RHZ$^+$13, ZY13] or with noisy network coding [DOS11b, DOS12b]. By focusing on the particular model of IRC and based on insight learned from proving the inner bound, we are able to prove *new* capacity and approximate capacity results for discrete memoryless and Gaussian channels. For the discrete memoryless channel, our results extend the optimality of hash-forward, which originally holds for the semideterministic relay channel in [CK07, Kim08], to certain classes of relay channels with interference. The established capacities also generalize and unify the

[1]Note that in the literature of relay channels, the term *binning* and *hashing* often have the same meaning.

capacity results of some relay and interference channels. One of the main technical contributions of this chapter is various outer bounds that help characterize the exact or approximate capacity regions. Further, unlike [WT11a] and [ZY13] which study only Gaussian channels, we study both discrete memoryless and Gaussian channels in this chapter. The results of our study show that a "simple" combination of rate-splitting with the quantize-bin(or hash)-forward protocol is optimal or nearly optimal for complex interference networks like the IRC. This has an important engineering implication because such a relaying scheme, which frees the intermediate nodes from the decoding burden, is highly desired in real networks.

The chapter is organized as follows: Section 6.2 presents an achievable rate region for the discrete memoryless IRC. Section 6.3 characterizes the capacity region of a class of discrete memoryless IRC and a class of discrete memoryless compound multiple access relay channels. Section 6.4 studies the Gaussian IRC, wherein a general outer bound to the capacity region is derived and used to show the capacity region or sum-capacity to within a constant number of bits for different type of channels. This section also characterizes the capacity region of a class of Gaussian compound multiple access relay channel to within a constant number of bits. Conclusions are presented in Section 6.5 and long proofs are relegated to the appendices.

In this chapter we will use the concept of k-bit gap to the capacity region, as defined in Definition 4.1.

6.2 Discrete Memoryless IRC and a General Achievability

6.2.1 Problem Formulation

Refer again to the IRC depicted in Fig. 6.1. Transmitter Tx_i wants to send a message M_i to receiver Rx_i, $i \in \{1, 2\}$. Relay R_i helps receiver Rx_i decode its desired message by sending aiding signals via a noiseless link with finite capacity of C_i bits. The channel is memoryless in the sense that

$$p(y_{1i}, y_{2i}, y_{r1i}, y_{r2i}|m_1, m_2, x_1^i, x_2^i) = p_{Y_1,Y_2,Y_{r1},Y_{r2}|X_1,X_2}(y_{1i}, y_{2i}, y_{r1i}, y_{r2i}|x_{1i}, x_{2i}). \tag{6.1}$$

A $(2^{nR_1}, 2^{nR_2}, n)$ code, where n denotes the codeword length, for the discrete memoryless IRC consists of the following elements, for each $k \in \{1, 2\}$:

1. a message set $\mathcal{M}_k = [1 : 2^{nR_k}]$;
2. an encoding function $f_k : \mathcal{M}_k \to \mathcal{X}_k^n$ that assigns a sequence x_k^n to each message $m_k \in \mathcal{M}_k$;
3. relaying functions $r_{ki} : \mathcal{Y}_{rk}^{i-1} \times [1 : 2^{nC_k}]^{i-1} \to [1 : 2^{nC_k}]$ that maps $(y_{rk}^{i-1}, v_k^{i-1})$ to a symbol v_{ki} for all $i \in [1 : n]$, where v_{ki} denotes the ith symbol sent via the noiseless link of capacity C_k;

4. a decoding function $g_k : \mathcal{Y}_k^n \times [1 : 2^{nC_k}] \to \mathcal{M}_k$ that produces an estimate $\hat{m}_k$ from a received sequence y_k^n and an index from the set $[1 : 2^{nC_k}]$.

We assume that the message pair (M_1, M_2) is uniformly distributed over $\mathcal{M}_1 \times \mathcal{M}_2$. The average probability of error is defined as

$$P_e^{(n)} = \mathsf{P}\left\{(\hat{M}_1, \hat{M}_2) \neq (M_1, M_2)\right\}. \tag{6.2}$$

As usual, achievable rate pairs and the capacity region of the channel are defined as in Section 2.1.

6.2.2 An Achievable Rate Region for the Discrete Memoryless IRC

Motivation of the achievable scheme: As discussed in Section 6.1, each relay should balance between relaying the useful signal and forwarding the interference, to help its intended receiver decode the desired message and mitigate the interference. In order to gain insight into possible coding strategies for the IRC, let us start with the primitive relay channel, first studied in [CK07], which can be considered as a component of the IRC. A primitive relay channel (PRC) is a 3-node relay channel with a noiseless link between the relay and the receiver. In [CK07] and subsequent papers [Kim08, Kim07] these authors have studied different relaying techniques for the PRC such as decode-forward, compress-forward, and introduce a new technique called hash-forward. This new technique was shown to be capacity achieving under a condition on the relationship between the relay output, the transmitter input, and the receiver output [CK07, Kim08]. For the general PRC, the hash-forward protocol can be extended to include a quantization stage at the relay, an approach named *extended hash-forward* (EHF) [Kim07]. Subsequently, for more complex channel models with interference [WT11a, ZY13], it was shown that a combination of rate-splitting at the transmitter with EHF (also called quantize-bin-forward in [WT11a]) or generalized hash-forward (GHF) can achieve to within a constant number of bits to the capacity region of the respective channel. This fact, together with a close relation between our channel model and those in [CK07, WT11a, ZY13], have motivated us to propose a coding strategy based on those known techniques to our current problem, which leads to the following inner bound.

Theorem 6.2.1 (Inner bound for DM-IRC)**.** *Let us define*

$$a_1 = I(X_1; Y_1 | X_{1c}, X_{2c}, Q) \tag{6.3}$$

$$a_1' = I(X_1; Y_1, \hat{Y}_{r1} | X_{1c}, X_{2c}, Q) \tag{6.4}$$

$$b_1 = I(X_1, X_{2c}; Y_1 | X_{1c}, Q) \tag{6.5}$$

$$b_1' = I(X_1, X_{2c}; Y_1, \hat{Y}_{r1} | X_{1c}, Q) \tag{6.6}$$

$$c_1 = I(X_1; Y_1 | X_{2c}, Q) \tag{6.7}$$

$$c_1' = I(X_1; Y_1, \hat{Y}_{r1}|X_{2c}, Q) \tag{6.8}$$
$$d_1 = I(X_1, X_{2c}; Y_1|Q) \tag{6.9}$$
$$d_1' = I(X_1, X_{2c}; Y_1, \hat{Y}_{r1}|Q) \tag{6.10}$$
$$\xi_1 = I(\hat{Y}_{r1}; Y_{r1}|X_1, X_{2c}, Y_1, Q), \tag{6.11}$$

and define $a_2, a_2', b_2, b_2', c_2, c_2', d_2, d_2', \xi_2$ *by exchanging indices* $1 \leftrightarrow 2$ *everywhere in* (6.3)–(6.11)*.* Q *is drawn from some finite set* $\mathcal{Q}$*. Let* $\mathcal{P}$ *denote the collection of joint distributions* P *of the form*

$$p(q)p(x_1, x_{1c}|q)p(x_2, x_{2c}|q)p(\hat{y}_{r1}|y_{r1}, q)p(\hat{y}_{r2}|y_{r2}, q)p(y_1, y_2, y_{r1}, y_{r2}|x_1, x_2), \tag{6.12}$$

where $p(y_1, y_2, y_{r1}, y_{r2}|x_1, x_2)$ *is set by the channel. For a fixed* $P \in \mathcal{P}$*, let* $\mathcal{R}(P)$ *denote the set of nonnegative rate pairs* (R_1, R_2) *satisfying*

$$R_1 < \min\{c_1 + (C_1 - \xi_1)^+, c_1'\} \tag{6.13}$$
$$R_1 < \min\{a_1 + (C_1 - \xi_1)^+, a_1'\} + \min\{b_2 + (C_2 - \xi_2)^+, b_2'\} \tag{6.14}$$
$$R_2 < \min\{c_2 + (C_2 - \xi_2)^+, c_2'\} \tag{6.15}$$
$$R_2 < \min\{a_2 + (C_2 - \xi_2)^+, a_2'\} + \min\{b_1 + (C_1 - \xi_1)^+, b_1'\} \tag{6.16}$$
$$R_1 + R_2 < \min\{a_1 + (C_1 - \xi_1)^+, a_1'\} + \min\{d_2 + (C_2 - \xi_2)^+, d_2'\} \tag{6.17}$$
$$R_1 + R_2 < \min\{b_1 + (C_1 - \xi_1)^+, b_1'\} + \min\{b_2 + (C_2 - \xi_2)^+, b_2'\} \tag{6.18}$$
$$R_1 + R_2 < \min\{d_1 + (C_1 - \xi_1)^+, d_1'\} + \min\{a_2 + (C_2 - \xi_2)^+, a_2'\} \tag{6.19}$$
$$2R_1 + R_2 < \min\{a_1 + (C_1 - \xi_1)^+, a_1'\} + \min\{d_1 + (C_1 - \xi_1)^+, d_1'\} \tag{6.20}$$
$$+ \min\{b_2 + (C_2 - \xi_2)^+, b_2'\} \tag{6.21}$$
$$R_1 + 2R_2 < \min\{b_1 + (C_1 - \xi_1)^+, b_1'\} + \min\{a_2 + (C_2 - \xi_2)^+, a_2'\} \tag{6.22}$$
$$+ \min\{d_2 + (C_2 - \xi_2)^+, d_2'\}. \tag{6.23}$$

The convex hull of the set $\bigcup_{P \in \mathcal{P}} \mathcal{R}(P)$ *is an achievable region for the IRC.*

Proof. The detailed proof is given in Appendix 6.A. Here we give a sketch of proof. Consider $i, j \in \{1, 2\}$ and $i \neq j$. We employ common-private rate-splitting [Car78, HK81] at the transmitters and extended hash-forward [Kim07] or quantize-bin-forward [WT11a], at the relays as follows: Transmitter i splits its message m_i into a *common* part m_{ic} of rate R_{ic} and a *private* part m_{ip} of rate R_{ip}. A superposition codebook is generated, with codewords $x_{ic}^n(m_{ic})$ and $x_i^n(m_{ic}, m_{ip})$ generated i.i.d. with distributions $p_{X_{ic}}$ and $p_{X_i|X_{ic}}$, respectively. Relay i's codebook has $2^{n\hat{R}_i}$ codewords $\hat{y}_{ri}^n$'s, each generated i.i.d. according to the marginal distribution $p_{\hat{Y}_{ri}}$. Each codeword in the quantization codebook is assigned to one of $\{1, \ldots, 2^{nC_i}\}$ bins by a uniformly generated random mapping.

Encoding: Transmitter i sends out a codeword x_i^n corresponding to its message index. Relay i quantizes its received sequence y_{ri}^n by choosing a quantization codeword $\hat{y}_{ri}^n$ jointly typical with y_{ri}^n, and then sends out the corresponding bin index to receiver i via the noiseless link of rate C_i.

Decoding: Receiver i finds a unique message pair (m_{ic}, m_{ip}) such that the sequences $x_{jc}^n(m_{jc}), x_{ic}^n(m_{ic}),\ x_i^n(m_{ic}, m_{ip}), \hat{y}_{ri}^n$, and y_i^n are jointly typical for *some* $m_{jc} \in \{1, \ldots, 2^{R_{jc}}\}$ and *some* $\hat{y}_{ri}^n$ whose bin index matches the index that receiver i received from relay i, $i \neq j$.

As a result, we obtain the following constraints on the partial rates so that the error probability vanishes when $n \to \infty$:

$$R_{1p} < \min\{a_1 + (C_1 - \xi_1)^+, a_1'\} \tag{6.24}$$

$$R_{1p} + R_{2c} < \min\{b_1 + (C_1 - \xi_1)^+, b_1'\} \tag{6.25}$$

$$R_{1p} + R_{1c} < \min\{c_1 + (C_1 - \xi_1)^+, c_1'\} \tag{6.26}$$

$$R_{1p} + R_{1c} + R_{2c} < \min\{d_1 + (C_1 - \xi_1)^+, d_1'\} \tag{6.27}$$

$$R_{2p} < \min\{a_2 + (C_2 - \xi_2)^+, a_2'\} \tag{6.28}$$

$$R_{2p} + R_{1c} < \min\{b_2 + (C_2 - \xi_2)^+, b_2'\} \tag{6.29}$$

$$R_{2p} + R_{2c} < \min\{c_2 + (C_2 - \xi_2)^+, c_2'\} \tag{6.30}$$

$$R_{2p} + R_{2c} + R_{1c} < \min\{d_2 + (C_2 - \xi_2)^+, d_2'\}. \tag{6.31}$$

Applying Fourier-Motzkin elimination to the above constraints and removing redundant inequalities we obtain (6.13)–(6.23). $\square$

Remark 6.1. Following the same line of proofs as in Sections 4.B and 5.D, we can show that the above region can also be achieved by noisy network coding [LKEC11], which does not use explicit binning, and by generalized hash-forward [RY10]. However, for this type of channels which have noiseless links, we find it easier to gain insight into the meaning of each term of the rate constraints if we employ the quantize-bin-forward strategy, see Remark 6.2 below. Such insight is useful in establishing approximate capacity results as will be shown in Section 6.4. Moreover, as shown in Section 5.4, noisy network coding incurs higher delay and encoding/decoding complexity than the coding schemes based on short-message like quantize-bin-forward.

Remark 6.2. Observe that each constraint in (6.24)–(6.31) is the minimum of two terms, in the same way as one can see in Theorem 4.3.1. The second term corresponds to the case when C_i, $i \in \{1, 2\}$, is large enough to convey the quantization $\hat{y}_{ri}^n$ correctly to receiver i. The first term corresponds to the case when the limited C_i allows receiver i to identify only a list of candidates of $\hat{y}_{ri}^n$. In this case, ξ_i plays the role of a "rate loss", similar to ξ_i in Theorem 4.4.1 and in [WT11a], whose value depends on the quantization distortion at the relay R_i. Further discussion on ξ_i can be found in Section 6.4.

Remark 6.3. If we replace Y_{ri} (resp. $\hat{Y}_{ri}$) in (6.3)–(6.11) by Y_j (resp. $\hat{Y}_j$), for $i, j \in \{1, 2\}$, $i \neq j$, and plug them into (6.24)–(6.31) we recover the achievable rates for the interference channel with limited receiver cooperation, derived in (4.67) of Chapter 4 and in [WT11a, bottom of page 2924] (one-round conferencing). This

is because such replacements make the current channel model coincide with the model studied in Chapter 4 and in the reference [WT11a].
On the other hand, by setting $\hat{Y}_{r1} = \hat{Y}_{r2} = \hat{Y}_r$ in (6.3)–(6.11) and in the expressions for $a_2, a'_2, b_2, b'_2, c_2, c'_2, d_2, d'_2, \xi_2$ we recover the achievable rate region for the IC with one degraded broadcasting relay in [ZY13].

6.3 Capacity Region of Some Classes of Discrete Memoryless IRCs

In this section we characterize the capacity region of some classes of discrete memoryless interfering relay channels. These channels have a common feature that the output at the relay R_i is a function of the input at transmitter Tx_i and the output at receiver Rx_i. As such, our results extend the optimality of the hash-forward scheme by Kim to more general settings than the one studied in [CK07]. In particular, the capacity results in this section generalize and unify the capacity regions of a class of semideterministic relay channels [CK07, Kim08], the strong interference channel [CE87], a class of deterministic interference channels [EC82], and partially generalize the semideterministic interference channel with common information [CM09].

6.3.1 Semideterministic IRC

Consider a semideterministic discrete memoryless IRC as depicted in Fig. 6.2. The channel has the following properties:
(i): There exist deterministic functions $t_1(\cdot)$, $t_2(\cdot)$, $f_1(\cdot,\cdot)$, $f_2(\cdot,\cdot)$ (not necessary invertible) such that:

$(i.a)$: $T_1 = t_1(X_1)$, $T_2 = t_2(X_2)$,

$(i.b)$: $Y_{r1} = f_1(X_1, Y_1)$, $Y_{r2} = f_2(X_2, Y_2)$.

(ii): Y_1 depends only on X_1 and T_2, via the conditional distribution $p(y_1|x_1, t_2)$. Similarly for Y_2, X_2, and T_1.
(iii): For all positive integers N and all input distributions of the form

$$p(w)p(x_1^N|w)p(x_2^N|w),$$

the following conditions hold:

$$I(T_1^N; Y_2^N, Y_{r2}^N|X_2^N, W) \geq I(T_1^N; Y_1^N, Y_{r1}^N|X_2^N, W) \tag{6.32a}$$
$$I(T_2^N; Y_1^N, Y_{r1}^N|X_1^N, W) \geq I(T_2^N; Y_2^N, Y_{r2}^N|X_1^N, W). \tag{6.32b}$$

The main result of this section is in the following theorem.

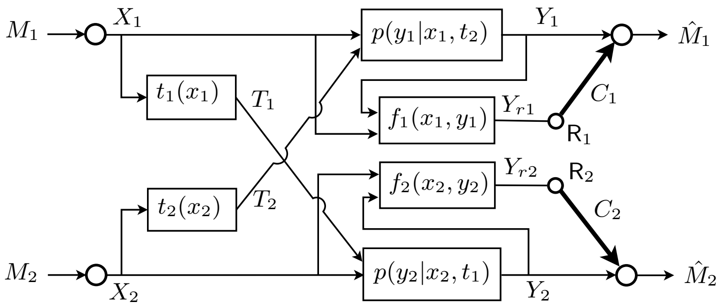

Figure 6.2: Semideterministic interfering relay channels.

Theorem 6.3.1. *The capacity region of the semideterministic IRC is the union of the set of nonnegative rate pairs* (R_1, R_2) *satisfying*

$$\begin{aligned}
R_1 &\leq \min\{I(X_1;Y_1|T_2,Q) + C_1, I(X_1;Y_1,Y_{r1}|T_2,Q)\} && (6.33)\\
R_2 &\leq \min\{I(X_2;Y_2|T_1,Q) + C_2, I(X_2;Y_2,Y_{r2}|T_1,Q)\} && (6.34)\\
R_1 + R_2 &\leq \min\{I(X_1;Y_1|T_1,T_2,Q) + C_1, I(X_1;Y_1,Y_{r1}|T_1,T_2,Q)\}\\
&\quad + \min\{I(X_2,T_1;Y_2|Q) + C_2, I(X_2,T_1;Y_2,Y_{r2}|Q)\} && (6.35)\\
R_1 + R_2 &\leq \min\{I(X_1,T_2;Y_1|T_1,Q) + C_1, I(X_1,T_2;Y_1,Y_{r1}|T_1,Q)\}\\
&\quad + \min\{I(X_2,T_1;Y_2|T_2,Q) + C_2, I(X_2,T_1;Y_2,Y_{r2}|T_2,Q)\} && (6.36)\\
R_1 + R_2 &\leq \min\{I(X_1,T_2;Y_1|Q) + C_1, I(X_1,T_2;Y_1,Y_{r1}|Q)\}\\
&\quad + \min\{I(X_2;Y_2|T_1,T_2,Q) + C_2, I(X_2;Y_2,Y_{r2}|T_1,T_2,Q)\} && (6.37)\\
2R_1 + R_2 &\leq \min\{I(X_1;Y_1|T_1,T_2,Q) + C_1, I(X_1;Y_1,Y_{r1}|T_1,T_2,Q)\}\\
&\quad + \min\{I(X_1,T_2;Y_1|Q) + C_1, I(X_1,T_2;Y_1,Y_{r1}|Q)\}\\
&\quad + \min\{I(X_2,T_1;Y_2|T_2,Q) + C_2, I(X_2,T_1;Y_2,Y_{r2}|T_2,Q)\} && (6.38)\\
R_1 + 2R_2 &\leq \min\{I(X_1,T_2;Y_1|T_1,Q) + C_1, I(X_1,T_2;Y_1,Y_{r1}|T_1,Q)\}\\
&\quad + \min\{I(X_2;Y_2|T_1,T_2,Q) + C_2, I(X_2;Y_2,Y_{r2}|T_1,T_2,Q)\}\\
&\quad + \min\{I(X_2,T_1;Y_2|Q) + C_2, I(X_2,T_1;Y_2,Y_{r2}|Q)\} && (6.39)
\end{aligned}$$

over all probability distributions of the form $p(q)p(x_1|q)p(x_2|q)$, *with* $|\mathcal{Q}| \leq 16$.

Proof. The achievability and converse proofs are deferred to Appendix 6.B. □

In the following we discuss some aspects of this capacity result.

Remark 6.4 (Deterministic IRC)**.** Suppose we replace the conditional distribution $p(y_1|x_1, t_2)$ (resp. $p(y_2|x_2, t_1)$) in Fig. 6.2 by some function $y_1(x_1, t_2)$ (resp. $y_2(x_2, t_1)$), and assume that there exist functions $k_1(\cdot, \cdot)$ and $k_2(\cdot, \cdot)$ such that $T_2 = k_1(X_1, Y_1)$ and $T_1 = k_2(X_2, Y_2)$. Then it can be verified that the resulting deterministic IRC, depicted in Fig. 6.3, satisfies all the properties of the semideterministic IRC in Fig. 6.2. Furthermore, one can show that the class of the former channel is strictly included in the class of the latter channel. Hence, the capacity region of the deterministic channel can be deduced from Theorem 6.3.1. What is more, due to the existence of $k_1(\cdot,\cdot)$ and $k_2(\cdot,\cdot)$, the capacity region of the deterministic IRC remains unchanged if we replace the condition $y_{r1} = f_1(x_1, y_1)$ (resp. $y_{r2} = f_2(x_2, y_2)$) by $y_{r1} = f_1'(x_1, t_2)$ (resp. $y_{r2} = f_2'(x_2, t_1)$), for some functions $f_1'(\cdot,\cdot)$, $f_2'(\cdot,\cdot)$. It is also easy to recognize that the deterministic IRC is in turn a generalization of the El Gamal-Costa deterministic interference channel [EC82]. In this sense the deterministic IRC includes the channel studied in [KLCS13], in which the two relays coincide, as a special case.

Remark 6.5. Of the properties of the semideterministic IRC, attribute $(i.b)$ is the same as that of [CK07]. We want to emphasize that in our converse proof, properties $(i.b)$ and (ii) play crucial roles in single-letterizing mutual information terms. In contrast, the converse for the deterministic IRC mentioned in Remark 6.4 can be proven *without* using $(i.b)$. As such, the semideterministic IRC can be considered as a non-trivial generalization of the deterministic IRC of El Gamal-Costa type.

Remark 6.6. The property (iii) is based on a feature of the semideterministic interference channel with common information [CM09]. We can interpret (6.32) as a partially strong interference condition: Knowing X_k^N, receiver k together with relay k can deduce more information about one part of the message sent by transmitter l (conveyed by T_l^N) than receiver l and relay l can, $k, l \in \{1, 2\}$, $k \neq l$. In Appendix 6.B we show that choosing T_k to represent the common message of transmitter k is capacity achieving. In the case when $T_k^N = X_k^N$, $k \in \{1, 2\}$, the condition (6.32) is an immediate generalization of the strong interference channel [CE87]. The capacity region of the semideterministic strong IRC will be given in Corollary 6.3.1 below.

Remark 6.7. We can easily obtain the capacity region of the semideterministic IRC when only one transmitter causes interference to its unintended relay and receiver, simply by setting $T_1 = \emptyset$ or $T_2 = \emptyset$ in Theorem 6.3.1. In a more extreme case, when there is no interference in the channel, namely $T_1 = T_2 = \emptyset$, the capacity region of the channel will reduce to the capacity region of two parallel deterministic relay channels of the type in [CK07]:

$$R_1 \leq \min\{I(X_1; Y_1) + C_1, I(X_1; Y_1, Y_{r1})\} \tag{6.40}$$

$$R_2 \leq \min\{I(X_2; Y_2) + C_2, I(X_2; Y_2, Y_{r2})\}. \tag{6.41}$$

This is because in this case we are employing the capacity achieving hash-forward coding technique [CK07] in two separate channels. The deterministic IRC in Remark 6.4 does not have this property.

Next, let us consider a DM-IRC which satisfies the following conditions:

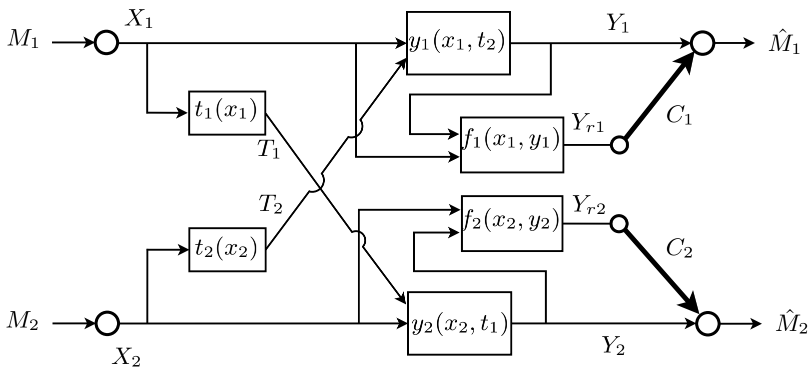

Figure 6.3: Deterministic IRC of El Gamal-Costa type. Due to the the property that $T_2 = k_1(X_1, Y_1)$ and $T_1 = k_2(X_2, Y_2)$, this channel has the same capacity region as the channel where $Y_{r1} = f_1'(X_1, T_2)$ and $Y_{r2} = f_2'(X_2, T_1)$.

(i) $Y_{r1} = f_1(X_1, Y_1)$ and $Y_{r2} = f_2(X_2, Y_2)$ for some functions $f_1(\cdot, \cdot)$ and $f_2(\cdot, \cdot)$ and,
(ii) The following conditions

$$
\begin{aligned}
I(X_1; Y_1, Y_{r1}|X_2, U) \leq I(X_1; Y_2, Y_{r2}|X_2, U) \\
I(X_2; Y_2, Y_{r2}|X_1, U) \leq I(X_2; Y_1, Y_{r1}|X_1, U)
\end{aligned}
$$

hold for all input distributions of the form $p(u, x_1, x_2) = p(u)p(x_1|u)p(x_2|u)$.
We call this channel the *semideterministic strong IRC* (SS-IRC). By the results in [CM09,MYK05] we have the conditions in (6.32) hold. Consequently the SS-IRC belongs to the class of the semideterministic IRC in Section 6.3.1. In particular, the SS-IRC is the semideterministic IRC with $T_1 = X_1$ and $T_2 = X_2$. Therefore the capacity region of the SS-IRC can be deduced from Theorem 6.3.1 as follows.

Corollary 6.3.1. *The capacity region of the semideterministic strong IRC is formed by the union of nonnegative rate pairs* (R_1, R_2) *satisfying*

$$R_1 \leq \min\{I(X_1; Y_1|X_2, Q) + C_1, I(X_1; Y_1, Y_{r1}|X_2, Q)\} \tag{6.42}$$

$$R_2 \leq \min\{I(X_2; Y_2|X_1, Q) + C_2, I(X_2; Y_2, Y_{r2}|X_1, Q)\} \tag{6.43}$$

$$R_1 + R_2 \leq \min\{I(X_1, X_2; Y_1|Q) + C_1, I(X_1, X_2; Y_1, Y_{r1}|Q)\} \tag{6.44}$$

$$R_1 + R_2 \leq \min\{I(X_1, X_2; Y_2|Q) + C_2, I(X_1, X_2; Y_2, Y_{r2}|Q)\}, \tag{6.45}$$

over all probability distributions of the form $p(q)p(x_1|q)p(x_2|q)$*, with* $|\mathcal{Q}| \leq 8$.

6.3.2 Compound Semideterministic Multiple Access Relay Channel

Recall that in the proof of achievability for the IRC in Section 6.2.2, each receiver can decode (non-uniquely) the common message sent by the unpairing transmitter. This motivates us to study a closely related setup, in which the message sent by a transmitter is required to be decoded uniquely by both receivers. It turns out that we can base the achievability for this new setup on the achievability of the IRC and moreover, the achievability is tight, as shown in the sequel.

Consider the same physical channel as in Fig. 6.1, but each receiver is required to decode both messages from the transmitters. We further assume that the following relations $Y_{r1} = f_1(X_1, Y_1)$ and $Y_{r2} = f_2(X_2, Y_2)$ hold for some deterministic functions $f_1(\cdot, \cdot)$ and $f_2(\cdot, \cdot)$. We call this channel *compound semideterministic multiple access relay channel* (CS-MARC). Then we have.

Proposition 6.3.1. *The capacity region of the deterministic CS-MARC is given by the union of the nonnegative rate pairs* (R_1, R_2) *that satisfy*

$$\begin{aligned} R_1 &\le \min\{I(X_1; Y_1|X_2, Q) + C_1, I(X_1; Y_1, Y_{r1}|X_2, Q)\} && (6.46)\\ R_1 &\le I(X_1; Y_2|X_2, Q) && (6.47)\\ R_2 &\le \min\{I(X_2; Y_2|X_1, Q) + C_2, I(X_2; Y_2, Y_{r2}|X_1, Q)\} && (6.48)\\ R_2 &\le I(X_2; Y_1|X_1, Q) && (6.49)\\ R_1 + R_2 &\le \min\{I(X_1, X_2; Y_1|Q) + C_1, I(X_1, X_2; Y_1, Y_{r1}|Q)\} && (6.50)\\ R_1 + R_2 &\le \min\{I(X_1, X_2; Y_2|Q) + C_2, I(X_1, X_2; Y_2, Y_{r2}|Q)\} && (6.51) \end{aligned}$$

over all distributions of the form $p(q)p(x_1|q)p(x_2|q)$*, where* $|\mathcal{Q}| \le 10$.

Proof. The proof is presented in Appendix 6.C. $\square$

Remark 6.8. The CS-MARC is one of the generalizations of the primitive relay channel studied by Cover and Kim in [CK07, Kim08]. Proposition 6.3.1 shows that the hash-forward scheme is not only generalizable to a relay channel with multiple primitive relays [Kim08], but also to the channel with one primitive relay and two senders, while the output at the receiver is a function of the relay's output and one of the senders' input.

By removing either Y_1 and Y_{r1} or Y_2 and Y_{r2} in Proposition 6.3.1 we can recover the capacity region of a class of MARC with orthogonal receive components established in [KS12].

6.4 Gaussian Interfering Relay Channels

The focus of this section is on the Gaussian IRC, depicted in Fig. 6.4. The received signals at the receiver Rx_i (denoted by Y_i) and at the relay R_i (denoted by Y_{ri}),

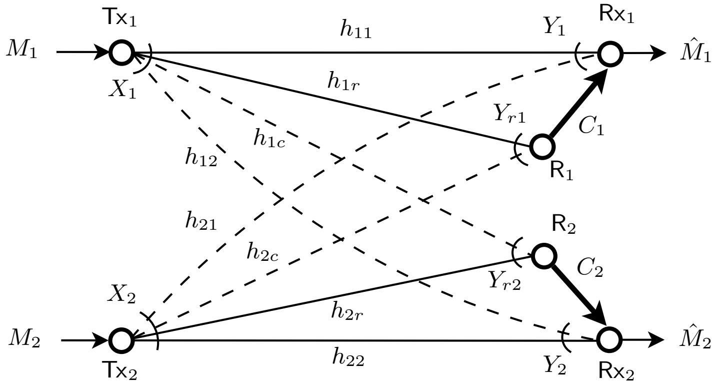

Figure 6.4: Gaussian interfering relay channels.

$i \in \{1, 2\}$, are given by

$$Y_1 = h_{11}X_1 + h_{21}X_2 + Z_1, \tag{6.52}$$

$$Y_2 = h_{12}X_1 + h_{22}X_2 + Z_2, \tag{6.53}$$

$$Y_{r1} = h_{1r}X_1 + h_{2c}X_2 + Z_{r1}, \tag{6.54}$$

$$Y_{r2} = h_{1c}X_1 + h_{2r}X_2 + Z_{r2}. \tag{6.55}$$

The noise processes Z_i's are independent and identically distributed $\sim \mathcal{N}(0, 1)$, $i \in \{1, 2, r1, r2\}$. Without loss of generality we assume an average unit power constraint at each transmitter, namely $\frac{1}{n}\sum_{i=1}^{n} |x_i|^2 \leq 1$, where n is the block length. For simplicity we only consider real-valued channel gains and input/output symbols, and assume that channel gains are known to all nodes in the network. For the convenience of notation, we define the following quantities, which will be used extensively hereafter:

$$\mathsf{SNR}_1 = h_{11}^2,\ \mathsf{INR}_1 = h_{21}^2,\ \mathsf{SNR}_{r1} = h_{1r}^2,\ \mathsf{INR}_{r1} = h_{2c}^2,$$
$$\mathsf{SNR}_2 = h_{22}^2,\ \mathsf{INR}_2 = h_{12}^2,\ \mathsf{SNR}_{r2} = h_{2r}^2,\ \mathsf{INR}_{r2} = h_{1c}^2,$$
$$\eta_1^2 = \left|1 - \frac{h_{1r}h_{21}}{h_{2c}h_{11}}\right|^2,\ \eta_2^2 = \left|1 - \frac{h_{2r}h_{12}}{h_{1c}h_{22}}\right|^2.$$

6.4.1 An Outer Bound to The Capacity Region

In the following we present an outer bound to the capacity region of the Gaussian IRC.

Theorem 6.4.1. *The following set of nonnegative rate pairs (R_1, R_2) forms an outer bound to the capacity region of the Gaussian IRC:*

$$
\begin{aligned}
R_1 &\leq \min\{\mathcal{C}(\mathsf{SNR}_1) + C_1, \mathcal{C}(\mathsf{SNR}_1 + \mathsf{SNR}_{r1})\} && (6.56)\\
R_2 &\leq \min\{\mathcal{C}(\mathsf{SNR}_2) + C_2, \mathcal{C}(\mathsf{SNR}_2 + \mathsf{SNR}_{r2})\} && (6.57)\\
R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + \mathcal{C}\left(\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + C_1 + C_2 && (6.58)\\
R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_2 + \mathsf{INR}_2) + \mathcal{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + C_1 + C_2 && (6.59)\\
R_1 + R_2 &\leq \mathcal{C}\left(\mathsf{INR}_1 + \tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + \mathcal{C}\left(\mathsf{INR}_2 + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + C_1 + C_2 && (6.60)\\
2R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + \mathcal{C}\left(\mathsf{INR}_2 + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + \mathcal{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) + 2C_1 + C_2 && (6.61)\\
R_1 + R_2 &\leq \mathcal{C}\left(\tfrac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right) + \mathcal{C}(\mathsf{INR}_2 + \mathsf{SNR}_{r2} + \mathsf{INR}_{r2} + \mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})) && (6.62)\\
R_1 + R_2 &\leq \mathcal{C}\left(\tfrac{\mathsf{SNR}_2+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right) + \mathcal{C}(\mathsf{INR}_1 + \mathsf{SNR}_{r1} + \mathsf{INR}_{r1} + \mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})) && (6.63)\\
R_1 + R_2 &\leq \mathcal{C}\left(\mathsf{INR}_1 + \mathsf{INR}_{r1} + \tfrac{\mathsf{SNR}_{r1}+\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right)\\
&\quad + \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{INR}_{r2} + \tfrac{\mathsf{SNR}_{r2}+\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right) && (6.64)\\
2R_1 + R_2 &\leq \mathcal{C}\left(\tfrac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right) + \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{INR}_{r2} + \tfrac{\mathsf{SNR}_{r2}+\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right)\\
&\quad + \mathcal{C}(\mathsf{INR}_1 + \mathsf{SNR}_{r1} + \mathsf{INR}_{r1} + \mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})) && (6.65)\\
R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2}) + \mathsf{SNR}_{r2} + \mathsf{INR}_2 + \mathsf{INR}_{r2})\\
&\quad + \mathcal{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right) + C_1 && (6.66)\\
R_1 + R_2 &\leq \mathcal{C}\left(\tfrac{\mathsf{SNR}_2+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right) + \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + C_1 && (6.67)\\
R_1 + R_2 &\leq \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{INR}_{r2} + \tfrac{\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right)\\
&\quad + \mathcal{C}\left(\mathsf{INR}_1 + \tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right) + C_1 && (6.68)\\
2R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + \mathcal{C}\left(\tfrac{\mathsf{SNR}_1}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right)\\
&\quad + \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{INR}_{r2} + \tfrac{\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right) + 2C_1 && (6.69)\\
R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1}) + \mathsf{SNR}_{r1} + \mathsf{INR}_1 + \mathsf{INR}_{r1})\\
&\quad + \mathcal{C}\left(\tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right) + C_2 && (6.70)\\
R_1 + R_2 &\leq \mathcal{C}(\mathsf{SNR}_2 + \mathsf{INR}_2) + \mathcal{C}\left(\tfrac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) + C_2 && (6.71)
\end{aligned}
$$

$$R_1 + R_2 \le \mathcal{C}\left(\mathsf{INR}_1 + \mathsf{INR}_{r1} + \tfrac{\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) + \mathcal{C}\left(\mathsf{INR}_2 + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right) + C_2 \quad (6.72)$$

$$2R_1 + R_2 \le \mathcal{C}(\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1}) + \mathsf{SNR}_{r1} + \mathsf{INR}_1 + \mathsf{INR}_{r1}) + \mathcal{C}\left(\mathsf{INR}_2 + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right) + \mathcal{C}\left(\tfrac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) + C_2 \quad (6.73)$$

$$2R_1 + R_2 \le \mathcal{C}\left(\tfrac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) + \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + \mathcal{C}\left(\mathsf{INR}_2 + \tfrac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) + C_1 + C_2 \quad (6.74)$$

$$2R_1 + R_2 \le \mathcal{C}\left(\tfrac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) + \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + \mathcal{C}\left(\mathsf{INR}_2 + \mathsf{INR}_{r2} + \tfrac{\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right) + C_1, \quad (6.75)$$

together with 6 *constraints on* $R_1 + 2R_2$ *obtained by exchanging indices* $1 \leftrightarrow 2$ *in the* 6 *constraints on* $2R_1 + R_2$.

Remark 6.9. By setting $\mathsf{INR}_{r1} = \mathsf{SNR}_{r2}$ and $\mathsf{INR}_{r2} = \mathsf{SNR}_{r1}$, which, in SNR and INR sense, is similar to letting the two relays in Fig. 6.4 coincide, the above outer bound reduces to the outer bound for the IC with one degraded broadcasting relay in [ZY13].

Sketch of proof of the outer bound. The bounds are proven by first giving different types of side information to the decoders (genie-aided bounds). We then leverage the outer bounds for the multiple input multiple output IC (MIMO IC) established by Telatar and Tse in [TT07], and the outer bound for the Gaussian MIMO IC [KV13] (which is equivalent to [TT07] for the current channel) for the ease of calculation. In fact, the outer bound in Theorem 6.4.1 is the intersection of 7 regions, each of which corresponds to a group of constraints whose interpretations and outline of proofs are given below. The detailed proofs are given in Appendix 6.D.

- Group 1: The bounds on individual rates in (6.56) and (6.57) are cut-set bounds.

- Group 2: The bounds in (6.58)–(6.61) are obtained by first upper bounding the sum-capacity gain due to the relays by $C_1 + C_2$. The remaining part is derived by optimizing the bounds established in [TT07] or directly applying [KV13, Lemma 1].

- Group 3: The bounds in (6.62)–(6.65) are genie-aided bounds. First we give Y_{ri}^n to decoder i, $i \in \{1, 2\}$, to turn the channel into a single input multiple output (SIMO) IC with two antennas at each receiver. Then we apply the bounds for MIMO IC in [TT07, KV13].

- Group 4: The bounds in (6.66)–(6.69) are genie-aided bounds. We upper bound the sum-capacity gain due to relay R_1 by C_1 and give Y_{r2} to decoder

2 to turn the channel into a SIMO IC. Then we apply the bounds for MIMO IC in [TT07, KV13].

- Group 5: The bounds in (6.70)–(6.73) are similarly derived as the bounds in Group 4, by symmetry.
- Group 6: The bound in (6.74): A genie gives (Y_{r1}^n, S_1^n, X_2^n) to one of the two decoders 1, and give S_2^n to decoder 2, where $S_1^n = h_{12}X_1^n + Z_2^n$, $S_2^n = h_{21}X_2^n + Z_1^n$. We also bound the sum-capacity gain due to the relays by C_1+C_2. The rest follows from the facts that conditioning does not increase entropy, the chain rule, and that Gaussian distribution maximizes differential entropy and conditional differential entropy subject to covariance constraint.
- Group 7: The bound in (6.75): A genie gives (Y_{r1}^n, S_1^n, X_2^n) to one of the two decoders 1, and gives (Y_{r2}^n, S_2^n) to decoder 2. We also bound the sum-capacity gain due to one of the relays R_1's by C_1.

□

6.4.2 Achievability

We can extend the inner bound for the DM-IRC in Theorem 6.2.1 to obtain an inner bound for the Gaussian channel. Specifically, we choose the following method to generate codebooks: First we set $Q = \text{const}$. Transmitter i, $i \in \{1, 2\}$, generates superposition Gaussian codebooks $X_{ic}^n(m_{ic})$ and $X_i^n(m_{ic}, m_{ip})$ with $X_{ic} \sim \mathcal{N}(0, P_{ic})$ and $X_i = X_{ic} + X_{ip}$, $X_{ip} \sim \mathcal{N}(0, P_{ip})$ and independent of X_{ic}, $P_{ip} + P_{ic} = P_i = 1$. The random variable used to generate the quantization codebook at relay R_i is specified by: $\hat{Y}_{ri} = Y_{ri} + Z_{qi}$ where $Z_{qi} \sim \mathcal{N}(0, \Delta_i)$, independent of everything else; Y_{ri} is given in (6.54) or (6.55). The quantization distortion Δ_i will be specified later, depending on which interference regime the channel belongs to. We also define the following quantities:

$$\mathsf{SNR}_{1p} = h_{11}^2 P_{1p}, \qquad \mathsf{SNR}_{2p} = h_{22}^2 P_{2p}, \tag{6.76}$$
$$\mathsf{INR}_{1p} = h_{21}^2 P_{2p}, \qquad \mathsf{INR}_{2p} = h_{12}^2 P_{1p}, \tag{6.77}$$
$$\mathsf{SNR}_{r1p} = h_{1r}^2 P_{1p}, \qquad \mathsf{SNR}_{r2p} = h_{2r}^2 P_{2p}, \tag{6.78}$$
$$\mathsf{INR}_{r1p} = h_{2c}^2 P_{2p}, \qquad \mathsf{INR}_{r2p} = h_{1c}^2 P_{1p}. \tag{6.79}$$

As demonstrated in [WT11a], the key parameters that determine the tightness of the inner bound for the Gaussian IC with limited receiver cooperation are: (i) power allocation for common and private messages at each transmitter, and (ii) quantization distortion at each relay. Reference [WT11a] shows that the power allocation rule of [ETW08] together with a new rule for designing the quantization distortion achieve the capacity region within a constant gap. In the remaining subsections of the current section, we will show the approximate optimality of such rules for some classes of Gaussian IRC. Especially, the results in Subsection 6.4.6

are obtained from the insight gained from the proof of the general achievability in Theorem 6.2.1. Before that, in the next subsection we will show that a slightly different method for allocating power achieves a bounded gap to the capacity region of the Gaussian IRC under a mild condition.

6.4.3 Capacity Region Within a Bounded Gap

As mentioned in Remark 6.3, our achievable scheme parallels the one-round conferencing scheme for the IC with conferencing decoders [WT11a]. Recall that in [WT11a] it was shown that a two-round conferencing scheme is necessary to achieve the capacity region within a constant number of bits, universally over the ensemble of channels, in the weak and mixed interference regimes. Therefore, one should expect that the achievability in Theorem 6.2.1 fails to attain a constant gap to the capacity region in the weak and mixed interference regimes, universally over the ensemble of the IRC. In this subsection, we instead look for a power allocation and quantization scheme that can guarantee a gap that is constant for a given channel and that can be bounded over all channels. In later subsections we will present stronger results, similar to that in [WT11a], for certain interference regimes.

To begin with, instead of directly applying the power allocation and quantization scheme of [WT11a], we use a similar strategy as in [ZY13], i.e., we fix the power allocation and optimize the quantization distortion at each relay. Specifically, following the insight of [ETW08] and [WT11a], each transmitter allocates power to its private message such that the corresponding interference induced at the unintended receiver is below the noise level, i.e.,

$$P_{1p} = \min\left\{1, 1/h_{12}^2\right\}, \quad P_{2p} = \min\left\{1, 1/h_{21}^2\right\}. \tag{6.80}$$

Δ_i, $i \in \{1, 2\}$, is optimized accordingly to minimize the gap between inner and outer bounds.

With the above choice of power allocation we have the following bounded-gap result.

Theorem 6.4.2 (Bounded gap to the capacity region). *For a given Gaussian IRC satisfying*

$$\theta := \max\left\{\frac{|h_{1c}|^2}{|h_{12}|^2}, \frac{|h_{2c}|^2}{|h_{21}|^2}\right\} < \infty, \tag{6.81}$$

the inner bound in Theorem 6.2.1, when extended to the Gaussian channel and with the power allocation specified in (6.80)*, is within a bounded gap of the capacity region. The gap is given by*

$$\delta R = \log\big(2 + \frac{1}{2}(\theta + \sqrt{\theta^2 + 16\theta + 16})\big) \text{ bits}, \tag{6.82}$$

and is obtained with quantization distortion $\Delta_1 = \Delta_2 = \frac{1}{4}(\sqrt{\theta^2 + 16\theta + 16} - \theta)$ *at both relays.*

Proof. See Appendix 6.E. □

For a given channel, the established gap is a constant. The astute reader may notice that this gap is similar to the gap derived in [ZY13, Theorem 3] for the IC with a degraded broadcast relaying. However, our gap depends only on how much interference each transmitter causes to the neighboring relay channel (via parameter θ), not on the channel gain between the transmitter and its intended relay. We emphasize that this is in contrast to [ZY13] where each interference-relay link carries both desired signal (with respect to one receiver) and interference (with respect to the other receiver).

Remark 6.10. This bounded gap result allows us to use the outer bound in Theorem 6.4.1 to study the generalized degrees of freedom behavior of the channel, as done in [WT11a, ZY13, CS12], although the definition and analysis are expected to be more involved due to the more complex structure of the channel. However we will only focus on non-asymptotic analysis in this work.

Remark 6.11. In [WT11a] the capacity region of the IC with conferencing receivers is characterized to within 2 bits for the complex-valued channel in the weak interference regime. The achievable scheme is a two-round conferencing protocol. In our channel model, however, the receivers do not talk to each other, therefore the two-round protocol is not feasible. Accordingly the quantize-bin-forward scheme does not guarantee a constant gap in the weak interference regime. This shows the advantage of having receiver conferencing compared with having dedicated relays in interference networks. For strong interference regime, however, we are able to show that the quantize-bin-forward scheme is still optimal to within a constant gap, as shown next.

6.4.4 Capacity Region Within $1/2$ Bit For Strong Interference

Let us focus on a class of Gaussian IRC in the strong interference regime. Recall that for the DM-IRC with strong interference in Section 6.3 we had the following conditions:

$$\begin{aligned} I(X_1; Y_1, Y_{r1}|X_2) &\leq I(X_1; Y_2|X_2) \\ I(X_2; Y_2, Y_{r2}|X_1) &\leq I(X_2; Y_1|X_1). \end{aligned}$$

We now consider the Gaussian counterpart. The Gaussian IRC is said to be in the strong interference regime if

$$\begin{aligned} \mathsf{SNR}_1 + \mathsf{SNR}_{r1} &\leq \mathsf{INR}_2 \\ \mathsf{SNR}_2 + \mathsf{SNR}_{r2} &\leq \mathsf{INR}_1. \end{aligned}$$

Let $\mathcal{R}^i_{S-IRC}$ denote the achievable region obtained by: specializing the general inner bound in Theorem 6.2.1 to the Gaussian IRC, as specified in Section 6.4.2; setting the whole message at each transmitter to be the common message, and setting the quantization distortion at each relay to be 1. Then we have the following results.

Proposition 6.4.1. *$\mathcal{R}^i_{S-IRC}$ is within $1/2$ bit to the capacity region of the Gaussian IRC with strong interference.*

Proof. The proof consists of deriving $\mathcal{R}^i_{S-IRC}$ and comparing it with the outer bound in Theorem 6.4.1. Details are in Appendix 6.F. □

6.4.5 Capacity Region Within $1/2$ Bit for the Compound Multiple Access Relay Channel

As can be seen in Section 6.3.2, the discrete memoryless compound MARC and the discrete memoryless IRC with strong interference have a close relation. We are drawing the same analogy here for the Gaussian channels. Consider the same physical channel as the Gaussian IRC but each decoder is required to recover both messages. We call this model the *compound multiple access relay channel* (C-MARC). Using the cut-set bound we can establish the following outer bound to the capacity region of the Gaussian C-MARC.

Lemma 6.1 (Outer Bound for the Gaussian C-MARC)**.** *The capacity region of the Gaussian C-MARC is contained in the region $\mathcal{R}^o_{C-MARC}$ defined as the set of nonnegative rate pairs (R_1, R_2) satisfying*

$$R_1 \leq \min\{\mathcal{C}(\mathsf{SNR}_1) + C_1, \mathcal{C}(\mathsf{SNR}_1 + \mathsf{SNR}_{r1})\} \tag{6.83}$$
$$R_1 \leq \min\{\mathcal{C}(\mathsf{INR}_2) + C_2, \mathcal{C}(\mathsf{INR}_2 + \mathsf{INR}_{r2})\} \tag{6.84}$$
$$R_2 \leq \min\{\mathcal{C}(\mathsf{SNR}_2) + C_2, \mathcal{C}(\mathsf{SNR}_2 + \mathsf{SNR}_{r2})\} \tag{6.85}$$
$$R_2 \leq \min\{\mathcal{C}(\mathsf{INR}_1) + C_1, \mathcal{C}(\mathsf{INR}_1 + \mathsf{INR}_{r1})\} \tag{6.86}$$
$$R_1 + R_2 \leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1) + C_1 \tag{6.87}$$
$$R_1 + R_2 \leq \mathcal{C}(\mathsf{SNR}_2 + \mathsf{INR}_2) + C_2 \tag{6.88}$$
$$R_1 + R_2 \leq \mathcal{C}(\mathsf{SNR}_1 + \mathsf{INR}_1 + \mathsf{SNR}_{r1} + \mathsf{INR}_{r1} + \mathsf{SNR}_1\mathsf{INR}_1\eta_1^2) \tag{6.89}$$
$$R_1 + R_2 \leq \mathcal{C}(\mathsf{SNR}_2 + \mathsf{INR}_2 + \mathsf{SNR}_{r2} + \mathsf{INR}_{r2} + \mathsf{SNR}_2\mathsf{INR}_2\eta_2^2). \tag{6.90}$$

We can derive an inner bound $\mathcal{R}^i_{C-MARC}$ for the Gaussian C-MARC from Theorem 6.2.1 by assigning the whole message at each transmitter as the common message and then follow the procedure in Section 6.4.2. Comparing $\mathcal{R}^i_{C-MARC}$ with $\mathcal{R}^o_{C-MARC}$ yields the following constant-gap result.

Proposition 6.4.2. *The inner bound $\mathcal{R}^i_{C-MARC}$ for the Gaussian C-MARC is within $1/2$ bit of the capacity region of the channel.*

Proof. See Appendix 6.G. □

6.4.6 Capacity Region Within $\log(7)$ Bits for IRC With High-Capacity Relay-Receiver Links

As we noted in Remark 6.2 in Section 6.2, each constraint in (6.24)–(6.31) is the minimum of two terms. The second term corresponds to the situation when the noiseless link has a high enough rate to describe the quantization codeword precisely to the receiver. This invokes a question of whether the coding strategy can guarantee a constant gap to the capacity region of the Gaussian channel in such a situation. The answer is positive, as will be shown next.

Definition 6.1 (IRC with high-capacity relay-receiver links)**.** Consider the Gaussian IRC as in Fig. 6.4, the channel is said to have *high-capacity relay-receiver links* if the following conditions are satisfied:

$$C_1 \geq \frac{1}{2}\log\left[\frac{\substack{2(\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_{r1}\eta_1^2)(1+\mathsf{INR}_1+\mathsf{INR}_{r1})\\ +2(4+4\mathsf{INR}_1+6\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}}{(4+4\mathsf{INR}_1+7\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}\right] \tag{6.91}$$

$$C_2 \geq \frac{1}{2}\log\left[\frac{\substack{2(\mathsf{SNR}_{r2}+\mathsf{INR}_{r2}+\mathsf{SNR}_2\mathsf{INR}_{r2}\eta_2^2)(1+\mathsf{INR}_2+\mathsf{INR}_{r2})\\ +2(4+4\mathsf{INR}_1+6\mathsf{INR}_{r2})(1+\mathsf{SNR}_2+\mathsf{INR}_2)}}{(4+4\mathsf{INR}_2+7\mathsf{INR}_{r2})(1+\mathsf{SNR}_2+\mathsf{INR}_2)}\right]. \tag{6.92}$$

Proposition 6.4.3. *For the Gaussian IRC with high-capacity relay-receiver links the quantize-bin-forward scheme can achieve the capacity region within* $\log(7)$ *bits.*

Proof. The detailed proof is given in Appendix 6.H. The proof also includes the derivation of the high-capacity relay-receiver links condition as stated in Definition 6.1. The key elements of the proof are as follows:

- Choosing a quantization distortion at each relay such that the rate loss terms ξ_1, ξ_2 are bounded so that one can characterize the conditions on C_1, C_2 for the second terms in the min in the RHS of (6.24)–(6.31) to be active.
- Choosing the suitable outer bounds among the ones in Theorem 6.4.1.
- Choosing a proper power allocation scheme at the transmitters such that the resulting achievable rates are within constant gaps to the corresponding chosen outer bounds.

Note that in order to achieve the above goals we use the following intuition: In the regime we are considering, we can think of receiver Rx_i and relay R_i collectively as a single receiver with output $(Y_i, \hat{Y}_{ri})$ for the inner bound and output (Y_i, Y_{ri}) for the outer bound. □

6.4.7 Sum-capacity Within 1 Bit in the Weak Interference-Relay Regime

In Section 6.4.4 we have seen that when the interference is strong, it is nearly optimal to set the whole message at each transmitter to be common. A relevant question is: When is it optimal or nearly optimal to treat interference as noise? In this section we characterize the conditions under which one can achieve to within 1 bit of the sum-capacity of the Gaussian IRC by treating interference as noise, i.e., by setting the whole message at each transmitter to be private. First, we need two lemmas, whose proofs are both given in Appendix 6.I.

Lemma 6.2. *For the Gaussian IRC, if*

$$C_1 \leq \frac{1}{2}\log\left(1+\frac{1+\mathsf{INR}_1+\mathsf{INR}_{r1}+\mathsf{SNR}_{r1}+\mathsf{SNR}_1(1+\mathsf{INR}_{r1}\eta_1^2)}{(1+\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}\right), \tag{6.93a}$$

$$C_2 \leq \frac{1}{2}\log\left(1+\frac{1+\mathsf{INR}_2+\mathsf{INR}_{r2}+\mathsf{SNR}_{r2}+\mathsf{SNR}_2(1+\mathsf{INR}_{r2}\eta_2^2)}{(1+\mathsf{INR}_{r2})(1+\mathsf{SNR}_2+\mathsf{INR}_2)}\right), \tag{6.93b}$$

then the sum-capacity is lower bounded by

$$\begin{aligned} C_{lb}^w = &\frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_1}\right)+\left(C_1-\frac{1}{2}\log\left(1+\frac{1}{1+\mathsf{INR}_{r1}}\left(1+\frac{\mathsf{INR}_{r1}}{1+\mathsf{INR}_1}\right)\right)\right)^+ \\ &+\frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_2}\right)+\left(C_2-\frac{1}{2}\log\left(1+\frac{1}{1+\mathsf{INR}_{r2}}\left(1+\frac{\mathsf{INR}_{r2}}{1+\mathsf{INR}_2}\right)\right)\right)^+. \end{aligned} \tag{6.94}$$

Further, C_{lb}^w is achieved by treating interference as noise at each decoder.

The next lemma establishes an upper bound on the sum-rate.

Lemma 6.3. *For the Gaussian IRC that satisfies*

$$\sqrt{\frac{\mathsf{INR}_1}{\mathsf{SNR}_2}}(1+\mathsf{INR}_2)+\sqrt{\frac{\mathsf{INR}_2}{\mathsf{SNR}_1}}(1+\mathsf{INR}_1) \leq 1, \tag{6.95}$$

the sum-capacity is upper bounded by

$$C_{ub}^w = \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_1}\right)+\frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_2}{1+\mathsf{INR}_2}\right)+C_1+C_2. \tag{6.96}$$

The above two lemmas lead us to the main result.

Proposition 6.4.4 (Sum-capacity within 1 bit)**.** *For the Gaussian IRC satisfying* (6.93) *and* (6.95), *called the* weak interference-relay regime, *the sum-capacity C_{sum}^w is bounded by*

$$C_{ub}^w - 1 \leq C_{sum}^w \leq C_{ub}^w, \tag{6.97}$$

where C_{ub}^w is given in (6.96)*.*

Proof. The proof follows directly from Lemma 6.2 and Lemma 6.3 and the fact that

$$\begin{aligned} &\frac{1}{2}\log\left(1+\frac{1}{1+\mathsf{INR}_{r1}}\left(1+\frac{\mathsf{INR}_{r1}}{1+\mathsf{INR}_1}\right)\right) \leq \frac{1}{2} \\ &\frac{1}{2}\log\left(1+\frac{1}{1+\mathsf{INR}_{r2}}\left(1+\frac{\mathsf{INR}_{r2}}{1+\mathsf{INR}_2}\right)\right) \leq \frac{1}{2}. \end{aligned}$$

□

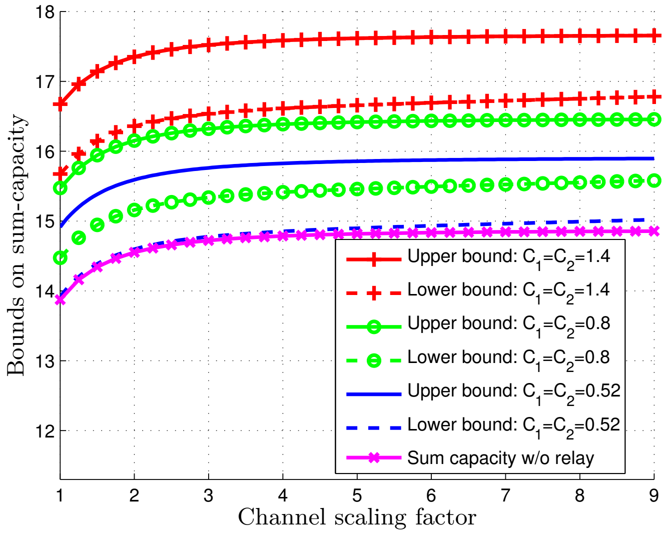

Figure 6.5: Sum-capacity upper and lower bounds in the weak interference-relay regime.

Our characterization of the weak interference-relay regime can be considered as a generalization of the one corresponding to the low-interference of noisy-interference regime of the Gaussian IC [SKC09, MK09, AV09]. In particular, in the extreme case of $C_1 = C_2 = 0$ the lower and upper bounds in (6.94) and (6.96) meet, and therefore we recover the *exact* sum-capacity of the Gaussian interference channel in the low-interference regime, established in [SKC09, MK09, AV09]. Proposition 6.4.4 shows that when the interference is weak and the capacities of the noiseless links are limited, a very simple coding scheme, where each relay quantizes the received signal and sends the bin index and each decoder treats the unintended signal as noise, is approximately sum-capacity optimal. This is a reasonable setup in multicell communication where the interference from the neighboring base stations is weak and the intra-cell backhaul link between a pico base station and the macro base station is capacity-limited.

Fig. 6.5 shows numerical examples for a Gaussian IRC under weak interference-relay condition. The plots use the following channel parameters

$$(h_{11}, h_{22}, h_{12}, h_{21}, h_{1r}, h_{2r}, h_{1c}, h_{2c}) = \alpha \cdot (200, 150, 1, 1, 350, 330, 0.05, 0.05),$$

where α is a parameter called the *channel scaling factor*. We plot the achievable sum-rate in (6.94), the sum-rate upper bound in (6.96) for different values of α and C_1, C_2 that satisfy the conditions (6.93) and (6.95). We also plot the sum-capacity of the IC without relays. As expected, one can see that the gap between the sum-rate upper bound and the lower bound (achievable sum-rate) is always less than one bit for each fixed (C_1, C_2). When C_1 and C_2 decrease, the two bounds move closer to the sum-capacity without relay. When $C_1 = C_2 = 0$ they both coincide with the sum-capacity of the IC without relay.

6.5 Chapter Conclusion

We propose and study an important setup which covers relevant practical scenarios such as the heterogenous wireless access networks. A general inner bound and multiple outer bounds are derived and shown to be tight or approximately tight for different classes of channels. The results generalize and unify the capacity results for a number of relay and interference channels. The obtained results improve the understanding of the optimality of the hash-forward scheme and its extension, when combined with the rate-splitting technique, for certain relay networks with interference. This also suggests that the same type of schemes can be optimal for more general multiple-unicast networks. Beside theoretical values, the results in this chapter have a good engineering implication because the proposed coding scheme can be implemented without requiring intermediate nodes in the network to decode the messages.

Appendices for Chapter 6

6.A Proof of Theorem 6.2.1

Codebook generation. Each message m_k is split into a common part m_{kc} and a private part m_{kp}: $m_k = (m_{kc}, m_{kp})$, $m_{kc} \in [1 : 2^{nR_{kc}}]$, $m_{kp} \in [1 : 2^{nR_{kp}}]$.
Codebook for transmitter k, $k \in \{1, 2\}$: Randomly and independently generate $2^{nR_{kc}}$ codewords $x^n_{kc}(m_{kc})$, each according to $\prod_{i=1}^n p_{X_{kc}}(x_{kci}(m_{kc}))$. For each $x^n_{kc}(m_{kc})$, randomly and conditionally independently generate $2^{nR_{kp}}$ codewords $x^n_k(m_{kc}, m_{kp})$, each according to $\prod_{i=1}^n p_{X_k|X_{kc}}(x_{ki}|x_{kci}(m_{kc}))$.
Codebook for relay k, $k \in \{1, 2\}$: Independently generate $2^{n\hat{R}_k}$ quantization codewords $\hat{y}^n_{rk}$, each according to the distribution $\prod_{i=1}^n p_{\hat{Y}_{rk}}(\hat{y}_{rki})$, where the marginal distribution $p_{\hat{Y}_{rk}}$ is obtained by the marginalization of (6.12). Uniformly partition the quantization codewords into 2^{nC_k} bins.

Encoding. Transmitter k sends out a codeword x^n_k corresponding to its message index $m_k = (m_{kc}, m_{kp})$. Relay k chooses a quantization codeword $\hat{y}^n_{rk}$ which is jointly typical with its received sequence y^n_{rk}, and then sends out the bin index l_k corresponding to the chosen quantization codeword, i.e., $l_k = B(\hat{y}^n_{rk})$, where $B(\cdot)$ is the binning function.

Decoding. We describe the decoding at receiver $\mathsf{R_{x1}}$. Decoder 1 looks for a unique index pair (m_{1c}, m_{1p}) such that

$$(x^n_1(m_{1c}, m_{1p}), x^n_{1c}(m_{1c}), x^n_{2c}(m_{2c}), \hat{y}^n_{r1}, y^n_1) \in \mathcal{T}^{(n)}_\epsilon \tag{6.98}$$

for some $m_{2c} \in [1 : 2^{nR_{2c}}]$ and some $\hat{y}^n_{r1}$ with $B(\hat{y}^n_{r1}) = l$, where l is the bin index received via the noiseless link from relay 1. If such a unique pair (m_{1c}, m_{1p}) is found, the decoder declares it as the transmitted codeword, otherwise it declares an error.

Error analysis. We will do analysis for the receiver 1, the analysis for the receiver 2 follows immediately by symmetry. In the following we use $\underline{m}$ to denote a message triple (m_{1c}, m_{1p}, m_{2c}) and $\underline{x}^n(\underline{m})$ to denote the corresponding codeword triple. $\underline{1}$ denotes a vector whose entries are 1's, with appropriate length.

Without loss of generality, assume that $(m_{1c}, m_{1p}, m_{2c}) = (1, 1, 1)$, i.e., $\underline{m} = \underline{1}$, was sent.

By the covering lemma [EK11, Lemma 3.3], the encoding at the relay succeeds with high probability if $\hat{R}_1 \geq I(Y_{r1}; \hat{Y}_{r1})$. By the conditional typicality lemma [EK11, Sec. 2.5], the transmitted codeword, the truly selected quantization codeword, and the received sequence are jointly typical as $n \to \infty$. Let K denote the index of the truly chosen quantization codeword by the relay 1, and L denotes its bin index. The decoding error happens if at least of the following two events occurs: (i) There exist wrong codewords of the transmitters and wrong quantization codewords of relay 1 that pass the joint typicality check (6.98); (ii) There

exist wrong codewords of the transmitters and correct quantization codewords of relay 1 that pass the joint typicality check (6.98). Hence, we define the following events:
$\mathcal{E}^1 := \{\exists\,(m_{1c}, m_{1p}) \neq (1,1), m_{2c} \in [1:2^{nR_{2c}}]$ and $\exists\,\hat{y}_{r1}^n(k), k \in [1:2^{n\hat{R}_1}], k \neq K$ such that $B(\hat{y}_{r1}^n(k)) = L$ and the corresponding sequences are jointly typical with y_1^n, as specified in (6.98).$\}$
$\mathcal{E}^2 := \{\exists\,(m_{1c}, m_{1p}) \neq (1,1), m_{2c} \in [1:2^{nR_{2c}}]$ such that the corresponding sequences and $\hat{y}_{r1}^n(K)$ are jointly typical with y_1^n, as specified in (6.98).$\}$
Conditioned on the successful encoding at the relay, the decoding error at the decoder 1 can be bounded by:

$$\mathsf{P}_{e1} \leq \mathsf{P}(\mathcal{E}^1) + \mathsf{P}(\mathcal{E}^2). \tag{6.99}$$

Next we bound $\mathsf{P}(\mathcal{E}^1)$ and $\mathsf{P}(\mathcal{E}^2)$.

Bounding $\mathsf{P}(\mathcal{E}^1)$. First note that the decoder 1 is not required to decode m_{2c} correctly. Hence, following the same argument as in [CMGE08] we have

$$\mathsf{P}(\mathcal{E}^1) \leq \sum_S \mathsf{P}(\mathcal{E}^1(S)), \tag{6.100}$$

where $\mathsf{P}(\mathcal{E}^1(S))$ is defined and bounded in the sequel with index set

$$S \in \{\{1p\}, \{1c, 1p\}, \{1p, 2c\}, \{1c, 1p, 2c\}\}.$$

We also define $\tilde{S} := \{1c, 1p, 2c\} \setminus S$. Further, let $\underline{m}_S$ denote the messages whose indices are given by S, and $R_S = \sum_{i \in S} R_i$.

$$\begin{aligned}
\mathsf{P}(\mathcal{E}^1(S)) &:= \sum_{\underline{m}:\underline{m}_S \neq \underline{1}, \underline{m}_{\tilde{S}} = \underline{1}} \sum_{k \neq K} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n(k), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}, B(\hat{Y}_{r1}^n(k)) = L\right\} \\
&\overset{(a)}{=} 2^{-nC_1} \sum_{\underline{m}:\underline{m}_S \neq \underline{1}, \underline{m}_{\tilde{S}} = \underline{1}} \sum_{k \neq K} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n(k), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\} \\
&\leq 2^{-nC_1} 2^{nR_S} \sum_{k \neq K} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n(k), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\},
\end{aligned} \tag{6.101}$$

where (a) follows due to the uniform binning. As an example, let us consider the case $S = \{1p, 2c\}$. For this case we have

$$\begin{aligned}
&\sum_{k \neq K} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n(k), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\} \\
&\qquad = \sum_{k \neq K} \mathsf{P}\left\{(X_{1c}^n(1), X_1^n(1, m_{1p}), X_{2c}^n, \hat{Y}_{r1}^n(k), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\} \\
&\qquad = \sum_{k \neq K} \sum_{(x_{1c}^n, x_1^n, x_{2c}^n, \hat{y}_{r1}^n, y_1^n) \in \mathcal{T}_\epsilon^{(n)}} p(x_{1c}^n, x_1^n, x_{2c}^n, \hat{y}_{r1}^n, y_1^n)
\end{aligned}$$

$$
\begin{aligned}
&\overset{(a)}{\leq} 2^{n\hat{R}_1} \sum_{(x_{1c}^n, y_1^n) \in \mathcal{T}_\epsilon^{(n)}} p(x_{1c}^n, y_1^n) \sum_{x_1^n \in \mathcal{T}_\epsilon^{(n)}(X_1|x_{1c}^n, y_1^n)} p(x_1^n|x_{1c}^n) \\
&\quad \cdot \sum_{x_{2c}^n \in \mathcal{T}_\epsilon^{(n)}(X_{2c}|x_{1c}^n, x_1^n, y_1^n)} p(x_{2c}^n) \sum_{\hat{y}_{r1}^n \in \mathcal{T}_\epsilon^{(n)}(\hat{Y}_{r1}|x_{1c}^n, x_1^n, x_{2c}^n, y_1^n)} p(\hat{y}_{r1}^n) \\
&\overset{(b)}{\leq} 2^{n\hat{R}_1} \sum_{(x_{1c}^n, y_1^n) \in \mathcal{T}_\epsilon^{(n)}} p(x_{1c}^n, y_1^n) 2^{-n(I(X_1, X_{2c}; Y_1|X_{1c}) + I(\hat{Y}_{r1}; X_1, X_{2c}, Y_1) - \delta(\epsilon))} \\
&\leq 2^{n\hat{R}_1} 2^{-n(I(X_1, X_{2c}; Y_1|X_{1c}) + I(\hat{Y}_{r1}; X_1, X_{2c}, Y_1) - \delta(\epsilon))},
\end{aligned} \tag{6.102}
$$

where $\delta(\epsilon) \to 0$ as $n \to \infty$. (a) follows due to the fact that $\hat{Y}_{r1}^n(k)$ and $X_{2c}^n(m_{2c})$ are independent of (X_{1c}^n, Y_1^n) for $k \neq K$ and $m_{2c} \neq 1$; (b) follows from the properties of jointly typical sequences [EK11, Sec. 2.5]. Substitute (6.102) back to (6.101) we conclude that for $S = \{1p, 2c\}$, $\mathsf{P}(\mathcal{E}^1(S)) \to 0$ as $n \to \infty$ if

$$
R_{1p} + R_{2c} < I(X_1, X_{2c}; Y_1|X_{1c}) + I(\hat{Y}_{r1}; X_1, X_{2c}, Y_1) + C_1 - \hat{R}_1. \tag{6.103}
$$

On the other hand, we can also bound $\mathsf{P}(\mathcal{E}^1(S))$ as follows:

$$
\begin{aligned}
\mathsf{P}(\mathcal{E}^1(S)) &= \sum_{\underline{m}: \underline{m}_S \neq \underline{1}, \underline{m}_{\bar{S}} = \underline{1}} \sum_{k \neq K} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n(k), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}, B(\hat{Y}_{r1}^n(k)) = L\right\} \\
&\leq \sum_{\underline{m}: \underline{m}_S \neq \underline{1}, \underline{m}_{\bar{S}} = \underline{1}} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\} \\
&\quad \cdot \mathsf{P}\left\{\exists\, k \neq K, B(\hat{Y}_{r1}^n(k)) = L, \left(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n, Y_1^n\right) \in \mathcal{T}_\epsilon^{(n)} \middle| (\underline{X}^n(\underline{m}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\} \\
&\leq 2^{nR_S} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\}.
\end{aligned} \tag{6.104}
$$

Similar to the derivation of (6.102) we can bound the probability in (6.104) and show that for $S = \{1p, 2c\}$, $\mathsf{P}(\mathcal{E}^1(S)) \to 0$ as $n \to \infty$ if

$$
R_{1p} + R_{2c} < I(X_1, X_{2c}; Y_1|X_{1c}). \tag{6.105}
$$

Combining (6.103) and (6.105) we conclude that for $S = \{1p, 2c\}$, $\mathsf{P}(\mathcal{E}^1(S)) \to 0$ as $n \to \infty$ if

$$
\begin{aligned}
R_{1p} + R_{2c} &< I(X_1, X_{2c}; Y_1|X_{1c}) + (I(\hat{Y}_{r1}; X_1, X_{2c}, Y_1) + C_1 - \hat{R}_1)^+ \\
&\overset{(a)}{=} I(X_1, X_{2c}; Y_1|X_{1c}) + (C_1 - I(\hat{Y}_{r1}; Y_{r1}|X_1, X_{2c}, Y_1))^+ \\
&= I(X_1, X_{2c}; Y_1|X_{1c}) + (C_1 - \xi_1)^+,
\end{aligned} \tag{6.106}
$$

where (a) follows from $\hat{R}_1 \geq I(Y_{r1}; \hat{Y}_{r1})$ and the Markov chain

$$
\hat{Y}_{r1} - Y_{r1} - (X_1, X_{1c}, X_{2c}, Y_1).
$$

Bounding $\mathsf{P}(\mathcal{E}^2)$. Similar to (6.100) we have

$$\mathsf{P}(\mathcal{E}^2) \leq \sum_S \mathsf{P}(\mathcal{E}^2(S))$$

where

$$\mathsf{P}(\mathcal{E}^2(S)) := \sum_{\underline{m}:\underline{m}_S \neq \underline{1}, \underline{m}_{\tilde{S}} = \underline{1}} \mathsf{P}\left\{(\underline{X}^n(\underline{m}), \hat{Y}_{r1}^n(K), Y_1^n) \in \mathcal{T}_\epsilon^{(n)}\right\}.$$

Notice that in this case the quantization codeword is the true one chosen by the relay. Hence, to bound $\mathsf{P}(\mathcal{E}^2(S))$'s we can consider $(Y_1^n, \hat{Y}_{r1}^n)$ as a single channel output sequence. The bounds then follow directly from the properties of the jointly typical sequences [EK11, Sec. 2.5]. In particular, for $S = \{1p, 2c\}$, $\mathsf{P}(\mathcal{E}^2(S)) \to 0$ as $n \to \infty$ if

$$R_{1p} + R_{2c} < I(X_1, X_{2c}; Y_1, \hat{Y}_{r1} | X_{1c}). \tag{6.107}$$

Combining (6.106) and (6.107) we have: For $S = \{1p, 2c\}$, $\mathsf{P}(\mathcal{E}^1(S)) \to 0$ and $\mathsf{P}(\mathcal{E}^2(S)) \to 0$ as $n \to \infty$ if (6.25) is satisfied. Applying the same analysis for other $S \in \{\{1p\}, \{1c, 1p\}, \{1p, 2c\}, \{1c, 1p, 2c\}\}$, we conclude that $\mathsf{P}(\mathcal{E}^1)$ and $\mathsf{P}(\mathcal{E}^2)$, and therefore P_{e1} in (6.99) vanishes as $n \to \infty$ if the constraints in (6.24)–(6.27) are satisfied. □

6.B Proof of Theorem 6.3.1

6.B.1 Achievability

Since $T_k = t_k(X_k)$, $k \in \{1, 2\}$, we can choose $X_{kc} = T_k$ in the general inner bound in Theorem 6.2.1. Furthermore, we set $\hat{Y}_{rk} = Y_{rk}$, i.e., we only do hash-forward. Using the properties of the channel we have:

$$\begin{aligned}
a_1 &= I(X_1; Y_1 | T_1, T_2, Q) \\
a_1' &= I(X_1; Y_1, Y_{r1} | T_1, T_2, Q) \\
b_1 &= I(X_1, T_2; Y_1 | T_1, Q) \\
b_1' &= I(X_1, T_2; Y_1, Y_{r1} | T_1, Q) \\
c_1 &= I(X_1; Y_1 | T_2, Q) \\
c_1' &= I(X_1; Y_1, Y_{r1} | T_2, Q) \\
d_1 &= I(X_1, T_2; Y_1 | Q) \\
d_1' &= I(X_1, T_2; Y_1, Y_{r1} | Q) \\
\xi_1 &= I(Y_{r1}; Y_{r1} | X_1, T_2, Y_1, Q) \\
&\overset{(a)}{=} 0,
\end{aligned}$$

where (a) follows because Y_{r1} is a function of (X_1, Y_1). By symmetry we easily obtain $a_2, a_2', b_2, b_2', c_2, c_2', d_2, d_2', \xi_2$. Substituting the above quantities into the inner bound in Theorem 6.2.1 we obtain an inner bound for the semideterministic IRC, which consists of the inequalities in Theorem 6.3.1 plus the following two inequalities, emanating from (6.14) and (6.16):

$$\begin{aligned} R_1 &< \min\{I(X_1;Y_1|T_1,T_2,Q)+C_1, I(X_1;Y_1,Y_{r1}|T_1,T_2,Q)\} \\ &\quad + \min\{I(X_2,T_1;Y_2|T_2,Q)+C_2, I(X_2,T_1;Y_2,Y_{r2}|T_2,Q)\} \qquad (6.108) \\ R_2 &< \min\{I(X_2;Y_2|T_1,T_2,Q)+C_2, I(X_2;Y_2,Y_{r2}|T_1,T_2,Q)\} \\ &\quad + \min\{I(X_1,T_2;Y_1|T_1,Q)+C_1, I(X_1,T_2;Y_1,Y_{r1}|T_1,Q)\}. \qquad (6.109) \end{aligned}$$

In the sequel we prove the redundancy of (6.108) and (6.109). First note that,

$$\begin{aligned} I(T_2;Y_2,Y_{r2}|X_1,Q) &= H(T_2|X_1,Q) - H(T_2|Y_2,Y_{r2},X_1,Q) \\ &\overset{(a)}{=} H(T_2|Q) - H(T_2|Y_2,Y_{r2},X_1,T_1,Q) \\ &\geq H(T_2|T_1,Q) - H(T_2|Y_2,Y_{r2},T_1,Q) \\ &= I(T_2;Y_2,Y_{r2}|T_1,Q), \qquad (6.110) \end{aligned}$$

where (a) follows because T_1 is a function of X_1, and X_1 is independent of T_2 given Q. By symmetry we have

$$I(T_1;Y_1,Y_{r1}|X_2,Q) \geq I(T_1;Y_1,Y_{r1}|T_2,Q). \qquad (6.111)$$

Next, we have

$$\begin{aligned} \min\{I(X_2,T_1;Y_2|T_2,Q)+C_2, &I(X_2,T_1;Y_2,Y_{r2}|T_2,Q)\} \\ &\geq I(X_2,T_1;Y_2|T_2,Q) \\ &\geq I(T_1;Y_2|X_2,T_2,Q) \\ &\overset{(c)}{=} I(T_1;Y_2|X_2,Q) \\ &\overset{(d)}{=} I(T_1;Y_2,Y_{r2}|X_2,Q) \\ &\overset{(e)}{\geq} I(T_1;Y_1,Y_{r1}|X_2,Q) \\ &\overset{(f)}{\geq} I(T_1;Y_1,Y_{r1}|T_2,Q), \qquad (6.112) \end{aligned}$$

where:

- (c) is because T_2 is a function of X_2, and Y_2 depends only on (X_2, T_1);
- (d) is because $Y_{r2} = f_2(X_2, Y_2)$;
- (e) is due to (6.32a) with $N = 1$, $W = Q$;
- (f) is due to (6.111).

Inserting (6.112) into the right hand side (RHS) of (6.108) gives:

$$\begin{aligned}\text{RHS of (6.108)} &\geq \min\{I(X_1;Y_1|T_1,T_2,Q)+C_1, I(X_1;Y_1,Y_{r1}|T_1,T_2,Q)\}\\ &\quad + I(T_1;Y_1,Y_{r1}|T_2,Q)\\ &\geq \min\{I(X_1;Y_1|T_2,Q)+C_1, I(X_1;Y_1,Y_{r1}|T_2,Q)\}\\ &= \text{RHS of (6.33).}\end{aligned}$$

Therefore (6.108) is redundant. By symmetry (6.109) is redundant. □

6.B.2 Converse

Before upperbounding the achievable rates we will prove some useful inequalities. Consider a sequence of $(2^{nR_1}, 2^{nR_2}, n)$ codes with vanishing error probability. Note that since the messages M_1 and M_2 are independent, $X_k^n(M_k)$ is independent of $X_l^n(M_l)$ and T_l^n, $k,l \in \{1,2\}$, $k \neq l$. Hence we have

$$I(T_2^n;Y_2^n,Y_{r2}^n|X_1^n) \geq I(T_2^n;Y_2^n,Y_{r2}^n|T_1^n), \tag{6.113}$$

whose proof is similar to the proof of (6.110), by replacing each random variable with the corresponding random n-sequence, and no Q is involved.

Let us denote the sequence that receiver k receives from relay k (via the noiseless link) by V_k^n, $k \in \{1,2\}$. We have

$$\begin{aligned}I(X_1^n;Y_1^n,V_1^n) &= I(X_1^n;Y_1^n) + I(X_1^n;V_1^n|Y_1^n) &(6.114)\\ &\overset{(a_1)}{\leq} I(X_1^n;Y_1^n) + H(V_1^n) &(6.115)\\ &\overset{(a_2)}{\leq} I(X_1^n;Y_1^n|T_2^n) + H(V_1^n) &(6.116)\\ &\leq I(X_1^n,T_1^n;Y_1^n|T_2^n) + H(V_1^n)\\ &= I(X_1^n;Y_1^n|T_1^n,T_2^n) + I(T_1^n;Y_1^n|T_2^n) + H(V_1^n) &(6.117)\\ &\leq I(X_1^n;Y_1^n|T_1^n,T_2^n) + I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n) + H(V_1^n), &(6.118)\end{aligned}$$

where: (a_1) follows from the nonnegativity of entropy, and conditioning does not increase entropy; (a_2) is because X_1^n is independent of T_2^n. We also have

$$\begin{aligned}I(X_1^n;Y_1^n,V_1^n) &\leq I(X_1^n;Y_1^n,Y_{r1}^n,V_1^n)\\ &\overset{(b_1)}{=} I(X_1^n;Y_1^n,Y_{r1}^n) &(6.119)\\ &\overset{(b_2)}{\leq} I(X_1^n;Y_1^n,Y_{r1}^n|T_2^n) &(6.120)\\ &\leq I(X_1^n,T_1^n;Y_1^n,Y_{r1}^n|T_2^n)\\ &= I(X_1^n;Y_1^n,Y_{r1}^n|T_1^n,T_2^n) + I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n), &(6.121)\end{aligned}$$

where: (b_1) is because V_1^n is a function of Y_{r1}^n; (b_2) is because X_1^n is independent of T_2^n. On the other hand, from (6.115),

$$\begin{aligned}
I(X_1^n;Y_1^n,V_1^n) &\le I(X_1^n;Y_1^n)+H(V_1^n)\\
&= I(X_1^n,T_2^n;Y_1^n)-I(T_2^n;Y_1^n|X_1^n)+H(V_1^n)\\
&\overset{(c_1)}{=} I(X_1^n,T_2^n;Y_1^n)-I(T_2^n;Y_1^n,Y_{r1}^n|X_1^n)+H(V_1^n)\\
&\overset{(c_2)}{\le} I(X_1^n,T_2^n;Y_1^n)-I(T_2^n;Y_2^n,Y_{r2}^n|X_1^n)+H(V_1^n)\\
&\overset{(c_3)}{\le} I(X_1^n,T_2^n;Y_1^n)-I(T_2^n;Y_2^n,Y_{r2}^n|T_1^n)+H(V_1^n),
\end{aligned} \tag{6.122}$$

where: (c_1) is because Y_{r1}^n is a function of X_1^n and Y_1^n; (c_2) is due to (6.32) with $N=n$, $W=\emptyset$; (c_3) follows from (6.113). Furthermore, using (6.119)

$$\begin{aligned}
I(X_1^n;Y_1^n,V_1^n) &\le I(X_1^n;Y_1^n,Y_{r1}^n)\\
&= I(X_1^n,T_2^n;Y_1^n,Y_{r1}^n)-I(T_2^n;Y_1^n,Y_{r1}^n|X_1^n)\\
&\overset{(d_1)}{\le} I(X_1^n,T_2^n;Y_1^n,Y_{r1}^n)-I(T_2^n;Y_2^n,Y_{r2}^n|X_1^n)\\
&\overset{(d_2)}{\le} I(X_1^n,T_2^n;Y_1^n,Y_{r1}^n)-I(T_2^n;Y_2^n,Y_{r2}^n|T_1^n),
\end{aligned} \tag{6.123}$$

where: (d_1) is due to (6.32); (d_2) follows from (6.113).

By symmetry we obtain the following inequalities, which are the duals of (6.122) and (6.123):

$$I(X_2^n;Y_2^n,V_2^n) \le I(X_2^n,T_1^n;Y_2^n)-I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n)+H(V_2^n), \tag{6.124}$$
$$I(X_2^n;Y_2^n,V_2^n) \le I(X_2^n,T_1^n;Y_2^n,Y_{r2}^n)-I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n). \tag{6.125}$$

We are now ready to bound the achievable rates.
Proof of (6.33)*:* By Fano's inequality,

$$\begin{aligned}
n(R_1-\epsilon_n) &\le I(W_1;Y_1^n,V_1^n)\\
&\le I(X_1^n;Y_1^n,V_1^n)\\
&\overset{(e_1)}{\le} I(X_1^n;Y_1^n|T_2^n)+H(V_1^n)\\
&\overset{(e_2)}{\le} \sum_{i=1}^n [I(X_{1i};Y_{1i}|T_{2i})+H(V_{1i})]\\
&\le \sum_{i=1}^n [I(X_{1i};Y_{1i}|T_{2i})+C_1],
\end{aligned} \tag{6.126}$$

where: (e_1) is due to (6.116); (e_2) is because conditioning does not increase entropy, and because Y_{1i} depends only on (X_{1i},T_{2i}).

Next,

$$
\begin{aligned}
n(R_1-\epsilon_n) &\le I(X_1^n;Y_1^n,V_1^n)\\
&\overset{(f_1)}{\le} I(X_1^n;Y_1^n,Y_{r1}^n|T_2^n)\\
&\overset{(f_2)}{\le} \sum_{i=1}^n I(X_{1i};Y_{1i},Y_{r1i}|T_{2i}),
\end{aligned}
\tag{6.127}
$$

where: (f_1) follows from (6.120); (f_2) is because Y_{r1i} is a function of Y_{1i} and X_{1i}, and Y_{1i} depends only on X_{1i}, T_{2i}.

We define a random variable Q, uniformly distributed over $\{1,\dots,n\}$ and independent of everything else. Further, for $k\in\{1,2\}$ we define $X_k = X_{kQ}$, $T_k = T_{kQ}$, $Y_k = Y_{kQ}$, $Y_{rk} = Y_{rkQ}$. Then (6.126) and (6.127) become

$$
\begin{aligned}
R_1-\epsilon_n &\le I(X_1;Y_1|T_2,Q)+C_1\\
R_1-\epsilon_n &\le I(X_1;Y_1,Y_{r1}|T_2,Q),
\end{aligned}
$$

where $\epsilon_n\to 0$ as $n\to\infty$, which proves (6.33).

The bound on R_2 *in* (6.34) *follows from* (6.33) *by symmetry.*

Proof of (6.35)*:* Again, starting with Fano's inequality,

$$
n(R_1+R_2-2\epsilon_n) \tag{6.128}
$$

$$
\begin{aligned}
&\le I(X_1^n;Y_1^n,V_1^n)+I(X_2^n;Y_2^n,V_2^n)\\
&\overset{(g_1)}{\le} \min\left\{\begin{matrix} I(X_1^n;Y_1^n|T_1^n,T_2^n)+I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n)+H(V_1^n),\\ I(X_1^n;Y_1^n,Y_{r1}^n|T_1^n,T_2^n)+I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n)\end{matrix}\right\}\\
&\qquad+\min\left\{\begin{matrix} I(X_2^n,T_1^n;Y_2^n)-I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n)+H(V_2^n),\\ I(X_2^n,T_1^n;Y_2^n,Y_{r2}^n)-I(T_1^n;Y_1^n,Y_{r1}^n|T_2^n)\end{matrix}\right\}\\
&=\min\{I(X_1^n;Y_1^n|T_1^n,T_2^n)+H(V_1^n), I(X_1^n;Y_1^n,Y_{r1}^n|T_1^n,T_2^n)\}\\
&\qquad+\min\{I(X_2^n,T_1^n;Y_2^n)+H(V_2^n), I(X_2^n,T_1^n;Y_2^n,Y_{r2}^n)\}\\
&\overset{(g_2)}{\le} \min\{\sum_{i=1}^n[I(X_{1i};Y_{1i}|T_{1i},T_{2i})+H(V_{1i})], \sum_{i=1}^n I(X_{1i};Y_{1i},Y_{r1i}|T_{1i},T_{2i})\}\\
&\qquad+\min\{\sum_{i=1}^n[I(X_{2i},T_{1i};Y_{2i})+H(V_{2i})], \sum_{i=1}^n I(X_{2i},T_{1i};Y_{2i},Y_{r2i})\}\\
&\le \min\{\sum_{i=1}^n[I(X_{1i};Y_{1i}|T_{1i},T_{2i})+C_1], \sum_{i=1}^n I(X_{1i};Y_{1i},Y_{r1i}|T_{1i},T_{2i})\}\\
&\qquad+\min\{\sum_{i=1}^n[I(X_{2i},T_{1i};Y_{2i})+C_2], \sum_{i=1}^n I(X_{2i},T_{1i};Y_{2i},Y_{r2i})\},
\end{aligned}
\tag{6.129}
$$

where we use (6.118), (6.121), (6.124), and (6.125) in (g_1); (g_2) is because conditioning does not increase entropy, Y_{ki} depends only on (X_{ki}, T_{li}), Y_{rki} is a function of (X_{ki}, Y_{ki}), for $k, l \in \{1,2\}$, $k \neq l$.
The bound on $R_1 + R_2$ *in* (6.37) *follows from* (6.35) *by symmetry.*
Proof of (6.36)*:* By Fano's inequality,

$$
\begin{aligned}
&n(R_1 + R_2 - 2\epsilon_n)\\
&\quad\le I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n)\\
&\quad\overset{(h_1)}{\le} \min\begin{Bmatrix} I(X_1^n, T_2^n; Y_1^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) + H(V_1^n),\\ I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) \end{Bmatrix}\\
&\qquad + \min\begin{Bmatrix} I(X_2^n, T_1^n; Y_2^n) - I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) + H(V_2^n),\\ I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) \end{Bmatrix}\\
&\quad= \min\begin{Bmatrix} I(X_1^n, T_2^n; Y_1^n) - I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) + H(V_1^n),\\ I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n) - I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) \end{Bmatrix}\\
&\qquad + \min\begin{Bmatrix} I(X_2^n, T_1^n; Y_2^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) + H(V_2^n),\\ I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) \end{Bmatrix}\\
&\quad\overset{(h_2)}{\le} \min\begin{Bmatrix} I(X_1^n, T_2^n; Y_1^n) - I(T_1^n; Y_1^n, Y_{r1}^n) + H(V_1^n),\\ I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n) - I(T_1^n; Y_1^n, Y_{r1}^n) \end{Bmatrix}\\
&\qquad + \min\begin{Bmatrix} I(X_2^n, T_1^n; Y_2^n) - I(T_2^n; Y_2^n, Y_{r2}^n) + H(V_2^n),\\ I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_2^n; Y_2^n, Y_{r2}^n) \end{Bmatrix}\\
&\quad\le \min\begin{Bmatrix} I(X_1^n, T_2^n; Y_1^n) - I(T_1^n; Y_1^n) + H(V_1^n),\\ I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n) - I(T_1^n; Y_1^n, Y_{r1}^n) \end{Bmatrix}\\
&\qquad + \min\begin{Bmatrix} I(X_2^n, T_1^n; Y_2^n) - I(T_2^n; Y_2^n) + H(V_2^n),\\ I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_2^n; Y_2^n, Y_{r2}^n) \end{Bmatrix}\\
&\quad\overset{(h_3)}{=} \min\begin{Bmatrix} I(X_1^n, T_1^n, T_2^n; Y_1^n) - I(T_1^n; Y_1^n) + H(V_1^n),\\ I(X_1^n, T_1^n, T_2^n; Y_1^n, Y_{r1}^n) - I(T_1^n; Y_1^n, Y_{r1}^n) \end{Bmatrix}\\
&\qquad + \min\begin{Bmatrix} I(X_2^n, T_1^n, T_2^n; Y_2^n) - I(T_2^n; Y_2^n) + H(V_2^n),\\ I(X_2^n, T_1^n, T_2^n; Y_2^n, Y_{r2}^n) - I(T_2^n; Y_2^n, Y_{r2}^n) \end{Bmatrix}\\
&\quad= \min\{I(X_1^n, T_2^n; Y_1^n|T_1^n) + H(V_1^n), I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n|T_1^n)\}\\
&\qquad + \min\{I(X_2^n, T_1^n; Y_2^n|T_2^n) + H(V_2^n), I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n|T_2^n)\}\\
&\quad\overset{(h_4)}{\le} \min\{\sum_{i=1}^{n}[I(X_{1i}, T_{2i}; Y_{1i}|T_{1i}) + C_1], \sum_{i=1}^{n} I(X_{1i}, T_{2i}; Y_{1i}, Y_{r1i}|T_{1i})\}
\end{aligned}
$$

$$+ \min\{\sum_{i=1}^{n}[I(X_{2i}, T_{1i}; Y_{2i}|T_{2i}) + C_2], \sum_{i=1}^{n} I(X_{2i}, T_{1i}; Y_{2i}, Y_{r2i}|T_{2i})\}, \quad (6.130)$$

where: we use (6.122), (6.123), (6.124), and (6.125) in (h_1); (h_2) is because T_1^n and T_2^n are independent; (h_3) is because T_1^n is a function of X_1^n, T_2^n is a function of X_2^n; (h_4) holds due to the same reasons that make (g_2) hold.

Proof of (6.38)*:*

$$\begin{aligned}
&n(2R_1 + R_2 - 2\epsilon_n)\\
&\leq I(X_1^n; Y_1^n, V_1^n) + I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n)\\
&\overset{(t_1)}{\leq} \min\begin{Bmatrix} I(X_1^n; Y_1^n|T_1^n, T_2^n) + I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) + H(V_1^n),\\ I(X_1^n; Y_1^n, Y_{r1}^n|T_1^n, T_2^n) + I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) \end{Bmatrix}\\
&\quad + \min\begin{Bmatrix} I(X_1^n, T_2^n; Y_1^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) + H(V_1^n),\\ I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) \end{Bmatrix}\\
&\quad + \min\begin{Bmatrix} I(X_2^n, T_1^n; Y_2^n) - I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) + H(V_2^n),\\ I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_1^n; Y_1^n, Y_{r1}^n|T_2^n) \end{Bmatrix}\\
&= \min\{I(X_1^n; Y_1^n|T_1^n, T_2^n) + H(V_1^n), I(X_1^n; Y_1^n, Y_{r1}^n|T_1^n, T_2^n)\}\\
&\quad + \min\{I(X_1^n, T_2^n; Y_1^n) + H(V_1^n), I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n)\}\\
&\quad + \min\begin{Bmatrix} I(X_2^n, T_1^n; Y_2^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) + H(V_2^n),\\ I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_2^n; Y_2^n, Y_{r2}^n|T_1^n) \end{Bmatrix}\\
&\overset{(t_2)}{\leq} \min\{I(X_1^n; Y_1^n|T_1^n, T_2^n) + H(V_1^n), I(X_1^n; Y_1^n, Y_{r1}^n|T_1^n, T_2^n)\}\\
&\quad + \min\{I(X_1^n, T_2^n; Y_1^n) + H(V_1^n), I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n)\}\\
&\quad + \min\begin{Bmatrix} I(X_2^n, T_2^n, T_1^n; Y_2^n) - I(T_2^n; Y_2^n) + H(V_2^n),\\ I(X_2^n, T_2^n, T_1^n; Y_2^n, Y_{r2}^n) - I(T_2^n; Y_2^n, Y_{r2}^n) \end{Bmatrix}\\
&= \min\{I(X_1^n; Y_1^n|T_1^n, T_2^n) + H(V_1^n), I(X_1^n; Y_1^n, Y_{r1}^n|T_1^n, T_2^n)\}\\
&\quad + \min\{I(X_1^n, T_2^n; Y_1^n) + H(V_1^n), I(X_1^n, T_2^n; Y_1^n, Y_{r1}^n)\}\\
&\quad + \min\{I(X_2^n, T_1^n; Y_2^n|T_2^n) + H(V_2^n), I(X_2^n, T_1^n; Y_2^n, Y_{r2}^n|T_2^n)\}\\
&\overset{(t_3)}{\leq} \min\{\sum_{i=1}^{n}[I(X_{1i}; Y_{1i}|T_{1i}, T_{2i}) + C_1], \sum_{i=1}^{n} I(X_{1i}; Y_{1i}, Y_{r1i}|T_{1i}, T_{2i})\}\\
&\quad + \min\{\sum_{i=1}^{n}[I(X_{1i}, T_{2i}; Y_{1i}) + C_1], \sum_{i=1}^{n} I(X_{1i}, T_{2i}; Y_{1i}, Y_{r1i})\}\\
&\quad + \min\{\sum_{i=1}^{n}[I(X_{2i}, T_{1i}; Y_{2i}|T_{2i}) + C_2], \sum_{i=1}^{n} I(X_{2i}, T_{1i}; Y_{2i}, Y_{r2i}|T_{2i})\}, \quad (6.131)
\end{aligned}$$

where: we use (6.118), (6.121), (6.122), (6.123), (6.124), and (6.125) in (t_1); (t_2) holds since X_2^n is a function of T_2^n, T_1^n and T_2^n are independent, and due to chain rule and the nonnegativity of entropy; (t_3) holds due to the same reasons that make (g_2) hold.

The bound on $R_1 + 2R_2$ *in* (6.39) *follows from* (6.38) *by symmetry.*

The bound on the cardinality of Q follows from the application of the support Lemma [CK81, Lemma 3.4, pp. 310].

6.C Proof of Proposition 6.3.1

Achievability: Note that in the general achievable rate region for the DMC-IRC, if we set the whole message at each transmitter to be the common message, then naturally the resulting achievable region is an inner bound for the SC-MARC. Note that in this case we retain the constraint on R_2 at Rx_1 (corresponding to (6.25)) and the constraint on R_1 at Rx_2 (corresponding to (6.29)) because each receiver is required to decode both messages correctly. In particular, by setting $X_{1c} = X_1, X_{2c} = X_2$, $R_{1c} = R_1$, $R_{1p} = R_2$, $\hat{Y}_{r1} = Y_{r1}$ in (6.3)–(6.11) we have

$$a_1 = a_1' = 0 \tag{6.132}$$
$$b_1 = b_1' = I(X_2; Y_1|X_1, Q) \tag{6.133}$$
$$c_1 = I(X_1; Y_1|X_2, Q), \quad c_1' = I(X_1; Y_1, Y_{r1}|X_2, Q) \tag{6.134}$$
$$d_1 = I(X_1, X_2; Y_1|Q), \quad d_1' = I(X_1, X_2; Y_1, Y_{r1}|Q) \tag{6.135}$$
$$\xi_1 = 0, \tag{6.136}$$

and symmetrically for $a_2, a_2, b_2, b_2', c_2, c_2', d_2, d_2', \xi_2$. Substituting the above values into (6.24)–(6.31) we obtain the desired rate regions.

Converse: First we note the following relationships: (a) X_1^n is independent of X_2^n; (b) the channel is memoryless; (c) V_k^n is a function of Y_{rk}^n, $k \in \{1,2\}$; (d) Y_{rk} is a function of X_k and Y_k, $k \in \{1,2\}$; (e) conditioning does not increase entropy. Using these observations in the equalities and inequalities below, we have:

By Fano's inequality

$$\begin{aligned} n(R_1 - \epsilon_n) &\leq I(X_1^n; Y_1^n, V_1^n) \\ &\leq I(X_1^n; Y_1^n) + H(V_1^n) \\ &\overset{(a)}{\leq} I(X_1^n; Y_1^n|X_2^n) + H(V_1^n) \\ &\overset{(b,e)}{\leq} \sum_{i=1}^{n} [I(X_{1i}; Y_{1i}|X_{2i}) + C_1]. \end{aligned} \tag{6.137}$$

Next, we have

$$\begin{aligned} n(R_1 - \epsilon_n) &\leq I(X_1^n; Y_1^n, V_1^n) \\ &\overset{(c)}{\leq} I(X_1^n; Y_1^n, Y_{r1}^n) \end{aligned}$$

$$
\begin{aligned}
&\overset{(a)}{\leq} I(X_1^n; Y_1^n, Y_{r1}^n | X_2^n) \\
&\overset{(b,e)}{\leq} \sum_{i=1}^{n} I(X_{1i}; Y_{1i}, Y_{r1i} | X_{2i}). \qquad (6.138)
\end{aligned}
$$

Combining (6.137) and (6.138) we have (6.46) proven. Again, by Fano's inequality

$$
\begin{aligned}
n(R_1 - \epsilon_n) &\leq I(X_1^n; Y_2^n, V_2^n) \\
&\overset{(c)}{\leq} I(X_1^n; Y_2^n, Y_{r2}^n) \\
&\overset{(a)}{\leq} I(X_1^n; Y_2^n, Y_{r2}^n | X_2^n) \\
&\overset{(d)}{=} I(X_1^n; Y_2^n | X_2^n) \\
&\overset{(b,e)}{\leq} \sum_{i=1}^{n} I(X_{1i}; Y_{2i} | X_{2i}), \qquad (6.139)
\end{aligned}
$$

which proves (6.47). By symmetry (6.48) and (6.49) follow.

For the sum-rate, let us also start with Fano's inequality

$$
\begin{aligned}
n(R_1 + R_2 - 2\epsilon_n) &\leq I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_1^n, V_1^n) \\
&\overset{(a)}{\leq} I(X_1^n; Y_1^n, V_1^n | X_2^n) + I(X_2^n; Y_1^n, V_1^n) \\
&= I(X_1^n, X_2^n; Y_1^n, V_1^n) \\
&\overset{(c)}{\leq} \min\{I(X_1^n, X_2^n; Y_1^n) + H(V_1^n), I(X_1^n, X_2^n; Y_1^n, Y_{r1}^n)\} \\
&\overset{(b,e)}{\leq} \min\left\{\sum_{i=1}^{n}[I(X_{1i}, X_{2i}; Y_{1i}) + C_1], \sum_{i=1}^{n} I(X_{1i}, X_{2i}; Y_{1i}, Y_{r1i})\right\},
\end{aligned}
$$

which proves (6.50). By symmetry (6.51) follows. The single letterization procedure is similar to the paragraph after (6.127).

The bound on the cardinality of Q follows from the application of the support lemma [CK81, Lemma 3.4, pp. 310]. □

6.D Proof of Theorem 6.4.1

Throughout the proofs we use V_i^n to denote the output of the digital relay link at receiver i, $i \in \{1,2\}$. In the inequalities below we utilize the following facts: (a) X_1^n is independent of X_2^n, (b) V_k^n is a function of Y_{rk}^n, $k \in \{1,2\}$, (c) i.i.d Gaussian distribution maximizes the differential entropy and the conditional differential entropy subject to covariance constraints.

Bounds on individual rates in (6.56) *and* (6.57)*:*
By Fano's inequality we have

$$\begin{aligned} n(R_1 - \epsilon_n) &\leq I(X_1^n; V_1^n, Y_1^n) \\ &\leq I(X_1^n; V_1^n, Y_1^n, X_2^n) \\ &\overset{(a)}{=} I(X_1^n; V_1^n, Y_1^n | X_2^n) \\ &= I(X_1^n; Y_1^n | X_2^n) + I(X_1^n; V_1^n | Y_1^n, X_2^n) \\ &\leq I(X_1^n; Y_1^n | X_2^n) + H(V_1^n). \end{aligned} \tag{6.140}$$

We have

$$\begin{aligned} I(X_1^n; Y_1^n | X_2^n) &= h(Y_1^n | X_2^n) - h(Y_1^n | X_1^n, X_2^n) \\ &= h(h_{11} X_1^n + Z_1^n) - h(Z_1^n) \\ &\overset{(c)}{\leq} \frac{n}{2} \log(1 + \mathsf{SNR}_1), \end{aligned} \tag{6.141}$$

and

$$\begin{aligned} H(V_1^n) &\leq nH(V_1) \\ &\leq nC_1. \end{aligned} \tag{6.142}$$

Putting (6.141) and (6.142) back to (6.140) we have

$$R_1 - \epsilon_n \leq \frac{1}{2} \log(1 + \mathsf{SNR}_1) + C_1, \tag{6.143}$$

where $\epsilon_n \to 0$ as $n \to \infty$. We also have

$$\begin{aligned} n(R_1 - \epsilon_n) &\leq I(X_1^n; V_1^n, Y_1^n) \\ &\overset{(b)}{\leq} I(X_1^n; Y_{r1}^n, Y_1^n) \\ &\overset{(a)}{=} I(X_1^n; Y_{r1}^n, Y_1^n | X_2^n) \\ &= I(X_1^n; Y_1^n | X_2^n) + I(X_1^n; Y_{r1}^n | Y_1^n, X_2^n) \\ &= h(h_{11} X_1^n + Z_1^n) - h(Z_1^n) + h(h_{1r} X_1^n + Z_{r1}^n | h_{11} X_1^n + Z_1^n) - h(Z_{r1}^n) \\ &\overset{(c)}{\leq} \frac{n}{2} \log(1 + \mathsf{SNR}_1 + \mathsf{SNR}_{r1}). \end{aligned} \tag{6.144}$$

Next, to prove the outer bounds on the sum-rate and weighted sum-rate, we will first transform the genie aided channel into a MIMO IC plus some bounded terms. We will then leverage the outer bounds for the MIMO IC which are developed by Karmakar and Varanasi in [KV13, Lemma 1], which can be shown to be the same as the bounds introduced by Telatar and Tse in [TT07] for the channel under consideration.

Bounds on $R_1 + R_2$ in (6.58)–(6.60)*.* By Fano's inequality we have

$$\begin{aligned} n(R_1 + R_2 - 2\epsilon_n) &\le I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &= I(X_1^n; Y_1^n) + I(X_1^n; V_1^n | Y_1^n) + I(X_2^n; Y_2^n) + I(X_2^n; V_2^n | Y_2^n) \\ &\le I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + H(V_1^n) + H(V_2^n) \\ &\le I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + nC_1 + nC_2. \end{aligned}$$

At this point we can use the bounds on $I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n)$ established in [TT07] or [KV13, Lemma 1] to obtain the bounds in (6.58)–(6.60). A similar procedure with more details will be shown shortly in the proof of (6.62).
Bounds on $2R_1 + R_2$ in (6.61)*.* By Fano's inequality we have

$$\begin{aligned} n(2R_1 + R_2 - 3\epsilon_n) &\le 2I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\le 2I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + 2H(V_1^n) + H(V_2^n) \\ &\le 2I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + 2nC_1 + nC_2. \end{aligned}$$

Applying the bounds on $2I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n)$ in [TT07, KV13] we complete the proof.
Bounds on $R_1 + R_2$ in (6.62)–(6.64)*.* By Fano's inequality we have

$$\begin{aligned} n(R_1 + R_2 - 2\epsilon_n) &\le I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\overset{(b)}{\le} I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_2^n; Y_2^n, Y_{r2}^n). \end{aligned}$$

By defining $\bar{Y}_i := (Y_i, Y_{ri})$, $i \in \{1, 2\}$, we can apply the bounds in [TT07, KV13] to obtain (6.62)–(6.64). In particular, we are having a MIMO interference channel where each transmitter has 1 antenna and each receiver has 2 antennas. With regard to the notation in [KV13] we have $M_1 = M_2 = 1$, $N_1 = N_2 = 2$, and the channel matrices given by

$$\mathsf{H}_{11} = [h_{11} \quad h_{1r}]^T \tag{6.145}$$

$$\mathsf{H}_{12} = [h_{1c} \quad h_{12}]^T \tag{6.146}$$

$$\mathsf{H}_{21} = [h_{21} \quad h_{2c}]^T \tag{6.147}$$

$$\mathsf{H}_{22} = [h_{2r} \quad h_{22}]^T. \tag{6.148}$$

We also have $\rho_{ii} = \rho_{ij} = 1$ for $i, j \in \{1, 2\}$, $i \ne j$. Putting the above parameters into [KV13, Eq. (13)] we have an outer bound on the sum-rate:

$$R_1 + R_2 \le \frac{1}{2} \log\det(\mathsf{I}_2 + \mathsf{H}_{12}\mathsf{H}_{12}^T + \mathsf{H}_{22}\mathsf{H}_{22}^T) + \frac{1}{2} \log\det(\mathsf{I}_2 + \mathsf{H}_{11}\mathsf{K}_1\mathsf{H}_{11}^T),$$

where $\mathsf{K}_1 = (1 + \mathsf{H}_{12}^T\mathsf{H}_{12})^{-1}$. Simple calculations give

$$\begin{aligned} \frac{1}{2} \log\det(\mathsf{I}_2 + \mathsf{H}_{12}\mathsf{H}_{12}^T + \mathsf{H}_{22}\mathsf{H}_{22}^T) &= \frac{1}{2} \log\begin{pmatrix} 1 + \mathsf{INR}_2 + \mathsf{SNR}_{r2} + \mathsf{INR}_{r2} \\ +\mathsf{SNR}_2(1 + \mathsf{INR}_{r2}\eta_2^2) \end{pmatrix} \\ \frac{1}{2} \log\det(\mathsf{I}_2 + \mathsf{H}_{11}\mathsf{K}_1\mathsf{H}_{11}^T) &= \frac{1}{2} \log\Big(1 + \frac{\mathsf{SNR}_1 + \mathsf{SNR}_{r1}}{1 + \mathsf{INR}_{r2} + \mathsf{INR}_2}\Big), \end{aligned}$$

which lead to (6.62). (6.63)–(6.64) follow similarly from [KV13, Eqs. (14)-(15)].
Bounds on $2R_1 + R_2$ *in* (6.65)*.* By Fano's inequality we have

$$\begin{aligned} n(2R_1 + R_2 - 3\epsilon_n) &\leq 2I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\overset{(b)}{\leq} 2I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_2^n; Y_2^n, Y_{r2}^n). \end{aligned}$$

Then we define $\bar{Y}_i := (Y_i, Y_{ri})$, $i \in \{1, 2\}$, and apply the bounds on $2I(X_1^n; \bar{Y}_1^n) + I(X_2^n; \bar{Y}_2^n)$ in [TT07, KV13].
Bounds on $R_1 + R_2$ *in* (6.66)*–*(6.68)*.* By Fano's inequality we have

$$\begin{aligned} n(R_1 + R_2 - 2\epsilon_n) &\leq I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\overset{(b)}{\leq} I(X_1^n; Y_1^n) + H(V_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n) \\ &\leq I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n) + nC_1. \end{aligned}$$

At this point we can use the bounds on $I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n)$ in [TT07,KV13] to end up with the bounds in (6.66)–(6.68).
Bound on $2R_1 + R_2$ *in* (6.69) By Fano's inequality

$$\begin{aligned} n(2R_1 + R_2 - 3\epsilon_n) &\leq 2I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\overset{(b)}{\leq} 2I(X_1^n; Y_1^n) + 2H(V_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n) \\ &\leq 2I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n) + 2nC_1. \end{aligned}$$

Then we can use the bound on $2I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n)$ in [TT07, KV13] to obtain (6.69).
Bounds on $R_1 + R_2$ *in* (6.70)*–*(6.72)*.* These bounds follow readily from the bounds in (6.66)–(6.68) by symmetry.
Bound on $2R_1 + R_2$ *in* (6.73) By Fano's inequality

$$\begin{aligned} n(2R_1 + R_2 - 3\epsilon_n) &\leq 2I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\overset{(b)}{\leq} 2I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_2^n; Y_2^n) + H(V_2^n) \\ &\leq 2I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_2^n; Y_2^n) + nC_2. \end{aligned}$$

Then we can use the bound on $2I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_2^n; Y_2^n)$ in [TT07, KV13].
Bounds on $2R_1 + R_2$ *in* (6.74): Let $S_1^n = h_{12}X_1^n + Z_2^n$, $S_2^n = h_{21}X_2^n + Z_1^n$. We see that $Y_1^n = h_{11}X_1^n + S_2^n$, $Y_2^n = h_{22}X_2^n + S_1^n$. Using Fano's inequality, conditioning does not increase entropy, chain rule we have that if a rate pair (R_1, R_2) is achievable, then

$$\begin{aligned} &n(2R_1 + R_2 - 3\epsilon_n) \\ &\quad \leq 2I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\quad \overset{(b)}{\leq} I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_1^n; Y_1^n) + I(X_1^n; V_1^n | Y_1^n) + I(X_2^n; Y_2^n) + I(X_2; V_2^n | Y_2^n) \end{aligned}$$

$$
\begin{aligned}
&\le I(X_1^n; Y_1^n, Y_{r1}^n, S_1^n, X_2^n) + I(X_1^n; Y_1^n) + H(V_1^n) + I(X_2^n; Y_2^n, S_2^n) + H(V_2^n)\\
&\overset{(a)}{=} I(X_1^n; Y_1^n, Y_{r1}^n, S_1^n | X_2^n) + I(X_1^n; Y_1^n) + H(V_1^n) + I(X_2^n; Y_2^n, S_2^n) + H(V_2^n)\\
&\le I(X_1^n; S_1^n | X_2^n) + I(X_1^n; Y_1^n, Y_{r1}^n | S_1^n, X_2^n) + I(X_1^n; Y_1^n) + h(Y_2^n, S_2^n)\\
&\quad - h(Y_2^n, S_2^n | X_2^n) + nC_1 + nC_2\\
&= h(S_1^n) - h(Z_2^n) + h(Y_1^n, Y_{r1}^n | S_1^n, X_2^n) - h(Z_1^n, Z_{1r}^n) + h(Y_1^n) - h(S_2^n)\\
&\quad + h(Y_2^n, S_2^n) - h(S_1^n, Z_1^n) + nC_1 + nC_2\\
&= h(S_1^n) - h(Z_2^n) + h(Y_1^n, Y_{r1}^n | S_1^n, X_2^n) - h(Z_1^n, Z_{1r}^n) + h(Y_1^n) + h(Y_2^n | S_2^n)\\
&\quad - h(S_1^n) - h(Z_1^n) + nC_1 + nC_2\\
&= h(Y_1^n, Y_{r1}^n | S_1^n, X_2^n) - h(Z_1^n, Z_{1r}^n) + h(Y_1^n) - h(Z_1^n) + h(Y_2^n | S_2^n) - h(Z_2^n)\\
&\quad + nC_1 + nC_2\\
&= h(h_{11} X_1^n + Z_1^n, h_{1r} X_1^n + Z_{r1}^n | h_{12} X_1^n + Z_2^n) - h(Z_1^n, Z_{1r}^n)\\
&\quad + h(h_{11} X_1^n + h_{21} X_2^n + Z_1^n) - h(Z_1^n) + h(h_{22} X_2^n + h_{12} X_1^n + Z_2^n | h_{21} X_2^n + Z_1^n)\\
&\quad - h(Z_2^n) + nC_1 + nC_2\\
&\overset{(c)}{\le} n(\text{RHS of (6.74)}).
\end{aligned}
$$

Bounds on $2R_1 + R_2$ *in* (6.75): We define S_1^n, S_2^n as above. Again, using Fano's inequality, conditioning does not increase entropy, and chain rule, we have: if a rate pair (R_1, R_2) is achievable then

$$
\begin{aligned}
&n(2R_1 + R_2 - \epsilon_n)\\
&\quad \le 2I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n)\\
&\quad \overset{(b)}{\le} I(X_1^n; Y_1^n, Y_{r1}^n) + I(X_1^n; Y_1^n) + I(X_1^n; V_1^n | Y_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n)\\
&\quad \le I(X_1^n; Y_1^n, Y_{r1}^n, S_1^n, X_2^n) + I(X_1^n; Y_1^n) + H(V_1^n) + I(X_2^n; Y_2^n, Y_{r2}^n, S_2^n)\\
&\quad \le h(S_1^n) - h(Z_2^n) + h(Y_1^n, Y_{r1}^n | S_1^n, X_2^n) - h(Z_1^n, Z_{1r}^n) + h(Y_1^n) - h(S_2^n) + nC_1\\
&\qquad\quad + I(X_2^n; Y_2^n, Y_{r2}^n, S_2^n).
\end{aligned} \tag{6.149}
$$

The last term in (6.149) can be bounded as follows:

$$
\begin{aligned}
I(X_2^n; Y_2^n, Y_{r2}^n, S_2^n) &= I(X_2^n; S_2^n) + I(X_2^n; Y_2^n, Y_{r2}^n | S_2^n)\\
&= h(S_2^n) - h(Z_1^n) + h(Y_2^n, Y_{r2}^n | S_2^n) - h(Y_2^n, Y_{r2}^n | X_2^n, S_2^n)\\
&\le h(S_2^n) - h(Z_1^n) + h(Y_2^n, Y_{r2}^n | S_2^n) - h(Y_2^n | X_2^n, S_2^n)\\
&\quad - h(Y_{r2}^n | X_2^n, S_2^n, Y_2^n, X_1^n)\\
&= h(S_2^n) - h(Z_1^n) + h(Y_2^n, Y_{r2}^n | S_2^n) - h(S_1^n) - h(Z_{r2}^n).
\end{aligned} \tag{6.150}
$$

Putting (6.150) back to (6.149) we have

$$
\begin{aligned}
n(2R_1 + R_2 - \epsilon_n) \le\ & h(Y_1^n, Y_{r1}^n | S_1^n, X_2^n) - h(Z_1^n, Z_{1r}^n) + h(Y_1^n) - h(Z_1^n)\\
& + h(Y_2^n, Y_{r2}^n | S_2^n) - h(Z_2^n) - h(Z_{r2}^n)
\end{aligned}
$$

$$\stackrel{(c)}{\leq} n(\text{RHS of (6.75)}). \tag{6.151}$$

□

6.E Proof of Theorem 6.4.2

Before proving Theorem 6.4.2 we need two lemmas, whose proofs follow in the sequel.

Lemma 6.4. *Given the power allocation in* (6.80)*, we have:*

$$\mathsf{SNR}_{1p} \geq \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}, \qquad \mathsf{SNR}_{r1p} \geq \frac{\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}, \tag{6.152}$$

$$\mathsf{INR}_{1p} \leq 1, \qquad \mathsf{INR}_{r1p} \leq |h_{2c}|^2/|h_{21}|^2. \tag{6.153}$$

Proof. Since we choose $P_{1p} = \min\left\{1, \frac{1}{|h_{12}|^2}\right\}$ and $P_{2p} = \min\left\{1, \frac{1}{|h_{21}|^2}\right\}$,

$$\begin{aligned}\mathsf{SNR}_{1p} &= |h_{11}|^2 P_{1p} = |h_{11}|^2 \min\left\{1, \frac{1}{|h_{12}|^2}\right\} \\ &\geq \frac{|h_{11}|^2}{1+|h_{12}|^2} \\ &= \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}.\end{aligned}$$

Similarly we can show that $\mathsf{SNR}_{r1p} \geq \frac{\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}$. Next,

$$\begin{aligned}\mathsf{INR}_{1p} &= |h_{21}|^2 P_{2p} = |h_{21}|^2 \min\left\{1, \frac{1}{|h_{21}|^2}\right\} \\ &\leq 1,\end{aligned}$$

and

$$\begin{aligned}\mathsf{INR}_{r1p} &= |h_{2c}|^2 P_{2p} = |h_{2c}|^2 \min\left\{1, \frac{1}{|h_{21}|^2}\right\} \\ &\leq \frac{|h_{2c}|^2}{|h_{21}|^2}.\end{aligned}$$

□

Lemma 6.4 enables us to prove Lemma 6.5 below.

Lemma 6.5. *Let us define:*

$$f_1(\Delta_1) = \frac{1}{2}\log\left(2 + 2\Delta_1 + |h_{2c}|^2/|h_{21}|^2\right) \tag{6.154}$$

$$f_2(\Delta_2) = \frac{1}{2}\log\left(2 + 2\Delta_2 + |h_{1c}|^2/|h_{12}|^2\right). \tag{6.155}$$

Given the power allocation in (6.80) *we have*

$$\xi_1 \leq \frac{1}{2}\log\left[1 + \frac{1}{\Delta_1}\left(1 + \frac{|h_{2c}|^2}{|h_{21}|^2}\right)\right] \tag{6.156}$$

$$a_1 \geq \mathcal{C}\left(\frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) - \frac{1}{2} \tag{6.157}$$

$$a_1' \geq \mathcal{C}\left(\frac{\mathsf{SNR}_1 + \mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) - f_1(\Delta_1) \tag{6.158}$$

$$b_1 \geq \mathcal{C}\left(\mathsf{INR}_1 + \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) - \frac{1}{2} \tag{6.159}$$

$$b_1' \geq \mathcal{C}\left(\mathsf{INR}_1 + \mathsf{INR}_{r1} + \frac{\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1}) + \mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) - f_1(\Delta_1) \tag{6.160}$$

$$c_1 \geq \mathcal{C}\left(\mathsf{SNR}_1\right) - \frac{1}{2} \tag{6.161}$$

$$c_1' \geq \mathcal{C}\left(\mathsf{SNR}_1 + \mathsf{SNR}_{r1}\right) - f_1(\Delta_1) \tag{6.162}$$

$$d_1 \geq \mathcal{C}\left(\mathsf{SNR}_1 + \mathsf{INR}_1\right) - \frac{1}{2} \tag{6.163}$$

$$d_1' \geq \mathcal{C}\left(\mathsf{INR}_1 + \mathsf{INR}_{r1} + \mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1}) + \mathsf{SNR}_{r1}\right) - f_1(\Delta_1). \tag{6.164}$$

Similar inequalities hold for $a_2, a_2', b_2, b_2', c_2, c_2', d_2, d_2'$ *by symmetry.*

Proof. The proof is similar to the proof of [ZY13, Lemma 1]. We will use the results of Lemma 6.4, indexed as follows: (a) $\mathsf{SNR}_{1p} \geq \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}$, (b) $\mathsf{SNR}_{r1p} \geq \frac{\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}$, (c) $\mathsf{INR}_{1p} \leq 1$, and (d) $\mathsf{INR}_{r1p} \leq \frac{|h_{2c}|^2}{|h_{21}|^2}$.

First, for the term related to the quantization at each relay:

$$\begin{aligned}
\xi_1 &= I(\hat{Y}_{r1}; Y_{r1}|X_1, X_{2c}, Y_1) \\
&= \frac{1}{2}\log\left[1 + \frac{1}{\Delta_1}\left(1 + \frac{\mathsf{INR}_{r1p}}{1+\mathsf{INR}_{1p}}\right)\right] \\
&\leq \frac{1}{2}\log\left[1 + \frac{1}{\Delta_1}\left(1 + \frac{\mathsf{INR}_{r1p}}{\mathsf{INR}_{1p}}\right)\right] \\
&= \frac{1}{2}\log\left[1 + \frac{1}{\Delta_1}\left(1 + \frac{|h_{2c}|^2}{|h_{21}|^2}\right)\right].
\end{aligned}$$

For other quantities in Lemma 6.5:

$$
\begin{aligned}
a_1 &= I(X_1;Y_1|X_{1c},X_{2c})\\
&= \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_{1p}}{1+\mathsf{INR}_{1p}}\right)\\
&\overset{(c)}{\geq} \frac{1}{2}\log\left(1+\mathsf{SNR}_{1p}\right)-\frac{1}{2}\\
&\overset{(a)}{\geq} \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_{1}}{1+\mathsf{INR}_{2}}\right)-\frac{1}{2}.\\
a_1' &= I(X_1;Y_1,\hat{Y}_{1r}|X_{1c},X_{2c})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_{1p})+\mathsf{SNR}_{r1p}+\mathsf{INR}_{r1p}(1+\eta_1^2\mathsf{SNR}_{1p})}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&= \frac{1}{2}\log\left((1+\Delta_1)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_{1p})+\mathsf{SNR}_{r1p}+\mathsf{INR}_{r1p}(1+\eta_1^2\mathsf{SNR}_{1p})\right)\\
&\qquad -\frac{1}{2}\log\left((1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}\right)\\
&\overset{(c,d)}{\geq} \frac{1}{2}\log\left(1+\mathsf{SNR}_{1p}+\mathsf{SNR}_{r1p})\right)-f_1(\Delta_1)\\
&\overset{(a,b)}{\geq} \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_{1}+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_{2}}\right)-f_1(\Delta_1).
\end{aligned}
$$

In the same way we can show that

$$
\begin{aligned}
b_1 &= I(X_1,X_{2c};Y_1|X_{1c})\\
&= \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_{1p}+\mathsf{INR}_{1}}{1+\mathsf{INR}_{1p}}\right)\\
&\geq \frac{1}{2}\log\left(1+\mathsf{INR}_1+\frac{\mathsf{SNR}_{1}}{1+\mathsf{INR}_{2}}\right)-\frac{1}{2}.\\
b_1' &= I(X_1,X_{2c};Y_1,\hat{Y}_{1r}|X_{1c})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_{1})+\mathsf{SNR}_{r1p}+\mathsf{INR}_{r1}(1+\eta_1^2\mathsf{SNR}_{1p})}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\geq \frac{1}{2}\log\left(1+\mathsf{INR}_1+\mathsf{INR}_{r1}+\frac{\mathsf{SNR}_{1}(1+\eta_1^2\mathsf{INR}_{r1})+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_{2}}\right)-f_1(\Delta_1).\\
c_1 &= I(X_1;Y_1|X_{2c})\\
&= \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_{1}}{1+\mathsf{INR}_{1p}}\right)\\
&\geq \frac{1}{2}\log(1+\mathsf{SNR}_1)-\frac{1}{2}.\\
c_1' &= I(X_1;Y_1,\hat{Y}_{1r}|X_{2c})
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_1+\mathsf{INR}_{1p})+\mathsf{SNR}_{r1}+\mathsf{INR}_{r1p}(1+\eta_1^2\mathsf{SNR}_1)}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\geq \frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{SNR}_{r1}\right)-f_1(\Delta_1).\\
d_1 &= I(X_1,X_{2c};Y_1)\\
&= \frac{1}{2}\log\left(\frac{1+\mathsf{SNR}_1+\mathsf{INR}_1}{1+\mathsf{INR}_{1p}}\right)\\
&\geq \frac{1}{2}\log(1+\mathsf{SNR}_1+\mathsf{INR}_1)-\frac{1}{2}.\\
d_1' &= I(X_1,X_{2c};Y_1,\hat{Y}_{1r})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}(1+\eta_1^2\mathsf{SNR}_1)}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\geq \frac{1}{2}\left(1+\mathsf{INR}_1+\mathsf{INR}_{r1}+\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})+\mathsf{SNR}_{r1}\right)-f_1(\Delta_1).
\end{aligned}
$$

Hence Lemma 6.5 is proven. □

Now we are ready to prove Theorem 6.4.2. First note that in order to show a bounded gap between an inner and an outer bound we must show that *every* constraint on individual rate or (weighted) sum-rate in the inner bound is within a bounded distance to *some* constraint of the same kind in the outer bound. Let us now define $\delta C_i := (C_i-\xi_i)^+$, $g_i(\xi_i) := 1/2+\xi_i$, $i\in\{1,2\}$. Comparing each constraint of the inner bound (6.13)–(6.23) with each corresponding constraint of the outer bound (6.56)–(6.75), and utilizing the inequalities in Lemma 6.5 (marked with $(*)$ below), we have:
For individual rate constraints:

- $\min\{c_1+\delta C_1, c_1'\}$ in the RHS of (6.13) is within $\max\{g_1(\xi_1), f_1(\Delta_1)\}$ of the RHS of (6.56).

  Proof. We have:

$$
\begin{aligned}
c_1+(C_1-\xi_1)^+ &\overset{(*)}{\geq} \frac{1}{2}\log(1+\mathsf{SNR}_1)-\frac{1}{2}+C_1-\xi_1\\
&= \frac{1}{2}\log(1+\mathsf{SNR}_1)+C_1-g_1(\xi_1),
\end{aligned}
$$

  which is within $g_1(\xi_1)$ of the outer bound $\frac{1}{2}\log(1+\mathsf{SNR}_1)+C_1$ in RHS of (6.13). The remaining gap follows straightforwardly from the lower bound on c_1' in Lemma 6.5. □

- $a_1+\delta C_1+b_2+\delta C_2$ (resp. $a_1+\delta C_1+b_2'$) in the RHS of (6.14) is within $g_1(\xi_1)+g_2(\xi_2)$ (resp. $g_1(\xi_1)+f_2(\Delta_2)$) of the first term in the RHS of (6.56).

Proof. We have:

$$
\begin{aligned}
a_1 + (C_1 - \xi_1)^+ + b_2 + (C_2 - \xi_2)^+ &\\
&\geq a_1 + C_1 - \xi_1 + b_2 + C_2 - \xi_2 \\
&\overset{(*)}{\geq} \frac{1}{2}\log\left(1 + \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) - \frac{1}{2} + C_1 - \xi_1 \\
&\quad + \frac{1}{2}\log\left(1 + \mathsf{INR}_2 + \frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) - \frac{1}{2} + C_2 - \xi_2 \\
&\geq \frac{1}{2}\log(1+\mathsf{SNR}_1) + C_1 - (\xi_1 + \frac{1}{2}) - (\xi_2 + \frac{1}{2}) \\
&= \frac{1}{2}\log(1+\mathsf{SNR}_1) + C_1 - g_1(\xi_1) - g_2(\xi_2),
\end{aligned}
$$

which is within $g_1(\xi_1) + g_2(\xi_2)$ of the outer bound $\frac{1}{2}\log(1+\mathsf{SNR}_1) + C_1$. Further,

$$
\begin{aligned}
a_1 + &(C_1 - \xi_1)^+ + b_2' \\
&\overset{(*)}{\geq} \frac{1}{2}\log\left(1 + \frac{\mathsf{SNR}_1}{1+\mathsf{INR}_2}\right) - \frac{1}{2} + C_1 - \xi_1 \\
&\quad + \frac{1}{2}\log\left(1 + \mathsf{INR}_2 + \mathsf{INR}_{r2} + \frac{\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2}) + \mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right) - f_2(\Delta_2) \\
&\geq \frac{1}{2}\log(1+\mathsf{SNR}_1) + C_1 - g_1(\xi_1) - f_2(\Delta_2),
\end{aligned}
$$

which is within $g_1(\xi_1) + f_2(\Delta_2)$ of the outer bound $\frac{1}{2}\log(1+\mathsf{SNR}_1) + C_1$. □

- $a_1' + b_2 + \delta C_2$ (resp. $a_1' + b_2'$) in the RHS of (6.14) is within $f_1(\Delta_1) + g_2(\xi_2)$ (resp. $f_1(\Delta_1) + f_2(\Delta_2)$) of the second term in the RHS of (6.56).

Proof. We have:

$$
\begin{aligned}
a_1' + b_2 + (C_2 - \xi_2)^+ &\overset{(*)}{\geq} \frac{1}{2}\log\left(1 + \frac{\mathsf{SNR}_1 + \mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right) - f_1(\Delta_1) \\
&\quad + \frac{1}{2}\log\left(1 + \mathsf{INR}_2 + \frac{\mathsf{SNR}_2}{1+\mathsf{INR}_1}\right) - \frac{1}{2} + C_2 - \xi_2 \\
&\geq \frac{1}{2}\log(1+\mathsf{SNR}_1 + \mathsf{SNR}_{r1}) - f_1(\Delta_1) - (\frac{1}{2} + \xi_2) \\
&= \frac{1}{2}\log(1+\mathsf{SNR}_1 + \mathsf{SNR}_{r1}) - f_1(\Delta_1) - g_2(\xi_2),
\end{aligned}
$$

which is within $f_1(\Delta_1)+g_2(\xi_2)$ of the outer bound $\frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{SNR}_{r1}\right)$. Next,

$$\begin{aligned} a_1'+b_2' &\overset{(*)}{\geq} \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2}\right)-f_1(\Delta_1) \\ &\quad +\frac{1}{2}\log\left(1+\mathsf{INR}_2+\mathsf{INR}_{r2}+\frac{\mathsf{SNR}_2(1+\eta_2^2\mathsf{INR}_{r2})+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right) \\ &\quad -f_2(\Delta_2) \\ &\geq \frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{SNR}_{r1}\right)-f_1(\Delta_1)-f_2(\Delta_2), \end{aligned}$$

which is within $f_1(\Delta_1)+f_2(\Delta_2)$ of the outer bound $\frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{SNR}_{r1}\right)$. □

Similar gaps for R_2 follow by symmetry.
For sum-rate constraints: Following the same line of the above proofs we can show that of the constraints on R_1+R_2 in (6.17)–(6.19):

- $d_1+\delta C_1+a_2+\delta C_2$, $a_1+\delta C_1+d_2+\delta C_2$, $b_1+\delta C_1+b_2+\delta C_2$ are within $g_1(\xi_1)+g_2(\xi_2)$ of the RHS's of (6.58), (6.59), and (6.60), respectively.
- $a_1'+d_2'$, $d_1'+a_2'$, $b_1'+b_2'$ are within $f_1(\Delta_1)+f_2(\Delta_2)$ of the RHS's of (6.62), (6.63), and (6.64), respectively.
- $a_1+\delta C_1+d_2'$, $d_1+\delta C_1+a_2'$, $b_1+\delta C_1+b_2'$ are within $g_1(\xi_1)+f_2(\Delta_2)$ of the RHS's of (6.66), (6.67), and (6.68), respectively.
- $b_1'+b_2+\delta C_2$, $a_1'+d_2+\delta C_2$, $d_1'+a_2+\delta C_2$ are within $f_1(\Delta_1)+g_2(\xi_2)$ of the RHS's of (6.70), (6.71), and (6.72), respectively.

For weighted sum-rate constraints: Of the inner bounds on $2R_1+R_2$ in (6.21), it can be shown that the bounds $a_1+\delta C_1+d_1'+b_2+\delta C_2$ and $a_1+\delta C_1+d_1'+b_2'$ are redundant. For the remaining bounds:

- $a_1+\delta C_1+d_1+\delta C_1+b_2+\delta C_2$ is within $2g_1(\xi_1)+g_2(\xi_2)$ of the RHS of (6.61).
- $a_1'+d_1'+b_2'$ is within $2f_1(\Delta_1)+f_2(\Delta_2)$ of the RHS of (6.65).
- $a_1+\delta C_1+d_1+\delta C_1+b_2'$ is within $2g_1(\xi_1)+f_2(\Delta_2)$ of the RHS of (6.69).
- $a_1'+d_1'+b_2+\delta C_2$ is within $2f_1(\Delta_1)+g_2(\xi_2)$ of the RHS of (6.73).
- $a_1'+d_1+\delta C_1+b_2+\delta C_2$ is within $f_1(\Delta_1)+g_1(\xi_1)+g_2(\xi_2)$ of the RHS of (6.74).
- $a_1'+d_1+\delta C_1+b_2'$ is within $f_1(\Delta_1)+g_1(\xi_1)+f_2(\Delta_2)$ of the RHS of (6.75).

The gaps for $R_1 + 2R_2$ follow directly from the gaps for $2R_1 + R_2$ by symmetry.

To this end, we loosen the gaps obtained from the above comparisons by replacing $|h_{2c}|^2/|h_{21}|^2$ in the definition of Δ_1 and ξ_1 in (6.154) and (6.156) with its upper bound θ defined in (6.81) (similarly done for Δ_2 and ξ_2). The final step is optimizing Δ_1 and Δ_2 to minimize the new (loosened) gaps. At this point we can apply the result in [ZY13] and conclude that the optimal values are $\Delta_1 = \Delta_2 = \frac{1}{4}(\sqrt{\theta^2 + 16\theta + 16} - \theta)$. Direct calculation leads to the optimized gap in (6.82).

6.F Proof of Proposition 6.4.1

First, by replacing X_{kc} by X_k, $R_{kp} = 0$, $k \in \{1, 2\}$, $Q = 1$ in Theorem 6.2.1, i.e., the whole message at each transmitter is set to be the common message, and noticing that each decoder does not interested in decoding the common message of the unpairing transmitter uniquely, we obtain the following achievable rate region $\mathcal{R}^i_{S-IRC}$:

$$R_1 < \min\{c_1 + (C_1 - \xi_1)^+, c_1'\} \tag{6.165}$$

$$R_2 < \min\{c_2 + (C_2 - \xi_2)^+, c_2'\} \tag{6.166}$$

$$R_1 + R_2 < \min\{d_1 + (C_1 - \xi_1)^+, d_1'\} \tag{6.167}$$

$$R_1 + R_2 < \min\{d_2 + (C_2 - \xi_2)^+, d_2'\}. \tag{6.168}$$

From the equations of the received signals Y_1, Y_2, Y_{r1}, Y_{r2} in (6.52)–(6.55), and by choosing the distribution of the quantization codebooks $\hat{Y}_{rk} = Y_{rk} + Z_{qk}$ with $Z_{qk} \sim N(0, \Delta_k)$, $k \in \{1, 2\}$ we have:

$$c_1 = I(X_1; Y_1|X_2) = \frac{1}{2}\log(1 + \mathsf{SNR}_1) \tag{6.169a}$$

$$c_1' = I(X_1; Y_1, \hat{Y}_{r1}|X_2) = \frac{1}{2}\log\left(1 + \mathsf{SNR}_1 + \frac{\mathsf{SNR}_{r1}}{1 + \Delta_1}\right) \tag{6.169b}$$

$$d_1 = I(X_1, X_2; Y_1) = \frac{1}{2}\log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) \tag{6.169c}$$

$$\begin{aligned} d_1' &= I(X_1, X_2; Y_1, \hat{Y}_{r1}) \\ &= \frac{1}{2}\log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \frac{\mathsf{SNR}_{r1} + \mathsf{INR}_{r1} + \mathsf{SNR}_1\mathsf{INR}_1\eta_1^2}{1 + \Delta_1}\right) \end{aligned} \tag{6.169d}$$

$$\xi_1 = I(\hat{Y}_{r1}; Y_{r1}|X_1, X_2, Y_1) = \frac{1}{2}\log\left(1 + \frac{1}{\Delta_1}\right), \tag{6.169e}$$

and $c_2, c_2', d_2, d_2', \xi_2$ follow by symmetry. Next, we choose the quantization level $\Delta_1 = 1$. This reflects the fact that in many networks with relays, it is near optimal for each relay to quantize its received signal at the noise level [ADT11, WT11a]. It then follows from (6.169e) that $\xi_1 = 1/2$. We proceed to compare the inner and outer bounds, keeping in mind that $\Delta_1 = \Delta_2 = 1$, $\xi_1 = \xi_2 = 1/2$, $\mathsf{SNR}_1 + \mathsf{SNR}_{r1} \le \mathsf{INR}_2$, and $\mathsf{SNR}_2 + \mathsf{SNR}_{r2} \le \mathsf{INR}_1$.

Individual rate constraints. Direct comparison reveals that

- $c_1 + (C_1 - \xi_1)^+ = \frac{1}{2}\log(1 + \mathsf{SNR}_1) + (C_1 - \xi_1)^+$ in (6.165) is within 1/2 bit of the outer bound $\frac{1}{2}\log(1 + \mathsf{SNR}_1) + C_1$ in (6.56).
- $c'_1 = \frac{1}{2}\log\left(1 + \mathsf{SNR}_1 + \frac{\mathsf{SNR}_{r1}}{1+\Delta_1}\right)$ in (6.165) is within 1/2 bit of the outer bound $\frac{1}{2}\log(1 + \mathsf{SNR}_1 + \mathsf{SNR}_{r1})$ in (6.56).

Therefore, the inner bound on R_1 is within 1/2 bit of the outer bound. The same gap for R_2 follows by symmetry.
Sum-rate constraints. Direct comparison reveals that

- $d_1 + (C_1 - \xi_1)^+ = \frac{1}{2}\log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + (C_1 - \xi_1)^+$ in (6.167) is within 1 bit of the outer bound
$$\frac{1}{2}\log\left(1 + \frac{\mathsf{SNR}_2+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1}\right) + \frac{1}{2}\log(1 + \mathsf{SNR}_1 + \mathsf{INR}_1) + C_1$$
in (6.67).
- $d'_1 = \frac{1}{2}\log\left(1 + \mathsf{SNR}_1 + \mathsf{INR}_1 + \frac{\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_1\eta_1^2}{1+\Delta_1}\right)$ in (6.167) is within 1 bit of the outer bound
$$\frac{1}{2}\log\left(1 + \frac{\mathsf{SNR}_2+\mathsf{SNR}_{r2}}{1+\mathsf{INR}_1+\mathsf{INR}_{r1}}\right) + \frac{1}{2}\log(1+\mathsf{INR}_1+\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1}))$$
in (6.63).

By symmetry, it follows that $\min\{d_2 + (C_2 - \xi_2)^+, d'_2\}$ is within 1 bit of the outer bounds in (6.71) and (6.62). □

6.G Proof of Proposition 6.4.2

Similarly to the Gaussian IRC in the strong interference regime, we also set the whole message at each transmitter to be the common message, i.e., replacing X_{kc} by X_k, setting $R_{kp} = 0$, $k \in \{1, 2\}$, $Q = 1$ in (6.24)–(6.31). As a result, the achievable rate region $\mathcal{R}^i_{C-MARC}$ consists of (6.165)–(6.168) plus two extra conditions given below, which follow from (6.29) and (6.25), due to the fact that each decoder is to decode both messages:

$$R_1 < \min\{b_2 + (C_2 - \xi_2)^+, b'_2\} \tag{6.170}$$
$$R_2 < \min\{b_1 + (C_1 - \xi_1)^+, b'_1\}. \tag{6.171}$$

By the same procedure for quantization as for the Gaussian IRC with strong interference in Appendix 6.F, we obtain $c_1, c'_1, d_1, d'_1, \xi_1$ as in (6.169), plus

$$b_1 = I(X_2; Y_1|X_1) = \frac{1}{2}\log(1 + \mathsf{INR}_1) \tag{6.172}$$
$$b'_1 = I(X_2; Y_1, \hat{Y}_{r1}|X_1) = \frac{1}{2}\log\left(1 + \mathsf{INR}_1 + \frac{\mathsf{INR}_{r1}}{1+\Delta_1}\right), \tag{6.173}$$

and symmetrically for $b_2, b'_2, c_2, c'_2, d_2, d'_2, \xi_2$. Note that we also choose $\Delta_1 = \Delta_2 = 1$, which make $\xi_1 = \xi_2 = 1/2$ (see (6.169e)). Keeping these in mind, let us compare the inner and outer bounds.
Individual rate constraints.

- $c_1 + (C_1 - \xi_1)^+ = \frac{1}{2}\log(1+\mathsf{SNR}_1) + (C_1 - \xi_1)^+$ in (6.165) is within 1/2 bit of the outer bound $\frac{1}{2}\log(1+\mathsf{SNR}_1) + C_1$ in (6.83).

- $c'_1 = \frac{1}{2}\log\left(1+\mathsf{SNR}_1 + \frac{\mathsf{SNR}_{r1}}{1+\Delta_1}\right)$ in (6.165) is within 1/2 bit of the outer bound $\frac{1}{2}\log(1+\mathsf{SNR}_1+\mathsf{SNR}_{r1})$ in (6.83).

- $b_2 + (C_2 - \xi_2)^+ = \frac{1}{2}\log(1+\mathsf{INR}_2) + (C_2 - \xi_2)^+$ in (6.170) is within 1/2 bit of the outer bound $\frac{1}{2}\log(1+\mathsf{INR}_2) + C_2$ in (6.84).

- $b'_2 = \frac{1}{2}\log\left(1+\mathsf{INR}_2 + \frac{\mathsf{INR}_{r2}}{1+\Delta_2}\right)$ in (6.170) is within 1/2 bit of the outer bound $\frac{1}{2}\log(\mathsf{INR}_2+\mathsf{INR}_{r2})$ in (6.84).

Therefore we conclude the active inner bound on R_1 is within 1/2 bit of some outer bound. By symmetry we obtain the same result for R_2.
Sum-rate constraints.

- $d_1 + (C_1 - \xi_1)^+ = \frac{1}{2}\log(1+\mathsf{SNR}_1+\mathsf{INR}_1) + (C_1 - \xi_1)^+$ in (6.167) is within 1/2 bit of the outer bound

  $$\frac{1}{2}\log(1+\mathsf{SNR}_1+\mathsf{INR}_1) + C_1,$$

  in (6.87).

- $d'_1 = \frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{INR}_1 + \frac{\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_1\eta_1^2}{1+\Delta_1}\right)$ in (6.167) is within 1/2 bit of the outer bound

  $$\frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{INR}_1+\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_1\eta_1^2\right),$$

  in (6.89).

Similarly we obtain the same gaps for the two remaining bounds on $R_1 + R_2$. Accordingly we conclude that the active inner bound on sum-rate is within 1/2 bit of some outer bound.

Finally, since the inner bound is within 1/2 bit of the outer bound, the inner bound is within 1/2 bit of the capacity region of the channel. □

6.H Proof of Proposition 6.4.3

We will prove the theorem in the reverse direction, i.e., we fix the power allocation at the transmitters and the quantization distortion at the relays, and then find the conditions on C_1, C_2 such that the second terms in the min in the RHS of (6.24)–(6.31) are active.

Since we are focusing on the case when the quantization codewords can be recovered correctly at that destination, i.e., $\hat{Y}_{rk}$ is a channel output at the receiver Rx_k, $k \in \{1,2\}$, it is reasonable to think of relay and receiver as single receiver. As such, we expect that the rule of thumb for power allocation in [ETW08, WT11a] should perform well. Accordingly, we allocate transmit power for the private signal such that it arrives at the unintended receiver at the level below the noise level. Specifically, we choose

$$P_{1p} = \min\left\{1, \frac{1}{\mathsf{INR}_2 + \mathsf{INR}_{r2}}\right\} \tag{6.174}$$

$$P_{2p} = \min\left\{1, \frac{1}{\mathsf{INR}_1 + \mathsf{INR}_{r1}}\right\}. \tag{6.175}$$

Moreover, as learned from the linear deterministic model in [WT11a], it is reasonable for each relay to choose the quantization distortion at the level of undesired signals plus noise perceived by the associated receiver. In particular we choose

$$\Delta_1 = 1 + \mathsf{INR}_{r1p}, \quad \Delta_2 = 1 + \mathsf{INR}_{r2p}. \tag{6.176}$$

Then we have the "rate-loss" terms to be bounded

$$\begin{aligned} \xi_1 &= I(\hat{Y}_{r1}; Y_{r1} | X_1, X_{2c}, Y_1) \\ &= \frac{1}{2}\log\left[1 + \frac{1}{\Delta_1}\left(1 + \frac{\mathsf{INR}_{r1p}}{1 + \mathsf{INR}_{1p}}\right)\right] \\ &\leq \frac{1}{2}. \end{aligned} \tag{6.177}$$

Similarly, $\xi_2 \leq \frac{1}{2}$. In the same line of the proof of Lemma 6.4 in Appendix 6.E, we can show the following bounds:

$$\frac{\mathsf{INR}_{r1}}{1 + \mathsf{INR}_1 + \mathsf{INR}_{r1}} \leq \mathsf{INR}_{r1p} \leq 1 \tag{6.178a}$$

$$\frac{\mathsf{INR}_{1}}{1 + \mathsf{INR}_1 + \mathsf{INR}_{r1}} \leq \mathsf{INR}_{1p} \leq 1 \tag{6.178b}$$

$$\frac{\mathsf{SNR}_{r1}}{1 + \mathsf{INR}_2 + \mathsf{INR}_{r2}} \leq \mathsf{SNR}_{r1p} \tag{6.178c}$$

$$\frac{\mathsf{SNR}_{1}}{1 + \mathsf{INR}_2 + \mathsf{INR}_{r2}} \leq \mathsf{SNR}_{1p}. \tag{6.178d}$$

Now, in order for the second terms in the min in the RHS of (6.24)–(6.31) to be active we need

$$\begin{aligned}(C_1-\xi_1)^+ &\geq \max\{a_1'-a_1, b_1'-b_1, c_1'-c_1, d_1'-d_1\}\\ &= d_1'-d_1.\end{aligned}$$

Since $\xi_1 \leq \frac{1}{2}$, a sufficient condition would be

$$\begin{aligned}C_1 &\geq \frac{1}{2}+d_1'-d_1\\ &= \frac{1}{2}+I(X_1,X_{2c};\hat{Y}_{r1}|Y_1).\end{aligned} \tag{6.179}$$

Now we have

$$\begin{aligned}I(X_1,X_{2c};\hat{Y}_{r1}|Y_1) &= \frac{1}{2}\log\left[\frac{1+\Delta_1+\frac{\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_{r1}\eta_1^2}{1+\mathsf{SNR}_1+\mathsf{INR}_1}}{1+\Delta_1+\frac{\mathsf{INR}_{r1p}}{1+\mathsf{INR}_{1p}}}\right]\\ &\overset{(a)}{\leq} \frac{1}{2}\log\left[\frac{1+\Delta_1+\frac{\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_{r1}\eta_1^2}{1+\mathsf{SNR}_1+\mathsf{INR}_1}}{1+\Delta_1+\frac{\mathsf{INR}_{r1p}}{2}}\right]\\ &\overset{(b)}{\leq} \frac{1}{2}\log\left[\frac{\substack{(\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_{r1}\eta_1^2)(1+\mathsf{INR}_1+\mathsf{INR}_{r1})\\+(4+4\mathsf{INR}_1+6\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}}{(4+4\mathsf{INR}_1+7\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}\right],\end{aligned}$$

where (a) follows from the upper bound on INR_{1p} in (6.178b), (b) follows from the lower bound on INR_{r1p} in (6.178a). Hence, a sufficient condition for C_1 is

$$C_1 \geq \frac{1}{2}\log\left[\frac{\substack{2(\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}+\mathsf{SNR}_1\mathsf{INR}_{r1}\eta_1^2)(1+\mathsf{INR}_1+\mathsf{INR}_{r1})\\+2(4+4\mathsf{INR}_1+6\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}}{(4+4\mathsf{INR}_1+7\mathsf{INR}_{r1})(1+\mathsf{SNR}_1+\mathsf{INR}_1)}\right]. \tag{6.180}$$

Due to this condition on C_1 (and similarly for C_2), the achievable rate region in Theorem 6.2.1 reduces to

$$R_1 < c_1' \tag{6.181}$$
$$R_1 < a_1'+b_2' \tag{6.182}$$
$$R_2 < c_2' \tag{6.183}$$
$$R_2 < a_2'+b_1' \tag{6.184}$$
$$R_1+R_2 < a_1'+d_2' \tag{6.185}$$
$$R_1+R_2 < b_1'+b_2' \tag{6.186}$$
$$R_1+R_2 < d_1'+a_2' \tag{6.187}$$
$$2R_1+R_2 < a_1'+d_1'+b_2' \tag{6.188}$$
$$R_1+2R_2 < b_1'+a_2'+d_2'. \tag{6.189}$$

Let us define

$$g_1(\Delta_1) := \frac{1}{2}\log((1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p})$$
$$g_2(\Delta_2) := \frac{1}{2}\log((1+\Delta_2)(1+\mathsf{INR}_{2p})+\mathsf{INR}_{r2p}).$$

Using the bounds in (6.178a)–(6.178d) we have

$$g_1(\Delta_1) \le \frac{1}{2}\log(7) \tag{6.190a}$$
$$g_2(\Delta_2) \le \frac{1}{2}\log(7), \tag{6.190b}$$

and

$$\begin{aligned}
a'_1 &= I(X_1;Y_1,\hat{Y}_{1r}|X_{1c},X_{2c})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_{1p})+\mathsf{SNR}_{r1p}+\mathsf{INR}_{r1p}(1+\eta_1^2\mathsf{SNR}_{1p})}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\ge \frac{1}{2}\log\left(1+\mathsf{SNR}_{1p}+\mathsf{SNR}_{r1p})\right)-g_1(\Delta_1)\\
&\ge \frac{1}{2}\log\left(1+\frac{\mathsf{SNR}_1+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right)-g_1(\Delta_1).
\end{aligned}$$

In the same way we can show that

$$\begin{aligned}
b'_1 &= I(X_1,X_{2c};Y_1,\hat{Y}_{1r}|X_{1c})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_{1p}+\mathsf{INR}_1)+\mathsf{SNR}_{r1p}+\mathsf{INR}_{r1}(1+\eta_1^2\mathsf{SNR}_{1p})}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\ge \frac{1}{2}\log\left(1+\mathsf{INR}_1+\mathsf{INR}_{r1}+\frac{\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})+\mathsf{SNR}_{r1}}{1+\mathsf{INR}_2+\mathsf{INR}_{r2}}\right)-g_1(\Delta_1).\\
c'_1 &= I(X_1;Y_1,\hat{Y}_{1r}|X_{2c})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_1+\mathsf{INR}_{1p})+\mathsf{SNR}_{r1}+\mathsf{INR}_{r1p}(1+\eta_1^2\mathsf{SNR}_1)}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\ge \frac{1}{2}\log\left(1+\mathsf{SNR}_1+\mathsf{SNR}_{r1}\right)-g_1(\Delta_1).\\
d'_1 &= I(X_1,X_{2c};Y_1,\hat{Y}_{1r})\\
&= \frac{1}{2}\log\left(\frac{(1+\Delta_1)(1+\mathsf{SNR}_1+\mathsf{INR}_1)+\mathsf{SNR}_{r1}+\mathsf{INR}_{r1}(1+\eta_1^2\mathsf{SNR}_1)}{(1+\Delta_1)(1+\mathsf{INR}_{1p})+\mathsf{INR}_{r1p}}\right)\\
&\ge \frac{1}{2}\left(1+\mathsf{INR}_1+\mathsf{INR}_{r1}+\mathsf{SNR}_1(1+\eta_1^2\mathsf{INR}_{r1})+\mathsf{SNR}_{r1}\right)-g_1(\Delta_1).
\end{aligned}$$

The preceding inequalities lead to the following gap between inner and outer bounds:

- *Constraints on R_1*:

  The RHS of (6.181) is within $g_1(\Delta_1)$ of the RHS of (6.56).

  The RHS of (6.182) is within $g_1(\Delta_1) + g_2(\Delta_2)$ of the RHS of (6.56).

- *Constraints on R_2 follow by symmetry.*

- *Constraints on $R_1 + R_2$*:

  The RHS of (6.185) is within $g_1(\Delta_1) + g_2(\Delta_2)$ of the RHS of (6.62).

  Similarly for the RHS of (6.187).

  The RHS of (6.186) is within $g_1(\Delta_1) + g_2(\Delta_2)$ of the RHS of (6.64).

- *Constraint on $2R_1 + R_2$*:
  The RHS of (6.188) is within $2g_1(\Delta_1) + g_2(\Delta_2)$ of the RHS of (6.65).

- *Constraint on $2R_1 + R_2$ follows by symmetry.*

In summary, we conclude that inner bound on the individual rates are within $g_1(\Delta_1)+g_2(\Delta_2)$ bits of the outer bound, inner bound on R_1+R_2 is within $g_1(\Delta_1)+g_2(\Delta_2)$ of the outer bound, inner bound on $2R_1 + R_2$ is within $2g_1(\Delta_1) + g_2(\Delta_2)$ of the outer bound, and inner bound on $R_1 + 2R_2$ is within $g_1(\Delta_1) + 2g_2(\Delta_2)$ of the outer bound. Using (6.190) we obtain the desired gap. Finally, recall that the outer bounds we have just used are derived by assuming each receiver cooperates with its associated relay. □

6.I Proofs of Lemma 6.2 and Lemma 6.3

Proof of Lemma 6.2: By setting the whole message at each transmitter to be private, i.e., setting $X_{kc} = \emptyset$, $R_{kc} = 0$, $k \in \{1, 2\}$, and $Q = \text{const}$ in Theorem 6.2.1, the achievable rate region for the IRC reduces to a rectangle given by

$$R_1 < \min\{I(X_1; Y_1) + (C_1 - I(\hat{Y}_{r1}; Y_{r1}|X_1, Y_1))^+, I(X_1; Y_1, \hat{Y}_{r1})\}$$
$$R_2 < \min\{I(X_2; Y_2) + (C_2 - I(\hat{Y}_{r2}; Y_{r2}|X_2, Y_2))^+, I(X_2; Y_2, \hat{Y}_{r2})\}.$$

Extending the above achievable region to the Gaussian IRC, as described in Section 6.4.2, with the quantization distortion $\Delta_k = 1+\mathsf{INR}_{rk}$, $k \in \{1, 2\}$, we can easily show that $I(X_k; Y_k) + (C_k - I(\hat{Y}_{rk}; Y_{rk}|X_k, Y_k))^+ \leq I(X_k; Y_k, \hat{Y}_{rk})$ if the conditions in (6.93) are satisfied. Hence, the resulting achievable region is given by

$$R_1 < I(X_1; Y_1) + (C_1 - I(\hat{Y}_{r1}; Y_{r1}|X_1, Y_1))^+$$
$$R_2 < I(X_2; Y_2) + (C_2 - I(\hat{Y}_{r2}; Y_{r2}|X_2, Y_2))^+,$$

which yields the achievable sum-rate $C_{lb}^{w} = R_1 + R_2$ in explicit form in (6.94). □

Proof of Lemma 6.3: Recall that we denote the sequences sent over the noiseless links by V_1^n and V_2^n. By the Fano's inequality

$$\begin{aligned} n(R_1 + R_2 - 2\epsilon_n) &\le I(X_1^n; Y_1^n, V_1^n) + I(X_2^n; Y_2^n, V_2^n) \\ &\le I(X_1^n; Y_1^n) + H(V_1^n) + I(X_2^n; Y_2^n) + H(V_2^n) \\ &\le I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) + nC_1 + nC_2, \end{aligned} \tag{6.191}$$

where $\epsilon_n \to 0$ as $n \to \infty$. Under the condition (6.95) we can proceed to bound $I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n)$ in the same way as done in [SKC09, MK09, AV09]. Specifically, we can find a useful and smart genie, which gives us

$$I(X_1^n; Y_1^n) + I(X_2^n; Y_2^n) \le \frac{n}{2} \log\left(1 + \frac{\mathsf{SNR}_1}{1 + \mathsf{INR}_1}\right) + \frac{n}{2} \log\left(1 + \frac{\mathsf{SNR}_2}{1 + \mathsf{INR}_2}\right). \tag{6.192}$$

Combining (6.191) and (6.192) completes the proof. □

Chapter 7

Conclusion

7.1 Concluding Remarks

In this thesis we have investigated the role of relaying and cooperation in interference management for wireless networks. We followed an information theoretic approach, and we addressed several problems in network information theory having their practical motivation in modern wireless networking. In so doing, we have been able to propose new coding strategies, establish fundamental inner and outer bounds on the capacity regions of multiple scenarios. These bounds have been shown to be the capacity regions in different cases. We illuminated the connection among coding strategies and showed how to tailor them to achieve better performance. In particular, we showed how to combine two different versions of superposition coding with classical relaying protocols to efficiently mitigate interference while keeping the analysis simple. We showed analytically and numerically the benefit of having layers in coding strategies for interference-relay channels. We also demonstrated the optimality of a simple relaying protocol for certain interfering relay networks.

An interesting finding of this thesis is the equivalence of different relaying strategies, with and without binning, in terms of achievable rates, for several interference-relay networks. This provides a more complete view on the many existing relaying strategies in the literature. A comparison of long- and short-message relaying also reveals a trade-off between complexity, delay, and achievable rates, thereby providing a deeper understanding of those relaying strategies. Another feature of the thesis is the study of networks which have both in-band and out-of-band links. This is a very important model for modern wireless systems, where the backhaul links are implemented to connect master nodes such as base stations. One such example is the HetNet introduced in Chapter 1, cf. Fig. 1.1. We showed how to analyze those networks, for both the discrete memoryless and the Gaussian models.

To sum up, this thesis conducted a comprehensive and in-depth study of coding strategies for cooperative interference management. Through concrete "toy problems" serving as abstractions of important practical scenarios, we developed a clear overview on the strategies and to provided an in-depth analysis. The methods, results, and insight obtained here are expected to be useful for further studies of the topic and for benchmarking and inspiration for implementation in practical systems.

7.2 Future Work

There are certainly many open questions remaining to answer and many directions to extend the work of this thesis. One example is to further optimize the parameters in the proposed coding schemes, particularly finding smarter power allocation and quantization schemes so as to approach the capacity regions more closely. Recall from Chapter 6 that a "rule of thumb" for signal quantization at the relays is to quantize at the noise level of the receivers. This, however, is not necessarily always optimal as recently shown in [KO13]. Therefore, finding a smarter quantization rule seems to be reasonable and may lead to significant improvements of the results.

An important immediate extension of our work is to analyze more complex interference networks, ideally with an arbitrary number of nodes. Within this topic, one natural question is whether we can generalize the analysis of quantize-bin-forward in Chapter 6 to such general networks, in a similar way that noisy network coding generalizes quantize-forward and network coding to the general discrete memoryless multicast networks. The question is particularly relevant to heterogeneous networks where in-band and out-of-band links coexist. It also has an important engineering implication because quantize-bin-forward is more favorable than noisy network coding in terms of delay and complexity.

Another important topic for future work is to study the interaction of the coding techniques in this thesis with other forms of cooperation, in particular the cooperation induced by feedback. Research has shown that a few bits of feedback can considerably improve the performance of many coding schemes in network information theory. For the IC, the recent works in [JXG07, ST11, TU11, SAYS09, VSA12] also showed promising results. Extending these knowledge to interference networks with relays is clearly an appealing topic.

Although briefly touched upon in Chapter 3, the asymptotic behavior of Gaussian networks at high signal-to-noise ratios is not covered in the thesis. In such cases the notion of degrees of freedom (DoF) appears to be the most relevant one, which might deserve further investigation. Another interesting topic that has not been covered is the use of deterministic models [ADT11] (DM) to approximate the capacity regions. We, however, note that some of the results in Chapter 6 are indirectly obtained from insight based on the DM.

Additionally, enhancing the coding schemes to include fading channels is always an important step towards real-world implementations.

Finally, using coding techniques that have previously been suggested for the relay and/or interference channel may not be sufficient to efficiently manage interference in general wireless networks. Therefore, an important challenge for future work may be to invent completely new coding strategies.

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